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A Proportional Solution in Economies with Envy

By

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Abstract

Here we reconstruct the Proportional Solution for Economies with both public and private ownership of productive assets, proving the existence of a competitive equilibrium assuming non-monotonic utility and actors behaving with envy. With a possibility for excess at no additional cost, our aim is to provide a mechanism under which envious individuals, competing for a commonly pooled resource, can reach a stable equilibirum. In other words, we derive a proportional solution with envious agents in a pure exchange economy.

Background

Although much work in the context of envy has been discussed in welfare economics, the behavioral implications at the intersection between public and private agents have not received much attention. Roemer and Silvestre 1993 outline a planner's problem in allocating resources in an economy with public as well as private endowments. Private ownership includes the right to use, destroy, and prevent other people from using, as well as the right to transfer these rights. In a public setting, such rights no longer exist; no one has the right to destroy the common resource and if it can be used without depletion, then everyone has the right to use the resource freely. Roemer and Silvestre 1993, when constructing the proportional solution assume free disposal, in that when resources must be divided fairly, there is the possibility for excess. This excess has no additional cost. Our aim is to provide a mechanism under which envious individuals, competing for a commonly pooled resource, can reach a stable equilibirum. In other words, we derive a proportional solution with envious agents in a pure exchange economy.

It is often heard that socialism is built upon envy. A witnessing of immense inequality in the 21st century has bred a kind of contempt for those with greater fortune than the masses. Thus, a discussion of the redistribution of existing wealth among the world's most affluent has become a policy topic that has yet to fain its flames. We in economics embrace the individualism associated with the desire for trade and upward mobility, provided that in pursuing ones own self-interest, they are not inhibiting another's similar pursuits. But in a society that does not fully govern itself solely from the culmination of expressive inputs from the individual (the United States has a mixed economy), we must introduce a more realistic yet complex element into the proportional solution put forth by

Roemer and Silvestre (1993). How does a planner regulate its own allocative strategies when dealing with a non-fully-cooperative body of consumers with a unique set of preferences that are constantly at flux? Brenner (1987) presents evidence to show that individuals care about their relative economic statuses, and demonstrates that such concern may lead to risk-taking behaviour. Elster (1991) observes that people may abstain from becoming superior to others so as to avoid provoking their envy. Kirchsteiger (1995) incorporates envy into the standard bargaining model to explain findings in ultimatum game experiments.

In this paper, I explore the role of envy in provoking retaliation against others using the proportional solution framework, taking into account the costs of such actions. A major difference between the current paper and previous research on envy is that I explicitly consider the conditions under which a common resource pool is drawn from multiple agents contingent upon their individual contributions (or labor inputs) and how any disruption in consumption of the social planner's designated and proportional allotment can result in envious retaliation behavior by other agents. The expanded decision model provides a means for analysing the interactions among people's propensities for envy and how an equilibrated state might be restored when retaliatory actions are a continual threat.

We follow closely the work of Mui (1992) in that people compare their economic well-being with others', and that if an agent improves their well-being as a, what we will refer to as an "innovation", that improves their relative economic status, the agents that are below in status may become envious. In a proportional solution, it is not unreasonable to believe if apportioned your fair share of the common resource, that an individual may not consume it, but merely discard of it. This form of excess to which we will refer henceforth as a "reservation" can be reallocated for use and put into circulation at the social planners' discretion. The change of policy as a result of this excess has aggregate consequences as we will show. Roemer and Silvestre introduce a free-disposal assumption that we use as the underlying force compelling an individual to become envious.

Net metering is a policy that lets people with solar panels sell excess power back to utilities at market rates. This allows for individuals to reintroduce the surplus of energy back on to the market to be distributed to those facing a higher demand. The proportional solution in its general framework does not allow for this type of application and simply restricts its occurence. "Efficiency requires that some public rights be transferred to individuals when what is publicly owned is not a pure public good. But the right to use may be transferred to individuals without full privatisation, i.e., without the individual acquiring the right to waste or sell".

We note that without a public endowment, we have the Walrasian equilibirum as the local solution for the economy. Given that envy in the absence of a public endowment has been sufficiently researched, we focus only on the public endowment effect within the proportional solution framework.

In this environment we have non-monotonic utility functions, whereby an agent receives an endowment from the social planner yet due to some unobservable cause, experiences no change in utility. This could be an appetite change, a hiccup in preference or a mistake in allocation.

Proportional Solution

Cooperative production with shared technology and convex production functions

Model

Suppose that we have a simple environment where $M = 1, 2$ agents, both of whom are engaging in independent production activities. Each agent has two types of goods: good 2 is a private good produced by a publicly owned technology, defined by a production function f, which has good 1 as an input.
For $i = 1, 2$, (x_{i1}, x_{i2}) denotes i's consumption vector. Also, both agents are endowed with one unit of labour, which can be used to as an input to produce good 2, denoted as y. It is assumed that the agents do not derive any utility from leisure. The economic utility of agent i is given by a function $V_i(y_i)$ with the given endowment of good one given as ω_{i1} and $L_i := \omega_{i1} - x_{i1}$ is the amount of input contributed by i. This assumption, together with the assumption below on how an agent compares his opportunity set to the other's, implies that each agent will always devote all of his labour endowment to production of good 2. Using Roemer and Silvestre (1993), we have a proportional solution defined by efficiency showing that the amount of output consumed by a person is proportional to the amount of input that the agents contribute. So we have,

$$L_i/x_{i2} = L_h/h_{h2} \qquad \text{for } i, h = 1, 2$$

Our focus here is in cooperative production of the shared technology, all of which agents contribute their labor. This is a non-market production process where agent i contributes the amount of input L_i and consumes the amount of output x_{i2}. Furthermore, the proportional solution is a Pareto optimal allocation in which output is distributed according to labor contribution.

Because of efficiency $\Sigma_i x_{i2} = f(L)$ and thus we have

$$L_i/x_{i2} = L/f(L)$$

where f is a production function. Since efficiency requires same marginal rates of substitution we set $-\frac{(\partial U_i/\partial x_{i2})}{(\partial U_i/\partial x_{i1})} := p$ and get

$$px_{i2} = L_i + [x_{i2}/f(L)][pf(L) - L], i = 1, ..., M$$

Now it is not unreasonable to conjure up a situation in which an agent unexpectedly consumes less of what there current allocation becomes. Even with a zero-expiration assumption, a single lag in the first period let's say, would result in an effectual disproportional consumption vector of the agent. Roemer and Silvestre (1963) emphasize that for a non-produced public good, "use" can be isolated from the transfer of property rights because use does not imply nor require depletion. Furthermore, although we assume there are no transaction costs in allocating proportional output, one might consider an agent's contribution to the shared technology in period 1, receiving their share in period 2, but are too ill to consume.

Existence Theorem

The existence theorem for proportional solution requires guaranteeing expenditure minimization implies utility maximization. So we can use Arrow and Hahn's "indirect resource relatedness". Agent 1 is resource related to person Agent 2 if for every feasible allocation s there exists an allocation s' and a vector

$\Delta\omega$ such that by increasing the endowments of some of the commodities that agent 1 initially owns, it is possible to make agent 2 better off without making anybody worse off.

For our general definition we rewrite our expression from above to get

$x_{i1} = \omega_{i1} + \frac{x_{i2}}{\Sigma_h x_{h2}} \Sigma_h (x_{h1} - \omega_{h1})$ i.e. i's consumption of the privately owned good equals their endowment plus a fraction $x_{i2}/\Sigma_h x_{h2}$ of the inputs (costs) of the public sector. $\frac{x_{i2}}{\Sigma_h x_{h2}}$ is the ration of i's consumption of the public sector output to the total output of the public sector. The vectors of the net outputs and inputs $t^+(s)$ (as well as $t^-(s)$) are used as the vector of net quantities from shared technology, we can aggregate a vector of efficiency prices to get a proportional share $\theta_i(p, s)$ for each agent i and obtain a feasible allocation s, defined for $j \in \{1, ..., N\}$,as

$t_j^+(s) = max\left\{0, \Sigma_{i=1}^M x_{ij} - \Sigma_{i=1}^M \omega_{ij} - \Sigma_{j=1}^F y_{ij}\right\}$,

$t_j^-(s) = min\left\{0, \Sigma_{i=1}^M x_{ij} - \Sigma_{i=1}^M \omega_{ij} - \Sigma_{j=1}^F y_{ij}\right\}$

With pareto optimality shown using standard competitive Walrasian equilibrium assumptions.

Proportional Allocative Share: With Envy

The reserve from the proportional output, as already mentioned can induce a type of envy from the other agent as the relative status has changed from one allocation to another.

We assume convexity, in that

$$lim_{y_i \to 0} V_i(y_i) = \infty, V_i' > 0, V_i'' < 0.$$

All production functions are considered common knowledge by both agents to the shared technology. In other words, i's production function $f_i(L_i) = Y_i L_i, 0 \leq L_i \leq 1$. Then $Y_i = f_i(1)$ is the maximum output of agent i. With two agents, we define relative status-based envy as follows:
(i) - Agent i will compare Y_i to Y_j. If $Y_2 > Y_1$, agent 1 will perceive agent 2 as having a better conditional status than themselves. As a result, agent 1 will then suffer from envy. If $Y_1 \geq Y_2$, we assume that agent 1 will not have any additional satisfaction from being economically better off than agent 2.

Let $E_1(Y_2/Y_1)$ be 1's envy of 2 given the observed allocative output of the two agents. $E_1(\cdot)$ is 1's propensity for envy. Envy enters negatively into the utility function. Following Adams (1963) in the theory of equity in psychology it is assumed that the propensity towards envy can be viewed as,

$$E_1\left(\frac{Y_2}{Y_1}\right) = 0 \text{ if } \frac{Y_2}{Y_1} \leq 1$$

$$E_1\left(\frac{Y_2}{Y_1}\right) = 0, E_1' > 0, E_1'' > 0, \text{ if } \frac{Y_2}{Y_1} > 1$$

As displayed, in our cooperative production based-economy the concept of envy is relative to the proportional output received from the social planner, based upon each agent's contribution to the shared technology. The proportional output of agent 1 to their input is equal to the proportional output relative to their input of agent 2. In the model, the outputs, and the envy that may emerge from the allocation, are fully exhausted maxima outputs provided by the social planner and the inputs are the maximum labour inputs of the agents. That is to say there is no waste in production so we assume free disposal.

Strategic Interaction between Agents with Envy

A Game Theoretic Approach

Given our conditions from the previous section the agents recognize the economic environment as being equitable if and only if $Y_1/1 = Y_2/1$. When $Y_1 < Y_2$, agent 1 perceives that they are in an unfavourable position (or in an inequitable relationship), and may take actions necessary to either increase Y_1 or to decrease Y_2 so as to restore the equitable state. Given these two strategies (only one of which can be chosen at a time), the agent must decide whether they will improve their lot, by investing in things that may enhance their inputs and as a result their allotment, or to retaliate against agent 2. Any change in the relative economic well-being of either agent will cause the other agent to suffer from envy. In order to bring about some form of restoration, they may inflict different kinds of punishment on the innovator. Hence we have what we will refer to as retaliation.

Retaliation

In a proportional solution setting, retaliation occurs as a result of the non-monotonic utility function of the agent whereby they receive an endowment from the social planner, yet due to some unobservable cause, experiences no change in utility. This could be an apetite change, a hiccup in preference or a mistake in allocation. Either way, the non-contributory surplus elicits envy from the other agent.

Let R_1 be the level of retaliation agent 1 inflicts on agent 2. Due to the opportunity cost of taking such an action against agent 2 we require one unit of labor for each unit of retaliation. Thus, $0 \leq R_1 \leq 1$. The infliction of retaliation reduces agent 1's envy by a fraction $F_1(R_1)$. So, $E_1(Y_2/Y_1)F_1(R_1)$, becomes the reduction of envy from retaliation. Because of this relationship there is a point at which one additional unit spent on retaliation reduces the utility of agent 2.

Let $G_1(R_2)$ be the disutility that 1 suffers from 2's retaliation. I will assume that $G_1(0) = 0, G_1' > 0, G_1'' > 0$.
In sum, the utility function for agent 1 is

$$U_1(Y_1, Y_2, R_1, R_2) = V_1(Y_1(1 - R_1)) - E_1\left(\frac{Y_2}{Y_1}\right)(1 - F_1(R_1)) - G_1(R_2)$$

similarly for agent 2 we have,

$$U_2(Y_1, Y_2, R_1, R_2) = V_2(Y_2(1 - R_2)) - E_2\left(\frac{Y_1}{Y_2}\right)(1 - F_2(R_2)) - G_2(R_1)$$

Model

Let x_2 be a nonproduced public good initially available in ω_2 units (i.e., $f(x_1) = \omega_2$ for all x_1) and let the endowment ω_2 be publicly owned. The proportional solution here requires

$$(x_{i1}, x_{i2}) = (\omega_{i1}, \omega_{i2}), i = 1, ..., M$$

i.e. that there are no voluntary transfers of the privately owned good x_1. Note the slight deviation in assumption of private transfers noted in Roemer and Silvestre (1993). Now suppose in a proportional solution equilibrium there is some exogenous break in the equilibrated state creating for envy. For example, an agent becomes ill and the aggregate consumption is reduced per capita as their workload inputs increase to compensate (and aid) the sick agent. This could stir up envy. Due to common knowledge of all labor inputs, each agent knows the utility function of the other agent, and thus, can anticipate the best response function of the other agent, due to a change in the economic environment. Given this structure, subgame-perfect equilibrium is the natural solution concept for the model. In this case, agent 1 maximizes his utility by solving

$$\text{Max}_{Y_1 \in \{\bar{Y}, a\}} V_1(Y_1(1-R_1)) - E_1\left(\frac{Y_2(Y_1)}{Y_1}\right)(1 - F_1\left[R_1(Y_1, Y_2(Y_1))\right]\Big)$$

$$-G_1\left[R_2(Y_1, Y_2(Y_1))\right]$$

such that

$$R_1(Y_1, Y_2) \in argmax_{R_1} V_1(Y_1(1-R_1)) - E_1\left(\frac{Y_2}{Y_1}\right)(1 - F_1(R_1)) - G_1(R_2(Y_1, Y_2))$$

$$R_2(Y_1, Y_2) \in argmax_{R_2} V_2(Y_2(1-R_2)) - E_2\left(\frac{Y_1}{Y_2}\right)(1 - F_2(R_2)) - G_2(R_1(Y_1, Y_2))$$

$$Y_2(Y_1) \in argmax_{Y_2} V_2((Y_2(1-R_2)) - E_2\left(\frac{Y_1}{Y_2}\right)(1 - F_2(R_2(Y_1, Y_2))) - G_2(R_1(Y_1, Y_2))$$

Here we have for agent 1 and agent 2 the best course of action which maximizes their respective utilities. Denote the equilibrium by $(Y_1^*, Y_2^*, R_1^*, R_2^*)$.

Aggregate Innovation

Overall to maintain this steady state of equilibrium aggregate levels of input per capita must be reduced than otherwise. The larger the spillover effect the larger the benefits and cost of retaliation once experienced by agent 1. In the proportional solution context, the larger spill over effect causes a reduction in the per capita output and thus the per capita accumulation from the shared resource is reduced, resulting in our equilibrated state.

Maintaining this equilibrated state and upholding efficiency in the distribution of the common resource from the social planner, the proportional solution must hold.

What if the proportional solution does not hold? When there is a surplus of output such that per capita consumption can increase but fails to do so, we are in a state at which the production possibilities are not being fully utilized and waste is created. This waste will result in a reduction in labor inputs, as less input is required, so as to reduce the waste. This transitional phase is a pareto improvement in social planing in the distribution of the common resource, thus, the proportional solution must hold.

Best Response Function

I assume that if $Y_j > Y_i$, then holding Y_j constant, i's utility is increasing in Y_i. So with an innovation by agent 1 we have our lemma.

Lemma

Given an innovation with a normal spill-over effect, if agent 1 inherits some deinnovative strategy, agent 2's best response function is $Y_2(a) = \alpha a$, where $0 < \alpha < 1$.

This is due to the fact that since $\bar{Y} < \alpha a < a$, in equilibrium, agent 1 will never be envious of agent 2, hence $R_1^* = 0$. If agent does indeed inherit this deinnovative strategy, compared to any $Y_2 < \alpha$ a, α a gives agent 2 a lower level of economic utility and a higher level of psychological disutility.

Equilibrium

The non-monotonic utility impacts aggregate returns when multiple agents are consuming significantly less than "necessary", affecting the output received at their respective level of work. The social planner will decide that they deserve less and can provide less at the same level of input, without disturbing the utility of the agents. Instead of an innovation, we have, let's call it deinnovation, which alters the relative status of the other agents. So to the same spill-over effect as innovation but moving in the opposing direction.

Observation

For notational simplification, define $\Pi_1(Y_1, Y_2) \equiv U_1(Y_1, Y_2, R_1(Y_1, Y_2), R_2(Y_1, Y_2))$. Now consider an innovation $(a, \alpha a)$ with a normal spill-over effect.
(i) if agent 1's resulting execesses from their innovation is sufficiently low, such that it can more than offset the threat of agent 2's envious retaliation, as agent 2 bears the costs of such envious imposition, agent 1 will continue to adopt a strategy such that their innovation results in a sufficiently low output. That is,

$$Pi_1(a, \alpha a) - \Pi_1(\bar{Y}, \bar{Y}) = [V_1(a) - G_1(R_2(a, \alpha a))] - V_1(\bar{Y}) \geq 0$$

$$Y_1^* = a, Y_2^* = \alpha a, R_1^* = 0, R_2^* = R_2(a, \alpha a) > 0.$$

(ii) If agent 1's resulting execesses from their innovation is not sufficient to offset the threat of agent 2's envious retaliation, agent 1 will adjust their labor inputs such that there are zero excesses. Thus, there will be neither innovation nor observed incidents of retaliation.

That is,

$$V_1(a) - V_1(\bar{Y}) < G_1(R_2(a, \alpha a))Y_1^* = Y_2^* = \bar{Y}, R_1^* = R_2^* = 0$$

So the threat of other's envious retaliation can deter a person from adopting an innovation. So in the context of a proportional solution, they may reduce their overall output so as to reduce the excess surplus that they would normally discard, lest their be an envious backlash from other agents.

At the optimum, the marginal reduction of disutility from envy due to retaliation equals the marginal cost of retaliation, where the marginal cost of retaliation is the opportunity of foregone income (including the expected punishment by the authority, which we do not model explicitly).

Empirical Evidence and Applications

Energy System Mechanism

In the heavily regulated energy sector individuals are able to sell back power, energy, and/or electricity back to the local utility company (the grid) when there exists a surplus of stores reserved from the period's expected amount. Net metering is one mechanism by which this system is ran. Solar energy system owners are given credits for the electricity they add back to the grid. For example, solar panels on top of roofs may generate more electricity than the home uses during daylight hours. If the home is net-metered, the electricity meter will run backwards to provide a credit against what electricity is consumed at night or other periods when the home's electricity use exceeds the system's output. Customers are only billed for their "net" energy use. On average, only 20-40 percent of a solar energy system's output ever goes into the grid, and this exported solar electricity serves nearby customers' demand for energy usage.

Net metering in the context of a proportional solution allows for individuals who contribute their excesses to the grid resulting in a payout of zero, to other users in the solar energy system to overconsume at a reduced rate. This could induce jealousy from the individual who sells back their energy. Conversely, if an individual chooses not to sell back their excesses which could have the potential (if sold) to reduce the total labor-input and thus the per-capita credits, this also might induce jealousy. The existence of a surplus signals the disequilibrated state which induces the potential for retaliation by the agent upon the agent bearing the surplus. The proportional solution offers a means to mitigate such risk by reallocating from the shared pool such that individuals who seek retaliation receive spill-over benefits plus the additional allotment that would discourage such acts. Similarly, such a reallocation would result in the agent who initially was faced with a surplus, to receive a reduced payout driving the surplus and their proportional labor-input to zero.

Public Buyback Programs

Buyback programs consist of agents that have a surplus of goods sell back the excess to put back on the market. This could be viewed as a rebate on an item or simply a discount on the initial selling price.

What makes buyback programs unique and relevant to the proportional solution is that the value of the shared resource increases proportionally the quantity sold. This change in value will result in an increase in per-capita ownership of the resource involuntarily. This action may result in a required larger labor input from all other agents (or just the agents not selling back to the market). Because of this, envy may be induced by the agent's whose reallocative plan is now changed.

There are many types of buyback mechanisms, one being gun back programs. In this instance, the value of the remaining weapons in circulation are increased, and thus the punishment for taking part in a gun-related crime is more severe. This increase in penalty may induce a form of envy on other individuals participating in the buyback program. Other mechanisms have similar effects. Buyback recycling centers, or stock buybacks in financial markets. All of that these examples have in common is related to the proportional solution that we have outlined in the paper.

Buy back recycling centers allow for individuals who have excess materials that are of value to "sell" back to the center. As in the proportional solution, an agent will recognize that the amount they get in return is less than the amount they paid for the good and will be encouraged to reduce the quantity of goods purchased, such that the expected return from their waste will be zero.

Discussion

The above analysis has focused on how agents' propensities for envy may substantially complicate their strategic interactions related to innovation. A natural direction for future research is to generalize the model to a multiple-agent setting. This generalization will allow us to study how a community may engage in some form of collective action to ensure that an innovator shares the result of his innovation in a way desired by the community. Such collective action may take the form of formal legislation, informal social customs, or even blatant violence. One would also want to know whether the presence of envy will magnify or mitigate the free-rider problem that the community may experience in orchestrating this kind of collective action. Moreover, since an innovator can choose his gift to each agent in the community strategically, it will be useful to find out whether such strategic behaviour on the part of the innovator can undermine the efficacy of collective action against him by others.

Useful Extensions

Sharing Mechanism as Proportional Solution

The discussion so far has excluded the possibility of side payments between the agents. Yet one way to cope with the threat of envious retaliation may be to have the innovator share part of his new wealth with those who lag behind. With such voluntary redistribution, it may be possible for the innovator to innovate and then carefully choose a transfer scheme to reduce others' envious retaliation while still making himself better off than without innovation.

Suppose that it is costless to for an agent to receive gifts from others. Let t_1 be the amount of voluntary transfer that 1 gives to 2. 2's utility after the agents have chosen their technologies (Y_1, Y_2) can be written as

$$\Pi_2(Y_1, Y_2) = V_2(Y_2(1 - S_2 - R_2) + t_1) - E_2\left(\frac{Y_1 - t_1}{Y_2 + t_1}\right)(1 - F_2(R_2))$$
$$-C_2(R_2, \gamma) - D_2(S_2, \gamma)$$

For any $Y_2 \in \{\bar{Y}, \alpha a\}$, let $t_i(a, Y_2)$ be the optimal transfer that agent i will give to the other agent if the incomes of 1 and 2 are a and Y_2 respectively. Since I am interested in a subgame perfect equilibria, define $Y_2(a)$ to be 2's best response to a subject to the condition that $Y_2 \in \{\bar{Y}, \alpha a\}$. It is obvious that $Y_2(a) = \alpha a$.

Let $t_i(a, \alpha a)$ be the optimal transfer that agent i chooses to give the other agent in the post-innovation subgame. Since $\alpha a < a$, it can immediately be observed that $t_1(a, \alpha a) > 0$ which $t_2(a, \alpha a) = 0$. Define $t_1^* \equiv t_1(a, \alpha a)$. (i) For the innovation opportunity $(a, \alpha a)$, with voluntary sharing, $Y_l^* = a$ can always be supported as the equilibrium.

Supposing that the innovation opportunity $(a, \alpha a)$ only has a small spill-over effect, so that, in the absence of sharing, 2 will be made worse off by l's innovation and will choose to inflict sabotage on 1. Then it is possible that even with the possibility of voluntary sharing, 2 will still choose to inflict sabotage on 1 so as to prevent 1 from succeeding in innovating.

Although (i) suggests that voluntary sharing can fully mitigate the threat of envious retaliation, this result only holds when we restrict our attention to the special case of "non-stochastic" innovation. In a more general model with stochastic innovation, in which an innovator needs to invest resources in innovation and with a positive probability may fail in innovating. With stochastic innovation, conditional on 1 succeeding in innovating, 2's retaliation will decrease l's utility. Thus, 1 will have the incentive to give a voluntary transfer to 2 so as to reduce 2's retaliation. Since 1 will not capture the full return on his investment in innovative activity, this constitutes an incentive to restrict investment. Thus, the sharing mechanism cannot fully mitigate the threat of others' envious retaliation

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