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The great difficulty here is not to represent the matter as if there were something one couldn't do.

Wittgenstein, *Philosophical Investigations* §374

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## 0. Introduction

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The aim of this dissertation is to vindicate Wittgenstein’s critique of Frege’s and Russell’s conceptions of logic in his *Tractatus Logico-Philosophicus*.<sup>1</sup> I call this a critique because Wittgenstein’s purpose is not merely to correct a mistake, but to uncover the sources of a deep philosophical confusion. What Wittgenstein calls “the cardinal problem of philosophy” — the failure to appreciate the true nature of logic — is also a failure to know who and what we are.<sup>2</sup>

The nature of the critique advanced in Wittgenstein’s *Tractatus* has gone lost on many interpreters, since it is commonly assumed that Wittgenstein’s views are much closer to Frege’s and Russell’s than they really are. Frege and Russell, the founders of analytic philosophy and Wittgenstein’s mentors, were themselves proponents of a radically new program of philosophical critique. Frege develops a powerful formal language by means of which the ambiguities of natural language can be avoided and the true logical form of our judgments can be revealed. This allows Frege to resolve longstanding philosophical disputes; for example, disputes over the nature of number are resolved by attending to the logical form of the claims in which we deploy this concept. One telling way in which Frege points the way to Wittgenstein’s critique is found in his polemic against the psychologistic logicians, who attempt to treat logic as an object of

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<sup>1</sup> Wittgenstein 1922 (henceforth abbreviated as *Tractatus* or *Tr.*)

<sup>2</sup> Ludwig Wittgenstein, “Letter to Bertrand Russell, August, 19, 1919” in Wittgenstein 1998. p. 121

empirical study. Frege likens the confusion of the psychologists to an attempt to “jump out of one’s own skin” — a form of self-alienation fostered by an incoherent use of language.<sup>3</sup> Russell points the way to Wittgenstein by demonstrating that longstanding philosophical puzzles, which may initially seem to turn on substantive metaphysical issues, originate from an abuse of the logical ambiguities of language. These puzzles are, in the last account, pseudo-problems, mere philosophical illusions, that can be made to completely disappear by means of logical analysis.<sup>4</sup>

And yet Wittgenstein finds that Frege and Russell fail to fully live up to their promise, that logical analysis will provide us with full philosophical clarity. In arguing for the logicist thesis, that mathematics is grounded in logic, Frege and Russell construe logic itself as a realm of substantive content, from which mathematical content follows. This idea, according to Wittgenstein, prevents them from recognizing the true nature of the relation between logic and thought. Logic is not the source of any content that we draw on in thought. It is the form of thinking.

My inquiry takes its start from a specific line of criticism advanced in the *Tractatus*, and shows how this criticism reflects the most fundamental differences between Wittgenstein’s, Frege’s and Russell’s respective conceptions of logic. In *Tractatus* 5.132, Wittgenstein says that Frege’s and Russell’s rules of inference are “senseless” and that they would be “superfluous”. Rules of inference are those rules which govern the operation of the formal systems of proof which Frege and Russell develop, and guarantee the validity of inferences drawn in those systems. Such rules are also to be found in all modern systems of proof. So why does Wittgenstein reject the rules of inference, and how does he think we can do without them?

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<sup>3</sup> Frege 2013, p. xvii (henceforth abbreviated as *Grundgesetze*).

<sup>4</sup> Russell 1905.

Existing interpretations of Wittgenstein's criticism in 5.132 fail to recognize the precise nature of the defect he finds in his predecessors' conception of inference, and the manner in which this criticism reflects his own conception of logic. Wittgenstein's rejection of rules of inference is often taken to consist in a restatement of the well-known regress argument presented by Lewis Carroll.<sup>5</sup> Carroll shows that if the manner in which inferences are justified requires additional inferential reasoning, there would be no end to the process of inferential justification. For, if the appeal to a justificatory logical principle consisted in adding it as a premise to the inference, then the act of justifying one inference would yield a second, different inference, which is yet to be justified — by means of a further appeal to a further logical principle — and so on *ad infinitum*. The justification of any single act of inference would require an infinite number of acts of justification.

The threat of regress can be avoided through the distinction, which is drawn in all modern axiomatic systems, between principles of logic which can serve as premises of inferences and principles of logic which govern inferences but do not form part of inferences, i.e. a distinction between axioms and rules of inference. It is widely recognized that Frege makes such a distinction; I argue for the more controversial claim that Russell does, too.<sup>6</sup> But this means that their conception of rules of inference is not open to the kind of criticism that is articulated in Carroll.

Could Wittgenstein be so uncharitable to his predecessors as to accuse them of failing to respond to the challenge of Carroll's puzzle? Or is he not really targeting their use of rules of inference? The current literature on the topic seems to treat this dilemma as unavoidable; I reject

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<sup>5</sup> Carroll 1895.

<sup>6</sup> E.g. Hacking 1979 and Ricketts 1985 think that Russell does not; Proops 2001 thinks that he does.

the assumption that any critique of rules of inference must take the form of Carroll's puzzle. Rather than dismiss Wittgenstein's criticism as a failure, I aim to uncover a deeper problem he identifies in their conception of inference.

Frege and Russell perceive a gap between logic and its expression in language: language is incapable of displaying — unable to put up for rational evaluation — the principles of reasoning that underlie its design and its use. Such principles must remain inexpressible within language; for the attempt to express them would give rise to circularity and regress. But it is these very principles that give a language the ability to express thought. A paradigmatic example is the distinction between concepts and objects, which Frege finds to be impossible to draw explicitly without at the same time violating it.<sup>7</sup> Rules of inference similarly represent a failure to make logic fully explicit. Frege says that the rules of inference cannot be expressed in the formal system because they form its basis;<sup>8</sup> and Russell concludes from his engagement with Carroll's regress that inference must be “simply perceived” and cannot be “guaranteed” by an explicit formal procedure.<sup>9</sup> In both of their systems of proof, the appeal to the rules of inference is not recorded as one of the premises. Thereby, Carroll's regress is thwarted.

Nonetheless, Frege and Russell continue to talk of the rules of inference as encoding substantive logical content, the appeal to which justifies the inferences. It is the rules, together with the axioms, that determine the content of the logical system, and together they form the foundation on which mathematical knowledge rests, according to the logicians.

Wittgenstein's critique aims to expose the incoherence of this conception of rules of

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<sup>7</sup> Frege 1984c (henceforth abbreviated as CO).

<sup>8</sup> Frege 1972d, p. 136 (henceforth abbreviated as *Begriffsschrift* or *BS*).

<sup>9</sup> Russell 1903, p. 41 (henceforth abbreviated as *Principles*).

inference, and more generally, of this construal of the relation between logic and language. The discovery that logical principles cannot be made explicit in a non-circular way does not signal the limits of our means of expression; it signals that something is confused in the idea of a logical principle. In treating the inexpressible rules of inference as contentful, Frege and Russell draw a picture according to which the realm of content extends beyond the realm of what can be said in language. To use the image which opens Wittgenstein's *Tractatus* — Frege and Russell think of logic as limiting language and thought from the outside. They pretend to be able to think both sides of the limit.<sup>10</sup>

Wittgenstein draws a fundamental distinction between what is said in language and what only shows itself in our use of language. The purpose of this distinction, on my account, is to spell out the relation between logic, language and thought in a way that reveals the incoherence of Frege's and Russell's views. This contrasts with the standard reading according to which Wittgenstein's distinction between saying and showing boils down to a distinction between two kinds of content — the expressible and the ineffable kind. The distinction, on this standard reading, is meant to enable Wittgenstein to intimate metaphysical truths that cannot be directly communicated.<sup>11</sup> But to strictly think through Wittgenstein's distinction is to see that what shows itself is not something of which we could say that, if only it could be said, would count as a contentful truth. To think that would be to think of what shows itself — logical form — as a kind of content (content of the non-expressible kind) rather than to truly distinguish the content and the form of thought.

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<sup>10</sup> Cf. the slightly different formulation in Conant 1992: Wittgenstein, unlike Frege, is not “making the space of thought wider than the space afforded by the logical structure of language” (p. 151).

<sup>11</sup> This is the approach advocated by Anscombe 1959 and Hacker 1972, among many others. I discuss their reading in Chapter 5.

Rules of inference, on Frege's and Russell's accounts, are precisely such things that show themselves but cannot be said. Thus to adopt the standard reading of Wittgenstein's distinction between saying and showing would debar us from explaining why Wittgenstein criticizes Frege's and Russell's conceptions of rules of inference. For on the standard reading, the whole purpose of the distinction is to justify Wittgenstein's own intimations of ineffable but contentful truths. It thus only becomes possible to appreciate why Wittgenstein rejects Frege's and Russell's rules of inference by coming to see that Wittgenstein's conception of the relation between logic and language eliminates the space that Frege and Russell carve out for the ineffable but contentful logical principles.

The most apt way I find to describe Wittgenstein's approach to logic is to say that he advances a logical hylomorphism. The rejection of rules of inference reflects the idea that logical form reaches all the way down, from reasoning through understanding to our mastery of sub-propositional components. Our grasp of any content always involves a grasp of its network of logical relations. Propositions cannot even be individuated in abstraction from the logical context in which they play a role — what Wittgenstein calls their “place” within “logical space”. Thus the appreciation of the internal, inferential relations between a whole array of propositions is presupposed in each of the propositions we understand.

Logic itself does not have a place in logical space — it articulates its dimensions and the relations between any of the points that occupy it. Logic, for Wittgenstein, is not an independent source of justification, added on to propositions from the outside. To understand a proposition and to see how it stands vis-a-vis other propositions that we understand must be one and the same act; the binding, normative force of logic must be internal to the propositions themselves. Thus the step from understanding to inferring is immediate, and there is no need for rules to

guide and justify us. That's why rules of inference would be superfluous.

To think that rules of inference are needed in order to justify inference reflects a philosophical confusion which is sustained by the indeterminacy that underlies the philosophers' use of language: to say that one has an understanding of the premises and the conclusion, and then to insist that one is not able discern the logical relations between them, is to implicitly deny that understanding has really taken place. The very claim that our reasoning depends on the application of rules of inference constitutes a form of philosophical *nonsense*.

It is not only mistaken, but also fatefully misleading to portray logic as a source of substantive knowledge. This opens a gap between logic and thought, and deepens our puzzlement about the very possibility of rationality. How can logic, understood as a realm of content (which we may or may not be able to make explicit) have any power over our thinking? And if it does have such power, how can we, finite reasoners, discern in what way its demands apply to our thought? This kind of questions beset both Frege's and Russell's accounts of logic. The hylomorphic conception of logic that I attribute to Wittgenstein in this dissertation avoids these puzzles, or rather, it provides a point of view from which such puzzles do not even begin to make sense.

### **Summary of the Dissertation**

The first half of the dissertation (chapters 1-3) is a study of Frege's and Russell's conceptions of inference and the way they articulate it in the context of their formal logical systems. I show that Frege and Russell conceive of rules of inference in ways that make their views vulnerable to Wittgenstein's criticism. The second half of the dissertation (chapters 4-6) provides an account of

Wittgenstein's criticism of Frege's and Russell's conception of rules of inference and of the conception of logic that it presupposes.

The first and second chapters concern Frege's conception of inference. Chapter 1 looks at Frege's understanding of the act of inference. I highlight aspects of Frege's account to which Wittgenstein's criticism poses no threat. These include the idea that basic logical notions such as inference can only be elucidated, but not defined; that the act of inference is indicated by signs but is not referred to by them; the idea that acts of inference confer a normative status on sequences of propositions; and the idea that the expression of inference exhorts an interlocutor to acknowledge the justification of the conclusion by retracing the inference on her own. Frege's recognition that inference "does not belong in the realm of signs"<sup>12</sup> thereby points the way for Wittgenstein's (albeit much more radical) conception of logical form, which shows itself in what we say but cannot itself be said.

Chapter 2 looks into Frege's treatment of the rules of inference of his system. I show that Frege is committed to the idea that rules of inference encode substantive logical content. I argue that Frege thereby blurs the contrast between the logical form of language and the content that the language is capable of expressing; by doing that, he invites Wittgenstein's criticism. In my discussion of Frege I distinguish between formal and substantive roles that Frege gives to rules of inference. The formal role of rules of inference is to secure the rigor of the system and to constrain its design in ways that ensure its reliability and perspicuity. The substantive role consists in the rules' putative contribution to the justificatory and explanatory powers of the system. In Frege's axiomatic system, the axioms encode logical content from which substantive theorems derive. Frege ascribes to the rules the same content that he ascribes to the axioms.

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<sup>12</sup> Frege 1984h, p. 318.

Rules are not only constitutive of the ways in which we frame content, but are also a source of content for the system — the content by virtue of which we can say that the theorems derive from the axioms. I contrast Frege’s austere picture of the act of inference that I presented in the first chapter with the more robust picture of inference being justified by contentful rules, that I develop in the second chapter. Rules play a dual role, being both contentful and formal, both justificatory and inexpressible.

Chapter 3 turns to Russell. Like Frege, Russell thinks that a logical formalism is more than a mere calculus — that it is a language, a vehicle of meaningful thought. But, like Frege, Russell is pushed to recognize that a formal language cannot fully and explicitly express all of the fundamental principles and logical distinctions that constitute its use. This does not prevent Russell from thinking that some of what language cannot express continues to play a substantive role in inferences, comparable to the one that the expressible axioms do. This opens him to the criticism that Wittgenstein makes in *Tractatus* 5.132.

The way in which Russell develops his view of inference is, however, quite different from Frege’s. The first part of chapter 3 is devoted to Lewis Carroll’s puzzle about inferential justification, and the ways in which Russell responds to it. It is often claimed that Russell lacks a proper distinction between rules of inference and the propositions of logic and that for this reason he fails to appropriately respond to the challenge posed by Lewis Carroll. I argue that this judgment is mistaken — the explicit distinction between rules and laws is not the only way, and certainly not the most fundamental way, in which one could respond to Carroll’s challenge. This is an important issue for my overall argument in this dissertation, since some of the interpreters who claim this are also wont to confuse Wittgenstein’s challenge in 5.132 with Carroll’s.<sup>13</sup> One

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<sup>13</sup> See Hacking 1979, Ricketts 1985.

of my tasks in this chapter is to show that the two challenges — Wittgenstein's and Carroll's — are independent. I proceed to offer a reading of Russell that shows him to provide an adequate response to Carroll's challenge, but leaves him open to Wittgenstein's criticism. Russell's solution to Carroll's challenge involves an appeal to an “extra-formal” act of “simply perceiving” the applicability of the rule. I show that there are important similarities between this and more recent attempts to respond to Carroll.<sup>14</sup> And I argue that Wittgenstein's criticism may be applied to these more recent accounts as well.

Chapter 4 introduces my interpretation of *Tractatus* 5.132 and sets the agenda for the chapters that follow. My main concern is to show that the actual targets of Wittgenstein's criticism in Tr. 5.132 are Frege's and Russell's conceptions of rules of inference. This is not a trivial claim; the unclarity of the language of 5.132 has given rise to a debate in the secondary literature in which I intervene.<sup>15</sup> My argument shows that Wittgenstein's concern with the status of rules is distinguishable from his concern with other aspects of Frege's and Russell's systems. Though Wittgenstein has ample reasons to object to the hierarchical, axiomatic approach to logic that Frege and Russell advocate, his objections to their conception of rules of inference calls for a separate explanation.

Chapter 5 motivates Wittgenstein's rejection of rules of inference by demonstrating that it is grounded in his distinction between saying and showing. What is *shown* — in logical propositions, as well as in ordinary, contentful propositions — is what we need to have mastery of if we are to be able to *say* and understand anything. What is shown, in other words, is the form of all representation — logical form. I argue that when this distinction is strictly drawn, it leaves

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<sup>14</sup> e.g. Boghossian 2014.

<sup>15</sup> I argue against the readings provided in Hacking 1979, Ricketts 1985, and Proops 2001.

no room for anything like Frege's and Russell's rules of inference, which cannot be said and yet are considered to encode substantive content.

Wittgenstein's entire conception of logic is contained, as in a nutshell, in the distinction between saying and showing, and the central debate in the scholarly literature on Wittgenstein's *Tractatus* is to a large extent a debate over the nature of this distinction. Wittgenstein scholars divide into two broad camps: the so-called "resolute" readers on one side, and the so-called "metaphysical", "ineffabilist", or simply, "traditional" readers on the other side.<sup>16</sup> The resolute reading, which I side with here, takes the framing remarks of the *Tractatus* with utmost seriousness, and refrains from attributing to Wittgenstein any intention to develop substantive theses, or to reveal any substantive metaphysical truths that can only be shown but cannot be said. What shows itself in our use of language cannot be thought of in terms of facts, properties of language, since these are precisely the kinds of things we can say. Logical form — what is shown in propositions — must not be thought of in terms of content.

I argue that Wittgenstein's critique of his predecessors conception of rules of inference can only be properly explained by a resolute reading of the *Tractatus*. Wittgenstein takes issue with Frege and Russell precisely because they adhere to an "ineffabilist" view of logic. For their account of rules of inference, in which logical content that is not expressible is taken to play a substantive role in reasoning, is an instance of the idea that something substantive can be shown, but cannot be said. A resolute distinction between saying and showing aims to rule this out. Frege's and Russell's accounts of rules of inference treat aspects of logical form as though they were substantive facts, ultimate grounds that justify our inferences. Frege's and Russell's rules

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<sup>16</sup> The term "resolute" was coined by Thomas Ricketts and first made public in Goldfarb 1997, to describe the reading of Wittgenstein advocated primarily by Cora Diamond and James Conant. See Conant and Diamond 2004; and for the opposing view see e.g. Hacker 2000.

are thus meant to achieve precisely what Wittgenstein denies can be done: to say, or quasi-say, what can only be shown. Frege's and Russell's rules of inference are rejected not because the work they are supposed to do belongs in the shadowy realm of what *shows*, and cannot be said, but rather because there is no such "work", and no substantive logical content sayable or showable — that is involved in inference.

Frege's and Russell's treatment of rules of inference are a case of what Wittgenstein diagnoses as philosophical nonsense. Philosophical nonsense arises from the tendency to misconstrue the nature of language and thought, to pretend that we can observe and discuss the form of our thought from an imaginary point of view that is external to it. Philosophers take it to be incumbent upon them to make contentful claims about the preconditions of making contentful claims.<sup>17</sup> They posit superfluous entities where none are needed; they take statements that are merely senseless to be deep and substantive; and they insist on using nonsensical forms of words that have not been assigned determinate meanings. This is the shape of Wittgenstein's threefold charge of rules of inference — that they are superfluous, senseless and ultimately, nonsense.

Chapter 6 seeks to respond to the following question: what kind of account of logic and reasoning must we have, if we are to get by without the need for rules of inference? One constraint that the rejection of rules poses on such an account is that there be no gap between inferring and understanding — the latter must already provide us with all that is needed for the former. A second constraint on the account is that logic not be treated as a source of content which is separate from the activity of grasping, thinking and reasoning — that logic provide no additional content to which one appeals within that activity. Logic, on my proposal, is the form of the activity itself, not the content with which this activity engages.

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<sup>17</sup> They attempt, to use Michael Kremer's phrase, to "take care of logic" rather than to let logic "take care of itself" — as Wittgenstein recommends in 5.473. See Kremer 2014.

The conception of logic articulated in the *Tractatus* is only capable of doing away with the need for rules of inference because it does not sunder inference off of understanding. If the propositions involved in an inference are truly understood, and one already has full mastery of their logical form, this includes an appreciation of the internal relations that propositions stand in to other propositions. One is then already in possession of the justification of the conclusion, and one already has all the permission to infer that one could ask for. But this way of speaking is still problematic, to the extent that it makes it seem as though logical form is something apart from the activity it informs. It is therefore more accurate to say that our understanding of the premises puts us in a position to infer. We do not need to *take* the premises to provide us with reasons — the premises *are* our reasons for inferring.

From this vantage point I restate Wittgenstein's reasons for rejecting the rules of inference. The account of the logically informed understanding of propositions renders the rules of inference *superfluous*; expressions of the rule of inference, such as the ones proposed by Russell and Frege, turn out to be *senseless*. Finally, I argue that Russell's and Frege's accounts of inference exemplify Wittgenstein's diagnosis of philosophical *nonsense*. It would be nonsensical to claim that one fully understands the propositions that stand in the proper internal relation of following, yet is unable to infer the one from the other without appeal to a rule which is independent of both. The insistence that rules of inference are needed for the justification of inference reflects an indeterminacy in the use of the terms proposition, inference, and justification.

## **Part I: Inference in Frege and Russell**

## 1. Frege's Conception of Inference

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Logic, for Frege, is universal and all-embracing: logical truths hold true without restriction, in all domains of objects, and any thought must be in conformity with them. Logic is at one and the same time the study of the principles of thought and the study of a realm of genuine knowledge. One of Frege's aims in constructing a formal system — what he calls a 'Begriffsschrift' —<sup>1</sup> is to make it possible to establish the validity of proofs. Another, more specific aim is to make it possible to establish the logicist thesis, that mathematical knowledge is grounded in logic. In line with these aims, Frege designs the Begriffsschrift to capture all and only the content that is actually relied on in proofs. Since the Begriffsschrift is governed by strict rules, it constrains and regiments our procedures of proof, restricting all inference to the simplest, most immediate transitions, which are authorized by the application of the rules.<sup>2</sup> The choice of these rules of inference complements the choice of the axioms of the system; together, they give the logical system its power and its content, which, according to Frege, is sufficient for showing that

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<sup>1</sup> 'Begriffsschrift' is Frege's term for the symbolic notation he devised for logic. It is also the name of his first major work in logic (Frege 1972d). I will use '*Begriffsschrift*' in *italics* to refer to Frege's work, and 'Begriffsschrift' in non-italicized letters to refer to the idea of such a logical system. In marginal citations of the work I will use the abbreviation BS.

<sup>2</sup> BS, p. 107; Frege 2013, p. VI. I will henceforth use '*Grundgesetze*' in referring to this work in the body of the text; in citing it, I will use the abbreviation GG.

mathematical truths are grounded in logic.

Inference, Frege holds, is the act whereby we justify one judgment by grounding it in other judgments.<sup>3</sup> However, inference forms a blindspot for the formal system; as Frege puts it, inference “does not belong in the realm of signs”.<sup>4</sup> One way to understand this idea is that inference is something that Begriffsschrift expressions may count as, not something that these expressions report on. But this conflicts with a central logicist concern, namely to make logic fully explicit in order to distinguish logical from non-logical sources of knowledge. If Frege is correct that arithmetic is analytic, then there must be no reliance on intuition in proofs that establish that fact — everything must be shown to depend on the content of logical principles.

The problem surfaces in a clearer way in Frege’s account of the rules of inference of his system. These rules, which guide our use of the formal system, cannot be expressed within the notation that they govern.<sup>5</sup> Nonetheless Frege thinks of these rules as principles that encode part of the content of the system; and so he is forced to acknowledge that some of the objective content that is relevant for the establishment of logical proofs cannot be made fully explicit within the Begriffsschrift.

The tension that underlies Frege’s account of inference reflects two conflicting commitments that are intertwined in his universalist approach to logic. On the one hand, logic is concerned to express the most general and most abstract truths, whose expression employs only logical vocabulary and unrestricted variables — this is what Thomas Ricketts calls Frege’s

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<sup>3</sup> Frege 1979b, p.3.

<sup>4</sup> Frege 1984h, p. 318. Henceforth abbreviated as FG2.

<sup>5</sup> BS, #13, p. 136.

“official construal of the content of the axioms”.<sup>6</sup> On the other hand, logic spells out constitutive laws that govern the acts of judgment, such that no judgments can be framed outside them — this is what Ricketts calls the view “that underlies the identification of the Begriffsschrift as logic”.<sup>7</sup> Frege pursues the first idea in setting up the formal axiomatic system, within which one is able to derive all other logical propositions and thereby to show that the content of mathematical statements is fundamentally logical. The second idea emerges most clearly in Frege’s elucidations of the basic logical notions, where he presents logic as the condition of possibility of our making sense of anything at all, as well as in Frege’s reflections on the value of the Begriffsschrift, which he takes to derive from its capacity to capture the universal principles that govern thought.<sup>8</sup>

The exceptional status that rules of inference end up having for Frege — being both inexpressible and contentful — indicates that ultimately, he acknowledges that the universality of logic cannot be accounted for in terms of the generality of content. Wittgenstein’s critique of Frege and Russell aims to undermine the idea that logic is such a realm of content. Logical axioms, which Frege takes to express substantive content, are for Wittgenstein senseless tautologies, completely empty of content.<sup>9</sup> Rules of inference, for their part, are senseless and

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<sup>6</sup> Ricketts 1985, p. 3f.

<sup>7</sup> Ibid; cf. Diamond 1991e. Putnam 2000 construes this tension in terms of the conflict between the Kantian thought (which Frege inherits) that there is no such thing as illogical thought, and the idea that the laws of logic are the utmost general truths. See Conant 1991, p. 127 for discussion. Conant himself discerns a different but related tension in Frege, between two the austere and the substantial conception of nonsense. See Conant 2001, p. 381, and my discussion in Chapter 5.

<sup>8</sup> Frege says, for instance, that his Begriffsschrift is capable of making explicit a large number of the laws of thought (BS, #13, p. 136). Cf. Frege 1979c (henceforth abbreviated as BLC), p. 10 and p. 35; Frege 1972c, p. 91, 1984e, p. 242, and Frege 1960 (henceforth abbreviated as FA), p. iv.

<sup>9</sup> Wittgenstein 1922, 6.1-6.111. Henceforth abbreviated Tr. or *Tractatus*.

superfluous.<sup>10</sup> I will argue, in the second part of this dissertation, that the underlying tension in Frege's construal of the universality of logic is overcome, in Wittgenstein, by rejecting the very idea of logical content, and proposing instead a hylomorphic conception of logic, where logic is understood as the form of thought.

Nonetheless, there are deep affinities between Frege's and Wittgenstein's approaches to logic, underlying these important differences. In this chapter I aim to develop these affinities and to consider the possibility of what I will call a "resolute reading" of Frege.<sup>11</sup> The idea is to show that we can find in Frege an austere conception of inference, where no content, beyond that of the premises and conclusion, is appealed to. We can also find in Frege a resolute conception of philosophical elucidation, according to which all of the claims which seem to ascribe substantive content to logical notions are in fact elements of a propaedeutic exposition, and they can all be thrown away once we have overcome our philosophical confusions and temptations. Even the judgment-stroke and the inference-stroke of Frege's *Begriffsschrift* can be understood as elucidatory devices, by comparison with which we can clarify our everyday notions of truth and inference. In the next chapter I will focus on the irresolute moments in Frege's account of inference. I will show that Frege's elucidations of the rules of inference betray the idea that there is a realm of logical content that extends beyond the realm of what can be said. This places Frege's view of inference within the target zone of Wittgenstein's critique.

The first section of this chapter (1.1) prepares the ground for this argument through an inquiry into Frege's notion of elucidation. Frege holds that basic logical notions, such as the notion of inference, can be elucidated but cannot be defined. This idea anticipates Wittgenstein's

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<sup>10</sup> Tr. 5.132.

<sup>11</sup> The point is to see to what extent one can apply to Frege the kind of reading that Conant and Diamond 2004 apply to Wittgenstein. The resolute reading of Wittgenstein is discussed in detail in Chapter 5, section 5.1.

distinction between what can be said in language, and what only shows itself in our use of language.<sup>12</sup> On my reading, however, Frege leaves open something that Wittgenstein aims to resolutely reject, namely that there is substantive content which elucidation points at, but must remain inexpressible.

In section 1.2 I discuss Frege's elucidations of inference. I argue that inference is a primitive logical notion, and that its elucidations reveal its holistic interrelatedness to other basic logical notions: judgment, truth and justification. None of these can be understood in abstraction from the others. This discussion allows me to situate Frege's view as an alternative to two opposing trends in the philosophy of logic — psychologism and platonism.

Section 1.3 focuses on a puzzling feature of Frege's account of inference. Frege says that inference lies beyond the realm of signs, and yet he seems to insist on indicating it within the Begriffsschrift symbolism, by means of signs that signal the transition from premises to conclusion. To save Frege from this inconsistency, I draw on a distinction he often makes between signs that *designate* and signs that merely *indicate*, but do not modify the content of what is said. What the expression of inference does instead is indicate the force of the conclusion, showing that it is a conclusion of the premises which precede it.

In section 1.4. I draw an analogy between judgment and inference. Like judgment, inference confers what we may call a *normative* status on propositions. It places a demand on the reader of Begriffsschrift to recognize and retrace for herself the justification of one judgment through other judgments. In section 1.5 I conclude that it is this normative status of inference that Frege thinks cannot be made explicit in the realm of signs, and cannot form the *content* of a Begriffsschrift expression.

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<sup>12</sup> See Geach 1976. And see my discussion of this claim in section 1.1 below, and in Chapter 5, section 5.2.

In section 1.6 of this chapter, I show that Frege has the resources to overcome the inner tensions that beset his work, even if he does not eventually take advantage of these resources. I sketch the shape of a “resolute” reading of Frege, that is a Wittgensteinian reading of Frege that takes the elucidations to be merely propaedeutic devices, a use of language that traffics in nonsense, but may eventually be thrown away.

### **1.1. Inference, Elucidation, and Useful Nonsense**

For Frege, inference is a primitive logical notion and as such, it cannot be defined.<sup>13</sup> Indeed Frege holds that the requirement that basic logical notions be defined is misguided. What definitions do is take us from the definiendum to something that is independent of it, by means of which we can fully grasp it. But in the case of basic logical notions, we do not have a way to grasp the definiens without having already (implicitly or explicitly) relied on the definiendum:

What is simple cannot be analyzed and hence not defined. If, nonetheless, one attempts a definition, the result is nonsense.<sup>14</sup>

On the introduction of a name for what is logically simple, a definition is not possible; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the word as it was intended.<sup>15</sup>

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<sup>13</sup> Admittedly, Frege does not explicitly assert that inference is primitive, but this can be gathered from what he says in BLC, p. 15 as well as FG2, p. 339. In what follows I will argue that inference must have this status — i.e. I will argue that what Frege does say about inference cannot count as a definition, and that his notion of inference is just as basic as the other logical notions which he takes to be primitive and undefinable. Thomas Ricketts seems to endorse this approach to Frege’s notion of inference in Ricketts 1986, p. 84.

<sup>14</sup> Frege 1979j, p.271.

<sup>15</sup> Frege 1984c, p. 183 (henceforth abbreviated as CO); cf. Frege 1979h and FG2, p. 300-301.

Instead of definition, Frege proposes to give an elucidation (*Erläuterung*).<sup>16</sup> A Fregean elucidation, such as the elucidations of inference I explore in the next section (1.2), is a non-reductive form of explanation, which puts one notion in the context of other basic logical notions and explores their interrelations.

Every science deploys primitive and simple notions which cannot be defined within it; in logic, the indefinability of the primitive terms is further complicated by the what is often termed the “logocentric predicament”: logical notions are presupposed in any act of thinking, explaining, and defining, and so they cannot themselves be properly defined.<sup>17</sup> The logocentric predicament is a direct consequence of Frege’s universalist conception of logic, understood as the idea that the laws of logic are the laws of thought as such. What the universalist cannot define within the formal system of logic, and must be presupposed by him in its design, is also something which he cannot define outside that system, since even there it is always-already employed and presupposed. Logic is all-embracing, and thus one cannot stand outside it and define its notions from an external vantage point.

However, any interlocutor can be presupposed to have some grasp of the indefinable logical notions, inasmuch as she has the ability to engage in rational discourse.<sup>18</sup> Frege makes this claim explicitly in his discussion of the difficulty of drawing the crucial distinction between concept

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<sup>16</sup> The English translations of Frege’s work often miss the importance of Frege’s notion of elucidation; for instance in Frege 1979h, p. 207, ‘*Erläuterung*’ is translated as ‘illustrative example’. See the discussion of the notion of elucidation in Conant 2001, p. 386; Weiner 1990, pp.227-280, as well as Weiner 2005a.

<sup>17</sup> The term “logocentric predicament” was first used in this sense by Harry Sheffer in his review of Russell and Whitehead’s *Principia Mathematica* (Sheffer 1926, p.228). Sheffer explains the idea in the following way: “In order to give an account of logic, we must presuppose and employ logic”. A forceful argument for the centrality of this predicament to an understanding of the work of Frege and his followers is presented by Ricketts 1985. Frege 1984d gives expression to this predicament in his remark on the indefinability of identity, since it is presupposed in every act of defining (p. 200).

<sup>18</sup> CO, p. 193. cf. FG2, p. 301, and Frege 1979h, p. 207: “we have to be able to count on a meeting of minds, on others guessing what we have in mind”.

and object:

I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway — who does not begrudge a pinch of salt.<sup>19</sup>

Not only is there no way to introduce terms into the formal language which express the distinction between concepts and objects, our very attempt to clarify these notions in our own language collapses into nonsense. For when we try to say how concepts differ from objects, we are bound to use sentences in which concepts take the grammatical subject position, and present themselves as objects. When we talk about concepts, our language forces us to treat concepts as objects; we thereby efface their predicative nature, so we do not manage to refer to concepts as concepts after all. To Frege, this indicates that language constrains what we can express. Some thoughts about logical principles are bound to remain inexpressible:

I admit that there is a quite peculiar obstacle in the way of an understanding with the reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought.<sup>20</sup>

As they stand — qua expressions of our natural language that present themselves as contentful assertions about the most basic logical notions — the elucidations fail to make sense.<sup>21</sup> Nonetheless Frege thinks that we can somehow grasp — though we have no way of asserting — that that which the elucidation cannot adequately express is in fact a logical distinction found “deep in the nature of things”.<sup>22</sup>

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<sup>19</sup> CO, p. 192. I discuss this text in more detail in Chapter 5, section 5.2.1

<sup>20</sup> Ibid., p. 193.

<sup>21</sup> GG, #35, p. 54; Frege 1979g, p.178; Frege 1984b, p. 148; and Frege’s “Letter to Russell, 29.6.1902” in Frege 1979k, pp. 135-137; (henceforth abbreviated as PMC).

<sup>22</sup> Frege 1984b, p. 156.

Elucidations may be useful, despite their being ultimately misleading. For example, in *Foundations of Arithmetic* Frege says that expressions of number refer to self-subsistent objects. This elucidation seems to involve a substantive claim. Indeed, many readers of Frege elevate these claims to the rank of an official ontological doctrine.<sup>23</sup> And yet Frege warns us against doing that:

The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes.<sup>24</sup>

The elucidation of the concept of number is meant to help us see how to recognize that the propositions of Frege's system are adequate substitutes for the claims we make in our ordinary talk of numbers. Such elucidations convince us that we are not losing any of the content of our ordinary talk when we switch to the formal language, nor adding anything that was not already there. As Frege observes, his elucidation of number guards us from confusion by reminding us that in natural language we do not treat number words as predicates. But elucidations of number only succeed in guarding us from the nonsense that would result from that by propounding another form of nonsense — that of the object-like self-subsistence of numbers.

In a sense, it is inevitable that in the propaedeutic elucidation of a logical system one would have to resort to imprecise expressions, and in certain cases, to nonsensical ones. Nonetheless, elucidations can be helpful to us to the extent that, despite their being misleading, they have some salutary effect. Their point is very often negative: they warn us against treating concepts as objects; against treating truth as a property; and against treating the laws of logic as psychological laws. At the same time, issuing such warnings requires that Frege himself use

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<sup>23</sup> For a defense of this reading of Frege, see Burge 1992. For a response, see Weiner 1995.

<sup>24</sup> FA, p. 72.

modes of speech that present concepts as objects, truth as a property, and thought and judgment as mental events.

The elucidations of inference present a similar problem. Frege says that inference lies beyond the “realm of signs”.<sup>25</sup> He thereby indicates that inference is not something about which we can make proper assertions — that an acknowledgment of a valid inference is not the possible content of judgment.<sup>26</sup> But in speaking about inferences, e.g. as objective and valid, our language creates the impression that we are dealing with facts and make robust claims about them. Further, Frege acknowledges that the rules of inference cannot be expressed within the formal system that they govern: “These rules... cannot be expressed in the Begriffsschrift because they form its basis”.<sup>27</sup> But when he discusses the rules, and explains how, by operating the Begriffsschrift properly, the validity of inference is guaranteed, Construes these rules as a source of logical *content*.<sup>28</sup> Wittgenstein’s critique of Frege’s conception of rules of inference targets this very issue: the idea that something which is inexpressible in language is appealed to as the source of content.

For Wittgenstein, Frege’s acknowledgment that an explication of basic logical notions requires elucidation, rather than definition or description, is a step in the right direction; but it is

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<sup>25</sup> FG2, p. 318.

<sup>26</sup> See Ricketts 1986, p. 84.

<sup>27</sup> BS, #13, p. 136 (translation emended); cf. Frege BLC, p. 37.

<sup>28</sup> This claim is established in the next chapter.

not radical enough.<sup>29</sup> According to the resolute interpretation of Wittgenstein's *Tractatus* that I develop in this dissertation, Wittgenstein resists the temptation to treat elucidation as capable of conveying ineffable truths.<sup>30</sup> In Frege, by contrast, we often get the feeling that in elucidating the logical features of language we reveal *something* that resists expression — some inexpressible insight about things that cannot be meaningfully said.

Wittgenstein, on the reading I will offer in Chapter 5, draws a stricter distinction between what is said by means of language and what only shows itself in our use of language. This distinction rules out the idea that what shows itself, but cannot be said, can nonetheless be contentful and substantive.<sup>31</sup> According to Wittgenstein, what shows itself in our use of language is logical form. But this means that logic is not a realm of contentful truths, and that language cannot be thought of as restricted in its ability to express anything. Elucidations, according to Wittgenstein, are useful, albeit misleading forms of expression, by means of which we remove

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<sup>29</sup> The term elucidation (Erläuterung) and its cognates are used by Wittgenstein in two ways, both of which can be traced back to Frege:

1. In Tr. 3.263, elucidation is the clarification of the meanings of primitive symbols of a formal language; In Frege, it is a recurring theme that meanings of primitive and basic logical notions are not susceptible to definitions, and must be clarified by other means (e.g. GG, p. 4, Frege 1979h, p.207, FG2, p. 300, CO, p. 182).

2. In Tr. 4.112 'elucidation' characterizes the nature of philosophical activity, which aims at the "logical clarification of thought". And in Tr. 6.54, Wittgenstein talk of his sentences as bringing about elucidation ('erläutern') to the extent that the reader learns to recognize them as nonsense and subsequently throws them away. Frege himself makes a similar point in his reflections on what an elucidation of basic logical notion can and cannot do. Frege admits that sentences that concern basic logical notions are useful for generating mutual understanding, even if they ultimately fail to make sense, and should only be taken as hints. (CO p. 193; Frege 1979h, p. 207).

I compare Wittgenstein's notion of elucidation with Frege's in Chapter 5, section 5.2.

<sup>30</sup> According to the resolute interpretation of Wittgenstein's distinction developed in Chapter 5, Wittgenstein makes no substantive use of elucidatory nonsense; all nonsense is mere nonsense, and is eventually to be thrown away. This line of interpretation stems from Diamond 1991c.

<sup>31</sup> See e.g. Tr., 2.172, 4.121 and 4.1212. Geach 1976 assimilates Wittgenstein's approach to elucidation to Frege's. An important corrective to Geach's reading is offered by Conant 2001. I develop the contrast between Geach's reading of Wittgenstein's distinction, and Conant's "resolute reading" in Chapter 5, section 5.2.

philosophical confusions; they are not capable of disclosing anything of substance, not even indirectly. At most, they can be seen as rungs on the ladder that leads us to a state of greater clarity — but such a ladder must eventually be discarded. This is how Wittgenstein famously characterizes his elucidatory activity in the *Tractatus*:

6.54 My propositions are elucidatory (erläutern) in this way: he who understands me ultimately recognizes them as nonsense, when he has climbed up through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it).<sup>32</sup>

From Wittgenstein's perspective, Frege does not manage to throw away the ultimate rung in the ladder of elucidation. He is implicitly committed to the possibility of saying (or quasi-saying) that which can only be shown; for him language is sometimes a constraint we run up against, and he seems to imagine a point of view from which we would not be constrained in this way. Wittgenstein thinks that once we get clear on what it is we do in using language, we would no longer regard ourselves as running up against any limit to expression, and therefore we would no longer feel that we are barred from saying anything that we might genuinely wish to say.<sup>33</sup> In particular, the fact that rules of inference cannot be given expression in the language they govern without giving rise to nonsense would no longer be taken to indicate that some logical content remains beyond our means of expression; rather, it would be taken to indicate that the very idea that there is some ineffable content that needs to be appealed to in inferring is illusory.

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<sup>32</sup> Tr. 6.54. Translation emended.

<sup>33</sup> As Cora Diamond puts it, in Wittgenstein there remains no residual saying — no glimpses of ineffable quasi-facts — in the talk of what is shown. See Diamond 1991d, p. 120.

## 1.2. The Holism of Frege's Elucidations of Inference

Inference, according to Frege, involves a *judgment* of the conclusion, an acknowledgement of its *truth*, made *on the basis of* premises and *in accordance with* logical laws.<sup>34</sup> But, as I will argue in this section, the notion of inference is just as primitive as the other fundamental notions which occur in its elucidations— judgment, justification, truth and law. None of these notions can be understood apart from the others, and apart from the notion of inference itself. In other words, inference can only be elucidated from within the holistic context which it forms together with these other notions.<sup>35</sup>

Here are the four main elucidations of inference Frege offers:

- INF1 To make a judgment because we are conscious of other truths as providing a justification is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of correct inference is the goal of logic.<sup>36</sup>
- INF2 An inference... is the pronouncement of a judgement made in accordance with logical laws on the basis of previously passed judgments. Each of the premises is a determinate thought recognized as true; and in the conclusion, too, a determinate thought is recognized as true.<sup>37</sup>
- INF3 [W]e can only infer something from true sentences. Thus if a group of sentences contains a sentence whose truth is not yet known, or which is certainly false, then this sentence cannot be used for making inferences. . . . When we infer we recognize a truth on the basis of other previously recognized truths according to a logical law.<sup>38</sup>
- INF 4 We justify a judgement either by going back to truths that have been recognized already or without having recourse to other judgments. Only the

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<sup>34</sup> See the elucidations of inference quoted below.

<sup>35</sup> For a discussion of the idea that Frege's basic logical notions presuppose each other see Ricketts 1996, p. 131 and Tolley 2011.

<sup>36</sup> Frege 1979b, p.3.

<sup>37</sup> FG2, p. 318.

<sup>38</sup> Letter to Dingler (31.1.1917) in PMC, pp. 16-17.

Inference is elucidated in a few additional places, but these do not add anything significant to the ones I review here.

first case, inference, is the concern of Logic.<sup>39</sup>

I divide my discussion of these elucidations into two. In 1.2.1 I discuss the relation Frege draws between inference and the content of judgment; in 1.2.2 I turn to the relations between inference, truth and justification.<sup>40</sup>

### 1.2.1. Inference and Content

That the elucidations of inference presuppose the notion of judgment is obvious — in all four elucidations, INF1- INF4, inference is elucidated as a way of combining several judgments. INF4 distinguishes inference from other forms of justification by the fact that in inference of a judgment that is achieved by appeal to other judgments. But it is not trivial to claim that the dependence is reciprocal — that the notion of judgment itself presupposes the notion of inference. Judgment, for Frege, is the act in which we transition from the grasp of a content (a thought) to the recognition of its truth.<sup>41</sup> Thus in order to justify the claim that it would be circular to define inference in terms of judgment (e.g. as a *species* of judgment, with some special features added to it), I'd like to show that according to Frege, the content of judgment is individuated in terms that presuppose the inferential nexus in which the judgment stands.

In Frege's early work, the "conceptual content" of any two judgments is said to be identical

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<sup>39</sup> Frege 1979f, pp. 174-175.

<sup>40</sup> The role of the notion of laws, or rules, in the elucidation of inference is the topic of the next chapter.

<sup>41</sup> On Frege's distinction between the logical *act* and its *content* See e.g. Frege 1984b, p. 149, and GG, #5, p. 9. For the idea that in judgment we progress from content to recognition of truth-value, see Frege 1984i, p. 355-356, as well as Frege 1997, p. 159: "Judgements can be regarded as advances from a thought to a truth-value. Naturally this cannot be a definition. Judgement is something quite peculiar and incomparable".

And see the illuminating discussion of Frege's conception of judgment in Smith 2009.

if and only if the same judgments may be inferred from them.<sup>42</sup> This criterion of inferential potential guides Frege in identifying the grammatical distinctions that must be reflected in his Begriffsschrift. All those distinctions that have any significance for inference must be reflected in the formal expression of judgment:

...the only thing considered in a judgment is that which influences its *possible consequences*. Everything necessary for a correct inference is fully expressed; but what is not necessary is usually not indicated; *nothing is left to guessing*.<sup>43</sup>

An example for the application of this criterion can be found in Frege's rejection of the grammatical distinction between subject and predicate, and of the distinction between active and passive verbs. Take for instance the following two sentences:

At Plataea the Greeks defeated the Persians.

At Plataea the Persians were defeated by the Greeks.

Each of these sentences entails the same judgments and is derivable from the same judgments. For Frege, this means that the very same Begriffsschrift judgment must stand for both.<sup>44</sup>

The idea of the inferentially articulated conceptual-content of judgment undergoes a major transformation in Frege's middle and later work, following the introduction of the sense-reference distinction.<sup>45</sup> The bifurcation in the notion of content does not alter the relation

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<sup>42</sup> BS, #3, p. 112f.

<sup>43</sup> Ibid; emphasis in the original.

<sup>44</sup> Ibid.

<sup>45</sup> A thorough discussion of this issue is found in Kremer 2010, p. 275ff.

between inference and the truth-value of the judgments, i.e. between inference and reference.<sup>46</sup> But the bifurcation allows Frege to account for more fine-grained differences in the inferences endorsed by different cognitive agents, in terms of the different senses they associate with judgments that concern the same objects. Such differences stem from the cognitive value that the constituents of judgments have for different agents, rather than from the reference of these constituents. So to a certain extent they do not reflect inferential potential. But inference continues to be presupposed in the cognitive notion of sense, in the sense that the finitude of the agents' cognitive points of view is measured in terms of the inferences they are willing to draw.<sup>47</sup> Frege's new, stratified notion of content thus continues to reflect, at both levels, an inferential articulation.

Another sense in which the notions of content and inference are interrelated can be gathered from Frege's remarks on his choice of the primitive logical connective of the *Begriffsschrift*, the material conditional. Frege says he chooses the conditional because of its "close affinity with the important relation of ground and consequence".<sup>48</sup> The inner articulation of all logically complex content, within the language of Frege's *Begriffsschrift*, thus reflects the patterns of inference that it may be involved in. Of course other connectives could have been

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<sup>46</sup> The idea that truth-value directly tracks inferential relations is made explicit in this remark Frege wrote shortly after the introduction of the distinction between sense and reference (Frege 1979d, p.122):

They [the "intensionalist logicians"] forget that logic is not concerned with how thoughts, regardless of truth value, follow from thoughts, that the step from thought to truth-value — more generally, the step from sense to reference — has to be taken. They forget that the laws of logic are first and foremost laws in the realm of reference and only relate indirectly to sense.

(I modified the translation by replacing "meaning" with "reference", where Frege uses *Bedeutung*). Frege's immediate aim here is to counter the view of those he calls "the intensionalist logicians", but the general point he is making is independent of that polemic and reflects his considered view of logic following the sense-reference distinction.

<sup>47</sup> See Frege 1997, p. 171.

<sup>48</sup> BLC, p.37. This claim will be discussed at greater length in the next chapter, as it relates to the choice of *modus ponens* as the basic rule of inference of Frege's system.

chosen as primitive, and with them, other modes of inference would have become more natural. Any such choice would have had the same result: the inner logical articulation of the proposition would mirror the patterns of inference through which it could be exploited.<sup>49</sup>

### 1.2.2. Truth and Justification

Let's now look at the way in which the notions of truth and justification figure in the elucidation of inference. In each of the four elucidations quoted above Frege says that in inference judgments are *justified* because they relate to previous judgments — the justification of the conclusion derives from the *truth* of the premises (INF1), and the conclusion is judged true “on the basis of” or “by recourse to” the judgments previously judged as true (INF2, INF3, INF4). In elucidations 1-3 he says that inferences are justified because they are made “in accordance with” logical laws (INF2, INF3) — the “laws governing this kind of justification” (INF1). The question in what way the justification of inference depends on “laws” and which laws are at issue will be discussed in full detail in the next chapter, where I will argue that in speaking about laws, Frege has the rules of inference in mind, rather than the explicit axioms of his system.<sup>50</sup> I will also show that the introduction of explicit rules of inference plays a crucial role in Frege's epistemological project.<sup>51</sup> It guarantees that all Begriffsschrift proofs fit into the narrow mold of logical inference. Given such a restriction, it becomes possible to scrutinize the sources of knowledge relied on in the proof of mathematical statements, and to tell whether they depend on

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<sup>49</sup> In GG #13, p.25, Frege acknowledges the arbitrariness of this choice, and he adds that what speaks in favor of the conditional is “the ease and perspicuity with which one can thereby represent inference”. I return to this issue in the next chapter, section 2.5.

<sup>50</sup> See in particular Chapter 2, section 2.2.

<sup>51</sup> See Chapter 2, section 2.3.

non-logical premises.

In the present section I'd like to focus on a different epistemological aspect of Frege's conception of inference: the idea that inference is a transition from truths to truth (INF3). This idea reflects a central principle of Frege's epistemology, and marks a crucial difference between Frege's approach to logic and the modern, post-Tarskian semantic approach. Let me begin by describing this difference, before I return to discuss the epistemological role of Fregean inference.

The fundamental difference between Frege's and the modern, post-Tarskian approach to logic is often construed in terms of the absence, in Frege, of a semantic meta-perspective.<sup>52</sup> Frege's conception of logic is *universalist*, in the sense that logic is taken to be all-embracing: there is no standpoint from which one could survey it and define its basic concepts. A Fregean logical system is a language of fully-interpreted signs, and the axioms of that system are universal truths that hold without restriction, of all objects and all domains of discourse.

The modern semantic approach to logic, by contrast, considers languages as systems of uninterpreted signs, and sets up a hierarchy of such languages, in which the semantic properties of one language can be spelled out within languages that are higher than it on the hierarchy. It thereby becomes possible to define a truth-predicate (relative to a given language) and a relation of consequence (between the sentences of a given language). Let us look more closely at the latter notion, and contrast it with Frege's notion of inference. According to Tarski's definition of logical consequence, a sentence  $p$  follows from  $q$  if any model that makes  $q$  true — any interpretation of the non-logical vocabulary of the language and any permutation of the domain

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<sup>52</sup> This difference has been stressed in Van Heijenoort 1967; cf. Goldfarb 1979 and Ricketts 1985. Objections to the claim that Frege lacks a semantic meta-perspective are made by Stanley 1996 and Heck 2007; but see the replies to Stanley in Weiner 2005b and Weiner 2008. A full discussion of these issues is beyond the scope of this chapter; I discuss it in a little more detail in Chapter 2, section 2.2.

of objects — is also a model that makes  $p$  true.<sup>53</sup> On this modern view of consequence, there can be no hierarchy between logical propositions: logically-true sentences are true in all models and hence they all entail, and are entailed by, all other logically-true sentences.

By contrast, Frege thinks of logic as a hierarchical system of knowledge in which the axiomatic starting points support the theorems. The form of scientific explanation which Frege applies to logic is euclidean: explanation is achieved when we show how knowledge of a small number of judgments — unprovable, but self-evident axioms — justifies us in holding a larger number of judgments to be true.<sup>54</sup> Inferences increase our understanding, since through them we expose the interrelations and dependencies between judgments that we already affirm. Through inference we descend down the hierarchy of true judgments, in a route that leads from the foundations to the derived theorems; our success in proving all true theorems reinforces our confidence that we have identified the utmost foundations of a science.<sup>55</sup> According to this conception of explanation, it would be nonsensical to say that a theorem, e.g. the last theorem of the *Begriffsschrift*, entails the axioms of the system, though both are logical truths. In Frege's

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<sup>53</sup> See Tarski 1983.

<sup>54</sup> BLC, in PW, p. 36; BS, #13, p. 136; GG, p. VI.

<sup>55</sup> This feature of Frege's conception of logic has been explored by Currie 1987, as well as Burge 1998.

picture of knowledge, justification goes in one way only.<sup>56</sup>

The peculiar epistemological nature of Frege's conception of inference puts him in contrast to two trends in the philosophy of logic — platonism and psychologism. This has given rise to two diametrically opposed lines of interpretation of Frege. Certain readers have assimilated Frege to the psychologists and accused him of confusing the psychology of belief with the epistemology of logic.<sup>57</sup> Others have sought to avoid this conclusion by assimilating Frege to the platonists, who perceive an objective hierarchy of knowledge, and conceives of inference as tracking this objective hierarchy.<sup>58</sup> But neither of these two alternatives is satisfying, for neither approach can account for the centrality Frege gives to the acts of judgment and inference in his account of the nature of logic.

An alternative to these two opposing approaches is proposed by Thomas Ricketts. On Ricketts's view, the ontological notions that are deployed in Frege's hierarchical view of knowledge are supervenient on the logical notions with which he articulates the realm of the activities of judgment and inference.<sup>59</sup> Frege's talk of an objective hierarchy has a merely

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<sup>56</sup> See the discussion in Kremer, 2010, p. 227ff and Smith 2009.

Another reason why Frege's restriction of inference to reasoning from true premises might seem peculiar to the modern reader is that it rules out proofs by *reductio*, and more generally, inference from assumptions — practices that are ingrained in scientific and mathematical work. But Frege finds ways to emulate indirect proofs without renouncing his restriction. Instead of asserting unconfirmed hypotheses, Frege expresses them as antecedents of conditionals. *Reductio* proofs are similarly be restructured such that at no point does the false assumption appear as asserted. On this see, in addition to the Letter to Dingler mentioned above (in PMC, pp. 16-17), Frege 1979h, p. 246.; FG2, p. 335; Letter to Jourdain (undated), in PMC, p. 79; Frege 1984j, p. 375; and Frege 1984k, p. 402.

There are moments in Frege's writings in which he seems more willing to consider a more general notion of justification that does not exclude inference from false premises; for example, he says that "[t]he task of logic is to set up laws according to which a judgment is justified by others, irrespective of whether these are themselves true" (Frege 1979f, p. 175; and cf. FA, #14, p.20-21, where the procedure of *reductio* is allowed). This relatively early view is later rejected by Frege; Michael Kremer argues that this rejection reflects the discovery of the sense-reference distinction. See Kremer 2010, p. 230 fn. 19.

<sup>57</sup> Currie 1987 raises the same suspicion.

<sup>58</sup> Burge 1998.

<sup>59</sup> Cf. Ricketts 1986 as well as Diamond 2003.

elucidatory role; through it we clarify the logical practices that reflect our concern with truth. In next two sections I propose a reading of Frege’s conception of inference that builds on and radicalizes Ricketts’s proposal. I do that by arguing that many of the things Frege says about judgment apply to inference as well. In particular, the distinction between force and content which Frege draws in his elucidation of judgment will be used to explain why inference is inexpressible; I will then argue that Frege’s elucidations of inference, like his elucidations of judgment, aim to capture the normative role of logic.

### 1.3. Inexpressible or Expressible?

Frege claims that inference “does not belong to the realm of signs”,<sup>60</sup> and yet he also claims that what a *Begriffsschrift* is supposed to do is to enable us to inspect the validity of inferences:

[The *Begriffsschrift*’s] chief purpose should be to test in the most reliable manner the validity [*Bündigkeit*] of a chain of inferences [*Schlusskette*] and expose each presupposition which tends creep in unnoticed, so that its source can be investigated.<sup>61</sup>

Frege goes on to say that a proper *Begriffsschrift* avoids expressing “everything which is without importance for the chain of inference [*Schlussfolge*]”.<sup>62</sup> In a *Begriffsschrift* proof, we write down one proposition below another, in accordance with the specified rules, and the result is supposed to amount to an inference.

In what sense, then, does inference remain beyond the realm of signs? Indeed, this claim

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<sup>60</sup> In its original context, this polemical remark is directed against the formalist logicians — those for whom logic is the investigation of the properties of systems of uninterpreted signs. See FG2, p. 318.

<sup>61</sup> BS, p. 104, translation emended.

<sup>62</sup> BS, *ibid.* (emphasis in the original); and see BLC, p.35 where Frege prides himself on the greater capacity of the *Begriffsschrift* to give expression to thought and inference, compared to the logical calculi of his predecessors.

seems inconsistent with the fact that in the *Begriffsschrift*, there is a sign dedicated to the expression of the act of inference. Corresponding to natural language expressions such as ‘hence’ and ‘therefore’, Frege’s formal language employs signs that signal the drawing of conclusions — a horizontal line separates the conclusion of an inference from the premises that are listed above it.<sup>63</sup> In *Grundgesetze*, where more than one rule of inference is employed, a variety of such horizontal lines is put to use, each marking the specific type of inferential transition that is being used.<sup>64</sup> These signs are meant to indicate “how” an inference is made — i.e. starting from what premises and according to which rule. But if inference is inexpressible, what role should these signs be taken to have?

The consistency of Frege’s elucidations of inference will be restored if we notice the distinction Frege draws between two functions signs may have. There are signs whose purpose is to designate [*bedeuten*], and signs that merely indicate [*andeuten*].<sup>65</sup> For example, in his account of natural language, Frege points out that some distinctions, such as the one between ‘but’ and ‘and’, do not consist in a difference in what is designated, but merely in what is indicated, which does not affect the logical content of a thought. For in terms of what is designated, the two

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<sup>63</sup> The analogy to mathematical symbolism is obvious. See Frege 1972b, p. 88:

The arithmetic language of formulas is a conceptual notation [*Begriffsschrift*] since it directly expresses the facts without the intervention of speech. ... Such contents — here equations or inequalities — as they follow from one another are written under one another. If a third follows from two others, we separate the third from the first two with a horizontal stroke, which can be read “therefore”. In this way the two-dimensionality of the writing surface is utilized for the sake of perspicuity.

<sup>64</sup> In Appendix A of the next chapter I present these formal devices in full detail.

<sup>65</sup> One application of this distinction (albeit of a different sort than the one under discussion here) concerns signs for indeterminate (free) variables, marked by Latin letters in GG, #8, p. 11, and #17, p. 31. On p. 313 of FG2, Frege objects to the formalist Korselt’s way of borrowing the distinction between *bedeuten* and *andeuten* from him. Korselt attempts to apply the distinction to the axioms, saying that (on his own formalist conception) mathematical axioms do not *designate* empirical facts, but “at best *indicate* them”. Importantly, Frege also appeals to the distinction between indicating and designating signs when he discusses the function of the judgment stroke, in Frege 1984b, p. 149, fn. 7. I discuss this in more detail in the next section.

expressions' contributions are identical.<sup>66</sup>

Along similar lines, the horizontal inference stroke should not be taken as a *designating* Begriffsschrift expression — in this sense it is true to say that it does not express inference. It merely indicates it:

How, and according to which rule, an inference is drawn is indicated [*deutet an*] by the sign between the formulas, while the sign —·— marks the termination of a chain of inference.<sup>67</sup>

When we *designate* something through signs, we say something about that thing. But in *indicating* something, we do not transmit an additional fact. Like the punctuation marks of our language, an indicating sign merely communicates to the reader *how* she is to take the content that is being expressed by the designating signs.<sup>68</sup> In the case of inference, the judgments themselves provides all the content that can be *designated*, whereas their arrangement on the page, and the horizontal stroke placed between them hint, by way of *indicating* signs, at the fact that these judgments constitute an inference. It is left to the reader to ascertain, by means of her own activity of reasoning, that it is, indeed, an inference, since this is nowhere said. In the next section I will provide further support for this proposal, by drawing an analogy between Frege's conception of inference and his conception of judgment.

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<sup>66</sup> See Frege 1984i, p. 357f. What the word 'but' indicates has to do, for instance, with a contrast between, on the one hand, the expectations that the content of the first part of the sentence creates in him and, on the other hand, the content of the second part.

<sup>67</sup> GG, p.VI; cf. *ibid.*, #26, pp. 43-44.

<sup>68</sup> Wittgenstein compares all logical connectives to punctuation marks in Tr. 5.4611. Frege would probably object to comparing his judgment stroke to a punctuation mark; his view is that the judgment stroke has a more substantive role. See "Letter to Jourdain" (undated), in PMC, p. 79.

#### 1.4. The Force of Inference

Frege takes one of his main achievements as a philosopher of logic to be the separation of assertoric force from the content of sentences. This separation is at the same time an acknowledgment of an inability to fully capture assertoric force by means of signs:

Now the thing that indicates most clearly the essence of logic is the assertoric force with which a sentence is uttered. But no word, or part of a sentence, corresponds to this; the same series of words may be uttered with assertoric force at one time, and not at another. In language assertoric force is bound up with the predicate.<sup>69</sup>

In natural language, assertoric force remains implicit. In the *Begriffsschrift*, however, a special sign is introduced for indicating it: the vertical component of the sign ‘ $\vdash$ ’, which Frege calls the judgment stroke.<sup>70</sup>

In the case of judgment, too, there is an apparent tension between Frege’s claim that the force of judgment does not form part of the content of a proposition and his proposal to introduce a *Begriffsschrift* sign for it. And here, too, the tension can be resolved by means of the distinction between designation and indication that I discussed in the previous section. The judgment stroke does not form a designating sign, and it does not contribute to the content of propositions. Indeed, The rules that lay down the use of the judgment stroke in the *Begriffsschrift* give it a special status, exempting it from the principle of compositionality that governs signs that designate content. Judgment strokes, for Frege, are not functional expressions:

The assertion sign [*Urteilsstrich*] cannot be used to construct a functional expression; for it doesn’t serve, in conjunction with other signs, to designate [*bezeichnet*] an object. ‘ $\vdash 2+3=5$ ’ does not designate anything; it asserts something.<sup>71</sup>

...[W]e do still sometimes want to assert something, and for this reason I have

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<sup>69</sup> Frege 1979i, 252

<sup>70</sup> GG #5, p. 9.

<sup>71</sup> Frege 1984b, p. 149, fn. 7.

introduced the judgment-stroke. This is a manifestation of my endeavor to have every objective distinction reflected in symbolism. With this judgment stroke I close off a sentence, so that each condition necessary for its holding is also effectively to be found in it, and by means of the self-same sign I assert the content of the sentence thus closed off as true.<sup>72</sup>

If we subjected the judgment stroke to compositionality, and allowed embedding it in larger contexts, it could no longer fulfill the function of closing off the content of thought and putting it forth as a single unit of significance. Moreover, in forming part of the content, it would itself stand in need of an act of acknowledgement; it would no longer have the role of indicating the force of the judgment, since its own force will become open to questioning.

To be sure, Frege's talk of an "objective distinction" that is marked by the judgment stroke should not be taken to express a psychologistic conception of judgment. Frege is not concerned with facts about the mental state of any particular thinker. As he sees it, the logician is always and everywhere concerned with "the investigation of *the* mind; of *the* mind, not of minds".<sup>73</sup> On the other hand, the recoil from psychologism must not lead too far in the opposite direction of platonism. What is objective in the distinction Frege aims to indicate cannot be a matter of fact — judgment does not assert a property of the proposition; it alters its status.

Frege's conception of judgment and assertoric force steers clear of the Scylla and Charybdis of psychologism and platonism. Judgment belongs to a dimension that is orthogonal to the dimension of content, and it is only on that dimension that the judgment stroke can be said

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<sup>72</sup> Frege 1984e, p. 247. It is worth noting that here Frege treats as one sign something which he elsewhere treats as two separate components — the horizontal stroke that closes off the thought and the vertical stroke that indicates judgment.

<sup>73</sup> Frege 1984i, p. 369. Wittgenstein condemns Frege's judgment stroke for only playing such a psychological role (in Tr. 4.442). Indeed, in BS, #2, p. 111, Frege says that it is the *writer's* acknowledgement that is indicated by the judgment stroke. On the other hand, Frege's anti-psychologistic approach to logic is quite explicit; the rejection of psychologism is announced, prominently, as one of his three fundamental principles (in the "Introduction" to FA, p. xxii). Whether there is a sense in which Wittgenstein's criticism of the judgment stroke is nonetheless justified is discussed below, in section 1.5.

to reflect an “objective” distinction. An act of judgment transforms the mere supposition of content into the acknowledgment of its truth.<sup>74</sup> For it to alter the status of the content, it must not combine with the content, but must rather close it off as a single unit whose status can then be evaluated.

To say that the act of judgment does not modify the content also means that the recognition of the status of the content that it effects is not something that can be directly communicated from one thinker to another. All we can do is indicate to our interlocutor the to-be-recognized-ness of the proposition, its being binding for any reader who understands us.<sup>75</sup> And in this sense the expression of judgment amounts to something more than the mere gesture at the writer’s own private psychological state. But it is up to the interlocutor to assess its status for herself.

It is illuminating to compare what Frege says about the judgment stroke and what he says about the way in which the truth-predicate functions in natural languages. Despite its misleading grammatical form, Frege holds that the truth predicate must not be taken to ascribe any property to the sentences it attaches to, on pain of regress. This is a point he makes in the context of arguing that truth cannot be defined at all, since in all definition, characteristic properties are being specified:

And any other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it was true that the characteristics were present. So we should be going round in a circle. So it seems likely that the content of the word ‘true’ is *sui generis* and indefinable.<sup>76</sup>

If acknowledging truth involved the identification of certain characteristics, the assertion of the

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<sup>74</sup> Frege 1984b, p. 149.

<sup>75</sup> This way of understanding the purpose of Frege’s judgment stroke is heavily influenced by Taschek 2008.

<sup>76</sup> Frege 1984i, p. 352.

truth-predicate would be tantamount to generating a new content, that of the ascription of the truth-property to the first sentence. But the ascription of this truth-property — like the ascription of any property — would itself stand in need of acknowledgement. So if this second acknowledgment itself involved the ascription of the truth-property, and would itself stand in need of acknowledgment, a regress would seem to arise.<sup>77</sup>

Analogously, if we treated the judgment stroke as a *designating*, functional expression, rather than an *indicating* expression, and if we further assumed that the acknowledgment of the force of the sentence required that we ascribe to it some property by means of this designating expression, a similar regress would arise. In the process of acknowledging the force of a sentence, we would find ourselves ascribing a property to it; the ascription of that property would itself stand in need of acknowledgment, which would again involve the ascription of the same property, and so on and so forth. Such regress is avoided because the judgment stroke does not designate anything, and is not taken to modify the content of what is said.<sup>78</sup>

We can now draw some analogies between Frege's treatment of the truth predicate and of the judgment stroke and his treatment of the inference stroke. Frege says that inference does not belong in the realm of signs, and yet he employs the inference stroke. In doing so, he seems to be relying on the distinction between designating and indicating expressions. And indeed, we can imagine the following regress argument, which would show that the inference stroke must be a merely indicating expression. Suppose that the inference stroke were a designating sign, and its

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<sup>77</sup> Michael Dummett objects that the regress, if there is one, is not vicious. He argues that we are not required to settle the question of the truth of the second thought before we judge the truth of the first. See Dummett 1973, Chapter 13 (pp. 442-470).

<sup>78</sup> There is a certain disanalogy between the judgment stroke and the truth predicate that I do not wish to deny. Frege himself says that the attempt to use 'is true' as an indication of force ultimately miscarries (Frege 1979i, p. 252), precisely because natural language allows us to embed it in contexts that cancel its force, whereas the rules governing the judgment stroke prevent the possibility of its embedding in such contexts.

function were to express content, to ascribe some inference-property to some object (to the conclusion, or the entire inference, etc.). Such ascription would amount to the generation of new content. But adding this content to the original inference would result in the generation of a new inference, in which the new content now serves as an additional premise. And the new inference would require its own inference stroke, that is, the ascription of its own inference-property. Thus if we insist that any inference (including the newly formed one) requires an inference stroke, a regress would arise, consisting of ever growing inferences, and the process of inferential justification would never come to an end.

This regress argument is similar to the one described by Lewis Carroll in his dialogue, “What the Tortoise Said to Achilles”.<sup>79</sup> When faced with a valid inference, Carroll’s Tortoise complains that accepting its premises is not enough to justify the conclusion. He claims that something else needs to be acknowledged, namely the validity of the sequence, and he further suggests that compensating for this missing acknowledgment is analogous to compensating for a missing (or doubtful) premise.<sup>80</sup> But the addition of further premises does not solve the Tortoise’s problem, it merely reduplicates it. Rather than effecting the acknowledgment of the validity of the first inference, it results in generating a new inference, whose validity is yet to be acknowledged.

The regress argument I proposed above similarly targets someone who takes Frege to introduce the inference stroke in order to compensate for something that would otherwise be lacking in the proofs, and who takes it that the inference stroke compensates for that lack that by

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<sup>79</sup> Carroll 1895; I discuss Carroll’s dialogue in Chapter 3, section 3.1.

<sup>80</sup> I am here indebted to Cora Diamond (private correspondence) for this point. In Chapter 3 (Section 3.1) I return to discuss Diamond’s suggestion that Carroll’s regress only arises because the dialogue succeeds in creating the illusion that compensating for the lack of acknowledgement of the validity of an argument is analogous to compensating for a missing premise.

*designating* some content. To avoid the regress that this setup would give rise to, the inference stroke must not be taken to be a designating expression. Indeed, this argument shows that what the inference stroke indicates — inferential justification — remains beyond the realm of (designating) signs.

### 1.5. Inference and Normativity

Let's take stock. Frege's inference stroke does not describe, designate, or refer to facts, and does not contribute any content to the expressions it attaches to. In this sense inference remains beyond "the realm of signs". Nor are the signs which indicate inference meant to express psychological states of mind. What Frege's judgment-stroke, truth-predicate and inference stroke indicate belongs outside the descriptive realm of *facts*, whether objective or subjective. Whether a certain fact about a proposition holds, and whether a fact about our minds holds — e.g. that the proposition is true, or that we take it to be true — does not in itself determine whether we *ought* to hold it to be true. It is the latter, *normative* demand which the acts of judgment and inference concern.<sup>81</sup>

It would be a misunderstanding of the nature of the transition from Frege to post-Tarskian logic to take the latter to provide adequate answers to the questions with which Frege struggles, and in particular, it would be confused to take modern logic to aid us in explaining the normative aspects of judgment and inference which Frege is concerned with here. Rather, with the transition from the universalist to the semantic approach, the concern with the phenomena of

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<sup>81</sup> A similar point is emphasized by Taschek (2008, p. 391), who thinks that the evidence for the claim that Frege thinks of logic as constitutively normative lies in Frege's insistence that Begriffsschrift proof involve asserted proposition only, as well as in Frege's distinction between rules of inference and logical laws (axioms). But Taschek does not discuss the normativity of inferences and the status of the inference stroke.

judgment and inference seems to simply drop out.<sup>82</sup> Though the semantic approach gives us the means of stating (in the meta-language) the relation of consequence that holds between premises and conclusion, such a meta-linguistic assertion is not equivalent to an act of inference; to entitle ourselves to assert the desired conclusion, we would still need to infer it from our meta-linguistic assertions. Reasoning in a meta-language involves inference and judgment no less than reasoning in the object language, but the semantic approach is unable to address the normative claim that these acts make without relativizing it.

By contrast, Frege's universalist conception of logic puts the emphasis on our ultimate concern with truth, and the essential role that logic plays in the pursuit of this end.<sup>83</sup> For him it makes no sense to speak of a relativized notion of truth and of justification. Like the act of judgment, an inference situates the content of our thoughts in the normative sphere of commitments and demands for acknowledgment. Such acknowledgment cannot be transmitted, or handed over, from one interlocutor to the next, by means of designative expressions (of either the object or the meta-language); they are not facts that we can acquire by hearsay. Indicative expressions, like elucidatory hints, can only signal what each thinker must do for himself, namely judge and infer. The acknowledgment of truth and acknowledgment of validity do not form the content that a speaker transmits to her interlocutor; they are the fulfillment of the activity that each rational thinker must engage in by herself.

Like expressions of judgment, expressions of inference place demands on us: they require that we recognize the validity of the inference. To use a Wittgensteinian expression, they do not

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<sup>82</sup> Relatedly, consider Gilbert Harman's argument that modern logic (conceived of as the Tarskian study of logical consequence) can be taken to play any normative role for thought and reasoning. See Harman 1986; and the attempt to bridge logic and reasoning in MacFarlane 2004.

<sup>83</sup> As Taschek 2008 puts it: "the essence of logic is revealed, in the first instance, by reflecting on its constitutive normative status vis-à-vis our practice of judgement assertion, and inference." (p. 388).

pretend to *say* that a sentence “has” truth, or that a certain sequence of sentences consists of an inference — they only *show* it. It is in this sense that what they indicate remains beyond the realm of signs. The normative statuses which Frege’s signs indicate must be acknowledged by each judger for herself, and cannot be transmitted directly.<sup>84</sup>

And yet why must we indicate these normative statuses, if it is up to the interlocutor herself to realize their force? The insistence on the need for such symbols marks a moment at which the possibility of reading Frege resolutely threatens to collapse. As Wittgenstein will argue, even though such devices could have an elucidatory role, they are at bottom superfluous and meaningless. Unless we discard them, we stand the risk of reverting to one of the substantive conceptions of judgment and inference that Frege aims to reject — psychologism or platonism.<sup>85</sup> To preserve the austere conception of inference, Frege’s indicative signs, the inference stroke and judgment stroke, must ultimately be thrown away.

## **1.6. Conclusion: Towards a Resolute Reading of Frege**

In the central sections of this chapter I began to delineate the contours of an austere picture of inference. According to this austere picture, inference does not involve an appeal to any content beyond that which is found in the premises and the conclusion. Inference is an act that endows sequences of judgments with a normative status. By displaying these sequences of judgments within a *Begriffsschrift* and indicating their status, we exhort the reader to acknowledge the

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<sup>84</sup> I attribute a similar conception of the normativity of logic to Wittgenstein in Chapter 6, section 6.1.

<sup>85</sup> This seems to be the point of Wittgenstein’s rejection of the psychologism inherent in Frege’s and Russell’s judgment stroke at Tr. 4.442; the general principle underlying Wittgenstein’s criticism is his version of Occam’s razor — see Tr. 3.328 and 5.47321.

justification of the conclusions. This reveals an affinity between Frege's and Wittgenstein's conceptions of logic; indeed Wittgenstein's rejection of rules of inference in Tr. 5.132 takes its start from the observation that the justification of inference does not require any content beyond that which is given in the understanding of its premises and conclusions.

In this way, I sketched the shape of a resolute reading of Frege. Such a resolute reading would be one which strictly thinks through Frege's remarks on the elucidatory nature of his philosophy. Frege's philosophical elucidations only receive their value from their role as a propaedeutic for the use of the logical system. But once we have gained clarity on that system we should no longer be tempted to take its elucidations to convey anything substantive. In fact, we should discard them completely, since apart from their practical purpose, what they amount to is sheer nonsense.<sup>86</sup> Unfamiliar as this may sound to some readers of Frege, this is a picture Frege himself proposes:

If our language were logically more perfect, we would perhaps have no further need of logic, or we might read it off the language. But we are far from being in such a position. Work in logic just is, to a large extent, a struggle with the logical defects of language...<sup>87</sup>

The ideal of a language so clear that no confusion could ever occur, and no need for elucidation could arise — this is the standard against which we must measure our position as finite rational beings whose language is a natural language. The point is certainly not that an ideal language would enable us to engage in the kind of meta-linguistic reflection that is excluded by universalism; rather, the point is that no such reflection would be needed.

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<sup>86</sup> Weiner 2009 (p. xiii) considers the suggestion that all of the propaedeutic consists of elucidations that ultimately count as nonsense, and dismisses it.

<sup>87</sup> Frege 1979i, p. 252.

Like other elucidations, the elucidations of inference and of the rules of inference themselves harbor the danger of confusion.<sup>88</sup> Their grammar misleadingly treats inference as a fact, and makes it seem as though it can be captured as the content of a proposition (for instance, when we say that in inference, we recognize that this follows from that, etc.). Our talk of rules that govern inference, for its part, tempts us to think of inferential justification in terms of an appeal to an independent logical fact.

To see that the elucidations of inference, though helpful, must eventually be discarded, means to come to see that there is no such *thing* as inference, and no *fact* that justifies it. Somewhat less provocatively, we may say: ‘inference’ is an elucidatory term that only misleadingly seems to refer to a thing; ‘rule’ is an elucidatory term that belongs to the specification of a Begriffsschrift, not to the reasoning which the Begriffsschrift, understood as an elucidatory device, clarifies.

A major step in the direction of this resolute reading of Frege is made by Thomas Ricketts when he argues, against platonistic readings of Frege, that Frege’s apparent ontological commitments are supervenient on his logical commitments; that what is basic for Frege are the practices of judgment and inference. Logic is concerned with the *objectivity* of these practices, rather than with the *objecthood* of entities that the Fregean elucidations seem to be about.<sup>89</sup> But we must be careful not to misread Ricketts’s claims about these practices. Their point is not to merely shift the focus of the platonists’ metaphysical temptation from the realm of semantics to

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<sup>88</sup> CO, p. 193.

<sup>89</sup> Ricketts 1986, p. 66.

the realm of pragmatics; their point is to eliminate it.<sup>90</sup> The resoluteness I tentatively ascribe to Frege here, and to Wittgenstein in the following chapters, eliminates the impression that the elucidations of logical practices are tantamount to such substantive claims.

But to eliminate the misleading implications of these elucidations by insisting that ultimately, there is no such thing as inference or judgment might seem too extreme. Would it not rob us of the possibility of asserting our own rationality? Would we not, by throwing away these elucidations, also give up the possibility of reflecting on our own reasoning, and wasn't Kant right to point out that the possibility of such reflection is essential to reasoning as such?<sup>91</sup> I do not take there to be a real tension between the resolute proposal I pursued here and the Kantian commitment to the essential self-consciousness of judgment. What resoluteness requires, indeed, is that we refrain from thinking about self-consciousness as a species of consciousness, i.e. as a contentful state that involves introspective observation of mental matters of fact. On the resolute proposal, the concepts of inference, judgment and self-consciousness fail inasmuch as they are

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<sup>90</sup> Burge 1992 and Tolley 2011 take Ricketts to have these anti-platonistic, reductive aims. Tolley 2011 assimilates Ricketts's approach to Robert Brandom's, who is more explicitly seeking to reduce semantic notions to pragmatic notions. But Tolley also points out that Ricketts's later work move away from the earlier anti-platonism, in which there is an asymmetric explanatory relationship between semantic and pragmatic notions, towards a more holistic picture of the relation between these notions.

<sup>91</sup> The essential role of self-consciousness in the account of all objective consciousness is famously captured in Kant's dictum, "the 'I think' must be able to accompany all my representations". For otherwise, Kant explains, "something would be represented in me that could not be thought at all, which is as much as to say that the representation would either be impossible or else at least would be nothing to me." (Kant 1998, pp. B131-132)

construed as material and *contentful* concepts, not inasmuch as they can be used to clarify the *form* of thought.<sup>92</sup>

Admittedly, the proposal that we read Frege resolutely faces serious difficulties, not least of which that it would make it impossible to explain what prompts Wittgenstein's criticism of Frege. Indeed, the reason why Frege scholarship steers away from reading his philosophical elucidations resolutely is that Frege himself is not consistently resolute. There is a residual irresoluteness in Frege, which can be seen to underlie the thought that we need to attain a more perfect language before we can recognize that logical theory is superfluous. To be fully resolute, we must come to see that we are already in a position to throw away the ladder of logical theory. In the next chapter I will demonstrate the irresolute moment in Frege's account of inference — his idea that the justification of inference depends on the inexpressible rules of inference.

There is a deep tension in Frege's philosophy of logic, and the resolute reading sketched here exploits but one side of the tension. Wittgenstein himself senses the tension in Frege, and the point of his critique is to root out the moments in which Frege succumbs to the irresolute temptation.

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<sup>92</sup> A similar proposal can be found in Kimhi 2017 as well as in Rödl 2014, who puts it as follows:

Consider the central act of thought: judgment. In judging, I am conscious of judging. I do not judge anyway, and then, in a second act of the mind, am conscious of doing so. ... Judging and being conscious of judging are one act of the mind; any judgment applies the concept of judgment. I call this the self-consciousness of judgment. ... judging things to be a certain way and being conscious of so judging are one act of the mind. ... Now, the concept of judgment is the same as the formal concept of an object of judgment. (p. 484).

## 2. Frege's Rules of Inference and their Content

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In this chapter I aim to establish the claim that Frege's conception of rules of inference merits Wittgenstein's criticism. In the conclusion of the previous chapter I proposed that a resolute reading of Frege would resolve the inner tensions of his work by treating all of his elucidations as propaedeutic means, that introduce us to the use of a logical apparatus, but are ultimately to be discarded as nonsensical: only expressions of the *Begriffsschrift* would count as legitimate judgments that have sense.<sup>1</sup> In this chapter, the possibility of such a resolute reading will be bracketed. My aim here is to identify the irresolute moment in Frege, which gives rise to Wittgenstein's criticism. I will show that Frege construes his rules of inference as inexpressible, but contentful principles. This places Frege's view of inference within the target zone of Wittgenstein's critique. Moreover, it exposes a deep tension in his own conception of logic.

Frege makes a clear distinction between the laws and the rules of his logical system.<sup>2</sup> Logical laws are the axioms and theorems of the system; they are explicit judgments that convey content — the most general and abstract truths of logic. The rules form the foundation of the

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<sup>1</sup> On the Fregean notion of elucidation, see my discussion in Ch 1, Section 1.1. and in Ch. 5, Section 5.2.1, where I compare Frege's notion of elucidation with Wittgenstein's.

<sup>2</sup> The distinction can be gathered both from Frege's explicit pronouncements, discussed here, as well as from his terminological distinctions, which I review in Appendix A.

system, in the sense that they spell out its design and govern its use.<sup>3</sup> For example, there are rules that assign different types of signs to different logical roles: small Latin letters stand for first-order variables, Greek letters stand for propositions, etc. The distinctions that these rules express — e.g. the distinction between predicates and names, concepts and objects — cannot be stated in a fully determinate way, within or outside the formal system, since they reflect the most basic logical notions, and what is logically primitive is indefinable.<sup>4</sup> The rules for the use of signs can be elucidated, but they cannot be expressed within the Begriffsschrift. Rules of inference are grouped with these other rules for signs. They too cannot be expressed in the Begriffsschrift, since they “form its basis”,<sup>5</sup> and therefore “must be expressed verbally”,<sup>6</sup> that is, by means of elucidations.

According to what I called the austere picture of inference, inference is an act which endows propositions with a logical status. The objective distinction between a mere set of propositions and a valid argument cannot be designated by any expression; it can only be indicated.<sup>7</sup> The status that inference endows our propositions is not a property on which we report; it is something which the interlocutor needs to recognize for herself.<sup>8</sup> The only content which inference involves comes from the premises and the conclusions alone.

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<sup>3</sup> BS #13, p. 136.

<sup>4</sup> See CO and FG2, p. 300.

<sup>5</sup> BS, #13, p. 136.

<sup>6</sup> BLC, p. 37.

<sup>7</sup> Frege says of his decision to indicate assertoric force with the judgment stroke that it is “a manifestation of my endeavor to have every objective distinction reflected in symbolism”. In the last chapter, section 1.4, I argue that the same consideration applies to inference — that Frege thinks an objective distinction is made when we infer, and that such distinctions should be reflected in the symbolism (hence his use of non-designative, indicating signs such as the judgment stroke and the inference stroke). See Frege 1984e, p. 247.

<sup>8</sup> See Chapter 1, section 1.5.

To a certain extent, the very idea of formal rules of inference is not inconsistent with this austere picture of inference. Like the rules for the correct use of signs, they do not determine the content of what is said, but only guarantee that it takes a certain form. In the elucidations of rules of inference that I focus on in this chapter, however, Frege treats rules of inference in a way which conflicts with this austere picture. He takes them to be a source of justification, which provides part of the content of the logical system — the content from which the theorems derive.

Accordingly, we can distinguish between a formal and a substantive role that Frege assigns to the rules of inference. The formal role of rules is to secure the rigor of the system and constrain its design in ways that guarantee its logical purity and reliability. The substantive role consists in contributing to the justificatory and explanatory powers of the system.

In Frege's system, it is paradigmatically the axioms that encode logical content from which substantive theorems derive. However, as I will show, Frege takes the rules of inference to encode content as well. This is apparent from what he says about the possibility of transforming rules to laws and vice versa, and from his discussions of the systematic properties of his *Begriffsschrift*. Frege thereby makes clear that for him, rules of inference are on a par with axioms with respect to the task of determining the content of the logical system and its capacity to yield proofs. This explains why at certain points Frege simply counts the rules of inference among the system's "primitive sentences".<sup>9</sup>

Rules of inference are the manifestation of an inner tension in Frege's views which

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<sup>9</sup> BLC, p. 39. cf. BS, #13, p. 136.

Wittgenstein sets out to criticize.<sup>10</sup> We can frame the point of Wittgenstein's criticism in the following terms. Frege violates Wittgenstein's strict form-content distinction. For Frege, *content* stretches beyond the realm of the expressible; the *formal* features of thought and reasoning are treated by Frege as contentful. Given that this logical content is also taken to be inexpressible, language is seen as deficient in its expressive capacity. As I will argue in chapter 5, the basis for Wittgenstein's critique is his much more strictly drawn distinction between what is said in language and what only shows itself, but cannot be said.

The chapter proceeds as follows. In 2.1 I discuss Frege's reasons for claiming that the rules of inference evade formal expression. This parallels the problem of the inexpressibility of inference that I discussed in the previous chapter. In section 2.2 I present and discuss Frege's elucidations of the rules of inference. These elucidations indicate the prescriptive character of rules, the relation between rules of inference and the conditional form of judgment, and the difference between rules of inference and other rules for the use of signs. To articulate my interpretation of Frege's elucidations, I discuss and dismiss the suggestion that these elucidations can be taken to represent Frege's attempt to provide semantic proofs of soundness for his rules. In section 2.3 I turn to examine the systematic, formal role that Frege gives to rules, focusing on his assessment of their methodological importance and their contribution to securing the rigor of the system. In section 2.4 I consider Frege's idea of the economy of rules in a system of proof. The choice of the number of rules depends on the number of axioms that they complement; together they give the system an adequate expressive power. Such choices result in different, but

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<sup>10</sup> This is the tension that Thomas Ricketts identifies between Frege's "official construal of the content of the axioms" and his "view of judgment", in Ricketts 1985 p. 3f; cf. Diamond 1991e. Putnam 2000 construes this tension in terms of the conflict between the Kantian thought (which Frege inherits) that there is no such thing as illogical thought, and the idea that the laws of logic are the utmost general truths. See Conant 1991, p. 127 for discussion. Conant himself discerns a different but related tension in Frege, between two the austere and the substantial conception of nonsense. See Conant 2001, p. 381, and my discussion in Chapter 5.

equally legitimate and equally powerful systems, which differ in the degree of perspicuity and surveyability of proofs that are conducted in them. Underlying this economy is the idea that rules may be transformed into laws and vice versa. I inquire into the idea of the transformability of rules and laws in section 2.5, where I argue that, taken in combination with Frege's view of the contentfulness of the principles of the logical system, this principle results in ascribing substantive content to the rules.

The chapter is accompanied by four Appendices. Appendix A shows that Frege makes a clear terminological distinction between the application of rules of inference and appeals to the laws of logic. Frege also distinctly marks off inferences from transitions between judgments that he does not count as full-fledged inferences. This reflects the unique epistemological role he reserves for inference. Appendix B reproduces the three most explicit and detailed lists of rules of inference that Frege provides. Appendix C concerns types of logical transition that are employed in Fregean proofs, but are not fully acknowledged in the official lists of the rules of the system. Appendix D provides an example of the procedures through which Frege proposes to reduce the traditional, Aristotelian modes of inference to the rules and axioms of his system. This exemplifies the kind of transformations that connect rules and axioms in Frege's system and illustrates one of the senses in which rules are contentful.

### **2.1. The Inexpressibility of Rules of Inference**

The most distinctive mark of the difference between rules of inference and laws of logic is that rules cannot be expressed in a Begriffsschrift and hence "must be expressed verbally".<sup>11</sup> In his

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<sup>11</sup> BLC, p. 37

most explicit statement of the contrast between rules and laws, Frege writes:

INE We have already introduced in the first chapter several principles [*Grundsätze*] of thought in order to transform [*verwandeln*] them into rules for the application of our symbols. These rules and the laws to which they correspond [whose transforms, *Abbilder*, they are] cannot be expressed in our “conceptual notation” because they form its basis [*zu Grunde liegen*]. Now in this chapter, some judgements of pure thought which can be expressed in the “conceptual notation” are to be stated in symbols.<sup>12</sup>

This passage, which I label *INE* (for ‘inexpressibility’), will prove crucial for my argument; in section 2.5 I argue that the connection Frege draws here between rules and laws implies that rules are contentful. But for now, let me focus on the way this passage distinguishes rules and laws. What can be expressed in the *Begriffsschrift* are “judgments of pure thought” — axioms, derived laws of logic, and definitions. Rules, on the other hand, can only to be stated in words. The rules of inference, alongside the other prescriptions for the correct use of *Begriffsschrift* signs, as well as “the laws whose transforms they are”, must all be kept out of the *Begriffsschrift*, since they “form its basis”. By saying that the rules are not expressible in the system that they govern, Frege articulates his understanding of the idea of a formal system of logic.

The rules of inference are inexpressible *within* the *Begriffsschrift* they govern, but in a sense they are not inexpressible *tout court* — after all, Frege talks about them, e.g. when he reflects on how to choose the right number of them and when he specifies the rules themselves, in a mix of German and *Begriffsschrift* signs.<sup>13</sup> Why, then, does he say that the rules of inference are not expressible within the *Begriffsschrift*?

In saying that the rules are inexpressible in a *Begriffsschrift* whose basis they form, Frege groups the rules of inference with the other rules for the use of signs. Some of these rules capture

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<sup>12</sup> BS #13, pp. 136. Bauer-Mengelberg’s translation of this passage has a better rendition of the second sentence, “These rules and the laws whose transforms they are cannot be expressed...”. See Frege 1967, p. 28.

<sup>13</sup> The choices at issue here are discussed in section 2.4 below.

fundamental logical distinctions, about which Frege says that they cannot be made fully explicit; the only way to explicate them is by means of elucidatory hints.<sup>14</sup> For instance, the rule for the use of capital letters for predicates and small letters for object names and variables reflects Frege's strict distinction between concept and object. Any attempt to explicate this distinction ends up violating it: concepts end up in the subject position of sentences, and so are implicitly treated as objects. Frege sees no way around this, and concludes that the elucidatory attempts to articulate this distinction, though practically useful, are a covert form of nonsense.<sup>15</sup> A Begriffsschrift which is designed to respect this distinction would obviously not be able to express the nonsensical elucidatory expression in which the distinction is violated; nor would it be able to express the distinction in any other way, since it has no means of referring to what is being distinguished (concepts and objects) except in ways that already prevent us from comparing them or even differentiating them.

Frege does not say why exactly he thinks the rule of inference resembles the other rules for signs and in what sense the rule of inference, like those other rules, forms the basis of the Begriffsschrift. He seems to think that attempting to express the rule of inference in the Begriffsschrift would not allow it to function as a rule of inference. There are several issues Frege could have in mind. The rule of inference prescribes how we are to treat the expressions of the language: it issues in the assertion of a conclusion, and marks the distinction between a mere series of judgments and a valid inference. However, the Begriffsschrift is designed to convey content, to enable us to make assertions, but it is not designed to express prescriptions. So Frege's point might be the following: an attempt to express the rule of inference as a proposition

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<sup>14</sup> Frege 1979j, p.271; Frege 1979h; CO, p. 183; and FG2, p. 300-301. And see section 1.1 of the previous chapter.

<sup>15</sup> See CO., and my discussion in Chapter 5, section 5.1.

of the Begriffsschrift that it governs would efface its ability to dictate the force of other propositions. Begriffsschrift propositions, on their own, cannot tell us to what to do with other propositions.<sup>16</sup>

Moreover, if the rule of inference, like the other rules, is supposed to tell us how to use the Begriffsschrift, and using the Begriffsschrift in order to express the rule subjects its expression to other rules, then the application of the rule (if it involved the Begriffsschrift expression of the rule) would depend on the application of other rules and would potentially give rise to a regress (or a circularity) in the application of rules.<sup>17</sup> Alternatively, if applying the rule of inference does not involve the Begriffsschrift expression of the rule, such a regress would be avoided; but then the Begriffsschrift expression of the rule should not count as capturing the rule itself. It would not count as an expression of the rule qua rule.<sup>18</sup>

As we have seen in the previous section, Frege does introduce Begriffsschrift various signs that mark the inferential transition, which differ according to the rule of inference that is applied.<sup>19</sup> I have argued that if this is not inconsistent with his claim that inference “does not belong within the realm of signs” and with Frege’s claim that the rules are inexpressible within the Begriffsschrift, then these signs must not count as *expressing* anything.<sup>20</sup> Frege must take the inference stroke to merely indicate logical status, not to designate any content. Like the

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<sup>16</sup> The requirement that we explicitly cite such a proposition whenever we infer would give rise to a regress of the kind discussed in Carroll (1895). See Chapter 1, section 1.4 and Chapter 3, section 3.1.

<sup>17</sup> A similar line of thought is suggested in Kant 1998, A133/B172.

<sup>18</sup> We could additionally think of the following obstacle to expressing the rule in the Begriffsschrift: the Begriffsschrift does not have the resources that allow one of its expressions to make reference to another, or to quantify over judgments of a particular forms, and this would seem to be required by an expression of a general rule of inference, that says how to proceed from propositions of one kind to propositions of another kind.

<sup>19</sup> A detailed description of the use of these signs can be found in Appendix B.

<sup>20</sup> FG2, p. 318.

judgement stroke, it does not form part of the compositional structure of the content they attach to.<sup>21</sup> The strokes serves as mere hints, aiding us in recognizing the logical relations of the premises and the conclusion, when these are too complicated to perceive directly. They act as a form of elucidation.

Wittgenstein's view of elucidation radicalizes these ideas of Frege. Wittgenstein holds that rule-governed systems of notation are nothing but elucidatory aids for thinking (Tr. 6.126b). To the extent that rules for the operation of Begriffsschrift are not taken to contribute to its *content*, they should not be objectionable to Wittgenstein.<sup>22</sup>

Interpreters of Frege often take him to succeed in specifying purely formal rules for his system, similar to the syntactic rules of modern, post-Tarskian logic. Even van Heijenoort, who in other respects is sensitive to the difference between Frege and his successors, says that Frege's rules of inference anticipate the modern method of specifying purely syntactic rules that are "void of any intuitive logic".<sup>23</sup> Admittedly, Frege groups the rules of inference with the rules for the use of signs, and this does seem to indicate that they can be construed as concerned with syntactic manipulations alone.

However, as I will argue below, Frege does ascribe logical content to the rules of inference, and this is the issue which forms the target of Wittgenstein's criticism. Whereas Van Heijenoort takes Frege to identify his purely formal system with logic, I take Frege's rules of inference to reflect his idea that there is a gap that separates logic and its expression in any language. By

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<sup>21</sup> See my discussion of the analogies between the inference stroke and the judgment stroke, in Chapter 1, section 1.4.

<sup>22</sup> Indeed, in *Tractatus*, 6.126 Wittgenstein discusses the legitimate possibility of constructing symbolic systems governed by explicit rules for signs. These systems could be helpful and could be used to show important distinctions and thus to dispel confusions that arise in less perspicuous languages. But we could equally do without them.

<sup>23</sup> van Heijenoort 1967, p. 326.

arguing that Frege’s rules of inference are supposed to capture logical content, I aim to show that for Frege, logical content outstrips our means of expression.

## **2.2. The Elucidations of Rules of Inference**

In the elucidations of inference I discussed in the previous chapter, Frege refers to “laws governing this kind of justification”,<sup>24</sup> and he says that conclusions are drawn “in accordance with logical laws”.<sup>25</sup> What he means by “laws” in these contexts are not the explicit judgments that form the axioms and theorems of the *Begriffsschrift*, for these cannot, on their own, govern inferential justification. Frege’s axioms are universally valid judgments, generalizations that hold of any object whatsoever; they are the starting points for all inferences conducted within the axiomatic system; but they are not principles “in accordance with” which conclusions are drawn.<sup>26</sup> Judgments (such as the explicit axioms are) do not issue in further judgments, and the way in which the axioms govern judging and inferring must therefore be indirect — whatever we do judge and infer, we judge in light of our also holding the axioms to be true, for otherwise we would introduce inconsistencies into our set of judgments. Still, something apart from the explicit axioms has to be involved in this activity, and tell us how each given judgment fares in light of the axioms. This is role of the principles of reasoning and the “rules whose transforms they are” (as Frege says in *INE*), which cannot be stated as explicit axioms.

Let us examine three of the elucidations Frege offers for the rules of inference. The first is

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<sup>24</sup> Frege 1979b, p.3.

<sup>25</sup> FG2, p. 318. cf. “Letter to Dingler” (31.1.1917) in *PMC*, pp. 16-17.

<sup>26</sup> See my discussion of the locution “in accordance with” that is usually associated with the rules of inference, in section 2.1 above.

the statement of the rule of inference in *Begriffsschrift* #6:

IR1 From the elucidation [*Erklärung*] given in #5 it is clear that from the two judgments  $\vdash B \supset A$  and  $\vdash B$  the new judgment  $\vdash A$  follows. Of the four cases enumerated above, the third is excluded by  $\vdash B \supset A$  but the second and fourth [cases are excluded] by  $\vdash B$ , so that only the first [case] remain.<sup>27</sup>

One thing to note about this elucidation is that it is framed in non-prescriptive terms (except perhaps for the use of the judgment stroke). Instead Frege refers back to the elucidation of the conditional stroke he gives in section #5, where he specifies the conditions under which a conditional sentence is to be evaluated as true. There he shows that the truth of a conditional sentence depends only on the truth or falsity of the propositions combined in it; or rather, it depends on whether these propositions are “to be affirmed” or denied. Here is how Frege illustrates his account:

A	B	$B \supset A$
affirmed	affirmed	affirmed
affirmed	denied	affirmed
denied	affirmed	denied
denied	denied	affirmed

The conditional  $B \supset A$  is to be denied whenever, of the two propositions that it combines, the antecedent is to be affirmed but the consequent is to be denied — i.e. when the third of the four possible combinations of A and B is excluded.<sup>28</sup> Given this understanding of the conditional, what IR1 tells us is that when we have an inference, the first premise of which is the assertion of

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<sup>27</sup> BS, #6, p. 120 (Translation emended: I replaced “definition” with “elucidation” as a translation of ‘*Erklärung*’, thus reflecting the importance of Frege’s distinction between proper definitions and indirect elucidations, discussed in the previous chapter, section 1.1).

<sup>28</sup> This account of the conditional is sometimes taken to be the origin of truth-tabular semantics for propositional logic; but as is evident from the absence of a truth predicate in the table Frege produces, or even of any talk of truth-values in the elucidation, the thought that this elucidation offers a semantic justification of the rule is quite problematic. See Weiner 2005b and Weiner 2008. Weiner’s aim is to correct the proposal made by Stanley 1996. I discuss this debate in more detail, below.

the conditional  $B \supset A$ , and the second premise of which is the affirmation of B, the assertion of the first premise excludes the possibility represented in the third line (i.e. it excludes the possibility that when B is affirmed, A would be denied) and the assertion of the second premise excludes the situations represented in the second and fourth line. Thus given the affirmations of the two premises, only one possibility remains — the one accounted for in the first line, in which the conclusion, A, is affirmed.<sup>29</sup> This shows, according to Frege, that the conclusion *follows*. But it palpably depends on the reader’s ability to ascertain this fact for himself.

In his writings following the *Begriffsschrift* Frege’s elucidations of rules more strongly emphasize the prescriptive nature of rules.<sup>30</sup> Thus Frege offers two different elucidations of the rule of inference in *Grundgesetze* #14; the first resembles the elucidation of *Begriffsschrift* #6:

IR2        From the propositions ‘ $\vdash \Delta \supset \Gamma$ ’ and ‘ $\vdash \Delta$ ’ one can infer ‘ $\vdash \Gamma$ ’; for if  $\Gamma$  were not the True, then, since  $\Delta$  is the True,  $\Delta \supset \Gamma$  would be the False.<sup>31</sup>

Frege shows that the judgments involved in a *modus ponens* inference match the truth conditions of a conditional. But there are important differences between this elucidation and the one given in *Begriffsschrift*. Here Frege uses the expression “is the True”, rather than talk of affirmation and denial; he carefully maintains a distinction between mention and use of judgments, by means of quotation marks; and he omits the judgment stroke from the names of the objects referred to in the second part of the explication. I will return to discuss these features below.

There is another elucidation of the rule given just a few lines below, which is also a prescription, but of a very different kind:

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<sup>29</sup> BS, #5, p. 114-117.

<sup>30</sup> In addition to the elucidations from *Grundgesetze*, discussed her, see FG2, p. 328, which I discussed in the previous section.

<sup>31</sup> GG #14, p. 25.

IR3      If a subcomponent of a proposition differs from a second proposition only in *lacking the judgment stroke*, then one may infer a proposition which results from the first by *suppressing that subcomponent*.<sup>32</sup>

The prescriptive nature of the first elucidation consists in the permission to infer the conclusion; in the second elucidation the prescription concerns acts of recognizing and manipulating sign patterns. The features we are here told to keep an eye on include the distinction between subcomponent and main component of the conditional; the occurrence of the judgment stroke, and the repetition of the subcomponent in asserted and unasserted form.<sup>33</sup>

Frege's intention in presenting these elucidations is subject to debate. Richard Heck and Jason Stanley have taken Frege to intend to offer genuine semantic proofs of the soundness of his rules of inference.<sup>34</sup> Others, including Jean Van Heijenoort, Thomas Ricketts, and Joan Weiner, object that no such proof is given here.<sup>35</sup> They argue that a semantic proof would presuppose a standpoint outside the system from which a non-circular account of logical notions can be given, but on Frege's universalist conception of logic, no such standpoint exists.<sup>36</sup> In the remainder of this section I would like to argue for a variant of the latter view.

A proof of soundness for a rule of inference is normally given in the context of the post-Tarskian, semantic conception of logic, where the question is whether patterns of an

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<sup>32</sup> GG #14, p. 26 (my emphasis).

<sup>33</sup> Frege refers to the occurrence of other sign patterns — in particular, of the sign of negation — when he elucidates the other rules of inference of *Grundgesetze*. See GG, #15 and #16, pp. 26-31, and the complete lists of rules given in Appendix B of this chapter.

It is interesting to note that Bynum considers IR1 (from *Begriffsschrift*) to be a “purely syntactic” definition of the rule; it is clear that in comparison with IR3, the non-syntactic nature of IR1 stands out. See Bynum “Editor's Introduction”, p. 71.

<sup>34</sup> See Heck 2007 and Stanley 1996.

<sup>35</sup> See Van Heijenoort 1967; Ricketts 1996; Weiner, 2005b.. And see the contrast I draw between Frege's universalist conception of logic and the semanticist approach to logic, in section 1.2.2 of the previous chapter.

<sup>36</sup> Van Heijenoort 1967 understands universalism as the idea that “nothing can be, or has to be, said outside of the system” (p. 326).

uninterpreted formal system of proof match the semantic relations of logical consequence, defined in the meta-language. As Joan Weiner argues, however, it would be anachronistic to attribute to Frege the kinds of distinctions that would be needed for such proof. Frege does not properly distinguish between object-language and meta-language. He does not have a way to generalize over sentences — the Greek letters employed by him in elucidating the rule are indeed variables, but they do not range over linguistic entities such as sentences — they range over all objects. And Frege does not define a truth-predicate for an object language; his use of “is the true” applies to interpreted sentences in use, not to sentences of the object language that are merely named, or mentioned.<sup>37</sup> Frege never gives a purely syntactic characterization of the system of *Begriffsschrift*, independently of its semantics. Even though there is a noticeable shift between *Begriffsschrift* and *Grundgesetze* (compare IR1 and IR3), even in his later system Frege cannot be taken to draw a clear distinction between a formal deduction and the act of inference, and he therefore cannot be taken to sense any gap between syntactic and semantic consequence. Soundness, in the modern sense, cannot be a pressing question for him. As I have argued in the previous chapter, inference, for Frege, is a basic logical notion which cannot be defined or reduced to other notions without circularity.<sup>38</sup> It is not a merely syntactic operation, the writing of one string of signs after another; it is a form of justification, which places one judgment in relation to other judgments.

The debate has mostly revolved around the elucidations of rules of inference given in *Grundgesetze*, but it would be helpful to also consider IR1, the elucidation given in *Begriffsschrift* #6. At first sight, it might seem to provide the semanticist with a strong footing.

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<sup>37</sup> See Weiner 2005b, p. 328 ff.

<sup>38</sup> In section 1.1. Frege has good, theoretical reasons to dismiss any attempt to reduce, justify, or define one basic logical notion in terms of the other.

IR1 itself is a descriptive statement, which seems to report a fact about what *follows* from what. However, on closer inspection, there is an alternative, and more convincing reading IR1. Frege’s elucidation of the mode of inference itself consists in an act of inference. Immediately following IR1, Frege writes:

We could write this inference perhaps this way:

$$\begin{array}{l} \vdash B \supset A \\ \hline \vdash B \\ \vdash A.^{39} \end{array}$$

Frege seems to refer to IR1 itself as “this inference”, rather than as a proof of the inference. If that is so, then in IR1, Frege is not *mentioning* propositions; he is *using* propositions, performing an *act* of inference that involves acts of judgments. When Frege talks in IR1 of the “case” which remains — the first line in Frege’s table — what he has in mind is not the *justification* for the affirmation of the conclusion; this “case” *consists in* the affirmation of the conclusion. Thus he makes no separation between the semantics of propositions and the syntactic manipulation of signs. Frege’s elucidation of a mode of inference is performed by means of *displaying* an inference; not by means of talking about independently specifiable properties of an inference.

Let us now turn to the more controversial cases, the elucidations of rules of inference in *Grundgesetze*. In the first (IR2) we have an explicit use of the predicate “is the true” and a distinction between mention and use of judgments, by means of quotation marks, along with a conspicuous absence of the judgment stroke; in the second elucidation (IR3) we find references to sign patterns such as the judgment’s ‘subcomponents’. All this raises the suspicion that here we *do* have an attempt at a semantic justification of a rule of inference of a purely formal

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<sup>39</sup> BS, #6, p. 117. I replaced Frege’s Begriffsschrift notation with modern notation, but kept the judgment stroke.

system.<sup>40</sup> But there are ways to draw the later and earlier elucidations closer and thus to avoid this conclusion.

The absence of the judgment-stroke from the first elucidation of *Grundgesetze* reflects the fact that Frege's reasoning takes the form of a *reductio*. Frege is generally opposed to the employment of *reductio* proofs within his formal system, and holds that all proper proofs should always have true judgments as their starting points.<sup>41</sup> Thus the very use of *reductio* in this elucidation may be taken as a signal that he is not attempting to provide a proper proof.<sup>42</sup> The *reductio* begins by stating the false hypothesis, which cannot really be judged: if the conclusion were not true, then given the truth of the second premise, the first premise would have to be false. Frege thus presupposes our prior understanding of inference as an act that takes its start from a recognition of the truth of the premises. If the conclusion of an inference that conforms to the rule were judged to be false, we could not judge both premises and acknowledge them as true, since that would conflict with the commitments we make in judgments. What we have here is not a *semantic* justification of the rule by means of an appeal to assignments of truth-values to the premises and the conclusion; it is an *elucidation* of the commitments one makes in making the judgments that make up the inference.

IR3 involves an even more explicit appeal to the use of signs of the *Begriffsschrift*. But this does not mean that Frege thinks that *Begriffsschrift* propositions are mere signs, pure syntactic objects, with no semantic value attached to them. Frege's resistance to the idea of a purely formal

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<sup>40</sup> See Heck 2007, p. 48, fn. 37.

<sup>41</sup> See Section 1.2.2 of the previous chapter.

<sup>42</sup> Weiner 2005b, p. 340 has a different take on this issue. She attempts to argue that Frege here explain the rule *modus ponens* by means of an appeal to *modus tollens*, since this mode of inference seems more self-evident to him. This is a dubious claim, but even on Weiner's reading, the overall form of the proof must be that of a *reductio*.

calculus is a major theme through all his writings.<sup>43</sup> It is not without reason that Frege titles his work a *concept-script*, and subtitles it “A formula language of pure thought, modeled on that of arithmetic”. He holds that the design of the *Begriffsschrift* reflects the form of genuine thought, and that this is where it gets its value from:

It is possible, of course, to operate with figures mechanically, just as it is possible to speak like a parrot: but that hardly deserves the name of thought. It only becomes possible at all after the mathematical notation has, as a result of genuine thought, been developed so that it does the thinking for us, so to speak.<sup>44</sup>

The rules for the use of the signs of the *Begriffsschrift* mirror the fundamental logical principles that we already command, as rational thinkers, and it is for this reason that the signs have the ability to express the content of thought.<sup>45</sup> Frege’s ultimate concern is not to specify a formal system of signs; the appeal to signs patterns in IR3 is meant to aid us in facilitating the procedure of inference, but it only gets its significance from its relation to thought.

To sum up, Frege’s elucidations of rules aim to display to the reader the correspondence between the formal procedures of a BS and the logical form of thought that the reader already possesses. This allows the reader to adopt the *Begriffsschrift* as a significant language. There is no question here of a semantic proof of soundness, and there is also no need for one.

### **2.3. The Methodological Role of the Rules of Inference**

Rules of inference are a central building block of Frege’s system of proof — a system within

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<sup>43</sup> BLC, p. 10 and p. 35; Frege 1972c, p. 91, and 1984e, p. 242.

<sup>44</sup> FA, p. iv.

<sup>45</sup> This is also the contrast Frege draws between Jevons’s calculating machine and his own *Begriffsschrift* in BLC, p. 35.

which we can track the logical justification of conclusions by deriving them from explicit premises in a gapless, stepwise manner. It is by virtue of the strict application of well-defined rules of inference that the rigor of the proofs is guaranteed, and this makes the vindication of the logicist thesis possible. My aim in this section is to specify those features of Frege's idea of a rule-guided formal system that do *not* merit Wittgenstein's criticism. On the picture I sketch in this section, the rigor and perspicuity achieved by the introduction of explicit rules of a formal system make the Begriffsschrift into a useful means of elucidation (a point with which Wittgenstein, in *Tr.* 3.325, agrees). Indeed, it is not the very idea of formal rules that Wittgenstein would object to (see *Tr.* 6.126), but rather the thought that these rules are needed in order to justify our conclusions (*Tr.* 5.132). But as I show in the following sections, Frege *does* ascribe to the rules of inference substantive logical content, and that this is what makes his view vulnerable to Wittgenstein's criticism.

Frege prides himself on the innovativeness of his conception of a logical system as governed by rules of inference. In the Foreword to *Grundgesetze* he suggests that by making the rules of the system explicit he has taken a decisive step beyond Euclid, advancing the ideal of a "strictly scientific method" in mathematics, i.e. the ideal of an axiomatic system of proof:

The ideal of a rigorous scientific method for mathematics that I have striven to realize here, and which could be named after Euclid, can be characterized as follows. It cannot be required that everything be proven, as this is impossible; but it can be demanded that all propositions appealed to without proof are explicitly declared as such, so that it can be clearly recognised on what the whole construction rests. One must strive to reduce the number of these fundamental laws as far as possible by proving everything that is provable. Furthermore, and in this I go beyond Euclid, I demand that all methods of inference and [derivation] which are used be listed in advance. Otherwise compliance with the first demand cannot be secured. This ideal I believe I have now essentially achieved.<sup>46</sup>

The crucial point in this passage is the following. The demand that modes of inference be

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<sup>46</sup> GG, p. VI. I replaced Ebert and Rossberg's 'consequence' with 'derivation'.

specified in advance is not a mere embellishment added onto the axiomatic method of Euclid. Rather, it reflects a transformation in the very method of proof, since it guarantees that the only notion of grounding employed would be that of logical inference. The specification and correct employment of rules of inference thus make a crucial contribution to Frege’s logicist project — his endeavor to show that mathematics is fully grounded in logic. The logicist’s aim is to show that logical axioms are sufficient to yield arithmetic. This requires that we be able to scrutinize our proofs and ascertain what sources of knowledge are drawn on at each step.<sup>47</sup> All “gaps” in proof must be avoided, meaning that the only steps that would be allowed are the simplest inferential steps. By ensuring that all the relations of grounding can be reduced to stepwise applications of logical rules of inference we guarantee that, given our knowledge of the axioms, no other source of knowledge is required for the establishment of arithmetic.

Logicism thus gives rise to two methodological requirements. One concerns the axioms. Frege restricts them to “laws of thought that transcend all particulars”, i.e. to logical laws. The other requirement concerns the rules, whose role is to ensure that the proofs are made “by means of inferences alone” and that they are “free of gaps”. Both requirements are reflected in the way in which Frege frames the official purpose of the *Begriffsschrift*:

[The *Begriffsschrift*’s] chief purpose should be to test in the most reliable manner the validity [*Bündigkeit*] of a chain of inferences [*Schlusskette*] and expose each presupposition which tends creep in unnoticed, so that its source can be investigated.<sup>48</sup>

What the specification of the rules guarantees is the purity of the inferences, of which Frege here

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<sup>47</sup> The logicist proceeds as follows. Once a mathematical concept has been cast as a logical concept, and a mathematical claim is expressed in logical symbols, the logicist shows that it is derivable from logical laws — and that it only depends on such laws. She shows that by showing that to proceed from the logical laws to the mathematical statements, we only require stepwise applications of the explicit rules of inference. An account along these lines is found in the first chapter of Blanchette 2012.

<sup>48</sup> BS, p. 104, translation emended.

speaks in terms of ‘validity’ (*Bündigkeit*) — a non-technical term that is perhaps better translated as ‘correctness’.<sup>49</sup> Frege is even more explicit about the role of rules when he compares his conceptual notation to Peano’s:

In my conceptual notation inference is conducted like a calculation. I do not mean this in the narrow sense, as if it were subject to an algorithm the same as or similar to that of ordinary addition and multiplication, but only in the sense that there is an algorithm there are all, i.e. a totality of rules which govern the transition from one sentence or from two sentences to a new one in such a way that nothing happens except in conformity with these rules.

What I am aiming for, then, is uninterrupted rigour of demonstration and maximal logical precision, together with perspicuity and brevity.<sup>50</sup>

Frege here reflects (indirectly, at least) on the very idea of proof, by reflecting on the properties of his system. But he is not worried about the meta-theoretical question of the soundness of the system: the pressing question for him is not whether proofs in the system only result in validities. This is because his system, though it does reduce proof to an algorithm of rules, is not a system of uninterpreted signs. What he says is that the design of the rule-governed Begriffsschrift makes sure our demonstrations would be rigorous because it restricts what could count as proof.

Let us look more closely into the epistemological role that the rules of inference have, on Frege’s view. While the ultimate point of conducting a proof is to present us with the justification of the conclusion, each proof also forms part of the larger epistemological project of the axiomatization and systematization of all knowledge. It is the essence of explanation that a small number of principles are shown to be sufficient for the derivation of a multitude of more complex ones. The reduction in the number of axioms marks the goal for any science; the fact that we can make do with a few of them is tantamount to the claim that these few principles *explain* all of our

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<sup>49</sup> The translation of the German ‘*Bündigkeit*’ as ‘validity’ is slightly misleading. The term seems to have been in common use among mathematicians and philosophers since at least Kant’s times. Kant uses it to describe the firmness and correctness of logical proofs. See Kant 1968. p. 6.

<sup>50</sup> Frege 1984e, p. 237.

theorems:

Merely knowing the laws is obviously not the same as also understanding how some are implicitly contained in others. In this way, we obtain a small number of laws in which (if we add the laws contained in the rules) is included, though in embryonic form, the content of all of them. And it is an advantage of the deductive mode of presentation that it teaches us to recognize this kernel.<sup>51</sup>

But it is a fundamental principle of science to reduce the number of axioms to the fewest possible. Indeed the essence of explanation lies precisely in the fact that a wide, possibly unsurveyable, manifold is governed by one or a few sentences.<sup>52</sup>

The aim of proof is, in fact, not merely to place the truth of a proposition beyond all doubt, but also to afford us insight into the dependence of truths upon one another. ... The further we pursue these inquiries, the fewer become the primitive truths to which we reduce everything; and this simplification is in itself a goal worth pursuing.<sup>53</sup>

This epistemological goal of science — systematization and minimization of the number of axiomatic starting points — can only be achieved by means of a rigorously designed system. And such rigor, Frege thinks, can only be achieved if we join him in taking the “step beyond Euclid”, by means of specifying a restricted number of rules of inference.<sup>54</sup> Without the specification of rules of inference, and the concomitant restriction of their number, inferential relations are bound to remain obscure.

Conversely, once we specify the rules, we force all our expressions into a perspicuous mold:

A strictly defined group of modes of inference is simply not present in language, so that on the basis of linguistic form we cannot distinguish between a “gapless” advance ... and an omission of connecting links. We can even say that the former almost never occurs in language, that it runs against the feel of language because it would involve an insufferable prolixity. In language, logical relations are almost always only hinted at —

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<sup>51</sup> BS, #13, p. 136. I discuss the rest of this passage in more detail in sections 2.4 and 2.5, below.

<sup>52</sup> BLC, p. 36.

<sup>53</sup> FA #2 p. 2.

<sup>54</sup> GG, p. VI, cited above.

left to guessing, not actually expressed.<sup>55</sup>

The defects of natural language are avoided in a formal system governed by a small number of rules of inference. And such a system allows us to expose “the epistemological nature” of each theorem:

The gaplessness of the chains of inferences contrives to bring to light each axioms, each presupposition, hypothesis, or whatever one may want to call that on which a proof rests; and thus we gain a basis for an assessment of the epistemological nature of the proven law.<sup>56</sup>

Some logical theorems cannot be recognized immediately — i.e. they cannot be recognized as binding for all rational thought without proof within a rigorous logical system which shows their relation to the axioms and to their consequences.

The use of rules improves our epistemological position by helping us avoid defective and impure proofs. There are three kinds of defect Frege has in mind here. The first is the slipping in of unaccounted for presuppositions. The second is a defect in the “validity [*Bündigkeit*] of a chain of inferences”.<sup>57</sup> The third is the threat of ambiguity and imprecision of language.

The first defect, that of covertly relying on presuppositions that are not accounted for is quite easy to remove by means of adhering to a system of proof that is governed by rules of inference. For any transition that occurs according to the rules of Begriffsschrift cannot fail to have its premises clearly stated:

We have to mark out in advance a few methods of inference and [derivation] [*Schluss- und Folgerungsweise*] and no step is allowed to occur that is not in accordance with one of them. Thus, in the transition to a new judgment, one is no longer to be satisfied, as mathematicians up until now nearly always have been, with its obvious correctness, rather it must be analysed into the simple logical steps of which it consists, and these

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<sup>55</sup> Frege 1972b, p. 85.

<sup>56</sup> GG, p. VII.

<sup>57</sup> BS, p. 104. See footnote 49, above, for a discussion of the meaning of *Bündigkeit*.

are often not particularly few. Herein, no presupposition can remain unnoticed; every required axiom must be uncovered.<sup>58</sup>

The restriction to a small number of explicit rules of inference guarantees that all presuppositions are exposed. For, to the extent that the rules constrain us to only make logically simple steps, the requirement that all proof be conducted by means of rules guarantees that no judgments would slip in whose content derives from intuition, rather than from the explicit premises.

The second defect Frege seeks to avert involves the appeal to intuition as a means for guaranteeing the justification of inferential steps.<sup>59</sup> As Frege writes in the preface to the *Begriffsschrift*, the formal system is meant “to provide us with the most reliable test of the validity [*Bündigkeit*] of a chain of inferences ...”.<sup>60</sup> The *Begriffsschrift*, by help of devices such as the rule of inference, replaces all reliance on intuition with reliance on reliable logical procedures. In this way, the *Begriffsschrift* shows us that proofs of mathematical propositions do not depend on intuitions at all (even though they might be helpful in practice). But Frege is not suggesting something like Jevons’s calculating machine: his aim is merely to offer a language of pure thought, not to replace thought with a mechanically reliable, but ultimately empty, procedure.<sup>61</sup>

The third defect that afflicts non-*Begriffsschrift* proofs arises from the ambiguity and imprecision of natural language, which tends to mislead us into drawing invalid inferences. A *Begriffsschrift* prevents this defect inasmuch as the construction of an artificial language can be

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<sup>58</sup> GG, p. 1. I replaced Ebert and Rossberg’s ‘consequence’ with ‘derivation’.

<sup>59</sup> Within Frege’s target range we find the Kantian idea that a synthesis in intuition provides justification for arithmetical proofs; Descartes’ *Regulae* is another example for a way of thinking about intuition as guiding us in transitioning between the simplest steps in proofs.

<sup>60</sup> BS, *ibid.*

<sup>61</sup> BLC, p. 35.

guaranteed not to involve any ambiguous terms. This is achieved not only through the choice of vocabulary — it also depends, indirectly at least, on the choice of rules. Take a well-known example. Frege refuses to allow the grammatical distinctions between subject and predicate, and between active and passive voice, to play any role in the *Begriffsschrift* language. Such distinctions mislead us into thinking that two different judgments are involved; this confusion can easily be dispelled by means of an inference test. For when it comes to drawing inferences, two propositions that may seem different in English would behave in the exact same way in the *Begriffsschrift*: “At Plataea the Greeks defeated the Persians” grounds the same judgments, and is derivable from the same judgments, as “At Plataea the Persians were defeated by the Greeks”.<sup>62</sup> The inference test, considered as a criterion of equivalence in content, guarantees that we do not mark a distinction where there is no difference. In this way, a *Begriffsschrift* governed by formal rules of inference helps us exploit all and only the logical content that can be made use of in inference. The specification of the rules of inference of the system indirectly guides our transcriptions.

These observations on the methodological and epistemological roles of rules of inference allow us to supplement the picture of Frege’s epistemology that is given by Ricketts. Here is the core of Ricketts’s account:

It is Frege’s conception of justification as axiomatic systematization that motivates his demand for gap-free proofs, for it gives these proofs an epistemic point. The notationally enforced rigor of the *Begriffsschrift* forces the explicit articulation of any premise on which an inference depends, and automatically separates the logical laws that are applied in the inference from the non-logical premises. In the setting of a suitably economical axiomatization of a restricted body of knowledge or of unified science, such proofs provide explanations of their conclusions, revealing the truths on which these conclusions depend.<sup>63</sup>

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<sup>62</sup> BS, #3, p. 112. I discussed this in Chapter 1, section 1.2.1.

<sup>63</sup> Ricketts 1997, p. 173f.

Ricketts is right to point out that the guiding principle of Frege's *Begriffsschrift* is the epistemological ideal of the axiomatization of all knowledge. This epistemological goal yields the demand for gap-free proofs, since only gap-free proofs can allow for the rigorous scrutiny of the sources of our belief. What goes missing from Ricketts's picture, however, is an account of the role of rules of inference in the elimination of such gaps. The specification of rules is, for Frege, a crucial step that guarantees that all the elements of the system hang together in the right way. For first, it determines what counts as a simple, "gap-free" step. And second, as I argue in the next section, our choice of rules determines the number of axioms needed for our system. Thus the economy of axioms that Ricketts emphasizes is ultimately an economy of axioms *and* rules. Underlying this economy, I will show, is Frege's thought that rules of inference can encode substantive logical content.

#### **2.4. The Economy of Rules**

Frege acknowledges that the strictness and rigor of the formal system may be achieved in more than one way. We may vary the basic elements of the system, i.e. the laws and the rules, without hampering the overall explanatory power of the system — its power to effect a reduction of a large number of derived proposition to a small group of axioms. This is how Frege construes the tradeoffs that such variation involves:

Now, of course, it must be admitted that the reduction is possible in other ways besides this particular one. Thus, one such mode of presentation will not elucidate all the interconnections of the laws of thought. Perhaps there is yet another series of judgements from which (with the addition of those contained in the rules) all the laws of thought can be derived.<sup>64</sup>

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<sup>64</sup> BS, #13, p. 136.

Frege's point is not simply that axioms could be added beyond the minimum that allows for the derivation of all the desired results. Rather, the necessary minimum number of axioms needed depends on the choice and number of rules. When we compare equally powerful systems, we discover that what in the one system counts as explicit axioms, are in the other system "contained" in the rules.

Let us consider some examples that illustrate this point. In *Begriffsschrift* sections #13-#22 Frege aims to show that the traditional Aristotelian modes of inference are reducible to the laws and rules of his own system, and to thereby show that these traditional devices are redundant. In the process, he also proves the soundness of the Aristotelian modes, since by showing that they are reducible to the rules and axioms of his system, he shows that these traditional modes do not go beyond what logic (as it is captured in the *Begriffsschrift*) allows.<sup>65</sup> The procedure is the following. Take an inference that employs an unreduced, Aristotelian mode of inference. Replace all the constants with variables. Then introduce as an axiom a judgment asserting that the conjunction of the premises implies the conclusion, or, if possible, derive this judgment from the existing axioms. From now on, whenever you wish to infer in accordance with the unreduced rule, simply cite the new axiom (or theorem) alongside the premises, instantiate the axiom (or theorem) appropriately, and apply *modus ponens* until you arrive at the desired conclusion (in Appendix D I provide a detailed example of this sort of reduction of an Aristotelian mode of inference).<sup>66</sup>

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<sup>65</sup> By the use of the modern quantification methods which he pioneered, Frege succeeds in reducing all the traditional modes of the syllogisms to a single rule of inference, applied to a small number of axioms, whereas in his *Prior Analytics* Aristotle had to stop at a much larger number of irreducible syllogistic forms. Cf. Aristotle, 1984a, A, 7, 29b1-26.

<sup>66</sup> The correspondence between the resulting axiom (or theorem) and the mode of inference that it replaces is not trivial; I return to it below and in section 2.5.

Frege proposes to do the same for all the modes of inference that might initially seem to be peculiar to mathematics. By increasing the number of our axioms, we can reduce these mathematical methods to the operation of a purely logical system. This might require that we introduce axioms that are mathematical, rather than purely logical in nature.<sup>67</sup> By converting implicit modes of inference to explicit axioms in this way, we guarantee the logical validity of our method of proof.

The advantage gained from a decrease in the number of rules of inference, so long as it does not reduce the expressive power of the system, resides in the greater perspicuity of proofs:

Since it is possible to manage with a single mode of inference, perspicuity demands that we do so... This restriction to a single mode of inference, however, is in no way intended to express a psychological proposition, but merely to settle to question of the most expedient form [of our “conceptual notation”].<sup>68</sup>

Perspicuity makes the system into a rigorous tool for ascertaining the sources of knowledge that we draw on in proving a certain result. For example, when only one rule of inference is followed (as is the case in the *Begriffsschrift* system), all inferential transitions conform to the same pattern, and so the transitions between premises and conclusions are easier to scrutinize. Perspicuity, achieved through a restriction on the number of rules, might not be a requirement of logic, but it does not merely reflect a psychological consideration either. The expediency that Frege has in mind lies somewhere in between the logical and the psychological; Frege elsewhere calls it a “demand of scientific parsimony” [*Das Gebot der wissenschaftlichen Sparsamkeit*].<sup>69</sup> It has to do with our nature as finite cognitive agents as well as with our epistemological goals — to establish the logicist thesis in the strictest way.

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<sup>67</sup> Frege 1979h, pp. 203-204.

<sup>68</sup> BS #6, p.120.

<sup>69</sup> GG, p. 26. I discuss this passage below.

Precisely for this reason, the decrease in the number of rules is not without disadvantages. Consider how Frege's introduction of the two additional rules of inference in *Grundgesetze* affects the length of proofs.<sup>70</sup> The first of these rules is the Hypothetical Syllogism: from  $(a \supset b)$  and  $(b \supset c)$ , infer  $(a \supset c)$ . It is quite easy to show that the *Begriffsschrift* system has the resources to reduce this rule of inference to the *modus ponens* (see Appendix D for the details). The advantage of adding the Hypothetical Syllogism as a rule, in *Grundgesetze*, lies in the significant shortening of proofs. A larger number of rules of inference thus improves the *surveyability* of proofs, at the cost of reducing their "perspicuity and order". Frege already anticipates the possibility of this tradeoff in *Begriffsschrift*:

The restriction, declared in §6, to a single mode of inference is justified by the fact that in laying the *foundation* of such a conceptual notation the primitive components must be chosen as simple as possible if perspicuity and order are to be created. This does not preclude the possibility that, *later*, transitions from several judgements to a new one, which are possible by this single mode of inference in only an indirect way, be converted into direct ways for the sake of abbreviation. Indeed, this may be advisable in a later application. In this way, then, further modes of inference would arise.<sup>71</sup>

A second example for the parsimony of the *Begriffsschrift* system is that it includes an axiom that allows Frege to manage without a rule of permutation (alteration of the order of antecedents in a complex conditional):

In accordance with my guiding principles, I also had to assume formulae which merely express the different ways in which you may alter the order of a number of conditions. Instead of giving a general rule that conditions may be ordered at random, I only introduced a much weaker axiom that two conditions may be interchanged, and then derived from this the permissibility of other transpositions.<sup>72</sup>

In *Grundgesetze*, Frege reverses course and chooses to emphasize the practical benefits that a

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<sup>70</sup> See discussion in Appendix B, below.

<sup>71</sup> BS, pp. 107.

<sup>72</sup> BLC, p. 38. At another point in BLC (p. 29) Frege discusses the possibility of turning theorems 52 and 53 of the *Begriffsschrift* into a rule of substitution of logically equivalent symbols.

laxity in the number of rules results in:

This is the sole mode of inference that I used in my *Begriffsschrift*, and one can even manage with it alone. The demand of scientific parsimony would now usually require this; but considerations of practicality pull in the opposite direction, and here, where I will have to form long chains of inferences, I will have to make some concessions. For an inordinate length would result if I were not to allow some other modes of inference, as already anticipated in the preface of my little work.<sup>73</sup>

Despite the “demand of scientific parsimony”, there are clear *practical* advantages to increasing the number of rules: we gain in surveyability what as we lose in perspicuity. These economical tradeoffs do not affect the overall logical content of the system. It is a practical question which system to prefer.

A related question arises in a later paper, “Negation”, where Frege argues that negating a proposition does not effect a change in the force of judgment from affirmation to denial, but rather alters the content which is asserted in the judgment.<sup>74</sup> But Frege’s stated reasons for rejecting the alternative view is not that it is implausible or unworkable. A system that employs two primitive acts — affirmation and denial — could be made to work if only we introduced additional rules of inference to handle the transitions between negated and affirmed judgments, etc..<sup>75</sup> The result could have the same expressive power as Frege’s original system. Thus Frege’s only reason for preferring his own system is practical; his system is much less complicated:

Thus the assumption of two different ways of judging must be rejected. But what hangs on this decision? It might perhaps be regarded as valueless, if it did not effect an economy of logical primitives and their expressions in language. ... There hangs together with this an economy as regards a principle of inference; with our decision we

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<sup>73</sup> GG, #14, p. 26.

<sup>74</sup> Frege 1984j.

<sup>75</sup> Pace what Peter Geach calls the “Frege point”, which he takes to be a decisive objection to Gilbert Ryle’s account of the conditional. See Geach 1965; Ryle 1950; and see my discussion of a related issue at the end of **section 2.5**.

can make do with one, where otherwise we need two. If we can make do with one way of judging, then we must...<sup>76</sup>

Even though Frege says “we must” prefer the more frugal system, his preference is based on considerations of scientific parsimony and economy only. There is no independent criterion with which to justify the fundamental choices that determine the shape of his system and the number of its rules, if the alternative system is equal in terms of its logical capacities.

There is one other example of this economy that I will discuss in the next section — the fact that what in *Begriffsschrift* counts as an axiom — judgment #28 — assumes in *Grundgesetze* the form of a rule — called Contraposition. I will use this example to illustrate the process whereby axioms can be transformed into rules, and vice versa.

The kind of economy discussed in this section consists in a compensation for the decrease in the number of rules by means of an increase in the number of axioms, and vice versa. Underlying this economy there is a principle that endows rules of inference with substantive logical content. For what is preserved throughout the economical tradeoffs that increase perspicuity at the price of surveyability (and vice versa) is the overall logical *content* of the systems, and that content is divided between the axioms and the rules of the system. In the next section I zero in on this idea of the transformable content of rules and laws.

## **2.5. The Content of Rules and Laws**

In *Begriffsschrift* Frege proposes that rules can be seen as projections or mappings (*Abbilder*) of the most fundamental principles of thought. The idea is that we can “transform” (*verwandeln*) these principles into rules. Let me quote the passage in full, again:

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<sup>76</sup> Frege 1984j.

INE We have already introduced in the first chapter several principles [*Grundsätze*] of thought in order to transform [*verwandeln*] them into rules for the application of our symbols. These rules and the laws to which they correspond [whose transforms, *Abbilder*, they are] cannot be expressed in our “conceptual notation” because they form its basis [*zu Grunde liegen*]. Now in this chapter, some judgements of pure thought which can be expressed in the “conceptual notation” are to be stated in symbols.<sup>77</sup>

Frege here draws a positive connection between the rules of the system and the “fundamental principles of thought”, the “laws whose transforms they [the rules] are”.<sup>78</sup> It is my contention that in drawing such a connection, Frege assigns substantive content to the rules of his system. Immediately following this remark, Frege makes clear that rules contribute to the content of the entire system:

In this way, we obtain a small number of laws in which (if we add the laws *contained* in the rules) is included, though in embryonic form, the content of all of them. ...Perhaps there is yet another series of judgements from which (with the addition of those *contained* in the rules) all the laws of thought can be derived.<sup>79</sup>

The power of the logical system to yield the content of all theorems of logic resides in the basic laws *in conjunction with* the rules of inference. Frege speaks of what is “contained” in the rules and of the *content* of derived laws which derives from the axioms taken together with the rules. Thus in the last account, the rules of inference do more than merely dictate correct manipulation of signs, and they do more than ensure the rigor, perspicuity and surveyability of the proofs of the system. Rules of inference encode content.

In these passages from *Begriffsschrift* #13 there are two transformations at issue. Rules are first said to be transforms of the “fundamental principles of thought” — i.e. principles which

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<sup>77</sup> BS #13, p. 136.

<sup>78</sup> This point has gone missing in the debates surrounding Frege’s method of introducing the rules of his system. See my discussion of van Heijenoort, Weiner and Heck in section 2.2 above.

<sup>79</sup> BS, #13, p. 136. My emphasis.

themselves are said to be inexpressible within the Begriffsschrift. But Frege then alludes to the transformability of rules and axioms, i.e. expressible judgments. It is not entirely clear how we should interpret the idea of the “fundamental principles of thought” that Frege appeals to in INE (and whether or not this means the same as “laws of thought” in the second quote). This is the only place I know of in which Frege uses the term *Grundsatz* in this specific way, i.e. with reference to principles of thought which cannot be expressed in the Begriffsschrift. Such fundamental principles are neither the laws, nor the rules of Frege’s system, but they are the source of the content of both.

With regard to the second kind of transformation, recall that in the previous section we have seen that in order to reduce other rules of inference to the *modus ponens*, we must add axioms and theorems to the system. The form that these added laws take is that of a conditional in which the antecedent is the conjunction of the premises and the consequent is its conclusion. This procedure can be further generalized, as Frege himself notes:

That, after all, is generally the use of a theorem: It keeps the result of a series of inferences ready for use whenever we wish.<sup>80</sup>

The idea is that the content of a theorem reflects the inferential process through which it has been derived, and so we could use it in further inferences.<sup>81</sup> For instance, in a system that does not already employ the rule Hypothetical Syllogism, we can prove a theorem of the form

$$((a \supset b) \ \& \ (b \supset c)) \supset (a \supset c)$$

We can then make inferences shorter by using this theorem as a premise, instantiating it appropriately, and, alongside premises of the form  $a \supset b$  and  $b \supset c$ , derive conclusions of the

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<sup>80</sup> FG2, p. 319.

<sup>81</sup> A related idea is expressed by Wittgenstein when he says that “every proposition of logic is a *modus ponens* presented in signs” (Tr. 6.1264).

form  $a \supset c$ .

Transformations can also go in the opposite direction — in the previous section I have already shown how this procedure results in the introduction of the rules of inference and the reduction in the number of axioms. Let us look at an example. In *Grundgesetze*, Frege introduces the rule of Contraposition (*Wendung*) and thereby eliminates the need for one of the axioms of *Begriffsschrift*. When in his earlier work Frege wishes to perform a contraposition of the conditional — transitioning from  $a \supset b$  to  $\sim b \supset \sim a$  and vice versa — he has to appeal to an axiom, his judgment number 28, which simply asserts

$$\vdash (b \supset a) \supset (\sim a \supset \sim b).^{82}$$

The *Grundgesetze* system no longer counts this judgment as an axiom. The introduction of a rule of Contraposition makes it entirely redundant. Indeed, the rule takes over the content that that axiom had. Note that Frege makes no mention of the fate of his old axiom in introducing the rule in *Grundgesetze*.<sup>83</sup> Instead, the new rule is given a quasi-semantic elucidation. This elucidation is almost identical to the one Frege offers when he introduces the axiom, judgment 28 of *Begriffsschrift*.<sup>84</sup> In both cases, the elucidations show us that our grasp of the conditional and of negation already commits us to the truth of the axiom and to the soundness of the rule.<sup>85</sup> Thus the fundamental principles of thought that underly judgment 28 in the system of *Begriffsschrift* are

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<sup>82</sup> BS #17, p. 154.

<sup>83</sup> GG, #15, p. 27.

<sup>84</sup> BS #17, p. 154. And see my discussion of such elucidations in section 2.2.

<sup>85</sup> There are some obvious differences between the early elucidation of the axiom and the later elucidation of the rule. The elucidation of judgment 28 uses the same terms that the *Begriffsschrift* elucidation of *modus ponens* did. It includes sentences like “The denial of  $(\sim a \supset \sim b)$  means that  $\sim a$  is affirmed and  $\sim b$  is denied” (BS #17, p. 154). The elucidation of contraposition in *Grundgesetze*, on the other hand, includes the sentence “ $(\sim \Gamma \supset \sim \Delta)$  is the false if  $\sim \Gamma$  is the true and  $\sim \Delta$  is not the true” (GG, p.27). These differences have to do with the overall development Frege’s system undergoes in the interim; they parallel the differences between the elucidations of *modus ponens* in each of these works, which I discussed above, in section 2.2.

carried over to the rule of contraposition of *Grundgesetze*. The content of the *Grundsätze* whose transforms these laws are simply shifts, from the law to the rule.

There is an important disanalogy, however, between the two directions that these transformations can take.<sup>86</sup> When we transform a rule into a law, we reduce a system in which the rule operates alongside other rules and axioms, to a more frugal system in which the rule is absent, but the added axioms, in combination with the remaining rules, do its work. In the other direction, when we reduce an axiom by the introduction of a new rule, the application of that new rule is independent of any of the rules and axioms of the system. In this direction, the transformation is complete, whereas in the transformation that goes in the other direction, the added judgment does not capture all the aspects of the eliminated rule on its own. On its own, it does not have the capacity to effect the assertion of any other judgment, in the way that the rule does. Instead, it has to rely on one of the remaining rules of inference. While the transformation of a rule to a law captures the “content” of the rule, it does not capture its “rulish” normativity — its power to guide us in inferring.

Frege further clarifies the idea of the proximity of the content of axioms and rules of inference in his discussion of the choice of the conditional as the sole binary connective of his system.<sup>87</sup> Frege justifies his choice as follows:

I chose the denial of the third case, because of the ease with which it can be used in inference, and because its content has a close affinity with the important relation of ground and consequence.<sup>88</sup>

Frege’s choice derives from his observation of the internal relation between the conditional

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<sup>86</sup> I was made aware of this disanalogy in conversation with Amichai Amit.

<sup>87</sup> Frege’s own system includes as its primitive logical connectives the conditional, negation and the universal quantifier.

<sup>88</sup> BLC, p. 37; cf. GG #13, p.25, BS, #7, p. 123.

connective and the relation of inferability.<sup>89</sup> This relation is what allows the reduction (through transformation) in the number of rules of inference. For the judgments we add to our system as axioms capture, in conditional form, the inferences that were authorized by the rule that was eliminated. However, these considerations in favor of the choice of the conditional presuppose the choice of *modus ponens* as the fundamental rule of inference of the system.

That we can alter that choice as well is an insight that Wittgenstein builds on, in *Tractatus* 5.1311, which leads up to his rejection of the idea of rules of inference in 5.132. Wittgenstein points out that different designs of logical systems render different forms of inference trivial, and not others. None of these systems take priority over the others; they are all equally valid expressions of thought. And this shows that there is nothing essential or contentful about any of the rules of inference.

## **2.6. Conclusion: The Tension in Frege**

Frege identifies logic with the all-embracing principles of thought. At the same time, the goal of Frege's logicist project is to show that logic is a source of knowledge, providing us with content from which substantive mathematical claims can be derived. He designs the *Begriffsschrift* with these two goals in mind, aiming for it to be both a genuine language of thought, as well as an axiomatic system which captures all the content that is required in order to derive the

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<sup>89</sup> This relation is captured in modern systems of logic by means of the Deduction Theorem: If  $\Gamma, A \vdash B$  then  $\Gamma \vdash A \supset B$ , i.e. if a conclusion  $B$  is derivable from the set of axioms  $\Gamma$  combined with a set of premises,  $A$ , then the conditional that has  $A$  as its antecedent and  $B$  as consequent is derivable from the set  $\Gamma$  alone. Frege does not have the resources to frame this modern theorem explicitly (it requires, for instance, a distinction between object and meta-language). But in the passage quoted above from FG2 (p. 319) Frege frames something like the converse of the deduction theorem: he there claims that any theorem "keeps the result of a series of inferences ready for use whenever we wish".

conclusions he seeks. Rules of inference have a role to play in furthering both of these goals. The introduction of explicit rules of inference constrains what counts as a legitimate use of the Begriffsschrift, and ensures that the only steps being taken in it are logical steps. But Frege also thinks of the Begriffsschrift system in terms of the axiomatic foundation on which the proof rests, and in that context, the rules of inference, alongside the axioms, are treated as the sources of the basic logical content from which the theorems are derived.

The recognition that rules cannot be fully expressed within the Begriffsschrift whose basis they form reflects the tension between Frege's two goals, which pull in two opposite directions: the logicist pull is to treat every logical distinction as a source of content, and to strive to make all content explicit; the universalist pull is to acknowledge that we cannot step outside the logical principles of thought and hence that we cannot fully express them. The result of this tension is Frege's idea of inexpressible logical content.

I exposed two ways in which Frege speaks of the content of the rules of inference of his system. The first is his idea that in the transition between different systems of different designs, the content of axioms can be transmitted to the rules and vice versa. Both kinds of principles — rules and laws — share the task of endowing the system with the power to yield the theorems. In each given system only part of the logical content is explicitly expressible; the other part remains inexpressible, and belongs with the rules. In other axiomatic systems, the content of each of the rules can be made explicit by being transformed into the content of explicit axioms.

The second way in which the rules to encode content consists in the idea that rules of inference are ultimately transforms of fundamental, but inexpressible principles of thought. These principles are inexpressible as such; and yet the rules of the system mirror them. Here Frege groups the rules of inference with the other rules for signs that reflect the most

fundamental logical distinctions and notions. He holds that some logical content — some fundamental principles of thought — remain beyond expression.

In speaking of the rules and principles in terms of the logical content that remains beyond expression, Frege implies that there is something that we are prevented from doing — some content of an impossible act of saying. From Wittgenstein's perspective, this idea fails to make sense. Logic, for Wittgenstein, does not involve content at all; logic concerns the form of thinking, not the content of thought. Logical propositions do not merit the title of laws of thought; they are empty tautologies. They reflect logical form no more and no less than any senseful proposition, and to think that contradicting them is different from making any other contradiction is an illusion.<sup>90</sup> Logical form is required in order to recognize tautologies, no less than it is required in order to recognize the sense of any other proposition. But what is required here is not logical *content*; it is the logical form of understanding. In Frege, the realm of logical content extends beyond our means of expression; in Wittgenstein, what can only show itself and cannot be expressed, must not be thought of as content.

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<sup>90</sup> Ricketts 1985 (p. 11) explains the status of Frege's laws of logic in terms of the immediate role they play in inferences, by arguing that we cannot entertain the negation of a logical law, since entertaining it involves inferential reasoning, which itself presupposes the logical law, and so it would involve a contradiction. But it is not clear in what sense the presupposition of a logical law in all reasoning is different from the assumption of any other truth -- if all our beliefs are interconnected, any inconsistency between one of our belief and a proposition that contradicts it can be described as a case in which we cannot entertain one of them. Entertaining the negation of a logical law, from Wittgenstein's perspective, cannot be special in any way. To suggest that it is to be under the same illusion as Frege: that the logical form of thought can be captured in a proposition.

## Appendix A: Frege's Terminology

My purpose in this appendix is investigate into the terminology Frege deploys in his elucidations, and to see how it reflects his distinctions between the laws of logic and rules of inference, and further, finer distinctions between inference and other types of transition between judgments.

Nowhere in the *Begriffsschrift* does Frege use the full term 'rule of inference'. Instead, he uses the terms 'manner of inference' and 'mode of inference' (*Schlußart*, *Schlußweise*) — the terms traditionally used by logicians to refer to the Aristotelian syllogistic moods. Indeed, Frege himself uses these terms not only to describe the kinds of inferential transitions that his *Begriffsschrift* allows, but also when he talks about the traditional syllogistic forms, which he shows to be reducible to the single mode of inference of his own system, the *modus ponens*.<sup>91</sup>

Shortly after the publication of the *Begriffsschrift* (1879), Frege begins to refer to the *modus ponens* as a 'rule' (*Regel*).<sup>92</sup> The term 'mode of inference' (*Schlussweise*) persists, to some extent, in *Grundgesetze* (1893), but it is overshadowed in this work by the predominant use of the term 'rule' (*Regel*). In fact, '*Schlussweise*' only occurs five times in the entire *Grundgesetze*; three of these occur in the Table of Contents, in the titles of sections #14-16; and two occasions are in the sections introducing the first two rules of inference of the system. Of these two occurrences, one is a place where Frege refers back to "the sole mode of inference used in my *Begriffsschrift*".<sup>93</sup> By contrast, the text of sections #14-16 of *Grundgesetze* contains the term 'rule' numerous times, and it is the only term that appears in the recapitulation of the rules in

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<sup>91</sup> BS #22, pp. 162-166. The manner of reduction is discussed below, and an example is provided in Appendix D.

<sup>92</sup> Frege uses 'Rule' [*Regel*] prominently in the list of rules given in BLC, p.39. It should be noted that there is one important point in *Begriffsschrift* in which Frege does use the term 'rules', though he does that in a more general way, to refer to all the "rules for the use of our signs", including the rule of inference. See BS, #13, p.136.

<sup>93</sup> GG, #14, p. 26.

section #48.<sup>94</sup> There are a few other terminological variants that Frege occasionally uses to refer to rules of inference, e.g. ‘forms of inference’ (*Formen des Schliessens*) and ‘inference procedure’ (*Schlussverfahren*), but the occurrence of these variants is quite rare.<sup>95</sup>

Frege’s distinction between laws and rules is manifested in several ways. First, when Frege summarizes the rules and laws of his system in the *Grundgesetze*, he does so in two separate lists, titled the “Summary of Basic Laws” (#47) and the “Summary of Rules” (#48). Whereas the former list contains six propositions, all stated in Begriffsschrift notation, the latter contains extensive paragraphs that state rules in words. Second, in the table at the end of *Begriffsschrift* in which he specifies, for each derived theorem, all the basic laws from which it was derived, Frege does not make any mention of the rules used in the derivations.<sup>96</sup> Third, whenever Frege articulates the rules he frames them as permissions, using locutions such as “one can infer” (*kann geschlossen werden*)<sup>97</sup> and “it is allowed” (*es ist erlaubt*).<sup>98</sup> Elucidations of laws, on the other hand, are stated in the indicative mood. For example, Law IIa of *Grundgesetze* is paraphrased as follows: “What holds of all objects, also holds of any”.<sup>99</sup> Fourth, Frege maintains the distinction between rules and laws by using distinct verbs and prepositions as their complements: derivations

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<sup>94</sup> Thus there seems to be a deliberate shift in Frege’s terminology for rules between his early and later work. Frege does not make it an explicit issue.

<sup>95</sup> For the former, see Frege’s “Letter to Dingler, 31.1.1917”, in PMC pp. 16-17); for the latter, see 1972b, p. 85.

<sup>96</sup> In this sense, at least, Frege does not seem to think of rules as part of the justification of the inference, contrary to Wittgenstein’s criticism in 5.132.

<sup>97</sup> GG, #14, pp. 25-26.

<sup>98</sup> GG, #17, p. 33. Ebert-Rossberg’s translation paraphrases this expression. They have “A Roman letter may be replaced...” where the original (Frege 1893, S. 33) reads “It is allowed, whenever a Roman letter appears...”.

<sup>99</sup> GG, #20, p. 35. A similar indicative statement of a basic law is found in BS #14, “The case in which *a* is denied, *b* is affirmed, and *a* is affirmed is excluded”.

are said to be performed “according to” (*nach*) or “by” (*durch*) certain rules;<sup>100</sup> laws, on the other hand, are what a derivation starts “from” (*aus*) and “relies on” (*stützt sich auf*).<sup>101</sup>

On a few occasions, mostly in his later works, Frege’s terminology blurs the contrast between rules and laws. In “On the Foundations of Geometry”, for instance, Frege uses the term ‘laws of all inferring’ (*Gesetze alles Schließens*) to refer not to the rules of inference, but rather to the axioms of his system:

Some people may nevertheless be inclined to refrain from ascribing the name ‘axiom’ to these general laws of all inference [*allgemeinen Gesetze alles Schließens*] but rather wish to reserve it for the basic laws of a more restricted field.<sup>102</sup>

The axioms of logic are contrasted here with the axioms of more specialized sciences, such as geometry; and Frege aims to contrast the former with the latter type of axioms in terms of their generality. Inference, in this passage, stands for the activity of reasoning as such: the axioms of logic are distinguished by the fact that they are applied in all inferential activity, not only in this or that field. In fact, Frege speaks here in terms of inference happening “according to” laws, rather than their following “from” laws, and in this sense, this passage seems to assimilate laws to rules, rather than rules to laws.

Ian Proops considers this passage as evidence for his claim that Frege understands “laws of inference” in a way which conflates rules with logical laws, and this is taken to ground Proops’s further claim that when Wittgenstein rejects the “laws of inference” (*Schlussgesetze*) in *Tractatus* 5.132, he is not rejecting the rules of inference, but rather objecting to the appeal that Frege and

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<sup>100</sup> e.g. Frege 1893, S. 71 / GG, #54, p. 71.

<sup>101</sup> e.g. Frege 1893, S. 73 / GG, #54, p. 73.

<sup>102</sup> Frege 1984g, p.273. Three other texts in which Frege uses “laws” to refer to rules in the elucidation of inference is Frege 1979b, p.3; and Frege 1979f, p. 175 and “Letter to Dingler (31.1.1917)” in PMC, pp. 16-17.

Russell make to the laws of logic in their account of entailment.<sup>103</sup> But the blurring of the distinction between laws and rules in this passage cannot undermine the otherwise overwhelming evidence for the terminological clarity of Frege’s distinction that I’ve presented above.<sup>104</sup>

Certain logically justified transitions (*Übergänge*) between propositions do not amount to full inferences on Frege’s view.<sup>105</sup> Frege reserves the term ‘inference’ to a very specific form of transition between propositions: the one which starts from two propositions (the premises) and leads to a third proposition (the conclusion). This is a rather narrow conception of inference; logicians nowadays tend to speak of an inference whenever a transition is made from any number of premises to a conclusion. But for Frege, this reflects a distinction of some philosophical importance.

Let us take a closer look at the textual evidence for the distinction between inference and other forms of transition. The title Frege gives to sections #14-18 in *Grundgesetze* is “*Schlüsse und Folgerungen*”, indicating that two types of transition are to be introduced. In both of the English translations of *Grundgesetze*, Furth’s as well as Ebert-Rossberg’s, this title is translated as “Inferences and Consequences”. But the term “consequence” has a technical sense in modern logic which we should avoid reading into Frege’s work. Another reason why the rendition of ‘Folgerungen’ by ‘consequence’ is a poor choice is that the English noun does not have a cognate verb, whereas ‘Folgerung’ in German does — namely ‘folgen’. I will use ‘derivation’ and

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<sup>103</sup> Proops 2002, p. 284; I discuss and criticize Proops’s reading in Chapter 4.

<sup>104</sup> At times Frege does say that the axioms of his systems are *laws of thought*, and this might seem to imply that he thinks they do have a “rulish” role. What Frege seems to indicate by this is that the axioms are universally applicable judgments, which any thought must be consistent with; but they do not, of themselves, serve as the rules guiding our reasoning. See e.g. BS #13, p.136; I say more about this issue in section 2.5.

<sup>105</sup> In my inquiry into this question — what transitions count for Frege as full fledged inferences — I am indebted to Michael Kremer, who explores this issue in unpublished correspondence with the translators and editor of the new edition of Frege’s *Grundgesetze*, Philip A. Ebert, Marcus Rossberg and Crispin Wright.

‘derive’ to translate these terms. As I show in Appendix B, sections #14-18 of *Grundgesetze* list numerous types of transition between judgments, but only three of them are called modes of inference (*Schlussweisen*), and these three are transitions that involve two premises. In the summary list of rules of the system of *Grundgesetze* (paragraph #48), too, only these three modes of inference are termed *Schlüsse*, to the exclusion of other modes of derivation (for instance, contraposition). There are other places in which the distinction between inference and derivation is evident, and there is no reason to think that it is mere accident that Frege uses these two distinct terms in *Grundgesetze*.<sup>106</sup> In fact, this is the only charitable way to explain how Frege could have prided himself on having employed only one rule of inference in the system of *Begriffsschrift*.<sup>107</sup> For, as is evident from the first list I provide in Appendix B, there is more than one mode of *derivation* that is employed by Frege in that book.<sup>108</sup> Only rarely does Frege blur this distinction; on one occasion he writes “We may distinguish two kinds of inferences: inferences from two premises and inferences from one premise”.<sup>109</sup>

The terminological choice to use ‘inference’ to mark only those transitions that involve more than one premise echoes Aristotle’s definition of a syllogism.<sup>110</sup> But Frege is not merely adopting this traditional concept; he marks a genuine logical difference by it. Inferences differ

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<sup>106</sup> See for example Frege’s Letter to Dingler, 31.1.1917, in PMC p. 17. The distinction between *Schluss* and *Folgerung* in this letter has been obscured in the English translation which renders both “inference”; however, see the German original in Frege 1976, S. 30. The same problem affects the translation of Letter to Dingler, 6.2.1917, (p. 22 in PMC, S. 35 in Frege 1976). Another place in which inference is explicitly associated with a transition from two premises is Frege 1972b, p. 88. This passage is quoted in footnote 116, below.

<sup>107</sup> BS, #6, p. 120: “since it is possible to manage with a single mode of inference, perspicuity demands that we do so.” I discuss this remark in section 2.4.

<sup>108</sup> See also my discussion in Appendix C concerning the unstated rules of *Begriffsschrift*.

<sup>109</sup> Frege 1979h, p. 204. Another minor counterexample is a case in which Frege speaks of substitution of particulars names for general variables (Roman letters) as a “simple inference”, even though it starts from a single premise. See GG #48, p. 61ff., and see my discussion of rules of substitution in Appendix C below.

<sup>110</sup> See Aristotle 1984a, A. 2, 24b, pp. 18-22.

from derivations that only involve one “premise” and one “conclusion” inasmuch as in inferences, the conclusion cannot be said to have the same content as either of the premises, when each of them is taken on its own. Inferences, according to Frege, lead us to new content — they are fruitful; they increase our knowledge.<sup>111</sup> Derivations, however, do not introduce fundamental changes in our epistemological state. Contraposition, change in the scope of quantifiers, and change in the order of the antecedents of a conditional do not significantly alter the content of the propositions. Even an existential instantiation of a general statement can be said to leave unchanged the content of what is known through the premise — at least in the sense that it does not *increase* the knowledge that we already have through knowing the premise.<sup>112</sup>

The notion of the fruitfulness of inference mirrors Frege’s idea that logic is a source of substantive knowledge. Frege’s logicism consists in the claim that genuine mathematical knowledge rests on logical principles alone. On this view, logic forms the foundation of a hierarchical system of knowledge, through which the content of the axioms is gradually unfolded.<sup>113</sup> In the axiomatic system, the theorems do not directly follow from any one of the axioms, taken individually.<sup>114</sup> Only when the axioms are taken jointly, together with the rules, and are employed as a single system, can we derive the desired conclusion. In this sense the content of the conclusion is different from the content of the axioms. Moreover, this is a one-directional process, in which the axioms can be said to explain the conclusions, but not vice

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<sup>111</sup> Frege discusses fruitfulness of definitions in FA, #88, p. 100f, as well as in Frege 1979c, p. 33f. However, in FA, #17 as well as in Frege 1979h p. 214 it is inference that Frege takes to be fruitful. By means of this notion, Frege contrasts his view of analytic statements with Kant’s, arguing that analytic statements can play a role in extending our knowledge, namely as the premises of logical proofs. See Dummett 1991. This idea of fruitfulness is different from the fruitfulness of concepts of discussed by Tappenden 1995.

<sup>112</sup> Instantiation is treated as a *Folgerung* in the “Letter to Dingler”, 31.1.1917 in PMC, pp. 16-17.

<sup>113</sup> Frege’s conception of the hierarchical, axiomatic structure of logic has been discussed in Chapter 1, section 1.2.2.

<sup>114</sup> For this construal of the fruitfulness of inference, see Kremer 2010, p. 229.

versa. Our inferences reveal the special explanatory status that the axioms have vis-a-vis the theorems, by showing how the latter are derived from the former.<sup>115</sup>

The formal Begriffsschrift system perspicuously marks the terminological distinctions discussed above by means of various distinct signs. Appendix B explores the use of these signs in more detail; here I'd like to offer some general observations. Frege adopts from arithmetic the idea that one can represent inferences symbolically by writing the conclusion under the premises, and marking the inferential transition by means of a horizontal stroke between them (in *Grundgesetze*, where Frege has more than one rule of inference, he also has more than one transition sign.).<sup>116</sup> And Frege employs additional transition signs that mark derivations such as contraposition and universal generalization. All of these signs can be taken to indicate not only that an inference has been performed, but also that a *specific* rule of inference has been followed. Thus, though the rules of inference themselves are not stated as explicit premises of the inference, they are indicated and distinguished by means of these various transition signs .

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<sup>115</sup> I discuss the epistemological goals Frege assigns to his Begriffsschrift in section 2.3.

<sup>116</sup> Frege acknowledges this connection to mathematical symbolism in Frege 1972b, p. 88:

The arithmetic language of formulas is a conceptual notation [*Begriffsschrift*] since it directly expresses the facts without the intervention of speech. As such it attains a brevity which allows it to accommodate the content of a simple judgment in one line. Such contents — here equations or inequalities — as they follow from one another are written under one another. If a third follows from two others, we separate the third from the first two with a horizontal stroke, which can be read “therefore”. In this way the two-dimensionality of the writing surface is utilized for the sake of perspicuity.

## Appendix B: Three Lists of Rules in Frege

This Appendix includes three lists. The first list reproduces the rules Frege provides in the essay “Boole’s Logical Calculus and the Concept Script”, written closely following the publication of the *Begriffsschrift*.<sup>117</sup> The second list includes the rules found in sections #14-18 of *Grundgesetze*. The third list includes the rules specified in section #48 of *Grundgesetze*. For convenience, I converted Frege’s symbols to modern logical notation, and simplified his expressions where possible. I specify, where relevant, the name Frege gives to each of his rules, and the notational devices he associates with it.

### 1. Rules in BLC

The article “Boole’s Logical Calculus” includes the following list of five rules, which Frege takes himself to have articulated in the *Begriffsschrift*.<sup>118</sup>

1. “What follows the content stroke must be a content of possible judgement”
2. “The rule of inference”
3. “Different Gothic letters are to be chosen when one occurs within the scope of another” (in a footnote, Frege adds: “Strictly, this rule is implicit in the first”)
4. “A rule for replacing roman letters by gothic”
5. “A rule for exporting a condition outside the scope of a gothic letter”

The most conspicuous item on this list is no. 2, “the rule of inference”. As Frege repeatedly

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<sup>117</sup> It might seem peculiar that I do not begin with a list of rules of the *Begriffsschrift* itself. The reason for that is quite simple. Frege does not offer a list of the rules of *Begriffsschrift*. But the BLC list is supposed to be explanatory of the system of *Begriffsschrift*.

<sup>118</sup> BLC, in PW, p.39.

states,<sup>119</sup> his early system restricts itself to a single rule of inference, namely the *modus ponens*. The other rules given in this list do not count as full-fledged rules of inference, even though some of them involve transitions between propositions which, in our eyes, might count as inferential in nature. Indeed, the list is not restricted to the rules of inference and derivation. Rules 1 and 3 would more naturally be considered as formation rules, rather than rules of derivation. They concern the proper use of the Begriffsschrift inasmuch as it is capable of expressing content. Rule 5 is a rule for “exporting” a condition outside the scope of a gothic letter, i.e. it forms a transition from

$$\forall x (p \supset Gx)$$

to

$$p \supset \forall x Gx$$

This rule allows us to delimit the scope of the quantifier and to pull components which do not contain free variables outside of its scope.

Moreover, the list is not meant to be economical or minimal, as is evidenced by the footnote to the third rule. On the other hand, Frege does not mention in this list each and every one of the rules which govern his Begriffsschrift. For instance, he does not mention the rules for distinguishing function names from object names (BS, p. 129), the rule of instantiation (BS, p. 130) and the rules for definitions (BS 167ff.). And the list ignores the rule of substitution, which is wholly implicit in BS.

When Frege introduces the *modus ponens* in section #6 of *Begriffsschrift*, he also specifies the manner in which its application should be notated and the ways in which it may be abbreviated. Inference by *modus ponens* is marked by a horizontal line placed between the

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<sup>119</sup> In the Preface to BS (p. 107) as well as in paragraph #6 of BS, and then again in retrospect in the Foreward to *Grundgesetze* (GG, p. VI) and in paragraph #14, p. 26.

premises and the conclusion. For the sake of brevity, Frege allows that only one of the premisses of a given inference be mentioned, whereas the other be noted (by name or number) on the left side of the horizontal line. If the premise cited by number is a conditional whose antecedent is to be dropped, this is marked with a colon (:); if the cited premise is the one in which the antecedent is independently asserted, this is marked with a double colon (::). Thus the following three expressions are equivalent:

$$\begin{array}{ccc}
 \begin{array}{c} (1) \vdash B \supset A \\ (2) \vdash B \\ \hline \vdash A \end{array} &
 \begin{array}{c} (1) \vdash B \supset A \\ (2) :: \hline \vdash A \end{array} &
 \begin{array}{c} (2) \vdash B \\ (1) : \hline \vdash A \end{array}
 \end{array}$$

There is also a variant of *modus ponens* in BS, through which two applications of modus ponens are combined in one step (leading from three premisses to one conclusion).<sup>120</sup> This is marked by a double horizontal line between the premisses and the conclusion. That this variant of “the rule of inference” is not specified separately and does not count for Frege as multiplying the total number of rules of inference of his system is an important fact I return to in the discussion that follows list no. 2, and in Appendix C.

## 2. Rules in *Grundgesetze*, #14-18

Sections #14-18 of *Grundgesetze*, titled “Inferences and Derivations” (*Schlüsse und Folgerungen*), survey the rules and laws of Frege’s system. There is no ordered list of rules here, and not all of the rules surveyed are even given a title. But they are fairly easy to identify since each rule is discussed in a separate, indented paragraph. Following each rule, I discuss Frege’s manner of indicating its use.

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<sup>120</sup> BS, #6, p. 119.

1a. The first rule, *modus ponens*, prescribes:

From the propositions “ $\vdash \Delta \supset \Gamma$ ” and “ $\vdash \Delta$ ” we may infer “ $\vdash \Gamma$ ”; for if  $\Gamma$  were not the True, then since  $\Delta$  is the true,  $\vdash \Delta \supset \Gamma$  would be the False.<sup>121</sup>

Rule 1a governs the most basic inferential transition. The application of the rule is marked with a horizontal line placed between premises and conclusion. As in *Begriffsschrift*, Frege allows that only one of the premises be mentioned, in which case he notes the name or number of the second premise on the left side of the horizontal line (see the discussion of this procedure in List no. 1, above).

1b. Given the rule of the “permutability of the subcomponents” of a conditional (*Vertauschbarkeit der Unterglieder*),<sup>122</sup> the *modus ponens* may be applied even in cases in which the antecedent to be dropped is not the outermost antecedent of a complex conditional. Thus from

$$\Gamma \supset \Delta \supset \Lambda$$

and

$$\Delta$$

we may infer

$$\Gamma \supset \Lambda$$

The application of this rule is also marked by a single horizontal line placed between premises and conclusion, just as for rule 1a.<sup>123</sup>

1c. A combined application of *modus ponens* in which three premises are involved.<sup>124</sup> One of them is a complex conditional; the other two assert the antecedents that are to be dropped. Thus from

$$\Gamma \supset \Delta \supset \Lambda$$

and

$$\Gamma$$

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<sup>121</sup> GG, #14, p. 25.

<sup>122</sup> GG #12, p. 22

<sup>123</sup> GG #14, p. 26.

<sup>124</sup> Ibid.

as well as

$$\Delta$$

we may derive

$$\Lambda$$

The application of this rule is marked by a double horizontal line placed between premises and conclusion.

2a. The hypothetical syllogism, according to which from

$$\Gamma \supset \Delta$$

and

$$\Delta \supset \Lambda$$

we may derive

$$\Gamma \supset \Lambda$$

The application of this rule is marked by a dashed horizontal line placed between premises and conclusion.<sup>125</sup>

For brevity, one may mention just one of the premises, and cite the other one on the left side of the dashed line, marked by a colon or double colon. The single colon marks the conditional whose consequent is the consequent of the conclusion (in our example,  $\Delta \supset \Lambda$ ); the double colon marks the conditional whose antecedent is the antecedent of the conclusion (in our case,  $\Gamma \supset \Delta$ ).

2b. Frege allows the application of rule 2a even in cases in which the antecedent which is to be dropped is nested in more complex conditionals (relying again on the permutability of subcomponents).<sup>126</sup> Thus from

$$\Gamma \supset \Delta \supset \Lambda$$

and

$$\Pi \supset \Delta$$

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<sup>125</sup> GG, #15, p. 26.

<sup>126</sup> Ibid., p. 27.

we may derive

$$\Gamma \supset \Pi \supset \Lambda$$

3a. The rule of Contraposition (“Wendung”, GG, #15), that governs a transition (Übergang) from

$$\Gamma \supset \Delta$$

to

$$\sim \Delta \supset \sim \Gamma$$

The application of this rule is marked by an X placed between the premise and the conclusion.<sup>127</sup>

3b. A more complex application of 3a, which governs cases in which nested subcomponents are contraposed, such that from

$$\Gamma \supset (\Delta \supset \Lambda)$$

we may transition to

$$\Gamma \supset (\sim \Lambda \supset \sim \Delta).^{128}$$

3c. Through a repeated application of contraposition and the principle of permutability of subcomponents, Frege shows how to group all the antecedents into one unit, such that from

$$\Gamma \supset (\sim \Delta \supset (\Lambda \supset \Pi))$$

we may transition to

$$\sim(\Gamma \supset (\sim \Delta \supset \sim \Lambda)) \supset \Pi$$

4. The fusion (amalgamation) of identical subcomponents (*Verschmelzung gleicher Unterglieder*), i.e. from

$$\Delta \supset \Delta \supset \Gamma$$

we may transition to

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<sup>127</sup> Ibid.,, p. 27..

<sup>128</sup> Ibid., p. 28.

$$\Delta \supset \Gamma.^{129}$$

5. A rule which combines the Hypothetical Syllogism, Contraposition and Amalgamation, such that from

$$\Pi \supset (\Lambda \supset (\Delta \supset \Gamma))$$

and

$$\Sigma \supset (\sim \Xi \supset (\Delta \supset \Lambda))$$

we may derive

$$\Pi \supset \Lambda \supset \Sigma \supset \sim \Xi \supset \Gamma$$

where antecedents which are common to both premises (such as  $r$ ) only appear once. This inference is marked with a horizontal dashed line between premises and conclusions.<sup>130</sup>

6. Allowing for the combined application of more than one rule at once, we get inferences from two premisses or more in which the Hypothetical Syllogism may be applied twice, or applied in combination with an application of *modus ponens*. These complex transitions are either marked with a double horizontal dashed line or with a horizontal dashed line placed under a continuous horizontal line, between the premises and the conclusion.<sup>131</sup>

7. The rule which combines hypothetical syllogism, amalgamation and contraposition, resulting in getting rid of contradicting subcomponents, such that from

$$\Sigma \supset \sim \Xi \supset \Delta \supset \Lambda$$

and

$$\Delta \supset \Xi \supset \sim \Theta \supset \Lambda$$

we may conclude

$$\Sigma \supset \Delta \supset \sim \Theta \supset \Lambda$$

where in the conclusion we have fused the two appearances of  $p$  in the premises and

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<sup>129</sup> Ibid., p. 29.

<sup>130</sup> Ibid..

<sup>131</sup> Ibid., p. 30.

eliminated the contradictory pair  $q$  and  $\sim q$ .<sup>132</sup> The application of this rule is marked with a horizontal dot-dash line between the premises and the conclusion:

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8. A rule for the replacement of a latin with a gothic variable letter, i.e. for replacing a wide-scope bound variable with a narrow scope bound variable, e.g. for

$$(a > 2) \supset (a^2 > 4)$$

we may substitute

$$\forall x [(x > 2) \supset (x^2 > 4)]$$

The application of this rule is marked with a large concave mark placed between the premise and the conclusion.<sup>133</sup>

9. Similar to the *Begriffsschrift*'s rule for exporting a condition outside the scope of a gothic letter (Rule no. 5 in list no. 1) Frege specifies a rule of transition from

$$\forall x (\Gamma \supset Gx)$$

to

$$\Gamma \supset \forall x Gx$$

in all cases in which  $x$  does not occur free in  $\Gamma$ .<sup>134</sup>

A few comments on this list are in order. In the Furth translation, the titles “First Method of Inference”, “Second Method of Inference” and “Third Method of Inference” are added to the rules that I list as 1a, 2, and 7. These titles do not appear in the original German text, but Furth’s suggestion can be justified. First, Frege mentions these three rules as the only rules of inference in list no. 3 (which is taken from the same book). Second, each of these three rules is given a distinctive sign in Frege's notation— a variant of the horizontal line of *Begriffsschrift*. Each of

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<sup>132</sup> GG #16, p. 30.

<sup>133</sup> GG #17, p. 31. Frege refers back to the rule of introducing the universal quantifier in GG #8,

<sup>134</sup> Ibid., p. 33.

the other rules which appear in Sections 14-17, on the other hand, is symbolized by a combination of two of these horizontal signs, indicating that these rules abbreviate the application of more than one rule; except contraposition, which is symbolized by a different kind of sign altogether (an x between the premise and the conclusion). Third, the rules given in list no. 2 partly correspond to the titles given to sections #14-17 in the Table of Contents of *Grundgesetze*. There Frege give these sections the following titles:

#14 First Mode of Inference

#15 Second Mode of Inference, Contraposition

#16 Third Mode of of Inference

#17 Latin Letters, Transition from Latin to German Letters

Paragraph #14 of *Grundgesetze* discusses rules 1a, 1b and 1c of my list; all of these count for Frege as variants of the first mode of inference, *modus ponens*. Section #15 discusses rules 2-6. of my list Rules 2a-b as well as rule 5 and 6 are either variants of the Hypothetical Syllogism or combinations of Hypothetical Syllogism with other rules. Thus Hypothetical Syllogism is probably what Frege means by “Second Mode of Inference”, whereas rules 3a-c, which are variants of Contraposition, and rule 4, Fusion, are apparently not counted by Frege as rules of *inference* at all, but rather as mere transitions between propositions whose content is identical or weakened. “The Third Mode of Inference” mentioned in the Table of Contents is rule 7 of my list. Rules 8 and 9 on my list do not count as rules of inference in the Table of Contents, but are rather referred to there as “Transition”.

This strengthens the hypothesis that Frege does not count transitions from a single premise as inferences. His reason, as I argue in Appendix A, is that the concluded proposition does not

seem to have a distinctively new content vis-a-vis the premise.<sup>135</sup> This consideration helps shed light on Frege's idea of content in *Grundgesetze*, as well as on his suggestions that proofs yield genuine knowledge (see discussion in Section 2.2, above).

Another point to note is that the title *Schlüsse und Folgerungen* given in both the Table of Contents and in the main text governs sections #14-18, and hence applies to section #18 as well, of which I made no mention so far. That is because in this section Frege lists the Basic Laws, i.e. the axioms, rather than any rules of his system. It is not clear why Frege subordinates section #18 to this title.

### 3. Rules in *Grundgesetze*, Section #48

Section #48, titled "Summary of Rules" (*Zusammenstellung der Regeln*), includes several rules not mentioned in list no. 2 B. We find the following rules listed:

1. Fusion of horizontals, explicated as a transition in section #6, but not mentioned in the discussion of Inferences and Derivations in #14-18;
2. Permutation of subcomponents, which is relied on in several of the rules given in list no. 2, but not independently mentioned as a rule of inference or derivation;
3. Contraposition (3a, in the list above)
4. Fusion of equal subcomponents (4, in the list above)
5. Transformation of a Roman letter to a German Letter (8, in the list above)
6. Inferring (a), i.e. *modus ponens* (1a in the list above, and its variants).
7. Inferring (b), i.e. the Hypothetical Syllogism (2a in the list above, and its variants)
8. Inferring (c) (7 in the list above).
9. Citing propositions: replacement of Roman letters (a rule of instantiation)
10. Citing propositions: replacement of German letters (to avoid ambiguity)
11. Citing propositions: replacement of greek vowels
12. Citing definitions (conversion of definition into a judgment)

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<sup>135</sup> Frege makes the distinction explicit in "Logic in Mathematics", p. 204, which I discuss in Appendix A, above. Elsewhere, he does not seem to count transitions that take their start from one premise as genuine inferences.

I am indebted to Michael Kremer, who explores the issues discussed here in unpublished correspondence with the translators and the editor of the new edition of Frege's *Grundgesetze*, Philip A. Ebert, Marcus Rossberg and Crispin Wright.

13-18. Stipulations concerning the use of brackets.

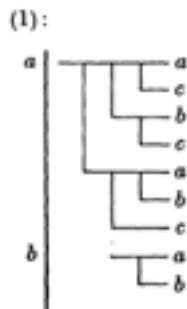
Some of the distinctions made in list no. 2 seem not to be important enough to be mentioned explicitly in this (no. 3). Rules 5 and 6 of list no. 2, in particular, are not given separate treatment here. The same attitude towards variants of modus ponens was evident in list no. 1, above, in which “the rule of inference” is mentioned without breaking it down to its different species.

## Appendix C: Frege's Rules of Substitution

Frege's *Begriffsschrift* has been criticized for covertly employing rules of inference that are not explicitly acknowledged. Primarily, the criticism targets the rule of substitution — the procedure whereby Frege instantiates general propositions by substituting the variable letters that appear in them.<sup>136</sup> For instance, in the proof of proposition (3) in section #15 of *Begriffsschrift*, Frege omits one of the premises and marks it in the margins, next to a single colon. The omitted premise is proposition (1), which was already proved in #14. It reads:  $a \supset (b \supset a)$ . But for the purposes of the proof of (3), (1) would not do. What is needed is the more complex, but structurally equivalent

$$\{[c \supset (b \supset a)] \supset [(c \supset b) \supset (c \supset a)]\} \supset \langle (b \supset a) \supset \{[c \supset (b \supset a)] \supset [(c \supset b) \supset (c \supset a)]\} \rangle$$

Thus for (1) to serve as the premise of the proof, we need to substitute  $a$  with  $\{[c \supset (b \supset a)] \supset [(c \supset b) \supset (c \supset a)]\}$ , and substitute  $b$  with  $(b \supset a)$ . To mark these substitutions Frege appends a clarificatory table, under the numbering of the missing premise. Here is how the table looks like:



The letters 'a' and 'b' on the left hand side indicate the components of the axiom; what they are substituted with are the formulas on the right hand side.

<sup>136</sup> In his Introduction to Frege's BS, Bynum shows that even though the rule of substitution is not fully stated, Frege does spell out a rule of instantiation (BS, p. 130). See Bynum 1972, p. 71.

To better understand this procedure, let us compare it with Frege's procedure in his later writings, in which he dispenses with the device of clarificatory tables altogether. Instead of the clarificatory tables, *Grundgesetze* employs an explicit rule to govern the instantiations. Rule 9 of section #48 states that when a proposition is cited by means of its number, any Latin letter occurring in it may be replaced by a different proper name (including names of propositions), or, in the case of function names, with names of functions. Frege describes this procedure as a "simple inference" (*Schluss*).<sup>137</sup>

The substitutions are stated in the elucidatory analysis (*Zerlegung*) that precedes each proof. The analysis stated, in words, each of the procedures Frege is about to follow in the constructive part of the proof (*Aufbau*). For instance, Frege explains how the different basic laws are instantiated so they can serve as convenient theorems that would be directly relevant to the proofs. Of these Analyses, Frege writes that they are only introduced for the convenience of the reader and could just as well have been left out entirely without thereby impairing the actual proof.<sup>138</sup> But since the status of these non-essential analyses is explicitly an elucidatory one, we may conclude that a similar elucidatory status attaches to the clarificatory tables of the *Begriffsschrift*.

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<sup>137</sup> GG #48, p.62. But see my discussion of Frege's distinction between inference and derivation in Appendix A, above.

<sup>138</sup> GG, #53, p. 70

## Appendix D: Frege's Reduction of Aristotle's Modes of Inference

Frege proposes the following method for reducing all syllogistic modes to the system in which a single rule of inference — *modus ponens* — is employed. To reduce any inference from two premises to an application of *modus ponens* within Frege's system, what is required is that we introduce as a theorem or as an axiom a judgment that instantiates the following schema:

$$(i) (N \supset (M \supset \Lambda))$$

where N, M and  $\Lambda$  have the form of the two premises and the conclusion. Then, by applying the relevant rules of substitution,<sup>139</sup> we would derive the conclusion from the two premises, along with the instance of (i), by applying *modus ponens* to them twice.<sup>140</sup>

Let us take as an example the aristotelian syllogistic form known as Barbara, which authorizes the following inference:

Premises:	(n)	All Mammals are Animals
	(m)	All Lions are Mammals
Conclusion	(l)	All Lions are Animals

In this syllogism one infers a universal affirmative proposition from two other universal affirmative propositions that share a “middle term”. In this case, the judgment instantiating schema (i) is derivable from the axioms that we already accept in Frege's system. We first transcribe the premises and conclusions as

$$(n) \forall x(Mx \supset Ax)$$

$$(m) \forall x(Lx \supset Mx)$$

$$(l) \forall x(Lx \supset Ax)$$

The instance of (i) that we then need is the following:

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<sup>139</sup> Frege relies on an implicit rule of substitution which allows him to replace each latin letter with a judgment of arbitrary content and complexity. See Appendix C for further discussion of Frege's rule of substitution.

<sup>140</sup> BS, p. 120.

$$(ii) \forall x(\theta x \supset \psi x) \supset ((\forall x(\phi x \supset \theta x) \supset \forall x(\phi x \supset \psi x)).$$

which is derivable from the other axioms.<sup>141</sup> We instantiate (ii) as follows:

$$(o) \forall x(Mx \supset Ax) \supset (\forall x(Lx \supset Mx) \supset \forall x(Lx \supset Ax)).$$

Them, by an application of modus ponens to (o) and (n) we get

$$(p) \forall x(Lx \supset Mx) \supset \forall x(Lx \supset Ax)$$

And by a second application of modus ponens to (p) and (m) we get the desired conclusion:

$$(l) \forall x(Lx \supset Ax)$$

This method of reduction can also be used to reduce other rules of inference to *modus ponens*. Both of Frege's major systems (*Begriffsschrift* and *Grundgesetze*) include *modus ponens* as the fundamental rule of inference. However, in *Grundgesetze* two additional rules of inference are introduced.<sup>142</sup> The first of these rules is Hypothetical Syllogism: from  $(a \supset b)$  and  $(b \supset c)$ , we infer  $(a \supset c)$ . But the *Begriffsschrift* system has the resources to reduce this rule of inference to the *modus ponens* without the introduction of any added axioms. The judgment that corresponds to schema (i), in this case, is Frege's theorem #5:<sup>143</sup>

$$BS\ 5 \quad (b \supset a) \supset ((c \supset b) \supset (c \supset a))$$

The only axioms of *Begriffsschrift* needed for the derivation of BS 5 are those expressed in

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<sup>141</sup> There is no exact parallel to the derivation I offer here in *Begriffsschrift*. Frege provides explicit reductions of other Aristotelian syllogistic forms there, that are slightly more complicated than the relatively simple example of Barbara that I gave here (e.g. BS, #22, p. 163f.), and he does not explicitly show how to reduce Barbara to his axioms. However, given Frege's judgment #5, which he uses to reduce the hypothetical syllogism (an issue I discuss in the next paragraph), as well as the rules and axioms which govern quantification, it is quite trivial to show that my judgment (o) can be derived from Frege's axioms. In *Grundgesetze*, where the hypothetical syllogism is treated as a primitive rule (the second rule of inference of the system), the reduction of Barbara to the system is, indeed, trivial. See Frege's demonstration of this claim in GG, #17, p.65f.

<sup>142</sup> See discussion in Appendix B, above.

<sup>143</sup> BS, #15, p. 140f.

Frege's judgments number #1 and #2:

BS 1       $a \supset (b \supset a)$

BS 2       $(c \supset (b \supset a)) \supset ((c \supset b) \supset (c \supset a))$

Thus the Hypothetical Syllogism reduces to a procedure in which we only apply a single rule of inference, alongside the numerous axioms of the *Begriffsschrift*.

### 3. Russell on Rules of Inference

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Russell's reflections on the nature of rules of inference bring to the surface a tension which underlies two of his fundamental commitments.<sup>1</sup> On the one hand, he is guided by the logicist idea that a system of proof can be constructed by means of which we can show that mathematical knowledge is grounded in logic. In such a system, logical truths are made explicit in the form of axioms, and mathematical theorems are derived from them. On the other hand, Russell holds the universalist view that logic concerns the all-encompassing principles of thought.<sup>2</sup> It is this that gives logic its self-evidence, and explains its foundational role for other forms of knowledge. The first line of thought demands that we view logic as concerned with the most general *content*, whereas the second thought demands that we view logic as concerned with the *form* in which all content is taken up and assessed. But Russell is pushed to recognize that the formal system cannot fully and explicitly express all of the fundamental principles and logical distinctions that govern thought; since he takes these principles to be contentful, it follows that not all logical content can be expressed.

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<sup>1</sup> This tension is similar to the one that besets Frege's work, as I argued in the previous chapters.

<sup>2</sup> The claim that Russell is a universalist in this sense is not uncontroversial. I discuss it in more detail in the concluding section of the chapter.

Russell's struggle to account for the rules of inference exemplifies this tension in the following way. The rules of inference, like the axioms, seem to get their justifying power from their general and universally applicable content; but as Russell learns from Lewis Carroll and F.H. Bradley, rules of inference cannot be treated as explicit propositions, on pain of regress.<sup>3</sup> Rules of inference thus signal a "limitation" or "breakdown" of the formalism.<sup>4</sup> Russell attempts to avoid and circumvent this breakdown, for it frustrates his aspiration that the formal system make the content of logic fully explicit. He is pushed to develop an account according to which rules of inference encode the content which justifies the inference, but they themselves remain inexpressible; they are applied to inferences and justify them by means of an extra-formal, quasi-intuitive act of perception.

In Carroll, Russell finds the argument that in order to justify an inference we must cite as evidence our belief that the premises support the conclusion; that belief then serves as a ground from which we infer that the first inference is valid. But this renders our acceptance of one inference dependent on our acceptance of another, and the new inference equally stands in need of justification, which is once again bound to take an inferential form. The justification of each act of inference thus gives rise to an infinite number of further acts.

Whereas Carroll's regress signals a problem with the idea that a rule is appealed to in the process of showing that a conclusion rests on the premises, Bradley's regress signals a problem with the idea that a rule is appealed to in the process of instantiating any general logical principle, including the axioms. General propositions seem to lend support to specific inferences because they can be instantiated in those specific ways. However, if the act of instantiation itself

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<sup>3</sup> Carroll 1895; Russell locates the second regress argument in Bradley 1883 (p. 227), but it is hard to find the argument he describes on that page.

<sup>4</sup> Principles, #18, p. 16 and #45 p. 41

requires justification, and the only kind of justification we have at our disposal is the appeal to some general rule, we would also need to show that that general rule applies to the specific case at hand, i.e. we would need to instantiate it. And so a regress is sure to arise.

It is often claimed that Russell lacks a proper distinction between rules of inference and the propositions of logic, of the kind we find in Frege, and that for this reason he fails to appropriately respond to the challenge posed by Lewis Carroll.<sup>5</sup> But this judgment is mistaken.<sup>6</sup> Frege's way of drawing the distinction between rules and laws is not the only way, and certainly not the most fundamental way, in which one could respond to Carroll's challenge. This is an important issue for my overall argument in this dissertation, since some of the same interpreters who claim this are also wont to confuse Wittgenstein's challenge with Carroll's.<sup>7</sup> One of my tasks in this chapter will therefore be to show that the two challenges — Wittgenstein's and Carroll's — are independent. I will offer a reading of Russell that shows him to provide an adequate response to Carroll's and Bradley's challenges, but leaves him open to Wittgenstein's criticism.

Russell's response to Carroll's and Bradley's puzzles has three components. First, he distinguishes the rules which guide us in inferring from the propositions that can be expressed in the formal system and serve as premises; second, he distinguishes the content of rules from the content of expressible propositions; third, he construes the application of rules as an immediate, non-formal act. Russell abandons the idea that the rules that justify inferences can and should be

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<sup>5</sup> See, e.g. Hacking 1979 and Ricketts 1985. An exception is Proops 2001, p. 286. And for further discussion see Chapter 4.

<sup>6</sup> It is mistaken with respect to Russell's two major philosophical works, with which Wittgenstein was acquainted — *Principles* and *Principia*. However, in chapter 7 in his popular book, *The Problems of Philosophy* (Russell 1912), Russell is much less careful in the way he treats the logical principle that underlies inference. I discuss at the end of section 3.2.4.

<sup>7</sup> E.g. Hacking 1979 and Ricketts 1985.

made explicit. Instead, he proposes that in addition to our capacity to register the content of the premises and conclusions of the inference, we have to acknowledge a further, irreducible capacity to “perceive” the applicability of the rule to an inference.<sup>8</sup> Giving rules of inference such a special non-formal status is all it takes to block Carroll’s and Bradley’s attack. In order to do that, however, Russell needs to modify the native conception of the relation between logic and language from which he starts — he must recognize the inability of language to express its own logic. This recognition, coupled with the logicist, contentful view of logic, is what is most distinctive of Russell’s (and Frege’s) universalist conception of logic. Russell continues to think of the application of rules on the model of the appreciation of contentful propositions. He speaks of the rules as the ultimate grounds of inferences, ineffable statements of general logical facts that are (ineffably) instantiated in each particular inference.

Wittgenstein, on the reading I will offer in the second part of this dissertation, rejects the idea that the logical form of thought can be captured in terms of the *content* of general propositions, no matter whether they are discursively grasped or directly perceived. He further rejects the idea that language is limited, i.e. that the realm of content stretches beyond our means of expression. What cannot be said — logical form — must not be construed as an in-principle sayable content that for some reason or other lies beyond our reach.

The chapter opens with a detailed discussion of Lewis Carroll’s dialogue (3.1). The task I set myself here is to delineate the conception of logic from which the puzzle arises and for which it poses an undeniable threat. In section 3.2 I trace the development of Russell’s account of inference. My main focus is on the works with which Wittgenstein was, or could have been acquainted. This includes the period in which Russell develops his response to Carroll’s puzzle,

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<sup>8</sup> Russell (1903) (henceforth, *Principles*), p. 41.

between *Principles of Mathematics* (1903)<sup>9</sup> and *Principia Mathematica* (1910).<sup>10</sup>

In section 3.3 I turn to Russell's 1913 manuscript, *The Theory of Knowledge*,<sup>11</sup> on which Russell worked while Wittgenstein was his student in Cambridge. Wittgenstein's criticism of the account of logical form that Russell develops in this work is closely related to his criticism of rules of inference.

An important part of Russell's response to Carroll's challenge is his discovery of the distinction between implication, understood as a truth functional connective, and what he calls the relation of "therefore". This distinction allows him to differentiate between the content of rules from the content which can be expressed by propositions in the logical formalism. Despite his acknowledgment of this distinction, Russell never stops using the term 'imply' in ambiguous ways, and he uses a single formal sign — the horseshoe — to indicate these two distinct relations. In section 3.4 I respond to Michael Kremer's suggestion, that the ambiguity in Russell's use of these terms exemplifies what Wittgenstein diagnoses as the source of philosophical nonsense. I argue that the precise target of Wittgenstein's criticism is not the ambiguity as such, but Russell's inability to fully distinguish logical form from content.

The chapter concludes with a discussion of the Russell's success in responding to Carroll and his failure to immune himself to Wittgenstein's objection. I draw a comparison between Russell's account and more recent accounts of inference, and conclude that Wittgenstein's criticism should be brought to bear on these contemporary accounts, since the problems he identifies have not been properly addressed.

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<sup>9</sup> Russell 1903 (henceforth *Principles*).

<sup>10</sup> Whitehead and Russell 1910 (henceforth *Principia*).

<sup>11</sup> Russell 2013 (henceforth ToK).

### 3.1. Carroll's Puzzle

Lewis Carroll's "What the Tortoise Said to Achilles" raises fundamental questions about the nature of logic: Does logic provide the ultimate starting points of all reasoning? And if so, do we need to make logic explicit in order to fully ground the beliefs that we reach by reasoning? If logical starting points are the ultimate underpinnings of all inference, there would be nothing more fundamental that we could appeal to in order to convince anyone to adopt them. If so, what would be the point of communicating them, and how could we help anyone who claims to stand in need of being informed of them?

Given the informal way in which Carroll presents his puzzle, however, it is not surprising to find that philosophers differ not only with regard to the nature of the proper solution that it requires, but also with regard to the nature of the problem that is being raised.<sup>12</sup> My aim in this section is to identify the naive conception of logic which Carroll's puzzle threatens. This will allow me to argue that Russell overcomes Carroll's challenge by developing a more sophisticated conception of logic. Seen in the larger context of my dissertation, this section has the additional aim of distinguishing two types of challenges to Russell's conception of rules of inference, which are often confused — Carroll's and Wittgenstein's.

Carroll's puzzle is cast in the form of a dialogue between two figures, Achilles and the Tortoise. The protagonists first consider a classic Euclidean inference, in which one draws a conclusion, (Z), from two premises, (A) and (B). The Tortoise purports to "accept" each of the

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<sup>12</sup> See e.g. Ryle 1950, Thomson 1960, Wisdom 1974, Stroud, 1979, Priest 1979, Smiley 1995, Blackburn 1995, and Hanna 2006, ch.3. For discussions of Carroll that situate him in his historical context see Moktefi 2008 and Marion 2012, pp. 70-97.

premises of this inference, and he orders Achilles to enter them in his notebook. And yet the Tortoise refuses to acknowledge the conclusion. His reason, he says, is that he does not yet “accept the sequence as a valid one”.<sup>13</sup> As Cora Diamond points out, it is already at this early point that a crucial move is made — the move which creates the appearance that something is missing in the seemingly valid inference which is being discussed:

[the dialogue] works to make it look as if we need to add something to the two original premises. It does this by first imagining a person who accepts the sequence from premise A and premise B to Z as valid, but rejects A and B. Carroll's argument works by getting us to treat that case as a model for the situation of a different person. The model we are being induced to see as relevant in both cases is that of a set of storable acceptances, such that rejection of one of the storable acceptances entitles you to not accept the conclusion. So in either case, the rejection of the conclusion can properly be responded to by adding a premise.<sup>14</sup>

Achilles takes the bait and offers to frame a statement asserting the validity of the inference; the Tortoise then adds that he would only be willing to reconsider the inference if this new statement if it is entered into the notebook — and thereby treated as one of the premises that are required in order to establish the conclusion. When Achilles does that, however, the sequence in the notebook no longer looks like the one whose validity was just asserted. And so the Tortoise's doubt is renewed, and the same process is repeated.

The Tortoise and Achilles refer to the missing premise as a ‘Hypothetical Proposition’. It is structured as a conditional whose antecedent is the conjunction of the premises (A) and (B) and whose consequent is the conclusion (Z):

(C) *If A and B are true, Z must be true.*

Adding (C) to the notebook, the inference assumes the following form:

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<sup>13</sup> Carroll 1895, p. 278.

<sup>14</sup> Diamond (private correspondence).

$$\begin{array}{c} (A) \\ (B) \\ \hline (C) \\ (Z) \end{array}$$

At this point, even though the Tortoise accepts (C), he still does not consider the task of justifying the conclusion of the inference to be complete. The validity of the new sequence is no longer guaranteed by (C) alone, since the sequence for whose validity (C) accounts — from (A) and (B) to (Z) — does not have the same shape as the new sequence, which goes from (A), (B) and (C) to (Z). The antecedent of the Hypothetical Proposition (C) does not include one of the premises of the new sequence, namely ‘(C)’. So the Tortoise suggests that we add a further premise, (D), that asserts the validity of the new sequence. This creates another, altered sequence, which requires its own justification. An infinite regress looms. None of the Hypotheticals we add have the power to “take the Tortoise by his throat” (to use Achilles’s phrase) and force him to acknowledge the conclusion.

Based on these initial observations, we can single out the following conditions which need to be fulfilled for Carroll’s regress to arise:

- (1) The justification of an inference depends on stating its validity, by means of a ‘Hypothetical’.
- (2) This Hypothetical is regarded as a missing part of the inference.
- (3) The Hypothetical must be accepted, and entered into the notebook alongside the other premises.
- (4) The inference is whatever we have written down in the notebook; so writing down the Hypothetical results in altering the inferential sequence.
- (5) A fully legitimating Hypothetical must refer to each and every proposition of the inference.
- (6) An inference derives all of its justification from its explicit premises alone. In other words, the Hypothetical has no force to affect an inference it does not appear in.

Let us look into these conditions and the way in which they contribute to the regress. According to assumption (1) and (2), the Hypothetical proposition provides justification by completing the inference. Carroll does not think that the statement of the validity of the sequence should be different in kind from any other statement. It can therefore be used as a premise. Nonetheless, it is noteworthy that Achilles' Hypothetical involves the modal 'must', unlike the other premises. This signals that it is not supposed to merely serve as a further premise, but also to convey a claim about the status of the argument itself.<sup>15</sup> Tortoise's failure to register this distinction has often been held to be the primary cause of the regress.<sup>16</sup> However, on my account, such a distinction would ultimately have no effect on the emergence of the regress, since no matter what kind of fact we appeal to when we justify an inference, so long as appealing to it gives rise to additional inferential reasoning, there will be a regress.<sup>17</sup>

Achilles and the Tortoise go on to treat the Hypothetical just like any other premise — they “accept” it, and write it down (3). This seemingly innocent move is in fact symptomatic of a deep

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<sup>15</sup> In a letter sent to the editor of *Mind* (Dodgson 1977, pp. 472-474) Carroll attempts to clarify the philosophical purport of his puzzle. He says that the Tortoise asks us to acknowledge that what makes the inference justified is “the validity of the sequence”; he then offers two possible formulations for the expression of this acknowledgment. In somewhat simplified form, the two variants are

- (C1)           The sequence “If A and if B then Z” is valid
- (C2)           If A and B are true, Z must be true

Premise (C) in the published dialog is the one here labeled (C2). It is a seemingly ordinary conditional, except for the appearance of the modal operator and the predicate ‘true’ — but Carroll does not make much of these features. The way (C1) works, on the other hand, is quite different. It mentions a proposition, a conditional, which supposedly captures the “sequence”, and it asserts of that sequence that it is valid. It is hard to tell with certainty what exactly Carroll means by “sequence”; in using this term he does not seem to clearly distinguish between the whole inference — a set of asserted propositions that stand in a certain relation of justification — and the conditional proposition, in which the premises of the inference are the antecedent, and the conclusion is the consequent.

Carroll is silent with regard to the differences between (C1) and (C2). He seems to take the two formulations to equally serve the same purpose. It is therefore safe to assume that he believes that in order to acknowledge the validity of an inferential sequence — as in (C1), we simply need to assert a proposition, as in (C2). Entering such a proposition into the notebook is all it means to acknowledge validity. It is to make one more assertion, to accept one more proposition.

<sup>16</sup> Thomson 1960, p. 100, and Moktefi 2008, p. 495.

<sup>17</sup> Wisdom 1974 makes a similar point.

problem, to which I will return momentarily. Condition (5) captures another subtle feature of the setup of Carroll's puzzle that is all too easy to ignore. It has to do with the *particularity* of the means of justification that the Tortoise considers to be at our disposal. Each Hypothetical proposition asserts the validity of a *particular* sequence, and can only play the role of justifying a specific inference because it seems to refer to the specific premises and conclusions that are used in it. A Hypothetical is not an expression of a general rule. We should not take the letters it contains as variable letters — they are names of actual propositions. This is not to say that if only Carroll was made aware of the idea of general rules of inference, he would conclude that his problem is illusory. Far from it. As Russell shows in his discussion of Bradley's puzzle (discussed in section 3.2.2, below), even if we do have general rules of inference in our system, a similar regress would arise so long as we do not block the demand that these rules be stated as explicit premises.

Conditions (5) and (6) demand that the Hypothetical premise be treated as belonging to the specific inference under consideration, and to have no force outside it.<sup>18</sup> Indeed for a regress to arise, the procedure of entering the Hypothetical proposition into the notebook (3) must result in altering the original inferential sequence (4). The line of thought that underlies this demand can perhaps be made plausible in the following way: at each point, the Tortoise may raise the seemingly pedestrian question: why must I draw *this* consequence?<sup>19</sup> And a legitimate reply

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<sup>18</sup> Note that every time Carroll's protagonists turn to consider what's in the notebook, only the most recent inference is considered. Perhaps the underlying thought here is that since the Hypothetical is only accepted for the purpose of justifying the conclusion of the inference, it must belong to that very same inference.

<sup>19</sup> Some readers of Carroll's take the possibility of raising this question to be highly contrived; Thomson, for instance, thinks that the Tortoise "changes his grounds" in raising the demand repeatedly; the regress is "an infinitely long red herring" (Thomson 1960, p.105). But Thomson merely assumes that which Carroll denies, namely that there are external standards for judging the validity of arguments, an appeal to which does not result in an expression that adds an explicit premise to the argument.

would take a form similar to that of the Hypothetical. In this sense the Hypothetical belongs to the inference under consideration, and may be always be stated alongside it. It is not the mere fact that it is stated alongside the inference that causes the trouble.

However, the sequence which the Hypothetical is about cannot be the sequence into which it is entered, because the sequence into which it is entered includes the Hypothetical, whereas the sequence that it is about does not. Each added Hypothetical concerns the whole of a specific inference, but the whole inference which it concerns is not the inference of which it forms part as a premise, and so it cannot justify the latter. Hypotheticals do not refer to themselves — in any case, Carroll does not consider this as a possibility. Each attempt to justify the inference thus leads to another, as yet unjustified inference. Condition (6) implies that propositions that are not premises of an inference cannot affect it. That means that even after we have “accepted” (C), we cannot consider the inference from (A) and (B) to (Z) as justified — even though that’s what (C) says, and we've accepted (C). The justification of inferences we considered at a previous stage cannot be settled by the premises of the current inference. The old inference lacks something; when we think we've added what it lacks to the inference, we find ourselves with a new inference, which is again found lacking.

Returning to condition (3), I consider the way in which Carroll’s protagonists treat propositions — what they mean by claiming to “accept” and by writing these propositions down in the notebook to be indicative of the most fundamental issue that underlies the puzzle. The only form of justification that the Tortoise seems to acknowledge is that of explicitly adding a premise to the one notebook, which then forms part of the one great inference that we are continually trying to justify. This offers a vivid picture of this naive, but very tempting picture of the relation between logic and language. Let us call this naive picture the ‘One Great Notebook’. According

to this picture, language is capable of expressing everything that is of relevance to logical justification — all justification consists in the explicit statement of supporting facts. Each expression of justification has the same form and status — it is entered into the one great notebook as a premise that supports the conclusion we aim to justify.<sup>20</sup> As the Tortoise says, “[w]hatever logic is good enough to tell me is worth writing down”.<sup>21</sup> On this naive picture, there’s only one kind of mental act involved in reasoning — the act of “accepting” a proposition. For the Tortoise, accepting a proposition just is the act of writing something down. Once they have been written down, the expressions in the notebook are completely *inert*; the acceptance of each of the propositions they express is independent from the acceptance of any other.<sup>22</sup> But this cannot be the way understanding and reasoning works. As Barry Stroud puts it,

[b]elieving something is more than having something written in a notebook...  
Understanding something requires seeing *some* (perhaps even a great many) of its  
obvious connections with other things.<sup>23</sup>

The tortoise reduces the interconnected acts of judgment and reasoning of a rational thinker to the handling of signs in the notebook. He seems to think that justification is a relation between otherwise independent and unrelated propositions. He ignores the dimensions of thinking and understanding that involve the recognition of internal relations between propositions that we

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<sup>20</sup> Moreover, like the antecedents of the material-conditional, the propositions the Tortoise accepts are indexed to the inference they form part of and do not count as having any force independently of it.

<sup>21</sup> Carroll 1895, p. 280.

<sup>22</sup> Both Wisdom 1974 and Stroud 1979 suggest that the Tortoise’s claim to ‘accept’ certain propositions, characterized in this way, does not satisfy the most basic requirements of rationality. Blackburn (1995) similarly argues that the Tortoise suffers from logical *akrasia*, since his acceptance of propositions does not lead him to engage in the appropriate inferential activities. Indeed, the Tortoise assumes an unduly thin notion of what understanding a proposition amounts to. In section 4, below, I argue that Wittgenstein criticizes Russell for endorsing a similarly thin conception of propositions; and cf. Valaris 2016, who argues that the target of Carroll’s argument is to expose the flaws of an atomistic view of beliefs.

<sup>23</sup> Stroud 1979, p. 191.

understand.<sup>24</sup> But given what is actually required in order to be entitled to assert propositions already puts one in a position to see how they are related, and which ones justify which. The Tortoise's repeated demand for further justification are therefore a symptom of irrationality: he claims to accept, but fails to manifest evidence of having understood.

### **3.2. Russell's Response: "Implication Must be Simply Perceived"**

Ian Hacking and Thomas Ricketts hold that Russell fails to appropriately respond to the challenge posed by Lewis Carroll, because he lacks a proper distinction between the rules of inference and the axioms of his system. As I will show here, this judgment is mistaken. There are two distinctions Russell makes which allow him to overcome Carroll's puzzle.<sup>25</sup> First, Russell distinguishes rules from axioms in terms of their logical content. Second, he draws the additional distinction between the way in which rules are deployed and the way in which axioms are appealed to. These distinctions signal Russell's rejection of the naive picture that is targeted by Carroll. In contrast to the Tortoise's assumptions, Russell holds that rules of inference are general, they do not form part of the inference, and they are not inert. There may be just One Great Notebook, but logic is not entirely contained in it.

Admittedly, Russell lacks an explicit terminological distinction between axioms and rules of inference. The single rule of inference of *Principles* (*modus ponens*) is indiscriminately included in the list of the axioms and equally referred to as a "principle" of inference;<sup>26</sup> in

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<sup>24</sup> On 'internal' relations, as relations that constitute the identity of their relata, see *Tractatus* 4.122. And see Chapter 6.

<sup>25</sup> See Hacking 1979, Ricketts 1985, and for an opposing view, see Proops 2001.

<sup>26</sup> *Principles*, #18 pp. 16-17.

*Principia*, the rule of inference is the first in the list of “Primitive Propositions” in which the axioms are included.<sup>27</sup> Axioms, on the other hand, are often called “principles of inference”<sup>28</sup> and “rules of inference”.<sup>29</sup> It is a burden of the following sections to show that despite the lack of explicit terminological distinctions, Russell distinguishes the use of rules from the appeal to propositions in a way which successfully blocks Carroll’s and Bradley’s regress.

My account of Russell’s developing views is quite elaborate, so it would be helpful to precede it with a quick summary. In *Principles* Russell diagnoses two problems that affect rules of inference. The first problem is the one he explicitly associates with Carroll’s puzzle, concerning the ability of the rule of inference to issue in the independent assertion of the conclusion. Russell observes that in a mere conditional (“implication”) such as Carroll’s Hypothetical, the consequent is dependent on the as yet unasserted antecedent, and thus an assertion of such a conditional does not of its own justify the independent assertion of the conclusion. Russell’s response to this challenge has three interrelated aspects:

- (1) Distinguishing the relation of implication that appears in conditional propositions, and the relation that rules of inference concern, and is captured by saying “therefore”.
- (2) Construing the distinction between asserted and unasserted propositions in a way that allows asserted propositions to figure as part of the content of rules of inference.
- (3) The content of rules — the relations of entailment between asserted propositions — cannot be expressed in the symbolism. Instead, it must be possible to appeal to rules in a non-formal way.

The shape of Russell’s proposal will become clearer once we consider the second problem that

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<sup>27</sup> *Principia*, p. 94.

<sup>28</sup> e.g. *Principles*, #18, p.16.

<sup>29</sup> *Principles*, #38, p. 34.

Russell takes to affect inference — the one he associates with Bradley.<sup>30</sup> The challenge here stems from the idea that rules have general content, whereas the inferences that they authorize are specific. It therefore seems that in order for the general rule to apply to an inference, it needs to be instantiated; but how is instantiation to be thought of, if not as an inferential step that itself requires justification, by means of an appeal to the general rule of inference? And if we are to attempt to provide such a justification, won't we have to first instantiate the rule so as to allow it to apply to the inference we are now concerned with? The very fact that Bradley's regress affects the rules of inferences (or at least initially seems to) is evidence that rules are thought of as having content, and that inferences are thought of as instances of these rules.

Russell's solution consists in denying that the instantiation of the rule of inference counts as a step in the formal deduction— it must be immediate and non-formal. Russell even speaks of the application of rules in quasi-intuitive terms: it “must be simply perceived”.<sup>31</sup> This might seem like an ad-hoc solution, but as I will argue, it is not without promise; at the very least it constitutes a good answer to Carroll's and Bradley's challenges. It preserves the thought that the justification of inference consists in an appeal to a logical fact, the content of the rule, which is taken to imply the inference. But the recognition of this justification, and the process whereby rules are applied to an inference, is construed as a non-formal process. We are taken to have the ability to “simply perceive” the way in which rules apply to specific cases, and we do not need to (in fact, cannot) make such application explicit by means of language.

Russell's account undergoes changes between *Principles* (1903) and *Principia* (1910), but

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<sup>30</sup> It is unclear, however, whether Bradley ever posed the problem in the way Russell poses it. In the contemporary literature on the topic, the problem Russell attributes to Bradley is sometimes lumped together with the problem that Russell attributes to Carroll, and both are commonly thought to stem from Carroll. Perhaps this conflation goes back to Russell, who in #38 attributes both problems to Carroll.

<sup>31</sup> *Principles*, #45, p. 41.

the overall shape remains constant. In the 1906 paper “Theory of Implication”,<sup>32</sup> Russell introduces an explicit rule of substitution into his system. But treating substitution as an official rule does not make the Bradley problem disappear — it only makes it more apparent, and that is perhaps the reason why Russell eventually abandons this rule in *Principia*. In *Principia* Russell notices that the problem with the application (by instantiation) of general propositions affects any appeal to axioms in proofs. The problem cannot be solved by introducing an explicit rule of substitution, since the application of such a rule would give rise to the same sort of regress. Instead, Russell suggests we must have the ability to make two different kinds of uses of any axiom. Axioms can be used as premises, or they can be applied as rules for inferring — rules according to which we justify conclusions. In the latter use, they can be immediately instantiated (by the premises and the conclusion) and justify the assertion of conclusions. In this account, the rules of inference are internalized within the “rulish” use of the axioms. Such rulish use is not expressible in the formalism, and would not result in adding premises to the inference; a regress such as Carroll’s or Bradley’s would not arise. This is not, to my mind, a radically different approach to rules from the one Russell proposed in *Principles*; it shares with it the two fundamental ideas that a rule of inference justifies only because it has the relevant logical content, and that it justifies in a non-formal way that avoids the threat of regress.

### **3.2.1. *Principles of Mathematics* (1): ‘Imply’ and ‘Therefore’**

Russell’s point of departure is the thought that a formal language should be able to give full expression to all of logic and that logical relations between propositions are not different in kind

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<sup>32</sup> Russell 1906.

from any other relation. The realization that the rule of inference and the logical relations it captures cannot be expressed in the formal system requires a modification of this initial picture:

A true hypothesis in an implication may be dropped, and the consequent asserted. This is a principle incapable of formal symbolic statement, and illustrating the essential limitation of formalism — a point to which I shall return at a later stage.<sup>33</sup>

There is only one rule of inference in *Principles*, namely *modus ponens* — the rule that allows us to ‘drop’ the antecedent of a conditional premise ( $p \supset q$ ), and to assert its consequent ( $q$ ), if we know that  $p$  is true. Russell’s reasons for holding that the rule of inference reveals the “essential limitation of the formalism” become explicit in the context of his discussion of Carroll’s challenge:

The independence of this principle is brought out by a consideration of Lewis Carroll’s “What the Tortoise Said to Achilles”. The principles of inference which we accepted led to the proposition that, if  $p$  and  $q$  be propositions, then  $p$  together with “ $p$  implies  $q$ ” implies  $q$ . At first sight, it might be thought that this would enable us to assert  $q$  provided  $p$  is true and implies  $q$ . But the puzzle in question shows that this is not the case, and that, until we have some new principle, we shall only be led into an endless regress of more and more complicated implications, without ever arriving at the assertion of  $q$ . We need, in fact, the notion of *therefore*, which is quite different from the notion of *implies*, and holds between different entities. In grammar, the distinction is that between a verb and a verbal noun, between, say, “ $A$  is greater than  $B$ ” and “ $A$ ’s being greater than  $B$ ”. In the first of these, a proposition is actually asserted, whereas in the second it is merely considered.<sup>34</sup>

Russell’s rule of inference differs from Carroll’s Hypothetical both in terms of its generality and in terms of its form — Russell’s statement of the rule employs propositional variables, and the rule is meant to be valid for any inference whose premises and conclusion have the same form, whereas Carroll’s Hypothetical is concerned with particular sentences, and is indifferent to their logical form. Nonetheless Russell takes Carroll to succeed in locating a problem that affects his

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<sup>33</sup> *Principles*, #18 p. 16. The promise to further discuss the non-formal character of the rule “at a later stage” must be taken to point ahead to the discussion of Carroll’s puzzle at section #38.

<sup>34</sup> *Principles*, #38 pp. 34-35.

own rule as well. If the content of the rule of inference is not distinguished from the content of the conditional proposition that “ $p$  together with ‘ $p$  implies  $q$ ’ implies  $q$ ”, the rule could never be enough to “enable us to assert” a conclusion. A conditional proposition can only count as a premise in an inference, but it cannot effect the assertion of a conclusion. But what the Tortoise ignores, according to Russell, are three issues:

1. The distinction between the relation denoted by ‘therefore’ and that denoted by ‘imply’.
2. The distinction between asserted and unasserted propositions.
3. The informal character of the rule.

The three points are internally related. What prevents us from clearly distinguishing the rule of inference from ordinary conditional propositions has to do with the way in which the distinction between the *therefore* relation and the relation of *imply* is covered up by the ambiguity of the way we often use the latter term. Russell himself often uses ‘imply’ ambiguously own use of this term, as we shall see below; in fact, he chooses to reduplicate this ambiguity when he decides to use the horseshoe sign to indicate both the sentential connective of implication and the *therefore* relation.<sup>35</sup>

Nonetheless, Russell explicitly notes the following difference:

- ‘imply’ is used as a sentential connective that relates two unasserted propositions (in modern terms, a material implication or conditional); an implication is true even when the antecedent is false.
- ‘therefore’ expresses a relation which holds between asserted premises and an asserted conclusion in a valid inference (in modern terms, it is a relation of *entailment* that holds between true propositions).

Russell’s initially draws a very sharp distinction: each relation holds between different kinds of

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<sup>35</sup> For the latter use, see Russell 1906, p. 165 and *Principia*, p. 9 and p. 106. I discuss this use of the horseshoe sign in the next subsections. Kremer 2014 suggests that this ambiguity is the cause of the philosophical confusion from which, according to Wittgenstein, Russell suffers. I discuss Kremer’s claims in section 3.4., below.

entities. A rule of inference, on this account, concerns the relation between asserted propositions, such as the ones we find in the premises and the conclusion of an inference. A proposition involving implication would immediately show itself to be inadequate to the task of justifying inferences, since what it says is plainly about different entities (i.e. unasserted propositions) than those that appear in the inference.

But this initial suggestion is too extreme. It drives a wedge between the two relations, and this makes it impossible to see how inference could ever lead us from an unasserted occurrence of a proposition (that which appears as the consequent of the conditional premise) to its asserted occurrence (when it appears independently, as the conclusion).<sup>36</sup> Russell himself realizes this, and he thereby stumbles upon the idea which Peter Geach called ‘Frege’s point’ about assertion.<sup>37</sup>

Here is how Russell frames his version of Frege’s point:

The question is: How does a proposition differ by being actually true from what it would be as an entity if it were not true? It is plain that true and false propositions alike are entities of a kind, but that true propositions have a quality not belonging to false ones, a quality which, in a non-psychological sense, may be called being asserted. Yet there are grave difficulties in forming a consistent theory on this point, for if assertion in any way changed a proposition, no proposition which can possibly in any context be unasserted could be true, since when asserted it would become a different proposition. But this is plainly false; for in “ $p$  implies  $q$ ”  $p$  and  $q$  are not asserted, and yet they may be true.<sup>38</sup>

To avoid the undesirable consequence — obscuring the relation between the premise (in which  $q$  is implied by  $p$ , but is unasserted) and the conclusion (where  $q$  is asserted), Russell revises his initial suggestion. The two relations should still be distinct, though the same propositional

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<sup>36</sup> *Principles*, p. 35.

<sup>37</sup> Geach 1965; Frege 1984j.

<sup>38</sup> *Principles*, #38, p. 35.

entities can enter them.<sup>39</sup> The difference between the relations consists in the fact that only asserted propositions enter the *therefore* relation. Russell does not have an account of the distinction between asserted and unasserted propositions; but assuming that such an account can be found, Russell believes that the distinction between implication and the *therefore* relation would be easy to specify:

Leaving this puzzle to logic, however, we must insist that there is a difference of some kind between an asserted and an unasserted proposition. When we say *therefore*, we state a relation which can only hold between asserted propositions, and which thus differs from implication. Wherever *therefore* occurs, the hypothesis may be dropped, and the conclusion asserted by itself. This seems to be the first step in answering Lewis Carroll's puzzle.<sup>40</sup>

Russell's evasiveness here is frustrating, but later in the book he attempts to remedy the lacuna in his account.<sup>41</sup> He does that in Appendix A of *Principles*, in which he reviews and criticizes Frege's philosophy.<sup>42</sup> Here Russell is much more careful about the difference between asserted and non-asserted propositions, and he refrains from claiming that it can be captured in terms of a property that propositions may have or lack. This is more in line with Frege's own way of

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<sup>39</sup> Referring to this initial suggestion, Dummett 1973 (p.304) thinks Russell "falls into confusion" since he lacks Frege's distinction between content and assertive force. But, as I argue below, Russell quickly becomes aware of the problem and offers a solution for it (which is made even more explicit in the Appendix, on p. 505).

<sup>40</sup> Ibid.

<sup>41</sup> Russell's excuse seems to be that as a mathematician, he should not be troubled by such issues. On #52, p. 49, Russell makes the similarly frustrating excuse for not offering a clearer verdict on the matter of assertion:

But there is another sense of assertion, very difficult to bring clearly before the mind, and yet quite undeniable, in which only true propositions are asserted. True and false propositions alike are in some sense entities, and are in some sense capable of being logical subjects; but when a proposition happens to be true, it has a farther quality, over and above that which it shares with false propositions, and it is this further quality which is what I mean by assertion in a logical as opposed to a psychological sense. The nature of truth, however, belongs no more to the principles of mathematics than to the principles of everything else. I therefore leave this question to the logicians with the above brief indication of a difficulty.

The irresolute attitude Russell takes in both cases stands in sharp contrast to Frege's and Wittgenstein's attitudes when confronted with manifestations of the logocentric predicament. On this see Conant 2002a.

<sup>42</sup> *Principles*, p. 505f.

thinking about assertion.<sup>43</sup> But Russell then makes the following objection to Frege's formulation of his "principle of inference":

Also a difficulty arises owing to the apparent fact, which may however be doubted, that an asserted proposition can never be part of another proposition: thus, if this be a fact, where any statement is made about  $p$  asserted, it is not really about  $p$  asserted, but only about the assertion of  $p$ . This difficulty becomes serious in the case of Frege's one and only principle of inference (Bs. p. 9) : " $p$  is true and  $p$  implies  $q$  ; therefore  $q$  is true\*." Here it is quite essential that there should be three actual assertions, otherwise the assertion of propositions deduced from asserted premises would be impossible; yet the three assertions together form one proposition, whose unity is shown by the word *therefore*, without which  $q$  would not have been deduced, but would have been asserted as a fresh premiss.<sup>44</sup>

If, as Frege holds, one cannot embed asserted propositions within other propositions, and if, moreover, the expression of the rule of inference is to count as a proper proposition (though by Frege's own lights, that is doubtful), then Frege's rule of inference cannot count as asserting any fact about asserted propositions. But if that is so, Russell continues, Frege's rule would not be able to justify inferences (which involve the *therefore* relation). The pseudo-fregean expression of the rule that Russell criticizes would merely be an assertion of implication. This would open the door, once again, to the Tortoise's objection.

Russell's solution to this apparent problem is to hold on to the idea that the rule of inference either is or has the content of a proposition, which asserts the *therefore* relation that holds between assertions, but to modify his conception of assertion in a way that would make it more wieldy and allow him to embed asserted propositions in the expression of the rule. All true propositions, Russell now proposes, are anywhere and always asserted, even when they are

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<sup>43</sup> "Thus no concept can be found which is equivalent to  $p$  asserted, and therefore assertion is not a constituent in  $p$  asserted. Yet assertion is not a term to which  $p$ , when asserted, has an external relation; for any such relation would need to be itself asserted in order to yield what we want." (*Principles*, p. 504).

The argument that underlies Russell's claim seems to be similar to the one Frege makes concerning the treatment of truth as a property. See my discussion of that argument in Chapter 1, section 1.4.

<sup>44</sup> *Principles*, #478, p. 504.

embedded:

We might then conclude that true propositions, even when they occur as parts of others, are always and essentially asserted, while false propositions are always unasserted, thus escaping the difficulty about *therefore* discussed above.<sup>45</sup>

Assuming that all true propositions are asserted, he thinks, makes it possible to explicitly frame propositions and propositional functions that assert the *therefore* relation which is what the rule of inference putatively does.<sup>46</sup> And this is an important step towards avoiding the regress that Carroll identified. For the rule can now be seen as justifying the assertion of the conclusion.

Before we move on to discuss why such a rule must remain inexpressible, let us take stock. The inexpressibility of the rule of inference within the formal system that it governs is the inexpressibility of a *sui generis* kind of content. Rules concern the *therefore* relation, which holds between asserted propositions. Denying that they can be expressed is denying that they can be expressed as implications.

### **3.2.2. *Principles of Mathematics* (2): The Problem of Instantiation**

Given Russell's distinction between the content of conditional propositions and the content of the rule of inference, the question immediately arises: if the rule has the content and unity of a relational proposition, how does our appeal to the rule differ from our appeal to any of the explicit premises of the inference? Having replaced Carroll's particular Hypothetical with Russell's general rule, and having distinguished the implication asserted in the premises from the

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<sup>45</sup> *Principles* #479, p. 505.

<sup>46</sup> One of the formulations of Russell's rule of inference enjoins us to "drop" the hypothesis in an implication when its antecedent is known to be true. This instruction seems less amenable to formalization as a proposition asserting the *therefore* relation in the way suggested here.

*therefore* relation that is indicated by the rule, wouldn't our application of such the rule to our inference still require some additional inferential reasoning? To use a term I introduced in order to make sense of Carroll's puzzle: isn't Russell's rule just as *inert* as any of the Tortoise's premises? If so, we would find ourselves in a regress again. Only this time, the regress would not start from the need to justify the relation between the premises and the conclusion without adding premises to the inference; it would start from the need to justify the applicability of the rules to the particular inference.

Russell's articulation of this worry emerges through his engagement with an argument he finds in Bradley, that casts doubt on the assumption that a general principle of inference can ever justify the endorsement of its particular instances.<sup>47</sup> Like Carroll's argument, Bradley's argument has the structure of a regress. It takes its start from a reflection on the inferential act through which we instantiate any general proposition. Consider the following inference:

- (1) All men are mortal
- (2) Socrates is a Man
- (3) Socrates is Mortal

The form of this inference (often called *universal modus ponens*) can easily be reduced to a conventional *modus ponens* inference, since the first premise can be understood to mean "if anything is a man, then it is mortal", and this can then be instantiated as

- (1\*) If Socrates is a man then Socrates is Mortal.<sup>48</sup>

We can then easily infer (3) from (1\*) and (2). In *Principles* Russell does not require that we

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<sup>47</sup> Russell 1903 (p. 41) refers us to Bradley 1883, p. 227 as the origin of this argument, but it is not clear that such an argument can be found there.

Mander 2008 (p. 704) asserts that Bradley's point is identical to Carroll's, but as my discussion shows, there are important differences.

<sup>48</sup> In modern notation, we instantiate (1)  $\forall x(\phi(x) \supset \psi(x))$  to get (\*1)  $\phi(a) \supset \psi(a)$ .

make this intermediate step explicit — he takes his single rule of inference to authorize such inferences as well. The single rule of inference of *Principles* is thus taken to do two things at once — both to justify the assertion of the conclusion and to allow the instantiation of general propositions:

Another form in which the principle is constantly employed is the substitution of a constant, satisfying the hypothesis, in the consequent of a formal implication. If  $\phi x$  implies  $\psi x$  for all values of  $x$ , and if  $a$  is a constant satisfying  $\phi x$ , we can assert  $\psi a$ , dropping the true hypothesis  $\phi a$ .<sup>49</sup>

The lack of distinction between instantiation and inference according to modus ponens might reflect the fact that in *Principles*, quantification is bound up with implication. Russell has primitive sentential connective he calls ‘formal implication’, which he borrows from Peano. A formal implication forms a proposition involving a universal generalization; for example,  $Fx \supset_x Gx$  means that any  $x$  which is an  $F$  is also a  $G$ . This explains why Russell thinks that his single rule of inference can also take care of instantiation. Instantiation can be thought of as an inference from a general premise,  $\phi x \supset_x \psi x$  taken together with a particular premise,  $\phi a$ , to a particular conclusion  $\psi a$ .<sup>50</sup>

Instantiation is required whenever we seek to apply general content — such as the content of axioms — for the purpose of grounding a specific proof:

This [procedure of substitution] occurs, for example, whenever any of those rules of inference which employ the hypothesis that the variables involved are propositions, are applied to particular propositions. The principle in question is, therefore, quite vital to any kind of demonstration.<sup>51</sup>

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<sup>49</sup> *Principles*, p. 34.

<sup>50</sup> Russell later realizes that there are two separate procedures involved, and he proposes (in the 1906 “Theory of Implication”, to be discussed below) to capture these in two separate rules of inference. In *Principia* (1910) he ceases to employ a rule of substitution, and prefers to keep instantiation implicit.

<sup>51</sup> *Principles* #38, pp. 34-5.

The axioms of Russell's system — here referred to as the “rules of inference” — involve free and bound variables that range over propositions and other terms.<sup>52</sup> The application of such axioms to any particular context must be made by means of the substitution of the variable letters with specific propositional signs. Take, for instance, the 7th axiom, called *Importation*: “If  $q$  implies  $q$  and  $r$  implies  $r$ , and if  $p$  implies that  $q$  implies  $r$ , then  $pq$  implies  $r$ .”<sup>53</sup> Suppose we have to prove  $F \& G \supset H$  and our premises are  $G \supset G$ ,  $H \supset H$  and  $F \supset G \supset H$ . We may add *Importation* as a premise, but that would not yet be enough to justify our conclusion — *Importation* first needs to be instantiated in the relevant way, i.e.  $p$ ,  $q$  and  $r$  are to be replaced with  $F$ ,  $G$  and  $H$  respectively.

Bradley's puzzle pushes Russell to recognize that such instantiation has to be treated as a separate step that requires its own justification, i.e. that the rule of inference itself is general in its content, and that it, too, needs to be instantiated if it is to apply to any concrete proofs:

In order to apply a rule of inference, it is formally necessary to have a premiss asserting that the present case is an instance of the rule; we shall then need to affirm the rule by which we can go from the rule to an instance, and also to affirm that here we have an instance of this rule, and so on into an endless process.<sup>54</sup>

To be clear, the pressing question raised here by Russell is not so much whether the rule actually

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<sup>52</sup> *Principles* #18, p. 16. To guarantee that the axioms pick out the properties of propositions only, by prefixing a “hypothesis” to the axioms that is only true for propositions, e.g. the hypotheses  $p \supset p$  and  $q \supset q$  are prefixed to them. E.g. the form of axiom (5) is (5),  $p.q \supset q$ . In this quote from #38, axioms (1)-(4) are distinguished from axioms (5)-(10) in that the latter employ such a hypothesis explicitly, whereas the former do not.

The first four axioms include the rule of inference as no. (4); the first three do not seem to be of any use in actual proofs, and may be seen as the rules of construction of propositions — what they state is what counts as a proposition.

<sup>53</sup> In Russell's notation, the conjunction of  $p$  and  $q$  is simply marked  $pq$ .

<sup>54</sup> *Principles*, p. 41. Perhaps a sign that Russell does not think of the looming regress as vicious can be detected in his calling it “an endless process” — According to Russell, not all endless processes are vicious (cf. Russell 1903, p. 50f). But he still seeks to avoid it, and this makes sense since his account of logic aims to capture the way in which we actually reason — where no endless processes have a place. Wisdom (1974) makes a similar point about Carroll's regress.

justifies the inference. Rather, the pressing question is whether we are ever entitled in taking it to apply to the particular inference. For if, in order to see that the general rule applies to a specific case we would need to appeal to another general rule, and another after that, how could this endless procedure ever justify our inferences? Russell concludes that the mediation between the rule and the inference must not be construed in a way that requires further inferential reasoning. This realization leads him to shift perspectives, from his preliminary concern with the justificatory content of the rule of inference to a concern with the nature of the act through which the rule is applied:

We may observe, however, that, in a particular inference, the rule according to which the inference proceeds is not required as a premiss. This point has been emphasized by Mr Bradley; it is closely connected with the principle of dropping a true premiss, being again a respect in which formalism breaks down. In order to apply a rule of inference, it is formally necessary to have a premiss asserting that the present case is an instance of the rule; we shall then need to affirm the rule by which we can go from the rule to an instance, and also to affirm that here we have an instance of this rule, and so on into an endless process. ... The fact is, of course, that any implication warranted by a rule of inference does actually hold, and is not merely implied by the rule. This is simply an instance of the non-formal principle of dropping a true premiss: if our rule implies a certain implication, the rule may be dropped and the implication asserted. But it remains the case that the fact that our rule does imply the said implication, if introduced at all, *must be simply perceived, and is not guaranteed by any formal deduction*; and often it is just as easy, and consequently just as legitimate, to perceive immediately the implication in question as to perceive that it is implied by one or more of the rules of inference.<sup>55</sup>

The first thing to note about this passage is the ambiguity of the term ‘imply’. Unless we read ‘imply’ and ‘implication’ in some of their appearances as referring to the *therefore* relation, making sense of what Russell says becomes impossible. And even so Russell’s point is obscure. But at least the following four points seem to come out with sufficient clarity. First, the rule of inference is taken to play a justificatory role — it warrants the inference — and it does that by virtue of its content. It is a *fact* that the rule “implies” the “implication” (that is, the inference) —

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<sup>55</sup> *Principles*, #45 p. 41 (emphasis added).

in fact, the rule implies (in the *therefore* sense), whether the relation is mentioned or not.

Second, the rule is construed as general in content, and hence as playing the role of a premise in a super-inference (“the implication... is implied by one or more of the rules”) which is being instantiated to apply to the rule. In this super-inference, the specific inference is thought of as a conclusion that is implied by the rule, first in the sense of ‘imply’ and then in the sense of the *therefore* relation; the process Russell describes can perhaps be put in the following (loose) way:

$$\frac{\text{rule of inference} \supset \text{specific inference}}{\text{specific inference}}$$

In Russell’s manner of speaking, the inference is first thought of as the sub-proposition that is implied by the rule of inference (in the first premise), and then (in the conclusion) as an independent act that holds independently of the rule. Russell equates the manner in which rules justify inferences with the manner in which propositions justify others: both are related in the *therefore* relation.

Third, Russell argues that in order to avoid the regress we must acknowledge that the act of applying the rule — a process which he himself pictures as quasi-inferential in form — is radically different from the act through which premises are appealed to in normal inferences. This act must stem from a capacity to ‘perceive’ the applicability of rules in an unmediated way. Such a capacity is independent from, and irreducible to the capacities that we draw on when we state premises (it is “not guaranteed by any formal deduction”).

Fourth, Russell goes on to claim that given that the same capacity to perceive the application of the rule allows us to perceive basic inferences “immediately”, without appeal to

the rules of inference.<sup>56</sup>

To conclude my discussion of Russell's *Principles*: despite the lack of a clear terminological distinction between the rule of inference and the other axioms, and although he conceives the rule as contentful, Russell distinguishes rules from propositions and prevents Carroll's regress from arising. The initial obstacle to thinking that the rule has content — that it asserts a relation between assertions — is cleared by the domestication of assertion. But since the rule is general in content, and the inferences it justifies are specific, it seems that the instantiation of the rule itself requires justification. More worrying still, instantiation seems to depend on the principle of “dropping” a premise, and the application of that principle seems to depend on instantiation, and so on ad infinitum. Regress is circumvented, however, because it is in fact possible to perceive basic inferential relations directly, and thus to apply the rule of inference immediately — even if in principle, something like inferential reasoning implicitly underlies every application of every rule.

In sum, Russell responds to Carroll's challenge by introducing rules of inference, whose application does not merely consist in an appeal to inert content. It is an act of logical perception, which is supposed to put a stop to all regress and effect the assertion of the conclusion.

### **3.2.3. “Theory of Implication”**

The changes in Russell's approach to rules of inference after *Principles* stem primarily from a

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<sup>56</sup> This idea of a capacity to grasp the *therefore* relation immediately is something Wittgenstein could in principle agree with (See Tr. 5.131-5.132, and the discussion in Chapter 6). But Wittgenstein generalizes the point beyond basic inferences, and that might be something Russell would not agree with. On Wittgenstein's account, we must be capable of perceiving the internal *therefore* relation between all the propositions that we understand, and once we perceive them, we no longer have a need to appeal to any general rules in order to justify the inference.

reconsideration of instantiation. This becomes an explicit theme in the 1906 paper “Theory of Implication”, which was originally intended to serve as Part 1 of *Principia Mathematica*.<sup>57</sup> Russell introduces an explicit “principle of substitution”, and this allows him to give a new formulation to Bradley’s puzzle.

“Theory of Implication” retains two of the conclusions reached in *Principles* — the idea that the content of rules is unique, asserting the *therefore* relation between asserted propositions, and the idea that the appeal to the rule is non-formal:

When this principle is used, we write

“ $\vdash p. \supset \vdash q$ ”

meaning “p, which is true, implies q, which is therefore true”, or “p is true; therefore q is true”. *Therefore* is distinguished from *implies* by being only applicable to implications between *true* propositions.<sup>58</sup>

One important innovation which appears here (and is retained in *Principia*) is the use of the horseshoe sign to indicate the *therefore* relation. Later, in *Principia*, Russell will explain that the expression “ $\vdash p. \supset \vdash q$ ” abbreviates an inference involving two premises —  $p$  and  $p \supset q$ , and the conclusion  $q$ .<sup>59</sup> But he continues to speak, as he does here, of this expression as meaning “ $p$  is true, *therefore*  $q$  is true”.<sup>60</sup> Russell uses the horseshoe sign ambiguously, to denote two kinds of relations, even though, given the context — the assertion signs that appear in the propositions that flank it — it is possible to tell which relation is being referred to.

In the 1906 paper a new type of argument is offered for the need to recognize the non-

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<sup>57</sup> Russell 1906 stems from a period in which Russell was trying out the so-called “substitutional theory”, which he eventually dropped in *Principia*. We do not need to get into the details of that theory here. See Hylton, “Russell’s Substitutional Theory”.

<sup>58</sup> Russell 1906, p. 165.

<sup>59</sup> *Principia*, p. 9.

<sup>60</sup> *Principia*, p. 92

formal status of the rule:

We cannot express the principle symbolically, partly because any symbolism in which  $p$  is variable only gives the *hypothesis* that  $p$  is true, not the *fact* that it is true.<sup>61</sup>

There is no precedent in *Principles* for the idea that the symbolic expression of rules fails because of how variables operate — that variables render propositions that employ them “hypothetical”. Perhaps the thought stems from Russell’s reconsideration of the idea that quantification and implication are bound together, which is indeed the case in *Principles*, where Russell employs Peano’s formal implication as a primitive connective.<sup>62</sup> But by 1906, Russell abandons the use of formal implication and is quite capable of accounting for bound variables apart from the conditional. To judge by the other things Russell says about the issue, it seems more likely that it is his reflection on Bradley’s puzzle of instantiation that leads him to make this cryptic statement about the effect of the use of variables.

Bradley’s puzzle is clearly at issue when Russell’s introduces the second primitive proposition of “Theory of Implication”, the new rule of substitution:

★2 · 2 If a propositional function ( $C \llbracket y$ ) is true for any value of  $y$ , it is true for such-and-such a value. Pp.<sup>63</sup>

This principle will be called the “principle of substitution”, and will be referred to as “Subst.” It allows us, for example, from “For any value of  $y$ , if  $y$  is a man,  $y$  is a mortal”, to infer “if Socrates is a man, Socrates is a mortal”. The value substituted is supposed to be a definite constant value, such as Socrates, or the fourth proposition of Euclid. This principle, like its predecessor, cannot be symbolized, because any possible symbolic expression would deal with any constant, and we should need the principle itself to deduce, from such a statement, that it holds also concerning Socrates or some other definite constant. “Any constant” is, in fact, not distinguishable from a variable: it

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<sup>61</sup> Russell 1906, p. 164. This passage is repeated, almost verbatim, in *Principia* (p. 94), and I will discuss it in more detail in the next subsection.

<sup>62</sup> See the discussion in the previous section (3.2.2)

<sup>63</sup> The sign  $\llbracket$  is not the one Russell originally uses for propositional functions; I am unable to reproduce Russell’s sign here.

is like what is called a parameter, i.e. a variable which we choose to regard as a constant.<sup>64</sup>

Recall the formulation of the puzzle associated with Bradley in *Principles*: “In order to apply a rule of inference, it is formally necessary to have a premiss asserting that the present case is an instance of the rule”.<sup>65</sup> The 1906 formulation of the problem, by contrast, is made in terms of the way in which variables operate alongside the principle of substitution.

The principle of substitution allows Russell to make Bradley’s problem more determinate in form, but it does not, on its own, solve the puzzle. Consider Russell’s example, where the rule of substitution is supposed to license the transition from “For any value of  $y$ , if  $y$  is a man,  $y$  is a mortal”, to “if Socrates is a man, Socrates is a mortal”. The rule of substitution tells us that the variable  $y$  may be replaced by the constant name ‘Socrates’ — or rather, that the particular claim about Socrates is an instance of the general claim. The rule of substitution itself, however, does not mention the individual Socrates — it can only tell us that  $y$  may be replaced by *any* constant. Any attempt to formulate the rule of substitution explicitly would result in a general statement that itself requires instantiation. Thus its instantiation could only give rise to a regress: we would need a rule of substitution to apply any rule of substitution to the instance under consideration. The regress can only be avoided by means of a “extra-formal” application of the rule.

The rule of substitution proposed in 1906 is not retained in *Principia*, and we can only speculate as to the reasons why Russell abandons it.<sup>66</sup> Perhaps Russell realizes that the explicit

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<sup>64</sup> Russell 1906, p. 165. This passage does not have a parallel in *Principia*; in fact, the rule of substitution is completely absent from the later work.

<sup>65</sup> *Principles*, #45, pp. 41.

<sup>66</sup> In Russell 1906 Russell was trying out a more wide-ranging “substitutional theory”, which he eventually dropped, before turning to write *Principia*. We do not need to get into the details of that theory here, except insofar as it has to do with the introduction of the rule of substitution. For discussion, see Hylton 1980.

application of the rule of substitution gives rise to the same circularity that it was designed to solve. But, as I argue below, Russell not only learns to live with the fact that instantiation of logical laws by means of substitution is something which cannot be formalized; this insight allows him to reconfigure his understanding of the relation between axioms and rules of inference.

### 3.2.4. *Principia Mathematica*

In the account of rules of inference given in *Principia* a major role is still played by the distinction between the relation of implication and the *therefore* relation; Russell continues to treat the rule as contentful; and emphasis is still put on the non-formality and immediacy of the application of rules. But there are also important shifts in the account of the nature of axioms and of rules. As Russell becomes aware of the ubiquity of the need for non-formal acts of instantiation, he is led to see that the same capacity that is needed to perceive the applicability of rules can be applied to any logical proposition, which is thereby transformed into a rule that guides and justifies inferences.

Here is Russell's initial characterization of inference:

*Inference.* The process of inference is as follows: a proposition “ $p$ ” is asserted, and a proposition “ $p$  implies  $q$ ” is asserted, and then as a sequel the proposition “ $q$ ” is asserted. The trust in inference is the belief that if the two former assertions are not in error, the final assertion is not in error. Accordingly whenever, in symbols, where  $p$  and  $q$  have of course special determinations,

“ $\vdash p$ ” and “ $\vdash (p \supset q)$ ”

have occurred, then “ $\vdash q$ ” will occur if it is desired to put it on record. The process of the inference cannot be reduced to symbols. Its sole record is the occurrence of “ $\vdash q$ .”

... An inference is the dropping of a true premiss; it is the dissolution of an implication.<sup>67</sup>

Like Frege, who claimed that inference “does not belong in the realm of signs”,<sup>68</sup> Russell points out that inference “cannot be reduced to symbols”. Conducting an inference within a formal system is not reflected in any contentful expression; it is only evident from the fact that we assert the conclusion after we asserted the premises.<sup>69</sup>

Section A of Part 1 of *Principia*, which contains materials that were already published in “Theory of Implication”, is titled “Theory of Deduction”, rather than “Theory of Implication”. This might indicate that Russell attempts to avoid the ambiguity of ‘implication’, but as we shall see, in another sense he only deepens his entanglement. In introducing the rule of inference as proposition \*1·1 of *Principia*, Russell still contrasts the rule with propositions that assert the implication of the conclusion by the premises — thus differentiating the rule from the Tortoise’s Hypothetical Propositions:

\*1 · 1. Anything implied by a true elementary proposition is true. Pp.

... It is not the same as “if  $p$  is true, then if  $p$  implies  $q$ ,  $q$  is true.” This is a true proposition, but it holds equally when  $p$  is not true and when  $p$  does not imply  $q$ . It does not, like the principle we are concerned with, enable us to assert  $q$  simply, without any hypothesis.<sup>70</sup>

The rule is here differentiated from the formally expressible implication  $p \supset q \supset q$ . It differs from it in content, since the rule connects assertions (this is covered up by the fact that Russell

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<sup>67</sup> *Principia*, p. 8-9.

<sup>68</sup> FG2, p. 318.

<sup>69</sup> I have offered a similar interpretation of Frege’s idea of inference in Chapter 1, section 1.5. This is also an idea that will be taken up by Wittgenstein; it will lead him to reject the thought that rules of inference have a substantive role to play in the justification of inferences

<sup>70</sup> *Ibid.*, p. 94.

uses the ambiguous ‘imply’ in the statement of the rule, to express the *therefore*). Moreover, Russell continues to also distinguish the rule from the axioms in terms of its expressibility. He frames this issue in the same he used in “Theory of Implication”, tying the threat of the Carroll regress to the threat of the Bradley regress:

We cannot express the principle symbolically, partly because any symbolism in which  $p$  is variable only gives the *hypothesis* that  $p$  is true, not the *fact* that it is true.<sup>71</sup>

In “Theory of Implication”, the explicit principle of substitution allowed Russell to articulate the regress in terms of the application of a rule (which itself stands in need of a justification). But the appeal to a rule of substitution itself could not put a stop to the regress; so it is not surprising that here, in *Principia*, Russell gives up the detour of rules of substitution.<sup>72</sup>

Russell still acknowledges the distinction between implication and the *therefore* relation, but he chooses to express both relations by means of the same sign (the horseshoe, ‘ $\supset$ ’), and it is only from the context — from the presence of assertion signs within the expression of the consequent and the antecedent — that we can tell which relation is intended:

Thus “ $\vdash : p \supset q$ ” means “it is true that  $p$  implies  $q$ ” whereas “ $\vdash . p \supset \vdash . q$ ” means “ $p$  is true; therefore  $q$  is true.” The first of these does not necessarily involve the truth either of  $p$  or of  $q$ , while the second involves the truth of both.<sup>73</sup>

Earlier in the book, Russell explains that the expression “ $\vdash . p \supset \vdash . q$ ” *abbreviates* an inference. It expresses three propositions — two premises,  $p$  and  $p \supset q$ , and the conclusion  $q$ .<sup>74</sup> But here he says the expression means “ $p$  is true, *therefore*  $q$  is true”, i.e. it is an expression of the relation of

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<sup>71</sup> Ibid. An identical passage is found in Russell 1906, p. 164, discussed in section 3.2.3 above.

<sup>72</sup> But Russell does mark the substitutions he makes, in marginal, parenthetical notes. See *Principia*, p. 98.

<sup>73</sup> *Principia*, p. 92. A footnote to this paragraph refers the reader to *Principles*, #38.

<sup>74</sup> *Principia*, p. 9.

*therefore*.

The reason why Russell does not distinguish the two relations more sharply might have to do with the fact that he takes the two relations to be internally related notions. This is evident in his definition of implication:

*Definition of Implication.* When a proposition  $q$  follows from a proposition  $p$ , so that, if  $p$  is true,  $q$  must also be true, we say that  $p$  implies  $q$ .<sup>75</sup>

To paraphrase what Russell says here, we use ' $p$  implies  $q$ ' when from the truth of  $p$  we may infer the truth of  $q$ , i.e. when an asserted  $p$  stands in the *therefore* relation to an asserted  $q$ . The *therefore* relation is in some sense implied in the expression of implication; inferential role is part of what defines the meaning of the sentential connective:

The meaning to be given to implication in what follows may at first sight appear somewhat artificial; but although there are other legitimate meanings, the one here adopted is very much more convenient for our purposes than any of its rivals. The essential property that we require of implication is this : "What is implied by a true proposition is true." It is in virtue of this property that implication yields proofs. But this property by no means determines whether anything, and if so what, is implied by a false proposition. What it does determine is that, if  $p$  implies  $q$ , then it cannot be the case that  $p$  is true and  $q$  is false, i.e. it must be the case that either  $p$  is false or  $q$  is true. The most convenient interpretation of implication is to say, conversely, that if either  $p$  is false or  $q$  is true, then "implies" is to be true. Hence " $p$  implies  $q$ " is to be defined to mean: "Either  $p$  is false or  $q$  is true."<sup>76</sup>

Implication "yields proofs". It expresses the possibility of an inference of the consequent starting from the antecedent. The *therefore* relation, for its part, is a stricter, more fundamental relation which grounds the inference or proof that holds only where the propositions are asserted:

Now in order that one proposition may be inferred from another, it is necessary that the two should have that relation which makes the one a consequence of the other. When a proposition  $q$  is a consequence of a proposition  $p$ , we say that  $p$  implies  $q$ . Thus

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<sup>75</sup> Ibid., p. 94.

<sup>76</sup> Ibid. This passage draws the converse relation to the one under discussion in Frege's elucidation of the rule of inference in terms of the semantics of the conditional, which I discuss in section 2.2 of the previous chapter.

deduction depends upon the relation of *implication*, and every deductive system must contain among its premisses as many of the properties of implication as are necessary to legitimate the ordinary procedure of deduction.<sup>77</sup>

The *therefore* relation is what ultimately justifies and grounds our inferential practices. But Russell thinks that it is grounded in implication, and hence, since the “properties” of the latter are articulated by the axioms of the system, we can see the axioms as indirectly encoding inferential relations.

This signals a new way of thinking about the nature of the axioms and their relation to the rules of inference. Russell’s new idea is that all the explicit principles of logic — all of its axioms and theorems — can serve in two distinct roles. Logical Propositions are not merely general statements of facts, they also serve as non-formal rules, that guide us in inferences. In their role as rules, the propositions are self-instantiating and directly applicable to inferences, presumably by means of the same perception that is appealed to in the application of the rules of inference:

The use of a general principle of deduction, such as either form of “Syll”, in a proof, is different from the use of the particular premisses to which the principle of deduction is applied. The principle of deduction gives the general rule according to which the inference is made, but is not itself a premiss in the inference. If we treated it as a premiss, we should need either it or some other general rule to enable us to infer the desired conclusion, and thus we should gradually acquire an increasing accumulation of premisses without ever being able to make any inference. Thus when a general rule is adduced in drawing an inference, as when we write “[Syll]  $\vdash \cdot (1) \cdot (2) \cdot \supset \vdash \cdot \text{Prop.}$ ,” the mention of “Syll” is only required in order to remind the reader how the inference is drawn.

The rule of inference may, however, also occur as one of the ordinary premisses, that is to say, in the case of “Syll” for example, the proposition “ $p \supset q \cdot \supset \cdot q \supset r \cdot \supset \cdot p \supset r$ ” may be one of those to which our rules of deduction are applied, and it is then an ordinary premiss.<sup>78</sup>

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<sup>77</sup> *Principia*, p. 90.

<sup>78</sup> *Principia*., p. 106. Cf. p. 98: “The proofs of the earlier of the propositions of this number consist simply in noticing that they are instances of the general rules given in #1. In such cases, these rules are not premisses, since they assert any instance of themselves, not some thing other than their instances.”.

Russell widens the scope of the solution he offered for Carroll’s and Bradley’s puzzles in *Principles*. There he suggested that we can “simply perceive” the informal relation between the general rule and its instance. Here he talks in terms of two uses. By using axioms in one way, we cite premises; by using them in another way, we show what rules apply to inferences. In this second use, we transform the *content* of the axioms (which involves implication) into the content of rules of inference (which employs the *therefore* relation).<sup>79</sup>

This is what allows Russell to solve, in one stroke, both of the challenges posed by Carroll and Bradley. Russell now allows that all axioms can be used in a “rulish” way. This use of axioms as rules is still extra-formal, and does not count as altering the content of the inference, thus avoiding both the Carroll and the Bradley regress. But the non-formal appeal to the logical proposition is nonetheless an appeal to a determinate logical content. In taking an axiom to serve as a rule we convert one of the implications that the axiom asserts into the *therefore* relation. Consider Russell’s example, the theorem he calls “Syll”:

$$\vdash (p \supset q) \supset ((q \supset r) \supset (p \supset r))$$

Qua proposition, what Syll says is “if  $q$  follows from  $p$ , then if  $r$  follows from  $q$ ,  $r$  follows from  $p$ ”.<sup>80</sup> In this formulation, both ‘imply’ and ‘if... then...’ are to be taken to mean ordinary implication. But in order to treat it as a rule, we need to take the main connective, ‘if... then...’, to denote the relation of *therefore*. In other words, the theorem-turned-rule indicates that the assertion of the consequent follows from the assertion of the antecedent. Transforming *Syll* into a rule allows us to immediately draw the following inference:

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<sup>79</sup> In Chapter 2, section 2.5 I discuss Frege’s similar idea that rules may be transformed to laws and vice versa.

<sup>80</sup> *Principia*, p. 28.

$$\begin{array}{c}
(p \supset q) \\
\frac{(p \supset q) \supset ((q \supset r) \supset (p \supset r))}{(q \supset r) \supset (p \supset r)}.
\end{array}$$

Using Russell’s way of abbreviating inferences, we could perhaps capture the transformation of the theorem *Syll* into a form of inference in the following way:

$$\begin{array}{l}
\text{Syll} \quad \vdash (p \supset q) \supset ((q \supset r) \supset (p \supset r)) \\
[\text{Syll}] \quad \vdash (p \supset q) \supset \vdash ((q \supset r) \supset (p \supset r))
\end{array}$$

The main connective in the first expression case is implication, whereas in the second case, which on Russell’s official account is an abbreviation of an entire inference, the horseshoe sign can be taken to indicate the *therefore* relation.<sup>81</sup> And a similar transformation can be applied to any logical proposition: any axiom can be taken to encode both a relation of implication, which it explicitly asserts, and the *therefore* relation, which it conveys indirectly.<sup>82</sup>

This novel way of using axioms has a tremendous effect on the length of proofs. Here’s a comparison of two proofs, one using the method of “Theory of Implication”, and the other using the new method of proof of *Principia*:<sup>83</sup>

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<sup>81</sup> Russell explains this use of the horseshoe flanked by assertion signs as an abbreviation of the inference in *Principia* p.9.

<sup>82</sup> Transforming an axiom into a rule might involve various other types of transformation of sentential connectives to inferential structure, besides the one I discuss here. For instance it could require that we interpret conjunctions as indicating the assertion of two independent premises.

<sup>83</sup> I do not offer a comparison the way inference would be represented in *Principles* because it is hard to tell what its exact form should be; it is plausible to assume that it would resemble the inferences of "Theory of Implication", except for the fact that steps of substitution would not be made explicit in *Principles*

**“Theory of Implication”**

1. $\vdash : p \supset q, \supset : q \supset r, \supset : p \supset r$	Syll
2. $\vdash : F \supset G, \supset : G \supset H, \supset : F \supset H$	[Subst.] <sup>84</sup>
3. $F \supset G$	Premise
4. $G \supset H$	Premise
———	[m.p., 2, 3] <sup>85</sup>
5. $G \supset H, \supset : F \supset H$	
———	[m.p., 5,4]
6. $F \supset H$	

***Principia***

1. $F \supset G$	
2. $G \supset H$	
———	[Syll]
3. $F \supset H$	

*Principia* is obviously a more economic system. Indeed, as we can see in this comparison, the single use of [Syll] as a rule of inference in *Principia* replaces three applications of rules in “Theory of Substitution”. But the increase in the number of rules of inference comes, as Frege already predicted, at the price of perspicuity — one has to master a large number of complicated rules and be able to tell, by inspection of complex propositions, how they apply.<sup>86</sup>

Importantly, the two kinds of use of propositions receive different expressions in the notation. When used as a proposition, the axiom is either stated as an explicit premise or marginally mentioned by name or number, as which indicates the source of the missing premise. When it is used as a rule, however, Russell encloses the marginal note which specifies the rule within square brackets. I take this to indicate that the bracketed expressions do not form actual parts of the proof, or of its justification. So Russell is not contradicting his repeated claims that the rule cannot be expressed formally, and that its application must be simply perceived. The bracketed expressions do not count as giving explicit, formal expression to the rules of inference,

<sup>84</sup> This is not the precise manner of in which substitution is carried out in “Theory of Implication”, where a slightly more complicated symbolism is employed. Perhaps the two first steps I offer in my representation of the syllogism of “Theory of Implication” could be captured in a single premise (stating the substitution of the variables that appear in Syll with the particular propositions needed for this specific inference). But the proof would still be longer than the one that results from the method of *Principia*.

<sup>85</sup> Russell does not mark the use of *modus ponens*; I have chosen to make it explicit here for the sake of comparison.

<sup>86</sup> Frege himself chooses to increase the number of the rules of his system in order to shorten the length of proofs; compare *Grundgesetze* (p.26) with *Begriffsschrift* (p. 107).

because all they do is indicate something about the proof that we must be able to directly perceive — the way in which the inference instantiates the *therefore* relation that is implicit in the axiom.

Assuming that the reader will eventually find this procedure natural, and master the axioms and theorems of the system and their use as rules, Russell eventually ceases to use square brackets in these marginal notes, and thus ceases to mark the distinction between propositions used as premises and propositions used as rules.<sup>87</sup> This does not mean that Russell gives up on the distinction between rules and axioms; rather, it reflects his complete reliance on the reader's ability to 'simply perceive' the proper way in which the proposition is to be applied:

The distinction between the two uses of principles of deduction is of some philosophical importance, and in the above proofs we have indicated it by putting the rule of inference in square brackets. It is, however, practically inconvenient to continue to distinguish in the manner of the reference. We shall therefore henceforth both adduce ordinary premisses in square brackets where convenient, and adduce rules of inference, along with other propositions, in asserted premisses, i.e. we shall write e.g.

“ $\vdash .(1).(2).\text{Syll}.\supset\vdash .\text{Prop}$ ”

rather than “ $[\text{Syll}]\vdash .(1).(2).\supset\vdash .\text{Prop}$ ”<sup>88</sup>

The marginal citation of rules and of axioms used as rules is exchanged for an in-line use within the proof itself.

But why should Russell bother to state the rules at all, if what is actually happening is done by means of perceiving, not formally deducing? Russell's insistence to specify the rules (albeit in brackets) reflects the lingering thought that rules contribute content, by virtue of which the inferences are justified, and that if we fail to mention that content, the justification of the

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<sup>87</sup> *Principia*, p. 98n. & p. 106.

<sup>88</sup> *Principia*, p. 106.

inference would be incomplete.

This concludes my discussion of the developments in Russell's account of inference between *Principles* and *Principia*. Throughout this period, Russell construes the justification of inference in terms of the extra-formal appeal to the rule of inference, whose content differs in kind from the content of logical propositions, but is internally related to it. His discussions of inference reflect the gradual dawning of the insight, that in order to adequately account for inferential justification it cannot be enough to specify the content that justifies our inferences (whatever that content might be) — we must also consider the way in which this content is used. Rules as involving a *sui generis* application of logical content, which is not otherwise expressible. In *Principles*, he spells this out in terms of the mental capacity to perceive the applicability of a rule, distinguished from the capacity to grasp and entertain propositions; In *Principia* he distinguishes the use of propositions as premises and the use of propositions to evoke the rules that apply to the inference. In both cases, the idea is that the application of the rule does not give rise to further inferential reasoning — though in both cases it involves an inexpressible appeal to logical content.

I have so far left out of my account one text which might seem to indict Russell for failing to respond to Carroll's challenge. In his popular book, *The Problems of Philosophy* (1912) Russell speaks about our knowledge of a "general logical principle" whose "concrete instances" are involved in all argument. It is the "indubitable knowledge" of this principle which makes proofs possible:

[A trivial valid argument] is not hard to follow; and if it is granted that its premisses are true in fact, no one will deny that the conclusion must also be true. But it depends for its truth upon an instance of a general logical principle. The logical principle is as follows: 'Suppose it known that if this is true, then that is true. Suppose it also known that this is true, then it follows that that is true.' ... In other words, 'anything implied by a true proposition is true', or 'whatever follows from a true proposition is true'.

This principle is really involved—at least, concrete instances of it are involved—in all demonstrations. ...If any one asks: ‘Why should I accept the results of valid arguments based on true premisses?’ we can only answer by appealing to our principle. In fact, the truth of the principle is impossible to doubt, and its obviousness is so great that at first sight it seems almost trivial.<sup>89</sup>

It seems that here Russell effaces all distinction between the rule of inference and the axioms, and fails to note any difference between inference and implication. Indeed, he seems to entertain a scenario in which an appeal to this general principle would be an adequate reply to someone who — like the Tortoise — fails to admit the force of sound arguments to compel belief. Perhaps the explanation for the naivety of Russell’s handling of the issues here is that his writing does not address the logician or the philosopher, but a more general public. Another possibility is that Russell is assuming the account of *Principia*, which allows us to cite an axiom in order to indirectly refer to the rule to which it can be transformed; this could justify him in saying (quite ambiguously) that the appeal to the principle of inference is an appeal to an indubitable truth, an axiom. In any case, the fact that here Russell fails to preempt Carroll’s puzzle does not seem to me to detract from his explicit attempts to address it in his other major writings of the period. I take those attempts to form an adequate response to Carroll’s challenge.

### **3.3. Russell vs. Wittgenstein on Logical Form**

I’d now like to turn to Russell’s 1913 *Theory of Knowledge*. Even though inference does not come up as an explicit topic in the surviving parts of this manuscript, this work is of special significance for my inquiry. The manuscript was written in the period in which Wittgenstein had daily interaction with Russell, and as we know from Russell’s correspondence, Wittgenstein’s

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<sup>89</sup> Russell 1912, p. 72.

criticism of the account of judgment proposed in this manuscript eventually led Russell to abandon the entire project.<sup>90</sup> There are illuminating connections between Wittgenstein's early criticism of Russell's theory of judgment and Wittgenstein's later criticism of Russell's account of rules of inference. It is the aim of this section to explore these connections.

The central concern in Russell's 1913 manuscript is the analysis of judgment, understood as the fundamental logical act. Judgment, according to Russell's analysis, is a "multiple" relation, holding between the judging subject and the components of the proposition.<sup>91</sup> The account gives a pivotal role to the idea of logical form.<sup>92</sup> Russell initially introduces the idea of logical form in terms of the results of a process of abstraction. The abstraction from the content of propositions ultimately stops when we reach an expression containing only variables and logical constants, and that is the expression of a logical form.<sup>93</sup> Nonetheless, logical form is not a mere abstraction, for Russell. It is presupposed in the very act of judging, even if we are not explicitly aware of it. Logical form is one of the terms that enter the relation of judgment, alongside the thinker and the other elements of the proposition. Treating form as a term in this relation means that our ability to make judgments that have a certain logical form presupposes that we have an acquaintance with that form.

There is, however, a fundamental problem with thinking of logical form as a term, as an

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<sup>90</sup> Russell planned to write a third part of the book, dedicated to the theory of inference; unfortunately he gave up the project before he got to that part. See the editors' Introduction to ToK. On Wittgenstein's objection to Russell, see Hanks 2007.

<sup>91</sup> ToK, p. 115ff.

<sup>92</sup> Some features of the account of logical form of this manuscript already appear, in crude form, in Russell 1912 Chapter 7; and in a short paper draft Russell began in 1912 but never finished, titled "What is Logic?" (Russell 1992b).

<sup>93</sup> The constants are considered as pure form, rather than as entities that enter the proposition as one of its constituents. (p. 98). Cf. Wittgenstein's "fundamental thought" that the logical constants do not refer, in *Tractatus*, 4.0312.

object that enters into relations. For if judgment is not a mere heap of elements but a unity that is structured in a determinate way (e.g. a function-argument unity), how could logical form, understood as a term in the relation of judgment, also have the role of determining the order and the unity of the other terms that enter the relation? What would unite the unity of the form and the other constituents, and what would order the order in which we take the logical form to combine with the elements? Can this be done by the same form that occurs as relatum in this relation, or do we need a different form to govern it? And if the latter, would we need to appeal to yet another form to explain how the second form unites the first form with the other elements of judgment, and then would we need to appeal to a third one, and so on and so forth?

Russell is already aware of this objection in 1912, when he attempts to write a paper he titles “What is Logic”:<sup>94</sup>

If we made [the form] a constituent, it would have to be somehow related to the other constituents, and the way in which it was related would really be the form; hence an endless regress. Thus the form is not a constituent.<sup>95</sup>

In the 1913 *Theory of Knowledge*, Russell considers a similar objection:

It would seem that logical objects cannot be regarded as “entities”, and that, therefore, what we shall call “acquaintance” with them cannot really be a dual relation.<sup>96</sup>

...the form is not a “thing”, not another constituent along with the objects that were previously related in that form.<sup>97</sup>

Even though Russell here seems to be aware of deep difficulties (to which he confesses not to have a proper solution), he continues to employ the term “acquaintance” to describe the relation

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<sup>94</sup> According to Klement 2015 (p. 261), this line of thought is traceable to texts dating from 1904 (Russell 1994a); and see my discussion in Chapter 5, section 5.3.7.

<sup>95</sup> Russell 1992b, p.55

<sup>96</sup> ToK, p. 97.

<sup>97</sup> ToK, p. 98.

to logical form, treated as an object, and as a term in the multiple relation of judgment.<sup>98</sup> Perhaps he thought that his informal caveats against treating logical form as entity are enough to block the looming regress.<sup>99</sup> This is one of the central points targeted by Wittgenstein in his 1914 objections to Russell's multiple-relation theory of judgment.

Wittgenstein makes two complaints, which I take to be internally related. Admittedly, my treatment here simplifies the very complex issues that are at stake in this debate; my aim is merely to indicate the general shape of Wittgenstein's critique.<sup>100</sup> First, Wittgenstein notes that the account is not capable of excluding nonsensical judgments, such as “the table penholders the book” — judgments in which a logical form might be a term, but one which in fact fails to perform the job it was assigned.<sup>101</sup> Second, Wittgenstein argues that logical form cannot be treated as a mere object that enters relations as any other terms:

There is no thing which is the form of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell's theory of judgment.<sup>102</sup>

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<sup>98</sup> That he is at least partly aware of the possibility of a problem here only becomes evident when Russell admits that he does not know how a full analysis of the act of acquaintance with a logical form would look like, and what it would show logical forms to be. See ToK, p. 99.

<sup>99</sup> But the account Russell gives of ordered relations, on p. 146, does not seem to me to be immune to this type of objection.

There is another place in ToK in which Russell addresses a similar issues. He says that a third-man regress would arise if we attempted to define a logical form in terms of the class of propositions that share that same form (p. 113).

<sup>100</sup> I am ignoring the larger issue of whether Wittgenstein's objection to Russell concerns an inconsistency between the theory of judgment and the theory of types or not. But even someone who rejects the former might wonder whether the two lines of objection that I am considering are one, and if not, which of them was the one Russell counted as “devastating” to his theory. See Peters Hanks's argument against the identification of the two issues, as well as his interpretation of the issues underlying the critique in terms of the problem of the unity of the proposition, in Hanks 2007, esp. p. 133 and p. 139.

<sup>101</sup> NB, p.103; cf. Tr. 5.5422.

<sup>102</sup> NB, p. 105.

Whether or not Russell already had this objection in mind in writing the *Theory of Knowledge*, in 1914 Wittgenstein still thinks it invalidates his account of judgment.<sup>103</sup> Indeed, it seems to me that with this second charge Wittgenstein singles out the more fundamental issue, which underlies the first charge, i.e. the nonsense objection. Wittgenstein thinks that Russell continues to treat logical form as a “thing”, despite his denials that this can be done. When logical form is thought of as a term in the relation of judgment, it becomes an inert object, and is no longer able to bring order into the propositional unity that the judgment is supposed to express. This cannot be solved by means of appealing to a higher-level form, which would regulate the relation of logical form with the other elements. Because any such higher level form could only unite the judgment if it enters into some (higher-ordered) relation with it — a relation which itself would require ordering, by some other form which is not one of its constituent. At no stage in this regress would we find assurance that logical form excludes nonsensical judgments. The conclusion is that so long as it is treated as a term, what we are dealing with cannot count as *logical form*.<sup>104</sup>

There is a strong analogy to be drawn between Wittgenstein’s objection to Russell’s appeal to logical form and Wittgenstein’s objection to Russell’s appeal to rules of inference as contentful quasi-propositions. In both cases Russell seems to be driven by a desire for ontological parsimony, which makes him unable to properly distinguish form from content. In the case of the theory of judgment, this results in a lack of distinction between terms and the ordered

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<sup>103</sup> It is not impossible that this objection originates with Wittgenstein, even though the earliest text by Wittgenstein in which this objection appears is the 1914 Notebooks, composed well after Russell deserted his ToK manuscript.

<sup>104</sup> There is an additional problem in the vicinity — if logical form is not a “thing”, then how can we say anything about it, that is, how can we place logical form in the subject position of elucidatory sentences? See the discussion of this issue by Conant 2002a, p. 102f.

combinations into which they enter; in the case of inference, this results in a lack of distinction between the content of propositions and the formal, inferential relations in which they stand. In the former case, Wittgenstein criticizes the failure of Russell's logical form to truly inform the content of judgments. In the latter case, Wittgenstein's criticism similarly aims to show that the idea that rules are contentful (albeit inexpressible) quasi-propositions prevents Russell from explaining how logic informs thought and understanding. This is, at any rate, the shape of the account of Wittgenstein's criticism that I develop in the second part of this dissertation.

### **3.4. The Lingering Ambiguity of 'Imply'**

In a recent unpublished paper Michael Kremer argues that the ambiguity of Russell's notion of implication is the true cause for Wittgenstein's criticism of Russell in *Tr.* 5.132.<sup>105</sup> According to Kremer, Wittgenstein's criticism of rules of inference can be explained as an instance of his more general diagnosis of the philosophical nonsense, which he takes to stem from an indeterminate use of terms. I find Kremer's account of Wittgenstein's diagnosis illuminating; However, the ultimate problem Wittgenstein identifies in Russell goes deeper than this ambiguity, and concerns Russell's inability to distinguish logical form from content.

Let me very briefly spell out the shape of Kremer's account of the Wittgensteinian diagnosis of philosophical nonsense. Philosophical puzzles arise when we "misunderstand the logic of our language", imposing on logic a mode of explanation that does not fit it and which can only seem to make sense because of our equivocal and indeterminate uses of words. The only antidote to this tendency is to let logic "take care of itself" — as Wittgenstein recommends

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<sup>105</sup> Kremer 2014.

in Tr. 5.473. One way to help us do that is by showing us how our expressions behave in different notations; such a comparison would lead us to realize the ambiguities that underly the statement of our original puzzlement, and the superfluity of the accounts we attempted to offer in response to it. Nothing substantive corresponded to the expressions that we initially found puzzling, and nothing substantive needs explaining in them — we were only confused by our own misuse of language.

Kremer goes on to argue that in Wittgenstein's eyes, Russell's account of inference is one such case in which a superfluous logical device — the rule of inference — is introduced in response to an ill-placed puzzlement. Rules seem to Russell to be needed, on this account, only because of Russell's failure to clearly distinguish the equivocal uses of 'imply'. This ambiguity is manifest in Russell's use of the horseshoe sign to signify the two relations (implication and *therefore*), as well as in what Kremer considers to be Russell's equivocal use of axioms in *Principia*, where they are cited both as premises and as rules.

However, despite the ambiguity of 'imply' and despite Russell's permission to treat propositions as rules, we have seen that Russell distinguishes the two relations of imply and *therefore*; in *Principles* he correlates each of them with a separate mental faculty and in *Principia* he correlates them with two distinct *uses* of propositions, which are marked in the symbolism. Russell's hope is that the distinction between the two uses would be sufficient to avoid the Carrollian objection; I argued that this hope is not unfounded. The distinction Russell needs in order to avoid Carroll's challenge is a distinction between, on the one hand, the propositional use of axioms, and on the other hand, the appreciation of modes of inference that correspond to these axioms. Wittgenstein would agree with Russell that this is not a distinction that could be properly expressed formally, by means of a disambiguation of one of the signs. The distinction concerns

the different uses of a notation, rather than different contents one expresses through it. Russell marks this by the use of brackets, that indicate that what they enclose should not be read as a proposition.

Russell is further justified in emphasizing the close correspondence between the modes of inference and the content of the axioms, and perhaps this is why he does not draw any explicit terminological distinctions between them. The same correspondence is noted by Wittgenstein when he says that “every proposition of logic is a *modus ponens* presented in signs” (though it makes all the difference in the world that what Wittgenstein means by “presented” here is not that in the propositions of logic, rules are indirectly revealed).<sup>106</sup> Modes of inference, on the account I will develop in chapters 5 and 6, are *shown*, meaning that they inform the activities through which we understand and infer, but they cannot be thought of as the content that we entertain. Russell’s two uses of the horseshoe sign can be contextually distinguished; thus the ambiguity of this sign is only apparent.

On my account, Russell is not badly misled by the ambiguity of ‘imply’; he retains it without being led into the confusion which Kremer’s Wittgenstein blames him for. In fact, it would not be of much help to Russell to disambiguate the two senses of ‘imply’, so long as he continues to think of both kinds of relations which this term designates in the same way, i.e. as substantive relations. What justifies an inference, according to Russell, is the appeal to a logical fact that, unlike other facts, cannot be properly expressed. The philosophical confusion Wittgenstein ultimately finds in Russell’s conception of rules of inference consists in a lack of

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<sup>106</sup> Tr. 6.1264 (the emphasis is mine). I discuss this passage in Chapter 6, section 6.3.

distinction between form and content, between what shows itself and what can be said.<sup>107</sup>

I am however in complete agreement with Kremer's deeper and more general point, that Wittgenstein's distinction between what shows itself and what is said is meant to put a stop to the insatiable tendency of philosophy to seek for justification even where no such justification is needed, and none can be found. It is this picture of logic that Wittgenstein aims to criticize — a picture in which logic is in the business of providing grounds, explanations and justifications.<sup>108</sup> Russell's construal of rules of inference as inexpressible, and yet contentful principles which we must perceive if we are to justify our inferences is a prime example of the attempt to treat logic as a theory that provides such justifications. A resolute distinction between content and form, such as the one proposed by Wittgenstein, shows that such rules are senseless and superfluous.<sup>109</sup>

### 3.5. Conclusion

Russell's fully developed answer to Carroll's and Bradley's puzzles consists in the conjunction of three ideas. The first is the idea that to avoid Carroll's regress, the rules of inference must remain inexpressible within the system they govern. The second is the idea of a distinction between the content of rules of inference (the *therefore* relation) and the content of propositions (which may involve implication but does not involve the *therefore* relation). Only rules can justify the assertion of conclusions, since the relation which ordinary implications express holds between

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<sup>107</sup> Kremer 2014 also argues that Wittgenstein would view the attempt to use of a connective such as '⊃' to indicate the internal relation that holds between propositions that follow from each other (i.e. to indicate the *therefore* relation) as nonsensical. Internal relations such as these cannot be expressed by language at all. This point would only become clearer on the background of an account of Wittgenstein's say-show distinction and his conception of nonsense, which I discuss in chapter 5.

<sup>108</sup> Kremer 2001, pp.51–52.

<sup>109</sup> This is the topic of chapters 5 and 6.

propositions regardless of whether these propositions are asserted or not. Given Bradley's regress, it is clear that the application of these general rules of inference to specific inferences must not require further inferential reasoning. Russell's third idea is that we must have the capacity to "simply perceive" the applicability of rules. What we thereby perceive, the *therefore* relation, is internally related to the content of the logical axioms and theorems. But in perceiving the content of a rule in the logical proposition, we use it in a distinct way, unlike the use we make of it when we treat it as the premise of an inference. When the latter kind of use is made, the proposition is inert; the former kind of use effects the assertion of the conclusion.

Let us reevaluate the extent to which this account succeeds in answering the challenges posed by Carroll and Bradley. The justification of an inference, for Russell, cannot involve adding new premises to the inference. For the fact that the content of the rule justifies the inference cannot be made explicit within the formal language. But to make room for the very idea of such inexpressible logical content, Russell must reject the naive picture of logic that serves as the backdrop to Carroll's puzzle, and specifically he must reject the idea that the formal language is all embracing in its expressive capacity. Securing the justification of inference by appeal to inexpressible rules still leaves room to doubt whether *we* can ever be justified in our inferential practices, since, so long as rules are thought of as contentful, they are no less inert than any other premise we can appeal to; the way in which we apply the content of rules still seems to call for additional inferential reasoning, even if the rules themselves do not appear as explicit premises in such inferences. To answer this residual challenge requires that we reject another one of the ideas that form the backdrop to Carroll's puzzle: the idea that reasoning consists in a single type of activity, which Carroll's dialogue captures in the image of entering the proposition into the notebook. If the applicability of rules can be appreciated directly, there is

no need for the mediation of further inferential reasoning. By positing a capacity to immediately *perceive* logical relations, which is distinct from the capacity to grasp the inert premises of our inference, Russell puts a final stop to Carroll's and Bradley's regress.

We can better assess the value of Russell's account by gauging the extent to which it points the way to more recent developments in the philosophy of logic. At least since Tarski, we treat questions about the validity of inferences by means of statements in a meta-language, which are inexpressible in the object-language. Russell lacks the resources to make a principled distinction between object and meta-language (among other things, Russell lacks Tarski's distinction between syntax and semantics, and the related idea that a language is merely a system of uninterpreted signs).<sup>110</sup> Nonetheless, Russell's account points the way to Tarski's in the following sense. It shows that rules that govern a system can only be spelled out outside of that formal system. For Russell, we are only capable of indicating the application of rules in the marginal notes of the proofs. In these marginal notes propositions are mentioned, not used; they are a form of commentary on the proof, in which we indirectly communicate the rule that applies to our inference by pointing out the axiom to which the rule may be transformed.

But Russell's account of inference has a further feature, which is lacking from this strictly Tarskian picture, and which I take to be crucial for answering the real worry that Carroll and Bradley raise. Russell takes these puzzles to concern the rationality of reasoning, not the justification of deduction in a formal system. For this reason, to retort to Carroll's puzzle by saying that from Tarski's perspective the regress is not vicious would miss the point entirely.<sup>111</sup> The Tarskian distinction can help us in giving a more accurate and explicit account of the

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<sup>110</sup> See Tarski 1943.

<sup>111</sup> Wisdom 1974 makes a similar point.

regress. But it does not eliminate the regress, and a proper account of rationality, which is capable of doing justice to our naive concepts of justification and truth, requires just that. The assertion of the truth of a conclusion is not made relative to a language, but absolutely. Russell's account aims to restore our confidence that this can be done. This is what leads him to argue that we must have the capacity to perceive, once and for all, the applicability of the rule of inference.

The way Russell's argument proceeds is by moving from an undisputed phenomenon to its conditions of possibility. He assumes that there is such a thing as inferential justification, and that we are actually capable of justifiably inferring. He further assumes that justified inference is an inference grounded in logical facts. But our recognition of such facts must not form part of the inference, lest it give rise to a regress. Thus if justified inference is to be possible, it must be possible for us to see that these facts justify inferences without it requiring from us further acts of inference. Such a capacity should at least have the property of not giving rise to further inferential acts; it must be unmediated, simple, direct. For these reasons, I take it, Russell sees himself compelled to declare that we must have the capacity to *perceive* that a general rule of inference applies.

We can think of this quasi-transcendental argument as a response to a kind of skeptic — Carroll's Tortoise, if you will — who is able to cast doubt on the possibility of rationality only because he draws on a very thin conception of the mind, which is the conception I tried to capture in terms of the One Great Notebook. Russell's response can then be taken to spell out what an account of rationality must consist in, if such skepticism is to be ruled out. It requires the rejection of the image of the One Great Notebook, and a revision of the idea of the relation between logic and language that this image implies. Reasoning involves capacities beyond the mere capacity to accept and express content; logic operates in ways that language cannot

express.

This is sufficient for avoiding Carroll's and Bradley's regress. And yet it might seem rather ad-hocish. Indeed, Russell does not do enough to develop and motivate this account, or to rule out alternative solutions. More specifically, it is not clear that we must conceive of the justification of rules in terms of a capacity to perceive logical relations. This can be seen by comparing Russell's account to a much more recent, and quite influential account of inference, namely Paul Boghossian's. Boghossian argues, like Russell, that if we do not acknowledge a primitive and unanalyzable capacity to follow rules, our account of inference would either involve a regress, or it would undermine the normativity of reasoning:

[W]e face a stark choice between attempting to account for our mental lives without something that looks like the traditional notion of person-level reasoning, on the one hand, and being willing to take the notion of following a rule as an unanalyzable primitive, on the other.

Since I have no idea how to dispense with the notion of reasoning, and since there are independent considerations that favor thinking that rule-following cannot be analyzed, I incline firmly towards the latter option.<sup>112</sup>

There are important differences between Boghossian's and Russell's arguments, which I cannot get into here. But there are two important points of commonality that should be highlighted. First, whereas the unanalyzable primitive capacity Russell appeals to is a capacity to perceive the applicability of the rule, in Boghossian's case it is the capacity to follow rules. Thus for Russell, inference is fundamentally a matter of recognizing a fact (i.e. seeing that the conclusion is justified by the content of the rule), whereas for Boghossian, it is fundamentally a matter of doing something (i.e. drawing a conclusion according to a rule).<sup>113</sup> Boghossian, in other words,

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<sup>112</sup> Boghossian 2014, p. 17.

<sup>113</sup> This difference might not seem as great if we consider that Russell is concerned with the justification of the *act* of asserting the conclusion; in this sense the causal element does not go entirely missing from Russell's account. I owe this point to Nick Koziolk.

emphasizes the causal aspects of inference rather than its evaluative aspect.<sup>114</sup>

Second, given the structure of their argument, and absent additional considerations, Boghossian and Russell are only able to offer us sufficient conditions for a proper account of rationality, but not necessary ones. Boghossian goes much further than Russell in terms of trying to rule out alternative explanations.<sup>115</sup> As Boghossian himself admits, even his case remains inconclusive, and alternative candidates for the role of the capacity that guarantees our rationality have been proposed by others.<sup>116</sup> But Russell's account can perhaps be defended from this charge. His description of the capacity to perceive logical relations is so thin — all that is required is that it be a simple and unmediated act — that it can in fact be taken as a stating a general condition that any candidate account must satisfy if rational inference is to be possible.

Taking another step back, we can bring into view a framing assumption that is shared by Russell as well as by proponents of similar accounts of inference, such as Boghossian: inference is taken to require a mediation between the premises and conclusion. In Russell's case, the mediation is effected by the rule of inference, even though appeals to the rule cannot be made explicit and must be explained in terms of a non-discursive capacity. But the very idea that mediation is needed — whatever form it ends up taking — is enough to put Russell and Boghossian in agreement with Lewis Carroll, that without mediation, the propositions themselves are inert, and hence that they do not, on their own, entitle us to treat the conclusion as

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<sup>114</sup> Eric Marcus proposes that inference involves two philosophically puzzling components: a causal and an evaluative one. See Marcus 2017.

<sup>115</sup> For instance, Russell does not even consider the need to rule out dispositionalist and sub-personal accounts of applying rules, whereas Boghossian 2014 does.

<sup>116</sup> Crispin Wright responds to Boghossian with an argument of his own to show that what we must treat as primitive is the capacity to act for a reason. See Wright 2014.

justified.<sup>117</sup>

We can trace the rejection of this assumption back to Wittgenstein's *Tractatus*, where Wittgenstein criticizes and rejects the need for anything like Russell's rules of inference. Instead, Wittgenstein urges that our understanding of the propositions themselves (the premises and conclusion) must be sufficient for the justification of inferences (*Tr.* 5.132). Such an approach to inference requires a much richer account of what it takes to truly understand a proposition than the one we find in Russell, Carroll, or even Boghossian. On such an account, rules are not needed, because propositions are *not* inert — understanding propositions already gives us all of the justification we need in order to infer.<sup>118</sup>

In thinking of inference as mediated by a separate act through which we perceive the inexpressible applicability of logical content to expressible inferences, Russell is committed to a thought that Wittgenstein finds deeply problematic: that we can make sense of the act of appreciating the content of a proposition as something that is completely isolated from the act of appreciating the logical relations that that proposition bears to others. Let me mention one difficulty this thought raises. If the two kinds of act are truly irreducible, how do they ever combine? In what sense do they all form part of one mind? Russell is far from clear on these issues. For Wittgenstein, there is no separation between the inert and the dynamic aspects of understanding.

On Russell's account, logically valid inferences are justified by reference to contentful rules, even though the content of those rules is not formally expressible. Throughout, Russell

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<sup>117</sup> This framing assumption has recently come under direct attack. See Hlobil 2014, McHugh and Way 2016, and Koziolk (unpublished manuscript).

<sup>118</sup> In Chapter 6 I develop the idea that for Wittgenstein, we cannot even individuate a proposition in abstraction from its role in activities of inferring.

insists on thinking of rules as something that *fails* to be susceptible to formal expression, and that means, as something that *should* in principle be given such expression. Rules, for Russell, are thought of as contentful quasi-propositions, and the justification that they are thought to provide is the same kind of justification that premises provide to their instances — hence the danger of regress. According to the reading of Wittgenstein’s critique that I develop in the second half of the dissertation, the main issue is that Russell thinks of content as extending beyond the realm of the expressible. What is at stake here is Russell’s construal of the universality of logic.<sup>119</sup>

Frege and Russell are both concerned to capture, within their formal systems, the substantive logical content that underlies all thought.<sup>120</sup> But the ultimate inexpressibility of rules signals to them that the realm of logic extends beyond our means of expression, and they both think of what evades expression as a species of content. Thus their universalism is one in which language is capable of expressing all that can be expressed, but logic is conceived of as the realm of content which extends beyond the expressive means of any possible language. This is the picture that Wittgenstein sets out to criticize in the *Tractatus*.

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<sup>119</sup> But cf. Proops 2007 and Blanchette 2013, and Korhonen 2013 who deny that Russell was a universalist in the sense proposed in Van Heijenoort 1967.

Here are two pieces of evidence that, in my mind, show that Russell is a universalist in the relevant sense, which rules out a meta-systematic, semantic reflection on logic itself:

1. Russell’s discussion of the impossibility of independence proofs for logical axioms — proofs in the process of which we assume the negation of one of our axioms. Russell holds that such an assumption is sure to throw our thought into confusion, since it would lead us to adopt invalid forms of reasoning (this follows from the account, proposed here, of the way in which axioms can be transformed to rules of inference). Russell’s discussion of the impossibility of independence proofs for the axioms of his system (*Principles*, #17; *Principia*, #4; as well as Russell 1992a). Proops 2007 and Blanchette 2013 try to play down the weight of Russell’s remarks on independence proofs, and they hold that it does not reflect his overall conception of logic.
2. Russell’s claim that we cannot meaningfully think of the falsity, or invalidity, of a logical form, in ToK, p. 133 and 141.

<sup>120</sup> Russell agrees with Frege in thinking that the logical formalism amounts to a genuine language, a vehicle of truly meaningful thought, and is not a mere uninterpreted calculus. See *Principia*, p. 115. An influential discussion of the Fregean contrast between a language and a calculus is found in Van Heijenoort 1967.

## **Part II: Inference in Wittgenstein's *Tractatus***

## 4. Wittgenstein's Rejection of the Rules of Inference

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In *Tractatus* 5.132 Wittgenstein attacks Frege's and Russell's conception of inference; but the precise nature of his critique is still debated.<sup>1</sup> In this chapter I argue that Wittgenstein's target is Frege's and Russell's idea of *rules* of inference. To say that Wittgenstein rejects the rules of inference is, in a sense, not a novel claim. Indeed the standard reading of *Tractatus* 5.132, endorsed by Mounce, Kenny, and Baker,<sup>2</sup> takes the rules of inference to be Wittgenstein's target. This standard reading construes Wittgenstein's rejection of rules of inference along the lines of the challenge posed by Lewis Carroll in his dialogue "What the Tortoise said to Achilles".<sup>3</sup> Carroll, and supposedly Wittgenstein, argue that any attempt to justify inference by appeal to rules would give rise to additional inferential reasoning, and so a vicious regress would arise.<sup>4</sup>

But the standard reading faces a devastating objection. To avoid Carroll's regress, it is

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<sup>1</sup> Wittgenstein 1922 (henceforth abbreviated as Tr. or *Tractatus*). In citing the *Tractatus* I refer to paragraph number (e.g. 5.132), and where needed I indicate the line number using letters (e.g. 5.132d). Unless otherwise noted, I will use the Ogden translation (Wittgenstein 1922); I also consult the Pears-McGuinness translation (Wittgenstein 1974).

<sup>2</sup> Mounce 1981, pp. 44-47; Kenny 2006, p. 78; and Baker 1988, pp. 130-132.

<sup>3</sup> Carroll 1895.

<sup>4</sup> To be precise, this is more or less how Russell understands Carroll's challenge to the idea of rules of inference, not how the original challenge is framed by Carroll, who makes no distinction between rules and propositions; but for present purposes this will do. See Russell 1903, #38, pp.34-35 (henceforth abbreviated as *Principles*).

widely agreed, it is sufficient that a sharp distinction be drawn between the use of rules that govern inference and the use of propositions that serve as premises and conclusions. If the application of rules does not alter the content of the inference, a regress of the kind Carroll imagines does not arise. In other words, if the application of rules does not belong within the language whose use it governs, there should be no vicious regress (though there might still be a non-vicious one). I have argued in Chapter 2 that such a distinction between rules and premises is already to be found in Frege, who independently of Carroll's puzzle worried about the circularity that would arise if we attempt to express the rules in the system they govern. I have argued in Chapter 3 that a similar distinction can be found in Russell, who responds to Carroll directly. So Carroll's regress can not pose a real threat to Frege's and Russell's conception of rules of inference.

Could Wittgenstein be so uncharitable to his predecessors as to accuse them of failing to respond to the challenge of Carroll's puzzle? Or is he not really targeting their use of rules of inference? Ian Hacking suggests that Wittgenstein was simply wrong to criticize Frege along these lines, though he was right to criticize Russell.<sup>5</sup> Thomas Ricketts and Ian Proops seek a more charitable reading of Wittgenstein, which leads them to what I call a non-standard reading of 5.132.<sup>6</sup> They assume that Wittgenstein had a fairly good knowledge of Frege's and Russell's logic, and that he knew about their distinction between rules and propositions. Ricketts concludes that *Tractatus* 5.132 does not target Frege's rules of inference; Proops agrees, but he adds that it cannot be targeting Russell's rules of inference either. On their reading, the critique

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<sup>5</sup> Hacking 1979.

<sup>6</sup> Ricketts 1985; Proops 2001.

concerns a different aspect of Frege's or Russell's conceptions of proof.<sup>7</sup>

In this chapter I seek to avoid the conclusion that rules of inference are not Wittgenstein's target. Even though I accept the non-standard point that Wittgenstein's criticism in 5.132 cannot be charitably read as merely repeating Carroll's puzzle, I do not believe that we should give up on the idea that the rules of inference are its target. My account rejects the assumption that seems to guide the non-standard line of interpretation, namely that any critique of rules of inference must take the form of Carroll's puzzle.

Wittgenstein has substantive grounds to object to Frege's and Russell's conceptions of rules of inference, even if they are immune to Carroll's regress. Frege and Russell both escape the regress by treating the rules of inference as principles that belong outside of the formal language which they govern. Nonetheless, these inexpressible rules of inference are taken to be endowed with substantive content. For both thinkers, the content of rules can be converted into the content of propositions, and vice versa. Frege thinks of the rules as a source of content for the axiomatic system; and Russell thinks of the application of these rules as an act of recognizing a fact that justifies the inference. What Wittgenstein aims to reject, on my reading, is precisely this idea of inexpressible and yet substantive logical content, an appeal to which is required for our inferences to be justified.

Rules of inference are for Wittgenstein a paradigm case of the failure to properly draw the distinction between what can be said in language, and what must show itself in our use of

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<sup>7</sup> To be precise, they conclude that that is not the primary, or the only, target of Tr. 5.132.

language but cannot form its explicit content.<sup>8</sup> Wittgenstein's rejection of the idea of inexpressible but contentful rules of inference signals how strict his distinction is meant to be. What *shows* itself in our use of language cannot be construed as a species of content. Everything that *must* be made explicit if our linguistic practices are to make sense *can* be made explicit within language. What shows itself is not the content, but the form of thought. Recall that Frege and Russell themselves make a distinction between what is said in a language and what cannot be expressed in it — the rules of inference are precisely such principles that remain inexpressible in the system they govern. But on Frege's and Russell's way of drawing the distinction, there's substantive content on both sides of the divide. For both of them, rules of inference are inexpressible, but contentful — whereas for Wittgenstein, as I read him, what is shown is pure form, what is said is content, and there is no grey area of content that is shown but cannot be said. So if Frege's and Russell's rules are indeed contentful, in the relevant sense, this would constitute a good reason for Wittgenstein to reject them.

The present chapter proceeds as follows. In section 4.1 I look closely at *Tractatus* 5.132 and discuss the textual grounds for holding that its target is Frege's and Russell's conceptions of rules of inference. In section 4.2 I look at surrounding texts that lend support to my reading. In Section 4.3 I show why the standard reading of 5.132 is unsatisfactory, and I discuss the alternative readings proposed by Hacking, Ricketts and Proops. Then, in section 4.4, I argue that by rejecting the assumption that the critique of rules must take the form of Carroll's regress, it becomes possible to see the deeper connections between Wittgenstein's critique of rules and his

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<sup>8</sup> The proper way to construe Wittgenstein's distinction will be discussed in detail in Chapter 5. This issue forms one of the main fault lines that divide contemporary Wittgenstein scholarship. The reading I offer here for Wittgenstein's rejection of rules of inference provides further support for the line of interpretation pursued in the resolute reading of the *Tractatus*. See Bronzo 2012.

For a recent account of Wittgenstein's distinction between showing and saying, with which I am in general agreement, can be found in Narboux 2016.

overall conception of the status of logic, captured in the distinction between what can be said and what shows itself.

#### 4.1. ‘*Schlussgesetze*’: Rules or Laws?

Let me start by looking at the precise wording of the charge Wittgenstein makes against his predecessors in *Tractatus* 5.132. Since the two English translations of the *Tractatus* obscure some of the crucial issues in this passage, I offer a translation of my own:

5.132 If  $p$  follows [*folgt*] from  $q$ , I can infer [*schließen*] from  $q$  to  $p$ ; derive [*folgern*]  $p$  from  $q$ .

The mode of inference [*Art des Schlusses*] is to be understood from the two propositions alone.

Only they themselves can justify the inference [*Schluss*].

‘Laws of inference’ [„*Schlussgesetze*“], which — as in the works of Frege and Russell — are supposed to justify inferences, are senseless [*Sinnlos*], and would be superfluous.<sup>9</sup>

At issue in the first three lines of 5.132 is the justification of inferences: the fit between the idea that two propositions are objectively related in the internal relation of ground to consequence, and the idea that we can infer the latter from the former. This discussion sets the stage for Wittgenstein’s criticism, which is voiced in the last sentence (5.132d). Here Wittgenstein rejects Frege’s and Russell’s conception of “*Schlussgesetze*”, inasmuch as they are supposed to *justify* inferences. It is said that such “*Schlussgesetze*” are “senseless”, and “would be superfluous”.

There are three important respects in which the translation I just offered differs from the Ogden translation as well as from the Pears and McGuinness translation:

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<sup>9</sup> In the next paragraph I discuss the differences between my translation and the ones provided by Ogden and by Pears and McGuinness.

1. I use the verb ‘infer’ as a translation of ‘*schliessen*’, since that is the cognate of the noun ‘*Schluss*’, which the translators of the Tractatus agree on rendering as ‘inference’. Ogden obscures this cognate relation by using ‘conclude’ for ‘*schliessen*’, whereas Pears and McGuinness use the cumbersome ‘make an inference’.
2. I choose ‘derive’ for a translation of ‘*folgern*’, where Ogden has ‘infer’ and Pears and McGuinness use ‘deduce’.<sup>10</sup>
3. I render ‘*Art des Schlusses*’ as ‘mode of inference’ rather than Ogden’s “method of inference” and Pears-McGuinness’s “nature of inference”. These translations obscure the fact that this is a Fregean technical term, which is often used in the *Begriffsschrift* to refer to the *modus ponens*.<sup>11</sup>

It should also be noted that the term ‘*Schlussgesetze*’ appears in 5.132 in quotes, but these do not seem to indicate citation, for the term does not appear in Frege, and it is not an obvious translation of any Russellian term.<sup>12</sup> Frege often uses the term ‘law’ in a way that does not preclude reading ‘*Schlussgesetze*’ as referring to his rules of inference.<sup>13</sup> Moreover, the second sentence of 5.132 employs the distinctly Fregean technical term ‘mode of inference’ (the other term he uses is *Schlussweise*). And this strongly speaks in favor of reading “*Schlussgesetze*” in the fourth sentence (5.132d) as referring back to Frege’s conception of rules such as *modus ponens*, rather than to the axioms of his system.

Recall that for Frege the distinction between laws and rules is quite explicit. Laws are the logical propositions which serve as the ultimate grounds of particular proofs. They can be made fully explicit and included as actual premises, or they can be cited marginally, by number, when

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<sup>10</sup> Pears-McGuinness reserve ‘derive’ for Wittgenstein’s ‘*ableiten*’ (which Ogden renders ‘deduce’).

<sup>11</sup> On Frege’s terminology, see Chapter 2, Appendix A.

<sup>12</sup> The German term literally means “laws of inference”.

<sup>13</sup> In Appendix A of Chapter 2 I discuss the places in which Frege uses “Gesetze” to refer to rules inference, or to rules and axioms together. These include Frege 1979b, p.3; Frege 1979f, p. 175; Frege 1984g, p.273; and “Letter to Dingler (31.1.1917)” in PMC, pp. 16-17.

a premise is an instantiation of one of them. Frege holds that rules of inference, like the other rules for the operation of the formal system, cannot be expressed within that system without circularity.<sup>14</sup> Unlike laws, rules cannot be cited as the premises of an inference; they are merely indicated by transition signs that stand between the premises and the conclusion.

It is a little more difficult to determine what, in Russell's writings, can be referred to as "*Schlussgesetze*", since Russell himself very often uses terms that indeterminately refer to both the axioms and the rules. The rules of inference of his systems are included as equal members in the official lists of the most basic logical principles, along with the axioms; Russell refers to all of the members of these lists equally as "principles of inference",<sup>15</sup> "principles of deduction",<sup>16</sup> "primitive propositions",<sup>17</sup> and even "rules of inference".<sup>18</sup> Nonetheless, as I've argued in chapter 3, Russell clearly marks the special status of the rule of inference. Like Frege, Russell further holds that there are logical principles that serve as rules and cannot be cited as premises (either explicit or implicit); they are inexpressible, a case in which "the formalism breaks down".<sup>19</sup> In *Principia*, Russell distinguishes two kinds of uses that can be made of logical principles — as premises and as rules. But on Russell's account, the application of rules differs from the appeal to axioms in that it does not alter the content of the inference; it is an extra-formal act. *Schlussgesetze* may therefore be taken to refer to his rules of inference, as distinct from the axioms.

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<sup>14</sup> Frege 1972d #13, p. 28 (henceforth abbreviated as BS or *Begriffsschrift*).

<sup>15</sup> *Principles*, #18, pp. 16-17.

<sup>16</sup> Whitehead and Russell 1910, p. 106 (henceforth abbreviated as *Principia*).

<sup>17</sup> *Principia*, p. 94.

<sup>18</sup> *Principles*, #38, p. 34.

<sup>19</sup> *Principles*, #45, p. 41.

Since *Schlussgesetze* is not a technical term used by either Frege or Russell, and since Russell's (and to some extent, Frege's) use of the terms 'law', 'rule' and 'principle' is somewhat slippery, Wittgenstein's term may be read as referring to the rules of inference of their systems (by contrast with the axioms), as standard readers of 5.132 have it. But it could also be read as indicating the axioms, understood as the laws which ground all inference. In fact, Given Wittgenstein's very condensed style of writing, it is not impossible to take 5.132 to advance two separate lines of criticism at once, targeting separate features of the account of inference in each of the two thinkers. Thomas Ricketts thinks that such a reading of 5.132 is not only possible, but in fact preferable: he takes Wittgenstein to criticize Frege's idea of the role that axioms serve as implicit premises, and at the same time to criticize Russell's conception of the rules of inference.<sup>20</sup> In section 4.3 I will argue against this nonstandard reading. There is a core view of rules of inference which is common to both Frege and Russell, and which makes them both susceptible to the same kind of criticism. As I have shown in the previous chapters, Frege and Russell both take the rules of inference to play a justificatory role, and they both think of these rules as inexpressible but contentful; they are therefore both open to the criticism Wittgenstein directs at them.

#### **4.2. The Rejection of Rules of Inference: Texts and Contexts**

There are several passages in Wittgenstein's pre- and post-tractarian manuscripts which lend further support to my reading of 5.132. Consider Wittgenstein's 1913 *Notes on Logic*, where a draft version of 5.132 is found:

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<sup>20</sup> Ricketts 1985, p. 12.

Logical inferences can, it is true, be made in accordance with Frege's or Russell's laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If  $p$  follows from  $q$ , it can also be inferred from  $q$ , and the "manner of deduction" is indifferent.<sup>21</sup>

Here is why I take the term "laws of deduction" in the opening of this remark to refer to the rules of inference.<sup>22</sup> Wittgenstein assumes that there is such a thing as a set of primitive propositions of logic, and then excludes from this set certain things, namely what he calls "laws of deduction".<sup>23</sup> He says that these "laws" are *not* "primitive propositions of logic", and it would be absurd to say something like that about the axioms of the system: for what propositions would then count as the primitive propositions of logic? Note, moreover, that the point Wittgenstein makes about "laws of deduction" in the first sentence is the same point that is made about "manner of deduction" in the second sentence — namely that they make no significant contribution to the justification of the inference. The term "manner of deduction" echoes Frege's "mode of inference" (*Schlussart*), with which he refers to the *modus ponens*. This second sentence reads quite naturally as a restatement of the first. The overall point Wittgenstein is making therefore seems to be that the justification of an inference does not depend on a rule being employed, but rather on something else — which Wittgenstein terms the relation of *following* that holds between  $p$  and  $q$ . Indeed, this is also the point made in 5.132 regarding the *Schlussgesetze* — that they have no justificatory role, and are therefore senseless and superfluous.

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<sup>21</sup> Wittgenstein 1984b, p. 100 (henceforth abbreviated as NL).

<sup>22</sup> Both the texts of the 1913 notes (NL) and the 1914 notes (Wittgenstein 1984c, henceforth abbreviated NM) are based on English transcripts, and the original German texts on which they are based (if there were any) are now lost. For discussion see Biggs 1996.

<sup>23</sup> Admittedly, Wittgenstein does criticize the very idea of a logically primitive proposition in the *Tractatus* (5.43 and 6.1271). But his argument is distinguishable from the one at issue here. See my discussion of Wittgenstein's revolutionary approach to logic in the next paragraph.

Another early passage in which Wittgenstein struggles with the same issues is found in the *Notes Dictated to G. E. Moore* (1914). Here Wittgenstein forcefully contrasts “rules of deduction” with “primitive propositions”:

This is the actual procedure of [the] old Logic: it gives so-called primitive propositions; so-called rules of deduction; and then says that what you get by applying the rules to the propositions is a logical proposition that you have proved. The truth is, it tells you something about the kind of proposition you have got, viz. that it can be derived from the first symbols by these rules of combination (= is a tautology).<sup>24</sup>

This passage appears in the context of a larger argument which spells out the revolutionary aspects of Wittgenstein’s conception of logic. On Wittgenstein’s view, “so-called primitive propositions” are “tautologies” (cf. *Tr.* 5.43), but no more and no less than any of the logical theorems; they are combinations of signs that exhibit logical form, but say nothing (cf. *Tr.* 4.46f.). In Frege and Russell, by contrast, logical theorems are derived from primitive propositions (axioms) by means of “rules of deduction”. There are two separate issues that their “old Logic” is accused of being wrong about here. The first issue is that it marks an illusory distinction between the “primitive” and the “derived” propositions of logic, where in fact, “all the propositions of logic say the same thing, namely nothing” (*Tr.* 5.43). The second issue is that “so-called rules of deduction” are taken to substantially contribute to proofs. It is this second issue that the 1914 passage is primarily concerned with (Indeed, the justificatory contribution is also what is at issue in the *Tractatus* 5.132). Wittgenstein says that the “so-called” rules do not contribute anything to the justification or establishment of the truth of the conclusions.

Wittgenstein’s ongoing concern to criticize his predecessors’ view of rules of inference is

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<sup>24</sup> NM, p. 109; cf. *Tr.*, 6.126; and especially *Tr.*, 6.1262: “Proof in logic is only a mechanical expedient to facilitate the recognition of tautology, where it is complicated.” I discuss this contrast between the innocent idea of rules as involved in “mechanical” procedures of proof and the idea that rules have a justificatory role in chapter 6.

also evident in his later remarks on the issue.<sup>25</sup> Let me briefly discuss one such example, from his lectures from the 1930s:

What justifies the inference is seeing the internal relation. No rule of inference is needed to justify the inference, since if it were I would need another rule to justify the rule and that would lead to an infinite regress.<sup>26</sup>

This passage shows that Wittgenstein has a clear grasp of a problem that specifically affects rules of inference, which is reminiscent of Carroll's regress. Both here and in *Tractatus* 5.132, the idea that rules play a justificatory role is contrasted with the idea that the entire justification of the conclusion comes from observing the internal relations between the propositions. However, given that Frege and Russell have adequate answers to Carroll's regress, it is crucial for my reading of *Tractatus* 5.132 that Wittgenstein's critique of his predecessors is not to be equated with Carroll's puzzle. When he criticizes Frege's and Russell's conception of inference in 5.132, he targets the rules on different grounds.

If we look at the Tractarian context of 5.132, things might seem less clear cut: it might seem that Wittgenstein is less interested in rules of inference, and more interested in Frege's and Russell's proofs and the role that axioms play in them. In 5.131 Wittgenstein argues:

5.131 If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a

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<sup>25</sup> The relevance of these later texts to deciphering 5.132 is suggested in Kremer 2002b, as well as in Kremer 2001, esp. p. 56 and p. 68., fn. 17; and Kremer 2014.

<sup>26</sup> Wittgenstein 1982, p. 56. Wittgenstein's discussion on pp. 56-57 strongly echoes his remarks in the 5.13s of the *Tractatus*, which I discuss below.

However, Wittgenstein's discussion of rules on p. 88 seems to me to break away from the *Tractatus* view of inference and to point towards the arguments of *Philosophical Investigations*. Here rules are thought of as part of an arbitrary system, comparable to the rules of a game. And in this context, Wittgenstein says that only an appeal to the rule justifies our inferences.

Another passage which explicitly concerns rules of inference is found in Wittgenstein 1979, p. 137f. The remarks on rules and tautologies in these lectures are in line with the reading of the 5.13s I offer here; I discuss them in more detail in Chapter 6.

proposition; for these relations are internal, and exist as soon as, and by the very fact that, the propositions exist.

What Wittgenstein objects to here is a feature of Frege's and Russell's conception of proof, namely that to prove  $q$  from  $p$  requires that we show that  $p \supset q$  is a theorem, derivable from the axioms. This is what Ricketts (and to an extent, Proops) takes to be the target of 5.132.<sup>27</sup> What is important for present purposes is to note that in 5.131 this line of criticism is pursued without mention of "*Schlussgesetze*", a term which only appears in 5.132. That's because there are two separate lines of criticism that we must keep apart:

- 5.131 shows that the internal relation of entailment is more fundamental (for our grasp of propositions) than the relation of provability in a formal system. Whereas the relation of provability gives a mediating role to the logical *axioms* and to equations, the internal relation of entailment is direct.<sup>28</sup>
- 5.132, by contrast, concerns the act of inferring. It argues that inference gets its justification directly from the understanding of propositions, and that since understanding is informed by internal relations such as entailment — that in understanding the premises and conclusion, we must already recognize the justificatory relations between them — inference does not depend on the appeal to rules of inference, or any other mediating content. "*Schlussgesetze*" are senseless and superfluous.

Both arguments, taken together, reflect two complementary aspects of Wittgenstein's unique perspective on logic. For Wittgenstein, we do not construct a formal system of proof in order to fix our grasp of logical notions such as inference, proposition, entailment etc.. These notions are

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<sup>27</sup> Ricketts 1985 does not even mention 5.131, where this conception of proof is clearly on Wittgenstein's mind. Proops discusses it on pp. 288 & 294.

<sup>28</sup> I will return to discuss 5.131 in the next section, and treat it in more detail there.

already fixed by reference to the practices of thinking and reasoning (5.5563); we construct formal notations in order to clarify our thought, not in order to discover hitherto unknown properties of it.

### **4.3. Standard and Non-Standard Readings of 5.132**

I claimed that *Tractatus* 5.132 targets Frege's and Russell's rules of inference, rather than their conception of the role logical axioms play in proofs. This puts my reading in *prima facie* agreement with what I call the Standard Reading of 5.132.<sup>29</sup> But the standard line of interpretation takes Wittgenstein's criticism to merely reiterate the objection made by Lewis Carroll in his dialogue "What the Tortoise said to Achilles". As I've shown in the previous chapters, Frege's and Russell's conceptions of rules of inference are not vulnerable to Carroll's objection. Given that Wittgenstein had sufficient acquaintance with Frege's and Russell's work, ascribing this line of criticism to Wittgenstein is highly uncharitable. This observation forms the starting point of the non-standard reading of 5.132 that is offered by Ricketts and Proops.<sup>30</sup> Non-standard readers turn to seek an alternative target which Wittgenstein could coherently be taken to object to.

Even though I fully embrace the non-standard point that Wittgenstein's criticism in 5.132 cannot be charitably read as a mere repetition of Carroll's puzzle, I do not believe that we should

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<sup>29</sup> For example, Mounce 1981, pp. 44-47; Kenny 2006, p. 78; and Baker 1988, pp. 130-132.

<sup>30</sup> Hacking 1979; Ricketts 1985; Proops 2001.

give up the idea that rules of inference are its target.<sup>31</sup> Wittgenstein has other, substantive reasons to reject Frege's and Russell's rules of inference, apart from those that stem from Carroll's puzzle.

Let me highlight some of the conclusions of my discussion in the previous chapters. Carroll's puzzle shows that as long as the principle responsible for justifying the inference is treated as a premise, we would fail to provide the conclusion with full justification. For the whole inference cannot be justified by one of its parts; and in seeking its ultimate justification, we would be led to add more and more parts to the original inference, but continue to fail to justify it as a whole. What is needed to avoid the regress is a clear distinction between rules of inference and other propositions that guarantees that rules can be applied without altering the inference. In Frege such a distinction is quite explicitly made, and in Russell it is (arguably) also present. To this extent Carroll's point cannot be used as an objection to Frege and Russell. The standard reading does not bother to note this, and as soon as one raises the issue — as Ian Hacking does — the standard reading loses its plausibility. Here is how Hacking reasons:

*Tractatus* 5.132 says that “Laws of inference which — as in Frege and Russell — are to justify the conclusions, are senseless and would be superfluous.” This may be a legitimate objection to the primitive propositions \*1.1 and \*1.11 of *Principia Mathematica*. But if one had the view that anything not expressible in concept-writing is *sinnlos* and can at best be elucidated, one would find less of a difference between *Begriffsschrift* and *Tractatus* than Wittgenstein asserts.<sup>32</sup>

According to Hacking, Wittgenstein's criticism in 5.132 reiterates Carroll's point by targeting the lack of proper distinction between rules of inference and laws of logic. This is a problem which, according to Hacking, affects Russell, but does not affect Frege. Wittgenstein, according to

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<sup>31</sup> Michael Kremer offers an account of the criticism made in 5.132 that treats it as concerned with rules of inference, without attributing to Wittgenstein the Carroll-style objection. See Kremer 2014 and Kremer 2001, and cf. Chapter 3, section 3.3.

<sup>32</sup> Hacking 1979, p. 290.

Hacking, is a surprisingly bad reader of Frege.

Hacking is correct that Frege draws an explicit distinction between rules and laws, and explicitly notes that the former cannot, whereas the latter can be expressed in propositions of the formal language; and it is true that giving the rules of inference this extra-linguistic status blocks the possibility of Carroll's regress. But Hacking thinks that Frege and Wittgenstein are much closer than Wittgenstein is willing to admit on the issue of the distinction between what a formal language can say and what cannot be expressed in it.<sup>33</sup> Hacking correctly takes Wittgenstein to reject the idea that, when it comes to logical form, what lies beyond the bounds of the expressible can play a justificatory role vis-a-vis our proofs. Indeed, Hacking's own conception of logic aims to respect "Wittgenstein's strictures" by insisting that the rules of a natural deduction system "are not justifications of transitions".<sup>34</sup> And yet when we look closer at Hacking's reading of Frege, we find that he simply ignores those features of Frege's conception of rules of inference which commit him to the opposite view. Hacking cites as evidence for Frege's immunity to Carroll's regress paragraph #13 of the *Begriffsschrift*, where Frege says the following of his rules of inference: "[t]hese rules and the laws whose transforms they are cannot be expressed in the *Begriffsschrift* because they form its basis".<sup>35</sup> But the claim Frege makes here is not merely that rules are inexpressible in the *Begriffsschrift*; he also says that rules are transforms of laws, and thereby implies that they have content. It is precisely this issue that Wittgenstein targets, on my reading: the idea of justificatory, logical content that remains beyond

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<sup>33</sup> Hacking's reading of the relation between Frege and Wittgenstein resembles the one proposed by Geach 1976.

<sup>34</sup> Hacking 1979, p. 290.

<sup>35</sup> BS #13, p. 136, translation emended. Hacking cites Bynum's translation: "These rules and the Laws to which they correspond cannot be expressed in our "conceptual notation" because they form its basis". This translation masks some important issues, such as the important "transformation" relation between rules and laws that Frege alludes to. For that reason, perhaps, Hacking fails to see that Frege's rules of inference do not lack content and that they therefore must be rejected by Wittgenstein. And see my discussion of this paragraph in Chapter 2, section 2.1.

the bounds of expression.

Hacking's claim that Russell fails to offer an adequate response to Carroll's worry is also problematic. Admittedly, Russell indiscriminately refers to the rules of inference as well as to the axioms as "primitive propositions".<sup>36</sup> He also makes an ambiguous use of 'imply', that wavers between material implication and entailment, and in his formal language, the horseshoe sign ' $\supset$ ' suffers from a similar ambiguity.<sup>37</sup> But this does not mean that Russell is incapable of making a proper distinction between rules and laws, and that he therefore lacks a response to Carroll.<sup>38</sup> Like Frege, Russell takes the rules of inference to be a case in which "the formalism breaks down"; rule are inexpressible in the system which they govern.<sup>39</sup> Moreover, despite the ambiguities in his use of the term 'implication', Russell distinguishes two logical relation: whereas axioms and theorems involve material implication, rules express the relation of "therefore" which holds between true propositions only.<sup>40</sup> Russell's distinction between rules and laws involves two different employments of the formal system, rather than a difference between the expressions of the formal system. The application of rules of inference ultimately rests on an

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<sup>36</sup> *Principia*, p. 94.

<sup>37</sup> This seems be the issue Hacking points out by citing \*1.1 and \*1.11 of *Principia* as evidence.

Michael Kremer finds similar ambiguities in Russell's concept of implication, but his reasons for objecting to them are different from Hacking's. On Kremer's account, Wittgenstein's critique targets the way in which these ambiguities lead Russell to philosophical nonsense (See Kremer 2001, fn. 17 p. 68 as well as Kremer 2014). Unlike Kremer, I do not believe that Russell's proposal, that a single proposition may be used either as a premise or to indicate a rule of inference, is beyond hope. I discuss these issues in Chapter 3, section 3.4.

<sup>38</sup> I argue that Russell succeeds in answering Carroll in Chapter 3. Proops 2001 (p. 285) agrees that Russell is immune to Carroll's regress, though the case he makes is slightly different from mine.

<sup>39</sup> *Principles*, #45 pp. 40-41.

<sup>40</sup> *Principles*, pp. 34-35

act of recognition that is different in kind from the appreciation of propositional content.<sup>41</sup> This Carroll's regress does not arise.

Though it is not entirely impossible that Wittgenstein had made a mistake in taking both Frege and Russell to fail to block Carroll's regress of justification, it would be uncharitable to assume so. At least in the case of Frege, the distinction between rules and laws is explicit enough to allow us to assume that Wittgenstein could not have missed it.<sup>42</sup> This point is granted by Thomas Ricketts, and it leads him to modify Hacking's reading in the following way. Ricketts concurs that 5.132d can be read as using the Carroll puzzle to make a justified objection against Russell, and hence as calling for a sharper distinction between rules and laws. But Ricketts takes this criticism to apply to Russell only. To avoid ascribing to Wittgenstein a gross misreading of Frege, Ricketts reads 5.132d as a condensed expression of two separate lines of criticism — one against Russell and a different one against Frege.<sup>43</sup>

This is where the ambiguity of Wittgenstein's term '*Schlussgesetze*' turns into a resource for the non-standard reader. The charge against Frege, on Ricketts's reading, concerns the use of logical axioms in proofs, not the use of rules of inference. In fact, he takes both Russell and Frege to be vulnerable to this secondary line of criticism, inasmuch as they both treat axioms as implicit premises of all inference.<sup>44</sup> Frege's and Russell's conception of proof, according to

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<sup>41</sup> *Principia*, p. 98 & p. 106. Admittedly, Russell holds that rules and laws are internally related. Axioms can be converted into rules of inference, and vice versa. But the two modes of use of a logical principle are distinguishable: the indication of the rule applied is done by means of a marginal citation the corresponding axiom in square brackets; when the axiom is used as a premise, it appears without the brackets.

<sup>42</sup> On Wittgenstein's acquaintance with Frege's works, see Diamond 2010 (in particular, the appendix, pp. 592ff.) and Goldfarb 2001b.

<sup>43</sup> Ricketts 1985, p. 11ff.

<sup>44</sup> As Ricketts puts it, "the last sentence of 5.132 criticizes Frege's and Russell's attempts to give an axiomatic formulation to logic" (ibid.).

Ricketts, requires that “to establish that  $q$  implies  $p$  is to prove the conditional ‘If  $q$ , then  $p$ ,’ from basic logical laws”.<sup>45</sup> Wittgenstein’s criticism, according to Ricketts, works as follows:

Suppose  $p$  and  $q$  are propositions of a special science, and suppose that the assertion of  $q$  provides a basis for the assertion of  $p$ , as Frege would say. Then, contrary to Frege, in order to infer  $p$  from  $q$ , there is never any need first to deduce the conditional ‘if  $q$ , then  $p$ ’ from “logical laws”. The inference does not have to be mediated by the knowledge of the truth of any proposition. Frege urges the replacement in science of enthymatic [*sic*] reasoning with completely explicit, gap-free proofs. Wittgenstein’s point is that logical laws are irrelevant for this goal.<sup>46</sup>

Frege’s ideal of perspicuity in logical proofs means that ordinary proofs, before they are subjected to the rigor of Frege’s methods, are incomplete (enthymematic), and they remain so until one exposes all their implicit premises. To show that the explicit premises of the inference (call these premises ‘ $p$ ’) ground its conclusion (‘ $q$ ’), Frege requires that we proceed stepwise, appealing only to axioms and rules of the system, and thereby revealing whether the need for any additional premise arises, which has so far been relied on implicitly. The rules of the system will then guide us in deriving a conditional ‘if  $p$ , then  $q$ ’ from the axioms, and from it, alongside the premises ( $p$ ), the conclusion will follow. When the inference is valid, this proof-supporting conditional is either derivable from the axioms of the system or is an instance of one of them.

Ricketts reading is insightful, and shed much light on Wittgenstein’s criticism of his predecessors’ conception of proof, but taking 5.132 to be concerned with these issues is problematic on several counts. First, it is unclear why Ricketts takes both Frege and *Tractatus* 5.132 to be particularly concerned with enthymematic proofs *in the special sciences*, which logic indirectly grounds. The context of 5.132 makes no indication that its criticism fails to apply to Frege’s logical proofs, and to proofs where all the premises are already explicit. When

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<sup>45</sup> Ibid., p. 7. I have switched between ‘ $q$ ’ and ‘ $p$ ’ in this quotation from Ricketts to render it consistent with the use of propositional variables in the next quotation.

<sup>46</sup> Ibid., p. 12.

Wittgenstein discusses the notion of proof in the 6.126s, he does distinguish proof in logic from the logical proof of a significant, empirical proposition (6.1263). But there is no indication that extra-logical proofs pose a different sort of problem.<sup>47</sup>

Moreover, Ricketts fails to note that it is in *Tractatus* 5.131 that Wittgenstein criticizes Frege's way of construing the relation between ground and consequence in proof; in particular, Wittgenstein rejects the idea that this relation must be made explicit in a proposition that connects the ground and the consequent (such as the proof-supporting conditional in Ricketts's account). Let me quote 5.131 again and offer some additional clarification:

5.131 If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another, *and we do not need to first put them in these relations by connecting them with one another in a proposition*; for these relations are internal, and exist as soon as, and by the very fact that the propositions exist.<sup>48</sup>

In the highlighted sentence Wittgenstein rejects the very idea that Ricketts finds in Frege, namely that the validity of proof is secured by deriving a conditional that connects the premises and the conclusion, from the logical axioms.<sup>49</sup> This follows from Wittgenstein's underlying thought that inferential relations are internal relations, i.e. that they constitute the identity of the proposition, and thus that the full understanding of propositions makes this relation evident to us, such that no separate act of derivation is required. The same idea is drawn on in the rejection of rules in

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<sup>47</sup> Similarly, although Frege might have things to say about how to apply logic in the sciences, the procedures he devises are equally applicable for logical proofs, where all the premises are explicit.

<sup>48</sup> My emphasis. This is Ogden's translation, slightly emended by me. Admittedly, 5.131 can also be read as a rejection of Carroll's suggestion that in order to infer  $q$  from  $p$ , we first add the Hypothetical premise that 'if  $p$  is true, so is  $q$ '. If this is Wittgenstein's intention in 5.131, this does not change the overall shape of the dialectic, as I see it. It does not require that we take 5.132 to accuse Frege and Russell of having no response to Carroll's charge, and in particular, it does not require that we take Wittgenstein to think that Frege's and Russell's rules of inference give rise to Carroll's regress.

<sup>49</sup> Proops 2001 (p. 294) does notice that 5.131 rejects this Fregean view. However, Proops then goes on to assimilate the point made in 5.132 to the point made in 5.131. I discuss Proops's reading, below.

5.132: since the inferential relations are internal, an appeal to rules of inference is superfluous.<sup>50</sup> But that does not mean that the target of 5.132 and 5.131 are the same.

By taking 5.132 to be concerned with Frege's ultimate appeal to axioms in proofs, rather than with his conception of the rules of inference, Ricketts turns 5.132 into a fifth wheel in Wittgenstein's attack on his predecessors view of logic. There's no denying that Wittgenstein also has reasons to object to Frege's and Russell's conception of the axioms as laws that justify other propositions, as well as to the idea that every proof must ultimately lead back to the axioms. At various places in the *Tractatus* Wittgenstein explicitly rejects this hierarchical, axiomatic construal of logic and proposes instead that all logical propositions enjoy the same status (6.12, 6.122, 6.127, 6.1271). They are all tautologies; they are all equal in that they say nothing, and in that they all have the structure that allows them to reveal the logical relations that hold between their sub-propositions. This means that logical theorems do not require a derivation in order to be justified. And so the axioms have no real role to play in an account of the logical relation between propositions; nor do axioms have a role to play in justifying inferences in the sciences. The logical relations between material, contentful propositions can be appreciated by close inspection without appealing to abstract logical propositions.<sup>51</sup> This is the point that we have seen Wittgenstein make in Tr. 5.131. On my reading, 5.132 aims to make a separate point, rather than anticipate or repeat a point that is also made elsewhere.

Moreover, Ricketts's reading opens a huge gap between the surface structure of 5.132 and its actual meaning: *prima facie*, Wittgenstein does not advance two separate lines of criticism, the one against Frege and Russell, concerning their appeal to laws as implicit premises, and the other

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<sup>50</sup> The plausibility of this view will be discussed in greater detail in Chapter 6; for now, let us focus on the cogency of Ricketts's reading.

<sup>51</sup> I develop this interpretation of Wittgenstein's account of understanding in Chapter 6.

against Russell only, concerning his vulnerability to Carroll's regress.<sup>52</sup> A proposal for dissolving this tension comes from Ian Proops.

Proops takes 5.132 to be concerned with a single line of criticism that applies to both Frege and Russell, in almost the same way. Proops seems to assume, as Hacking and Ricketts do, that since Frege's conception of rules of inference is not vulnerable to Carroll's regress, it is not vulnerable to any other criticism either. In other words, the failure of Carroll's objection vis-a-vis Frege is a reason, for Proops, to think that Wittgenstein could not criticize Frege's conception of rules. However, Proops argues, against Ricketts (and implicitly, against Hacking), that Russell, too, has an adequate response to Carroll's worry. Thus according to Proops it would be equally gratuitous of Wittgenstein to attack Russell by appeal to Carroll's regress.<sup>53</sup> And so there is no need to claim that two separate charges are made in 5.132; on Proops's reading, neither Russell nor Frege are accused of having a flawed conception of rules of inference.

Proops holds that Wittgenstein's concern is not with rules of inference inasmuch as they "are supposed to justify inferences" (5.132d), but rather with a different feature of Frege's and Russell's conception of inference. Proops takes the term '*Schlussgesetze*' to refer to the laws of logic, rather than rules, and he holds that it is the role assigned to these laws in the account of the justification of inferences that is Wittgenstein's genuine target.<sup>54</sup> On Proops's reading of Wittgenstein, the justification of an inference derives from the internal relation of entailment that

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<sup>52</sup> To be sure, that this is the *prima facie* reading of Wittgenstein does not exclude the possibility of the other readings — readings according to which Wittgenstein pursues two separate lines of criticism in tandem. A similar problem arises in other passages in which Wittgenstein attacks Frege and Russell together, such as 5.521.

<sup>53</sup> Proops 2001, p. 286.

<sup>54</sup> Proops 2001, p. 283f. As I've argued above, this way of reading '*Schlussgesetze*' conflicts both with the immediate context of 5.132 as well as with pre- and post-tractarian evidence that the true target of 5.132 are rules of inference. Proops simply fails to consider the evidence that comes from texts outside the *Tractatus*. This failure has already been noted by Kremer 2002b.

holds between the premises and the conclusion — the relation Wittgenstein speaks of in terms of “following”.<sup>55</sup> 5.132, on Proops’s reading, consists of an objection to Frege’s and Russell’s own attempts to account for this relation of entailment:

What Wittgenstein means to be challenging at 5.132 is a view about what it is for two propositions to stand in the relation of logical entailment. Wittgenstein takes logical entailment to consist in an “internal” relation ... The question what justifies the inference from  $p$  to  $q$  is equivalent to the question what would justify us in asserting that  $q$  on the basis of the supposition that  $p$ . And this question is answered by citing whatever fact one takes the relation of logical entailment to consist in. If these thoughts are on the right lines, then Wittgenstein would seem to be criticizing Frege and Russell for having given accounts of logical entailment that somehow involve an essential appeal to logical laws (and possibly to inference rules).<sup>56</sup>

But it is not at all clear that there is an account of entailment in Frege and Russell, nor that Wittgenstein thinks there is a different “fact” to which we ought to appeal to justify a proof. Even though Frege and Russell are expressly concerned with securing the validity of proofs, it is a mark of their universalist conception of logic that they lack the kind of semantic perspective from which questions about validity and entailment are nowadays raised and answered.<sup>57</sup> Elsewhere, Proops explicitly denies that Frege and Russell are committed to this universalist position, and that they are prevented from giving entailment a proof-theoretic account.<sup>58</sup> He holds that in both Frege and Russell think of the relation of entailment is thought of in proof-theoretic terms. Since all proof within their systems depends on an appeal to the axioms, the rejection of “*Schlussgesetze*” in 5.132 should be read as a rejection of the role these proof-

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<sup>55</sup> E.g. in 5.132a. Proops seems to blur the differences between Wittgenstein’s and the post-Tarskian notion of entailment.

<sup>56</sup> Proops 2001, p. 288.

<sup>57</sup> According to Van Heijenoort 1967, from Frege’s universalist point of view, we can only respond to metatheoretical questions “experimentally”, by actually conducting proofs, and so a general account of entailment cannot be given. Ricketts 1985 follows Van Heijenoort in attributing such a point of view to Frege.

<sup>58</sup> Proops 2007

theoretic accounts of entailment give to the axioms. Wittgenstein, according to Proops, criticizes his predecessors' attempt to reduce the internal relation of entailment to the relation of provability from axioms.

Russell's account of entailment, according to Proops, is stated in terms of the *possibility* of deducing a conditional proposition that connects the conclusion and the premises from logical laws:

... Russell's view would seem to be that the relation that justifies inference is just the relation of logically grounded implication. (I shall call an implication "logically grounded" just in case it is derivable as a theorem of Russell's system.)<sup>59</sup>

Frege's account of entailment similarly arises from his conception of proof:

[For Frege] The idea would be to say that a thought  $q$  objectively depends on another  $p$ , if  $q$  can be derived from  $p$  together with primitive logical laws, using only primitively logical rules of inference.<sup>60</sup>

Proops understands Frege's and Russell's conception of proof along the same lines highlighted by Ricketts. Proops and Ricketts also agree that the element that grounds a Fregean or a Russellian proof is the derivation of a proof-supporting theorem from logical laws, a conditional proposition whose antecedent contains the premises and whose consequent contains the conclusion. Proops's perspective differs from Ricketts's, however, in that for him the issue is not whether logical laws actually serve as implicit premises in this procedure; it is the question whether an account of entailment can be given in terms of the *possibility* of this procedure.<sup>61</sup>

Whereas Ricketts focuses on justification, Proops focuses on the role of the notion of entailment

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<sup>59</sup> Proops 2001, p. 290.

<sup>60</sup> Proops 2001, p. 292. Note that rules of inference play a role in this account, but Proops does not think they are singled out in a way that would make them plausible candidates for Wittgenstein's criticism.

<sup>61</sup> See Proops 2001, p. 287; and cf. Van Heijenoort 1967, p. 327.

in their conception of justification; Ricketts thinks that Wittgenstein rejects the claim that justification can only be achieved through deductions that lead back to the axioms; Proops thinks that Wittgenstein rejects the claim that justification can be *defined* in terms of deducibility. Ricketts's picture is more in line with Frege's and Russell's commitment to the universalist conception of logic, which is well-documented in their elucidatory and programmatic remarks. Frege and Russell almost never seem concerned to develop a meta-theoretic notion of entailment, of the kind Proops ascribes to them.

The evidence Proops marshals for his view is far from robust. The texts he cites are minor or unpublished works, and the ideas of deducibility, provability and entailment that seem to be at play in them play no role in Frege's and Russell's central works. Proops finds that in one of Frege's later works, the justification that one proposition gives to another is accounted for in terms of the "provability" of the conclusion from the premises.<sup>62</sup> And Proops also finds that in one of Russell's unpublished papers the justification of a proof is said to depend on the relation of "deducibility" holding between propositions.<sup>63</sup> Even though Proops is at pains to show that Wittgenstein might have been acquainted with these texts, he admits that there is no direct evidence for it.

What seems to motivate Proops in seeking such an account of entailment in Frege and Russell is both the fact that such an account plays a foundational role in modern semanticist approach to logic, as well as the fact that Wittgenstein's account of internal relations that hold between propositions that imply one another seems to Proops to anticipate this modern semantic

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<sup>62</sup> Frege 1984h (FG2) develops the notion of "dependence", which Proops understands in terms of "provability". But, as Ricketts 1997 argues, Frege's tentative foray into metalogic in 1984h is ultimately declared to be a theoretical dead-end.

<sup>63</sup> Russell develops the notion of "deducibility" in a 1905 paper which was never published. See Russell 1994b.

account. Only towards the end of his paper does Proops admit that Wittgenstein's talk of internal relations is subject to other things Wittgenstein says about how we should understand his aims in the *Tractatus*, which, as Proops admits, is best understood along the lines proposed by the Resolute Reading. The notion of internal relations is deployed as a means of elucidation, but it should not be taken to amount to an attempt at a meta-logical theory.<sup>64</sup> Nonetheless, in the body of his paper Proops proceeds as though Wittgenstein's objection to Russell and Frege concerns a substantive difference of opinions — as though the point is to replace one mistaken account with another, more correct account.

In order to attribute to Wittgenstein the concern to criticize Frege's and Russell's accounts of entailment, Proops must downplay those features of 5.132 that clearly indicate that what Wittgenstein is concerned with are rules of inference, rather than laws of logic. Perhaps Proops thinks, like Ricketts and Hacking, that Wittgenstein has no call to criticize Frege's and Russell's conceptions of rules of inference. It is my contention that Wittgenstein has ample reason to reject Frege's and Russell's ideas of rules of inference, even though his reasons have little to do with Carroll's puzzle and are not blocked by the kind of distinctions between laws and rules that Frege and Russell draw. Even if Proops is right that Wittgenstein's view differs from Frege's and Russell's in that Wittgenstein does, and they do not, acknowledge the difference between the concepts of entailment and the concept of provability, and even though Proops and Ricketts are right that Wittgenstein would reject Frege's and Russell's conception of inferential justification— this does not change the fact that Wittgenstein also has reasons to criticize Frege's and Russell's conception of rules of inference. And this he does in 5.132.

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<sup>64</sup> Proops 2001, p. 300. The Resolute Reading of Wittgenstein's *Tractatus* will be extensively discussed in chapters 5-7; Proops cites Diamond 1991a and Goldfarb 1997 as proponents of this reading. I discuss this literature in Chapter 5.

#### 4.4. The Real Problem in Frege's and Russell's Accounts of Inference

The exceptional status that rules of inference end up having in Frege and Russell, as inexpressible but contentful logical principles, exposes an underlying tension between their own fundamental commitments.<sup>65</sup> On the one hand, there's the commitment to the idea that logic captures the most fundamental laws that guide our thought and judgment everywhere; on the other hand the commitment to logicism, that logic has content of its own, which we need to make fully explicit and thereby show that other kinds of knowledge are grounded in it.

Both Frege and Russell assert that rules and laws stem from a single source; in their systems, rules are transformable to laws and vice versa, and the general logical content which they ascribe to laws is thereby also ascribed to rules of inference. They construe rules in terms of logical contentfulness — terms that are taken from the logicist account of the generality of the axioms.<sup>66</sup> Hence, given the realization that rules of inference cannot be expressed in formal propositions, both thinkers stretch the idea of logical content beyond the realm of the sayable.

Wittgenstein, on the reading I develop in the next chapters, targets this aspect of Frege's

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<sup>65</sup> This is the tension that Thomas Ricketts identifies between Frege's "official construal of the content of the axioms" and his "view of judgment", in Ricketts 1985 p. 3f; this tension in Frege and Wittgenstein's response to it is also discussed in Diamond 1991e. Putnam 2000 construes this tension in terms of the conflict between the Kantian thought (which Frege inherits) that there is no such thing as illogical thought, and the idea that the laws of logic are the utmost general truths. See Conant 1991, p. 127 for discussion. Conant himself discerns a different but related tension in Frege, between two the austere and the substantial conception of nonsense. See Conant 2001, p. 381, and my discussion in Chapter 5.

I believe that a version of the very same tension can be ascribed to Russell, and I discuss this point in Chapter 3 .

<sup>66</sup> Frege explains the ability of the logical system to yield arithmetical truths in terms of the content encoded by the axioms taken together with the rules of the system; and Russell is comfortable speaking of rules of inference in terms that assimilate them to propositions; he says rules play the role of a general premise from which the inference itself (not its conclusion) follows. See Chapter 2, section 2.5 and chapter 3, section 3.212 for a detailed discussion of these points.

and Russell's idea of content — the idea that it makes sense to think of logic in terms of content that lies beyond the reach of language. To put Wittgenstein's position in very broad terms, he thinks that the universality of logic cannot be accounted for as long as the axioms are treated as having a substantive logical role. Wittgenstein's task is not to deny that logic extends beyond the limits of language; his aim is to show that this is neither a true nor a false claim, but rather a disguised form of nonsense. Wittgenstein does not resolve the tensions in the thought of his predecessors by picking and choosing the views that can be salvaged from their philosophy of logic; he responds to these tensions by rethinking the very terms in which these views are cast — the very idea of logic, language, content, understanding, judgment and inference.

On my reading of the *Tractatus*, there is no such thing as inexpressible content; in fact, there is no such thing as *logical* content at all — universal or not, expressible or not. This is the main point of Wittgenstein's distinction between saying and showing — to expose the incoherence of thinking about logic as a realm of substantive content.<sup>67</sup> Frege's and Russell's treatment of the rules of inference is a prime example of such an attempt to “say” what can only “be shown” in our use of a language. It is an attempt, to use Michael Kremer's phrase, to “take care of logic”, rather than to let logic “take care of itself” — as Wittgenstein recommends in 5.473.<sup>68</sup>

We are now in a position to bring into clear view the framing assumption that separates Frege and Russell from Wittgenstein. Frege and Russell assume that inference requires a mediation between the premises and conclusion. This assumption is also the one to which

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<sup>67</sup> This distinction forms the main topic of the next chapter.

<sup>68</sup> See Kremer 2014. For further elaboration of the Wittgensteinian idea that philosophers tend to seek grounding even where no grounds could be given, see Kremer 2001, esp. p. 70, fn. 24: “*Logic* itself is, I maintain, “beyond grounds.” It is neither a source of justification nor something to be justified.”

Carroll's puzzle poses a challenge; but it survives intact even where Carroll's objection cannot be raised. In fact it is still in force in many recent accounts of inference.<sup>69</sup> Russell assumes this kind of mediation in thinking of the justification of inference in terms of a *sui-generis* act of appreciating the applicability of rules. Frege's case is harder to compare; but for him, too, inference is only valid if it is performed "*in accordance with logical laws*", and the restrictions placed by the system, that all inference be carried out by means of one of the official rules, is meant to guarantee this mediation.<sup>70</sup>

The very idea that mediation is needed — whatever form it ends up taking — puts Frege and Russell in agreement with Carroll on this one point: that without mediation, the premises and conclusion are inert, in the sense that merely understanding each of these propositions does not yet entitle us to treat the conclusion as justified. Compare this to Wittgenstein's claim in 4.243 — that understanding two internally related expressions means that "it is impossible for me not to know" that they are so related. On Wittgenstein's view, understanding each of the propositions in an inference prevents us from treating the conclusion as unjustified.<sup>71</sup>

If understanding propositions and appreciating their inferential relations are truly separate acts, as Frege and Russell assume, how do these acts ever combine? In what way do these acts form part of one single mind? Frege and Russell are far from clear on these issues. Wittgenstein, by contrast, is not open to such criticism, since for him there is no real separation between the inert and the dynamic aspects of understanding. Admittedly, Wittgenstein speaks of "perceiving" the internal relations between propositions, and that might seem to put him in proximity to

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<sup>69</sup> For example, in Boghossian 2014, discussed in the previous chapter, section 3.5.

<sup>70</sup> Frege 1984h, p. 318.

<sup>71</sup> Note that these points do not depend on Wittgenstein's view of the truth-functionality of all logical complexity, or on the account of entailment in terms of truth-grounds containment proposed in *Tractatus* 5.1ff.. For inferences would be grounded by internal relations even if these relations are not truth-functional.

Russell, who speaks of “perceiving” that the rule applies.<sup>72</sup> But in Wittgenstein, there is no third element (e.g. the contentful rule) which is being perceived, and which is external to the understanding of the propositions themselves (to use terms which I will develop in the next chapters, internal relations are perceived inasmuch as our use of propositions reflects them — not inasmuch as we acknowledge a further fact about the propositions). Wittgenstein thinks that our understanding of propositions is sufficient, on its own, for the justification of our inferences. This conception of inference requires a much richer account of what it takes to truly understand a proposition than the one we find in Frege, Russell, or Carroll.

For Wittgenstein, formal rules are only needed where we do not have the full resources of understanding at our disposal. He grants (at 6.126), that we *can* construct a rule-governed calculus of proof, and that having such a calculus at our disposal would make our life much easier. It would allow us to simplify proofs — so long as in designing the calculus, we make sure that the rules mirror the logical form of our thinking (5.514).<sup>73</sup> But Wittgenstein adds that the relative usefulness of such a calculus does not change the fact that in principle we could do without it. The justification of inferences does not depend on the relations between *signs* that this calculus governs; it has to do with relations between propositions, so far as these propositions are fully understood. In fact, we cannot even individuate a proposition in abstraction from the role it plays in the full-fledged activity of understanding of a competent thinker. Only as situated in a logical space is the proposition fully determinate; only then is it properly understood. (3.42). By

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<sup>72</sup> See *Tr.* 5.13, 6.122 and 6.232.

<sup>73</sup> Frege makes a similar point in the introduction to *The Foundations of Arithmetic*:

It is possible, of course, to operate with figures mechanically, just as it is possible to speak like a parrot: but that hardly deserves the name of thought. It only becomes possible at all after the mathematical notation has, as a result of genuine thought, been developed so that it does the thinking for us, so to speak. (Frege 1960, p. iv)

rejecting the need for rules of inference, Wittgenstein is rejecting the idea that propositions are inert objects, into which we breathe life, by means of separately specifiable logical rules.<sup>74</sup>

#### 4.5. Conclusion

In this chapter I argued for reading 5.132 as a criticism of Frege's and Russell's conception rules of inference. But I only hinted at the general shape of the account of logic and language that grounds Wittgenstein's criticism. I have not, thus far, shown why Wittgenstein should think that Frege's and Russell's rules of inference are *senseless* and *would be superfluous*. As I will argue in the next two chapters, the criticism of 5.132 forms part of a much larger endeavor to root out a deep confusion that underlies philosophical theorization. Such confusion takes the shape of an attempt to *say* what can only *show* itself. This happens, for instance, whenever one tries to treat the logical forms of thought as substantive facts that must be made explicit, put up for considerations, and treated as though they themselves are in need of justification.

Frege's and Russell's accounts of rules of inference are a case of philosophical confusion in this sense. They realize that at least some aspects of logic cannot be treated as the content of our judgments, and they take that to indicate the inexpressibility of some logical content. But a proper response to this realization, according to Wittgenstein, would require that they renounce the temptation to speak of logic as content, and instead, learn to speak of it as pure form. Because Frege and Russell are committed to the idea that the logic provides the foundation of mathematical knowledge, they continue to think of logic as contentful, while cleaving the realm of logical content into two parts: the expressible and the inexpressible.

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<sup>74</sup> This conception of understanding will be elaborated in Chapter 6.

We may put the difference between Wittgenstein and his predecessors in the following, rough form: Frege and Russell want to assure us that every content relevant to logical justification is expressible in the logical formalism, and they consider the inexpressibility of rules as a lamentable, but unavoidable defect of our language. Their treatment of rules bespeaks of a feeling of resignation; Wittgenstein, on the other hand, treats such cases as cause for *satisfaction*. As he says: “When I say: Here we are at the limits of language, that always sounds as if resignation were necessary at this point, whereas on the contrary complete satisfaction comes about, since *no* question remains”.<sup>75</sup> That we cannot express logical form separately from the activity in which we actually put it to use is not a lamentable predicament, for Wittgenstein, for there is nothing substantive — no content — that we are thereby barred from expressing. This follows from Wittgenstein’s distinction between saying and showing, to which I turn in the next chapter.

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<sup>75</sup> Wittgenstein 2005, p. 310. On this remark, and its relation to the Tractarian critique, see Diamond 2012.

## 5. Saying and Showing

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The goal of the *Tractatus* is to rid philosophy of the fundamental confusions which plague it. According to Wittgenstein, philosophical puzzles ultimately rest on indeterminacy in our use of terms (3.324; 4.003; 5.473). By exposing this indeterminacy, we make the appearance of philosophical depth disappear; it becomes clear to us that the puzzle, as well the suggested solutions, were mere nonsense. Wittgenstein inherits this critical task from Frege and Russell, who themselves sought to resolve philosophical problems by attending to the underlying logical form of our expressions.<sup>1</sup> However, in devising their logical systems, Frege and Russell fail to appreciate “the cardinal problem of philosophy”, according to Wittgenstein.<sup>2</sup> They fail to recognize the special status of the form of thought, and so they end up blurring the distinction between what propositions *say* and what *shows* itself in our use of propositions. It is in this context that I wish to place Wittgenstein’s rejection of Frege’s and Russell’s conception of rules

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<sup>1</sup> This task, in its various manifestations, can be seen at work in Frege’s *Foundations of Arithmetic* (Frege 1960, abbreviated as FA) as well as his “On Concept and Object” (Frege 1984c, abbreviated as CO) and in Russell’s “On Denoting” (Russell 1905).

Another important source of inspiration for Wittgenstein’s understanding of the task of philosophy is found in Heinrich Hertz’s idea that the term “force” is used indeterminately by physicists, and that this indeterminacy gives rise to a confused wish to discover the “nature” of force. The removal of the indeterminacy will not give us an answer to the physicists’ question; it will show us that this question is not one we need to answer. See *Tr.* 3.323-3.325; Hertz 1973; and the discussion in Kremer 2007, p. 150 and Kremer 2012.

<sup>2</sup> “Letter to Russell, August 19, 1919”, in 1905 2008, p. 98.

of inference.

The charge made in *Tractatus* 5.132 — that Frege’s and Russell’s rules of inference are *senseless* and *would be superfluous* — is deeply rooted in Wittgenstein’s view of language and logic. According to the account of the understanding that I develop in Chapter 6, to *understand* a proposition is to engage in a logically informed activity.<sup>3</sup> It is, in fact, a precondition of the understanding of any proposition that we be able to tell whether it follows from any other proposition we understand. If the propositions that figure in an inference are truly and fully understood, their logical forms and relations must already be manifest. We must already be able to see that the premises justify the conclusion, and we must already have all the permission needed to infer it. Where full understanding is available, no work is left for rules of inference to do. Given that the mastery of logical form is a precondition for any significant use of language, it is *nonsensical* to insist that one can fully understand two propositions that stand in the proper internal relation, yet is unable to infer the one from the other before she appeals to an independently given principle, a logical fact, namely the rule of inference.

My aim in this chapter is to inquire into the distinction which provides the ultimate grounds for Wittgenstein’s critique of Frege and Russell — the distinction between what can be said and what only shows itself in what we say. Wittgenstein holds that the preconditions for the use of language must show forth of themselves whenever we actually use language. The preconditions of understanding are not only something that it would be superfluous to state, by means of a rule; there is no such thing as representing these preconditions. The idea that we can represent logical form is a covert form of *nonsense*; for, as Wittgenstein puts it in 4.1212, “[w]hat *can* be shown *cannot* be said”.

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<sup>3</sup> I develop this Wittgensteinian conception of understanding in the next Chapter.

What is *shown* — in logical propositions, as well as in ordinary, contentful propositions — is what we need to have mastery of if we are to *say* and understand anything. What is shown, in other words, is logical form. This is the *only* object of showing that figures in the arguments I develop in this dissertation. I do not wish to rule out the possibility of a resolute interpretation of the remarks made in the *Tractatus* concerning the way in which ontology, ethics, aesthetics and faith are subject to *showing*.<sup>4</sup> I simply wish to bracket these issues here.

Wittgenstein's distinction between what is said and what is shown is not novel; in fact there is a sense in which it is already at work in Frege and Russell. It is the strictness with which Wittgenstein draws this distinction that is unprecedented. What shows itself cannot be said; and yet philosophers are repeatedly and confusedly tempted to think of it as something which fails to be said, as a matter of coincidence, as it were. But logical form is not an object or a fact, and it is not even an ineffable object or a fact. To truly appreciate logical form as *form*, we must avoid treating it as a kind of *content*. We must learn to appreciate it by looking at it in action, as it informs the activities through which we make contentful claims. For instance, we can allow it to show itself by comparing the ways in which different languages fulfill the same task. But we must not get attached to the elucidatory modes of speaking through which we clarify logical form. The grammar of these elucidations is inevitably misleading. It portrays logical form in substantive terms, presenting it as a fact, or as an object that stands in a certain relation to other objects. These elucidations ultimately amount to nonsense, and they must eventually be discarded.

Frege's and Russell's construal of the rules of inference is a violation of Wittgenstein's distinction. Though they both acknowledge that the rules of inference cannot be *said*, they take

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<sup>4</sup> But see e.g. the attempts to make sense of the remarks on ontology and ethics, in Friedlander 2001 and Kremer 2001.

them to have some kind of quasi-propositional content, which grounds and justifies the inference. Frege and Russell thus stretch the realm of logical content beyond the realm of expression; they attempt to achieve precisely what Wittgenstein denies: to *say*, or quasi-say, what can only be *shown*. Frege and Russell treat that which can only *show* — the logical form of inference — as something which, even though it cannot be *said*, can be spoken of in terms of the content of that impossible saying. Their rules are un~~sayable~~ sayables.

Wittgenstein thinks that this is a confused idea, which derives from a tendency to misconstrue the nature of language and thought, to pretend we can observe and discuss the form of our thought from an imaginary point of view that is external to it.<sup>5</sup> For all their sensitivity to the logocentric predicament, Frege's and Russell's contentful view of rules of inference succumbs to this illusion.

By articulating the way in which Wittgenstein's distinction is drawn and by explaining why one needs to draw it in this way, the present chapter aims to reveal the inadequacy of Frege's and Russell's approaches to rules of inference. What they should have been led to see, in their reflections on inference, is not that some logical content cannot be captured in their symbolism, but rather that logic as a whole has no content. Logical form cannot be expressed by any means; nor does it need to be. Frege's and Russell's rules of inference are rejected not because the work they are supposed to do belongs in the shadowy realm of what *shows*, and cannot be said, but rather because there is no such "work", and no substantive logical content, sayable or showable, that is involved in inferring.

In the first section of this chapter (5.1) I contrast two ways of interpreting the distinction

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<sup>5</sup> Cf. Diamond 1991e, p. 185. Wittgenstein's conception of philosophical nonsense is further discussed in the next section; in section 6.6 of the next chapter I conclude that Frege's and Russell's treatment of rules of inference amounts to a form of philosophical nonsense.

between saying and showing that prevail in the scholarly literature: the resolute reading and the metaphysical or ineffabilist reading. I endorse a form of the resolute reading; and I argue that the ineffabilist reading is incapable of making sense of Wittgenstein's criticism of his predecessors' conception of rules, precisely because it takes the idea of inexpressible but contentful logical principles to be one which Wittgenstein himself endorses.

The following two sections motivate my version of the resolute reading. It is widely held that Frege's distinction between elucidatory discourse and descriptive discourse, in the context of his attempt to distinguish concepts and objects, is the source of inspiration behind Wittgenstein's distinction between saying and showing. In section 5.2 I argue that Wittgenstein's distinction is best understood as the result of his criticism and rejection of the irresolute aspects of the Fregean conception of elucidation. Section 5.3 reconstructs the Tractarian argument that shows that the distinction between saying and showing is necessary in order to make sense of the very idea of representation. I discuss some of the objections raised against Wittgenstein's distinction, which take the form of rejecting the assumptions that make up Wittgenstein's conception of representation. Finally, I conclude that the distinction between saying and showing, insofar as it applies to logical form can be given a coherently resolute reading.

### **5.1. Resolute and Irresolute Approaches to the Distinction Between Saying and Showing**

The distinction between saying and showing stands at the center of a debate that divides contemporary Wittgenstein scholarship into two camps: the so-called "resolute" readers,<sup>6</sup> and the

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<sup>6</sup> The term "resolute" was coined by Thomas Ricketts and first made public by Warren Goldfarb, in Goldfarb 1997; and see the discussions of the various manifestations of the Resolute approach in Conant and Diamond 2004 and in Goldfarb 2011. A review and extensive bibliography of the debate may be found in Bronzo 2012.

so-called “ineffabilist”, “metaphysical” or simply “standard” and “traditional” readers.<sup>7</sup> I will argue that Wittgenstein’s critique of his predecessors’ conception of rules of inference can only be properly explained by a resolute reading of the *Tractatus*. The purpose of this section is to lay out the terms of the debate.

Standard readers take Wittgenstein to find himself in a bind, faced with what Geach calls “Ludwig's self-mate”.<sup>8</sup> He supposedly wishes to advance substantive metaphysical theses, but he is also committed to the unintelligibility of the expressions which these theses take. The distinction between saying and showing is taken to be crucial for saving Wittgenstein’s from this bind. For if we are to do as Wittgenstein asks in 6.54, and “recognize [Wittgenstein’s own] propositions as nonsense”, what would be the status of the theory of meaning that the *Tractatus* itself (allegedly) propounds? But without that theory, how would we be able to recognize whether anything is senseful, senseless or nonsensical? And given how this theory determines what counts as nonsense, how should we make sense of the propositions in which that theory itself is pronounced? It is by means of the doctrine of showing that Wittgenstein supposedly succeeds in conveying the crucial ineffable truths that save him from inconsistency.

The Tractarian theory of meaning, on this account, consists in true insights whose

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<sup>7</sup> Anscombe 1959; Hacker 1972; Kenny 1973; Geach 1976 Fogelin 1976; Pears 1987; McGuinness 1988; and Baker 1988.

There are other types of reading that I leave out of discussion here. One line reading which is quite common in popular accounts of the *Tractatus*, though less so in contemporary scholarship: the “mystical” reading suggested by Bertrand Russell in his “Introduction” to Wittgenstein 1922 (discussed in section 5.3.6, below). Another influential reading is the one advanced by the logical positivist followers of Wittgenstein, particularly Carnap 2000 (discussed below). A more recently developed line of reading, which attempts to overcome the divide between the resolute and the ineffabilist readings, is found in McGinn 2007 and Hutto 2003. For a discussion and critique, from a resolute standpoint, see Hutchinson and Read 2006.

<sup>8</sup> Geach 1976, p. 54f. Hacker pictures the situation as the sawing of the branch on which one is sitting. See Hacker 2000, p. 361.

expression fails to make sense. But, even though the putative theory of meaning of the *Tractatus* condemns itself to being unsayable, it continues to be treated as true. This is why this reading is sometimes called an “ineffabilist” reading — it takes Wittgenstein to propound deep but ineffable truths about language and the world. Importantly, the say-show distinction, on this account, turns on itself: the distinction itself is something true that can only *show*, and cannot be said. It too is ultimately ineffable.

The idea of ineffable content is the least common denominator of the various interpretations proposed by standard readers of the *Tractatus*. Anscombe’s proposal is exemplary:

An important part is played in the *Tractatus* by the things which, though they cannot be ‘said’, are yet ‘shewn’ or ‘displayed’. That is to say: it would be right to call them ‘true’ if, per impossibile, they could be said; in fact they cannot be called true, since they cannot be said, but ‘can be shewn’, or ‘are exhibited’, in the propositions saying the various things that can be said.<sup>9</sup>

Anscombe holds that the distinction between saying and showing allows legitimate sentences to both say whatever it is they do say, and in addition to that, to show ineffable truths. If we tried to express these directly, our sentences would amount to nonsense. Nonetheless, for Anscombe it makes sense to say that what shows itself are “things that would be true if they could be said”.<sup>10</sup>

There are different variants of ineffabilism, differing with respect to the question what does the showing — is it the nonsensical propositions that purport to say the things that turn out to be ineffable? Can the senseless tautologies of logic and the pseudo-propositions of mathematics have that secondary function? Or is it the sensical, empirical propositions which do that? Anscombe herself allows the nonsensical propositions to do at least part of the showing. She

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<sup>9</sup> Anscombe, 1959, p. 162.

<sup>10</sup> Ibid.

talks of the sentences of the *Tractatus* as being “helpful, in spite of their being strictly nonsensical according to the very doctrine that they propound”.<sup>11</sup> The idea is that the only way in which the nonsensical expressions of the *Tractatus* could be considered as helpful is if they somehow pointed towards ineffable truths, though they cannot count as communicating or showing them. That they are helpful in this way is evident, Anscombe thinks, from the fact that asserting the negation of at least some of the Tractarian sentences would bring about “more error, and more darkness” whereas as they stand, i.e. as supposedly attempting to say the things that are shown, they point us in the right direction.

Peter Hacker advances a similar reading. On the one hand he criticizes Max Black for thinking that the Tractarian nonsensical propositions do some of the work of showing:

Wittgenstein’s own propositions, which Black called ‘formal statements’, are, by the lights of the *Tractatus*, nonsensical pseudo-propositions. They show nothing at all. The propositions that are held to show the ineffable truths which the *Tractatus* seems to be trying to say are not the pseudo-propositions of the book but well-formed propositions (including the senseless propositions of logic).<sup>12</sup>

Hacker denies that nonsense succeeds in showing what we fail to say. He holds instead that the ineffable truths show themselves in features of the symbolism, i.e. in the use of non-nonsensical sentences:

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<sup>11</sup> Ibid.

<sup>12</sup> Hacker 2000, p. 356. Hacker’s objection to Black 1964 assumes that he takes the set of nonsensical statements of the *Tractatus* (which Black discusses on p. 382) to include within it the statements of “logical syntax” (which he discusses on p. 381, and equates with logical and mathematical statements). As Cora Diamond points out (in private correspondence) this is not a correct account of Black’s view.

Be that as it may, it is true that Black takes nonsensical statements to be capable of showing, whereas Hacker denies that (and this puts him closer to the resolute readers, who similarly deny it).

The difference between Black’s and Hacker’s view on the status of nonsensical expressions is discussed in Kremer 1992, p. 417f.

In the course of the book, Wittgenstein asserts many different kinds of truths that *stricto sensu* cannot be said, but that are held to show themselves in features of the symbolism.<sup>13</sup>

At the same time, and like Anscombe, Hacker holds that when we utter nonsense, what we *mean* to say is correct; it is *merely* our modes of expression that fails us:

What one *means* when one tries to state these insights is perfectly correct, but the endeavour must unavoidably fail. For the ineffable manifests itself, and cannot be said.<sup>14</sup>

This is a Fregean-sounding thesis, and I will scrutinize its Fregean pedigree in the next section; I will also argue that Wittgenstein repudiates this very thought. The crucial point to note, for the moment, is that the failure of the attempt to *say* the so-called ineffable truths does not indicate to Hacker that there is something amiss with thinking of what we fail to express as contentful truths. Perhaps Hacker would deny that he gives nonsense the role of showing ineffable truths; but nonsense continues to play a positive function for him, since what it fails at is in directing us to see true (but ineffable) insights. Nonsense is conducive, however indirectly, to our seeing truths that only show.

It would be useful to summarize the main features of the ineffabilist line of interpretation and contrast them with the resolute reading. There are five points I'd like to highlight:

1. According to the ineffabilist readings, the *Tractatus* contains a theory of meaning, a consequence of which is the claim that some propositions are nonsensical. But this theory of meaning also has the consequence that the propositions in which it itself is propounded — the entirety of the *Tractatus*, perhaps — are considered by it to be nonsensical. Resolute readings deny that the *Tractatus* aims to offer a theory of meaning. On the contrary, resolute

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<sup>13</sup> Hacker 2001, p.146.

<sup>14</sup> Hacker 2000, p. 382.

readers point to remarks in which Wittgenstein flatly rejects the need for such a theory: “logic must take care of itself” (5.472); “Every possible proposition is legitimately constructed” (5.4733).<sup>15</sup>

2. Ineffabilist readers tend to think that there are two concepts of nonsense Wittgenstein deploys — mere nonsense and substantial nonsense. The latter form of nonsense arises when intelligible words, which supposedly belong to fixed grammatical categories, are combined in so-called illegitimate ways. This forms “deep” or “illuminating” nonsense, which succeeds in showing something which language would not allow us to say. Indeed, such is supposedly the nature of Tractarian propositions, by contrast to the mere nonsense of “piggle wiggle tiggie”.<sup>16</sup> Resolute readers object to treating ‘nonsense’ as a well defined grammatical category of propositions — a category for the identification of which we would indeed require a theory of meaning. According to the resolute reading of 5.473 and 6.53, we are never prevented from making new assignments of meaning to any of our signs; grammar does not fix the meaning of a sign independently of a specific context of use. We cannot rule out in advance that “piggle wiggle tiggie” might, like any other expression, be meaningfully used. Wittgenstein’s term ‘nonsense’, according to the Resolute reader, does not attach to any form of words independently of the use and misuse that one makes of it. The resolute reading thus denies that there are two types of nonsense: all nonsense is mere nonsense, the result of a failure, or an insistence on the part of the speaker, to make arbitrary assignments of meaning to some of the signs she uses. If it is nonsense, it is only

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<sup>15</sup> Ineffabilists resist reading paragraphs such as 5.4733 as evidence against their view; in Anscombe 1959, p. 68 and in Frascolla 2007, p.215 it is under discussion but it does not lead to the anti-theoretical conclusion.

<sup>16</sup> The distinction between two kinds of nonsense is upheld in Hacker 1986, pp. 18ff.

because the philosopher using it “had given no meaning to certain signs in his propositions” (6.53).<sup>17</sup> But in that case, it has no meaning at all, direct or otherwise. Qua mere nonsense, a Tractarian proposition cannot be taken to “*mean*” something “perfectly correct”. Anything we can mean, we can also say.

3. Metaphysical propositions, according to *Tractatus* 6.53, are cases in which a failure to assign meaning to certain signs occurs; in 5.4733 Wittgenstein diagnoses the same flaw as the source of nonsense more generally. And in 4.003 he says: “Most propositions and questions that have been written about philosophical matters are not false but nonsensical”. Thus there is indeed a strong connection made in the *Tractatus* between metaphysical propositions and nonsensical uses of language — the former are cases of the latter. Whereas the ineffabilist reader takes this to indicate that metaphysical *insights* are inexpressible, (except indirectly, by means of nonsense) the resolute reader sees this as an indication of the meaninglessness of metaphysical claims, and as reason to treat its expressions with suspicion. So-called metaphysical insights are nothing but illusions brought forth by the misuse of nonsensical expressions. The illusion they create is that something important seeks to find expression in a certain form of words, even if it eventually fails to. Nonsense, on the resolute reading, is not the result of our running against the limitations of grammar, something that a philosopher can choose to make willingly in order to be able to show her metaphysical insights. Rather, nonsense is the result of an often unconscious refusal, on the part of the philosopher, to make a determinate claim. Drawing on Wittgenstein’s distinction between the perceptible sign and the significant symbol (3.32) we may say: perceptible

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<sup>17</sup> See Diamond 1991b and Diamond 1991c. For further discussion of this point, see Conant 2002b, p. 176 f.; as well as Diamond 2005, pp. 78-89.

signs that appear to be nonsense can always be given meaning such that the appearance will disappear; but there is no such thing as a nonsensical symbol. Taking a certain combination of words on its own, we could always find ways to assign words with meanings so as to dispel their nonsensicality; or we could recognize the confusion that underlies a given piece of nonsense and thereby lose our attraction to this form of words and avoid it altogether.

4. The closest Wittgenstein gets to saying that Tractarian nonsense can have any significance is in 6.54. He there says that the elucidatory propositions of the *Tractatus* fulfill their purpose only once they are recognized to be simply nonsense. The kind of success they are supposed to have, however, should not be conceived of on the model of the success of informative, senseful propositions. Nonsense has no oracular function; it plays no positive role, not even the one Anscombe and Hacker ascribe to it — of getting us closer to (if not all the way to) the ineffable truths. On the resolute reading, nonsense only has a therapeutic use. In elucidation, it is used to loosen the grip that other kinds of nonsense have on us. By means of nonsense, we fight our attraction to nonsense — just as one allows oneself to indulge in one fantasy as part of a therapy whose ultimate aim is to teach us to let go of another fantasy.<sup>18</sup>
  
5. The least common denominator of the ineffabilists is the thought that there is such a thing as ineffable content: truths or facts or features that are not sayable, but are showable. According to the resolute reading the idea of an ineffable insight has to be thrown away; everything that can be thought clearly can also be said clearly (4.116). For some resolute readers this means that the dialectical strategy of the *Tractatus* demands that the say/show

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<sup>18</sup> This is a point of contact between the earlier and the later Wittgenstein, according to the resolute reading.

distinction itself be eventually thrown away; for others it means that it needs to be handled with care.<sup>19</sup> I follow Cora Diamond in taking the latter view, that in making the distinction between saying and showing, the grammar of our language forces us to express ourselves misleadingly, in terms of “what is said” and “what is shown”, but in fact there is no “what” in the realm of the shown, no substantive content that lies beyond the realm of expression. Wittgenstein remarks, in the context of his criticism of Russell’s theory of judgment, that “[t]here is no thing which is the form of a proposition, and no name which is the name of a form”.<sup>20</sup> Thus even though he himself uses the term ‘form’ in our elucidations, he thinks there is something misleading in the elucidatory use of this term (which has indeed misled the ineffabilists). As Diamond puts it, “Really to grasp that what you were trying to say shows itself in language is to cease to think of it as an expressible *content: that which* you were trying to say.”<sup>21</sup> According to the resolute reading, the distinction between *saying* and *showing* ultimately discredits the attempt to misrepresent logic as though it involved substantive content (albeit of the non-sayable kind). Logical form, on the resolute reading, shows itself — though not by means of nonsense, to be sure. Every sentence of our language shows its logical form; logical form is grasped (to the utmost extent to which such grasp can be had) by anyone who has mastery of language. But to have such mastery is not to know that anything is the case; it is to know how to go on in our reasoning.

The manner in which each side in this debate approaches the distinction between saying

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<sup>19</sup> Bronzo 2012, p. 57f. gives a good typology of extreme and mildly resolute readings of the distinction between saying and showing.

<sup>20</sup> NB, p. 105. And see discussion in section 3.3. of Chapter 3.

<sup>21</sup> Diamond 1991e, p. 198.

and showing has clear implications for the way in which Wittgenstein's critique of rules of inference in 5.132 would be read. Frege's and Russell's account of inference reflects a certain recognition that logic is a precondition of any expression of content. To the extent that they place the rules of inference beyond the realm of propositional signs, Frege and Russell agree that the justification of inference cannot be one of the things that are *said*. But as I have argued in Chapters 2-3, Frege and Russell nevertheless ascribe to these rules of inference substantive content, and they take us to appeal to this content in inferences.

This account of rules of inference commits Frege and Russell to a variant of the metaphysical reading of the say-show distinction. The distinction they implicitly make between what can be said and what can only be shown does not overlap with their distinction between the contentful and the non-contentful. They think that some content — important, justificatory content — extends beyond the limits of language. They thereby endorse the idea of ineffability, of there being unsayable content which must nonetheless be quasi-said, in order for our ordinary practices of saying (inferring) to make full sense.

By contrast, on the reading of Wittgenstein that I advance in Chapter 6, the justification of inference is found in the propositions themselves, since the full understanding of these propositions is constituted by the internal relations between the premises and the conclusions, which only show themselves but cannot be said. Justification shows through in our actual use of language, but it does not consist in any implicit appeal to ineffable logical content. The very idea of an unsayable but justificatory logical content is illusory. All of the justification of the inference we draw comes from understanding in the propositions it involves.

Wittgenstein's rejection of rules of inference can therefore be thought of as a rejection of

the ineffabilist reading's distinction between saying and showing, and as an endorsement of the resolute distinction. If the metaphysical reading truly made sense, Wittgenstein would have no cause to reject Frege's and Russell's accounts of rules. But since he does just that, the ineffabilist reading must be rejected as well.

## 5.2. Showing and Elucidating

A major source of inspiration for Wittgenstein's distinction between saying and showing is found in Frege's distinction between fact-stating discourse and elucidatory discourse.<sup>22</sup> But at the hands of the irresolute reader, the claim that Wittgenstein inherits his distinction from Frege could be quite misleading. My aim in this section is to distinguish between ineffabilist and resolute moments in Frege, and to show that Wittgenstein rejects the former. His own distinction is meant to be consistently resolute.<sup>23</sup>

Elucidation (Erläuterung) is mentioned in the *Tractatus* three times. Wittgenstein uses the term in two interrelated ways, both of which can be traced back to Frege. In Tr. 3.263, elucidation denotes the clarification of the meanings of primitive symbols of a formal language; In Frege, it is a recurring theme that the primitive signs of a formal system, and the fundamental logical distinction they capture, are not susceptible to formal definitions, and must rather be

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<sup>22</sup> Goldfarb (unpublished manuscript) conjectures that Wittgenstein could also have acquired his distinction between saying and showing from a different source — through his engagement with Russell's attempts to draw a similar distinction (e.g. in Chapter 7 of *Principles*). In Chapter 3, section 3.3., I have shown that in reflecting on his appeal to the idea of logical form, Russell senses the problem that eventually leads Wittgenstein to draw a strict distinction between what is said and what is shown, i.e. between the content and form of language and thought. I return to discuss this in section 5.3.7, below.

<sup>23</sup> Resolute readings of the contrast between Wittgenstein's and Frege's distinctions are found in Diamond 1991d, p. 120, and Conant 2001, p. 386.

clarified by elucidations — expressions that already employ these signs and display their use.<sup>24</sup>

In Tr. 4.112, however, ‘elucidation’ characterizes philosophical activity more generally, which aims at the “logical clarification of thought”. And in Tr. 6.54, Wittgenstein talks of his sentences as bringing about elucidation (‘erläutern’) to the extent that the reader learns to recognize them as nonsense and subsequently throws them away. This sense of elucidation can be traced back to Frege, as well, although in this case the genealogy is more complicated.

Frege holds that basic logical notions articulate the shape of all thought, and the design of the Begriffsschrift is meant to reflect them in a vivid way. But such notions are so basic that they cannot be defined. They may only be elucidated, by means of a transitional mode of discourse which is useful for generating mutual understanding, but is not itself to be taken to convey substantive assertions.<sup>25</sup> However, at least when it comes to some of these notions, elucidation itself proves to be problematic. This realization arises with full force in the context of Frege's discussion of the distinction between concepts and objects.<sup>26</sup> In trying to articulate this distinction, Frege recognizes that the grammar of language stands in his way, forcing him to frame misleading expressions — since whenever he attempts to explain what a concept is, he is forced to place the expression “concept” in the grammatical subject position, and thereby treat it as an object of which predicates are asserted. The result, Frege thinks, is a nonsensical

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<sup>24</sup> *Grundgesetze*, p. 4, Frege 1979h, p.207, 1979h, p. 271, FG2, p. 300f., Frege 1984b, p. 18, CO, p. 182. I discuss the nature of Fregean elucidations in Chapter 1, section 1.1.

<sup>25</sup> On the idea that elucidations involve a “transitional” mode of discourse, see Diamond 2000 and Conant 2001, p. 385ff.; the elucidatory mode of discourse is described as “didactical” by Geach 1976, p. 58; Weiner's preferred term is “elucidatory”. See Weiner 2005a and Weiner 1990, pp.227-280.

<sup>26</sup> CO, pp. 182-194. The term ‘elucidation’ itself does not appear in Frege's discussion in “On Concept and Object” (CO), but the same idea is clearly deployed there. See Weiner 2005a, p. 201. Frege's discussion of the distinction between concept and object forms the topic of the next subsection, 5.2.1.

expression that frustrates the attempt to respect the strict distinction between concept and object.<sup>27</sup> Frege thinks that in recognizing that these attempts fail to make sense, we gain a deep insight. The second sense of elucidation deployed by Wittgenstein arises from this Fregean realization. But there are different the way in which Wittgenstein inherits from Frege.

One influential version of the ineffabilist way of spelling out the claim that Wittgenstein's distinction between saying and showing stems from the Fregean reflection on the elucidation of basic logical notions is proposed by Peter Geach. Geach suggests that Wittgenstein adopts Frege's distinction between elucidatory and constative uses of language, but whereas Frege applies it in the narrow realm of logic only, Wittgenstein expands its scope beyond logic. Geach's construals of both Frege's and Wittgenstein's distinctions favor the ineffabilist reading of the *Tractatus*. To recall, such a reading usually involves (among other things) the following claims:<sup>28</sup>

1. there are important features of language and reality that cannot be *said*,
2. we can have true insights into these ineffable matters;
3. elucidatory nonsense helps us see these truths.<sup>29</sup>

Unlike other ineffabilists, Geach himself does not claim that Frege's or Wittgenstein's distinctions between what is said and what is shown are meant to allow us to express *truths* — i.e. to express something resembling the content of true propositions. Its purpose, for Geach, is to express logical “category distinctions”, “logical structure” and “aspects of reality” (ibid., p. 68).

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<sup>27</sup> See e.g. CO, p.204; GG, #35, p. 54; Frege 1979g, p.178.

<sup>28</sup> I am not claiming that Geach's reading involves these claims; as I go on to discuss in the next paragraph, Geach is careful not to treat that which shows in terms of ineffable truths.

<sup>29</sup> See my discussion of these claims in section 5.1, above.

These are not necessarily construed as propositional in nature.

On a resolute reading, by contrast, even this way of spelling out the content of the realm of showing is problematic.<sup>30</sup> What Frege's struggle with the concept-object distinction should teach us, on the resolute reading, is to avoid speaking of the inevitable failure to express this distinction as a failure to express *something*. There is only one kind of nonsense — that which results from indeterminacy in the meanings of the signs we use; elucidatory nonsense is no more revelatory than mere nonsense.

In Chapter 1 I attempted to counteract the ineffabilist line of reading by sketching the shape of a possible resolute reading of Frege. Such a resolute reading takes Frege at his word when he says that if we had a logically proper language, we would have no need for logic at all.<sup>31</sup> This indicates that Fregean elucidations should not be taken to do anything more than to serve as a propaedeutic to the correct *use* of the formal system. Elucidations help us see the accord between features of a formal system and our ordinary uses of language — but this does not mean that what elucidation thereby clarify has the form of the effable, i.e. that it can be spoken of as the content of a true claim.<sup>32</sup> Moreover, even elucidatory notions such as judgment and inference should ultimately be recognized as elucidatory nonsense, and eventually be discarded. In a state of full clarity, there would be no talk of inference, nor would there be any need to talk of its structure, content, etc.

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<sup>30</sup> Sullivan 2002 aims to defend Geach from the claim that talk of such “features” commits one to there being ineffable states of affairs and “quasi-truths” that capture them; Michael Kremer replies to Sullivan that Geach's talk of “features” cannot be as innocent as Sullivan proposes, in Kremer 2007, p. 159ff.

<sup>31</sup> Frege 1979i, pp. 251-252.

<sup>32</sup> Ricketts 1986 suggests that Frege's ontology is supervenient on the grasp we have of the practices of judgment, inference, etc. It helps us get a clearer grip on these practices, but it does not inform us of anything substantive. I discuss this in Chapter 1, section 1.5.

Leaving aside the question whether Frege actually held an ineffabilist or a resolute view, it can be argued that Wittgenstein distances himself from the ineffabilist construal of Frege's distinction.<sup>33</sup> Wittgenstein's own distinction can then be taken as a response to the mere possibility of reading Frege along the ineffabilist lines. In other words, we can take the ineffabilist reading of Frege as a foil, against which the resolute reading of Wittgenstein will emerge with clarity. To do that, section 5.2.1 takes a closer look at Frege's struggle to articulate the distinction between concepts and objects. In 5.2.2. I reconsider the Tractarian method of elucidation; and in 5.2.3. I look at Wittgenstein's alternative to Frege's elucidations of the basic logical concepts.

### 5.2.1. On Concept and Object

The distinction between concepts and objects forms one of the three fundamental principles Frege lists in *The Foundations of Arithmetic*:

[A]lways to separate sharply the psychological from the logical, the subjective from the objective;

never to ask for the meaning of a word in isolation, but only in the context of a proposition;

never to lose sight of the distinction between concept and object.<sup>34</sup>

The three principles work in conjunction. Concepts and objects can only be distinguished properly when we consider them as the working constituents of full sentences, rather than

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<sup>33</sup> Conant 2001 (p. 381) argues that there is a tension in Frege's thought between the substantial and the austere conception of nonsense, and that this is the tension which Wittgenstein discerns and aims to resolve in the *Tractatus*.

<sup>34</sup> FA, p. xxii.

through reflection on isolated words, in abstraction from their propositional context. Ignoring context, we are in danger of confusing the psychological with the logical, allowing the ideas that we associate with a term to obscure the changing logical roles that this term can play in each particular context.<sup>35</sup>

Thus we can only be properly presented with concepts when we understand what is said by a complete proposition, in a given context. Concepts, in the strictly logical sense of the word, are “predicative”; objects are those things of which concepts are predicated. Concepts are “unsaturated” — they await completion by an object of the appropriate type — whereas objects, being saturated and capable of saturating, cannot serve in predicative roles.<sup>36</sup> Concepts and objects thus form two mutually exclusive logical categories. Failure to respect the distinction between them results in unintelligible propositions.

However, Frege observes that the very attempt to express this logical category distinction contravenes the distinction, and gives rise to a cross-category confusion. By asking what concepts are, we place them in the subject position of our sentences, and thereby treat them as objects, to which properties apply. Similarly to say that there is a difference between concept and object, we already treat concepts as the objects to which the relation of not being identical applies. By framing such expressions, we produce nonsense:

Hence what is here said concerning a concept can never be said concerning an object; for a proper name can never be a predicative expression, though it can be part of one. I do not want to say it is false to say concerning an object what is here said concerning a concept; I want to say it is impossible, senseless, to do so.<sup>37</sup>

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<sup>35</sup> Cora Diamond brings out this point by comparing the sentences “Vienna is the capital of Austria” and “Trieste is no Vienna”. In the second sentence ‘Vienna’ no longer functions as a proper name. See Diamond 1991b.

<sup>36</sup> CO, p. 183.

<sup>37</sup> Ibid., p. 189; cf. Frege 1979d, pp. 119-120.

The difficulty cannot be avoided by negating the sentences which misapply concepts and objects in such ways, since the negation of these sentences would be just as senseless. The negated elucidations would violate the distinctions they aim to affirm no less than the original ones.<sup>38</sup> It is an unavoidable predicament for the logician, according to Frege, that he must use language in such misleading ways:

The proposition ‘A is a function’ is such an expression: it is always imprecise; for A stands for a proper name.... While I am writing this, I am well aware of having again expressed myself imprecisely. Sometimes this is just unavoidable.<sup>39</sup>

Frege finds himself in a bind of wanting and not wanting to use the expressions ‘concept’ and ‘function’ in the ways which end up violating the logical roles he ascribes to them.

But the ineluctability of Frege’s predicament might not be immediately obvious. Indeed, one of Frege’s early critics, Benno Kerry, rejects the idea that the categories of concept and object are mutually exclusive. According to Kerry, concepts *can* sometimes take the role of objects, and it is just as well that grammar poses no obstacles to that.<sup>40</sup> Kerry gives an example: the sentence “the concept *horse* is a concept easily attained”. In this true sentence, Kerry thinks,

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<sup>38</sup> This point is made by Conant 2002b, p. 187.

<sup>39</sup> Frege “Letter to Russell, 29.6.1902” in PMC, pp. 135-137. A similarly unavoidable misuse of the term ‘concept’ occurs in laying out the principle “as regards concepts, that, for any argument, they shall have a truth-value as their value; that it shall be determinate, for any object, whether it falls under the concept or not”. See Frege 1984b, p. 148. Russell himself offers a very lucid formulation of the problem in his letter to Frege, dated 10.7.1902:

Concerning function names, there still seems to me to be a difficulty. If we leave aside names altogether and speak merely of what they mean, then we must admit that there is no proposition in which a function takes the place of a subject. But the proposition ‘A function never takes the place of a subject’ is self-contradictory... (PMC, pp. 137-138).

<sup>40</sup> As Frege points out, there are two separate claims involved in his distinction, only one of which is attacked by Kerry. The one Kerry does not attack is that no object can play the predicative role of a concept. The claim Kerry does argue against is that no concept can play the role of an object. But even here Kerry’s treatment is imprecise; Frege does not deny that there are legitimate sentences in which concepts are the grammatical subject, e.g. when concepts are subsumed under other, higher-level concepts. This would not threaten Frege’s distinction. See CO, 182f and 187f.

a concept plays the non-predicative role of an object.

Kerry's objection violates all of Frege's three principles of the *Foundations* — not just the distinction between concept and object, but also the context principle and the distinction between the logical and the psychological. By observing the context in which the word 'concept' appears in Kerry's example, we can see that if it refers to anything at all, that thing cannot be a concept.<sup>41</sup> The claim can be made sense of, however, if we take Kerry to have surreptitiously changed the subject, i.e. if the sense he had given to the term 'concept' is the *psychological* capacity of a subject, rather than the logical sense of the term. Assuming that such a switch has been made, the sentence "the concept *horse* is a concept easily attained" acquires sense — it might even be a true claim concerning the acquisition of a perceptual capacity. Accordingly, Frege proposes that the dispute can be resolved if each party acknowledges the difference between each other's intentions — Kerry will be allowed to make the psychological point, whereas Frege will assert his right to the logical point, and be allowed to reply that "the concept *horse* is not a concept".<sup>42</sup>

I do not at all dispute Kerry's right to use the word 'concept' and 'object' in his own way, if only he would respect my equal right, and admit that with my use of terms I have got hold of a distinction of the highest importance.<sup>43</sup>

Nevertheless, this response to Kerry — to point out to him that he is merely changing the subject, or that he does not use his terms determinately — does not solve Frege's own difficulty in expressing the distinction *he* is concerned with. His challenge remains to explain how we can

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<sup>41</sup> Ibid., p. 184.

<sup>42</sup> Ibid., p. 196. It is probably more historically accurate to say that Kerry would not take himself to merely want to make this psychological point — after all, Kerry takes himself to contradict what Frege has claimed about the logical use of the term. By contrasting Kerry's use of the term to his own Frege exposes an indeterminacy in Kerry's use; and he offers Kerry a remedy, namely allowing Kerry to give a non-ambiguous sense to his words.

<sup>43</sup> Ibid., p. 193.

convey this important logical distinction without at the same time transgressing it.

The notion of elucidation offers a way out of the bind Frege is in. By according the expressions of the distinction an elucidatory status, Frege treats them as mere “hints” and “figures of speech”,<sup>44</sup> rather than self-standing acts of communication, and the success of elucidation requires that the reader will not “begrudge a pinch of salt”.<sup>45</sup> Elucidation serves to pick out something indefinable, which can nevertheless be assumed to be grasped by every reader, and to show how it is reflected in a language with which the reader might not yet be familiar:

On the introduction of a name for what is logically simple, a definition is not possible; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the word as it is intended.<sup>46</sup>

Hints cannot count as full-fledged assertions, and so they are neither true nor false. In the case of the distinction between concepts and objects, the hints amount to nonsensical expressions. Nonetheless, Frege does not think that he is trafficking in *mere* nonsense: the elucidations serve a purpose, they convey something deep. Thus he says, regarding the logical category distinction between first-level and second-level concepts, that it is founded “deep in the nature of things”.<sup>47</sup> By hinting at these ineffable features of reality, Frege wishes to state a truth, or quasi-truth, that, in principle, should be impossible to state. A similar appeal to an ineffable realm of truths is found in the passages, discussed in Chapter 2, in which Frege presents the rules of inference of his system as transforms of fundamental laws of thought that cannot be expressed in the

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<sup>44</sup> Ibid., p. 183, 194, and *passim*.

<sup>45</sup> Ibid., p. 193. This passage is further discussed below.

<sup>46</sup> Ibid., p. 182.

<sup>47</sup> Frege 1984b p. 156.

Begriffsschrift, “because they form its basis”.<sup>48</sup>

It is not insignificant that Frege expresses an air of resignation when he blames the awkwardness of language for his inability to express himself clearly — “[b]y a kind of necessity of language, my expressions, taken literally, sometimes miss my thought”.<sup>49</sup> Frege thereby implies that there is something out there to be thought of, even though its expression inevitably fails. In the *Big Typescript* Wittgenstein sketches two ways of understanding the talk of the limits of expression:

When I say: Here we are at the limits of language, that always sounds as if resignation were necessary at this point, whereas on the contrary complete satisfaction comes about, since *no* question remains.<sup>50</sup>

Wittgenstein suggests that even an expression such as “we cannot draw the limits of language” can be given a resolute reading. Its point would then be to indicate the end of questioning, the dismissal of a puzzlement. For the ineffabilist, by contrast, the sentence would be taken to indicate the continuing pull of an unsolvable puzzle. It is of the latter attitude that Frege’s reflections bespeak.

Unless these irresolute moments in Frege are dismissed as aberrations, or unless we find a way to interpret them resolutely, we must ascribe to Frege an ineffabilist distinction between saying and showing. On such an ineffabilist reading, elucidatory nonsense is an attempt to communicate ineffable truths, or quasi-truths. For present purposes, I do not resolve the question

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<sup>48</sup> BS, #13, p. 136.

<sup>49</sup> CO, p. 193.

<sup>50</sup> Wittgenstein 2005, p. 310. I am only citing this (relatively late) passage to point out an alternative to Frege’s way of thinking about his predicament — not in order to claim that this is Wittgenstein’s position in the *Tractatus*. For a discussion of this passage, see Diamond 2012.

whether Frege still stands on a rung in the ladder of elucidation from which it seems that ineffable truths are indirectly expressed via elucidations, or whether he willingly throws away that ladder. What is crucial for my aims is to show is that the ladder that is formed by Wittgenstein's own elucidations is meant to be thrown away for good, once we climb up it.<sup>51</sup>

### 5.2.2. Tractarian Elucidation

The irresolute reading of Wittgenstein takes him to follow Frege's lead in taking elucidatory nonsense to be capable of revealing ineffable insights. But if we look at how Wittgenstein actually speaks about nonsense, we can see that he makes no reference to ineffable insights; what he aims to do is to dispel the appearance that nonsense conveys anything at all. Nonsense reflects a failure to make meaningful use of language; since such failure can in principle be remedied, in any case in which we refuse to do so, the fault must be our own — not the fault of language. Here's an example for this way of treating nonsense:

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<sup>51</sup> There have been attempts to make sense of Frege's elucidations of the distinction between concepts and objects as a robust, meta-linguistic theory. Frege supposedly entertains the idea that he can escape the bind he is in by means of a more sophisticated account language in which he speaks of concepts. His elucidations of concepts take the form of statements about objects, but these objects, according to this account, have the role of "going proxy" for concepts. Regardless of the question whether this reading can count as a correct interpretation of Frege's text, I find the proposal to be ultimately unconvincing. Expressions can serve a double purpose of the sort considered here only if we have some way of reading into them that content which is not their official and explicit content — we have to be able to read the elucidations both as being *about* the proxy-objects and as being *about* concepts. So it is only by means of an independently given theory that we determine the secondary sense of these expressions, and that theory would either be clearly expressible, which would make the elucidatory sentences superfluous, or inexpressible, which would lead back to the same problem, rather than offer a solution.

See Parsons 1986 and Proops 2013. They both rely on CO, p. 186 & p. 189.

The reason why “The property of not being green is not green” is nonsense, is because we have only given meaning to the fact that “green” stands to the right of a name; and “the property of not being green” is obviously not that.<sup>52</sup>

Here nonsense depends merely on whether or not we have given meaning to some type of fact — the fact that the constituent part “is not green” stands to the right of something which does not name an object. But this can be overcome in multiple ways: we can assign a new meaning to the sign “green”; we can look for a different way to cut up the string of signs into functional parts, and assign these parts a meaning which would render the whole meaningful.<sup>53</sup> Of course this might contravene the conventions of the language one is using (or depend on the introduction of a new convention), and in practice, we might have no motivation to take such drastic measures.<sup>54</sup> The important point is that it is only our insistence not to do so that makes a proposition nonsensical. We only find ourselves thinking that “the property of not being green is not green” is nonsense because we have some sense of what “the property of x” usually means, and we assume that we *must* carry this meaning over from one context to another without making any adjustments. But in the example Wittgenstein discusses, we make conflicting contextual demands on the expression “the property of x”, and the result is an indeterminacy in our use of signs.

When we insist that this is how we *must* treat the signs of our language we make an

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<sup>52</sup> MN, p. 116.

<sup>53</sup> For instance, if the constituent part “not being green” is used as a name, we could understand the sentence “The property of not being green is not green” as an instance of the claim that someone’s property — someone’s land — is not green.

<sup>54</sup> Whether or not we can disregard the role of convention in facilitating communication may be a controversial matter, but the point I am making is not that linguistic use always involves originality, and that conventions have no role to play. The point is that one may always introduce new ways to make sense of signs whose use is well established.

unwarranted assumption about the logic of language. Wittgenstein thinks that logical roles cannot be assigned to signs in complete abstraction from the context of use in which they appear:

5.4732 We cannot give a sign the wrong sense.

5.4733 Frege says: Every legitimately constructed proposition must have a sense; and I say: Every possible proposition is legitimately constructed, and if it has no sense this can only be because we have given no meaning to some of its constituent parts. (Even if we believe that we have done so.)

Sentences of our language can lack sense only if *we* insist on making conflicting demands on the words we use, which leads us to refuse to assign these words with unambiguous meaning. In doing that, we attempt to treat something that only shows — the sense of an expression in use — as something that we can say, determine rules for, and manipulate. Having abandoned this insistence, we are free to go in one of two directions: we can either assign our words the meanings that would render the whole proposition into an ordinary, senseful one; or we can just leave the nonsensical expression alone, exclude it from our discourse. Once we realize that the nonsensicality does not derive from some essential limitation on our mode of expression, we lose the feeling that something deep is being revealed by the nonsensical expressions. In particular, we see that these sentences cannot be taken as “deep” nonsense, that arises from a putative violation of logical distinctions, and whose nonsensicality is indicative of the logical features of language.<sup>55</sup>

5.4733 cannot be taken as a criticism of those places in Frege in which he advances the more austere, Wittgensteinian approach to nonsense. I suggested above that in “On Concept and Object” Frege draws on the possibility of doing what 5.4732 and 5.4733 suggest, i.e. the

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<sup>55</sup> With these remarks I aim to exclude Hacker’s view, that nonsense (for Frege as well as for Wittgenstein) is the result of combining words in a specific way that creates a clash between logical categories (in Hacker’s terminology, the rules for logical syntax). See Hacker 2000, and the criticism of his view in Diamond 2005. I return to this issue below.

possibility of assigning determinate meaning to words that so far lacked it so as to avoid nonsensicality. He does that when he proposes to acknowledge Kerry's equal right to use the word 'concept' as he pleases; this implies that there are ways in which Kerry can use the words "the concept horse is a concept easily attained" that would render them intelligible, and perhaps even true.<sup>56</sup> The resulting sentence might no longer be about what Kerry originally thought it was about, namely about concepts, in the logical sense of the term. Rather, the words 'the concept horse', in Kerry's mouth, would then be taken to refer to a psychological capacity, and to say something about it — that it is basic, and is attained easily. But given the two determinate and non-ambiguous uses of the word 'concept', Kerry would face the choice of either opting for one of them, or giving up on using the word altogether.

In 5.4733 Wittgenstein is concerned with the view expressed elsewhere in Frege, namely in paragraph §32 of the *Grundgesetze*, where Frege responds to the worry that there might be a gap between the expressions constructed in accordance with the rules of formation of the system, and the sensefulness of the resulting expressions.<sup>57</sup> The issue for Wittgenstein seems to be whether what Frege says about a Begriffsschrift can make any sense as a general claim about language and thought. Drawing such an analogy, according to Wittgenstein, would be confused. When we think about language as such, sensefulness and propositionhood cannot be taken to result from a "legitimate", i.e. rule-abiding, construction; the components of a proposition do not make sense

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<sup>56</sup> CO, p. 1931; and section 5.2.1, above.

<sup>57</sup> Wittgenstein does not cite Frege's *Grundgesetze* §32, but the words he ascribes to Frege seem to be taken from that passage. Wittgenstein says "Frege sagt: Jeder rechtmäßig gebildete Satz muss einen Sinn haben." Frege, for his part, says "so haben wir in jedem rechtmäßig Begriffsschriftsatze ein Urtheil, das ein Gedanke wahr sei" as well as "Aber nicht nur eine Bedeutung, sondern auch ein Sinn kommt allen rechtmäßig aus unsern Zeichen gebildeten Namen zu. Jeder solche Name eines Wahrheitwerthes drückt einen Sinn, einen Gedanken aus". (S. 50-51 / p. 50-51).

in abstraction from the entire context in which we use them (*Tr*: 5.5563). Thus the possibility Frege aims to exclude, that there might be some legitimately constructed propositions that do not have a sense, is a chimera, at least if this is meant to apply to our language.<sup>58</sup> Construction of signs does not precede the assignment of meanings, and is not independent from it. This goes to show that for Wittgenstein, there can be no such thing as deliberate violations of the rules of syntax of our language, for the purpose of hinting at ineffable distinctions (as Hacker, for instance, holds), since any such “violation” can be made to disappear by the appropriate assignments of meaning, and if we refuse to do that, the fault would merely be in us.<sup>59</sup>

Our attraction to philosophical nonsense — our feeling that such expressions harbor a solution to deep problems — dissipates once we realize that it is our own doing, that it is *we* who have created the air of mystery, by insisting on not making the proper assignments of meaning, both in our statement of what we take to be deep philosophical problems, and in our statement of the proposed solutions. The true method of philosophy, according to Wittgenstein, consists in showing that philosophical puzzles can be made to disappear, by showing that the only thing that sustains our puzzlement is our equivocal use of terms:

4.003 Most propositions and questions, that have been written about philosophical matters, are not false, but nonsense. We cannot, therefore, answer questions of this kind

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<sup>58</sup> The reading of 5.4733 I am suggesting here might seem to result in attributing to Wittgenstein a somewhat uncharitable reading of Frege. In *Grundgesetze* §32, Frege is not making any sweeping claim about language as such. What he says there applies to *Grundgesetze* expressions only. And at first sight, what Frege says is not very different from something which Wittgenstein himself seem to advocate in 3.325, namely the idea of a symbolism governed by a logical grammar which excludes ambiguities (this has been pointed out to me by Cora Diamond, in private correspondence). However, 3.325 does not directly concern the question whether the grammatically correct expressions of such a symbolism would or would not have sense. That would depend, according to 3.326, on the significant use. Thus Wittgenstein’s point, in 5.4733, might be more accurately stated as follows: Frege is wrong to think that the question whether an expression has sense depends on whether that expression is grammatically well-formed.

<sup>59</sup> See Hacker 2000.

at all, but only state their nonsensicality. Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language. (They are of the same kind as the question whether the Good is more or less identical than the Beautiful.)  
And so it is not to be wondered at that the deepest problems are really no problems.

This is the paradigmatic work of elucidation, as Wittgenstein sees it: to compare and contrast different modes of expressions in ways that bring to light that what at first seemed to be of substance is in fact completely empty, a merely accidental feature of the signs of our language (cf. 6.53). For example, consider the various systems of notation that Wittgenstein proposes in the *Tractatus*. Through their comparison, Wittgenstein establishes his *Grundgedanke* — that the logical constants do not contribute any *content* to representation (4.0312, 5.441). The philosophical confusion Wittgenstein aims to thereby remove is the failure to appreciate that logic is *formal*, not *contentful*. Frege's and Russell's construal of the rules of inference is a manifestation of such failure.

### 5.2.3. Formal Concepts

The stretch of passages running from 4.12 to 4.128 of the *Tractatus* proposes a strategy for elucidating basic logical concepts that forms an alternative to Frege's. This is also where Wittgenstein spells out the idea that the logical form of all representation cannot itself be represented, and yet it shows itself in our expressions (4.1211). The argument he offers for this claim will be explored in detail in the next section; here I am concerned merely with the status it gives to those expressions of our language that purport to do precisely what 4.1212 seems to disallow — to represent logical form. Words such as 'object' and 'predicate', and their

combination in sentences like ‘no concept is an object’ are what Wittgenstein has in mind. His idea is that when we treat these terms as though they designate proper concepts, that contribute to the *content* of sentences, fundamental philosophical confusions arise. Their true function, according to Wittgenstein, is to designate “formal concepts”.

Wittgenstein thinks that in most cases, we can translate sentences employing formal-concept words into a logically perspicuous language, in which these words are not replaced by predicates or object names, but rather by variables:

4.1272 ... Wherever the word “object” (“thing”, “entity”, etc.) is rightly used, it is expressed in logical symbolism by the variable name. For example in the proposition “there are two objects which ...”, by “ $(\exists x,y) \dots$ ”.

The use of a variable highlights a certain formal feature (*Zug*) that expressions can have in common (4.126). In translating a sentence such as “there are two objects which...” into the formal language, it becomes clear that ‘object’ does not add content to what we say. In particular, it no longer appears to function as something of which we can assert anything — it is neither represented as an object, nor as a predicate. One variable cannot be predicated of another; indeed the variable itself is not something of which we predicate anything. It only helps us pick out those values of which we want to predicate something. This shows us that in some contexts, at least, words like ‘object’ and ‘predicate’ only indicate the form of the expression, not its content (4.1272), and in those contexts the use of these words does not give rise to philosophical nonsense.

The trouble is that in philosophy we are tempted to treat these words differently. Recall the Fregean expressions with which the distinction between concept and object is spelled out. There, formal concepts were treated as though they are proper concepts, or names of objects, etc.. This

insistence, Wittgenstein holds, gives rise to nonsense:

4.1272 ... Wherever it is used otherwise, i.e. as a proper concept word, there arise senseless pseudo-propositions.

So one cannot, e.g. say “There are objects” as one says “There are books”.

4.1241 One cannot distinguish forms from one another by saying that one has this property, the other that: for this assumes that there is a sense in asserting either property of either form.

Wittgenstein does not assert or deny that a single property can range over two different logical forms — for that would imply that for Wittgenstein, it makes sense to assert any property of any logical form. Rather, he says that what corresponds to our talk of logical form is not something of which any claims can be made at all. To think otherwise is to be misled by the grammar of our language. If we insist that the use of the formal concept words in the context of the philosophical statement must have certain features, which we recognize and attach to them in other contexts, we are bound to be puzzled by them. Similarly, if we insist on understanding the context in which these words appear as segmented in a particular way (e.g. taking “no concept is an object” as belonging to the same paradigm as “no dog is a fish”) and refuse to make the proper adjustments that would allow it to make sense, we would find ourselves in the same trouble — where the trouble is not that we violate the grammar of language, but rather that we refuse to allow language to take care of itself (cf. 5.473).

The philosopher that Wittgenstein has in mind here (Frege, perhaps) could not respond by granting that in ordinary contexts our use of formal concept does correspond to variables, but insisting that in the special philosophical context he is concerned with, these words mean something completely different. For the philosopher’s claims concerning this second role would be of no interest unless they help explain the way the formal concept functions in the ordinary

contexts. The philosopher wants to be able to say, for instance, that in those ordinary contexts, no concept ever takes the place of an object. At the same time, the philosopher wants to pursue this intuition, embedded in the grammar of natural language, that each formal concept word means a predicate, and hence that it forms a class, comprising e.g. of objects, concepts, etc. Wittgenstein aims to eliminate the ambiguity and indeterminacy that underlies the philosopher's approach. The translation into a language that employs variables gives us a determinate grip on one function that formal concept words have; nothing in the ordinary use of formal concept words requires that we *also* think that they have the additional functions that the philosopher wants them to have.<sup>60</sup>

This applies not only to someone like Kerry, but also to Frege — his nonsensical elucidations could be made sense of by appropriate assignments of meaning, but then they would lose their interest for us. The temptation to treat the nonsensical expression as mysteriously indicating something which cannot be coherently said about concept words would begin to dissipate. Indeed elucidatory nonsense is not useless; in certain dialectical contexts it might be helpful to counter one form of philosophical nonsense with another, and in such cases, we might just want to discard the nonsensical expression once it has served its purpose, rather than seek to assign it an anodyne sense. Ultimately, there should not seem to be any substantive content at which the philosophical elucidation counts as getting at; the ladder of elucidation is completely thrown away.

The philosophical use of formal concept words as proper concepts is an attempt to treat

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<sup>60</sup> Narboux 2014 (p. 243) points out that the philosopher is wrongly thinking that apart from the way in which formal features of sentences shows themselves in some symbolism, we can have a grasp on what the formal concept word signifies. Natural language expressions foster the illusion that there is something more that is involved; but there is nothing determinate, and yet ineffable, that this illusion is attesting to.

what *shows* itself in the use of a language as being of the kind of what we say in language — an attempt to treat form as content. In the next section I explore a different route by which Wittgenstein aims to show that such attempts can only result in nonsense. It will be my claim, in Chapter 6, that a similar attempt to blur the distinction between saying and showing underlies in Frege’s and Russell’s account of rules of inference.

### 5.3. Why Can’t What is Shown be Said? The Third-Man Argument

Wittgenstein holds that the distinction between what can be said and what can only show itself can be recognized by reflection on the form-content structure of representation.<sup>61</sup> His argument proceeds in two stages. He first establishes (what I will call) the “relative distinction” between the form of any given mode of representation and the content of the representations we can make by its means. Within any specific medium of representation, we cannot represent its own form of representation. Wittgenstein then leads his reader from the appreciation of the distinction in this relative and limited context, through a consideration of the idea that all forms of representation are ultimately founded on the logical form of representation, to the realization of the need for an “absolute distinction” (AD, for short) between saying and showing as such:

4.1212. What *can* be shown *cannot* be said.

In reconstructing this argument, my aim is to provide a resolute reading, i.e. one which aligns the distinction between what is said and what is shown with the distinction between the content and

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<sup>61</sup> The distinction itself is independent from this argument, and from the Tractarian account of representation more generally. When the distinction appears for the first time, in MN (1914), it is stated independently of the idea of picturing and its form. The two issues are not connected until the later *Notebooks*.

form of language, and thereby rules out the thought that what is shown can be thought of as ineffable *content*. On my reading, the distinction does not lend any support to the ineffabilist thought that there are substantive truths about language and the world that can be illuminated but cannot be expressed. The argument shows that the very idea of asserting a truth concerning logical form is nonsense.

In order to shed light on Wittgenstein's argument I will explore an analogy between it and the platonic Third Man Argument (TMA, for short).<sup>62</sup> The analogy might initially seem surprising — after all, Wittgenstein is concerned with logic and language, whereas the TMA, in its original context, concerns a problem that arises in Plato's epistemology and metaphysics. But as I will argue, the analogy can be quite illuminating. Plato's TMA shows that given the way in which the theory of ideas construes the relation between abstract universals and concrete particulars, a regress is bound to arise; this reveals a tension between two roles that ideas are required to play in that theory: they are supposed to explain the predication of properties, and at the same time, they should count as substantive entities which exemplify such properties.

Wittgenstein's argument, too, seeks to reject a certain way of construing the relation between the abstract and the concrete. Wittgenstein's target is the platonistic conception of logic that he finds in Frege and Russell. On the conception Wittgenstein attacks, logic is at one and the same time formal and substantive. It spells out the laws of thought, but it is also construed as the content of the most general and abstract truths. On the conception Wittgenstein argues for, logic informs representation, but cannot serve as an object of representation, on pain of regress. In

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<sup>62</sup> The argument appears in Plato's *Parmenides* (Plato 1997, 132a–133a); it is also discussed repeatedly by Aristotle in *Sophistical Refutations* (Aristotle 1984b, 178b36ff.) and in *Metaphysics* (Aristotle 1984c, 990b17, 1039a2, 1079a13). The analogy between Wittgenstein's Tractarian argument and the TMA has rarely been noted. Fogelin 1987 makes a general, offhand remark (p. 19).

order to avoid such regress, the distinction between logical form and representational content must be drawn much more radically. To acknowledge that logical form cannot be said is not to acknowledge that a peculiar kind of content remains beyond expression; it is to recognize that the very idea of treating logic as content is nonsense. Logical form can only be appreciated from within the actual activity of thinking. It manifests itself in *how* we think, not in *what* we think of.<sup>63</sup>

The structure of this section is as follows. In 5.3.1. I trace the shape of Plato's TMA. In 5.3.2. I reconstruct Wittgenstein's argument that pictures are unable to represent the formal features of their specific mode of picturing. In 5.3.3 I trace Wittgenstein's argument that the account of picturing can be generalized into an account of language as such, where the relative distinction is replaced with the absolute distinction. In section 5.3.4 I compare Wittgenstein's and Plato's arguments. In 5.3.5 and 5.3.6 I consider objections to Wittgenstein's absolute distinction, which consist in the rejection of one or more of the assumptions that underlie his account of representation, and in 5.3.7. I conclude with a discussion of the resolute reading of Wittgenstein's distinction.

### **5.3.1. Plato's TMA**

Let us briefly recall the shape of Plato's TMA. Consider a plurality of concrete particular men,

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<sup>63</sup> It is possible that Wittgenstein found inspiration for his distinction, as well as for his third-man argument, in Russell's reflections on the idea of logical form (see e.g. Russell 1994a, p. 98, Russell 1992b, p.55 and ToK, p. 94, discussed in section 5.3.7, below). It might also be the case that the origin of Russell's doubts concerning his own deployment of the idea of logical form stem from his early conversations with Wittgenstein. On this possibility see Klement 2015.

(The set {Plato, Socrates} in Column 1 in the table below). The plurality shares a certain property, which makes it into a single unit — the property of being a man (see Column 2). According to Plato, concrete particulars are mutable, and so any property that applies to them can only be truly known by reference to an ideal entity — the idea of man (in Column 3). However, the idea itself exemplifies the very same property — the idea is what it really means to be a man. So by introducing it, we are also introducing a second plurality, different from the first one. Go down one line in the table, and note that the second plurality we reach, {Plato, Socrates, Idea of Man}, is only unified because it shares some property (namely, being a man<sup>2</sup>) and so there must be a further idea — a Third Man, by reference to which the property is known. And we can repeat this process indefinitely.

(1)	(2)	(3)
{Plato, Socrates}	{x   x is a man <sup>1</sup> }	Idea of Man
{Plato, Socrates, Idea of Man}	{x   x is a man <sup>2</sup> }	Third Man
{Plato, Socrates, Idea of Man, Third Man}	{x   x is a man <sup>3</sup> }	Fourth Man

Of course such a regress only arises given certain assumptions about predication. So let's spell out the assumptions of the Platonic theory that the TMA threatens; this will give us some points of comparison when we come to evaluate Wittgenstein's argument. There are three such assumptions:<sup>64</sup>

- (1) One-over-Many: For any set of entities that resemble each other with respect to a certain property, there is an idea that paradigmatically exemplifies that property

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<sup>64</sup> The classic reconstruction of Plato's argument and its presuppositions is found in Vlastos 1954; for further elaboration see Geach 1956; Strang 1963; and more recently, Meinwald 1992; Frances 1996; Pelletier and Zalta 2000; and Rickless, 2015. My own reconstruction is closest to Pelletier and Zalta's.

(2) Self-Predication: As exemplary, ideas are *self-predicative*, i.e. they are bearers of the property that they exemplify

(3) Non-Identity: Ideas must be *separate* from particulars, i.e. an idea that serves as a paradigm for a certain property is not identical with any of the entities which resemble each other with respect to the property that it is the paradigm of.

Plato's assumptions are motivated by epistemological concerns. Ideas serve to fix the meaningfulness of predicates; they are "one-over-many" in the sense that they unify a plurality of concrete and changeable things. The ideas subsist separately from the changeable individuals that fall under the predicate, and, being unchangeable, ideas are those things to which the predicates apply in an exemplary fashion.

But taken together, these assumptions generate a regress that would force us to posit an infinite number of ideas grounding each of the predicates we meaningfully use. Here's how. Suppose that all the great things resemble each other with respect to Greatness. Then by assumption (1) there is an idea that exemplifies this predicate, the idea of The Great. This idea is, by assumption (2), itself great. But by assumption (3), it is not identical with any of the other things we call great. And yet this idea, and all of the other great things, resemble each other with respect to some property; so by (1), there must be a second idea, call it The Very Great, which stands over and above all of them. And over it, we would have The Very Very Great, and The Very Very Very Great, and so on. Whatever relation holds between ideas and particulars, the TMA shows that that relation cannot be explained in terms of the idea itself. A separate idea is required; and this gives rise to a regress. Importantly, if we take Plato to be additionally

committed to the thought that ideas must be unique, the third-man regress gives rise to a contradiction.<sup>65</sup>

### 5.3.2. The Argument for the Relative Distinction

Let's now turn to Wittgenstein. What I call Wittgenstein's Relative Distinction is the claim that no picture can represent its own form of representation. Otherwise, Wittgenstein reasons, a regress would arise. For if a picture is to count as a representation, two conditions must be met.

The first is the condition of Common Form. Wittgenstein writes

2.16 In order to be a picture a fact must have something in common with what it pictures.

2.17 What the picture must have in common with reality in order to be able to represent it after its manner—rightly or falsely — is its form of representation.

A form of representation is the medium within which certain elements of the picture become significant. Within this form, we correlate the structures of pictorial elements with the structures of states of affairs. Thus in the simple case, Wittgenstein writes,

2.171 The picture can represent every reality whose form it has. The spatial picture, everything spatial, the colored everything colored, etc.

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<sup>65</sup> There is wide disagreement in the literature over whether the TMA is truly threatening to Plato's theory of ideas, and over whether Plato ever provides a proper response to it. Geach 1956, for example, takes Plato's TMA to be "a record of honest perplexity" (p. 72). By contrast, Meinwald 1992 suggests that given the distinction Plato has at his disposal, between two different kinds of predication — a form of predication which applies to pluralities of objects, and a form of predication which applies only to ideas, and articulates their essence — Plato is in a position to easily overcome the TMA.

Rickless 2015, following Frances 1996, doubts that Meinwald's distinction can put to rest all the variants of the Third Man; he argues that what Plato ends up rejecting are not the assumptions that lead to the regress, but rather the assumption of the uniqueness of ideas, which makes such a regress seem problematic. Once Plato rejects this assumption, he learns to live with infinite hierarchies and infinitely complex ideas in his theory.

This correlation may be a very complex matter, for example when we take a two dimensional picture as a representation of three dimensional space, or when the relative hue and sharpness of patches of color in an oil painting are taken to signify the relative depth of objects in space. Another example is the correlation between the order of frames on film and the flow of time in the narrative of a movie. Wittgenstein gives the following example: the physical shape of the grooves on a vinyl record *picture* the music. It would give us a better grip on the idea of form of representation to consider the fact that the converse is true as well: we can take the music to be the picture of the groove (4.0141). What we learn from these examples is that the common form of representation consists of the rules of correlation between two structures, even if each of these structures is ordered in a completely different way (2.151).<sup>66</sup>

The second condition that must be met by any representation, according to Wittgenstein, is what I will call the Outsideness Condition. He states it at 2.173:

2.173 The picture represents its object from outside (its standpoint is its form of representation), therefore the picture represents its object rightly or wrongly.

The idea is that the capacity of a picture to truly or falsely depict something depends on there being a distinction between two facts — the picture and the pictured. Nothing can count as its own representation. This seems to be such a trivial point that one might think it is not worth mentioning; but as we shall see, it is not inconsequential.

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<sup>66</sup> Moreover, not every formal aspect of the depicting object has to have representational significance. The form of representation dictates what counts as significant, and what doesn't. Some elements of pictures have no pictorial role to play: in traditional painting, for instance, the weight, hardness or smell of the depicting objects are insignificant, even though they are features whose form is shared with the objects in reality. Thus even though there is something non-arbitrary about the role which the spatial form of pictures traditionally plays in painting, it is arbitrary that this, rather than other shareable features of the pictorial object and the reality it depicts, are made significant in a painting.

Outsideness is not completely independent of the principle of Common Form. It is clear from 2.173 that a “standpoint” can only count as “outside” another to the extent that they belong to the same medium — that they share the form of representation. The form of representation is the neutral grounds, the plane on which two separate facts may stand in the relation of depicting one another.

These two principles, taken together, capture the distinction between the form of representation of a picture and the content that the picture depicts. Given Common Form and Outsideness, the Relative Distinction results: it turns out that it is impossible for a picture to represent its own form of representation; for, whatever a picture represents, it must be “outside of”. At the same time, the picture must be “within” the common form of representation that it shares with its object. The two demands are in tension; as Wittgenstein puts it:

2.174 The picture cannot place itself outside of its form of representation.<sup>67</sup>

A form of representation that enables a picture to depict certain objects does not allow that picture to represent its own form. Nonetheless, Wittgenstein writes, were it not for the form of representation, the picture would not be able to depict anything; even though a picture does not represent the form, the form must somehow be present. The form of representation, Wittgenstein holds, “shows forth” in the picture, even though it is impossible for a picture to depict — i.e. to *say* — what is its own form of representation:

2.172 The form of representation, however, cannot be represented by the picture; it shows it forth.

This is Wittgenstein’s statement of the Relative Distinction, the claim that a picture which has a

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<sup>67</sup> My emphasis.

certain form of representation cannot depict its *own* form, even though it shows it. This claim must be distinguished from the stronger claim, that logical form cannot be represented at all. At this point, Wittgenstein does not deny that some forms of representation could be represented — as long as this is done by means of other forms of representation.<sup>68</sup>

We can get clearer on the point of the Relative Distinction by considering some illustrative examples. Take the Necker Cube — does it represent one cube, or two? It is only from the perspective of a determinate form of representation that the marks on the page that make it up can be taken to represent a cube at all. On one way of seeing it, it is one cube, and on another way, it is another cube; on yet another way of looking at it, what we see is a hexagon. When we say that there are different ways of looking at this object, that yield different cubes, we are in fact admitting that the form of representation is not to be found within the picture. For if it were, we could not transition from seeing it as representing the one cube to seeing it as representing the other. Consider another case. When analyzing and teaching perspectival painting, we use geometrical diagrams that indicate the vanishing point and perspective lines emanating from it. Don't these diagrams depict the form of representation? Here's the sense in which I think they do not. It is only by virtue of our having the ability to appreciate perspectival form that we can see the two dimensional lines of these diagrams as representing depth. The form of representation is what allows us to take the diagram to have depth, and it is only in a derived sense that depth is being depicted by the diagram.

The form of representation that allows pictures to depict spatial states of affairs is not itself

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<sup>68</sup> For instance, we seem to be able to describe formal properties of a spatial picture by algebraic means; and we seem to be able to verbally describe how musical notation works. Whether these descriptions would truly count as a representation of the form of representation (rather than description of the picture and of what is pictured) is a complicated question, which I cannot pursue here in detail.

a spatial state of affairs — even in the simple case in which both the picture and the pictured are spatial, and even in sculpture, where the depicting object itself is a three dimensional object. It is true that we can use a spatial picture to represent other pictures, in the sense that we can picture the physical elements of a picture and their arrangement. But we thereby picture these elements qua spatial objects, not qua picture.<sup>69</sup>

Given that the form of representation is a correlation between two structures, in order to represent its own form of representation, a picture that has a certain structure would need a form of representation that correlates the picture's structure with a correlation between structures. But then the picture would need to have a form of representation which is different from the one it already has, and so it would no longer be the same picture — for it cannot have two forms of representation at once. Hence Wittgenstein's Relative Distinction.

### **5.3.3. The Argument for the Absolute Distinction**

The argument for the Absolute Distinction extends the form / content distinction that we just drew in the realm of pictures to the realm of thought and representation as such. For Wittgenstein, thought is virtually defined as “the logical picture of facts” (Tr. 3). Moreover it is crucial that in Wittgenstein's argument, there is no distinction between language and thought: logical form is the form of language AND of thought. Hence Wittgenstein writes

(3.032) To present in language anything which “contradicts logic” is as impossible as in geometry to present by its co-ordinates a figure which contradicts the laws of space; or to give the co-ordinates of a point which does not exist.

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<sup>69</sup> Cf. MN p.116.

This indicates a crucial new assumption, which I will call the principle of the Universality of Logic. It is in effect a generalization of the principle of Common Form:

2.18 What every picture, of whatever form, must have in common with reality in order to be able to represent it at all—rightly or falsely—is the logical form, that is, the form of reality.

2.182 Every picture is also a logical picture.

Universality consists in the claim that logical form is the least common denominator of any representation whatsoever. By claiming this, Wittgenstein makes no substantive claim about the content and shape of logic. Logical form just is the form of representation — it is the medium in which it is possible for certain structures to depict others (2.151). To say that it is the universal form of representation means that any picture of any fact by whatever means is also a logical picture of that fact (2.2).<sup>70</sup> The Absolute Distinction is the claim that logic cannot be depicted at all, since it is the form of any picture whatsoever.

Before I proceed, let me pause to reflect on some inescapable ambiguities in my use of the terms ‘logic’ and ‘logical form’ here. I sometimes use ‘logic’ and ‘logical form’ interchangeably, and I believe Wittgenstein does the same, where he discusses language taken as a whole. Logic is commonly spoken of as the study of logical form, rather than that form itself. For Wittgenstein, ‘logic’ is very often the name for the subject matter, not the discipline. But the term ‘logical form’ also means the *specific* structure of an expression, i.e. when we say that a proposition has the form of a conditional, or a disjunction, etc. In the context of 2.18, “logical form” means the

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<sup>70</sup> Wittgenstein talks about the degree to which a form of representation succeeds in complying with logical form, in terms of the combinatorial possibilities or the “multiplicity” of the states of affairs that a notation is capable of representing (4.0412). The thought that the capacity of a notation to capture logical form comes in degrees only makes sense in light of the idea that there is one universal logical form that the different notations can fail or succeed in capturing.

*global* form of representation, i.e. the medium or the dimension in which it is possible for any representation to depict anything. The global and the specific are internally related, of course:

2.15 That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another.

This connexion of the elements of the picture is called its structure, and the possibility of this structure is called the form of representation of the picture.

2.151 The form of representation is the possibility that the things are combined with one another as are the elements of the picture.

It is the *global* logical form that provides the representation with what is *common* to it and to the thing represented (2.2). It thereby opens up the space (the “possibility”) in which a structured fact of one sort — e.g. the linguistic proposition, with its *specific* logical form — can be taken to stand for another structured object — the state of affairs which it depicts.

Wittgenstein argues is that the three principles of Universality of Logic, of Common Form and of Outsideness lead to a contradiction, and that this is to be avoided by means of the Absolute Distinction. Here is how Wittgenstein puts it:

4.12 Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it—the logical form. To be able to represent logical form, we should have to be able to put ourselves with the proposition outside logic, that is outside the world.

...

4.1212 What can be shown cannot be said.

Let’s examine this argument in detail.<sup>71</sup> We have three assumptions:

(1) Common Form, that any representation must have the form in common with what it represents (2.161).

(2) Outsideness, i.e. that in order for a proposition to say anything about its object of representation, it must be separate from it (2.173).

(3) Universality, i.e. the principle that logic is the least common denominator, the form of representation that is shared by any picture and any reality represented (2.18).

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<sup>71</sup> I discuss a different variant of this argument, which is found in MN, p. 108, below. And cf. 5.4731.

Our primary target for *reductio* is the following claim:

(A) we can make a representation of logical form.

According to our Assumption (A), the object we purport to represent is logical form; it results from (A), together with Outsideness that

(A2) the representation of logical form needs to be “outside” its putative object, i.e. outside logical form

On an even stronger reading of Outsideness, which takes "being outside x" to mean “being different from x”, (A2) entails:

(A3) the representation cannot have a logical form

But given Common Form, it follows that

(A4) this representation would not have anything in *common* with logical-form

and so we can conclude:

(A5) it could not be a *representation of* logical form after all — in other words, it could not *say* anything about logical form.

In other words, if we could take the object of a certain representation to be its own form of representation, satisfying the condition of Outsideness would involve overriding the condition of Common Form, and vice versa. A representation of logical form would at once have to be *within* the logical form that is *common* to the representation and to what it represents, and *outside* its object, that is, outside logical form. If, as (A3) states, what this amounts to is the demand that we be able to represent logic “illogically”, then the very idea that we could have such a representation is incoherent.<sup>72</sup> Logical form cannot be thought of as an object; the very idea of

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<sup>72</sup> Cf. Tr. 5.4731.

representing the universal form of representation breaks down. We must reject Assumption (A), and adopt the Absolute Distinction between saying and showing: insofar as logical form is concerned, it can be shown but cannot be said.

Absent any further disambiguation of the terms “outside” and “within”, the two demands placed on any picture purporting to say anything about the *logical* form of representation are conflicting. This is the point Wittgenstein makes when he says

4.12 To be able to represent logical form, we should have to be able to put ourselves with the proposition outside logic, that is outside the world.

In the early *Notes Dictated to Moore* Wittgenstein consider the very idea of representing logic “from outside” as an impossibility:

LOGICAL so-called propositions *shew* [the] logical properties of language and therefore of [the] Universe, but *say* nothing.

...

It is impossible to *say* what these properties are, because in order to do so, you would need a language, which hadn't got the properties in question, and it is impossible that this should be a *proper* language. Impossible to construct [an] illogical language.<sup>73</sup>

To say that there is something “impossible” here is problematic, since it can be read as making a substantive claim, which sets a limit against which we run up in thought, while at the same time indicating what is ineffable and remains beyond that limit. But there is a way to read it resolutely, as indicating that the very idea (of representing logic from outside logic) is nonsense. Nothing substantive is denied by ruling out such a pseudo-possibility.<sup>74</sup> This is the way in which the distinction between saying and showing itself is to be read: the denial that what shows itself can be said should not be treated as leaving any content beyond the realm of the sayable; the point is

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<sup>73</sup> These are the first and third paragraphs of MN, p. 108. Cf. Tr. 5.4731.

<sup>74</sup> For an illuminating discussion of Wittgenstein’s dialectical use of the idea of the “impossibility” of illogical thought, see Conant 1992.

to see that things that once seemed to us to be contentful (for example, formal concept words) are in fact purely formal. I will return to this point at 5.3.7, below.

The reductio argument shows us that if we could take the object of representation to be a form of representation, satisfying the condition of Outsideness would imply overriding the condition of Common Form, and vice versa. For, when the represented object is logical form, stepping outside it leads to having nothing in common with it, and having something in common with it means that we have not stepped outside it. Wittgenstein concludes that Assumption (A) must be rejected, and that instead we must draw the Absolute Distinction between saying and showing, since insofar as logic is concerned, it can be shown but cannot be said.

#### **5.3.4. Wittgenstein's vs. Plato's TMA**

There are illuminating analogies between Wittgenstein's and Plato's arguments. Take the idea that we can make a representation of any state of affairs; this would be parallel to Plato's One-over-Many principle. Wittgenstein also has something like Plato's Self-Predication principle, in the sense that Common Form takes the representation and the represented to share a form.<sup>75</sup> In Plato's TMA, Nonidentity tells us that for there to be something in common to the idea and the particulars that resemble it, there must be a separate idea that is different from all of them. The same effect is achieved by Wittgenstein's Outsideness, although as we shall see the setup is a

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<sup>75</sup> However, there is a crucial difference in the structures of the arguments, that renders Wittgenstein's self-predication assumption somewhat more plausible than it seems to be in the platonic context. In Plato, self-predication applies to the idea, and non-identity applies to the relation between particulars and the self-predicated idea; in my reconstruction of Wittgenstein's argument, self-predication applies to representations, and non-identity applies to the relation between particulars (representations) and the *form* of representation.

little different.

In Wittgenstein's argument for the Relative Distinction, the trouble begins when we try to make a picture that depicts its own form of representation. Given Outsideness, in order to depict its own form, the picture would have to "place itself outside its form of representation" (2.174); but how can a picture both have one form of representation and be outside it?

Wittgenstein denies that a picture could have two forms of representation at once. Let us call this the assumption of the Exclusivity of Form — it is basically a variant of the principle of non-contradiction. There's a parallel assumption at work in Plato, namely, that a form cannot unite the plurality that includes, apart from the various particulars, that form itself.

Wittgenstein's argument for the Relative Distinction does not result in the denial of the possibility of depicting specific forms of representation. All it tells us is that *any given* picture cannot depict its own forms of representation. This opens the possibility of a regress in which the form of representation of one picture is represented by a second picture, whose own form of representation can only be represented by a third picture, etc.. This is not a vicious regress, since unlike Plato's theory, Wittgenstein's theory does not give representations of forms of representation the epistemological role that platonic ideas have. In Plato, the infinite regress threatens the thought that an idea is a unity that explains plurality, a fully determinate object, the knowledge of which is required for the understanding of indeterminate particulars.

When it comes to logic, however, Wittgenstein considers the possibility of regress to be much more problematic; to thwart it, Wittgenstein concludes that logic, understood as the form of all representation as such, cannot be represented at all. The very idea that there is a logic outside logic contradicts the explicit assumption of Universality. In a platonic idiom,

Universality can be expressed as the claim that ideas are unique.<sup>76</sup> Given the assumption of Uniqueness, the possibility of a Third Man regress makes the theory of ideas contradictory. An infinite regress would threaten the thought that an idea is a unity that explains plurality, that it is that determinate entity the knowledge of which is required for the understanding of the indeterminate particulars.

The lesson for Plato is (or at least, should be) that there is a deep confusion in the thought that we can treat abstract properties as substantive entities. The unity of predication is not itself a bearer of properties. For Wittgenstein, the lesson is that to think of logical form as an object of representation is no longer to think of it as logical form. Logical form is not a fact, or a thing, or a principle, but rather the way in which any fact, thing or principle can be expressed. It cannot be said, but it must show itself in everything we say.

### **5.3.5. Alternative 1: Rejecting Outsideness**

To the extent that Wittgenstein's TMA has the form of an argument by *reductio*, the rejection of Assumption (A) — that we can make a representation of logical form — is not the only response open to us. We could also reject either (1) Common Form, (2) Outsideness, or (3) Universality. In the next two subsections I consider the shape of such alternatives. I then return to discuss the upshot of Wittgenstein's own response.

Let us begin with the objection that sees (2) as the culprit in the *reductio* — the principle of

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<sup>76</sup> The platonic assumption of Uniqueness is discussed by Pelletier and Zalta 2000, p. 169, and Rickless 2015. Another platonic notion that would be threatened is that of the highest idea, in which all other ideas partake — the idea of the good.

Outsideness, that we cannot make logical form into an object we stand outside of. The objection would run as follows: for a picture to be “outside” its object, it suffices that it differ from its object in terms of its matter, but it is not necessary that it differ from it in terms of its form. This would be consistent with the parenthetical remark in 2.173: “A picture represents its object from without (Its standpoint is its form of representation).” The objection would thereby seek to show that no third-man regress arises, since it would be possible for a representation to differ from logical form in terms of its matter, but not in terms of its form. So (A2) would seem to be false.

This objection can also be put as follows. Outsideness is ambiguous, in the sense that a representation’s being outside<sup>1</sup> logical form (by making it into its object) does not necessarily mean that it has to be outside<sup>2</sup> its own logical form (by not having a logical form). A representation could be both logically informed, i.e. not outside<sup>2</sup> its form, AND have logical form as its object, i.e. be outside<sup>1</sup> that logical form.

One trouble with this thought is that the objector would have to grant that representations of logical form, understood in this way, are uninformative. They presuppose logical form (i.e. their own form) rather than inform us of it. To judge by what Wittgenstein has to say about tautologies, he would deny that an uninformative representation can count as a representation at all (4.462). And moreover, according to Wittgenstein’s way of applying Occam’s razor (3.328, 5.47321) , such superfluous representations are logically meaningless, and could be excluded from a proper language, as they play no significant role.

Another trouble with this objection is that it presupposes that we can make sense of the idea of differing *in matter* from logical form. But it is not clear at all that there is such an idea to make sense of. For on the one hand, logical *form* has no “matter” from which to differ; and on

the other hand, the sense in which the *matter* of the representation differs from logical form is completely trivial. Every representation differs from logical form in this sense. It is not even clear that in treating logical form as an object of representation (which this objection assumes is possible) we would truly capture those very features which make it into a *form*. This is related to a point that came up in my discussion of Frege's distinction between concepts and objects. When we try to talk of concepts, and we assign them to the object position in sentences, we efface their predicative nature.<sup>77</sup> The objection under discussion here assumes the possibility of that which Frege finds to be impossible: that we can fix our grip on the sense of a term (in this case, 'logical form'), regardless of the context in which it is deployed.

On the account I will develop in the next chapter, logical form informs an activity — the activity through which we reason and understand. To think of logical form as an object of representation, apart from this activity which it informs, is not really to think of it as logical *form*. It is to think of an abstraction, a mere fact, but no fact can do the work of logical form, since to see how any logical fact bears on any other fact, we would need to know how to apply it — and that application would itself have to be logically informed. Thus to represent logic as something standing apart from the activity which logic informs leaves what is most crucial to logic unexplained.

Wittgenstein's Absolute Distinction, strictly thought through, casts doubt not only on the idea that we can make logical form into an object of representation, but also on the intelligibility of what he himself says about logical form. The grammar of language tempts us into confusion here — it invites mischaracterizations that result in taking the term 'logical form' to signify a

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<sup>77</sup> See section 5.2.1

kind of ineffable content. Wittgenstein's strategy in dealing with this thought is dialectical and therapeutic, not substantive. After the ladder of the *Tractatus* has been climbed, even Wittgenstein's use of the term is to be recognized as nonsense (6.54). There is something useful in this sort of nonsense — not because it is more correct and illuminating than other nonsense, but because, when it succeeds, it helps us realize that other forms of philosophical nonsense to which we were previously attracted have lost their charm. But if we learn to look for logic in the activity of our mind rather than its products, we will no longer be tempted to think of logic in the substantive ways that philosophers have, and we will no longer be bothered by their puzzles. Our use of the term 'logical form' would be merely elucidatory, and we should be ready to discard it if we find out that it gets us into trouble.

### 5.3.6. Alternative 2: Rejecting Universality

Russell, in his Introduction to the *Tractatus*, rejects Wittgenstein's argument by denying Premise (3), Universality.<sup>78</sup> As a result, he knowingly unleashes the Third-Man regress:

These difficulties suggest to my mind some such possibility as this: that every language has, as Mr. Wittgenstein says, a structure concerning which *in the language*, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit. Mr. Wittgenstein would of course reply that his whole theory is applicable unchanged to the totality of such languages. The only retort would be to deny that there is any such totality. The totalities concerning which Mr. Wittgenstein holds that it is impossible to speak logically are nevertheless thought by him to exist,

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<sup>78</sup> Russell, "Introduction" in Wittgenstein 1922, pp. 7-23.

Carnap's *Logical Syntax of Language* responds to Wittgenstein by rejecting Universality as well (Carnap 2000, pp. 282ff). The distance between Carnap's conception of language and logic and Wittgenstein's is great and goes beyond the scope of the present inquiry. For discussions of Carnap's project as a response to Wittgenstein's AD, see Awodey and Carus, 2007; Kuusela 2012; and Beni 2015.

and are the subject-matter of his mysticism. The totality resulting from our hierarchy would be not merely logically inexpressible, but a fiction, a mere delusion, and in this way the supposed sphere of the mystical would be abolished. Such a hypothesis is very difficult, and I can see objections to it which at the moment I do not know how to answer. Yet I do not see how any easier hypothesis can escape from Mr. Wittgenstein's conclusions.<sup>79</sup>

This discussion is the first appearance of the idea of the hierarchy of languages that is later systematized and made more rigorous in Carnap's and Tarski's semantics.<sup>80</sup> The core of Russell's suggestion is that logic no longer applies universally, no longer counts as the study of the most general truths and no longer aims to account for the basic laws of thought and inference. Logic is to be fragmented into a hierarchy of metalinguistic theories, each of which is relative to an object language, and is itself only specifiable by higher meta-languages.

Russell respects the principle of Outsideness (that within any given language, its own logical form cannot be represented), and only denies the principle of Universality. Hence the need for a hierarchy of languages. Russell then denies that the infinite hierarchy of languages can form a totality, and takes himself to thereby block Wittgenstein's attempt to express substantive ineffable truths about it. Were it not for the fact that Wittgenstein denies the need to set up such a hierarchy in the first place, he would perhaps agree with Russell on this specific point — that nothing is being said when we try to speak of such totalities. So this point cannot count as an objection to Wittgenstein, resolutely read.<sup>81</sup>

Giving up the principle of Universality comes at a very high price; together with it, we also

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<sup>79</sup> Russell, "Introduction", p. 23.

<sup>80</sup> Carnap 2000; Tarski 1944.

<sup>81</sup> Russell's reading suffers from all the drawbacks of the Metaphysical reading — Wittgenstein is taken to be concerned to assert the truth of ineffable but substantive claims. But the point of Wittgenstein's AD, on my reading, is to deny that there is anything that can be said about the totality of a language, and so a hierarchy of the kind Russell proposes could not even begin to arise.

give up on the identification of logic with the form of thought. We no longer talk about logic as the form of representation as such, but only as a set of rules which is relative to a specific formal language, specifying how its signs, taken as mere signs, are to function. Thus once the Russellian move is taken, the resulting, semantic approach to logic no longer provides answers to the questions Wittgenstein (or Frege, and the earlier Russell) is asking about the form of thought — it completely changes the subject. A relativized conception of logic is no conception of logic at all, and a separation of syntax from semantics that allows us to talk of the grammatical form of the object-language without regard for meaning and truth would no longer be recognized by Wittgenstein as an account of logical form.

Sure enough, Wittgenstein would concede that we can construct logical notations that help us clarify our thought. Indeed large parts of the *Tractatus* are devoted to doing just that. And he might also concede that there could be value in constructing a meta-theoretical semantics, in which the concepts of truth (relativized to a given language), and logical consequence (relativized to a given language) can be defined. Wittgenstein would see such reflections on formal language as aids for the clarification of thought and of language. However, and crucially, Wittgenstein would not think of these reflections as providing an adequate model of our thought, or of our language.

Russell's criticism of Wittgenstein prompts us to notice that Wittgenstein's distinction between saying and showing is meant to be a distinction concerning language as such, not this or that language. Wittgenstein does not think that the expressions that *show* logical form belong to a different language or are in some other way segregated from the expressions that *say* things. It is in our own language — in using it to say what we want to say — that logical form shows itself.

In order to make sense of Wittgenstein's absolute distinction we must abandon the thought that logical form is an object of representation, apart from the activity which it informs. The grammar of language tempts us into this confusion, and invites mischaracterizations that ultimately result in taking the idea of logical form to signify a kind of ineffable content.

### 5.3.7. The Resolute Reading of the Absolute Distinction

In laying out Wittgenstein's argument for AD I played down its dialectical nature. What I mean is that Wittgenstein's *reductio* argument is not meant to culminate in asserting the negation of (A), but rather in realizing the incoherence of the view (A) implies. If this is the case, then our rejection of (A) must not take the form

(A6) We cannot make a representation of logical form.<sup>82</sup>

Otherwise, the argument for the absolute distinction between saying and showing would truly cut the branch it is sitting on, committing what Geach calls "Ludwig's self-mate".<sup>83</sup> For to say that we are impeded from saying something would imply that there is such a thing which cannot be said — and how could we say *that*?

The proper way to avoid this predicament is to seek a different way of responding to it — to abandon the temptation that led to it, not to deny that we can fulfill it. This is one of the guiding ideas of the resolute reading of Wittgenstein's *Tractatus*. Wittgenstein's argument, on

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<sup>82</sup> Nor would it be much help to switch (A6) with

(A7) There's no such thing as a representation of logical form.

<sup>83</sup> Geach 1976, p. 54f.

this reading, does not aim to show that representation is somehow limited in its capacity to represent certain things. Rather, it aims to show that form is radically different from content, and that logical form cannot be thought of as an object — not even a special, ineffable object. But it is not a limitation on representation that we cannot force the form of representation into the box of the object of representation; there is no coherently specifiable possibility that is being ruled out here (any more than we rule out something by saying that an event cannot be put through a hole).<sup>84</sup> The philosophical desire that underlies assumption (A) is nonsensical; it places conflicting demands on the term form. As a result, we end up not having anything determinate in view that we can affirm or deny the possibility of.

The standard reading of the distinction sees it as a means through which Wittgenstein aims to bring out features of language and reality that cannot be said. There are truths, it is held, that can only be shown. On the kind of resolute reading I advance, the purpose of Wittgenstein's distinction is primarily negative. There are confused ways of thinking, such as the thought that there are logical truths; But when we think through this idea, we realize that we end up empty handed, that we did not succeed in saying anything, and that instead we have only deepened our confusion. According to Wittgenstein's strongly contextual approach to meaning, terms do not mean anything outside of a determinate context; and if the context in which we try to deploy the term 'form' is itself ambiguous — if there is more than one role that we ask this term to play in a given context, then the meaning of the term is left undetermined. The result is nonsense

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<sup>84</sup> cf. MN, p. 108.

(5.473).<sup>85</sup>

When Wittgenstein says that what can be shown cannot be said, he is not saying that there is a thing, a fact or a truth, of which we can say that nothing can be said about *it*; he is saying that *we* sometimes find ourselves using words in an indeterminate way that allows us to imagine we have singled out an ineffable fact, an unsayable sayable.

An example might help. Consider the Principle of Non Contradiction (PNC) — the principle that a thing cannot both have and not have the same property, in the same respect, at the same time.<sup>86</sup> What are we telling ourselves when we think of the PNC as a truth? We might think that by recognizing it as a truth, we place a constraint that everything needs to respects somehow, or else... But if this principle is truly part of logical form, can we really think of it as a constraint that excludes anything — can we even imagine what anything would be like were it not subject to this “constraint”? Would we still have a grip on any individual thing, qua individual thing, in imagining the PNC as something that the thing (individuated how?) would otherwise be lacking? That “thing” which the PNC is supposed to tell us something about — can the PNC apply and not apply to it at the same time? Don’t we already adhere to the PNC in thinking of those things to which the PNC may or may not apply, as things?

We were trying to say something about what shows itself in language, about PNC as part of logical form, but in attempting to say it, to treat it as a truth, we cast it in a role that fits truths — that they may be true or false, or that they are necessarily true or necessarily false. This leads us

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<sup>85</sup> The temptation to hold on to (A6) stems, perhaps, from a projection of our understanding of the argument for the relative distinction, given in the context of picturing, on our understanding of language as such. What we should do instead is project the radical conclusions reached here backwards, and recognize that even in the case of pictorial media of representation, it is not just that a picture cannot represent its own form of representation; nothing can count as a representation of that form, without distorting its formality; it would be nonsense to insist, in that case, that what we are talking about is the form.

<sup>86</sup> My discussion is inspired by Aristotle’s *Metaphysics*, Book IV, ch.4 (Aristotle 1984c).

to treating it as a constraint, i.e. a condition that excludes certain possibilities and affirms others. But there is no such thing as a constraint that excludes possibilities that cannot even be thought. And so there is nothing that we can actually manage to say when we attempt to treat the form of thought as a fact.

This is the line of thought that underlies Wittgenstein's response to Russell's conception of logical form: "There is no thing which is the form of a proposition, and no name which is the name of a form".<sup>87</sup> Wittgenstein objects that Russell uses the term "thing" in a way that indeterminately covers both form and content; the trouble is not that he asserts the something false about logical form, but that he treats it as something of which one could assert something true or false — for example that it does, or does not, enter into a relation with the other constituents of a judgment. Russell is not completely unaware of the problem; though this does not stop him from persisting in using terms in problematic ways, he does express certain doubts:

If we made [the form] a constituent, it would have to be somehow related to the other constituents, and the way in which it was related would really be the form; hence an endless regress. Thus the form is not a constituent.<sup>88</sup>

...the form is not a "thing", not another constituent along with the objects that were previously related in that form.<sup>89</sup>

These remarks might reflect Russell's acknowledgement of his acquaintance with the Wittgensteinian worries, but they might also reflect a Russellian source of inspiration for

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<sup>87</sup> NB, p. 105. See discussion in section 3.3. of Chapter 3. And Recall Cora Diamond's formulation: "Really to grasp that what you were trying to say shows itself in language is to cease to think of it as an expressible *content*: *that which* you were trying to say" in Diamond 1991e, p. 198.

<sup>88</sup> Russell 1992b, p.55.

<sup>89</sup> ToK, p. 98; Cf. the much earlier (1904) Russell 1994a:

The mode of combination of the constituents of a complex is not itself one of the constituents of the complex. For if it were, it would be combined with the other constituents to form the complex; hence we should need to specify the mode of combination of the constituents with their mode of combination . . . (p.98).

And see the discussion in Klement 2015.

Wittgenstein's Third-Man argument and for his conclusion that we must draw a strict distinction between what can be said, and what can only show itself. Be that as it may, the trouble is that Russell himself does not draw this distinction resolutely enough, and his account of judgment suffers from it, as he himself comes to appreciate.<sup>90</sup>

To conclude: what shows in our use of language, according to Wittgenstein's TMA, is the logical form of representation. This is not a fact, or a thing, or a principle, and the distinction between saying and showing cannot be taken as a constraint that prevents us from doing something, when it tells us that logical form cannot be said. Recall the lesson from Plato's TMA: there is a deep confusion in the thought that we can treat properties as substantive entities. Similarly, the lesson from Wittgenstein's TMA is that to think of logical form as an object of representation, apart from the activity which it informs, is no longer to think of it as the logical form of representation.

#### **5.4. Conclusion: Logical Form Shows Itself, But It Cannot Be Said**

In this chapter I aimed to shed light on Wittgenstein's distinction between saying and showing in three ways. First, I spelled out the terms of the debate between resolute and ineffabilist readings of the *Tractatus* and suggested that the resolute reading of the distinction between saying and showing can give us the tools in terms of which we could also understand Wittgenstein's rejection of Frege's and Russell's rules of inference, whereas the ineffabilist reading cannot do

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<sup>90</sup> In a letter to Lady Morrell Russell writes (regarding the composition of Russell 1992b):

I can't get on with "what is logic?", the subject is hopelessly difficult, and for the present I am stuck. I feel very much inclined to leave it to Wittgenstein. (Quoted in the editor's introduction to Russell 1992b, p. 54).

that. This is so because the ineffabilist can make no objection to the idea that rules are inexpressible but contentful, whereas for the resolute reader this signals a deep confusion. Second, in 5.2 I tried to show that Wittgenstein's distinction should be seen as an alternative to Frege's distinction between elucidation and ordinary discourse, and that only in this way can the rejection of rules of inference make sense. Finally, in 5.3 I have shown that the distinction between saying and showing is deeply rooted in Wittgenstein's conception of representation. It articulates the form / content structure of the activities that make up our cognitive life. And it exposes the incoherence of the idea that we would be lacking anything by lacking a representation of logical form.

Here is what remains to be argued, in chapter 6, in order to fully justify Wittgenstein's rejection of rules of inference. The idea that we need rules of inference reflects a misunderstanding of understanding. Once we see how logic informs our understanding of propositions, we see that in inference, all of the justification we need is already given with the premises and conclusion. The source of the idea that rules of inference are needed is the tendency to think of logical form as separate content, and to treat as sayable that which can only show itself within what we say. As I have shown in the earlier chapters of this dissertation, Frege's and Russell's approach to rules is an attempt to do that; their rules are the expression of philosophical nonsense.

## 6. Logical Form and the Rejection of Rules of Inference

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The goal of this chapter is to justify Wittgenstein's rejection of rules of inference by showing how it is grounded in his conception of logic. I argue that in order to be able to coherently deny the need for rules of inference, Wittgenstein must think of logic in a way that closes the gap between inferring and understanding. Wittgenstein advances a hylomorphic view, in which logical form pervades the entire realm of sense. On such a view, the step from understanding to inferring requires no mediation. Rules of inference are, as Wittgenstein says in 5.132, "senseless and would be superfluous".

Moreover, the rejection of rules of inference is a rejection of the idea that logic is a source of content that we appeal to in the course of reasoning, as well as the rejection of the idea that in principle, we can conceive of logic apart from the activity of reasoning and we can conceive of this activity apart from logic. This follows from Wittgenstein's distinction between saying and showing, as I construed it in Chapter 5. I there argued that Wittgenstein's distinction undermines the very idea of inexpressible content. And in Chapter 4 I argued that both Frege and Russell think of the rules of inference of their system as inexpressible but contentful. But on Wittgenstein's view, logical form is not an additional fact to be appreciated, alongside the premises and the conclusion — it is the form of the appreciation of facts. It *shows* forth, but it

cannot, and need not be *said*.

The conception of logic articulated in the *Tractatus* is capable of doing away with rules of inference only because it does not sunder inference from the understanding of the proposition. In Chapter 3 I argue that Russell's idea of rules of inference arises on the background of a naive picture of logic which he is initially tempted to hold, on which propositions are inert objects, each of which can be grasped independently of the others; rules of inference are called for in order to infuse life into them. In his attempt to think explicitly about the idea of logical form, Russell ends up assimilating it, too, to an object to which the other constituents of judgment relate. By contrast, I will here show that on Wittgenstein's account, propositions take part in a logically-informed activity. Logical form pervades the entire realm of sense, such that in understanding a proposition, we form a new network of logical relations between that proposition and the other propositions that we understand. This network of internal relations spells out the demands and obligations that the affirmation of the proposition imposes on us. In particular, to understand the premises of a valid inference is to already be in possession of the justification of the conclusions that one might wish to draw from them, and to have all the permission to infer it. A rule of inference (if there was such a thing) could not affect the justification of the inference in the least bit, for the justification is nothing apart from the propositions in their internal relations to each other.

Thomas Ricketts suggests that Wittgenstein seeks to endow the entirety of logic with the status which Frege and Russell preserve for rules of inference — understood as inexpressible norms, that govern our practices of reasoning and understanding.<sup>1</sup> But Ricketts understates the depth of the transformation in the conception logic that Wittgenstein seeks to effect. In particular,

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<sup>1</sup> Ricketts 1985, p. 4.

he does not see that Frege's and Russell's rules of inference are not a model of imitation, for Wittgenstein, but an instance of the cardinal problem of philosophy. In saying that logical form is shown, but cannot be said, Wittgenstein is not denying the expressibility of something that operates like rules or plays the substantive role that rules do, according to Frege and Russell. He is denying that it makes sense to talk of logical form as something which is separate from our thought and governs it from the outside. That would be to treat form as a special kind of content — the ineffable kind. It would be to efface the difference between content and form.

The chapter proceeds as follows. In section 6.1 I articulate Wittgenstein's idea that understanding is logical informed. Sections 6.2 and 6.3 focus on the ways in which Wittgenstein's account of tautologies reflects the idea that logic informs our understanding of propositions as well as their sub-propositional components. Tautologies do not *say* what logical forms are; they are senseless, and say nothing. However, precisely for this reason, tautologies are occasions on which the logical form shows forth with particular clarity. It manifests the inspection and calculation that is presupposed in all understanding — the articulation of the logical network of commitments that each act of understanding must involve.

In sections 6.4 I turn to Wittgenstein's criticism of rules of inference. I first articulate the sense of "senselessness" that rules of inference suffer from. I argue that it is the expression of the rules of inference that Wittgenstein calls "senseless". In section 6.5 I articulate the sense in which rules of inference are superfluous. And in the concluding section 6.6, I argue that Russell's and Frege's accounts of inference are a case of philosophical nonsense. This charge of nonsensicality is not made explicitly in *Tractatus* 5.132. But it follows from the way in which Wittgenstein understands philosophical nonsense. It is nonsensical to claim that one has truly understood propositions that stand in the internal relation of following, yet is unable to infer the

one from the other without appeal to a rule which is independent of both. And it is nonsensical to think that a system of notation that employs such rules is an adequate model of the activity we engage in when we think.

## 6.1. Logical Form and Understanding

To understand a proposition is to make sense of something. When we do that, we put ourselves in a certain relation to the way in which the world is like. We enter the relation of being able to see whether the proposition agrees or disagrees with reality, i.e. the relation of being answerable to the world, in some particular respect:

4.021 The proposition is a picture of reality, for I know (*kenne*) the state of affairs presented by it, if I understand the proposition.

4.024 To understand a proposition means to know (*wissen*) what is the case if it is true.

To take a proposition to be making a truth-evaluable claim requires that our understanding of it be determinate (5.156c). But how is a single proposition capable of providing us with a complete and determinate picture? A claim is determinately true or false only if it allows whoever understands it to see what it excludes and what it affirms. This means that to affirm a proposition, its inferential relations to other propositions must be manifest to us.

Wittgenstein expresses this in terms of the position of a proposition in “logical space” (3.4). The relations between propositions in this logical space are its “logical coordinates”, or “scaffolding”:

3.42 Although a proposition may only determine one place in logical space, the whole logical space must already be given by it. ... (The logical scaffolding round the picture determines the logical space. The proposition reaches through the whole logical space.)

4.023 The proposition determines reality such that one only needs to say “yes” or “no” to it, and nothing more, to make it agree with reality. Reality must therefore be completely described by the proposition.

...The proposition constructs a world with the help of a *logical scaffolding*, and therefore one can actually see in the proposition all the logical features possessed by reality if it is true.<sup>2</sup>

“Logical space” might be a somewhat misleading metaphor, since the logical relations between propositions in logical space are, according to Wittgenstein, *internal* relations. Being an internal property (or relation) means that the identity of the object it belongs to is constituted by those features (4.123). For instance, Wittgenstein says the following about the internal relation between two colors:

4.123b (This blue colour and that stand in the internal relation of brighter and darker *eo ipso*. It is unthinkable that these two objects should not stand in this relation).

Thus the proposition and its logical place (its internal relations to other propositions) are one and the same thing. When propositions are internally related, any one of them involves the recognition of a whole array of others. These relations are implicit. To say that they only show themselves (4.122), means they are unfolded and revealed when we infer, reject, and assert propositions, but they are not specifiable separately from this activity. In the activity through which we unfold our grasp of the proposition’s internal relations, we show its logical form.

A proposition is not to be identified with any one of its various expressions, according to Wittgenstein. Seemingly different expressions say the same thing if the same thing must be the

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<sup>2</sup> My emphasis. I emended Ogden’s translation, in accordance with Wittgenstein’s suggestions Wittgenstein 1973, p. 17, but with one minor change — I replaced ‘so far’ with ‘such’. Wittgenstein’s proposal for a translation, as well as the Ogden and the Pears-McGuinness translations stray quite far from the original German: “Die Wirklichkeit muss durch den Satz auf ja oder nein fixiert sein.” Indeed, Wittgenstein writes to Ogden that he (Wittgenstein) does not know how to translate this sentence; but then he proceeds to offer the translation I used above. Before offering that translation, Wittgenstein also offers the following: “a proposition determines reality such that by merely affirming or denying it one can make it agree with reality”. Perhaps a better translation would be the following: “the proposition restricts reality to a yes or no answer”.

case in order for any of them to be true. Logically equivalent propositions (e.g.  $\sim\sim p$ , or  $p \vee p$ , or  $p \& (q \vee \sim q)$ , etc.) occupy the same logical *place*, inasmuch as they make the same claim about reality (5.44-5.441; cf. 3.344). Moreover, we can have more and less complete understanding of a proposition, since according to Wittgenstein, it is not clear what atomic propositions there are, and how a complete analysis of the ordinary propositions we use will ultimately look like (5.55).

To some traditional readers, Wittgenstein seems to endorse two separate accounts of propositional sense — a picture theory which accounts for the sense of atomic propositions, and the truth-functional account of molecular propositions. I follow Anscombe and Ishiguro in denying that Wittgenstein has two such separate accounts of representation.<sup>3</sup> Indeed, elementary propositions — the propositions that stand in the immediate picturing relation — must already involve logical complexity, or at any rate, they must already involve the possibility of their logical combination:

5.47c Whenever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants.

Logical complexity is presupposed in all propositional sense — even in the simplest, atomic propositions. Consider the case of negation. Every proposition presupposes its negation (5.5151), in the sense that the two propositions exhaustively divide the space of possibilities, the space of “what is the case if (each of them) is true”, in the same way:

4.0621c The propositions “p” and “ $\sim p$ ” have opposite senses, but to them corresponds one and the same reality.

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<sup>3</sup> Anscombe 1959 attacks this view, citing Popper as an example, in her *Introduction to Wittgenstein's Tractatus* p. 25ff. Another example for the approach she rejects is von Wright 1955, p. 533. More recently, Zalabardo 2015 has offered a reading of the *Tractatus* which takes there to be two such separate elements in Wittgenstein's theory of representation; his view is criticized by Bronzo 2016.

The view has also been rejected by Ishiguro 1969, who shows that Wittgenstein could not think that there is a way to fix the reference of names outside the context of the complex propositions in which they occur.

Wittgenstein takes this point to be generalizable to any form of logical complexity:

5.47b For all logical operations are already contained in the elementary proposition.  
For “fa” says the same as “ $(\exists x).fx.x=a$ ”.

Moreover, Wittgenstein says that any propositional symbol presupposes all the truth-functional combinations in which it can take part (5.515b). To understand a proposition — even an atomic proposition — always means to place it within a whole network of propositions; and to be in such a position is to have the capacity to infer consequences, to check for inconsistency, and to revise our beliefs accordingly. We never deal with atomic propositions in a logical vacuum.

Consider what understanding a proposition commits us to: to the extent that we are competent thinkers, we must be able to recognize that the same claim is being made in all of the various expressions of each proposition that we understand. But we only count as having a full understanding of the proposition once we know its full analysis, which means, in the case of empirical propositions, that we can only hope that one day science will provide us with it. This might seem to mean that Wittgenstein holds the implausible view that we must be logically omniscient to have any understanding. I discuss this objection towards the end of this section. I propose there that the fact that propositions place demands that we are not yet able to recognize does not mean that we are unable to recognize any of the logical demands that they place on us. It only means that the internal relations between propositions we understand might outstrip our current understanding.

Logical form governs the articulation of what we understand. This applies to our grasp of sub-propositional components as well. Wittgenstein says that the meanings of sub-propositional expressions are fixed not only with respect to the context of the proposition in which the expression does appear, but also to the class of all propositions in which that expression *might*

appear while retaining its logical identity (3.311). An expression, individuated in this way, consists in the contribution that it makes to the logical nexus of our language taken as a whole. We only identify an expression as a determinate logical unit — as a symbol — once we recognize the logical roles it takes. Wittgenstein's distinction between sign and symbol is crucial here: it is in the activity of articulating and unfolding our understanding that signs have their life and truly serve as symbols. A sign — a mere mark on a page — is only identifiable as an abstraction from a symbol, i.e. by reference to the role the sign plays when it forms part of a symbol-in-use:

3.326 In order to recognize the symbol in the sign we must consider the significant use.

3.327 The sign determines a logical form only together with its logical-syntactic application.

Understanding has to do with symbols, i.e. with the significant *use* of signs. On this basis,

Wittgenstein places a very high bar for understanding of names:

4.243 Can we understand two names without knowing (*wissen*) whether they signify the same thing or two different things? Can we understand a proposition in which two names occur, without knowing (*wissen*) if they mean the same or different things?

If I know (*kenne*) the meaning of an English and a synonymous German word, it is impossible for me not to know that they are synonymous, it is impossible for me not to be able to translate them into one another.

Meanings of names and the relations of identity between them is something we need to have mastery of (*kennen*); without it, we cannot understand the propositions containing them. I am here following Michael Kremer, who convincingly argues that in speaking of how understanding presupposes the knowledge of the meaning of signs, Wittgenstein uses the term '*kennen*' to indicate a practical *mastery* of the linguistic instrument, rather than to indicate propositional knowledge, or to indicate a Russellian kind of acquaintance with meanings.<sup>4</sup>

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<sup>4</sup> Kremer 2002a, pp. 272–303, p. 290 and pp. 282ff. *Kennen* is similarly used in Tr. 2.01231 and 4.021, cited above.

Understanding our language, on Kremer's suggestion, consists in a practical know-how, a mastery of language use. This is an idea that is more commonly associated with the later Wittgenstein's *Philosophical Investigations*, where he says, for instance:

150. The grammar of the word “knows” (“*wissen*”) is evidently closely related to that of “can” (“*können*”), “is able to” (“*imstande sein*”). But also closely related to that of “understands” (“*verstehen*”). (‘Mastery’ (*‘beherrschen’*) of a technique).

199. To understand a sentence means to understand a language. To understand a language means to have mastered a technique.<sup>5</sup>

There are important differences between the earlier and the later Wittgenstein, e.g. with respect to the conception of language and what is involved in its use, which I will not get into here. What I'd like to emphasize is the following continuity between the *Tractatus* and the later Wittgenstein: that already in the *Tractatus* Wittgenstein construes understanding in terms of the activity of using language.

The remark about the relation between understanding and mastering a language in 4.243 articulates the constitutive role of logical form. To have mastery of language — in this case, a mastery of synonymies — is to be sensitive to internal logical relations, which dictate what I can know, and what I must not be ignorant of. It might seem surprising that our mastery of the vocabulary of sub-propositional expressions is here considered as part of *logical form*.<sup>6</sup> But this only goes to show how central the context principle is in Wittgenstein's conception of language (3.3). To say that it is an aspect of logical form means that the identity of meaning of two expressions can only *show* itself in the way we use these expressions. It cannot be asserted, since

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<sup>5</sup> Wittgenstein 2009 (Henceforth abbreviated as PI).

<sup>6</sup> It contrasts sharply with the custom of logicians, going back to Frege, to treat synonymy as part of the extra-logical excess baggage of natural language, a phenomenon that presents an obstacle for the formalization of language, rather than an aspect of the logical form of language. If Wittgenstein's view is correct, puzzles such as the one Frege raises regarding the identity of meaning of two names should only arise for someone with a deficient grasp of the symbolism.

it is the precondition of making sense of any assertion that includes these expressions:

6.2322 The identity of the meaning of two expressions cannot be asserted. For in order to be able to assert anything about their meaning, I must know (*kennen*) their meaning, and if I know their meaning, I know (*weiss*) whether they mean the same or something different.

Meaning-identity statements, Wittgenstein holds, are not assertions: a statement of identity cannot count as an informative and contentful saying. The ability to recognize the relations of equivalence of meaning is a part of our mastery of language. Without it, identity statements could make no sense; and with it, they are superfluous.

Kremer's point — that understanding language is akin to having a practical know-how — can be extended to what Wittgenstein says about the logical relations of consequence, contradiction, etc.. In 4.1211 and 5.13, Wittgenstein claims that relations of following between ground and consequence *show* themselves, so the competent thinker must be able to appreciate them by inspection of the propositions:

4.1211 Thus a proposition “*fa*” shows that in its sense the object *a* occurs, two propositions “*fa*” and “*ga*” that they are both about the same object. If two propositions contradict one another, this is shown by their structure; similarly if one follows from another, etc.

5.13 That the truth of one proposition follows from the truth of other propositions, we perceive [*ersehen*] from the structure of the propositions.

“Perceiving” (*ersehen*) the structure involves not only perceiving the structural properties of each proposition, but also the structural relations between them. This point is relied on in 5.131:

5.131 If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a proposition; for these relations are internal, and exist as soon as, and by the very fact that, the propositions exist.

This passage, which leads up to the rejection of rules of inference, contrasts the view that we

need to “put” propositions in relations that trace out inferences, with Wittgenstein’s own view.<sup>7</sup> According to Wittgenstein, we do not count as truly having propositions in our grasp except by recognizing the internal relations that make them into the propositions that they are. When we understand a proposition, we acquire the ability to use it; our entire use of language must be informed by our affirming it. It is then *impossible* for us (to echo 4.243) not to know whether or not it is logically related to other propositions that we understand. Logical form is not a property of our propositions. It is the role these propositions take within the activity of reasoning of a competent thinker.

It would be helpful to consider another paragraph which elaborates the point made in 5.13-5.131. In his discussion of arithmetical equations in paragraph 6.232, Wittgenstein writes

6.232b But what is essential about an equation is that it is not necessary in order to show that both expressions, which are connected by the sign of equality, have the same meaning: for this can be perceived [*ersehen*] from the two expressions themselves.

Here too the point is that a certain formal method of proof — by appeal to equations — is superfluous. For the competent thinker, equations have no informative value. To be a competent thinker — to truly have an understanding of language — is to be able to recognize (*ersehen*) internal relations of meaning by mere inspection of the expressions. For that kind of thinker, equations are not necessary.

The same claim that Wittgenstein makes with regard to equations, in 6.232, and with regard to identity in 4.243, is made in 5.131 with regard to the appeal to axioms in proofs. Such an appeal is superfluous because the internal relations between  $p$  and  $q$  must show themselves directly in the reasoning of the competent thinker who understands them. They constrain what

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<sup>7</sup> I discussed 5.131 in Chapter 4 and distinguished the point made in it from the point made in 5.132.

she can know and what she must not be ignorant of.<sup>8</sup>

For a competent thinker, understanding a proposition is not a passive intake of content and the content which is understood is not inert and isolated. Understanding a proposition is modifying our overall ability to reason. Arguably, we cannot even individuate a proposition in abstraction from the role it plays in the activity of the thinker. Recall that on Wittgenstein's view, to understand a proposition is to know what is the case when it is true (4.024), but only as situated in a logical space is the proposition determinate (3.42; 4.023) — only when taken together with its “logical co-ordinates” is the propositional sign truly understood (3.41). Even though in any given notation, the internal relations between propositions may remain obscure, different notations, and different choices of primitive connectives make certain inferences obvious and immediate. 5.1311 exemplifies this approach:

5.1311 When we conclude from  $p \vee q$  and  $\sim p$  to  $q$  the relation between the forms of the propositions “ $p \vee q$ ” and “ $\sim p$ ” is here concealed by the method of symbolizing. But if we write, e.g. instead of “ $p \vee q$ ” “ $plq.lplq$ ” and instead of “ $\sim p$ ” “ $plp$ ” ( $plq$  = neither  $p$  nor  $q$ ), then the inner connexion becomes obvious.

To someone closely familiar with the sheffer-stroke notation, certain modes of inference become trivial. This shows something essential about thought (3.3441) — namely that the logical relations that hold between propositions are internal.<sup>9</sup> It is only due to the limitations of each specific notation that we require external assistance, such as applying formal rules (5.132), or

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<sup>8</sup> Further support for this line of reading can be found in Jinho Kang's account of Wittgenstein's conception of propositional sense (Kang 2004). According to Kang, in the 1915 notebook Wittgenstein develops an inferential account of sense, which Kang calls the “class-theory”. Thinking through the implications of this account is what eventually leads Wittgenstein to treat logic in an entirely “anti-theoretical and anti-metaphysical” way, i.e. in the way the resolute reading understands him (p. 184). In the class-theory, the sense of a proposition is constituted by the relations of implication that it stands in to other propositions. But Wittgenstein realizes that there can be no independently-given semantic account of these relations. They are only revealed properly in the acts of inference and assertion, and these acts — inference and assertion — must be taken as primitive (p. 198f.)

<sup>9</sup> As Wittgenstein says in 3.3421, “A particular method of signifying may be unimportant, but it always important that this is a *possible* mode of signifying”.

deriving the conditional from some general principles (5.131), in order to recognize the relation between premises and conclusion.

This is what 5.132b tells us, namely that the mode of inference can in principle be gathered by inspection of the premises and conclusion alone (cf. 6.122). The relation of logical entailment which grounds inference is internal to and constitutive of the premises and conclusion, qua the propositions that they are; it is not some additional fact apart from them, that we need to be informed of in any other way. And though for any given inference, within the context of any specific notation, the external manifestation of the internal properties may be obscure, we can always devise alternative notations that would render it perspicuous. These notations would not tempt us into thinking that the relation is something over and above the propositions.<sup>10</sup> Internal relations *show* themselves: what this means is that formal properties and relations are constitutive of the propositions, and (equivalently) that they are constitutive of what counts as understanding. Having a full grasp of a proposition is manifested by the fact that we can do things with it, that manifest the internal properties and relations that it stands in — we can infer, assert, or reject propositions in light of our other beliefs.

Wittgenstein has another way to elucidate the internal, inferential relations which hold when one proposition “follows” from the others, namely by means of the metaphor of sense-containment:

5.11 If the truth-grounds which are common to a number of propositions are all also truth-grounds of some one proposition, we say that the truth of this proposition follows from the truth of those propositions.

A proposition follows from one or more propositions when the truth of the latter guarantees the

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<sup>10</sup> This is analogous to the way Wittgenstein establishes his *Grundgedanke* — that the logical constants do not contribute any *content* to representation (4.0312, 5.441).

truth of the former. Thus affirming the propositions that serve as grounds commits one to also affirming the truth of their consequence (5.124).<sup>11</sup> And in this sense, we can say either that the sense of the consequence is “contained” in the sense of the grounding propositions, or that the truth grounds of the premises are “contained” in the truth grounds of the consequence:

5.121 The truth-grounds of q are contained in those of p; p follows from q.

5.122 If p follows from q, the sense of “p” is contained in that of “q”.

But to say that the sense of the consequence is *contained* in the sense of the propositions we understand (5.122) might be taken to mean that in entertaining the containing proposition we explicitly entertain all the contained propositions. Let me turn to consider this objection.

Wittgenstein might seem to be committed to the implausible view that we must be logically omniscient, i.e. that for thinkers to fully count as understanding any proposition, they must already recognize an infinite number of complex logical forms. Indeed, it might seem that rules of inference cannot be superfluous, precisely because we are not logically omniscient — the manipulation of signs in formal systems saves us the trouble of having to ascertain inferential relations directly. What Marie McGinn has to say about Wittgenstein’s view of inference in the *Tractatus* paints him as raising such an implausible demand:

Thus, it follows from Wittgenstein’s treatment of inference that anything that is logically entailed by a proposition is already contained in it as part of its sense. To entertain a proposition is to entertain everything that is entailed by it; the thought of what it entails is intrinsic to the thought of the proposition.<sup>12</sup>

McGinn adds that Wittgenstein later recognized a need to abandon the principles that lead him to

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<sup>11</sup> Compare this to Frege’s elucidation of the *modus ponens* in terms of the conditions of affirmation and denial that make up his account of the conditional, in BS, #6, p. 120. I discuss it in Chapter 2, section 2.2.

<sup>12</sup> McGinn 2007, p. 214

this view.<sup>13</sup>

But here is a much more plausible alternative to McGinn's account, inspired by Ramsey.<sup>14</sup> If we *do* understand certain propositions that stand in internal relations, we are able to untangle their complexity, and hence to see the internal relations that hold between them. Thus to truly understand a proposition is to be clear on the logical relations between it and any other proposition *which we actually understand*. This point does not extend to those propositions that we have not yet entertained — only to those with which we are already concerned. Similarly, what the talk of sense-containment is supposed to make clear is the fact that whenever a question become salient, whether certain internal relations between propositions which we understand hold, we *ought* to be able to answer it, since we already possess the answer. We can speak of propositions as giving us *pro tanto* reasons to infer; these reasons do not override all other concerns we might have as finite beings. Though we may be obligated to perfect our knowledge and fully articulate the relations between our beliefs, we are very often more motivated to engage in other pursuits until the occasion arises in which inferring becomes expedient. We are not under a strict obligation to derive all the consequences of the beliefs that we already have.<sup>15</sup> There is no reason to think that Wittgenstein's account in the *Tractatus* conflicts with this platitude.

Thus the response to the charge that Wittgenstein implausibly presupposes logical omniscience requires a distinction between, on the one hand, being logically *committed* to what

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<sup>13</sup> Ibid., p. 293: "We've already seen that he is quickly forced to recognize that this idealized picture is oversimplified, even in respect of what would ordinarily be called logical inference. Thus, he comes to see that the inference from  $Fa$  or  $FavFb$  to  $\exists xFx$ , for example, has nothing to do with tautologies." A similar problem is addressed by Ramsey, whom I briefly discuss next.

<sup>14</sup> See Ramsey 1927, p. 154, which I cite and discuss below.

<sup>15</sup> MacFarlane 2004 proposes that the norms that bridge logic and reasoning have such a pro-tanto, conditional form.

is implied by what we understand and, on the other hand, *actually* entertaining all that is implied by it. This kind of distinction has been suggested by Ramsey, in dealing with a variant of the omniscience objection:

I anticipate that objection will be made [to Wittgenstein's view of general propositions] ... on the following lines: ... it will be said that *a* cannot enter into the meaning of "For all *x*, *fx*", because I can assert this without ever having heard of *a*.

To this I answer that this is an essential part of the utility of the symbolism of generality, that it enables us to make assertions about things we have never heard of and so have no names for. Besides, that *a* is involved in the meaning of "For all *x*, *fx*" can be seen from the fact that if I say "For all *x*, *fx*", and someone replies 'not-*fa*', then, even though I had not before heard of *a*, he would undoubtedly be contradicting me.<sup>16</sup>

If we truly understand the general proposition and we understand any one of its instances, we must be able to see the inferential relations that show in them; but that does not mean that to understand the general proposition is to "entertain everything that is entailed by it". If we interpret Wittgenstein's view of understanding along the lines suggested here, nothing in it commits him to the implausible view that requires that we have an infinite mind. McGinn fails to distinguish what is shown — the inferential relations that inform propositions — with what is said by a proposition, and counts as its actual content.<sup>17</sup> Ramsey, by contrast, is looking at the right place, the right phenomena — the actual uses and activities in which we employ propositions. It is there that our understanding of propositions is found.

Wittgenstein's account of understanding does not picture us as perfect reasoning machines, and does not exclude the possibility of various kinds of breakdown. Indeed, one of the ways in which Wittgenstein brings out the constitutive role of logic is in his reflections on the possibility of nonsense. According to Wittgenstein, nonsense reflects the failure of an interlocutor to engage

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<sup>16</sup> Ramsey 1927, p. 154. The objection to which Ramsey responds is raised again in Potter 2009, pp. 179f.

<sup>17</sup> McGinn might be misled by the sense-containment metaphor Wittgenstein appeals to; but its purpose is elucidatory, not substantive.

in the project of perfecting her understanding of the world. Instead, the interlocutor insists on using signs in such a way that no determinate meaning is being assigned to them (cf. Tr. 5.4733). What the interlocutor thereby does cannot count as a proper act of thinking, since no determinate sense is being entertained. We do not want to say that the interlocutor has no mastery of language; it is because we impute such a mastery to him, that we expect him to give up his insistence once he is confronted with the ambiguity.

Logical form is constitutive of the activity through which we unfold the commitments implicit in the propositions we understand. This means that in making sense of ourselves and our world, we exhibit a responsiveness to such norms. These logical norms are best thought of as conditional, in the following sense: they tell us that in order to perfect our state of understanding, we have reasons to modify it in certain ways and not in other ways. As we have just seen, this conditional construal allows us to dismiss the appearance that logic makes excessive demands on finite reasoners like us, or that it completely excludes the possibility of breakdown.<sup>18</sup>

## **6.2. What Tautologies Say, and What they Show**

I have shown that logical form constitutes the activity through which we understand propositions and unfold the commitments implicit in them. Tautologies present a special case study for this account. Wittgenstein thinks that even though they do not represent anything, and in particular, even though they do not represent logical form, logical propositions show logical form.

In logical propositions the representational function of language is, as it were, suspended.

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<sup>18</sup> There are different ways in which a conditional account of logical normativity can be developed, in light of different puzzles about the normativity of logic for thought. This approach is developed in MacFarlane 2004.

These propositions do not *say* anything — they have no sense — since they do not single out any determinate state of affairs. For instance, in the proposition “Socrates is mortal or Socrates is not mortal” we cannot take ourselves to have said anything in particular about Socrates. Thus unlike propositions with sense, tautologies are not pictures:

4.462 Tautology and contradiction are not pictures of reality. They present no possible state of affairs. For the one allows every possible state of affairs, the other none.

In the tautology the conditions of agreement with the world — the presenting (*darstellenden*) relations — cancel one another, so that it stands in no presenting relation to reality.

Logical propositions thus present a “limiting cases” of the meaningful combination of signs — Wittgenstein says they bring about the “dissolution” of meaningful combination (4.462).

To recognize the sense of any proposition we have to engage in inspection and calculation, attempting to put the subcomponents into a significant whole. In a tautology, this attempt misfires. It is not that we fail to come up with a determinate whole — tautologies are not nonsense. But in trying to see what significant whole they make up, we reveal that the subcomponents form an “equilibrium”:

6.121b ...In a logical proposition propositions are brought into equilibrium with one another, and the state of equilibrium then shows how these propositions must be logically constructed.

In this state of equilibrium, the logical structures of the subcomponents combine in such a way that their representational relations to the world cancel each other. Taken together, the proposition that results turns out to represent everything (tautology) or nothing (contradiction),

and this, for Wittgenstein, means that they do not represent at all.<sup>19</sup>

Wittgenstein explains this idea by analogy with the status of ‘0’ in arithmetic (4.4611). The introduction of the number zero expands the concept of number that we acquire from the practices of counting and measuring. Unlike other natural numbers, it does not indicate an amount. And yet it is included in the set of natural numbers, because a simpler system of rules results from that inclusion (for instance, sense can now be given to the function  $x-x$ ). The same can be said for tautologies: they result from the fact that logical form allows us to combine expressions which *do* represent facts into larger expression that no longer represent *any* fact.<sup>20</sup>

Wittgenstein holds that in logical propositions, it is the fact that they do not represent anything which shows something:

6.12 The fact that the propositions of logic are tautologies *shows* the formal—logical—properties of language, of the world.

That its constituent parts connected together *in this way* give a tautology characterizes the logic of its constituent parts.

In order that propositions connected together in a definite way may give a tautology they must have definite properties of structure. That they give a tautology when *so* connected shows therefore that they possess these properties of structure.

Logical propositions can’t be *about* the logical relations between their subcomponents, because

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<sup>19</sup> Wittgenstein seeks a way to reconcile the principle that for anything to be a picture at all, it must have the capacity to be true or false (NB p. 55, 5.6.1915), with the realization that tautologies can only have the truth-value ‘true’. In the early *Moore Notes* Wittgenstein writes that, since they cannot be false, “logical propositions are neither true nor false” (MN, p. 109). But in the *Tractatus* his solution is to treat tautologies as true (6.113 and 4.464) but deny that tautologies are pictures, i.e. that they are genuine propositions. Thus he writes tautology “is not a picture of reality, in the sense that it does not PRESENT anything; it is what all--mutually contradictory--pictures have in common” (NB, p.56). Since “there are no pictures that are true a priori” (2.225), it follows that tautologies are not “pictures of reality” (4.462); they are merely the limiting case of the meaningful combination of signs (4.466). There’s evidence for this change in view in the *Tractatus*; for instance, at several places the word ‘true’ only appears in scare-quotes when it is applied to logical propositions (6.125, 6.1223).

<sup>20</sup> I have already drawn such a connection in the previous section, as well as in Chapter 4. In these contexts, as well as here, I have been following Michael Kremer “Mathematics and Meaning in the *Tractatus*”, p. 293. It should be noted that there are important differences between Wittgenstein’s characterization of tautologies and his characterization of equations, which must not be ignored. In calling mathematical equations “Scheinsätze” (6.2) Wittgenstein is placing them in one basket with nonsensical misuses of formal concepts (4.1272) and with identity statements which give rise to philosophical puzzles, and which a correct notation will completely eliminate (5.534).

in order to understand the logical propositions to be tautologies, we must already be able to recognize these logical relations. In realizing that a proposition is a tautology we deploy logical form, and the realization that it is a tautology, that it has no sense, *shows* that we recognize the subcomponents to have the logical forms that create an equilibrium. What happens in our engagement with a tautology is not the discovery of latent logical content. Rather, tautologies make it apparent that our understanding of language is logically informed — that to grasp the sense of one subcomponent of a tautology, we need to already be able to see how it is related to the others. We cannot recognize the proposition as a tautology without engaging in inferential activity; it is our ability to do that which is put on display in tautologies. This is entirely consistent with the idea that logical propositions are senseless: logical form is not something that is *said* by logical propositions. Indeed, the capacity to discern such relations must already be there, and hence we could in principle do without tautologies (6.122).

### 6.3. “Every proposition of logic is a *modus ponens* represented in signs”

6.1264 spells out the connection between Wittgenstein’s say-show distinction, his conception of tautologies, and his rejection of rules of inference. It reads as follows:

6.1264 The significant proposition asserts something, and its proof shows that it is so; in logic every proposition is the form of a proof.  
Every proposition of logic is a *modus ponens* represented (*dargestellt*) in signs (and the *modus ponens* can not be expressed (*ausgedrückt*) by a proposition).<sup>21</sup>

Wittgenstein draws a contrast between what is represented (*dargestellt*) in signs and what cannot

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<sup>21</sup> I emended Ogden’s translation of the term ‘*dargestellt*’. I discuss my reasons below.

be expressed (*ausgedrückt*) by them.<sup>22</sup> I read this contrast as an expression of the say-show distinction.

Let me begin by removing a possible misunderstanding. In 6.1264 rules of inference — or something very much like rules of inference, namely the “*modus ponens*” — might seem to be rehabilitated, and this is in apparent conflict with my suggestion that rules of inference are rejected in 5.132. Is there an inconsistency here? Here’s one way to deal with the apparent tension between 5.132 and 6.1264 that I do *not* endorse. One could take 6.1264 to mitigate the attack on rules of inference that was made in 5.132, by telling us that we must reject rules, for they cannot be “said”, but rules are O.K., as long as we remember that they only “show”. Keep rules of inference implicit, and you are fine!<sup>23</sup> This approach would be a case of what Cora Diamond calls “chickening out”.<sup>24</sup> It grants that logical form — in this case, the form of inference — is something that we are impeded from giving expression to, but it also insists that we can say what shape and function the form should have, were it to be expressible. This is not very far from what Frege and Russell themselves say about rules of inference. As I’ve argued in the previous chapters, for Frege and Russell, although the rules of inference are not expressible in the formalism, they are taken to have determinate logical content which is drawn on in the

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<sup>22</sup> ‘*Darstellen*’ is a term Wittgenstein uses quite flexibly, and it can denote either *saying* or *showing*, depending on the context. In 4.12, for example, it denotes *saying*:

4.12 Propositions can represent [*darstellen*] the whole reality, but they cannot represent [*darstellen*] what they must have in common with reality in order to be able to represent [*darstellen*] it—the logical form.

At 6.124, however, he uses it in a way that cannot denote *saying*:

6.124 The logical propositions describe [*beschreiben*] the scaffolding of the world, or rather they represent it [*darstellen*].

Here logical propositions are said to *darstellen* the “scaffolding” — i.e. the logical form. But in 4.12 we were told that propositions cannot *darstellen* logical form. So here ‘*darstellen*’ must denote *showing*.

<sup>23</sup> Something like this seems to be what Max Black has in mind when he reads 6.1264. See Black 1964, p. 339.

<sup>24</sup> See Diamond 1991e, especially pp. 181–182, 194–195.

justification of the conclusion: at least part of this content can be captured in the logical proposition into which the rule may be transformed.<sup>25</sup>

A similar case of “chickening out” affects Anscombe’s approach to 6.1264. She speaks of tautologies as being “*as it were* a picture or a proposition with the *modus ponens* as its sense”.<sup>26</sup> This blurs the difference between saying and showing — tautologies are treated as senseful pictures, and what they show is treated as the object of an *as-it-were*-saying. This results in a peculiar reading of 5.132: as denying that rules of inference are sayable, but not denying that they are *as-it-were*-sayable.<sup>27</sup> But on such a reading, it would be quite surprising that Wittgenstein criticizes Frege and Russell, rather than praise them for having anticipated his own point — since Frege and Russell conceive of rules of inference as inexpressible, while maintaining that they have some quasi-content. Anscombe’s suggestion simply effaces the difference between Wittgenstein and his predecessors.

6.1264 says that *modus ponens* is shown by tautologies, but it also tells us that that which is shown (“represented”) cannot be said (“expressed”). Anscombe takes it that the words ‘*modus ponens*’ in 6.1264 manage to refer to that thing which cannot be said. But there is a different way to defuse the tension between 5.132 and 6.1264, which would retain the sharpness of the distinction between saying and showing, would not result in a rehabilitation of rules of inference, and would not involve any ineffable content.

First, we must distinguish the term *mode* of inference from the term *rule* of inference,

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<sup>25</sup> See my discussion in Chapter 2, sections 2.4-2.5, and in Chapter 3, section 3.2.4.

<sup>26</sup> Anscombe 1959, p. 115.

<sup>27</sup> Anscombe herself does not discuss 5.132 in her book at all, and she does not seem to be aware of the tension between 5.132 and 6.1264. It might well be that she does not take rules of inference to be the target of 5.132. In her list of concerns that Wittgenstein “had to deal with”, on page 79, the first item is: “Laws of inference, and, generally, logical truths.”

conceived of as giving guidance to the act of inference and authorizing or justifying it.<sup>28</sup> Indeed, at 5.132 Wittgenstein clearly distinguishes the *mode* of inference (“*Die Art des Schlusses*”) that we gather directly by inspecting the premise and conclusion of an inference, from the *rules* of inference (*Schlussgesetze*) which are senseless and superfluous. 5.132d does not reject the idea that internal relations between propositions exhibit structural patterns, i.e. modes. At 6.1264 we are told something about such a pattern — *modus ponens* — considered as the form of a valid inference. But it is not thought of as something we appeal to in order to justify inferences.

What 6.1264 aims to do is to get us to compare tautologies and inferences, and see both as manifestations of the same logical patterns. We are asked to compare the appreciation of tautologousness — the annulment of the sense of a logical proposition — with the appreciation of the justification of the conclusion in an inference. The appreciation of the internal relations between subcomponents that allows us to recognize that a tautology is a tautology is the same activity that we engage in when we recognize an inference as valid. The internal relations that we thereby recognize are not something that the tautology describes, and they are not something we have an independent grasp of, that we can refer to by name (e.g. ‘*modus ponens*’) and then apply mechanically to inferences. As Wittgenstein says with regard to Russell’s notion of logical form, a form cannot be treated as a thing.<sup>29</sup>

A tautology has the form of a proposition, and this tempts us to think of it as asserting a fact about logical form. But this is mistaken: logical form resides in the activity through which

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<sup>28</sup> Frege makes an implicit distinction between the two ideas — in *Begriffsschrift*, he talks of inference primarily in terms of modes and manners, whereas in *Grundgesetze*, he talks about it almost exclusively in terms of rules of inference. So it is not implausible that Wittgenstein would implicitly make a distinction between these terms, or at least that he would refer to *modus ponens* without thinking about it in terms of its serving as a justifying rule. On Frege’s terminology, see Chapter 2.

<sup>29</sup> NB, p. 105. And see my discussion in Chapter 3, section 3.3.

we discern the sense (or senselessness) of a proposition, and the tautology is a mere occasion for this activity. Logical form cannot be directly *expressed* by any proposition since it is not a fact or a thing, and it is neither sayable, nor “as it were” sayable. The only way to present it is by enacting it — for example, in our calculation of the senselessness of a tautology.

*Modus ponens* is but one example of a single mode of inference, and Wittgenstein’s point in 6.1264 is far more general. He here uses the term ‘*modus ponens*’ metonymically, as standing for any mode of inference, and for the variety of tautologies that relate to them. This is evident from the examples Wittgenstein lists in 6.1201:

6.1201 That e.g. the propositions “ $p$ ” and “ $\sim p$ ” in the connexion “ $\sim(p \cdot \sim p)$ ” give a tautology shows that they contradict one another. That the propositions “ $p \supset q$ ”, “ $p$ ” and “ $q$ ” connected together in the form “ $(p \supset q) \cdot (p) : \supset : (q)$ ” give a tautology shows that  $q$  follows from  $p$  and  $p \supset q$ . That “ $(x) \cdot fx : \supset : fa$ ” is a tautology shows that  $fa$  follows from  $(x) \cdot fx$ , etc. etc.

Tautologies show modes of inferring; though they say nothing, they are not entirely useless, for they can be used as reminders or guides for inferences. For instance,  $\sim(p \cdot \sim p)$  can be taken as a guide in reductio proofs;  $p \vee \sim p$  can be taken as a guide in argument by cases, etc.<sup>30</sup> This line of thought helps make sense of Wittgenstein’s claim that logical propositions are their own proof (6.1265). This is so since to understand tautologies *is* to engage in inferential activity, which requires that we appreciate the senses of subcomponents, realize their equilibrium, and as a

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<sup>30</sup> I owe this point to Kremer 2002a, p. 299, fn. 22. As Kremer notes, There is a close analogy between tautologies and equations that is only implicit in Wittgenstein’s writings. In his 1915 Notebooks, Wittgenstein writes, about equations, exactly what he says about tautologies in 6.1264:

Every mathematical proposition is a symbolic representation of a modus ponens. (And it is clear that the *modus ponens* cannot be expressed in a proposition.) (NB, 8.6.1915).

Like mathematical equations, senseless logical propositions can be put to use in non-trivial contexts— for instance, they can remind us how to substitute one complex logical form for another, and thus to simplify inferences that might be too complicated to directly recognize. In fact, this is how Wittgenstein thinks of logical proofs more generally — as expedients that are in principle superfluous, but sometimes useful (6.1262). And as we have seen, in 4.242 something quite similar is said about statements of identity.

result, realize that they are indeed tautologies.

In fact, logical form is just as manifest in sensible (i.e. non-logical) propositions and in inferences involving such propositions, as it is in tautologies. Tautologies do not have a privileged role; not only do they not represent anything, we do not need them in order to show anything. We could in fact do without them:

6.122 Whence it follows that we can get on without logical propositions, for we can recognize in an adequate notation the formal properties of the propositions by mere inspection.

That logical propositions are sometimes useful for evoking the recognition of logical form does not mean we cannot do without them. Indeed, we *must* be able to do without them. Otherwise, we would have to admit that they inform us of something we do not already have command of.

In his lectures in the early 1930s, Wittgenstein returns to discuss the relation between tautologies and rules of inference in a way that supports this reading. He says:

Now what is the use of all these propositions? Let us examine one which has played a role in logic:  $p \supset q.p. \supset q$ . Here we have a tautology. ... Although it seems to say something, inasmuch as we make inferences in accordance with it. By itself it is not a rule of inference, for a rule should say something, and  $p \supset q.p. \supset q$  says nothing. That it seems to say something is because the second sign of implication seems to say something the first does not, something having to do with the word 'follows'. Inferring is connected with the second sign, not the first.<sup>31</sup>

Towards the end of this paragraph Wittgenstein comes close to the idea that we found in Russell, that implication is ambiguous, and that an axiom can be converted into a rule if we treat one of the implication signs as indicating the transition from the assertion of the antecedent to the assertion of the consequent.<sup>32</sup> This is the idea captured in saying that tautologies “show” *modus ponens*. But as Wittgenstein puts the point here, the tautology only *seems* to express a rule,

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<sup>31</sup> Wittgenstein 1979, p. 137-138.

<sup>32</sup> See Chapter 3, section 3.2.4.

whereas in fact, it says nothing. What Wittgenstein recommends, then, is that we resist the temptation which lurks in the ambiguity of implication.<sup>33</sup>

The difference between Wittgenstein's account of logical form and the one that is presupposed in Frege's and Russell's rules of inference is significant. On Wittgenstein's account, the logical form of inference is something which our use of language exhibits everywhere. This explains how the rejection of rules of inference (5.132) can coincide with the recognition that *modus ponens* is ubiquitous (6.1264). By contrast, Frege and Russell treat the logical form of inference as a substantive principle, captured in the rule of inference, an informal reference to which is required in order to make an inference complete. The way they construe the relation between rules and axioms, the systematic role that they give to rules, and the insistence that they be mentioned in the margins of proofs, all indicate that rules are meant to achieve precisely what Wittgenstein denies can be done: to treat what is shown as something which can (and should) be said. What their admission of inexpressibility amounts to is a paradigm case of what Diamond calls "chickening out": Frege and Russell act as though they know what form and content the rule would have, were it to be expressible. To use a phrase coined by Michael Kremer, Frege's and Russell's rules of inference are an attempt to "take care of logic", rather than to let logic "take care of itself" — as Wittgenstein recommends in 5.473.<sup>34</sup>

What Wittgenstein's account of tautology teaches is that the logical form that underlies inference is not separable from the logical form which constitutes our grasp of any proposition.

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<sup>33</sup> See Kremer 2014. The fact that in this passage from the *Lectures*, Wittgenstein describes rules as "saying something" presents a possible difficulty for my reading. One reading this remark would take it to say of *rules of inference* that their nature is such that they should say something. This would be inconsistent with my reading of 5.132, in particular with my understanding of the charge that Frege's and Russell's rules are senseless, which I discuss in the next section. But the remark could also be read as saying that rules generally have content, whereas in inferring there are no contentful rules involved.

<sup>34</sup> See Kremer 2001, fn.17, p. 67-68.

In appreciating the sense of a propositions we already engage in inferential activity. In the case of tautologies, this is particularly apparent, since we cannot recognize them as tautologies without engaging in inferential activity. Logical form is not the content of what we come to recognize when we understand propositions; it is the form of the activity *through which* we recognize content — or, in the case of tautologies, the lack of content.

#### **6.4. The Senselessness of Rules of Inference**

We can now more fully appreciate the polemical intent of 5.132, and see how it targets Frege's and Russell's views of rules of inference. There are three components that make up Wittgenstein's critique. In this section I address the charge that rules of inference are senseless; in section 6.5 I address the charge that they are superfluous; and in section 6.6 I suggest that Frege's and Russell's accounts of inference fit Wittgenstein's diagnosis of philosophical nonsense.

Why, then, does 5.132 claim that rules are senseless? In the *Tractatus* senselessness is ordinarily associated with tautologies and contradictions — propositions that fail to make a truth-evaluable claim (4.461). But rules of inference, Frege and Russell hold, are not propositions of the formal system. So in what sense are they senseless? If one follows Hacking, Ricketts and Proops, in thinking that 5.132 does not object to rules of inference, but targets logical laws instead, then one has no problem explaining the charge of senselessness — logical propositions are senseless *par excellence*.<sup>35</sup>

Indeed, for some readers of the *Tractatus* it is enough to point at the occurrence of the word

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<sup>35</sup> See Hacking 1979; Ricketts 1985 and Proops 2001. I argue against this interpretation of *Tr.* 5.132 in Chapter 4.

“senseless” to settle the question to what ‘*Schlussgesetze*’ refers. Frascolla, for instance, writes:

As we know, what are senseless are tautologies and contradictions; as a consequence, in the case of our example, the law of inference which Wittgenstein is referring to is not to be conceived of as a rule for manipulating propositions (or formulae of a formalized language), .... but should be thought of in terms of the following scheme  $A.B.\supset.A$  wherein the uniform substitution of the metavariables A and B with propositions, or formulae, generates tautologies.<sup>36</sup>

Frascolla’s idea seems to be that senselessness is a property defined by Wittgenstein’s theory of meaning, and applied to all and only to tautologies.<sup>37</sup> Marie McGinn relies on the same line of thought. She writes of the “laws of inference” that are targeted in 5.132 that “[t]hey have no sense insofar as they are combinations of signs in which the representational relation to reality has been cut”.<sup>38</sup> She, too, seems to take the sense of ‘senselessness’ to be everywhere determined by what Wittgenstein says about tautologies.

However, if one takes the target of 5.132 to be the *rules* of inference, as I do, and one grants that Frege and Russell distinguish the inexpressible rules of inference from the expressible logical propositions; if moreover, one holds that Wittgenstein was well aware of these views of Frege’s and Russell’s, then one is required to explain the sense in which ‘senseless’ could apply to rules, and that sense must not depend on what Wittgenstein has to say about tautologies. It is crucial for my interpretation of 5.132 that even though logical propositions form the paradigmatic examples of senselessness, they do not exhaust the realm of the senseless. This point has been stressed by the resolute readers of the *Tractatus*. Conant and Diamond as well as

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<sup>36</sup> Frascolla 2007, p. 178.

<sup>37</sup> Frascolla goes on to make the interesting suggestions that 5.132 implies the application of the “zero-method” for analyzing tautologies, proposed in *Tr.* 6.1201 and 6.121. But Frascolla’s approach fails to do justice to Wittgenstein’s idea (which I find expressed in 5.132) that logical methods (including the appeal to “logical laws”) are superfluous, and that language is already perfectly logically in order, just as it is — see for instance 6.122, 5.473 and 5.5563.

<sup>38</sup> McGinn 2007, p. 72. The same claim is repeated on p. 217.

Kremer deny that a theory of sense is propounded in the *Tractatus* which would allow us to identify the logical properties of propositions by reading them off of the sign alone, in abstraction from the contexts in which a concrete use is made.<sup>39</sup> Moreover, they show that a wider variety of propositions, beyond that of tautologies, count for Wittgenstein as not-senseful-but-not-nonsense-either, including propositions such as “ $p$  entails  $q$ ”.<sup>40</sup> As I will show, it also includes propositions such as “I know that  $p \vee \sim p$ ”, “ $p$  is a tautology”, and “this argument is valid”. Such propositions are not tautologies; and they are clearly not senseless merely by dint of their grammatical structure. Moreover, they are not nonsense (unless one insists on abusing them). I will argue that for similar reasons, expressions of rules of inference, such as Frege’s “from  $p$  and  $p \supset q$ , one may infer  $q$ ”, count as senseless.

In order to show that rules of inference are senseless even though they are not tautologies, I must first dismiss the idea that senselessness is defined by reference to the special case of tautologies. One might be tempted to take this to be the point of 6.113, where it is said that we are able to perceive *in the symbols* of a tautology alone that they combine to form a tautology — it is “this *fact*” that “contains in itself the whole philosophy of logic”:

6.113 It is the characteristic mark of logical propositions that one can perceive in the symbol alone that they are true; and this fact contains in itself the whole philosophy of logic.

On my reading, what 6.113 tells us is that senselessness is something that we learn of in and through (attempted) uses of symbols, i.e. through the understanding inspection of the proposition, not through a mere mechanical inspection of the syntactic properties of *signs*.

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<sup>39</sup> In a slightly different context, see Conant and Diamond 2004: “It is central to the teaching of the *Tractatus* that there is no straightforward way to read off a propositional sign whether or not it is in accord with the system of the *Tractatus*”. (p. 71).

<sup>40</sup> See Conant and Diamond 2004, p. 67ff.; Kremer 2002a.

Senselessness shows forth in the attempt to make a significant use of symbols, or rather, in the failure of this attempt. This is nicely encapsulated in 4.461a: “the tautology and the contradiction *show* that they say nothing”.<sup>41</sup>

Let us call the proposal that senselessness can be identified with a purely syntactic property of tautologies, which can be read off of the signs alone, the *positivist* proposal.<sup>42</sup> I will argue that to adopt the positivist proposal is in effect to confuse what Wittgenstein calls “the whole philosophy of logic” (6.113) with what he takes to be a merely “unessential” part of it (6.126d). The first three paragraphs of 6.126 might seem to reinforce this positivist reading:<sup>43</sup>

6.126 Whether a proposition belongs to logic can be determined by determining the logical properties of the symbol.

And this we do when we “prove” a logical proposition. For without troubling ourselves about a sense and a meaning, we form the logical propositions out of others by mere symbolic rules.

We prove a logical proposition by creating it out of other logical propositions by applying in succession certain operations, which again generate tautologies out of the first. (And from a tautology only tautologies follow).<sup>44</sup>

But to give a fundamental role to the second paragraph of 6.126 is to ignore the asymmetry of the distinction Wittgenstein draws between sign and symbol. A sign, in Wittgenstein’s terminology, is only specifiable as an abstraction from a symbol (3.32), i.e. from a meaningful

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<sup>41</sup> My emphasis.

<sup>42</sup> This is the logical positivist construal of analytic statements, whose truth is guaranteed by the syntax of their signs. See e.g. Carnap 2000, p. 186.

It seems, as Osakri Kuusela argues, that historians of analytic philosophy have simply accepted Carnap’s claim that he inherits this approach from Wittgenstein, rather than realize the huge differences that separate them. See Kuusela 2012. And see Carnap’s account of his relation to Wittgenstein in Carnap 2000, p. 282ff.

<sup>43</sup> Another paragraph which supports the positivist reading is 3.317. Since it does not directly concern tautologies and senselessness, I leave it outside my discussion here.

<sup>44</sup> In the 6.126b *beweisen* appears within scare-quotes, indicating the reservation Wittgenstein has of considering formal systems to have the power to give genuine logical justification. This is in line with 6.126d, which I quote below.

context of use. Moreover, the positivist reading of 6.126b must ignore completely the fourth paragraph of 6.126:

6.126d Naturally this way of showing that its propositions are tautologies is quite unessential to logic. Because the propositions, from which the proof starts, must show without proof that they are tautologies.

This passage tells us that syntactic proof procedures are in fact superfluous; even though they can help us recognize tautologies where such recognition is difficult (6.1262) they do not reveal the “whole philosophy of logic” — thus they are “quite unessential”.

The positivist proposal thinks of notation as a set of mere signs whose behavior we stipulate, and then show the properties that result mechanically (e.g. tautologousness). But for Wittgenstein, a notation is a device that we design in order to clarify thought; we let the logical form of thought speak out through the design of the language. The fact *that* we can construct such formalisms *shows* what is truly essential in logic:

6.124 In logic it is not we who express, by means of signs, what we want, but in logic the nature of the essentially necessary signs itself asserts,

A notation can *show* sense (and senselessness) only because it is designed in a way that reflects logical form. The mechanical procedures a notation allows us to perform do not, by themselves, *say* anything, nor would they *show* anything. The capacity of such procedures to identify senselessness points beyond what is merely mechanical in them.<sup>45</sup> For this reason, mechanical procedures are inessential to logic (6.126d), whereas the fact that senselessness *shows* forth when *we* inspect a tautologous symbol reflects “the whole philosophy of logic” (6.113). The essence of

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<sup>45</sup> Frege makes a similar point in the introduction to *The Foundations of Arithmetic*:

“It is possible, of course, to operate with figures mechanically, just as it is possible to speak like a parrot: but that hardly deserves the name of thought. It only becomes possible at all after the mathematical notation has, as a result of genuine thought, been developed so that it does the thinking for us, so to speak.” (*FA*, p. iv)

logic is, in other words, the recognition that for any notation to serve as a language, its symbols must be able to *show* logical form (whether dimly or clearly) and so to allow a competent user to appreciate senselessness. Any language has to allow the construction of logical propositions, in which we can perceive what the *symbols* show, namely, the internal relation between other propositions.

I contrast this understanding of senselessness with what I called the positivist proposal in order to make clear in what sense senselessness is something which *shows* — something which is exhibited in our engagement with symbols. It is not a property of signs in certain arrangements, but rather something which shows forth in our use of symbols.

It might be helpful to compare the showing of senselessness in logical propositions with the showing one speaks of when one says that a proof ‘shows’, i.e. ‘demonstrates’ the truth of the conclusion.<sup>46</sup> The fact that a proof leads us from the premises to a conclusion is not something that the proof itself *says*. Without our capacity to see what is shown in the sequence of propositions produced by a procedure of proof, the product of that procedure would be a mere sign. But to see it as a proof is to go beyond what the signs themselves can say. Similarly, senselessness is not a property of the signs of a tautology — it is the role which we let the whole proposition play in our cognitive life.

We can now begin to see how to broaden the range to which the term ‘senseless’ applies. Wittgenstein gives an example for a case in which the recognition of senselessness arises in the

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<sup>46</sup> The idea that *zeigen* means the same in these contexts that it does in the context of Wittgenstein’s distinction between saying and showing has been rejected by Goldfarb (unpublished manuscript). But the very example Goldfarb refers to in order to illustrate the putative difference between the two uses of ‘*zeigen*’ seems to me to undermine his point, since it concerns a feature of logical symbolisms that cannot be expressed within any specific symbolism:

5.42 ...The interdefinability of Frege’s and Russell’s ‘primitive signs’ of logic is enough to *show* that they are not primitive signs, still less signs for relations. [my emphasis].

use of a non-tautological expression:

5.1362b (“A knows (*weiss*) that  $p$  is the case” is senseless if  $p$  is a tautology.)

It might be tempting to think that the reason why “A knows that  $p \vee \sim p$ ” is senseless is because this proposition is a tautology in disguise, since logic (supposedly) is something we have *a priori* knowledge of, and so all logical propositions are guaranteed to be “known”. But Wittgenstein rejects the idea that we can treat logic as an object of knowledge. Tautologies say nothing, and so there is nothing in them for us to know.

A more promising route is to look at the proposition itself, ‘A knows that  $p \vee \sim p$ ’, and see what is required in order to understand it. When we understand what the proposition says, we pull the tautology ‘ $p \vee \sim p$ ’ out of the seemingly “oblique” context of indirect speech in which it is positioned, and recognize it as a tautology. But then the idea that we ascribe anything to A by saying that she knows it begins to dissolve. It is not because A “already knows” the tautology  $p \vee \sim p$  that the proposition ascribing it to him is senseless. Rather, it is because we who try to meaningfully use “A knows that  $p \vee \sim p$ ” do not ascribe anything to A by uttering it. Compare 4.461e: “I know, e.g. nothing about the weather, when I know that it rains or does not rain.”. Our inspection of “A knows that  $p \vee \sim p$ ” shows us that *nothing* is being said by it.

Rules of inference are senseless for the same reason: even though they do not have a tautologous form, they say nothing. Take Frege’s rule of inference, “from  $p$  and  $p \supset q$  one may infer  $q$ ”.<sup>47</sup> What it prescribes is something which, if we have truly inspected the propositions involved, we already see for ourselves. The true permission to infer can only come from an appreciation of what shows itself in the propositions themselves —  $p$ ,  $p \supset q$  and  $q$ . The expression

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<sup>47</sup> Cf. Frege’s first rule of inference in *Grundgesetze*, #14, p. 58:

From the propositions “ $\vdash \Gamma \supset \Delta$ ” and “ $\vdash \Gamma$ ” we may infer “ $\vdash \Delta$ ”

of the rule cannot be appreciated before we mindfully inspect each of the propositions it involves.<sup>48</sup> And once we do, once we mindfully inspect the putative expression of the rule, we consider  $p$ ,  $p \supset q$  and  $q$  as symbols, and this already puts us in a position to see their internal relations and to see that we would be justified in inferring  $q$ . The expression “one may infer” adds nothing that is not already given to the competent thinker by the inspection of the propositions themselves. No work is being done by the explicit permission.

It is only if one takes the signs  $p$ ,  $p \supset q$  and  $q$  (or whatever results from substituting more complex propositions for  $p$  and for  $q$ ) to be in some sense opaque and inaccessible to the understanding that one might think that the rule is making a substantive claim.

## **6.5. The Superfluity of Rules of Inference**

Frege’s and Russell’s rules of inference form part of a rule-governed notation, and as we have seen, Wittgenstein does not deny that such formal systems of signs can be useful for us, as means for clarification. Moreover, the very possibility of constructing such systems shows something essential about logical form. But for Frege and Russell, rules of inference play a second role. In addition to prescribing the formal procedure by which, within a given notation, we produce one proposition out of others, the rules are taken to contribute something more to this procedure, to imbue it with significance. They are taken to ground and justify the conclusions, by virtue of their inexpressible logical content.

In the previous chapter I argued that the proper response to the question “why can’t what is

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<sup>48</sup> Again, I am denying that the propositions mentioned in the rule appear in an oblique context, even where Frege employs quotation marks, e.g. in the rule cited in the previous footnote.

shown be said” must take the form of denying that logical form gives us a genuine “what” of which we can ask whether or not it can be said. “What is shown” answers the question “how do we say?”, not the question “what do we say?”, and the proper answer to the former question would be to exhibit a way in which logical form shows itself in use — e.g. by giving an example, or by comparing our language to a notation in which some aspect of logical form shows itself more perspicuously. Rules of inference, understood in their first role as rules of a notation, can help us do that. But in their second role, where they are taken to be contentful, they misrepresent what shows — logical form — by making it seem to be something which is said — as providing the grounds for our conclusions.

We realize that something is amiss when we realize that we have posited a superfluous logical device (5.47321). According to Wittgenstein, it makes no sense to think that anything substantive is contributed by the application of rules, for this presupposes that we can have an understanding of propositions (or of sub-propositional parts) that is not already informed by logical form, and does not already reveal the inferential relations they stand in. We would stand in need of rules of inference only if our grasp of propositions was missing something, such that we could not see the inferential relations between them. According to Wittgenstein’s account, however, the ability to see inferential relations is internal to the understanding of propositions and is a precondition of our ability for making determinate claims. And it is internal to it not in the sense that logical form is a constituent of the *content* of what we understand, but rather in the sense that it is the *form* of the activity which we engage in, when confronted with this content: we situate propositions in the logical space of internal relations, we become capable of deploying them in inferences, evaluating their consistency with other propositions, etc. Only to the extent that our understanding is already sensitive to such internal relations do we have proper grasp of

propositions. When we truly know and master the use of a language, nothing is missing: we do not need anything more to see that the inference is justified (5.132b-c). For these reasons, rules of inference would be completely superfluous. Logic simply takes care of itself.

## **6.6. The Nonsensicality of Rules of Inference**

The Tractarian critique of traditional philosophy aims to expose and reject the philosophical obsession with pseudo-problems that arise from “a misunderstanding of the logic of our language” (Tr., 4.003 and Preface). This philosophical confusion is manifested in the use of nonsensical expressions — expressions of which no determinate sense can be made, since they involve an indeterminacy in the use of the signs (5.473; 4.003). The failure to assign unambiguous meanings to the terms that make up the nonsensical expression sustains the feeling that one has succeeded in saying something substantive even where no claim was made. This misuse of language allows us to imagine that we fulfill our philosophical desires; hence the philosopher’s attraction to nonsense. Solving the problems of philosophy, for Wittgenstein, does not consist in giving answers to the nonsensical questions that are posed by philosophers (4.003). It aims at uncovering the incoherence of these questions, by showing the philosopher that she has failed to make sense of her own words (6.53) — that the issue she thought she was explaining by means of her nonsensical expression has not even been made determinate.<sup>49</sup>

The philosophical confusion Wittgenstein is most concerned with in the *Tractatus* involves the status of logic. Philosophers set themselves the goal of transcending what they perceive as

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<sup>49</sup> Wittgenstein’s response to philosophical confusions can be fruitfully compared to Kant’s approach to “Transcendental illusion” in the Dialectic chapter of the *Critique of Pure Reason* (Kant 1998, p. 384ff.).

the logocentric predicament — the fact that logic is drawn on in every explanation, and therefore that it cannot itself be explained without circularity.<sup>50</sup> Wittgenstein’s distinction between saying and showing, resolutely read, shows that although we cannot give an explanatory account of logical form, this is no predicament at all, but merely a manifestation of our misguided philosophical desires. The philosopher wants to account for logical form in terms of expressions that are already logically informed, to subject logic to the treatment that fits objects of explanatory accounts — one which grounds one phenomenon in terms of another. But logic cannot be grounded in something deeper and more fundamental; logic takes care of itself (5.473).<sup>51</sup> To borrow a term that Cora Diamond borrows from McDowell, the philosophical illusion Wittgenstein tries to free us from is that we can achieve “a view from sideways on” on ourselves.<sup>52</sup>

In their elucidations of the rules of inference, Frege and Russell succumb to this kind of philosophical nonsense.<sup>53</sup> They aim to make contentful claims about the formal preconditions of making contentful claims, seeking a justification where none is actually needed, and none can be found. It is not the rules themselves that are nonsense, but the idea that no justification will be found without them that is nonsensical. In a certain sense, Wittgenstein responds to their

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<sup>50</sup> See Ricketts 1985.

<sup>51</sup> See Kremer 2014 for this account of philosophical nonsense.

<sup>52</sup> Diamond 1991e, p. 185.

<sup>53</sup> Michael Kremer suggests that in calling rules of inference senseless and superfluous in 5.132, Wittgenstein also takes them to be “meaningless nonsense”, and to manifest a philosophical confusion of the kind I described above. Kremer’s argument appeals to Wittgenstein’s version of Occam’s Razor, that superfluous signs are meaningless (Tr. 3.328 and 5.47321). See Kremer 2001, p. 53.

accounts as he would respond to Lewis Carroll's Tortoise.<sup>54</sup> If one claims to understand propositions that stand in internal relations, but continues to insist that the inference that they form is not justified, there is not much we can do to convince her. She is not truly entitled to claim that she understands each of the propositions on its own, and it is only by using the term 'understand' ambiguously that she convinces herself that something is missing. She asks us to give her an assurance that something apart from the propositions grounds the conclusion; but there is no such a thing as giving this kind of assurance, and logical form is not something that is ever given apart from our propositions.

## 6.7. Conclusion

I have shown that Wittgenstein's conception of inference is inseparable from his conception of understanding. All the justification that an inference needs is already present in our understanding of the premises, for the appreciation of the internal relations between propositions is constitutive of understanding. This applies equally to significant propositions and to logical propositions, as well as to sub-propositional components: in all these cases, understanding involves recognizing the relations between its parts and its relations to other propositions.

Logical form is the form of the activity through which we navigate through logical space. This image makes it clear why rules of inference are *superfluous*, since what they purport to contribute to an act of inference is already at play before we even begin to infer. If we treat the rule of inference as providing us with something which is separate from what is already found in

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<sup>54</sup> Carroll 1895. The response to the Tortoise that I ascribe to Wittgenstein here is reminiscent of the one that Stroud 1979 attributes to the later Wittgenstein. Indeed, I think there is a continuity between the earlier and the later Wittgenstein on these issues.

the propositions we understand, we would be left with an impoverished conception of propositions. This, for Wittgenstein, would no longer be a conception of propositions at all.

I accounted for the *senselessness* of the rules of inference by looking at Frege's and Russell's elucidatory expressions of the rules, rather than at the tautologies which seem to correspond to these rules. An expression of a rule of inference is senseless, for when we inspect it with understanding, we encounter in it a reference to propositions, or to forms of propositions, the understanding of each of which presupposes that we have recognized the internal relations between all of them. We thus find that nothing is being said by the expression of the rule.

Finally, to think that superfluous rules are needed in order to infer constitutes a form of philosophical *nonsense*. Logical form is not a source of justification, given to us apart from the activities of reasoning and understanding. To insist that logical form can go missing from our understanding, and needs to be provided by us, is to admit that no understanding has taken place.

## 7. Conclusion

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The aim of this dissertation has been to reevaluate the distance that separates Wittgenstein's conception of logic and the views of his predecessors, Frege and Russell. My point of departure was the criticism advanced in *Tractatus* 5.132, in which Wittgenstein rejects Frege's and Russell's rules of inference as "senseless" and "superfluous". I have shown that Frege and Russell treat these rules as a source of content on which the justification of inferences ultimately depends, even though they both acknowledge that to avoid circularity and regress, the rules of inference must be inexpressible in the formal symbolism which they govern. I then showed that on Wittgenstein's view, the idea of inexpressible but contentful logical principles reflects a misunderstanding of the nature of logic and of thought. Logic, on Wittgenstein's view, is neither a source of content, nor a source of justification for our thoughts. Logic is the form of thought.

The readings of Frege and Russell presented in the first half of the dissertation were carried out with an eye to uncovering those aspects of their philosophy of logic that one must have in view in order to appreciate the depth of Wittgenstein's criticism in 5.132. Two questions emerged, which set the task for the second half of the dissertation:

1. Why does Wittgenstein reject his predecessors' rules of inference?
2. How does Wittgenstein himself conceive of logic, given this rejection?

On my way of reading *Tractatus* 5.132, which I defend in Chapter 4, Wittgenstein rejects Frege's and Russell's conception of rules of inference since they take these rules to serve as sources of justification. Contrary to much of the existing secondary literature on this topic, I show that Wittgenstein's critique cannot be charitably read as a mere restatement of Lewis Carroll's puzzle. For, as I show in Chapters 1-3, Frege and Russell already have adequate answers to that puzzle. This leads me to the first of the two guiding questions: why does Wittgenstein reject his predecessors' rules of inference?

In response to this question I argued that Frege's and Russell's conception of the rules of inference reflects the inner tension that underlies their views concerning the nature of logic and its relation to language and thought. I proposed to articulate this tension in the following way. On the one hand, Frege and Russell take logic to be concerned with the constitutive laws that govern all thought and judgment, such that no thought can be framed outside them. Logic is the condition of possibility of making sense. On the other hand, they take logic to be an epistemological foundation, a realm of knowledge in which other realms of knowledge are grounded — in particular, logic grounds mathematical knowledge. Both Frege and Russell are concerned to set up formal axiomatic systems in which this dependence can be demonstrated. In their systems, logical laws take the form of explicit logical truths. On Frege's and Russell's way of understanding what a true proposition is, however, no proposition, no matter how general, can itself be a guide for reasoning. What guides us in reasoning are independent principles. This is the role that the rules of inference are called upon to play.

Frege and Russell distinguish the rules of inference from the propositions of logic in that unlike propositions, the rules of inference cannot be expressed within the language that they govern. They both think that any attempt to do that would lead to circularity and regress. According to Russell, our acknowledgment of these principles must be unmediated, and he describes it as an act of perceiving. And yet, in Chapters 2 and 3 I show that both Frege and Russell are tempted to continue to treat the rules of inference in ways that assimilate them to the logical laws that can be captured in the logical axioms. They both think that rules can be transformed into axioms, and vice versa, and they both speak of the rules as sources of logical content. This is the sense in which their rules of inference encapsulate the aforementioned tension between the idea that logic governs thought and the idea that logic is a realm of content: the rules of inference are both inexpressible and contentful. Frege's and Russell's response to the pressures that this tension puts on their conception of logic consists in assuming that the realm of logic exceeds the realm of expression. Ineffable but contentful logical principles such as the rules of inference can be recognized, even though they cannot be expressed.

It only becomes possible to appreciate why Wittgenstein rejects Frege's and Russell's rules of inference if one understands why Wittgenstein's own conception of the relation between logic and language eliminates the space that Frege and Russell carve out for ineffable but contentful principles. As I show in Chapter 5, many readers of the *Tractatus* erroneously take the purpose of Wittgenstein's distinction between what can be said and what only shows itself to allow him to carve out a similar space for ineffable, metaphysical truths. This ineffabilist reading of Wittgenstein makes it impossible to explain *Tractatus* 5.132. What is needed is a resolute reading of the distinction between saying a showing — specifically, a reading which maintains this

distinction while rejecting the idea that it leaves any room for ineffable content.

I advance such a resolute reading in Chapter 5. I argue that Wittgenstein's distinction between saying and showing radicalizes Frege's approach to the elucidation of basic logical notions. For Frege, elucidatory nonsense can help us grasp substantive true thoughts that are not expressible in language; whereas for Wittgenstein, elucidation's aim is to relieve us of the philosophical confusions that we are prone to, but it does not lead to the recognition of ineffable content. Given our state of philosophical confusion, the activity of elucidation can improve our grasp of the logical form of language. This does not mean that we learn some truth from such elucidation. It means that elucidation results in a modification of our practices of reasoning, through the emergence of an ability to make clearer sense of things, or through the realization that a certain form of words no longer satisfies us, and that another form of words is more apt to express what we wanted, or perhaps through the realization that we no longer know what it was that we wanted. This is the way in which elucidation lets logical form show itself.

I proposed to look at the distinction between saying and showing as the articulation of the relation between logic and language. The way in which logic informs our language shows forth when we use language. But logical form is not itself a thing; it is not something of which we can say anything substantive. The idea that logical form can itself be represented is confused. For, Wittgenstein argues, given the principle that logic is the universal form of all representation, and given the principle that all representation must occupy a standpoint outside its object, i.e. that a representation that presupposes what it purports to represent is not a representation at all, it results that in order to represent logic we would need to occupy a point of view outside logic, i.e. we would need to frame an illogical representation of logic, and that is patently nonsense.

Conversely, without a strict distinction between what is shown and what is said — without denying that logical form can be an object of representation — we would lose our grip on basic principles that spell out the idea of representation. I concluded from this that the only proper way to draw the distinction between saying and showing must rule out the very idea of treating logical form as an object of representation, and so it must rule out the possibility of rules of inference that are inexpressible but contentful.

The second guiding question of the dissertation concerns the shape of Wittgenstein's account of inference, given his rejection of the rules of inference. I responded to this question in Chapter 6, by developing the account of what I called Wittgenstein's logical hylomorphism. The appearance that rules of inference are needed has its mirror image in the appearance that propositions, taken on their own, are inert. But on the hylomorphic view, propositions cannot be understood in abstraction from the position they occupy in the context of an entire logical space, and they cannot be understood in abstraction from the active role they take in reasoning. Logic, for Wittgenstein, informs the entire realm of sense, such that what it is to have a grasp of a proposition *is* to appreciate the logical commitments and permissions it involves. Thus there is no gap between understanding propositions and inferring on their basis, particularly no gap which needs to be bridged by an appeal to logical principles. In this sense, rules of inference (if there were any) would be superfluous. Moreover, any attempt to encode such rules results in empty expressions that presuppose what they purport to say. In this sense, rules of inference are senseless.

Finally, I argued that the philosophical insistence that rules of inference are nevertheless needed in order to ground our inferences is a form of philosophical nonsense. Only by

equivocally using terms such as “understand”, “justify” and “proposition” can a philosopher insist that it is not enough to understand the premises and the conclusion in order to justify the inference, and that we need something in addition, namely the application of a rule of inference. Such insistence is a covert form of nonsense; and the temptation to think in this way reflects an inability to draw a strict distinction between content and logical form, between what can be said and what can only show itself.

To be clear, Wittgenstein does not reject the very idea of a rule-governed calculus of signs; what Wittgenstein warns us against is the idea that such formal systems provide us with adequate models of reasoning. He himself develops logical notations which are governed by rules of syntax. The purpose of these notations, according to Wittgenstein, is to serve as objects of comparison with our own language; they aid us in clarifying our grasp of the way logical form shows itself in our own use of language. But the success of these notations in carrying out complicated technical tasks should not tempt us to assimilate our own mindedness to the mechanical operation of a mere calculus.

Taking a look beyond this dissertation, many challenges to Wittgenstein’s hylomorphism remain to be explored, and their full treatment lies beyond the scope of this dissertation. To defend Wittgenstein’s view, one would need to respond to the worry that it places logic too far from psychology, and thereby excludes the possibility of error, illusion and ignorance. The shape of one possible response to this challenge is to argue that to see understanding and reasoning as logically informed is not to deny the possibility of error and illusion; it is to deny that error and illusion, however pervasive and prolonged they may be, are truly indistinguishable from correct and valid reasoning. Similarly, the hylomorphic view of logic must not be taken to deny the

ignorance and finitude of actual reasoners (an issue I discussed in Chapter 6); it only denies that ignorance of some logical relations can count as evidence against the idea that understanding presupposes the recognition of many other logical relations. The propositions that we understand have fully determinate positions in logical space, even though we may not yet know how to fully articulate all of their interrelations.

Stanley Cavell captures an essential concern of the later Wittgenstein in saying that whereas Frege aims to depsychologize logic, Wittgenstein's aim is to depsychologize psychology.<sup>1</sup> The roots of this project, according to my reading, are already to be found in the *Tractatus*. Wittgenstein aims to relieve us of our attachment to a philosophical desire that is equally present in psychologism as it is in platonism — the desire to treat the form of thought as though it were yet another kind of content. Frege and Russell succumb to this desire in their account of the rules of inference.

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<sup>1</sup> Cavell 2002, 91.

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## List of Abbreviations

### Works by Gottlob Frege

FA	1960. <i>Foundations of Arithmetic</i>
CN	1972a. <i>Conceptual Notation and Related Articles</i>
CP	1984a. <i>Collected Papers</i>
CO	1984c. "On Concept and Object"
BS, <i>Begriffsschrift</i>	1972d. <i>Conceptual Notation</i>
BLC	1979c. "Boole's Logical Calculus"
FG2	1984h. "Foundations of Geometry: Second Series"
GG, <i>Grundgesetze</i>	2013. <i>Frege: Basic Laws of Arithmetic</i>
PMC	1979k. <i>The Philosophical and Mathematical Correspondence</i>
PW	1979a. <i>Posthumous Writings</i>

### Works by Bertrand Russell

<i>Principles</i>	1903. <i>Principles of Mathematic</i>
PM, <i>Principia</i>	Whitehead and Russell 1910. <i>Principia Mathematica</i>
ToK	2013. <i>Theory of Knowledge</i>

### Works by Ludwig Wittgenstein

Tr., <i>Tractatus</i>	1922. <i>Tractatus Logico-Philosophicus</i>
NB	1984a. <i>Notebooks, 1914-1916</i>
NL	1984b. <i>Notes on Logic</i>
NM	1984c. <i>Notes Dictated to Moore</i>
PI	2009. <i>Philosophical Investigations</i>

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