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To the Founders who encouraged me to become a global citizen and live a contributive life

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ABSTRACT

Job Ladder over Production Networks (joint with Emmanuel Dhyne)

This paper studies the roles of firm-to-firm networks in explaining workers' movements between employers. Merging Belgian data on the universe of firm-to-firm sales relationships with a matched employer-employee dataset, we document the prevalence and characteristics of worker reallocation along the supply chain. Belgian workers are connected through the sparse networks of their employers, and more than 40 percent of job-to-job movers find their next employers among the buyers and suppliers of their current employers. The movers within production networks, on average, do not receive immediate gains in their earnings relative to other movers, and these movements are not explained by a random matching of workers and firms alone. Motivated by these findings that workers are disproportionately more likely to find job opportunities within production networks, we develop and estimate an equilibrium model of firm-to-firm trade and on-the-job search. We estimate a higher job-finding rate along production networks and find that workers direct around 30 percent of their job search toward buyers and suppliers, implying a considerable overlap between the set of potential employers in the labor market and the firm-to-firm linkages in the product market. Our results suggest that the network search channel reduces the diversification of workers' outside options against productivity shocks to production networks.

Foreign Demand Shocks to Production Networks: Firm Responses and Worker Impacts (joint with Emmanuel Dhyne, Ayumu Ken Kikkawa, Magne Mogstad, and Felix Tintelnot)

We quantify and explain the firm responses and worker impacts of foreign demand shocks to domestic production networks. To capture that firms can be indirectly exposed to such shocks by buying from or selling to domestic firms that import or export, we use Belgian data with information on both domestic firm-to-firm sales and foreign trade transactions. Our estimates of firm responses suggest that Belgian firms pass on a large share of a foreign

demand shock to their domestic suppliers, face upward-sloping labor supply curves, and have sizable fixed overhead costs in labor. Motivated and guided by these findings, we develop and estimate an equilibrium model that allows us to study how idiosyncratic and aggregate changes in foreign demand propagate through a small open economy and affect firms and workers. Our results suggest that the way the labor market is typically modeled in existing research on foreign demand shocks—with no fixed costs and perfectly elastic labor supply—would grossly understate the decline in real wages due to an increase in foreign tariffs.

CHAPTER 1

JOB LADDER OVER PRODUCTION NETWORKS

1.1 Introduction

Firms operate in a complex network of buyer-supplier relationships with other firms in the product market. Meanwhile, firms also interact with workers in the labor market, hiring job seekers from unemployment and from other firms. In the presence of labor market frictions, the movement of workers between employers accounts for a large fraction of aggregate worker flows (see, e.g., Haltiwanger et al., 2018 and Moscarini and Postel-Vinay, 2018), creating a complex network of current employers and potential next employers (Nimczik, 2023). While we can find well-known anecdotes of workers finding job opportunities through their business contacts, such as management consultants or temporary agency employees, the prevalence of such a search channel at the aggregate level is relatively less known. Yet, an overlap between these networks in the product market and labor market is key for understanding how workers' outside options are diversified against shocks in the product market. That is, if a large fraction of potential employers buys from or sells to the current employers, the shocks to production networks affect not only the current employers but also the majority of potential employers.

The aim of this paper is to examine the roles of firm-to-firm networks in explaining workers' movements between employers and assess their contributions to aggregate labor market flows. To this end, we first combine several administrative datasets from Belgium to study the prevalence and characteristics of worker reallocation along the supply chain. The VAT transaction database covers the universe of domestic firm-to-firm sales relationships, and we merge it with a matched employer-employee dataset based on social security records. The merged dataset allows us to observe worker mobility along firm-to-firm linkages over the period 2003-2014.

Equipped with the data, we provide several pieces of motivating evidence on the interaction between the Belgian labor market and production networks. First, Belgian workers are well connected through the firm-to-firm linkages of their employers. We compute the employment-based *labor market connectedness* of each firm—the share of total employment accounted for by the firms with which it is directly connected in the production network. Even though Belgian production networks are sparse, with the average firm having only 51 buyers and suppliers out of a total of around 100,000 firms, the average Belgian worker is connected to around 23 percent of total employment through the direct links of their employer. This difference arises because larger firms with more employment tend to be connected with more buyers and suppliers.

Next, we find that a sizable fraction of Belgian workers find their next employers among the buyers and suppliers of their current employers. The movements of workers along the firm-to-firm linkages, *B2B moves*, account for around 42 percent of job-to-job transitions. A part of these B2B moves can be rationalized just by chance, as movers at the firms with a high level of labor market connectedness are likely to move within networks by construction. However, this alone does not fully account for the disproportionately high share of B2B moves among Belgian workers. A simple statistical random benchmark suggests that only 20 percent of movers would move within networks if they were to be randomly matched with hiring firms.[1] Furthermore, these job-to-job transitions along supply chains are common whether workers move within or across the industries and geographic regions of their current employers, and regardless of their gender and worker types.

Workers may choose to move within networks if they find the buyers and suppliers of their current employers either more attractive or easier to move to than the other firms. Therefore, we then perform a movers analysis to examine the consequences of B2B moves on the earnings of workers. We find that, while both B2B movers and non-B2B movers

1. This random benchmark corresponds to the mover-level average of labor market connectedness based on job-to-job hires next period without any additional controls. See Section 1.3.1 for further discussion.

experience earnings gains upon moving, those who move along firm-to-firm linkages do not gain relatively more than those who find their next employers outside the networks. In fact, the earnings gains for B2B movers are slightly lower, and this difference is robust with respect to including various sets of controls that account for market-specific time trends and firm fixed effects.

Motivated and guided by these empirical findings, we construct an equilibrium model that features both firm-to-firm trade and on-the-job search. The goal of the model is to quantify the contribution of firm-to-firm linkages in the product market to aggregate worker flows. In order to incorporate these features of the product and labor markets into our model, we borrow from the models of firm dynamics with nonlinear production technologies and random on-the-job search, such as Elsby and Gottfries (2022) and Bilal et al. (2022), and present a parsimonious way to incorporate firm-to-firm trade in such models.

One important feature of our model is that workers can be matched with vacancies through two search channels. These channels are similar in spirit to the models of Carrillo-Tudela et al. (2023) and Lester et al. (2021) in which workers face multiple job-finding rates through different channels. However, we incorporate a novel channel by considering their interactions with firm-to-firm linkages in the product market. In our model, the standard constant returns to scale matching function allows all workers to meet all vacancies randomly according to the vacancy distribution (*market search*), while the employed workers can also meet vacancies at a constant rate if the vacancy poster is connected to their current employers in the product market (*network search*). The introduction of network search alters the pattern of worker flows in two ways compared to the standard on-the-job search model. First, the overall job-finding rate is now specific to each firm, as the additional job-finding rate from network search is proportional to the vacancy-based labor market connectedness of the firm. The more closely the current employers of workers are connected to the other vacancy-posting firms, the more likely they are to meet some vacancies through their search channels.

Second, network search directs part of workers' search effort into a set of connected firms in production networks. Therefore, the workers at different firms no longer meet a certain vacancy at the same rate, which makes the outside options of workers more concentrated and less diversified. This creates an overlap between the labor market and product market.

We take our model to the data with the goal of quantifying the contribution of the network search channel to worker flows and explaining the worker impacts of productivity shocks to production networks. The estimation of our labor market and product market parameters follows a two-step procedure. We first estimate all the labor market parameters using the simulated method of moments to match the observed characteristics of the Belgian labor market. In our model, firms' own productivity, intermediate input costs, and demand shifter in the product market all proportionally affect the marginal returns to hiring an additional worker. This allows us to estimate the labor market and product market parameters separately. In the second step, we then estimate the remaining product market parameters using the estimated labor market parameters. Conditional on the labor market parameters and employment distribution, we can consider the within-period decisions of firms in the product market as static. Therefore, we can apply the identification arguments similar to the ones of Bernard et al. (2022) and Huneeus et al. (2022) to the cross section of the Belgian economy and estimate the parameters in the product market. This two-step process allows the estimation of our model to be computationally feasible.

Our estimates suggest that workers utilize both market search and network search to find their next employers and that the standard search-and-matching technology of market search is not enough to generate a disproportionately high share of B2B moves that we observe in the Belgian economy. We estimate the parameter for the network search premium to be around 0.18, which, taken together with the labor market connectedness of each firm, implies that around 30 percent of workers' job search is directed toward the buyers and suppliers of their current employers on average. This suggests a considerable overlap between the set of potential employers in the labor market and firm-to-firm linkages in the product market.

In order to examine how the presence of this network search channel alters the patterns of worker flows, we also consider a counterfactual economy with no additional job finding through the firm-to-firm linkages, similar to the standard labor search model. When matching the same overall quarterly employment-to-employment rate of 5 percent, we find that this counterfactual economy can generate job-to-job transitions along firm-to-firm linkages of only around 17 percent. This number is even lower than the statistical random benchmark reported earlier because the absence of the network search channel contributes to the lower worker flows into and out of well-connected firms, where the B2B moves are more likely to happen.

Lastly, we take our estimated model to analyze the worker impacts of a 5 percent decline in productivity among Belgian manufacturing firms. The propagation of shocks in the production networks results in an immediate wage reduction of around 6 percent among manufacturing workers and 1 percent among non-manufacturing workers. Furthermore, these shocks also affect the wages of the potential employers. When matched with firms through the market search channel, workers find the average wage of the matched firms to be lower by around 2 percent. On the other hand, when matched with firms through the network search channel, workers at different firms find that the average wage decline of the matched firms ranges from 2 percent to 5 percent. We also find a positive correlation of around 0.6 between the wage decline at the current employer and the average wage decline of the firms matched through the network search channel. Therefore, our results suggest that the network search channel reduces the diversification of workers' outside options against shocks to the production networks.

Related literature. This paper contributes to several strands of literature. First, we contribute to the literature that analyzes the roles of networks in job search. A seminal work by Montgomery (1991) lays out a model in which social networks reduce search frictions.

Since then, a large number of papers have provided both empirical evidence and theoretical frameworks in which workers utilize their connection with other workers in their job search process and have analyzed the consequences of such connections (see, e.g., Dustmann et al., 2016, Lester et al., 2021, Glitz, 2017, Arbex et al., 2019, and Caldwell and Harmon, 2019). A recent work by Carrillo-Tudela et al. (2023) studies firms' and workers' use of multiple search channels and also finds that the networks of personal contacts play an important role in the matching process. Compared to these previous works that focus on worker-level networks, we emphasize the roles of employer-level linkages in production networks and investigate how the overlap between two networks in the labor market and product market affects aggregate labor market flows.

Next, this paper joins a set of recent papers that combine a firm-to-firm transaction database with matched employer-employee information from social security records. For instance, Adão et al. (2022) study how international trade affects earnings inequality in Ecuador, while Demir et al. (2024) use data from Turkish manufacturing firms and workers to document the positive assortative matching of skills between buyers and suppliers. Alfaro-Ureña et al. (2021) estimate the effects of foreign multinationals on Costa Rican workers, and Huneeus et al. (2022) show that firm heterogeneity arising from firm-to-firm linkages in production networks plays a substantial role in explaining the volatility of earnings among Chilean workers.

The closest in this literature to our empirical analysis is the work by Cardoza et al. (2023), which analyzes worker mobility along domestic supply chains using firm-to-firm trade data and matched employer-employee data from the Dominican Republic. They reach a similar conclusion that workers move disproportionately more into the suppliers or buyers of their current employers. Several differences are worth highlighting. First, Belgium is an advanced economy with smaller informal sector employment. As pointed out by Donovan et al. (2023), labor market flows are systematically different between advanced economies and developing economies, where the transitions to and from self-employment and informal sector employment play a major role. Second, while they find a positive wage premium upon moving within networks in the Dominican Republic, which they rationalize by the supply-chain-specific human capital, we find that this channel is less evident in the Belgian labor market. Given smaller wage gains for those who move within networks, our results are rather supportive of reduced search frictions along the supply chains in Belgium. Lastly, we construct a structural model of production networks and worker mobility to quantify the value of firm-to-firm linkages in the labor market.

This paper also contributes to the theory of firm-to-firm trade by incorporating labor market frictions and on-the-job search. A limited number of papers incorporate imperfect competition in the labor market into the models of domestic production networks. For instance, Huneeus et al. (2022) develop a structural model of heterogeneous firms and workers to quantify the contribution of network linkages to earnings inequality in Chile. Using the same Belgian firm-to-firm transactions data as ours, Dhyne et al. (2022a) find that accounting for an upward-sloping labor supply curve and fixed overhead costs in labor substantially alters the aggregate implications of foreign demand shocks to production networks. Both papers introduce monopsony power of employers through workers' idiosyncratic preferences over workplaces, such as in Card et al. (2018) and Lamadon et al. (2022). Compared to these studies, we construct a model of dynamic monopsony in the spirit of Burdett and Mortensen (1998), which allows employers to set lower wages because of search frictions.[2] Constructing a dynamic model in the labor market is important in our objective to study the job-to-job transitions and outside options of workers.

Similarly, we add to the theory of on-the-job search and the job ladder by incorporating firm heterogeneity arising from production networks. The majority of job ladder models that build on the seminal work of Burdett and Mortensen (1998) assume a linear production

2. See Manning (2021) for the review and taxonomy of different monopsony models.

technology in labor as well as the production of a single homogeneous final good. These assumptions make it difficult to introduce production networks into such models. A small set of exceptions propose a model of firm dynamics with nonlinear production technologies and on-the-job search, such as Schaal (2017), Elsby and Gottfries (2022), and Bilal et al. (2022). While their models are able to characterize the rich dynamics of firms and workers, they still abstract away from the intermediate input market and assume a single homogeneous good.[3] We propose a parsimonious way to add the intermediate input market to this class of models, and, to the best of our knowledge, we provide the first model that features both firm-to-firm trade and on-the-job search.

Outline. The remainder of the paper proceeds as follows. Section 1.2 describes our data, and Section 1.3 presents motivating empirical facts on the Belgian labor market and production networks. Motivated by those findings, we construct an equilibrium model of firm-to-firm trade and job-to-job transitions in Section 1.4. We discuss in Section 1.5 the estimation strategies to bring our model to the data. In Section 1.6, we perform a counterfactual exercise to quantify the contribution of firm-to-firm linkages to labor market dynamics. Section 1.7 concludes.

1.2 Data

Our analyses draw on several administrative datasets from Belgium over the period 2003-2014. These datasets allow us to combine the firm-to-firm linkages in the product market with worker flows in the labor market through the same firm identifiers. In this section, we briefly discuss data sources and the construction of our analysis sample; additional details are provided in Appendix A.1.

3. Other papers construct a model of a frictional labor market with firms producing unique differentiated products, such as Coşar et al. (2016) and Kaas and Kimasa (2021), but they do not consider on-the-job search.

1.2.1 Data on firm-to-firm linkages

We draw information on the Belgian domestic production networks from the Business-to-Business (B2B) transaction database provided by the National Bank of Belgium (NBB). As further explained in Dhyne et al. (2015), all firms in Belgium are assigned unique identifiers for the purpose of collecting value-added taxes (VAT). In each year, all VAT-liable firms in Belgium are legally required to report the amount of annual sales to each of their VAT-liable buyers, provided that the amount to a given buyer exceeds 250 euro. This allows us to accurately measure the firm-to-firm transaction linkages in the product market.

We then merge this dataset with firms' annual accounts in order to supplement the firm-level information. The annual accounts provide detailed information on firm-level sales, value added, cost of labor inputs, and ownership shares in other VAT-liable firms. In addition, we observe each firm's number of full-time equivalent (FTE) employees as well as its industry code at the NACE four-digit level and the postal code of its main economic activity. For the firms that operate in multiple geographic locations, we also have a list of sub-provinces (*arrondissements* of Belgium, at the NUTS two-digit level) where they have establishments.

It is important to note that all the information described above is recorded at the VAT identifier level. The VAT identifiers in Belgium do not always correspond to the notion of firms or establishments, as some firms may have several VAT identifiers for accounting or tax purposes. In these cases, we follow Dhyne et al. (2021) and aggregate all VAT identifiers into a firm identifier using information about their ownership structure.[4] See Appendix A.1.1 for further details on the aggregation procedure.

4. While the linkages and worker flows within the same firm may be of interest, this paper primarily focuses on inter-firm linkages and worker flows. See, for example, Giroud and Mueller (2019) and Huitfeldt et al. (2023) for discussions on firms' internal networks and internal labor market.

1.2.2 *Data on worker flows*

In order to observe worker mobility along firm-to-firm linkages in the product market, we then link information on the employment histories of individual workers using the matched employer-employee data for the period 2003-2014. The employer-employee data are based on the social security records provided by the Crossroads Bank for Social Security (CBSS) and then merged with our firm-level data by NBB. See Appendix A.1.2 for details on the merging procedure.

The data consist of a quarterly panel for the sample of 500,000 workers, drawn from the population of workers who have worked at least once at the non-financial private-sector firms that have 10 or more FTE employees during the period 2003-2014. For each employer-employee-quarter pair, we observe the status of the worker (blue collar or white collar) and a fraction of the FTE quarter she worked at the employer. When workers work at the non-financial private-sector firms that have 10 or more FTE employees, we also observe their quarterly earnings.[5]

1.2.3 *Constructing the sample of movers*

Our merged dataset allows us to observe both firm-to-firm linkages in the product market and employment histories in the labor market through the same firm identifiers. When analyzing worker flows along the supply chain, we impose a few restrictions on firms and workers to construct a suitable sample of movers. On the firms' side, we restrict our analysis to the non-financial private-sector firms that have at least one FTE employee and report positive sales, value added, and labor costs. We refer to these firms as the firms in the main analysis sample.

5. Because of the restrictions imposed by the Belgian social security administration, we cannot observe the earnings of workers when they work at small firms. These observations account for around 9 percent of employer-employee matches. However, we can still track the entire employment histories of our sampled workers regardless of the size and industry of their employers.

For workers, we first identify their main employers. In each quarter, workers can be observed at multiple firms. This happens when workers hold multiple jobs simultaneously or switch their employers in the middle of the quarter. In our main analysis, we define a worker's main employer in a given quarter as the firm where she spends the highest fraction of the FTE quarter.[6] We restrict our analysis to the workers who are currently employed at the firms in the main analysis sample.

Combining these, we define a *job-to-job mover* in a given quarter as the worker whose main employer changes to another firm in the main analysis sample in the next quarter. We impose two additional restrictions to construct our baseline sample of job-to-job movers. First, we drop the job-to-job movers who return to their previous employers within one quarter. This restriction excludes the workers who only temporarily move to another firm as well as the multiple job holders whose main employers keep switching every quarter. Second, we exclude a group of movers from the baseline sample if more than 500 workers in a given firm move to the same firm in the same quarter. These massive movements could be driven by the mere changes in employer identifiers, such as those due to outsourcing events (see, e.g., Goldschmidt and Schmieder, 2017), and may not necessarily reflect the actual job-to-job transitions of workers. Our analyses in the following sections are not significantly altered when we consider different thresholds or do not impose these restrictions.

Taken together, our sample selections lead to around 100,000 firms in the main analysis sample each year and a total of 446,343 job-to-job movements over the period 2003-2014. In Table A.1 of Appendix A.1.3, we present some summary statistics on the firms and workers in our main analysis sample.

6. Note that because of the limited availability of earnings information, we do not define workers' main employers based on their highest quarterly earnings. While it is more common in the existing works to look at the employers with the highest earnings, such as in Haltiwanger et al. (2018) and Lamadon et al. (2022), our definition based on the hours does not seem to be a major concern. Conditional on observing earnings, the employers with the longest hours correspond to the ones with the highest earnings more than 99.7 percent of the time.

1.3 Motivating empirical facts

Equipped with the data described in the previous section, we next provide several pieces of motivating evidence on the interaction between the Belgian labor market and production networks.

1.3.1 Connectedness of workforce through firm-to-firm linkages

We first document how connected the Belgian labor market is through the linkages of firms in the product market. As reported in Table A.1 of Appendix A.1.3, the Belgian product market can be described as the sparse networks of buyer-supplier relationships. In 2012, the average Belgian firm has only 51 buyers and suppliers out of a total of around 100,000 firms. Therefore, the average share of firms accounted for by the connected firms in production networks is less than 0.1 percent.

However, this does not necessarily mean that these sparse networks do not connect the Belgian workforce. In Figure A.1 of Appendix A.2.1, we show that the Belgian firms with a greater number of buyers and suppliers tend to be larger on average, consistent with the evidence from other countries (e.g., Bernard et al., 2019, for Japan and Arkolakis et al., 2023, for Chile). This implies that it is more likely for two randomly selected workers to find their employers to be connected in production networks than for two randomly selected firms to be connected.

To quantify the degree of connectedness from the perspective of workers, we define the labor market connectedness of each firm. The *employment-based labor market connectedness* of firm j, denoted by $\mathbb{C}^e_j$, is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. Let Ω be the set of all firms and Ω^B_j and Ω^S_j be the sets of firm j's buyers and suppliers, respectively. Then, firm j's employment-based labor market connectedness can be computed as follows:

$$\mathbb{C}_j^e = \frac{\sum_{i \in \Omega_j^B \cup \Omega_j^S} n_i}{\sum_{i \in \Omega \setminus \{j\}} n_i}, \tag{1.1}$$

where n_i is the employment of firm i.[7] This measure ranges between 0 and 1 and captures the degree of connectedness between workers at a given firm with workers at the other firms.[8]

Figure 1.1 plots the distribution of employment-based labor market connectedness in the Belgian economy. In 2012, an average firm is connected to around 4 percent of total employment through its firm-to-firm linkages. This number is significantly higher when we consider an average worker. An average Belgian worker is connected to around 23 percent of total employment through the direct links of their employer.

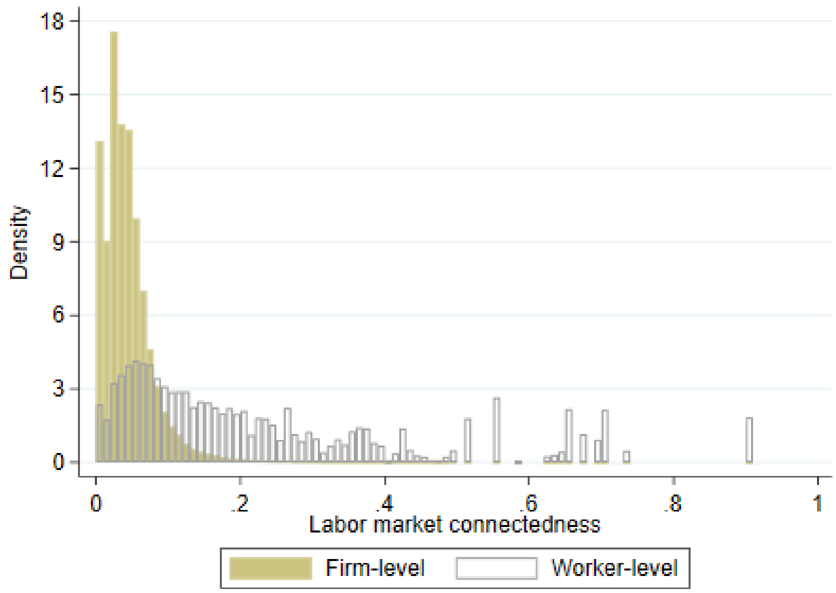
One potential concern with this measure is that our results might be driven by a certain set of firms that have small transactions with the majority of firms. For instance, some large wholesalers and utility companies supply to thousands of customer firms, each of which might have a small transaction volume. To address this concern, we also compute the labor market connectedness of firms after dropping the links from retailers, wholesalers, and utility companies to their buyers.[9] As shown in Figure A.3 of Appendix A.2.3, we find that an average Belgian worker is still connected to around 20 percent of total employment through the other links. Thus, we conclude that Belgian workers are well connected through the sparse networks of buyer-supplier relationships in the product market.

7. In a similar fashion, we can also define the hiring-based and vacancy-based labor market connectedness of firm j, denoted by $\mathbb{C}_j^h$ and $\mathbb{C}_j^v$, respectively. In Appendix A.2.2, we plot the distribution of hiring-based labor market connectedness in the Belgian economy.

8. This measure is essentially a weighted *degree centrality* in the network science, where each node (firm) is weighted by its size (employment).

9. This procedure drops around 45 percent of firm-to-firm linkages and 36 percent of transaction volume.

Figure 1.1: Distribution of labor market connectedness



Notes: This figure shows the distribution of employment-based labor market connectedness. The employment-based labor market connectedness of firm j, denoted by $\mathbb{C}^e_j$ and defined in equation (1.1), is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

1.3.2 Frequency of job-to-job transitions along the supply chain

We next describe the prevalence and characteristics of worker mobility along the supply chain in Belgium. To do so, we first compute the share of job-to-job transitions where the origin and destination firms are connected in production networks, which we call *B2B moves*. We can define the *share of B2B moves*, denoted by $\mathbb{B}$, as follows:

$$\mathbb{B} = \frac{\sum_i \mathbf{1}_{\left\{j(i,t+1)\in\Omega^S_{j(i,t),t}\cup\Omega^B_{j(i,t),t}\right\}}}{\sum_i \mathbf{1}_{\{j(i,t+1)\neq j(i,t)\}}}, \tag{1.2}$$

where we denote the employer of worker i at time t as firm $j(i,t)$. The denominator corresponds to the number of all job-to-job movers, while the numerator computes the number of movers who find their next employers to be in buyer-supplier relationships with their previous employers. In the baseline specification, we pool all job-to-job movers throughout our sample period 2003-2014 and compute the share of B2B moves in the Belgian economy.

Table 1.1 reports the shares of B2B moves out of all job-to-job movers in Belgium. As reported in the first column, we find that around 32 percent of movers find their next employers among the buyers of their current employers. Similarly, around 23 percent of movers move to the direct suppliers of current employers, and taken together, the movements to the directly connected firms of current employers account for around 42 percent of all job-to-job transitions. This suggests that a sizable fraction of Belgian workers move along the firm-to-firm linkages of their current employers.

A potential concern is that this high share of B2B moves can be fully rationalized by coincidence or other factors that are orthogonal to buyer-supplier relationships. As we discussed in Section 3.1, the Belgian labor market is well connected through firm-to-firm linkages, which makes it more likely for the randomly selected employer to be connected to the current employer. To alleviate this concern, we first plot in Figure 1.2 the share of B2B moves by the percentiles of labor market connectedness of current employers. As one would expect, the share of B2B moves increases as the firm-to-firm linkages of their current employers account for a larger share of total employment. Nonetheless, the observed shares of B2B moves are significantly higher than the 45 degree line. This finding is suggestive that workers move systematically and disproportionately more into the buyers and suppliers of their current employers.

To formalize this argument, we now conduct a simple simulation exercise to compute the statistical random benchmarks that we can compare our numbers to. The goal of this simulation is to compute the share of B2B moves if movers were to be randomly matched with hiring firms. We take the set of observed movers and hiring firms and randomize the matches between them, so that we can compute the share of job-to-job transitions that happened to be between two firms connected in the product market. We compute the average share

Table 1.1: Share of B2B moves

	Share of moves	
	Data	Random benchmark
All movers		
all B2B moves	0.42	0.20
moves to buyers	0.32	0.14
moves to suppliers	0.23	0.12
moves through key B2B links	0.11	0.02

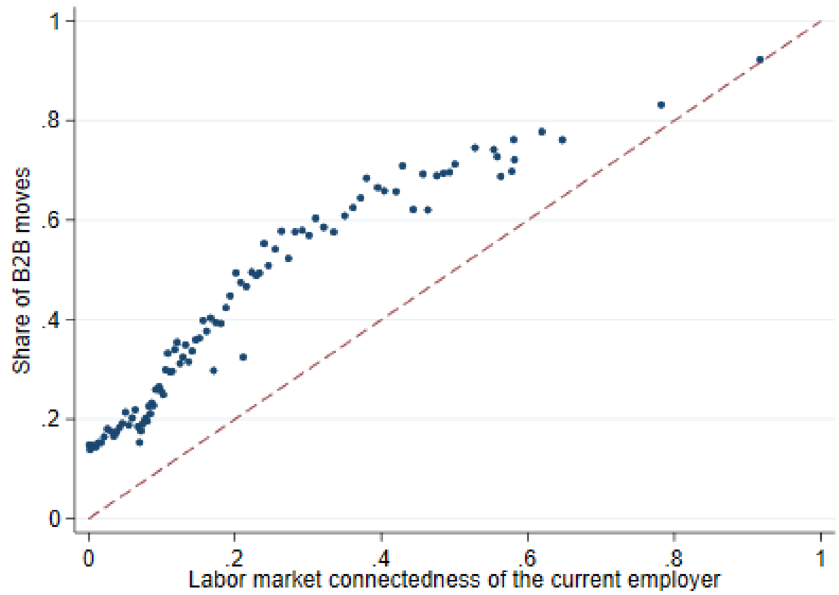
	Share of movers	Share of B2B moves	
		Data	Random benchmark
Industries			
movers within NACE 2	0.27	0.38	0.25
movers across NACE 2	0.73	0.43	0.18
Locations			
movers within NUTS 2	0.37	0.38	0.23
movers across NUTS 2	0.63	0.44	0.18
Markets (NACE 2×NUTS 2)			
movers within market	0.12	0.36	0.24
movers across market	0.88	0.42	0.19
Worker types			
male movers	0.66	0.42	0.19
female movers	0.34	0.41	0.21
blue-collar movers	0.53	0.43	0.16
white-collar movers	0.37	0.49	0.21

Notes: This table reports the shares of B2B moves among different sets of job-to-job movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks, and the share of B2B moves is defined in equation (1.2). In the first panel, we compute the shares of B2B moves among all movers. We report the observed shares in the data in the first column, while, in the second column, we report the results from simulation exercises to compute the statistical random benchmarks. The random benchmarks compute the share of B2B moves if movers were to be randomly matched with hiring firms (see Appendix A.4.1 for details). In the last row, we only consider the firm-to-firm linkages with transactions that exceed 5 percent of buyers' network purchases and/or suppliers' network sales. In the second panel, we compute the shares of B2B moves among different groups of movers. We also report the share of movers for each group in an additional column. The shares of blue-collar movers and white-collar movers do not add up to one because we did not include the movers who changed their worker classes upon moving. This table is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 1.2.3 for details).

after repeating this exercise 100,000 times.[10] The full details of this exercise are provided in

10. Without any additional controls, this share converges to the mover-level average of labor market connectedness based on the employment-to-employment hires next period, which we report in the last row of Table A.2 in Appendix A.2.2.

Figure 1.2: Share of B2B moves vs. labor market connectedness



Notes: This figure shows the share of B2B moves by the percentiles of labor market connectedness of the employers at the origin of job-to-job transitions. For each percentile of job-to-job movers, sorted by the employment-based labor market connectedness of their employers at the origin, we compute the share of B2B moves among those movers. The employment-based labor market connectedness is defined in equation (1.1), and the share of B2B moves is defined in equation (1.2). The dashed red line represents the 45 degree line. This figure is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 1.2.3 for details).

Appendix A.4.1.

The second column of Table 1.1 reports the simulation results. We find that only around 20 percent of job-to-job movers would move within networks if they were randomly matched with hiring firms. This result is in stark contrast with the B2B share of 42 percent that we observe in the data, implying that the random matching of all firms and workers alone cannot justify the movements of workers along the supply chain.[11]

Another concern is that the existence of firm-to-firm transactions may not necessarily reflect significant business ties between two firms in production networks. For instance, as

11. Admittedly, this simple simulation exercise does not take into account the endogenous mobility decision of workers and the hiring decision of firms. We will revisit this result using a full structural model in Section 1.6.

discussed above, some wholesalers and utility companies may report small transactions with many customer firms, each of which neither buyers nor suppliers consider to be an important connection. Therefore, we address this concern by considering the job-to-job moves along the *key links* in production networks, which at least one party considers to be an important part of its business. To be precise, we only consider the firm-to-firm linkages with transactions that exceed 5 percent of buyers' network purchases and/or suppliers' network sales. This procedure yields the subset of production networks, which accounts for 13 percent of firm-to-firm linkages and 78 percent of transaction volume in Belgium. As reported in the fourth row of Table 1.1, this restriction indeed does reduce the share of B2B moves to 11 percent. Nonetheless, this also cuts the random benchmark further down to 2 percent, suggesting an even stronger role of firm-to-firm linkages in explaining job-to-job transitions.

In the second panel of Table 1.1, we also consider the heterogeneity of firms and workers in the shares of B2B moves. In the first six rows, we compute the shares of B2B moves after splitting all the job-to-job movers into those who moved within or across the industries and geographic regions of their current employers. We find that around 38 percent of movers who stayed in the same two-digit industries moved along firm-to-firm linkages, while 43 percent of movers who switched industries moved within networks. Comparing these numbers with the random benchmarks, we find that both types of movers are still disproportionately more likely to move within networks compared to random moves, though the industry switchers are more likely to move within the linkages of their employers.[12] We find a similar result for the movers within or across the two-digit geographic locations: those who change their work location are more likely to move along the linkages of their current employers.[13] In the last

12. One might be concerned that our results for the industry switchers are driven by selected industries, such as consultants at the consulting firms moving to their former client firms or workers at temporary employment agencies moving to their customers. In Table A.3 of Appendix A.2.4, we report the B2B shares excluding movers from consulting firms and temporary employment agencies. While the movers from those firms have a higher share of movements to buyers, our result among the other movers is still robust in comparison to the random benchmark

13. Importantly, we do not observe the exact workplace of each worker if firms have multiple establish-

four rows, we also show that our results are robust regardless of workers' gender and their status as blue-collar or white-collar workers.

Table 1.1 also reveals that a large share of movers switches their industries and work locations: only around 12 percent of movers stay in the same narrowly defined market of two-digit industries and regions. Our results suggest that firm-level linkages play an important role in shaping the patterns of worker flows beyond the conventional boundaries of the labor market in terms of industries and regions. These patterns are consistent with worker flows found in other countries. For instance, Bjelland et al. (2011) find that around 60 percent of job-to-job transitions in the United States are across broadly defined 11 NAICS super-sectors, and Nimczik (2023) finds that industries do not serve as a good predictor of data-driven boundaries in the Austrian labor market. Similar to our setting, Cardoza et al. (2023) find that around one-fifth of job-to-job movers in the Dominican Republic move within production networks. While the magnitude of B2B moves is different, as two countries can differ in how connected their labor markets are, they reach a similar conclusion that workers move disproportionately more into the suppliers or buyers of their current employers.

1.3.3 Consequence of job-to-job transitions on earnings

The previous discussions point toward Belgian workers systematically moving to the firms that trade with their current employers. A natural question that arises from this observation is why they are more likely to move within networks. Intuitively, workers may be likely to move within networks if they find the buyers and suppliers of their current employers either more attractive or easier to move to than the other firms. In order to shed light on the potential mechanism and guide our theoretical analysis in the coming sections, we now perform a movers analysis to examine the consequences of B2B moves on the earnings of

ments. Therefore, the movers whose current and next employers report different geographic regions may not necessarily move across regions. To alleviate this concern, we checked whether two firms have any establishments that operate in the same regions. As reported in Table A.3 of Appendix A.2.4, our results are robust for the movers between two firms that have no overlapping business coverage.

workers.

We consider a sample of movers who switch their main employers between $t-1$ and t and have tenures of at least eight quarters at both the origin and destination firms. In order to track the trajectories of earnings, we also restrict our sample to be the employer-employee matches with observed earnings information. We then use the balanced panel of movers from $t-8$ to $t+7$ and estimate the effects of moving within or across networks by running the following regression:

$$\log w_{i,s} = \psi_i + \sum_{j \in \{0,1\}} \sum_{k=-8}^{7} \tau_k^j \mathbf{1}_{\{k=s, T(i)=j\}} + \epsilon_{i,s}, \tag{1.3}$$

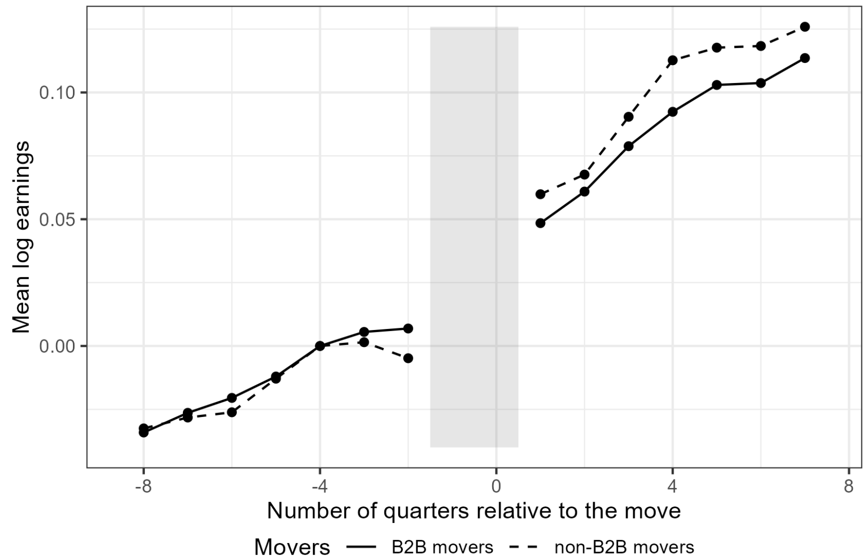
where $\log w_{i,s}$ denotes mover i's log quarterly earnings in quarter s relative to the quarter of the move, $T(i)$ is an indicator for the move along firm-to-firm linkages in the product market, and ψ_i is a worker fixed effect. In order to ensure that our estimates are not contaminated by partial-quarter employment spells in a given firm, we drop the observations in quarters $t-1$ and t.

Panels (a) and (b) of Figure 1.3 present a graphical representation of the gains in movers' earnings after the move. In Panel (a), we first show the trajectories of average log quarterly earnings from our data, normalized at $t=-2$. This figure shows that both B2B movers and non-B2B movers experience gains in their earnings, although the gains seem to be larger for non-B2B movers. To assess this difference in earnings gains, we then present in Panel (b) the results from our movers analysis. This figure tracks the quarter-by-quarter difference in earnings effects, denoted by $(\tau_k^1 - \tau_k^0)$ in equation (1.3). Our findings support common trends prior to the move and relatively smaller earnings gains for B2B movers.

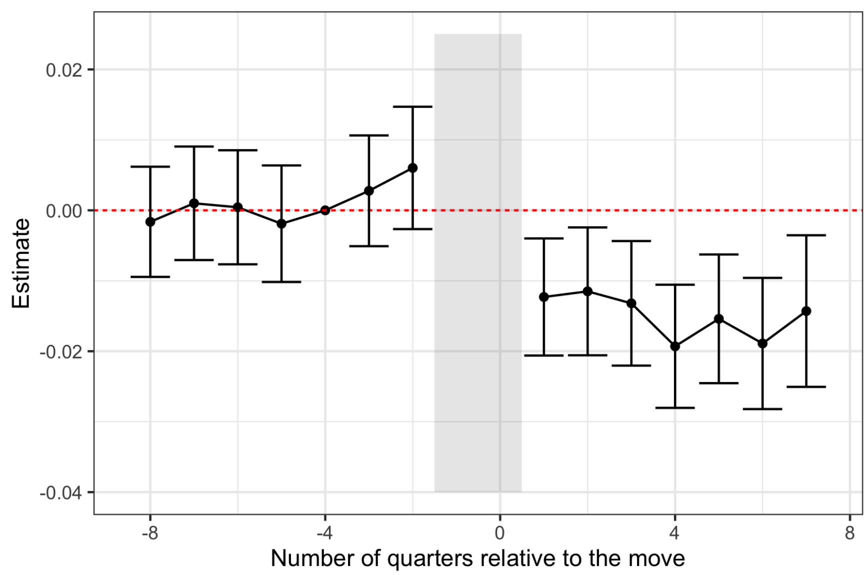
We also consider the sensitivity of our movers analysis in Table 1.2. We pool the quarter-specific coefficients τ_k^j in equation (1.3) into a single coefficient τ_{post}^j for all post-move periods $t \geq 1$ and consider the average effect. The difference is robust with respect to including

Figure 1.3: Trajectories of quarterly earnings before and after the move

(a) Raw quarterly earnings



(b) Movers analysis for the difference



Notes: These figures report the trajectories of job-to-job movers' quarterly earnings before and after the move for B2B movers and non-B2B movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks. In Panel (a), we show the trajectories of average log quarterly earnings from the data, normalized at $t = -2$. Panel (b) shows the results from a movers regression in equation (1.3). For each quarter k relative to the quarter of the move, we report the difference in earnings effects, denoted by $(\tau_k^1 - \tau_k^0)$. In both panels, we drop the observations in quarters $t-1$ and t to avoid the contamination by partial-quarter employment spells in a given firm. The figures are based on the balanced panel of 26,846 movers who have at least two years of tenure at both the origin and destination firms and whose quarterly earnings are observed throughout four years (see Sections 1.2.3 and 1.3.3 for details).

various sets of controls that account for market-specific time trends and firm fixed effects. We find that, while both B2B movers and non-B2B movers experience gains in their earnings upon moving, those who move along the firm-to-firm linkages do not gain relatively more than those who find their next employers outside the networks.

Table 1.2: Sensitivity of earnings differences

	(1)	(2)	(3)	(4)
Post×B2B move $\left(\tau^1_{post}\right)$	0.0973***	0.0133***	0.0121***	0.0135***
	(0.00204)	(0.00281)	(0.00271)	(0.00332)
Post×non-B2B move $\left(\tau^0_{post}\right)$	0.114***	0.0293***	0.0287***	0.0251***
	(0.00211)	(0.00305)	(0.00296)	(0.00378)
Difference $\left(\tau^1_{post} - \tau^0_{post}\right)$	-0.0163***	-0.0160***	-0.0165***	-0.0117***
	(0.00263)	(0.00257)	(0.00231)	(0.00341)
Worker FE	Yes	Yes	Yes	Yes
Calendar time FE		Yes		
Market×calendar time FE			Yes	Yes
Firm FE				Yes

Notes: This table reports the results from a movers regression in equation (1.3). For each column, we pool the quarter-specific indicators $\mathbf{1}_{\{k=s,T(i)=j\}}$ and the corresponding coefficients τ^j_k into a single post-move indicator $\mathbf{1}_{\{s>0,T(i)=j\}}$ and a coefficient τ^j_{post}. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks. In columns (3) and (4), market fixed effects are included at the interaction of NACE two-digit and NUTS two-digit level. The table is based on the balanced panel of 26,846 movers who have at least two years of tenure at both the origin and destination firms and whose quarterly earnings are observed throughout four years. We drop the observations in quarters $t-1$ and t to avoid the contamination by partial-quarter employment spells in a given firm (see Sections 1.2.3 and 1.3.3 for details). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Our results suggest that the observed patterns of job-to-job transitions along the supply chains in Belgium are, on average, not driven by the immediate gains in earnings compared to other movers. It is interesting to note that these patterns do not necessarily hold in any economy and can be at odds with the findings in other countries. For instance, Cardoza et al. (2023) find a positive wage premium upon moving within networks in the Dominican Republic, which they rationalize by the supply-chain-specific human capital. On the other hand, we find that this channel is less evident in the Belgian labor market. Given smaller gains in earnings for those who move within networks, our results are rather supportive of reduced search frictions in finding more job opportunities along the supply chains in Belgium.

Taken together, our findings point toward the important roles of firm-to-firm linkages in the product market in explaining the patterns of job-to-job transitions in the labor market. While these findings suggest that workers are disproportionately more likely to find job opportunities within the production networks, we have not yet taken into full considerations the endogenous hiring decisions of firms and the mobility decisions of workers. Therefore, in the next section, we will introduce a structural model of firm-to-firm trade and job-to-job transitions.

1.4 Model

Motivated by the empirical findings in the previous section, we now construct an equilibrium model that features both firm-to-firm trade and job-to-job transitions. The model serves to quantify the contribution of firm-to-firm linkages in the product market to aggregate worker flows. In order to introduce these features in the product market and labor market, we borrow from the models of firm dynamics with nonlinear production technologies and random on-the-job search, such as Elsby and Gottfries (2022) and Bilal et al. (2022), and present a parsimonious way to incorporate firm-to-firm trade in such models.

One important feature of our model is that workers can be matched with vacancies through two search channels, similar to the models of Carrillo-Tudela et al. (2023) and Lester et al. (2021). This class of models allows workers to face multiple job-finding rates through different channels, and we incorporate a novel channel by considering their interactions with firm-to-firm linkages in the product market. In addition to the constant returns to scale matching technology, which is standard in the models of job-to-job transitions, we allow workers to meet vacancies through their employers' firm-to-firm linkages in the product market. This creates an overlap between the labor market and product market, as we elaborate more in the coming sections.

In what follows, we first describe the model environment and introduce the firms' problem in product market. We then discuss the dynamic problems of firms and workers in the labor market and define our equilibrium. Lastly, we characterize the aggregate steady state where the distribution of workers remains unchanged.

1.4.1 Model environment

Time is continuous, and the economy consists of a mass L of households and a set of firms denoted by Ω. The set Ω has a measure of one, and the firm views itself as infinitesimal. Each firm produces a unique differentiated product by combining labor inputs and intermediate inputs from an exogenous set of suppliers, denoted by Ω_j^S for firm j. Within each period, the firm acts monopolistically competitively in the product market and sells its product to the households and customer firms in an exogenous set of buyers Ω_j^B. The firm takes as given the prices of its intermediate inputs and purchases them in a spot market, whereas hiring workers is subject to search frictions and wage bargaining. We now describe the details of each problem below, starting from the static problems in the product market and moving on to the dynamic problems involving the labor market. For notational convenience, we suppress the dependence on the aggregate state until the end of this subsection.

Final product demand. All households are risk-neutral, discount the future at rate ρ, and have the same preference for goods. The instantaneous utility from consuming the final goods, denoted by C, is given by the following constant elasticity of substitution (CES) aggregate of each firm's goods, $\{q_{jH}\}_{j\in\Omega}$:

$$C = \left(\int_{\Omega} \left(\beta_{jH} q_{jH}\right)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}, \tag{1.4}$$

where $\sigma > 1$ is the elasticity of substitution parameter, and $\beta_{jH} \geq 0$ denotes the weights that the households place on firm j's product. Given the CES structure that is common to all households, we can write the aggregate final consumer demand for firm j's product as follows:

$$q_{jH} = \beta_{jH}^{\sigma-1} \frac{p_{jH}^{-\sigma}}{P^{1-\sigma}} E, \tag{1.5}$$

where $P = \left(\int_{\Omega} \beta_{kH}^{\sigma-1} p_{kH}^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}$ is the aggregate price index, and E denotes the aggregate income.

Production technology. Each firm has a firm-specific technology to produce a unique differentiated product. Firm j produces its product q_j by converting labor inputs and intermediate inputs from each of its suppliers, denoted by n_j and $\{q_{ij}\}_{i \in \Omega_j^S}$, respectively. We write the production technology of firm j as the following Cobb-Douglas production function between labor inputs and the CES intermediate input bundle:

$$q_j = \phi_j n_j^{\alpha} m_j^{1-\alpha} \tag{1.6}$$

$$m_j = \left(\int_{\Omega_j^S} \left(\gamma_{ij} q_{ij}\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1.7}$$

where ϕ_j denotes firm j's own total factor productivity, and $\gamma_{ij} > 0$ governs the saliency of firm i's product in the production of firm j.[14]

Intermediate input market. Within each period, firm j takes as given the stock of its employed workers n_j and decides how much to purchase from each of its suppliers in a spot market. We assume that the suppliers, who produce unique differentiated products, have the power to set prices and that the buyers take the prices of intermediate inputs as given.[15]

14. Note that we assume the elasticity of substitution parameter σ to be the same in both the household utility and production function. This assumption is common in recent literature on production networks, such as Huneeus et al. (2022), Demir et al. (2024), Arkolakis et al. (2023), and Dhyne et al. (2022b), and implies that the demand elasticity does not depend on whether firms sell their outputs to the households or other customer firms.

15. The price-setting power of buyers is often assumed in the models of production networks, such as in Lim (2018) and Bernard et al. (2022), but other approaches are not uncommon. See, for example, Alviarez

With the CES structure of the intermediate input bundle in equation (1.7), we can solve the cost minimization problem of firm j to obtain the following demand function for supplying firm i's product given the level of intermediate input bundle m:

$$q_{ij}(m) = \gamma_{ij}^{\sigma-1}\frac{p_{ij}^{-\sigma}}{z_j^{-\sigma}}m \tag{1.8}$$

$$z_j = \left(\int_{\Omega_j^S} \gamma_{kj}^{\sigma-1}p_{kj}^{1-\sigma}dk\right)^{\frac{1}{1-\sigma}}, \tag{1.9}$$

where we call z_j the intermediate input cost index of firm j, which it takes as exogenous.

Output market. Firms sell their unique differentiated products to the households as well as to each of their customer firms in a monopolistically competitive fashion. Taking as given the demand curves from the household and customer firms described in equations (1.5) and (1.8), respectively, firm j sets the output prices $\{p_{jk}\}$. As we formally claim in Appendix A.3.1, the common demand elasticity across all firms and households implies that firms optimally charge the same price to all of their customers:

$$p_{jk} = p_j, \tag{1.10}$$

for all $k \in \Omega_j^B \cup \{H\}$. As a result, we can aggregate the demand curves across all customers and write the demand curve for firm j's product as follows:

$$q_j \equiv \int_{\Omega_j^B \cup \{H\}} q_{jk}dk = \chi_j p_j^{-\sigma}, \tag{1.11}$$

et al. (2023) for the model of buyer-supplier bargaining and Dhyne et al. (2023) for the model that features buyers' full bargaining power with their suppliers. In our setting, the presence of labor market frictions makes the static marginal cost increasing in intermediate inputs, which makes the problem of the buyer's price setting considerably complicated.

where

$$\chi_j = \int_{\Omega_j^B} (\gamma_{jk} z_k)^{\sigma-1} z_k m_k dk + (\beta_{jH} P)^{\sigma-1} E. \tag{1.12}$$

Firms maximize their profits facing this demand curve and the costs of intermediate inputs. In the presence of labor market frictions, the costs paid to workers are sunk at the time when firms decide how much to produce and how much to buy from their suppliers. Therefore, each firm maximizes its instantaneous revenue net of intermediate input costs given the level of current employment. Within each period, firm j with n workers solves the following static problem by choosing its output price and demand for the intermediate input bundle:

$$R_j(n) = \max_{p,m} \left(pq_j - z_j m \right) \tag{1.13}$$

such that equations (1.6) and (1.11) hold. Solving this problem, we can write firm j's optimal revenue net of intermediate input costs as follows:[16]

$$R_j(n) = \Phi_j n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}, \tag{1.14}$$

where

$$\Phi_j \propto z_j^{-\frac{(1-\alpha)(\sigma-1)}{1+\alpha(\sigma-1)}} \phi_j^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} \chi_j^{\frac{1}{1+\alpha(\sigma-1)}}. \tag{1.15}$$

As shown in equation (1.14), the revenue net of intermediate input costs exhibits decreasing returns to scale in labor, and its extent depends on both the saliency of labor inputs in the Cobb-Douglass production function (α) and the substitutability of goods in the product market (σ). Furthermore, equation (1.15) implies that the firm-specific labor productivity Φ_j—one of the key determinants of the firm size distribution—is not only determined by its own productivity ϕ_j but also depends on who it buys from and sells to in the production network. The latter information is summarized by the cost index z_j and demand shifter χ_j.

16. See Appendix A.3.1 for the derivation. We also provide the full expression of Φ_j in equation (A.6).

Labor market. Having described the within-period static problems in the product market, we now turn our attention to the labor market. Firm j with current employment n_j faces search frictions when increasing or decreasing its employment. It posts vacancies v_j when hiring additional workers and loses a part of its current workers through exogenous separation at a constant rate at δ_0 and endogenous separation. Posting a vacancy incurs a flow per-vacancy cost of c, and each vacancy can be matched with workers who search for vacancies while employed and unemployed. The unemployed workers search for vacancies while enjoying a flow payoff of b, whereas the employed workers at firm j receive a flow wage w_j, which is endogenously determined by a bargaining game discussed later. The matching between workers and vacancies can occur through two search channels, which we call *market search* and *network search*, and are described below.

Market search. In the *market search* channel, all job seekers and vacancies meet randomly through the standard search-and-matching procedure. The mass $u = L - \int_\Omega n_j dj$ of unemployed workers dedicates all of their search efforts into market search, while the employed workers $\{n_j\}_{j\in\Omega}$ search for vacancies through market search with exogenous relative search intensity ζ. The number of meetings between workers and vacancies through market search is governed by the following constant returns to scale aggregate matching function:

$$M(\tilde{u}, V) = A\tilde{u}^{\xi}V^{1-\xi}, \tag{1.16}$$

where $\tilde{u} = u + \zeta \int_\Omega n_k dk$ is the total mass of effective searchers, and $V = \int_\Omega v_j dj$ is the total mass of vacancies. We denote the matching efficiency and matching elasticity by A and ξ, respectively.

The constant returns to scale matching technology implies that the job-finding rate for unemployed workers through market search, denoted by λ^m, can be written as follows:

$$\lambda^m(\theta) = M(1, \theta) = \frac{M(\tilde{u}, V)}{u + \zeta \int_\Omega n_k dk}, \tag{1.17}$$

where $\theta = V/\tilde{u}$ denotes the labor market tightness in market search. Similarly, the job-finding rate for employed workers can be written as $\zeta\lambda^m(\theta)$. It is important to note that the job-finding rate through the market search channel is common across all employed workers regardless of their current employers. To ease notation, we make the dependence on the labor market tightness θ implicit in the coming sections.

Network search. In the *network search* channel, the employed workers have an additional chance to meet vacancies at a constant rate if the vacancy poster is connected to their current employers in the product market. To formalize the idea, let $\bar{\lambda}$ be the constant relative search premium from the network search, which is common across all employed workers. Then, the rate for a worker at firm j to meet a vacancy through network search, denoted by λ_j^n, is given by

$$\begin{aligned}\lambda_j^n &= \int_\Omega \bar{\lambda} \mathbf{1}_{\{k \in \Omega_j^B \cup \Omega_j^S\}} \tilde{v}_k dk \\ &= \bar{\lambda} \mathbb{C}_j^v, \end{aligned} \tag{1.18}$$

where $\tilde{v}_k$ is the density of vacancy distribution, and $\mathbb{C}_j^v$ is the vacancy-based labor market connectedness of firm j as defined in Section 1.3.1.

The introduction of network search alters the patterns of job search behavior in two ways compared to the models with market search alone. First, the overall job-finding rate, which combines the job finding rates from two search channels ($\zeta\lambda^m$ and λ_j^n), is now specific to each firm. The more closely the firms are connected to the other vacancy-posting firms, the more likely their workers are to meet vacancies through network search. Second, network search directs part of workers' search effort into a set of firms that buy from or sell to their current employers in the product market. This makes the distributions of potential employers different from the perspective of workers at different firms.

Separation rates. We next define the separation rate for each firm. The matches between firms and workers can be dissolved for three reasons. First, workers can be separated into unemployment at an exogenous separate rate δ_0. Second, workers may voluntarily quit and move to another firm when meeting its vacancy through one of the search channels and accepting its offer. Lastly, firms can also implement additional separations at no cost. We then define the separation rate for firm j, denoted by δ_j, as the rate at which workers are separated from firm j for the first or second reasons:

$$\delta_j = \delta_0 + \int_{k \in \Omega_j^A} \lambda_{jk} \tilde{v}_k dk, \tag{1.19}$$

where

$$\lambda_{jk} = \zeta \lambda^m + \bar{\lambda} \mathbf{1}_{\{k \in \Omega_j^B \cup \Omega_j^S\}}, \tag{1.20}$$

and Ω_j^A denotes the endogenous set of firms that workers at firm j are willing to accept the offers from when they meet the vacancies. The separation rate δ_j is specific to each firm due to the set of acceptable firms Ω_j^A as well as heterogeneous job-finding rates through network search, which is captured by the second term of equation (1.19).

Vacancy-filling rates. We can also define the vacancy-filling rate for each firm in a similar manner. When firm j post vacancies, each of its vacancies can be matched with unemployed workers through market search or workers at the other firms through one of the search channels, who then decide whether to accept the offer. Hence, the vacancy-filling rate for firm j, denoted by μ_j, can be written as follows:

$$\mu_j = \frac{1}{V}\left[\lambda^m u + \int_{j\in\Omega_i^A} \lambda_{ij} n_i di\right]. \tag{1.21}$$

Similar to the separation rate, the vacancy-filling μ_j is also specific to each firm due to the endogenous mobility decision of workers, which is captured by the set of acceptable firms $\{\Omega_i^A\}_{i\in\Omega}$, as well as the heterogeneous job-finding rates at the other firms $\{\lambda_{ij}\}_{i\in\Omega}$.

Wage setting. Lastly, we characterize the wage-setting protocol adopted by firms and workers. When firms and workers form successful matches through one of the search channels described above, the matches generate positive surplus. We model the wage to be determined by the bargaining game between a firm and its workers to split such surplus. We follow the model of Elsby and Gottfries (2022) that builds on the bargaining games of Stole and Zwiebel (1996) and Brügemann et al. (2019), such that wages are determined after all the successful matches are formed. In other words, a firm pays the same wage to all of its workers once employed.

We now describe the bargaining process in detail. In each period, a firm and each of its workers sequentially engage in bilateral bargaining over the marginal surplus to determine its flow wage, in which all workers have the bargaining power of $\eta \in (0, 1)$. As in Elsby and Gottfries (2022), we assume that an unsuccessful negotiation leads to a temporary disruption of the match, during which a worker receives the flow payoff of w^e and the firm pays the flow per-worker cost of w^f. This implies that the wage is determined by splitting the marginal flow surplus between a firm and each of its workers, and the flow wage at firm j solves the following differential equation:

$$(1-\eta)\left(w_j - w^e\right) = \eta\left(\frac{dR_j}{dn} - w_j - \frac{dw_j}{dn}n + w^f\right), \tag{1.22}$$

where R_j is firm j's optimal revenue net of intermediate input costs that we obtained in equation (1.14). Solving this differential equation, we can write the flow wage paid to workers at firm j that has n workers as follows:

$$w_j(n) = \frac{\eta}{1-\eta\left(1-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}\right)}\Phi_j \frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)} n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}-1} + \bar{w}, \tag{1.23}$$

where $\bar{w} \equiv \eta w^f + (1-\eta)w^e$.

1.4.2 Equilibrium and dynamic problems of firms and workers

Given the model environment described above, we now characterize the dynamic problems of firms and workers and define the equilibrium of this economy. From here on, we assume for simplicity that there is no shock to firms' own productivity $\{\phi_j\}_{j\in\Omega}$:

Assumption 1. *A firm's own productivity is time-invariant such that $\phi_{jt} = \phi_j$ for all $j \in \Omega$.*

Note that a firm's labor productivity Φ_j is still time-variant under this assumption, as it also depends on the cost index z_j and demand shifter χ_j, which can evolve according to the evolution of employment at other firms. For notational convenience, we follow Ahn et al. (2018) and use the time-dependent notation with respect to the distributions of labor productivity $\{\Phi_j\}_{j\in\Omega}$ and workers $\{n_j\}_{j\in\Omega}$.

Firms' value. We first state the dynamic problem of firms in the labor market. In a given moment, firms take as given their separation rate and vacancy-filling rate and decide the optimal amount of vacancies and additional separations, denoted by v and s, respectively. The dynamic problem for firm j that currently has n workers can be characterized by the following Hamilton-Jacobi-Bellman equation:

$$\rho\Pi_{jt}(n) = \max_{v\geq 0, s\geq 0}\left[R_{jt}(n) - w_{jt}(n)n - cv + \left(\mu_{jt}v - s - \delta_{jt}n\right)\frac{\partial \Pi_{jt}}{\partial n} + \frac{\mathbb{E}_t\left[d\Pi_{jt}(n)\right]}{dt}\right], \tag{1.24}$$

where $\mathbb{E}_t\left[dW_{jt}\right]/dt \equiv \lim_{\Delta t\downarrow 0}\mathbb{E}_t\left[W_{jt+\Delta t} - W_{jt}\right]/\Delta t$. The first three terms correspond to firm j's flow profits, which can be obtained from its revenue net of intermediate input costs, wages paid to its workers, and vacancy costs. The fourth term captures the gains and losses from the evolution of its own employment, while the last term summarizes the changes in the firm's value with respect to the evolutions of labor productivity and employment distributions.

Solving this problem, we obtain an optimal level of additional separation as:

$$s^*_{jt}\frac{\partial \Pi_{jt}}{\partial n} = 0, \tag{1.25}$$

whereas the optimal amount of vacancies is determined by

$$v^*_{jt}\left(c - \mu_{jt}\frac{\partial \Pi_{jt}}{\partial n}\right) = 0. \tag{1.26}$$

When firm j has excess employment, it dissolves the matches until its marginal value of labor becomes zero. On the other hand, when it is in need of additional workers, it posts vacancies until the marginal value of hiring an additional worker is equal to its marginal cost, which is determined by the ratio between vacancy cost c and vacancy-filling rate μ_{jt}.

Workers' value. The value for a worker employed at firm j with the level of current employment n, denoted by $W_j(n)$, is given by

$$\rho W_{jt}(n) = \max\Bigg\{w_{jt}(n) + \int_\Omega \lambda_{jkt}\tilde{v}_{kt}\left(W_{kt} - W_{jt}\right)^+ dk + \left(\delta_0 + \frac{s_{jt}^*}{n}\right)\left(U_t - W_{jt}\right) + \left(\mu_{jt}v_{jt}^* - s_{jt}^* - \delta_{jt}n\right)\frac{\partial W_{jt}}{\partial n} + \frac{\mathbb{E}_t\left[dW_{jt}(n)\right]}{dt}, \rho U_t\Bigg\}, \tag{1.27}$$

where U denotes the value for an unemployed worker such that

$$\rho U_t = b + \lambda_t^m \int_\Omega \tilde{v}_{kt}(W_{kt} - U_t)^+ dk + \frac{\mathbb{E}_t\left[dU_t\right]}{dt}. \tag{1.28}$$

Using these expressions, we can rewrite the separation rate at firm j in equation (1.21) as

$$\delta_{jt} = \delta_0 + \int_\Omega \lambda_{jkt}\tilde{v}_{kt}\mathbb{1}_{\{W_{kt} > W_{jt}\}} dk \tag{1.29}$$

and the vacancy-filling rate at firm j in equation (1.19) as

$$\mu_{jt} = \frac{1}{V_t}\left[\lambda_t^m u_t + \int_\Omega \lambda_{ijt} n_{it}\mathbb{1}_{\{W_{jt} > W_{it}\}} di\right]. \tag{1.30}$$

General equilibrium. To close the model, we define the aggregate income E_t. The aggregate income of the economy is the sum of firm profits and wages paid to workers, and firm profits are given by their revenue net of intermediate input costs, wage payments, and vacancy costs. We make the following assumption on how firms pay their vacancy costs:

Assumption 2. *The vacancy costs are paid in final goods C defined in equation* (1.4).

This assumption states that the amount spent by firms to post vacancies remains in the aggregate income of the economy. Therefore, given the distribution of workers $\{n_{jt}\}_{j\in\Omega}$, the aggregate income E_t can be computed as follows:

$$E_t = \int_\Omega R_{jt}(n_{jt})dj. \tag{1.31}$$

We can now define the general equilibrium of this economy. As an intermediate step, we first define the within-period product market equilibrium given the realized distribution of workers $\{n_{jt}\}_{j\in\Omega}$.

Definition 1 (Product market equilibrium)**.** *Given the distribution of workers* $\{n_{jt}\}_{j\in\Omega}$*, a product market equilibrium at time t is the set of prices* $\{p_{jt}\}_{j\in\Omega}$ *that satisfies equations* (1.4)-(1.15) *and* (1.31).

Because the problem in the product market is static, a product market equilibrium in each period is characterized independently of the evolution of the employment distribution. Furthermore, we claim that a product market equilibrium can be characterized as a solution to the following system of equations:

Claim 1. *Let* $\tilde{p}_j \equiv p_j^{1-\sigma}$*,* $\tilde{z}_j \equiv z_j^{1-\sigma}$*, and* $\tilde{m}_j \equiv z_j m_j / p_j^{1-\sigma}$*. Under Assumption 2, a product market equilibrium is characterized by the following system of equations:*

$$\begin{aligned}
\tilde{z}_j &= \int_\Omega f_{jk,Z}\left(\tilde{p}_j\right) dk = \int_\Omega \mathbb{1}_{\{k\in\Omega_j^S\}} \gamma_{kj}^{\sigma-1} \tilde{p}_k dk && (1.32)\\
\tilde{m}_j &= \int_\Omega f_{jk,M}\left(\tilde{z}_k, \tilde{m}_k, \tilde{p}_j\right) dk\\
&= \int_\Omega \left(\frac{1+\alpha(\sigma-1)}{\sigma} + \frac{(1-\alpha)(\sigma-1)}{\sigma} \mathbb{1}_{\{k\in\Omega_j^B\}} \gamma_{jk}^{\sigma-1} \tilde{z}_k^{-1}\right) \tilde{p}_k \tilde{m}_k dk && (1.33)\\
\tilde{p}_j &= f_{j,P}\left(\tilde{z}_j, \tilde{m}_j\right)\\
&= \left(\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_j n_j^\alpha\right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} \tilde{z}_j^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} \tilde{m}_j^{-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} && (1.34)
\end{aligned}$$

Derivations of these equations are presented in Appendix A.3.2.

Lastly, the general equilibrium of this economy features paths of prices $\{p_{jt}\}_{j\in\Omega}$, employment distributions $\{n_{jt}\}_{j\in\Omega}$, vacancy-posting decisions $\{v_{jt}\}_{j\in\Omega}$, firing decisions $\{s_{jt}\}_{j\in\Omega}$, and workers' mobility decisions such that (i) a set of prices in each period is a product market equilibrium given the employment distribution, (ii) firms and workers satisfy their HJB equations, and (iii) markets clear.

1.4.3 Aggregate steady state

Next, we characterize the aggregate steady state of the model, in which the distribution of workers remains unchanged. Under Assumptions 1 and 2, Claim 1 implies that the firm's labor productivity $\{\Phi_j\}_{j\in\Omega}$ remains constant under the stationary distribution of workers. Therefore, we can characterize the stationary distribution of workers using the distribution of labor productivity instead of the underlying product market equilibrium that generates this distribution:

Claim 2. *Under Assumptions 1 and 2, the stationary distribution of workers at the aggregate steady state can be characterized by the set of employment $\{n_j\}_{j\in\Omega}$ and labor productivity $\{\Phi_j\}_{j\in\Omega}$. In particular, the employment at firm j satisfies the following:*

$$n_j = \left(\frac{c}{\mu_j} + \frac{\bar{w}}{\rho+\delta_j}\right)^{\frac{1}{\tilde{\alpha}-1}} \left(\frac{\Phi_j\tilde{\alpha}}{\rho+\delta_j\tilde{\alpha}}\right)^{-\frac{1}{\tilde{\alpha}-1}} \left[1-\frac{\eta}{1-\eta\left(1-\tilde{\alpha}\right)}\tilde{\alpha}\right]^{-\frac{1}{\tilde{\alpha}-1}}, \tag{1.35}$$

where $\tilde{\alpha} \equiv \frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}$.

The vacancy-filling rate μ_j and separation rate δ_j in equation (1.35) follow the expressions in equations (1.30) and (1.29) and are determined by firms' vacancy-posting decisions $\{v_j\}_{j\in\Omega}$. Firms' vacancy-posting decisions are then characterized by the set of employment $\{n_j\}_{j\in\Omega}$ and labor productivity $\{\Phi_j\}_{j\in\Omega}$. We provide the derivation and further discussions in Appendix A.3.3.

Combining these results, we can now characterize the aggregate steady state of the model. Based on Claims 1 and 2, we characterize the aggregate steady state of the model as follows:[17]

17. We cannot formally establish the uniqueness of the steady state. The difficulty rises from the in-

Claim 3. *Under Assumptions 1 and 2, the aggregate steady state of the model is characterized by the set of prices* $\{p_j\}_{j\in\Omega}$ *and employment* $\{n_j\}_{j\in\Omega}$ *such that*

1. $\{p_j\}_{j\in\Omega}$ *is a product market equilibrium given the distribution of workers* $\{n_j\}_{j\in\Omega}$*;*

2. $\{n_j\}_{j\in\Omega}$ *is a stationary distribution of workers given the distribution of labor productivity* $\{\Phi_j\}_{j\in\Omega}$ *implied by the product market equilibrium.*

1.5 Estimation strategies

We now take our model to the data with the goal of quantifying the contribution of the network search channel to worker flows and explaining the worker impacts of productivity shocks to production networks. Taking advantage of Claims 1-3, we estimate the labor market and product market in a sequential manner: we first estimate the labor market parameters using the simulated method of moments to match the observed characteristics of the Belgian labor market. We then apply Claim 1 and estimate the product market parameters using the cross section of the Belgian economy. This two-step procedure allows the estimation of our model to be computationally feasible. The list of parameters and estimation strategies are summarized in Table 1.3.

1.5.1 Parameters calibrated outside the model

We first calibrate several parameters outside the model. We set the time span to be quarterly, and we take the year 2012 as our baseline year. We assume that the economy is at its steady state. We set the discount rate $\rho = 0.01$ to match the quarterly interest rate of 1 percent (or, equivalently, the annual interest rate of 4 percent). The total mass of workers L is fixed at 20.2 to match the average firm size of 18.6 under the steady-state unemployment rate of

tractability of workers' mobility decisions in closed form. Nonetheless, we confirm numerically that the employment distribution converges to the unique distribution for a wide range of parameters and for different initial guesses.

8 percent. We set the substitutability parameter $\sigma = 4$ following a common choice in the prior literature (see, e.g, Antràs et al., 2017 and Oberfield and Raval, 2021). We calibrate the returns to scale parameter for labor α to match the average labor cost share of 0.37. Lastly, we set the matching elasticity $\xi = 0.5$ as in the literature (see, e.g., Petrongolo and Pissarides, 2001).

Given the calibration for σ and α, we can directly compute each firm's labor productivity $\{\Phi_j\}_{j\in\Omega}$ in equation (1.15) using the observed levels of value added and employment. Figure A.4 in Appendix A.5.1 plots the distribution of $\log \Phi_j$ in the Belgian economy. We also normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero.

1.5.2 Labor market parameters

Given the calibrated parameters, we then estimate the other parameters of the model. As we know the distribution of firms' labor productivity $\{\Phi_j\}_{j\in\Omega}$, Claim 3 implies that we can solve for the stationary distribution of workers without knowing the product market parameters and product market equilibrium. Hence, we first estimate the set of labor market parameters by the method of simulated moments and move on to estimating the product market parameters in Section 1.5.3.

To proceed with solving for the stationary distribution of workers and estimating the labor market parameters, we first discretize the set of firms. Furthermore, as it is computationally infeasible to evaluate the mobility decisions of workers for every firm pair among 100,000 firms, we cluster firms into *firm groups*. We construct firm groups by first splitting firms based on their NACE two-digit industries and then clustering them by the quantiles of labor productivity $\{\Phi_j\}$ and labor market connectedness $\{\mathbb{C}^e_j\}$. This procedure results in clustering the Belgian firms into around 1,000 firm groups.

We assume that each firm is infinitesimal within its firm group and that all firms are homogeneous within their firm groups. For the network structure, we make the following assumption:

Assumption 3. *For a given pair of firm groups (J, K), a fraction ω_{JK} of firm-pairs (j, k), where $j \in J$ and $k \in K$, is randomly matched in the production network.*

Under Assumption 3, we replace the indicator functions for the firm-to-firm linkages with the continuous measure $\{\omega_{JK}\}$. For a given origin firm $j \in J$, a pair-specific job-finding rate λ_{jk} in equation (1.20) is now rewritten as $\lambda^{jk} = \zeta\lambda^m + \bar{\lambda}\omega_{JK}$ for all destination firms $k \in K$.[18]

Based on the network structure of firm groups obtained from the data, we now estimate the labor market parameters. Seven parameters are relevant in solving for the stationary equilibrium of workers. We first normalize the matching efficiency of market search A at one[19]. We then let $\Theta = \{\bar{\lambda}, \zeta, \delta_0, c, \bar{w}, \eta\}$ be the vector of the remaining six labor market parameters to be estimated. We estimate Θ using the method of simulated moments.

In order to estimate six labor market parameters, we select the six moments to be targeted. While we use all six moments jointly, each moment corresponds to and is more informative about different parameters. We target the share of B2B moves of 0.42 to identify the network search premium λ. Market search intensity ζ corresponds to the average quarterly employment-to-employment (EE) rate of 5 percent, while the exogenous separation rate δ_0 is set to target the average quarterly employment-to-unemployment (EU) rate of 4 percent. We estimate the vacancy cost c and constant in wage equation $\bar{w}$ by targeting the ratio of firm sizes between the 25th and 75th percentiles as well as the steady-state unemployment rate of 8 percent. Lastly, we estimate the worker bargaining power η to target the average wage gains of movers at 0.02, which corresponds to the average gains in detrended earnings

18. With these groupings, it is now possible for a worker to find a vacancy posted by the other firms in the same firm group. In this small probability event, I assume that workers decide randomly with an equal probability whether to stay or move to the matched firm. This choice is not quantitatively important to our findings.

19. This affects the level of total vacancies and vacancy-filling rates but does not alter the patterns of worker flows when the other parameters are appropriately scaled.

reported in column (2) of Table 1.2. The details for solving for the stationary distribution of workers and computing the model counterparts of these moments are provided in Appendix A.4.2.

Given the choice of these six moments, we estimate the parameters Θ as follows. We denote the vector of targeted moments in the data by $\hat{y}$, and let $y(\Theta)$ be the vector of model-implied moments at the parameter value Θ. We assume that the following moment condition holds at the true parameter value Θ^*:

$$E[\hat{y} - y(\Theta^*)] = 0 \tag{1.36}$$

Then, we estimate the labor market parameters by minimizing the following objective function:

$$\hat{\Theta} = \arg\min_{\Theta} [\hat{y} - y(\Theta^*)]'[\hat{y} - y(\Theta^*)]. \tag{1.37}$$

1.5.3 Product market parameters

Equipped with the labor market parameters estimated above, we proceed to estimating the remaining product market parameters.

Saliency in intermediate input production. We first describe our identification strategies for the saliency parameters in intermediate input production $\{\gamma_{jk}\}$. Following the discussions in Bernard et al. (2022) and Huneeus et al. (2022), we assume that γ_{jk} can be decomposed as follows:

Assumption 4. *The saliency of firm j's good in the intermediate input production of firm k, denoted by γ_{jk}, takes the following functional form:*

$$\log \gamma_{jk} = \log \gamma_j + \log \gamma_k + \log \tilde{\gamma}_{jk}, \tag{1.38}$$

where $\tilde{\gamma}_{jk}$ is independent across all firm pairs.

Assumption 4 states that γ_{jk} can be decomposed in a log-additive manner into the *relationship capability* of each firm as well as the firm-pair-specific *relationship residual*. It is useful to observe that we allow the relationship residuals to be asymmetric, such that $\log\tilde{\gamma}_{jk} \neq \log\tilde{\gamma}_{kj}$.

Under Assumption 4, rearranging equation (1.8) yields the following relationship:

$$\begin{aligned}
\log p_j q_{jk} =& (\sigma-1)\log\gamma_{jk} + (1-\sigma)\log p_j + \sigma\log z_k + \log m_k \\
=& \underbrace{(\sigma-1)\log\gamma_j + (1-\sigma)\log p_j}_{\equiv \log\Gamma_j^S} + \underbrace{(\sigma-1)\log\gamma_k + \sigma\log z_k + \log m_k}_{\equiv \log\Gamma_k^B} \\
&+ \underbrace{(\sigma-1)\log\tilde{\gamma}_{jk}}_{\equiv \log\tilde{\Gamma}_{jk}}.
\end{aligned} \tag{1.39}$$

Equation (1.39) gives the structural interpretations to the buyer and supplier fixed effects in the decomposition of firm-to-firm transactions. Practically, we take the observed log sales of firm j to firm k ($\log p_j q_{jk}$) and regress them on firm j and firm k fixed effects, recovering the *supplier fixed effect*, *buyer fixed effect*, and *buyer-supplier residual*, denoted by $\log\Gamma_j^S$, $\log\Gamma_k^B$, and $\log\tilde{\Gamma}_{jk}$, respectively. As further discussed in Bernard et al. (2022), the identifications of these two-way fixed effects can be achieved through cross-sectional variations alone when firms have more than one buyer and supplier.

Using the estimated buyer and supplier fixed effects and buyer-supplier residual, we then estimate the relationship capability γ_j of each firm. Rearranging equation (1.9), we show that the relationship capability must satisfy the following relationship with the intermediate input cost index implied by the product market equilibrium:

Proposition 1. *The product of firm j's relationship capability γ_j and its intermediate input* *cost index z_j is identified up to normalization, given by*

$$\gamma_j^{\sigma-1} = \frac{z_j^{1-\sigma}}{\int_{\Omega_j^S} \Gamma_k^S \tilde{\Gamma_{kj}} dk}. \tag{1.40}$$

In what follows, we estimate the relationship capability and solve for the product market equilibrium simultaneously. Further details for this procedure are provided in Appendix A.4.3.

Saliency in households' preference. We next provide our identification strategies for the saliency parameters in households' preference $\{\beta_{jH}\}$. We identify β's through the observed variations in firms' share of network sales out of their revenues. We define firm j's share of network sales, denotec by r_j^{net}, as

$$r_j^{net} \equiv \frac{p_j \int_{\Omega_j^B} q_{jk} dk}{p_j q_j} = \frac{\chi_j - \left(\beta_{jH} P\right)^{\sigma-1} E}{\chi_j}, \tag{1.41}$$

where the last equality follows from equations (1.10) and (1.12). We can then show that the ratio between the firm's relationship capability and the saliency of its goods in households' preference is as follows:

Proposition 2. *The ratio between firm j's relationship capability γ_j and the saliency of its goods in households' preference β_{jH} is identified up to normalization, given by*

$$\frac{\beta_{jH}^{\sigma-1}}{\gamma_j^{\sigma-1}} = E^{-1} \left(\frac{1 - r_j^{net}}{r_j^{net}} \right) \int_{\Omega_j^B} \Gamma_j^B \tilde{\Gamma_{ij}} dk. \tag{1.42}$$

Propositions 1 and 2 imply that the saliency parameters $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ are jointly determined by the same normalization. Intuitively, if we scale all γ's and β's by the same amount, it affects the levels of labor productivity $\{\Phi_j\}_{j\in\Omega}$ but does not alter the shape of distribution. Therefore, we normalize the saliency parameters $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ such that the average of log γ_{jk} is set at zero.

Firms' own productivity. Lastly, we estimate firms' own productivity $\{\phi_j\}$ to match their labor productivity $\{\Phi_j\}$ that we computed in Section 1.5.1. Equations (1.15) and (A.6) suggest that we can find a firm's own productivity ϕ_j that exactly fits its labor productivity Φ_j, using its cost index z_j and demand shifter χ_j implied by a product market equilibrium. Therefore, we iteratively estimate $\{\phi_j\}$ while solving for the product market equilibrium at the steady state. We provide further explanations of our estimation procedures in Appendix A.4.3.

Table 1.3: List of model parameters

(a) Externally set parameters

Parameter		Value	Reason
Discount rate	ρ	0.01	Annual interest rate of 4 percent
Returns to scale for labor	α	0.37	Average labor share
Substitutability of goods	σ	4	e.g., Antràs et al. (2017)
Total mass of workers	L	20.2	Average firm size of 18.6
Matching elasticity	ξ	0.5	e.g., Petrongolo and Pissarides (2001)

(b) Labor market parameters

Parameter		Value	Target	Data	Model
Matching efficiency	A	1	Normalization		
Network search premium	$\bar{\lambda}$	0.18	Share of B2B moves	0.42	0.42
Market search intensity	ζ	0.07	Average EE rate	0.05	0.05
Exogenous separation rate	δ_0	0.04	Average EU rate	0.04	0.04
Vacancy cost	c	3.85	Firm size ratio 25/75	0.22	0.12
Constant in wage equation	$\bar{w}$	0.16	Unemployment rate	0.08	0.08
Worker bargaining power	η	0.45	Avg. $\Delta \log w$ for movers	0.02	0.02

(c) Product market parameters

Parameter		Value	Identification strategy
Saliency in interm. input production	$\{\gamma_{jk}\}$	—	See Proposition 1
Saliency in households' preference	$\{\beta_{jH}\}$	Figure A.5a	See Proposition 2
Firm's own productivity	$\{\phi_j\}$	Figure A.5b	Match $\{\Phi_j\}$ in equation (A.6)

Notes: In this table, we report the list of model parameters and provide their estimated values as well as estimation strategies. In Panel (a), we list the parameters that are calibrated outside the model and report the rationale for the choice of each parameter value. See Section 1.5.1 for further discussions. Panel (b) shows the labor market parameters that we estimate using the method of simulated moments. For each parameter, we list the corresponding moment to be targeted and provide its value in the data and in the model. See Section 1.5.2 for further discussions. Panel (c) lists the product market parameters and their identification strategies. See Figure A.5 in Appendix A.5.2 for the estimated values and Section 1.5.3 for further discussions.

1.6 Application

We now use the estimated model to perform several counterfactual analyses. The goals of these exercises are to quantify the contributions of job search through firm-to-firm networks to labor market dynamics and to understand how the introduction of such a search channel would alter the impacts of shocks to production networks on workers.

1.6.1 Contribution of network search channel

We first quantify the contributions of the network search channel to aggregate worker flows. By doing so, we revisit the discussions in Section 1.3.2 and characterize the patterns of job-to-job transitions if movers did not take into account the firm-to-firm linkages.

We first solve for the steady-state worker flows in our baseline model using the parameters estimated in Section 1.5. As reported in Table 1.4, the estimated value for the network search premium $\bar{\lambda}$ is 0.18. Taken together with the vacancy-based labor market connectedness of each firm, this implies that the job-finding rate through network search in equation (1.18) is around 0.04 on average. We solve for the stationary distribution of workers and worker flows under this economy. The procedure to solve for the steady state is explained in detail in Appendix A.4.4.

We report the results in the first column of Table 1.4. The average quarterly employment-to-employment rate is 5 percent, out of which the job-to-job transitions through market search account for 3.5 percent. The remaining 1.5 percent comes from the matches through network search. Some of the matches through market search also result in the movement between buyers and supplier. Hence, the average employment-to-employment rate of B2B moves is 2.1 percent, while the rate of non-B2B moves is 2.9 percent, which yield the B2B move share of 42 percent. Taken together, we find that around 30 percent of the job-to-job transitions happen through the network search channel, suggesting a considerable overlap between the set of potential employers in the labor market and firm-to-firm linkages in the

Table 1.4: Contribution of network search channel

	Baseline	No Network Search
Network search premium $\bar{\lambda}$	0.18	0
Average EE rate		
overall	0.050	0.050
market search	0.035	0.050
network search	0.015	0
B2B moves	0.021	0.008
non-B2B moves	0.029	0.042
Share of network search	0.30	0
Share of B2B moves	0.42	0.17

Notes: In this table, we report the contribution of the network search channel to the worker flows in the baseline economy and counterfactual economy with no network search. In the first column, we provide the steady-state worker flows in the baseline economy using the parameters estimated in Section 1.5. The share of network search is computed as the ratio between the average employment-to-employment (EE) rate through network search and the overall average EE rate, and the share of B2B moves is defined in equation (1.2). In the second column, we report the steady-state worker flows in the economy with no market search. We reestimate the model parameters by setting $\bar{\lambda} = 0$ while targeting the same set of moments as in the baseline economy (including the overall average EE rate of 0.05). The procedure to solve for the steady state is explained in detail in Appendix A.4.4.

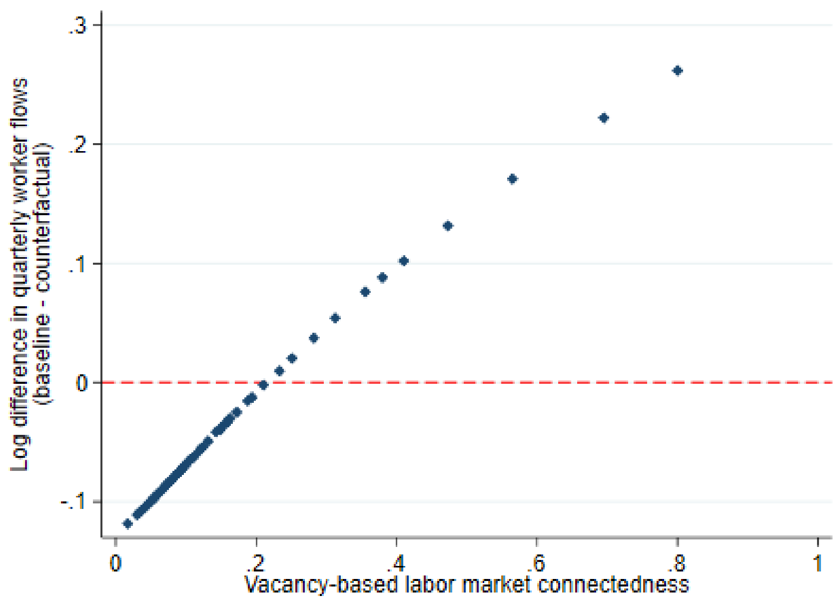
product market.

We then consider what would happen to the worker flows if we consider the class of models in which there is no network search. To match the overall EE rate, we reestimate the parameters in a counterfactual economy by setting $\bar{\lambda} = 0$ while targeting the same set of moments. We then consider the worker flows in this counterfactual Belgian economy with alternative parameterization.

The second column of Table 1.4 summarizes aggregate worker flows in the counterfactual economy with $\bar{\lambda} = 0$. In this counterfactual economy with no network search channel, all matches are formed through market search. Therefore, the B2B moves happen only if workers are randomly matched with the buyers or suppliers of their current employers through market search. The average employment-to-employment rate of B2B moves without network search is 0.8 percent, implying that only around 17 percent of job-to-job transitions would be considered movements along the supply chain.

It is worth noting that the share of B2B moves in this counterfactual economy is smaller than the statistical random benchmark for the B2B moves reported in Table 1.1 of Section 1.3.2. This difference comes from the fact that we now take into account the endogenous vacancy-posting decisions of firms as well as the mobility decisions of workers. Intuitively, in the absence of the network search channel, well-connected firms face both lower job-finding rates and lower separation rates. This is because workers at the other firms are less likely to be matched with the vacancies posted by these well-connected firms, while concurrently, their own workers are less likely to be poached by the other firms. Therefore, the absence of the network search channel contributes to the lower worker flows into and out of well-connected firms, where the B2B moves are more likely to happen.

Figure 1.4: Quarterly worker flows: baseline vs. no network search



Notes: In this figure, we report the log differences in quarterly worker flows between the baseline economy and the counterfactual economy with no network search. We use the parameters estimated in Section 1.5 and solve for the steady state in the baseline economy (see Appendix A.4.4 for details). In the counterfactual economy with no network search, we reestimate the model parameters by setting $\bar{\lambda} = 0$ while targeting the same set of moments as in the baseline economy. In each economy, we use the steady-state employment and separation rate to compute the quarterly worker flows for each firm group. For each bin of firm groups, sorted by the percentiles of the vacancy-based labor market connectedness in the baseline economy, we compute the average log differences in quarterly worker flows between two economies. The vacancy-based labor market connectedness is defined analogously to the employment-based labor market connectedness in equation (1.1).

In order to see this point that the absence of the network search channel alters the patterns of worker flows, in Figure 1.4, we compare the worker flows in the baseline and counterfactual economies based on the labor market connectedness of firms. To do so, we first use the steady-state employment and separation rate to compute the quarterly worker flows for each firm group in both economies. We then present the binscatter plot of log differences in quarterly worker flows based on the vacancy-based labor market connectedness in the baseline economy. Figure 1.4 reveals that there is a positive relationship between the labor market connectedness and the differences between the two economies. For the majority of the firms that have low levels of labor market connectedness, their worker flows are smaller in the baseline economy, whereas the firms with high levels of labor market connectedness exhibit greater worker flows, up to more than 20 percent in the baseline economy compared to the counterfactual economy with no network search. Thus, we find that not accounting for the network search channel would understate the worker flows in well-connected firms while overstating the worker flows at less-connected firms.

1.6.2 Wage change in response to the productivity shocks in production networks

Lastly, we use the estimated model to analyze the wage changes in response to the productivity shocks in production networks. Throughout this section, we consider a 5 percent reduction in firms' own productivity $\{\phi_j\}$ for all manufacturing firms in the Belgian economy. We then examine how the shocks propagated in production networks differentially affect workers at different firms.

When computing the worker impacts of a productivity decline among Belgian manufacturing firms, we start from the steady state of our baseline economy and only alter their own productivity $\{\phi_j\}$ while taking as given all the other model primitives and parameters estimated in Section 1.5. We then solve for the new distribution of labor productivity

$\{\Phi_j\}$, for both manufacturing firms and non-manufacturing firms, following the procedure in Appendix A.4.4. In order to consider the impacts on workers, we primarily focus on the instantaneous responses of firms' labor productivity and workers' wages.[20]

Panel (a) of Figure 1.5 shows the instantaneous responses in log labor productivity Φ_j. Even though only manufacturing firms are directly affected by their own productivity decline of 5 percent, the total decline in their labor productivity $\{\Phi_j\}$ can be larger than 5 percent, and non-manufacturing firms are also affected indirectly because of the propagation of shocks in the production networks. On average, we find that a manufacturing firm experiences a decline in its labor productivity of around 9.4 percent, while the labor productivity of a non-manufacturing firm drops by around 1.7 percent.

We then examine how these declines in labor productivity affect the wages of workers. Equation (1.23) allows us to compute how the change in labor productivity Φ_j can be translated into the change in wages w_j. We find that the shocks to manufacturing firms result in an immediate wage reduction of around 6.3 percent and 1.2 percent among manufacturing workers and non-manufacturing workers, respectively.

It is useful to note that computing the wage response at the current employers does not necessarily require the model of on-the-job search. Our model further allows us to compute the wage changes at the firms that workers are matched with. In Panel (b) of Figure 1.5, we present binscatter plots to show the relationship between firms' own wage changes and the average wage changes of the matched firms. Importantly, we compute the average wage changes of the firms matched through each of two search channels, so that the average changes are represented by a blue diamond for market search and a red marker for network search, respectively.

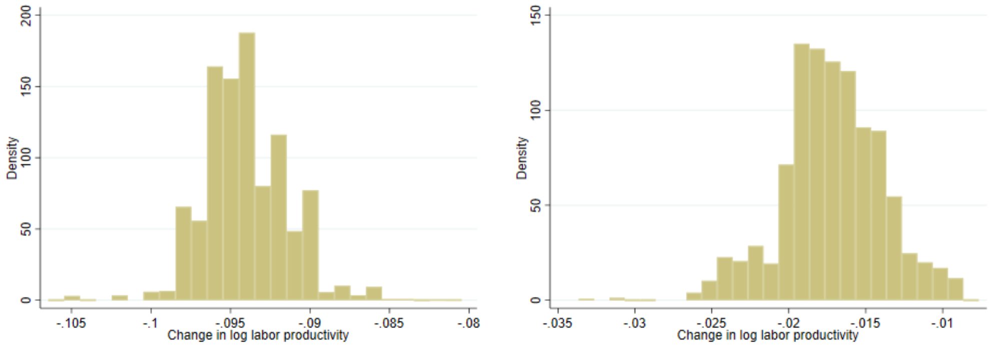
Several interesting observations arise. First, the average wage change of the firms matched through market search is constant regardless of their own wage change. This is because the

20. In Appendix A.5.3, we also provide the long-run response of firms by comparing them to the new steady state.

Figure 1.5: Instantaneous response to 5 percent reduction in manufacturing productivity

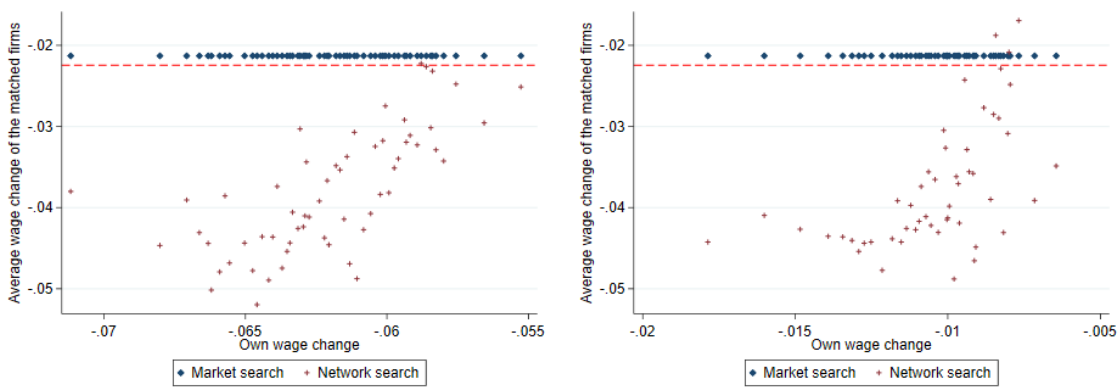
(a) Changes in log labor productivity

Manufacturing firms Non-manufacturing firms



(b) Changes in log wage: own vs matched firms

Manufacturing firms Non-manufacturing firms



Notes: In this figure, we report the changes in log labor productivity and log wage due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. In Panel (a), we show the distributions of instantaneous changes in log labor productivity $\{\Phi_j\}$ for both manufacturing firms and non-manufacturing firms, following the procedure in Appendix A.4.4. In Panel (b), we present the relationship between firms' own wage changes and the average wage changes of the firms with which workers are matched through the market search and network search channels. We first use equation (1.23) to compute the change in wage w_j for each firm. For each bin of firm group, sorted by the percentiles of the own wage changes, we then compute the average wage changes of the matched firms, weighted by the likelihood of the matching. The blue diamonds represent the average wage changes of the firms matched through the market search channel, whereas the red markers represent the average wage changes of the firms matched through the network search channel. The dashed red lines represent the employment-weighted average of the wage changes in the entire economy.

composition of matched firms is determined solely by the vacancy distribution and does not depend on the identity of the current employers. When matched with firms through the market search channel, workers find the average wage of the matched firms to be lower by around 2.1 percent.

On the other hand, the average wage decline is no longer homogeneous once we take into account the network search. When matched with firms through the network search channel, workers at different firms find that the average wage decline of the matched firms ranges from 1.8 percent to 5.2 percent. Quantitatively, we find that the average wage decline of the firms matched through the network search channel tends to be greater than that of the firms matched through the market search channel. This greater wage decline arises when well-connected firms tend to be hit harder by the shocks to production networks.

Furthermore, Panel (b) of Figure 1.5 also reveals a positive correlation between the wage decline at the current employer and the average wage decline of the firms matched through the network search channel. When workers experience a larger decline in their wages at the current employers, they also tend to meet the other firms that are hit harder by the productivity shocks. We find a positive correlation of 0.69 for manufacturing firms and 0.56 for non-manufacturing firms.

It is worth noting that this positive correlation does not necessarily appear in any economy. For instance, suppose there exists an economy whose production networks are complete, such that all firms are directly connected with all the other firms. In this economy, workers at any given firm could meet all the other firms through the network search channel, and thus, the average wage change of the firms matched through the network search channel coincides with that of the firms matched through the market search channel. The positive correlation between firms' own wage decline and the average wage decline of the matched firms appears when well-connected firms that are hit harder by the productivity shocks are more likely to trade with the other well-connected firms. Our estimated model implies that the structure of the Belgian production networks is such that it exhibits a positive correlation in wage responses among the current and potential employers. Therefore, our results suggest that the network search channel reduces the diversification of workers' outside options against productivity shocks to production networks.

1.7 Conclusion

The aim of this paper was to examine the prevalence of job-to-job transitions along the supply chain and quantify its contribution to the dynamics of the Belgian labor market. We find that a sizable fraction of Belgian workers find their next employers among the buyers and suppliers of their current employers, suggesting that the linkages of firms in the product market play a significant role in explaining workers' job search behaviors in the labor market. Quantifying the contribution of firm-to-firm linkages in aggregate worker flows through the lens of an equilibrium model, we show a considerable overlap between the set of potential employers in the labor market and the firm-to-firm linkages in the product market. Our results suggest that shocks to production networks are likely to affect wages not only at the current employers but also at the future employers.

While our results shed new light on the interaction between the production network and labor market, our model is still parsimonious and restrictive in several dimensions. For instance, we have not taken into account the endogenous link formations in buyer-supplier relationships as well as firm entry and exit. Incorporating the endogenous responses in the extensive margins of firm-to-firm trade could be important depending on the nature of shocks researchers are interested in. We have also abstracted away from worker heterogeneity in skills and preferences, which can introduce another source of labor market imperfection. Lastly, future works may incorporate the business cycle into our model and analyze the responses to temporary shocks.

CHAPTER 2

FOREIGN DEMAND SHOCKS TO PRODUCTION NETWORKS: FIRM RESPONSES AND WORKER IMPACTS

2.1 Introduction

The increased availability of micro datasets with information on both domestic firm-to-firm sales and foreign trade transactions has expanded the focus of research on international trade to include firms that only trade indirectly by buying from or selling to domestic firms that import or export (see, e.g., Huneeus, 2020, Adão et al., 2022, Demir et al., 2024, and Dhyne et al., 2021). An important insight from these data is that smaller and less productive firms often overcome the costs of entering foreign markets by selling to or buying from domestic firms that trade internationally. This finding raises several important questions that we seek to answer in this paper: How do changes in foreign demand transmit from one firm to the next in the domestic production network? How are firms responding to and workers affected by foreign demand shocks to direct exporters and their domestic suppliers? What are the aggregate implications of foreign demand shocks for output, input costs, and real wages?

We study these questions in the context of Belgium, a small open economy. As discussed in Section 2.2, our analyses employ a panel dataset of Belgian firms and workers, covering the period 2002-2014. This dataset is based on several micro data sources that we have linked. Annual accounts provide data on input factors and output; customs records and intra-EU declarations give information on imports and exports; a value-added tax (VAT) registry provides information on domestic firm-to-firm transactions; and social security records and employer-employee data give information on workers and their earnings, hourly wages, and work hours. Importantly, these data allow us to accurately represent firms' domestic production networks and measure their total import and export. These measures capture that firms may choose to access foreign markets both directly and indirectly by buying from or selling to domestic firms that trade internationally.

In Section 2.3, we use the panel dataset on firms and workers to develop three sets of empirical facts about the Belgian economy. First, we characterize the relationships in our data between (changes in) firm-level sales, labor costs, and intermediate input purchases. We find that input purchases respond nearly proportionally to changes in sales. In contrast, changes in sales are associated with less than proportionate changes in labor costs. These findings are consistent with firms facing fixed overhead costs in labor inputs, whereas intermediate inputs (such as energy and materials) are predominantly variable costs in production. Second, we build on the analysis of Dhyne et al. (2021) to show that even though direct exporters are rare, a majority of firms are indirectly exporting. This finding points to the importance of incorporating indirect export through the production network to measure firms' ultimate exposure to foreign demand.[1] Third, we show that firms that are more exposed to foreign markets are larger, more productive, and pay higher wages, and that these wage differentials cannot be entirely explained by observed or unobserved differences across workers. This finding suggests it is necessary to depart from the canonical model of a competitive labor market where wages depend only on the marginal product of workers and not the firm for which they work.

Motivated and guided by these empirical facts, we next develop, in Section 2.4, a small open economy model and then analyze the comparative statics properties of the relationships between key variables. One important feature of our model is that we allow for imperfect competition, in the form of monopsonistic competition in the labor market and monopolistic competition in the product market. Another important feature of our model is that we allow the production of goods to depend on both fixed and variable costs of labor and intermediate goods. Fixed labor costs may reflect tasks such as administration, worker management,

1. Dhyne et al. (2021) also demonstrate that what matters for the transmission of foreign demand shocks to a firm's revenue is how much it ultimately sells to foreign markets, not whether these sales are from direct or indirect export.

facility maintenance and any other work that does not translate directly into production and revenue. Examples of fixed intermediate input costs include waste management, accounting services, and electricity payments that occur irrespective of sales.

The model serves three purposes. First, the comparative statics of the model show the elasticities we need to recover to quantify and interpret how firms and workers respond to and are affected by changes in foreign demand. These elasticities include the labor supply elasticity facing the firm as well as the firm's elasticities of labor costs and intermediate input purchases with respect to demand-driven changes in sales. Second, the model helps to make explicit the data, instrument, and assumptions we need to identify and estimate these elasticities. The comparative statics of the model consider a change in the sales of a firm due to an exogenous change in the demand it faces. In the data, however, sales may change for many reasons other than demand, including shifts in input prices, technology, preferences, or amenities. To address this identification challenge, we draw on the work of Hummels et al. (2014) and Dhyne et al. (2021) to construct an instrument that intends to isolate the variation in sales that is induced by a change in foreign demand. We perform a number of robustness checks to examine various threats to the validity of the instrument. Lastly, the model makes it possible to perform counterfactuals to quantify and explain the aggregate welfare implications of changes in foreign demand (or productivity). In this analysis, we consider several counterfactual economies, defined by alternative parameterizations of the model. One of these economies is constructed to mirror the actual Belgian economy. In the other counterfactual economies, we instead assume no fixed costs or perfectly elastic labor supply (or both).

In Section 2.5, we take the model to the data with the goal of quantifying and explaining the firm responses and worker impacts of changes in firm sales that are induced by changes in foreign demand. To do so, we use the instrumental variables approach described in Section 2.4 to recover the elasticities defined by the comparative statics of the model. Our estimates of firm responses suggest that Belgian firms pass on a large share of a foreign demand shock to their domestic suppliers, face upward-sloping labor supply curves and, thus, have wage-setting power, and have sizable fixed overhead costs in labor.[2] To help interpret these estimates and gauge their economic implications, we next perform a simple model-based simulation of the direct and indirect effects of foreign demand shocks to production networks. The first step in the simulation is to use the estimated elasticities to predict the employment and wage responses of a direct exporter to a foreign demand shock. The next step is to simulate the spillover effects of the foreign demand shock to the domestic suppliers of the direct exporters.

We find that, on average, a direct exporter raises employment by 1.4 percent and the wages it pays by 0.4 percent in response to a foreign demand shock that increases the firm's direct export by 10 percent. This shock cascades through the production network as the direct exporters buy more inputs both directly from their own suppliers and indirectly from the suppliers' (direct and indirect) suppliers. These indirect demand effects increase, on average, the employment and wages of the direct exporter's key supplier by 0.3 and 0.08 percent, respectively. In other words, the key supplier experiences one-fifth of the percentage increases in wages and employment of the direct exporter. These indirect demand effects decay quickly with the distance to direct exporters in the supply chain. In fact, the foreign demand shock has little if any impact on the employment and wages of the key supplier's key supplier. An implication of the direct and indirect effects on wages and employment is that workers in the production network will get surplus or rents due to the foreign demand shock. On average, workers in the directly exporting firms get 75 percent of these rents. The remaining rents are shared among the other workers in the production network, with most of them going to the workers of the direct suppliers.

The simulation results in Section 2.5 point to the importance of production networks,

2. Our estimates of fixed overhead costs—that we infer from firm responses to demand shocks—are broadly comparable to the findings of Loecker et al. (2020) based on accounting data for publicly listed US firms.

upward-sloping labor supply curves, and fixed overhead costs to accurately predict how firms respond to and workers are affected by foreign demand shocks. However, this simulation relies on several simplifying assumptions. In particular, it considers a demand shock to a single of exporter, and it abstracts from general equilbrium effects and the need to adjust aggregate imports if aggregate exports fall. In Section 2.6, we relax these assumptions and analyze the aggregate effects of a five percent increase in foreign tariffs on all Belgian exports. Our results suggest that the increase in foreign tariffs produces a substantial 5.7 percent fall in the average real wage. By comparison, the reduction in real wages would be predicted to be as low as 3.3 percent if we assume the economy had no fixed costs and perfectly elastic labor supply. Thus, we conclude that the way in which the labor market is typically modeled in existing research on foreign demand shocks—with no fixed costs and perfectly elastic labor supply—may grossly understate the decline in real wages due to an increase in foreign tariffs.

The Lerner Symmetry Theorem is useful to explain these results. It establishes the equivalence between any intervention that increases the cost of import and export by the same amount. Thus, the five percent increase in foreign tariffs is equivalent to a five percent increase in tax on imports. An increase in the cost of imports has a larger impact on the firms' total variable costs, and, in turn, on output and real wages, if the economy has sizable fixed overhead costs in labor while imported inputs are predominately variable costs. Furthermore, in an economy in which the firms face upward-sloping labor supply curves, they pay lower wages than they otherwise would by hiring fewer workers, which effectively amplifies the share of labor costs that is fixed.

Our paper contributes to several areas of economics. Our first contribution is to the literature that analyzes how foreign demand shocks affect labor market outcomes. A large literature uses worker data at the firm or regional level to analyze the labor market effects of foreign demand shocks (see, e.g., Autor et al., 2013, Autor et al., 2014, Kovak, 2013, Dix-Carneiro, 2014, Dix-Carneiro and Kovak, 2017, Pierce and Schott, 2016, Traiberman, 2019,Kim and Vogel, 2020, 2021, Felix, 2022, Galle et al., 2023, and Costinot et al., 2022). Our paper joins a small set of papers that combine information on firm-to-firm transactions with micro data on firms and workers. Demir et al. (2024) document positive assortative matching between buyers and suppliers in terms of wages paid to workers. They analyze how this positive assortative matching affects the wage responses of exporters and their suppliers to a foreign demand shock. Adão et al. (2022) study the impact of trade on inequality in a framework with perfect competition, and Alfaro-Ureña et al. (2021) estimate the effects of foreign multinationals on workers. Huneeus et al. (2022) estimate that supply shocks transmitted through the production network can account for around 20 percent of the earnings volatility of Chilean workers.[3] All these studies analyze data on firm-to-firm transactions in developing countries.[4] To our knowledge, the country we study, Belgium, is currently the only developed country with such data. Another key difference between our study and prior work is that our analysis allows for imperfect competition in the labor market and distinguishes between fixed and variable costs in production. We find this distinction to be empirically important, as it materially affects the conclusions from both partial and general equilibrium analyses of the labor market effects of foreign demand shocks.

Our paper also contributes to the literature on general equilibrium effects in production networks. Baqaee and Farhi (2024) analyze the general equilibrium effects of trade shocks, and Bigio and La'O (2020), Baqaee and Farhi (2019a), and Baqaee and Farhi (2019b) analyze the transmission of shocks in closed economies with and without distortions. Baqaee and Farhi (2021) illustrate how free entry affects the long-run effects of shocks, vom Lehn and Winberry (2021) document the role of the investment network for sectoral comovement along the business cycle, and Atalay (2017) quantifies the importance of sectoral shocks along

3. In other work based on data with firm-to-firm transactions, Huneeus (2020) studies the pass-through of foreign shocks to Chilean firms during the Great Recession.

4. A large literature in development economics studies the role of intermediaries in trade. See, for example, the works by Atkin and Donaldson (2015), Chatterjee (2023), Dhingra and Tenreyro (2023), Bergquist and Dinerstein (2020), Grant and Startz (2022), and Zavala (2023).

the business cycle.[5] To our knowledge, our study is the first to incorporate estimates of fixed overhead costs and imperfect competition in the labor market in a general equilibrium analysis with production networks. Baqaee and Farhi (2021) discuss the relevance of fixed costs in a framework that abstracts from imperfect competition in the labor market.

Our paper is also related to a large literature on firm growth. A series of papers study the role of growth in the number of customers and products in overall firm-level growth (see, e.g., Einav et al., 2021, Argente et al., 2023, Argente et al., 2024, and Fitzgerald et al., 2019). Hottman et al. (2016) and Bernard et al. (2022) provide evidence that differences in demand are a key determinant of heterogeneity in firm performance. Sterk et al. (2021), Foster et al. (2016), and others examine the life-cycle patterns of firms. Exploiting our detailed data on firms, workers and production networks, we show how the firms use of and payments to labor and intermediate inputs change in response to demand shocks. A related literature in labor economics examines the rent sharing between firms and workers, exploiting changes in patents, demand, or taxes and subsidies (see, e.g., Van Reenen, 1996, Guiso et al., 2005, Kline et al., 2019, Friedrich et al., 2024, Berger et al., 2022, Kroft et al., 2023, Howell and Brown, 2023, **?**, and Chan et al., 2023). All of these papers focus on outcomes at directly affected firms. Our main contribution to this literature is to study the effects of firm-level demand shocks across the supply chain, including the firms that are only indirectly exposed to the demand shocks through their buyers.

2.2 Data and estimation sample

Our analyses combine multiple administrative data sources from Belgium for the period 2002-2014. Below we briefly describe our data and sample selection; additional details are

5. Several papers analyze endogenous production networks that allow for the creation or destruction of firm-to-firm links in response to economic shocks. See Oberfield (2018), Lim (2018), Taschereau-Dumouchel (2024), Elliott et al. (2022), Acemoglu and Tahbaz-Salehi (2020), and Arkolakis et al. (2023). For tractability, we abstract from this adjustment mechanism in our model.

given in Appendix B.1.

2.2.1 Data on firms

The National Bank of Belgium (NBB) provided us with three datasets on firms, each covering the period 2002-2014. These datasets can be linked through (anonymized) firm identifiers, assigned and recorded by the government for the purpose of collecting value-added taxes (VAT). For details on the linking procedure, we refer to Dhyne et al. (2015) and Dhyne et al. (2021).

The first dataset is the Business-to-Business (B2B) transactions database. Every year, all VAT-liable firms are legally required to report annual sales to every other VAT-liable firm in Belgium. This information must be reported to the tax authority provided that annual sales to a given buyer exceed 250 euro. Thus, the B2B dataset allows us to measure the firms' domestic production networks as well as their purchases from and sales to domestic suppliers and buyers.

We merge this dataset with information on the firms' international trade and their annual accounts. The information on international trade comes from the Belgian customs records and the intra-EU trade declarations, which contain the value of imports and exports (disaggregated by the EU's eight-digit coding system for products) and the countries of origin or destination. The annual accounts contain detailed information from the firm's balance sheets on sales, revenues, operating profits, ownership shares in other enterprises, costs of inputs (such as capital, labor, and intermediates), as well as four-digit (NACE) industry codes and geographical identifiers (at the postal code level).

Importantly, the annual accounts also include information about the number of full-time equivalent (FTE) workers. The calculation of FTE is an employee's actual scheduled hours divided by the regular scheduled hours for a full-time workweek. To measure the (average) hourly wage that each firm pays to its workers, we divide the firm's labor cost by the number of FTE workers.

A limitation with the Belgian data is that all the information is recorded at the level of the VAT identifier. This creates a challenge because a firm may have several VAT identifiers (for accounting or tax reasons). If a firm has multiple VAT identifiers, we follow Dhyne et al. (2021) in aggregating the data up to the firm level using information from the annual accounts about the ownership structure. Further details on the aggregation procedure are provided in Appendix B.1.1.

2.2.2 Data on individual workers

As described above, the firm data offers information on the number of FTE workers and the wages paid to these workers. This allows us to measure changes over time in the labor cost, hourly wages, and the size of the workforce of a given firm. The firm data do not, however, allow us to follow the same workers over time.

To do so, we add information from matched employer-employee data for the period 2003-2014. The employer-employee data are provided to NBB by the Banque Carrefour de la sécurité sociale (Crossroads Bank for Social Security, BCSS), and then linked by NBB to our firm data (see Appendix B.1.2 for details.) The linked data consist of a random sample of 500,000 workers, drawn from the population of firms with 10 or more FTE employees at least once during the period 2003-2014. We have to work with this subsample of workers in larger firms because of restrictions imposed by the Belgian social security administration.

As discussed in greater detail later, this subsample of workers is useful for two reasons. First, it lets us perform an analysis of the wage impacts of worker mobility across firms. This allows us to examine if the hourly wage paid to a given worker depends on the firm for which she works. Second, it lets us restrict the estimation sample to stayers, who are observed working for the same firm over several years. This allows us to examine whether changes in a firm's labor cost are a result of changes in the wage that it pays to a given worker or changes in the composition and quality of its workforce.

2.2.3 Estimation samples

Most of our analyses use only the firm data. In these analyses, we impose a few restrictions to construct a suitable estimation sample. We restrict our analysis to firms in the private, non-financial sectors with at least one FTE employee and positive labor costs and sales. Following De Loecker et al. (2014) and Dhyne et al. (2021), we also restrict our analysis to firms with tangible assets of more than 100 euro and positive total assets in at least one year during our sample period 2002-2014. In the remainder of the paper, we refer to this sample as the main estimation sample.

As evident from Appendix B.1.3, the main estimation sample covers a large majority of the aggregate value added, gross output, labor costs, exports, and imports in the Belgian economy. In this appendix, we also present summary statistics on the workers and the firms. These statistics show, for example, that about half of the workers are categorized as white-collar workers and that the wages are 70 percent higher than those of blue-collar workers.

In a few of our analyses, we will rely on the subsample for which we have additional information from the worker data (i.e., the firms with 10 or more FTE employees at least once from 2002 to 2014). Even though this subsample contains only about a quarter of all firms, it still makes up most of the total sales, inputs, and trade in the Belgian economy.

2.3 Motivating empirical facts

We next present three sets of empirical facts about the Belgian economy that we use to motivate and guide the choices of model and econometric specification.

2.3.1 Firm-level sales, labor costs, and input purchases

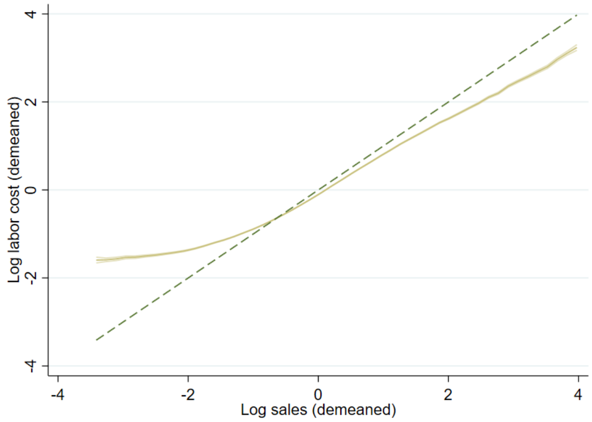
The first set of facts describes the relationships in our data between firm-level sales, labor costs, and intermediate input purchases. We find that input purchases respond nearly proportionally to changes in sales. In contrast, changes in sales are associated with less than proportionate changes in labor costs. This finding is difficult to reconcile with the usual homothetic production function with constant return to scale. Instead, it is suggestive of sizable fixed overhead costs in labor.

We begin by describing the cross-sectional relationships between firm-level sales, labor costs, and intermediate input purchases. We pool the cross-sectional data for the entire period 2002-2014. In an attempt to adjust for differences across industries, we first demean the log of each variable using the firm's four-digit industry average. Thus, a firm with reported log sales of zero has the sales of an average firm in its industry. Next, we use local polynomial regressions to non-parametrically estimate the relationships between log labor costs and log sales (panel (a) of Figure 2.1) and log input purchases and log sales (panel (b) of Figure 2.1). In these cross-sectional comparisons across firms, we find that both labor and intermediate input purchases are nearly proportionally aligned with sales.

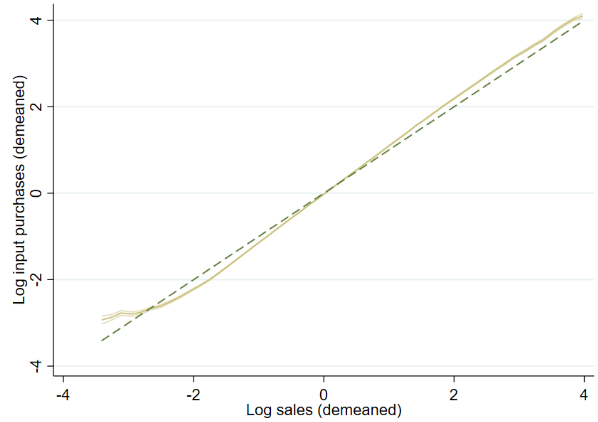
It is admittedly difficult to learn much from such cross-sectional comparisons. For example, large and productive firms may be systematically different from smaller and less productive firms in the production technology they use. A natural way to start addressing this concern is to examine changes over time within firms in sales, labor costs, and intermediate input purchases. In panels (c) and (d) of Figure 2.1, we perform the same analyses as in the two first panels, except that we now look at changes within firms over a four year period.

Figure 2.1: Relationship between firm-level sales, labor costs, and input purchases

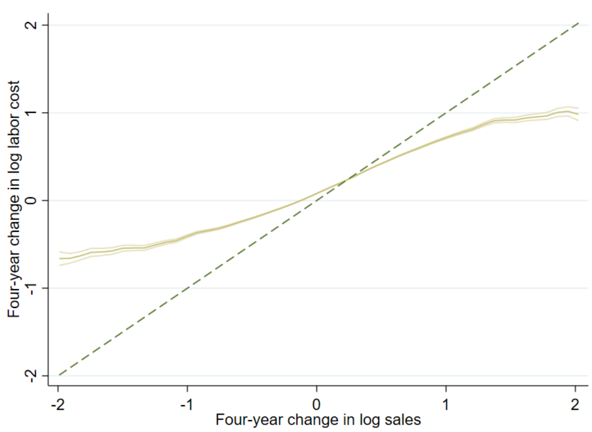
(a) Sales and labor costs



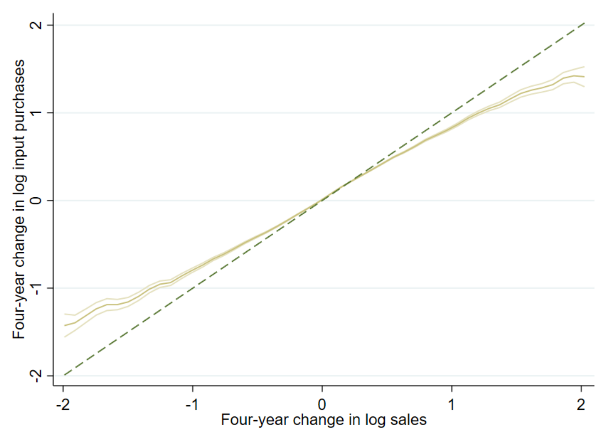
(b) Sales and input purchases



(c) Sales and labor costs (four-year change)



(d) Sales and input purchases (four-year change)



Notes: The figures use the main estimation sample of private-sector firms in Belgium from 2002 to 2014 (see Section 2.2.3 for details). They display the relationship between firm-level sales, labor costs, and input purchases, using the smoothed values of kernel-weighted local polynomial regression estimates with 95 percent confidence intervals. We use the Epanechnikov kernel function with kernel bandwidth of 0.05. In panels (a) and (b), each variable is demeaned with four-digit industry-year fixed effects. Log sales and four-year changes in log sales are trimmed at the top and bottom 1 percentiles.

We find that input purchases respond close to proportionally to changes in sales (a slope coefficient of 0.82). In contrast, changes in sales are associated with less than proportionate changes in labor costs (a slope coefficient of 0.57). While these results could be interpreted as suggestive evidence of sizable fixed overhead costs in labor, we need to be cautious. Input costs and sales are simultaneously determined and may be affected by many omitted variables. To draw conclusions about fixed versus variable costs we therefore will, in Section 2.4, develop an explicit model and construct an instrument that isolates the variation in sales that is induced by an exogenous change in demand.

2.3.2 Indirect export and exposure to foreign demand

The second set of facts build on the work of Dhyne et al. (2021). As in their analysis, we combine the data on domestic firm-to-firm sales with information on firms' foreign trade transactions to show that even though direct exporters are rare, a majority of firms are indirectly exporting. This finding motivates why our model and analysis will include indirect export through the production network to measure the firms' exposure to foreign demand.

To arrive at this conclusion, we first construct measures of the firms' total export. As in Dhyne et al. (2021), we assume the firm's composition of inputs in production does not vary across its buyers, so that we can measure the total export of a firm by the total share of output that it sells directly or indirectly to foreign markets (i.e., the total export share). Formally, the total export share of firm k, r_{kF}^{Total}, is defined as the share of revenue from direct export, r_{kF}, and the share of revenue coming from sales to other domestic firms, multiplied by the total export shares of those firms:

$$r_{kF}^{Total} = r_{kF} + \sum_{i \in W_k} r_{ki} r_{iF}^{Total}, \tag{2.1}$$

where W_k denotes the set of buyers of firm k, and r_{kF} and r_{ki} are the share of k's revenue that comes from direct export and from sales to domestic firm i, respectively. The denominator of the export shares is the total revenue of the firm, which consists of sales to other domestic firms, sales to households, and direct exports.

It is important to observe that the definition of the total export share is recursive. A firm's total export share is the sum of its direct export share and the share of its sales to other domestic firms multiplied by the total export shares of those firms. Thus, the total export share is high if a lot of the firm's output is exported directly to foreign markets or indirectly via sales to domestic buyers with high export shares.

Table 2.1: Descriptive statistics in 2012

(a) Direct and total export participation

	Exporters and non-exporters	Exporters only	Non-exporters only
Number of observations	98,745	11,892	86,853
Fraction of firms with total export participation	0.875	1.000	0.858
Average export shares:			
Total export	0.138	0.445	0.096
Direct export	0.039	0.322	0.000
Indirect export	0.100	0.122	0.096

(b) Firm characteristics

	Exporters and non-exporters	Exporters only	Non-exporters only
Log sales	13.6	15.5	13.3
Log TFP	10.7	11.3	10.6
Log value added	12.5	13.9	12.3
Log FTE employment	1.5	2.5	1.3
Log average wage	10.5	10.8	10.5

Notes: This table uses the main estimation sample of private-sector firms in Belgium in 2012 (see Section 2.2.3 for details). Panel (a): The total export share of firm k, r_{kF}^{Total}, is recursively defined as $r_{kF}^{Total} = r_{kF} + \sum_{i \in W_k} r_{ki} r_{iF}^{Total}$, which can be decomposed into direct export share, r_{kF}, and indirect export share, $\sum_{i \in W_k} r_{ki} r_{iF}^{Total}$. Panel (b): For each column, we report the averages of variables listed on the left for a set of firms noted at the top of the column. Firms' sales consist of their sales to other firms in the NBB sample (network sales), sales to households at home, and direct exports to foreign markets. Firms' TFPs are calculated using the estimation procedure of Wooldridge (2009). Firms' value added and FTE employment are from the reported values from the annual accounts. Firms' average wages are the reported labor costs divided by their FTE employment.

In panel (a) of Table 2.1, we compare the (direct and total) export participation and shares of firms that directly export to those that only export indirectly. While few Belgian firms are directly exporting, a majority of the firms are indirectly exporting through sales to domestic buyers that subsequently trade internationally. In fact, even the firms that do not directly export are, on average, selling nearly 10 percent of their output indirectly to foreign markets.

2.3.3 Wage differentials and firm effects

Using the firm and worker data, the third set of facts show that i) firms that are more exposed to foreign markets are larger, more productive, and pay higher wages, and that ii) these wage differentials cannot be entirely explained by observed or unobserved differences across workers. These findings motivate why we will depart from the canonical model of a competitive labor market where wages depend only on the marginal product of workers and not the firm that employs them.

A large body of previous work has documented that firms that export look very different from non-exporters along a number of important dimensions. This is also true in the Belgian data: the descriptive statistics reported in panel (b) of Table 2.1 show that the direct exporters not only are more productive and have higher sales but also have more employees and pay higher wages than other firms. This pattern in the data is consistent with an imperfectly competitive labor market where each individual firm faces an upward-sloping labor supply curve, implying that wages are an increasing function of firm size and productivity. However, several alternative explanations exist.

One alternative explanation is that workers could be paid differentially because of unobserved skill differences, not imperfect competition (see, e.g., Abowd et al., 1999, Gibbons et al., 2005). To investigate this possibility, we run a set of wage regressions on a sample of workers who switch firms (and have at least four years of tenure at both the origin and destination firms, to ensure that we can accurately measure their wages both before and after the move). This sample is based on the subset of firms for which we have additional information from the worker data (see Section 2.2.3 for details). The results are presented in Table 2.2. In the first column, we regress the log wages of workers on a dummy variable for being employed in a firm that directly exports, controlling only for calendar year effects. In the second and third columns, we add controls for observable worker characteristics and sector fixed effects, respectively. In the final column, we use the panel dimension of the data to add controls for worker fixed effects. By including these fixed effects, we control for any time-invariant (observed or unobserved) worker heterogeneity. Since aggregate shocks are absorbed by the time fixed effects, identification is achieved from a common trend assumption in the workers' wages in the absence of moving to firms that directly export. In Appendix B.3.1, we empirically assess this assumption, finding support for common trends prior to the move.

Table 2.2: Wage regressions on the sample of movers

	(1)	(2)	(3)	(4)
Exporter dummy	0.229***	0.131***	0.0639***	0.0258***
	(0.00375)	(0.00307)	(0.00361)	(0.00288)
Number of workers	10,179	10,179	10,179	10,179
Number of firms	7,101	7,101	7,101	7,101
Calendar year FE	Yes	Yes	Yes	Yes
Worker characteristics		Yes	Yes	Yes
Industry FE			Yes	Yes
Worker FE				Yes

Notes: This table uses the subsample of firms for which which we have additional information from the worker data (see Section 2.2.3 for details). For each column, we run a worker-level regression of log FTE wage on the sample of movers between any firms. Movers in year t are defined as workers who are employed by the origin firms at no later than $t-4$, switch their jobs between $t-1$ and t, and stay at their destination firms at least until $t+3$. The sample is balanced from $t-4$ to $t+3$. Observations in years $t-1$ and t are dropped from the regressions, to ensure we only use full-year employment spells in a given firm. Worker characteristics include worker class (blue collar or white collar)—which can vary across employers for the same worker—gender, and age bin-year effects. Industry fixed effects are included at the NACE four-digit level. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Taken together, the results in Table 2.2 show that controlling for (observed and unobserved) worker characteristics significantly reduces the differences in wages between workers in firms that do and do not directly export, highlighting the benefits of using panel data in our setting. Nevertheless, even after controlling for worker characteristics, workers in firms that directly export still earn about 2.6 percent more than workers in firms that do not directly export, consistent with imperfect competition in the labor market.

An alternative explanation to imperfect competition for why (even the same) workers are paid differently across firms is that observed wages may not necessarily reflect the full compensation that individuals receive from working in a given firm. Indeed, both survey data (e.g., Hamermesh, 1999, Pierce, 2001, Maestas et al., 2023) and experimental studies (e.g., Mas and Pallais, 2017, Wiswall and Zafar, 2018, Chen et al., 2020) suggest that workers may be willing to sacrifice higher wages for better non-wage job characteristics or amenities when choosing an employer. Thus, the wage premia in firms that directly export could reflect compensating differentials for unfavorable amenities, not imperfect competition. To distinguish between compensating differentials and imperfect competition as sources of wage differentials, we will, in Section 2.5, exploit changes in employment and wages within firms in response to plausibly exogenous foreign demand shocks.

2.4 Model

Motivated and guided by the empirical evidence presented in the previous section, we now develop a model and then analyze the comparative statics properties of the relationship between key variables.[6] The goals of the model are threefold. First, the comparative statics show the elasticities we need to recover to quantify and interpret how firms and workers respond to and are affected by changes in foreign demand. Second, the model makes explicit the data, instrument, and assumptions we need to identify and estimate these elasticities. Lastly, the model makes it possible to perform counterfactuals to quantify and explain the aggregate welfare implications of changes in foreign demand (or productivity).

2.4.1 Model environment

We model Belgium as a small open economy where firms take the prices in the foreign market as given. Our model is parsimonious and restrictive in several ways. For example, we take all buyer-supplier relationships as given, in terms of both the domestic firm-to-firm links and

6. The model is similar to the one in Huneeus et al. (2022), with a key distinction being that the cost structure of firms differs, and our model incorporates fixed overhead costs that lead to increasing returns to scale in production. We also consider a small open economy, whereas their model considers a closed economy.

the firms' direct export and import participation. However, in other important ways, the model is rich and flexible as compared to much of the existing work on production networks.

One important feature of our model is that we allow for imperfect competition, in the form of monopsonistic competition in the labor market and monopolistic competition in the product market. To let the labor supply facing the firm be imperfectly elastic, we assume that many firms compete with one another for workers who have heterogeneous preferences over amenities. This assumption gives rise to firms having an increasing marginal cost of labor, consistent both with the descriptive evidence in Section 2.3.3 and with recent empirical findings from countries other than Belgium (see, e.g., Almunia et al., 2021, **?**, and Kroft et al., 2023). In the product market, we assume a market structure in which firms have many competitors, but each one sells a single differentiated good. As a result, the product demand facing the firm may be imperfectly elastic. The firms may sell to households with heterogeneous preferences, to other domestic firms, and directly to foreign markets. Intermediate inputs may be purchased from other domestic firms or imported directly from abroad.

Another important feature of our model is that we allow the production of goods to depend on both fixed and variable costs of labor and intermediate goods. The reason we allow for fixed overhead costs is that both the descriptive evidence in Section 2.3 as well as previous studies indicate that such expenses can be important in matching key moments of the data on firms and workers in developed countries, at least in the short or medium run (see, e.g., Bartelsman et al., 2013, Traina, 2018, and Autor et al., 2020).

Product demand. The households in the economy consist of workers and firm owners. All households have the same preferences for goods. The utility of a household from final good consumption is denoted by C, which is a CES aggregator of the household's purchases of each firm's goods, q_{kH}:

$$C = \left(\sum_{k \in \Omega} \left(\beta_{kH} q_{kH} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.2}$$

where Ω denotes the set of all available products and H denotes domestic final demand from households. With the CES structure, one can write $P = \left(\sum_{k \in \Omega} \beta_{kH}^{\sigma-1} p_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ as the aggregate price index. The demand for product k (which is produced by firm k) can be expressed as

$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_k^{-\sigma}}{P^{1-\sigma}} E_H, \tag{2.3}$$

with E_H being the aggregate income of workers and firm owners, and similarly for exports, we have

$$q_{kF} = p_k^{-\sigma} D_{kF}, \tag{2.4}$$

where D_{kF} is the exogenous foreign demand shifter for firm k.

Labor supply. We consider an environment where labor is hired in a spot market, and each worker has idiosyncratic preferences over firms that are private information. Because of this information asymmetry, a firm cannot price-discriminate with respect to workers' reservation wages. Each worker n supplies one unit of labor and has the following preference for working at firm k:

$$v_{nk} = \log w_k - \log P + \nu_{nk}, \tag{2.5}$$

where w_k denotes the wage at firm k, P is the aggregate price index, and ν_{nk} denotes worker n's idiosyncratic taste for the non-wage attributes or amenities of firm k.

This specification of preferences allows for the possibility that workers are heterogeneous in their preferences over the same firm. The importance of this source of horizontal differentiation is governed by the variance of ν_{nk}. For empirical tractability, we assume that ν_{nk}

is distributed according to a Type-1 Extreme Value distribution with parameter ε.

Given the set of offered wages by all firms, worker n chooses a firm k to maximize her utility. Due to the distributional assumption on ν_{nk}, we obtain the following firm-specific labor supply curve:

$$\ell_k = A w_k^{\varepsilon}, \tag{2.6}$$

where $A = \frac{1}{\sum_j w_j^{\varepsilon}} L$. The term L is the aggregate labor supply.

Firms' production technology. Each firm produces a unique differentiated product and has a firm-specific production technology to convert variable labor, domestic inputs, and foreign inputs into output. It also has firm-specific fixed overhead input requirements for labor and inputs purchased from other firms. Finally, each firm has an exogenous set of domestic suppliers as well as exogenous access to the import or export market. We denote variables associated with variable inputs with superscript v and variables associated with fixed overhead inputs with superscript f.

Let the fixed overhead input requirement of firm k be denoted by $\bar{q}_k^f$. This fixed overhead input requirement can be fulfilled via a CES production technology by choosing inputs from a set of domestic suppliers Z_k as well as imports. The technology parameters ω_{jk}^f and ω_{Fk}^f are given. For all firms that do not directly import to fulfill the fixed input requirement, we have $\omega_{Fk}^f = 0$. Specifically, firm k's technology to fulfill its exogenous fixed overhead input requirements is as follows:

$$\bar{q}_k^f = \left(\sum_{j \in Z_k} \omega_{jk}^f \left(q_{jk}^f \right)^{\frac{\sigma-1}{\sigma}} + \omega_{Fk}^f \left(q_{Fk}^f \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{2.7}$$

Note that the term on the left-hand side is exogenously given, whereas the inputs on the right-hand side, q_{jk}^f and q_{Fk}^f, can be endogenously chosen. In addition, firm k has a fixed overhead labor input requirement $\bar{\ell}_k^f$. By letting the fixed overhead input requirements be firm-specific, our specification is flexible enough to capture that large firms may have higher overhead costs, as suggested by Traina (2018) and Loecker et al. (2020).

After fulfilling the fixed overhead requirements, the output production function has constant returns to scale. Firms combine variable labor inputs and a variable intermediate input bundle in a Cobb-Douglas fashion. The variable intermediate input bundle is a CES aggregate of variable inputs purchased from their suppliers and variable imports. We write the output production function of firm k as follows:

$$\begin{aligned} q_k &= \phi_k \left(q_k^v\right)^{1-\alpha_{\ell k}} \left(\ell_k^v\right)^{\alpha_{\ell k}} \\ q_k^v &= \left(\sum_{j \in Z_k} \omega_{jk}^v \left(q_{jk}^v\right)^{\frac{\sigma-1}{\sigma}} + \omega_{Fk}^v \left(q_{Fk}^v\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \end{aligned} \tag{2.8}$$

where $\alpha_{\ell k}$, ω_{jk}^v, and ω_{Fk}^v are the saliency parameters for labor inputs, inputs from individual suppliers, and imports. The term ϕ_k represents the exogenous Hicks-neutral productivity term of firm k. The left-hand side of equation (2.8), the total output of firm k, is an endogenous variable. Note that the CES substitutability parameters in firms' production technology are restricted to be the same as the substitutability parameter in final demand. This assumption, which is common in the literature (see Huneeus et al., 2022 and Demir et al., 2024 for example), implies that firms face the same demand elasticity regardless of who they sell their output to, and, thus, charge common markups under monopolistic competition.

Firm k's use of labor inputs, ℓ_k, inputs from supplier j, q_{jk}, and imports q_{Fk} are equal to the sum of their usage as variable and fixed inputs:

$$\begin{aligned}
\ell_k &= \ell_k^v + \bar{\ell}_k^f \\
q_{jk} &= q_{jk}^v + q_{jk}^f \; \forall j \in Z_k \\
q_{Fk} &= q_{Fk}^v + q_{Fk}^f.
\end{aligned} \tag{2.9}$$

Firm's problem. We now turn to the firm's profit maximization problem. Recall that the idiosyncratic preferences of workers are private information and thus unobserved to the firm. However, the firm knows the distribution of the idiosyncratic preferences. The firm views itself as infinitesimal within both the product and labor markets, acting monopolistically competitive in the output market and monopsonistic in the labor market. Firm k maximizes its profits by taking as given the labor supply curve (as in $w_k(\ell_k)$), the required fixed costs of $\bar{q}_k^f$ and $\bar{\ell}_k^f$, the intermediate input prices, as well as the price of imports p_{Fk}.

Given this environment, each firm k chooses demand for inputs and its output price to solve

$$\max_{\left\{q_{jk}^v\right\}, q_{Fk}^v, \left\{q_{jk}^f\right\}, q_{Fk}^f, \ell_k^v, p_k} p_k q_k\left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right) - w_k(\ell_k)\,\ell_k - \sum_{j \in Z_k} p_j q_{jk} - p_{Fk} q_{Fk}, \tag{2.10}$$

such that equations (2.6) to (2.9) hold.

Price. The first-order condition with respect to the firm's output price yields that firm k charges a firm-level markup of $\frac{\sigma}{\sigma-1}$ over its marginal cost, as it faces a common residual demand elasticity of σ. This result is a consequence of the assumption of firms engaging in monopolistic competition when they sell to other firms or sell to final demand.

Labor costs. The first-order condition with respect to variable labor inputs yields the following expression for firm k's variable labor cost share out of its total sales:

$$\frac{\ell_k^v w_k\left(\ell_k\right)}{p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}=\frac{\varepsilon}{\varepsilon+1}\frac{\partial\log q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}{\partial\log\ell_k^v}\left(1+\frac{d\log p_k}{d\log q_k}\right). \tag{2.11}$$

The first term in equation (2.11) represents the markdown of labor cost, which comes from the upward-sloping supply curve that has a constant elasticity of ε. The steeper the slope of the labor supply curve (low ε), the greater the markdown. The second term is the output elasticity with respect to variable labor inputs, which is summarized by the parameter $\alpha_{\ell k}$. The third term captures the inverse of the demand elasticity, which can be written as a constant term of $1+\frac{d\log p_k}{d\log q_k}=\frac{\sigma-1}{\sigma}$.

Intermediate input purchases. We now turn our attention to intermediate input purchases. Similar to equation (2.11), the first-order condition of firm k for variable input purchases from supplier j yields the following equation for the share of intermediate inputs from supplier j out of firm k's total sales:[7]

$$\frac{p_j q_{jk}^v}{p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}=\frac{\partial\log q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}{\partial\log q_{jk}^v}\left(1+\frac{d\log p_k}{d\log q_k}\right). \tag{2.15}$$

7. The profit maximization problem of (2.10) also yields the following relationships for the fixed input bundle. Firms minimize the cost of their fixed input purchases, $\sum_{j\in Z_k} p_j q_{jk}^f + p_{Fk} q_{Fk}^f$, such that equation (2.7) holds. We obtain the price index for the fixed input bundle:

$$c_k^f=\left(\sum_{j\in Z_k}\left(\omega_{jk}^f\right)^\sigma p_j^{1-\sigma}+\left(\omega_{Fk}^f\right)^\sigma p_{Fk}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{2.12}$$

The individual demand for the fixed inputs is expressed as

$$q_{jk}^f=\left(\omega_{jk}^f\right)^\sigma p_j^{-\sigma}\left(c_k^f\right)^\sigma \bar{q}_k^f \tag{2.13}$$

$$q_{Fk}^f=\left(\omega_{Fk}^f\right)^\sigma p_{Fk}^{-\sigma}\left(c_k^f\right)^\sigma \bar{q}_k^f. \tag{2.14}$$

The share of variable inputs from supplier j depends on two parameters. The first is the output elasticity with respect to the variable input, and the second is the inverse of the demand elasticity the firm faces.

General equilibrium. Equations for aggregate trade balance, labor market clearing, and aggregate income are presented in Appendix B.2.1. The general equilibrium features a set of firms' wages, $\{w_k\}$, and aggregate expenditure, E_H, such that firms and workers optimize and markets clear (see Appendix B.2.1 for details).

2.4.2 Comparative statics and target parameters

We now analyze the comparative statics properties of the relationship between key variables in the model. These comparative statics show the elasticities we need to recover to quantify and interpret how firms and workers respond to and are affected by exogenous changes in demand.

Elasticity of labor cost with respect to a demand-driven change in a firm's output. The change in the labor cost in response to a demand-driven change in a firm's output is informative about the share of labor inputs that is used for fixed overhead costs. Using equation (2.9) and taking the total derivative of equation (2.11) while holding firms' labor-specific technology parameter $\alpha_{\ell k}$, labor supply elasticity ε, and worker amenity draws $\{\nu_{nk}\}$ fixed, we obtain[8]

$$\frac{d\log\left(\ell_k w_k\left(\ell_k\right)\right)}{d\log\left(p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)\right)}=\frac{\ell_k^v}{\ell_k}\frac{\varepsilon+1}{\frac{\ell_k^v}{\ell_k}+\varepsilon}. \tag{2.16}$$

8. See Appendix B.2.2 for the derivation. Note that both a firm's foreign demand parameter, D_{kF}, and its productivity parameter ϕ_k are allowed to vary. Equation (2.16) is therefore consistent with both a demand-driven and a TFP-driven change in sales.

Equation (2.16) illustrates that the labor cost elasticity of firm k is a function of its variable share of labor inputs $\left(\frac{\ell_k^v}{\ell_k}\right)$ and the labor supply elasticity ε it faces. One extreme case is that all labor inputs are fixed $\left(\frac{\ell_k^v}{\ell_k} = 0\right)$, so that changes in sales do not get passed on to labor costs. The other extreme case is that all labor inputs are variable $\left(\frac{\ell_k^v}{\ell_k} = 1\right)$. Labor costs are then changing proportionally to changes in sales. Between these two extremes cases, some but not all labor is fixed. The response of labor costs to changes in sales is monotonically increasing in variable share of labor inputs (until the labor cost elasticity is equal to one).

More generally, the labor cost elasticity depends on both the variable share of labor inputs and the labor supply elasticity ε. If there is perfect competition in the labor market ($\varepsilon = \infty$), the labor cost elasticity increases linearly with the variable share of labor inputs. However, if firms face upward-sloping labor supply curves, they pay lower wages than they otherwise would by hiring fewer workers, which effectively amplifies the share of labor inputs that is fixed. Thus, for a given variable share of labor inputs, the elasticity of labor costs is declining in ε (i.e., as the labor supply curve gets steeper and labor markets become less competitive).

Elasticity of input purchases with respect to a demand-driven change in a firm's output. The change in intermediate input purchases in response to a demand-driven change in a firm's output is informative about the share of input purchases that is used for fixed overhead costs. Using equation (2.9) and taking the total derivative of equation (2.15) while holding firms' input-factor-specific technology parameters $\left(\left\{\alpha_{\ell k}, \omega_{jk}^v, \omega_{Fk}^v, \omega_{jk}^f, \omega_{Fk}^f\right\}, \sigma\right)$ and relative prices of the suppliers fixed, we obtain[9]

$$\frac{d \log \left(p_j q_{jk}\right)}{d \log \left(p_k q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right)\right)} = \frac{q_{jk}^v}{q_{jk}}. \tag{2.17}$$

9. See Appendix B.2.2 for the derivation.

Equation (2.17) implies a one-to-one relationship between the share of the firm's intermediate inputs that is variable costs in the production and the elasticity of intermediate purchases in response to a demand-driven change in the firm's output.

Labor supply elasticity and demand-driven changes in a firm's output. The firm-specific labor supply elasticity governs how much the firm's employment increases if it raises the wage it is paying. Leveraging equation (2.6), we obtain the following relationship between the labor supply elasticity and the ratio of the average response of labor costs and employment to an increase in sales:

$$\sum_k \frac{d\log\left(\ell_k w_k\left(\ell_k\right)\right)}{d\log\left(p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)\right)}\Bigg/\sum_k \frac{d\log \ell_k}{d\log\left(p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)\right)}=\frac{\varepsilon+1}{\varepsilon}. \quad (2.18)$$

Equation (2.18) provides the basis for estimating the labor supply elasticity in Section 2.5. The labor supply elasticity also determines the firm's wage-setting power and its markdown of wages relative to the marginal revenue product of labor.

Worker rents. In our model, worker rents are due to the idiosyncratic taste component ν_{nk}, giving rise to upward sloping labor supply curves and employer wage-setting power. We assumed that employers do not observe the idiosyncratic taste for amenities of any given worker. This information asymmetry implies that firms cannot price-discriminate with respect to workers' reservation wages. As a result, the equilibrium allocation of workers to firms creates surpluses or rents for inframarginal workers, defined as the excess return over that required to change a decision, as in Rosen (1986).

Intuitively, the area above the labor supply curve constitutes the rents of workers. These rents represent the willingness-to-pay to stay at the current firm which, on average, is greater when the labor supply curve is steeper. Following **?** and Kroft et al. (2023), the additional rents of workers due to a demand-driven increase in sales can be expressed as:

$$\begin{aligned}\frac{d\left(\ell_k w_k\left(\ell_k\right)\right)}{1+\varepsilon} = & \underbrace{\ell_k dw_k\left(\ell_k\right)}_{\text{incumbent workers}} \\ & + \underbrace{\left(w_k\left(\ell_k\right)+dw_k\left(\ell_k\right)\right)\left(\frac{\ell_k+d\ell_k}{1+\varepsilon}-\ell_k\right)+\frac{\varepsilon}{1+\varepsilon}\ell_k w_k\left(\ell_k\right)}_{\text{new workers}}. \end{aligned} \tag{2.19}$$

The additional rents for incumbents is the wage change multiplied by the number of incumbent workers. The additional rents for new hires is the wage bill of new hires minus the wage bill required to make them indifferent between the new and initial firm choices.

2.4.3 Instrumenting the change in firms' sales

The comparative statics of the model consider a change in the sales of a firm due to an exogenous change in the demand it faces. In the data, however, sales may change for many reasons other than demand, including shifts in input prices, technology, preferences, or amenities. To address this identification challenge, we now construct an instrument that intends to isolate the variation in sales that is induced by a change in foreign demand.

To obtain plausibly exogenous variation in the exports of firm k, $d\log X_{nF,t}$, we follow Hummels et al. (2014) and Dhyne et al. (2021) and construct a measure of the change in the world import demand for this firm ($d\log X^{shock}_{kF,t}$):

$$d\log X^{shock}_{kF,t} = \sum_{c,p} r^{\mathrm{EX}}_{k,c,p,t-1} d\log WID_{c,p,t}, \tag{2.20}$$

where c denotes countries and p denotes products. We denote the lagged share of firm k's exports to country-product c,p in k's total exports with $r^{\mathrm{EX}}_{k,c,p,t-1}$. The term $WID_{c,p,t}$ represents country c's imports of product p from all other countries excluding Belgium. We define firm k's direct export shock on its total sales using the lagged export share,

$r_{kF,t-1} d \log X_{kF,t}^{shock}$. We further define the firm's total export shock, which includes its own direct export shock as well as takes into account the firm's indirect exposure to the direct export shocks through its buyers, as $\sum_n \tilde{H}_{kn,t-1} r_{nF,t-1} d \log X_{nF,t}^{shock}$. The term $\tilde{H}_{kn,t-1}$ captures the share of firm k's total sales that are purchased by firm n directly and indirectly through firm k's buyers and their buyers, and so on.[10]

Given this measure of total export shock, we can construct our first stage regression equation, which instruments the change in sales of a firm with its total export shock:

$$d \log X_{k,t} = \alpha + \beta_{Et} \sum_n \tilde{H}_{kn,t-1} r_{nH,t-1} + \beta \sum_n \tilde{H}_{kn,t-1} r_{nF,t-1} d \log X_{nF,t}^{shock} + \varphi_{k,t}. \quad (2.21)$$

The ultimate purpose of this equation is to isolate plausibly exogenous, demand-driven variation in sales that we can use to identify the elasticities derived out above. Unobserved time-invariant heterogeneity across firms (e.g., in technology, amenities or input prices) is eliminated by looking at changes over time within firms. To eliminate changes in sales that reflect shifts in domestic household demand, we control for $\sum_n \tilde{H}_{kn,t-1} r_{nH,t-1}$, where $r_{nH,t-1}$ is firm n's lagged revenue share to households. Furthermore, we include both industry-year fixed effects and firm fixed-effects as controls in our difference specification. As a result, we allow for differential time trends in the outcomes of interest across firms (linearly at the firm level and unrestricted at the industry level). The motivation for doing so is that the foreign demand shock to a firm might covary with underlying secular trends in or shocks to the firm's outcomes, such as in its use of factor inputs (e.g., because of changes to input prices facing the firm or from shifts in the workers' valuation of the firm's amenities). In addition to including this set of controls, we perform a battery of robustness checks, discussed in Section 2.5.3.

10. The term $\tilde{H}_{kn,t-1}$ is defined as the k, n element of matrix $\tilde{H}_{t-1}$. The matrix $\tilde{H}_{t-1}$ is defined as $(I - R_{t-1})^{-1}$, which the k, n element of matrix R_{t-1} is the share of revenue of firm k that is sold to firm n, $r_{kn,t-1}$.

2.5 Firm responses and worker impacts of foreign demand shocks

In this section, we quantify and explain the firm responses and worker impacts of changes in firm sales that are induced by changes in foreign demand. To do so, we will use the instrumental variables approach described in Section 2.4.3 to recover the elasticities defined by the comparative statics in Section 2.4.2. Armed with these elasticities, we then perform model based simulations to understand the direct and indirect effects of foreign demand shocks to production networks.

2.5.1 Graphical evidence

We begin our presentation of results with a graphical inspection of the IV approach described in Section 2.4.3. To do so, we plot a set of first-stage and reduced-form estimates that allow us to visually inspect the pre-trends in that data and the timing of the responses to the foreign demand shocks.

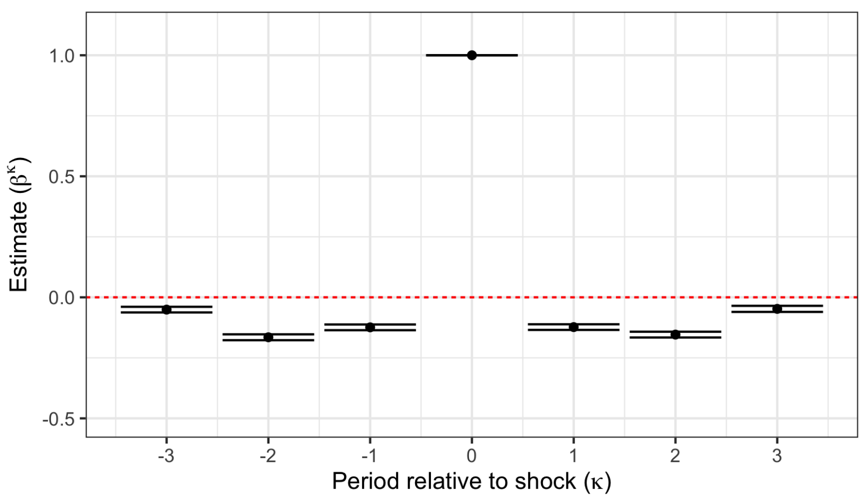
To make these plots, we run a series of first difference regressions. In these regressions, the dependent variable is the (percentage) change from year $t + \kappa - 1$ to year $t + \kappa$ in the outcome of firm k (denoted generically by $d \log W_{k,t+\kappa}$). The regressor of interest is our instrument, the (percentage) change from year $t - 1$ to t in the foreign demand the firm faces (as measured by the foreign demand shock, $\sum_n \tilde{H}_{kn,t-1} r_{nF,t-1} d \log X^{shock}_{nF,t}$). Our specification of the first difference regressions is given by

$$d \log W_{k,t+\kappa} = \alpha^{\kappa} + \beta^{\kappa}_{Et} \sum_n \tilde{H}_{kn,t-1} r_{nH,t-1} + \beta^{\kappa} \sum_n \tilde{H}_{kn,t-1} r_{nF,t-1} d \log X^{shock}_{nF,t} + \varphi^{\kappa}_{k,t}. \tag{2.22}$$

For each outcome variable we consider, we estimate this model separately for different choices of κ. The outcome is measured prior to the shock if $\kappa \in \{-3, -2, -1\}$, at the same time as the shock if $\kappa = 0$, and after the shock if $\kappa \in \{1, 2, 3\}$. As explained in Section 2.4.3, all specifications control for industry-year fixed effects, firm fixed effects, and shifts in domestic household demand as measured by the share of firm k's sales that is (directly or indirectly) sold to domestic households in the previous year, $\sum_i \tilde{H}_{ji,t-1} r_{iH,t-1}$.

Characterizing the foreign demand shock. The first set of outcome variables we consider is the past and future values of the foreign demand shock. In Figure 2.2, we report estimates (and 95 percent confidence intervals) of the coefficient β^κ for different choices of κ for this outcome.

Figure 2.2: Characterizing the foreign demand shock



Notes: This figure uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). We run seven firm-level regressions based on equation (2.22) for κ from -3 to 3 with total export shock as the outcome variable. Total export shocks are defined in Section 2.4.3. This figure shows the point estimates as well as 95 percent confidence intervals. Variables are winsorized at the top and bottom 0.5 percentiles. All specifications include industry-year fixed effects and firm fixed effects.

By construction, the estimate for $\kappa = 0$ is equal to one. The estimates for other values of κ can be small or large depending on the statistical dependence between the current foreign demand shock and past or future foreign demand shocks. The results suggest that foreign demand shocks to a firm are weakly correlated over time. A firm that currently experiences a demand shock is not much more likely to have experienced a demand shock in the past or experience a demand shock in the future. In other words, the changes in the foreign demand shocks over time are reasonably consistent with a unit root process. This suggests that we can infer lagged responses from regressions of future outcomes on current foreign demand shocks.

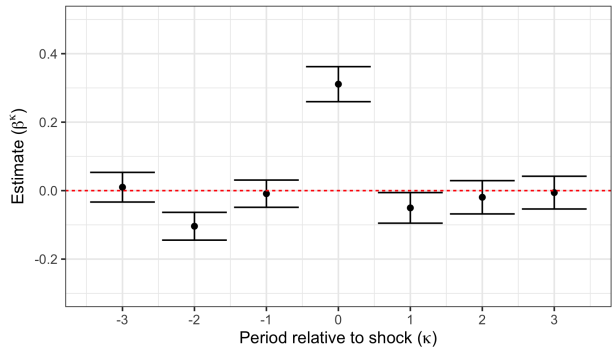
Examining the first-stage and reduced-form estimates. In Figure 2.3, we examine how the shocks to foreign demand affect a wide range of outcomes. For each outcome variable, this is done by plotting the estimate (and 95 percent confidence intervals) of the coefficient β^{κ} for different choices of κ. In panel (a), we focus on the outcome variable in the first stage regression of our IV model: total sales (which include sales to foreign markets as well as sales to domestic firms and households). This panel displays how past, current, and future total sales statistically depend on current foreign demand shocks.

We find that an increase in the foreign demand facing a firm leads to an instantaneous, sharp and fairly persistent increase in its total annual sales. The point estimates suggest that if foreign demand increases by 10 percent, the firm's sales are expected to instantaneously (i.e., for $\kappa = 0$) increase by 3.1 percent. Over time, the yearly increases in sales decline modestly. As of the fourth year after this shock (i.e., for $\kappa = 3$), the cumulative increase in sales is approximately 2.4 percent. Thus, the cumulative or total impact on sales is about three-quarters of the immediate effect. If, for example, a foreign demand shock leads to a 10 percent instantaneous increase in sales, the firm's cumulative sales are expected to increase by 7.6 percent.

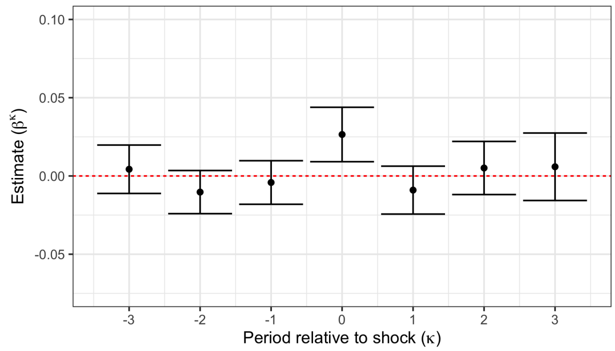
In panels (b)-(f) of Figure 2.3, we shift attention to the reduced-form estimates of the IV model. For all outcomes, we find significant instantaneous impacts of the foreign demand shocks. The delayed or lagged responses, however, vary across outcomes. Our estimates suggest that both the wage a firm pays and its use of intermediate inputs (from either domestic sources or from abroad) increase instantaneously and persistently in response to a foreign demand shock. By comparison, employment and labor cost increase gradually over

Figure 2.3: Examining the first stage and reduced form of the IV model

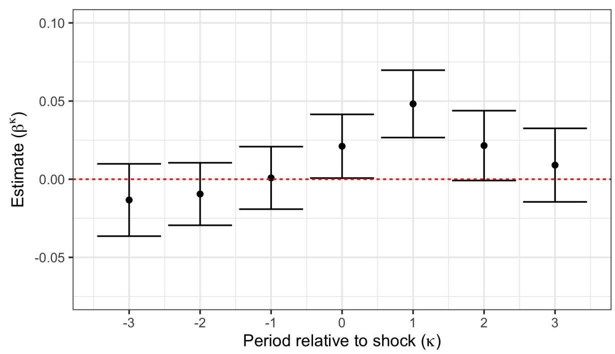
(a) LHS: total sales



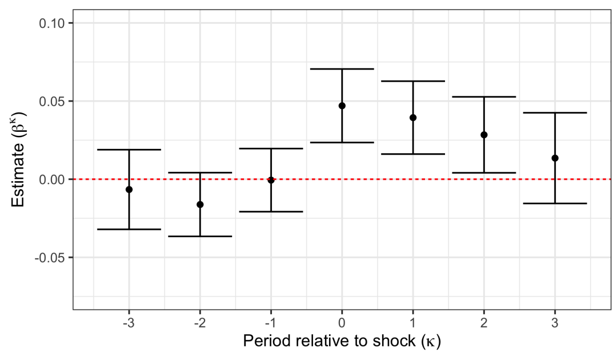
(b) LHS: average wage



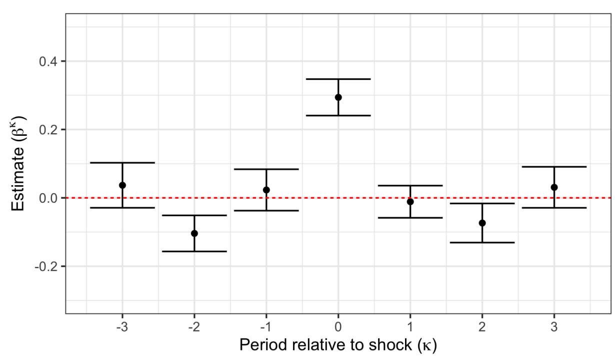
(c) LHS: FTE employment



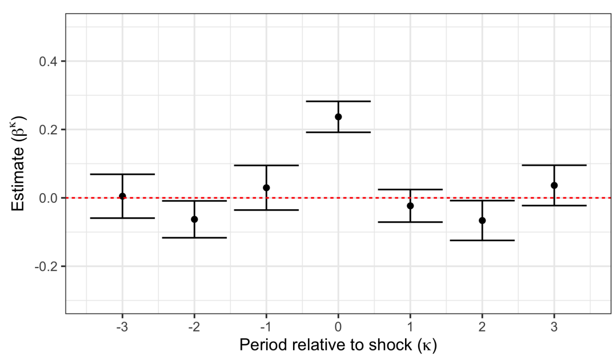
(d) LHS: labor cost



(e) LHS: input purchases



(f) LHS: domestic input purchases



Notes: The figures use the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). For each panel, we run seven firm-level regressions based on equation (2.22) for κ from -3 to 3 and report the responses of the outcome variable to the total export shock defined in Section 2.4.3. Each panel shows the point estimates as well as 95 percent confidence intervals. Variables are winsorized at the top and bottom 0.5 percentiles. All specifications include industry-year fixed effects and firm fixed effects.

time, consistent with some form of adjustment costs. The yearly increases in these variables decline over time, and there is little if any additional growth as of the fourth year after the shock. Guided by these estimates, we will be measuring the total responses to the foreign demand shocks by cumulating the impacts over time for κ equal to 0 to 3.

2.5.2 IV estimates

In Table 2.3, we present the IV estimates of the impacts of the changes in firm sales that are induced by the foreign demand shocks. The first-stage regression of the IV model is given by equation (2.21). Using the same notation, we can express the second stage as a regression in first differences of the outcome variable of interest, $d\log W_{k,t+\kappa}$, on the total sales of the firm, $d\log X_{k,t}$:

$$d\log W_{k,t+\kappa} = \tilde{\alpha}^{\kappa} + \gamma^{\kappa}_{Et}\sum_{n}\tilde{H}_{kn,t-1}r_{nH,t-1} + \gamma^{\kappa} d\log X_{k,t} + \tilde{\varphi}^{\kappa}_{k,t}. \qquad (2.23)$$

For each outcome we consider, we estimate the IV model by two-stage least squares regressions separately for each $\kappa \in \{0,1,2,3\}$. We report both the instantaneous response, γ^0, and the cumulative response, the sum of γ^0, γ^1, γ^2, and γ^3. As explained in Section 2.4.3, all specifications control for industry-year fixed effects, firm fixed effects, and shifts in domestic household demand as measured by the share of firm k's sales that is (directly or indirectly) sold to domestic households in the previous year, $\sum_i \tilde{H}_{ji,t-1}r_{iH,t-1}$.

Worker impacts, labor supply elasticity, and fixed overhead costs in labor inputs. The first two columns of Table 2.3 report the impacts on wages and employment of the increase in sales that is induced by the foreign demand shocks. The point estimates suggest that if sales increase by 10 percent, the firm's average wage and employment are expected to instantaneously (i.e., for $\kappa = 0$) increase by 0.9 percent and 0.7 percent, respectively. Over time, the employment effects cumulate. As of the fourth year after the demand shock (i.e.,

Table 2.3: IV estimates of the impact of changes in firm sales that are induced by the foreign demand shocks

	(1) Average wage	(2) FTE Employment	(3) Labor cost	(4) Input purchases	(5) Domestic input purchases
Instantaneous response	0.0868***	0.0679**	0.153***	0.946***	0.764***
(γ^0)	(0.0296)	(0.0322)	(0.0368)	(0.0770)	(0.0672)
Cumulative response	0.0915	0.320***	0.412***	0.781***	0.599***
($\sum_{\kappa=0}^{3}\gamma^{\kappa}$)	(0.0571)	(0.0273)	(0.0698)	(0.120)	(0.104)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). For each outcome variable, we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (2.23) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (γ^0) as well as the cumulative response (the sum of four coefficients $\{\gamma^{\kappa}\}_{\kappa=0}^{3}$). The first-stage F-statistics for excluded instruments is 142.3. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

for $\kappa = 3$), the cumulative impacts on wages and employment are about 0.9 percent and 3.2 percent, respectively.

The evidence that a foreign-demand-driven increase in sales causes the firm to bid up wages to hire more workers is at odds with the textbook model in which the labor supply curve facing the firm is perfectly elastic. Instead, it is consistent with the notion that firms face upward-sloping labor supply curves and, therefore, have wage-setting power in the labor market. Indeed, as shown formally in equation (2.18), we can recover the slope of the firm-specific labor supply curve, and thus the degree of imperfect competition in the labor market, from the employment and wage impacts of the foreign-demand-driven increase in sales.

This identification argument, however, requires that firms are shifted along their labor supply curve. A possible threat to identification is adjustment costs. Since our findings suggest that labor enters the firm slowly over time rather than immediately when the new wage is posted, the instantaneous impacts on wages and employment will understate the labor supply elasticity. Thus, we look at the cumulative responses to infer the labor supply elasticity. The 0.9 percent cumulative increase in wages relative to a 3.2 percent cumulative increase in employment is consistent with firms facing a labor supply elasticity of about 3.5. This labor supply elasticity suggests that wages in the Belgian economy are marked down

22 percent relative to the marginal revenue product of labor.

In the above estimation, we consider all the workers in our sample. However, we could also look at the incumbent workers who stay in the same firm. An advantage of doing so is that it keeps the composition of the workforce fixed, and thus, we are not confounding increases in the wages paid to a given worker with changes in the quality of the workers. In the first column of Table 2.4, we report the IV estimate on wages for the sample of stayers (who stay in the same firm before and after the demand shock, from $\kappa = -1$ to $\kappa = 3$). This sample is based on the subset of firms for which we have additional information from the worker data (see Section 2.2.3 for details). We find that their wages increase by 1.1 percent in response to a 10 percent increase in sales, which is similar to the impact for all workers.

Table 2.4: IV estimates on the wages and work rate of stayers

	(1) Stayer wage	(2) Stayer hourly wage	(3) Stayer work rate
Cumulative response	0.1170***	0.1093***	0.0091
$(\sum_{\kappa=0}^{3} \gamma^{\kappa})$	(0.0322)	(0.0317)	(0.0131)
Ind.-Year FE	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes

Notes: This table uses the subsample of firms for which we have additional information from the worker data (see Section 2.2.3 for details). For each outcome variable, we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (2.23) for $\kappa \in \{0, 1, 2, 3\}$ and report the cumulative response (the sum of four coefficients $\{\gamma^{\kappa}\}_{\kappa=0}^{3}$). The first-stage F-statistics for excluded instruments is 81.8. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level and computed using the bootstrap method. We compute firm-level average stayer wage, stayer hourly wage, and stayer work rate based on the balanced panel of stayers from $t-1$ to $t+3$. The analysis is based on 452,025 worker-year observations of stayers, which yield 75,849 firm-year observations of private-sector firms in Belgium from 2003 to 2014. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Another advantage of the stayers sample is that it allows us to examine if the increase in our measure of FTE employment reflects the hiring of new workers or an increase in the working hours of incumbent workers. To do so, we report, in Table 2.4, the IV estimates on hours of work (as measured as the share of full-time employment) and on hourly wages for the sample of stayers. The estimated impact on hours of work is close to zero, whereas the effect on hourly wages is close to what we find for all workers. Taken together, these findings suggest that Belgian firms mostly adjust to demand shocks by hiring additional workers, not by increasing incumbent workers' hours of work.

The evidence that both wages and employment increase notably in response to the increase in sales that is induced by the foreign demand shock implies that labor costs also go up. The third column of Table 2.3 reports the estimated impacts on labor costs. These estimates suggest that if sales increase by 10 percent, the firm's labor costs are expected to instantaneously (i.e., for $\kappa = 0$) increase by 1.5 percent. Over time, labor costs continue to grow as the employment effects cumulate. As of the fourth year after the demand shock (i.e., for $\kappa = 3$), the cumulative impact on labor costs reaches 4.1 percent.

As for the labor supply elasticity, we focus on the cumulative impacts to infer the elasticity of labor costs with respect to a demand-driven change in sales. The reason for doing so is that our comparative statics assume that firms are shifted along their labor demand curve. By comparing the cumulative impacts on total labor costs versus sales, we find an implied elasticity of labor costs of approximately 0.54.

As shown in equation (2.16), the elasticity of labor costs is informative about the presence of fixed overhead costs in labor inputs. In the absence of such costs, the elasticity would be equal to one in our model. If all labor is fixed, on the other hand, the elasticity would be zero. Our finding of a labor costs elasticity of 0.54 is therefore evidence of non-zero fixed overhead costs in labor inputs. However, it is not straightforward to quantify the magnitude of the fixed labor input share. Because of upward sloping labor supply curves, the relationship between the labor cost elasticity and the fixed shares of labor inputs is nonlinear. Thus, even if we know the average elasticity of labor costs, we do not necessarily know the share of labor costs that is fixed (on average or for any given firm).

This issue is resolved if one is willing to assume that the fixed share of labor inputs is homogeneous across firms. Under this restriction, we can plug our estimate of the labor cost elasticity into equation (2.16) and solve for the fixed share of labor inputs. We then find that 52 percent of the firm's labor is fixed. In Appendix B.3.2, we let the fixed shares of labor inputs vary across different types of firms, such as exporters versus non-exporters and manufacturing firms versus non-manufacturing firms. We find that the weighted averages of the fixed shares of labor inputs do not differ materially from the estimate under the restriction that the fixed shares of labor inputs are homogeneous across all firms.

Another special case in which one is easily able to recover the fixed share of labor cost is if the labor supply is perfectly elastic (i.e., $\varepsilon \to \infty$). In this case, the share of labor cost that, on average, is fixed would be pinned down directly by the estimated labor cost elasticity. If one imposes this restriction, which is admittedly at odds with the data, we would conclude that, on average, the fixed share of labor inputs is about 46 percent. This illustrates that the nonlinearities that arise because of the upward-sloping labor supply curves do not matter greatly for the estimates of the fixed shares of labor inputs.

Input purchases and fixed overhead costs in intermediate inputs. The fourth and fifth columns of Table 2.3 report the impacts on input purchases of the increase in sales that is induced by the foreign demand shocks. The estimates suggest that if sales increase by 10 percent, the firm's total (domestic) input purchases are expected to instantaneously (i.e., for $\kappa = 0$) increase by about 9.5 (7.6) percent. By comparison, the cumulative impact on total input purchases, domestic input purchases, and total sales is around 7.8 percent, 6 percent, and 7.6 percent, respectively. These findings suggest that firms pass on a large share of their foreign demand shocks to their domestic suppliers.

As shown in equation (2.17), the change in input purchases due to a demand-driven change in sales allows us to draw inferences about both the presence and importance of fixed overhead costs in intermediate inputs. As for labor inputs, we focus on the cumulative impacts to infer the elasticity of input purchases with respect to a demand-driven change in sales. The reason for doing so is that our comparative statics assume that firms are shifted along their input demand curve. By comparing the cumulative impacts on total input purchases versus sales, we find that the implied elasticity of total input purchases is approximately equal to one. In other words, the fixed share of intermediate inputs is roughly equal to zero.

As shown in Figure B.2 in Appendix B.3.3, the elasticities of input purchases, and thus, the fixed shares of intermediate inputs, vary systematically by the type of input. To reach this conclusion, we estimate the elasticities of (domestic and foreign) input purchases separately by the industry (as measured by the one-digit NACE code) of the firms' suppliers. Close to half of all input purchases in the Belgian economy come from the manufacturing industry (including imported manufactured goods). We find that purchases from this industry increase by nearly as much (7.5 percent) as the cumulative increase in total sales (7.6 percent). This suggests that the elasticity of input purchases from this industry is close to one. Thus, we conclude that inputs from the manufacturing industry are predominately variable and can be adjusted in response to demand shocks.

In contrast, Figure B.2 shows that input purchases from the service industry (a combination of NACE G to N one-digit sectors) have a larger share of fixed inputs. We compute the size-weighted average of the cumulative responses of these service inputs and find that purchases from this industry increase by around 4.8 percent. This implies that these service inputs—which supply around 30 percent of total input purchases in Belgium—have a fixed input cost share of 37 percent.

When interpreting our estimates of the fixed costs shares, it is useful to observe that the comparative statics in Section 2.4 hold the markups fixed. If one instead assumes that firms increase the markup on the goods that they sell in response to an increase in foreign demand, the elasticities of labor cost and input purchases would be lower than one even in the absence of fixed costs. In this case, however, all elasticities would be uniformly lower than one, and there should be no differences between the elasticities of labor cost and total input purchases or between different types of input purchases. In contrast, we find a labor cost elasticity of 0.54, a total input purchase elasticity that is roughly equal to one, and systematic heterogeneity in the elasticities of input purchases by supplier industry.

Comparison with existing evidence. Our estimate of the labor supply elasticity is broadly comparable to existing work from countries other than Belgium. Card et al. (2018) review this work and pick 4.0 as the preferred value in their calibration exercise. More recent evidence includes **?**, who estimate an average labor supply elasticity in the U.S. economy of 4.6, and Kroft et al. (2023), who estimate a labor supply elasticity in the U.S. construction sector of 4.2. By comparison, Huneeus et al. (2022) find labor supply elasticities that range from around 3 to 6 in Chile, and Chan et al. (2023) estimate the elasticity to be 5.7 in Denmark.

We are not aware of previous studies with directly comparable estimates of the fixed shares of labor or intermediate inputs. The closest comparison is arguably the results in Loecker et al. (2020). Using data on American firms (from Compustat), they compute the share of total costs that is fixed. They measure fixed costs as the reported spending in the category "Selling, General and Administrative Expenses." They conclude that the share of total costs that are fixed ranges from 18 to 22 percent during the period 2000-2016. By comparison, our estimates suggest that around 29 percent of total costs (including labor and intermediate inputs) are fixed in the Belgian economy.

2.5.3 Specification checks

We consider several alternative specifications to evaluate the sensitivity of the above estimates. These results are presented in Appendix B.3.4.

One possible concern with our baseline specification is that foreign demand shocks might be correlated with regional shocks that may directly affect both sales and input costs. To address this concern, we add location-year fixed effects to both the first stage and the reduced form equations. Location is measured using level 2 of the Eurostat NUTS classification. We find that the IV estimates relative to the cumulative increase in sales barely change when we include these controls.

A related concern is that foreign demand shocks might be correlated with changes in import prices that may directly affect both sales and input costs. We test this empirically, finding that the correlation between firm-level import price changes and the firm-level foreign demand change, $\sum_n \tilde{H}_{kn,t-1} r_{nF,t-1} d \log X^{shock}_{nF,t}$, is close to zero.

Another possible concern is that the differences between the instantaneous and cumulative responses may be confounded by non-random attrition of firms over time. To address this concern, we restrict our estimation sample to a balanced panel of firms that are observed for at least seven consecutive years (from κ equal to -3 to 3). It is reassuring to find that our IV estimates relative to the cumulative increase in sales are not substantially affected by this sample restriction.

We also conduct a robustness check that investigates the sensitivity of our results to weighting each firm by the level of employment (as measured in year $\kappa = -1$). By doing so, we assign more weights to larger firms. The IV estimates relative to the cumulative increase in sales do not materially change when we use these weights.

2.5.4 *Direct and indirect effects of foreign demand shocks to production networks*

The findings above suggest that Belgian firms face upward-sloping labor supply curves and sizable fixed overhead costs in labor inputs. We now perform a simulation to explore the implications of these findings for conclusions about firm responses and worker impacts of foreign demand shocks to direct exporters and their domestic suppliers. Our simulation considers a foreign demand shock that increases the export of a direct exporter by 10 percent.

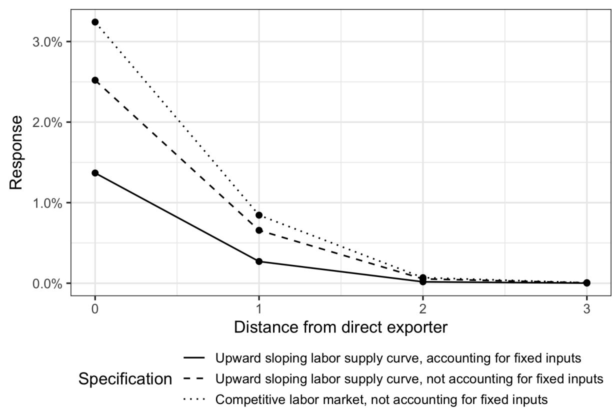
We draw 100 direct exporters from our estimation sample of firms in 2012 and then average the results from our simulation across these draws. In the simulation, we use a labor supply elasticity of 3.5, a fixed share of labor costs of 52 percent, and the input-specific fixed share of input costs (estimated in Appendix B.3.3).

The first step in the simulation is to use the elasticities of employment and wages with respect to sales to predict the employment and wage responses of a direct exporter to the demand shock. The next step is to simulate the spillover effects of the foreign demand shock to the domestic suppliers of the direct exporters. Most of these direct exporters have many suppliers. Instead of looking at the spillover or indirect effects to each of these suppliers, we focus on the supplier that sells most of its output to the direct exporter. For this key supplier, we simulate the employment and wage responses to the foreign demand shock by combining our estimates of the input purchase elasticities with data on the share of sales to the direct exporter. In the same way, we continue the simulation of the spillover effects through the production network by predicting the employment and wage responses of the key supplier's key supplier, and so on.

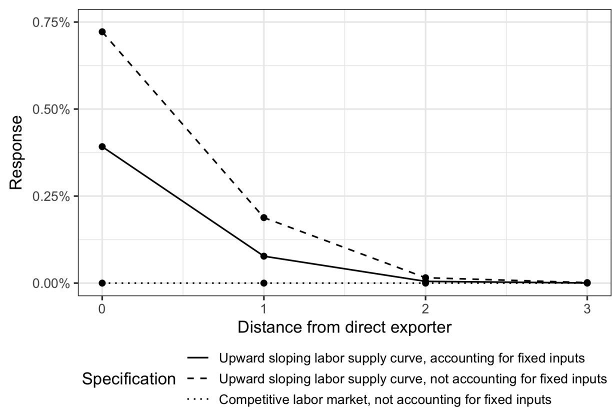
The results from this simulation exercise for employment and wages are reported in the solid lines of panels (a) and (b) of Figure 2.4, respectively. We find that, on average, a direct exporter raises employment by 1.37 percent and the wages it pays by 0.39 percent in response to a foreign demand shock that increases the firm's direct export by 10 percent. As evident in Figure 2.4, this shock cascades through the production network as the direct exporters buy more inputs both directly from their own suppliers and indirectly from the suppliers' (direct and indirect) suppliers. These indirect demand effects increase, on average, the employment and wages of the direct exporter's key supplier by 0.3 and 0.08 percent, respectively. In other words, the key supplier experiences one-fifth of the percentage increases in wages and employment of the direct exporter. These indirect demand effects decay quickly with the distance to direct exporters in the supply chain. In fact, the foreign demand shock has little if any impact on the employment and wages of the key supplier's key supplier.

Figure 2.4: Simulation results of foreign demand shock transmission along the supply chain

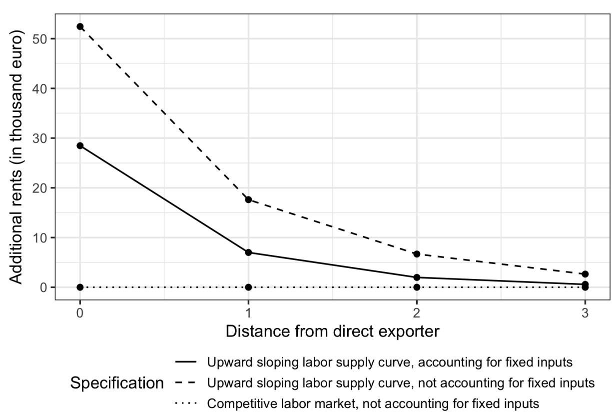
(a) Response of FTE employment



(b) Response of average wage



(c) Additional worker rents



Notes: For each panel, we report the simulation results of the transmission of foreign demand shocks along the supply chain (see the discussion in the text for how the simulation is done). The first two panels present the employment and wage response at the direct exporter, the direct exporter's key supplier, the key supplier of the exporter's key supplier, and so on. The bottom panel aggregates the rents to the workers in firms that direct export, to workers in their direct suppliers, to workers in their suppliers' suppliers, and so on (up to three links). In each line of every figure, we make different assumptions regarding the fixed shares of labor inputs. For the solid lines, we use our estimated labor supply elasticity $\varepsilon = 3.5$ as well as the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level); for the stippled lines, we assume that all inputs are variable; and for the dotted lines, we further assume that the labor market is perfectly competitive with $\varepsilon = \infty$.

An implication of the direct and indirect effects on wages and employment is that workers in the production network will capture rents as a result of the foreign demand shock. As evident from equation (2.19), these rents depend on the labor supply curve it faces and the increase in labor costs in response to the demand shock. We use our estimates of these quantities to compute the worker rents induced by the demand shock, capturing incumbent and new workers both in the firm that directly exports and in its direct and indirect suppliers. In total, workers earn 38,000 euro in additional rents as a result of a foreign demand shock that increases the firm's direct export by 10 percent. Panel (c) of Figure 2.4 shows how these rents are shared among the workers in the production network. On average, the workers in the directly exporting firm get 75 percent of the total rents (28,000 euro). The remaining rents are shared among the other workers in the production network, with most of it (18 percent of the total additional rents) going to the workers of the direct suppliers.

To help interpret the magnitudes and mechanisms of the direct and indirect effects we redo the simulations under the assumptions of a perfectly competitive labor market and no fixed overhead costs. The results from this simulation are shown by the dotted lines in the three panels of Figure 2.4. In each of these panels, we also report results from a simulation with imperfect competition in the labor market (same labor supply elasticity as we estimate) but no fixed overhead costs. These results are shown by the stippled lines. We find that the employment responses of Belgian firms to foreign demand shocks decrease by nearly half because of the upward sloping labor supply curves and the fixed overhead costs. Most of this reduction can be attributed to the fixed costs. In the absence of such costs, the increase in wages would be 84 percent larger for the direct exporters relative to what we find with fixed overhead costs and upward-sloping labor supply curves. As a result, workers would earn more rents if there were no fixed costs, but the sharing of the rents would be only marginally affected as the fixed costs do not vary much across direct exporters and their suppliers.

A potential concern with the simulations discussed above is that they assume that the direct exporters and their suppliers do not face different labor supply curves or fixed overhead costs. To address this concern, we re-estimate the labor supply curves and the fixed overhead cost shares in labor inputs separately for the direct exporters and for the firms that do not directly export. Interestingly, the labor supply curves and the fixed cost shares do not materially differ between the firms that do and do not directly export. Consistent with this finding, we show in Appendix B.3.5 that both the estimates of direct and indirect effects and the results on the (sharing of) the additional rents are robust to allowing for heterogeneity in the labor supply curves and the fixed shares of labor inputs.

Another potential concern with the simulations is that they assume the fixed labor cost share is homogeneous across all firms in the economy. To address this concern, we estimate both the labor supply elasticity and the labor cost elasticity separately for manufacturing and non-manufacturing firms. These separate estimates allow us to compute industry-specific fixed labor cost shares (under the assumption that the fixed labor cost shares do not vary across firms within these industries). As shown in Appendix B.3.5, the simulation results do not materially change when we allow the fixed labor cost shares to vary across industries.

2.6 Aggregate implications of the foreign demand shocks

The simulation results in the previous section point to the importance of production networks, upward-sloping labor supply curves, and fixed overhead costs to accurately predict how firms respond to and workers are affected by foreign demand shocks. However, this simulation relies on several simplifying assumptions. In particular, it considers a demand shock to a single (randomly drawn) direct exporter, and it abstracts from general equilbrium effects and the need to adjust imports if exports fall. We now relax these assumptions and consider the aggregate effects of foreign demand shocks to a production network. The primary goal of this analysis is to quantify how changes in foreign demand propagate through a small open economy and affect firms and workers.

Throughout this section, the foreign demand shock we consider is a five percent increase in foreign tariffs on all Belgian exports. We implement this shock by expressing it as a change in the foreign demand shifters. Let $\hat{x}$ denote the change in variable x, defined as the ratio of the post-shock value x' over the pre-shock value x. Given this definition, a uniform foreign demand shock of $\hat{D}_{kF} = 1.05^{-\sigma}$ is approximately equal to an 18 percent decline in the foreign demand shifters.

2.6.1 Counterfactual economies

We consider several counterfactual economies, defined by alternative parameterizations of the model outlined in Section 2.4.

One of these economies is constructed to mirror the actual Belgian economy. We then use the estimates we obtained in Section 2.5, including the estimated value of the labor supply curve ($\varepsilon = 3.5$) and the estimated values of the fixed cost shares in labor inputs and input purchases. For the fixed cost shares of input purchases, we use the estimates for each NACE one-digit industry obtained in Appendix B.3.3. One of the alternative counterfactual economies is constructed by shutting down the fixed overhead costs. This is done by imposing the restriction $\bar{\ell}_k^f = \bar{q}_k^f = 0$, so all the inputs of firms are variable inputs, as is standard in previous work. In another counterfactual economy, we allow for fixed costs but shut down imperfect competition in the labor market. This economy is constructed by setting $\varepsilon = \infty$ so that the labor supply is perfectly elastic. The last counterfactual economy we consider has a perfectly competitive labor market and no fixed costs.

2.6.2 Parameterization and solution of the model

For each of the counterfactual economies, it is necessary to parameterize the model in order to calculate key variables and predict the impacts of the foreign demand shocks. Prior work on trade and production networks highlight the importance of holding key variables fixed to meaningfully compare results across counterfactual economies (see, e.g., Baqaee and Farhi, 2024). We therefore impose the restriction that certain firm-level observables (i.e., firms' total labor costs, imports, exports, and purchases from and sales to other domestic firms) are identical across the alternative parameterizations of the model and equal to what we observe in the data (in 2012, our reference year throughout this section).

To rationalize that different parameterizations of the model are producing identical firm-level observables, we let certain parameters vary across the counterfactual economies, including the firm-level productivity parameters, ϕ_k, the technology parameters, ω^v_{jk}, ω^v_{Fk}, ω^f_{jk}, and ω^f_{Fk}, and the workers' preference parameters, ν_{nk}. For each counterfactual economy, these parameters are assumed to be invariant to the foreign demand shock. With this assumption, we can solve for the counterfactual changes without recovering these underlying parameters by implementing the technique developed by Dekle et al. (2007). We solve for the counterfactual outcomes using the system of equations described in Appendix B.2.3.

We further parameterize the model as follows. We calculate the Belgian trade balance as the difference between exports and imports in our reference year, 2012. We hold the trade balance fixed throughout the counterfactual analyses. We set $\sigma = 4$, a common choice in the prior literature (see, e.g., Antràs et al., 2017, Oberfield and Raval, 2021, and Dhyne et al., 2021). This choice implies that firms charge a common markup of $\frac{\sigma}{\sigma-1} = 1.33$ over marginal cost.

As shown in Appendix B.4.1, our parameterization of the model may create firm-level discrepancies between the theory-implied variable input costs, $\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k$, and the observed variable input costs, $w_k\ell^v_k + \sum_j p_j q^v_{jk} + p_{Fk}q^v_{Fk}$. A possible reason for these discrepancies is the firms' usage of inventories, which we do not model. To deal with these discrepancies, we follow Dhyne et al. (2022b) in imposing that the firm-level ratios of the theory-implied variable input cost over the observed variable input costs are invariant to the foreign demand shock. A natural interpretation of this assumption is that the amount of inventory the firm uses (or accumulates) relative to its inputs and sales does not change in response to the foreign demand shocks.[11]

11. In Appendix B.4.1, we show that this assumption is isomorphic to assuming that firm k charges a firm-specific markup of $\frac{p_k q_k}{w_k\ell^v_k + \sum_j p_j q^v_{jk} + p_{Fk}q^v_{Fk}}$.

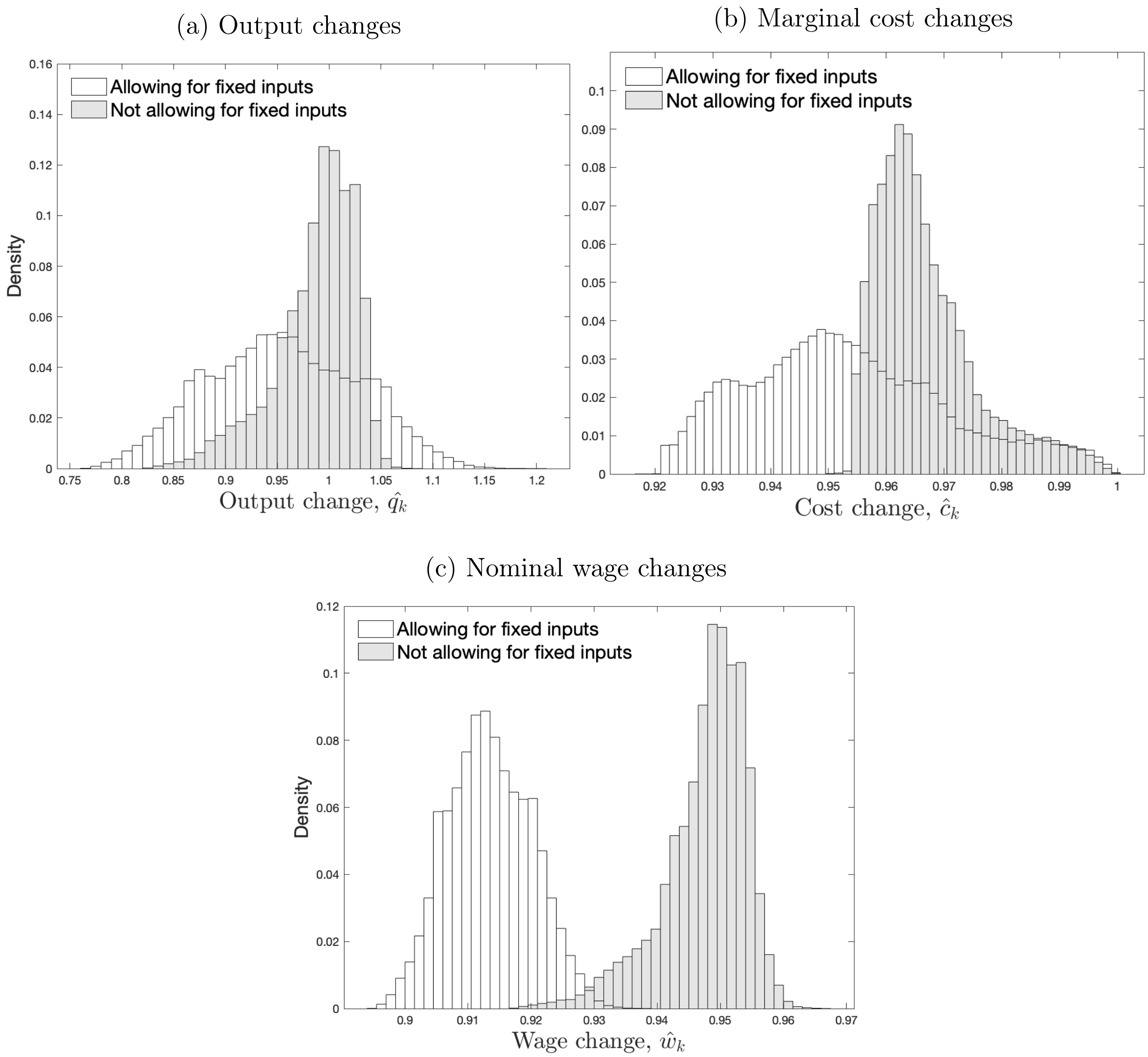
2.6.3 Impacts of foreign demand shocks in the actual economy

We begin by analyzing the impacts of the foreign demand shocks in our representation of the actual Belgian economy with firms that face fixed overhead costs and upward-sloping labor supply curves. The results are presented in the white bars in Figure 2.5. This figure shows the firm-level distributions of changes in output (panel (a)), marginal costs (panel (b)), and wages (panel (c)).

We find that most but not all firms decrease their output in response to the foreign demand shocks. The median firm reduces its output by 4.7 percent. The marginal costs of firms fall, with the median firm experiencing a cost reduction of 4.8 percent. Marginal costs fall primarily because the cost of labor decreases significantly for all firms. The median firm reduces its wage by 8.8 percent.

To interpret these results, it is important to observe that the foreign demand shocks have both direct and indirect effects. A reduction in foreign demand would directly reduce the output of the firms that sell directly or indirectly to foreign markets. This translates to a reduction in demand for both labor and intermediate inputs, thereby lowering the prices on these factors. Indirect equilibrium effects also influence how firms respond to and workers are affected by the foreign demand shock. Importantly, imports have to be reduced to ensure trade balance. The foreign price is exogenous and fixed, and the wage is the only price that can move to ensure trade balance. This decline in wages contributes to reducing marginal costs, increasing labor, and raising output, especially among firms that rely heavily on labor (instead of foreign inputs) in their production. Indeed, some of these firms experience an overall increase in output as a result of the foreign demand shock, as evident from the right tail of the distribution in panel (a) of Figure 2.5.

Figure 2.5: Firm-level distribution of changes in output, marginal costs, and wages in response to a 5 percent increase in foreign tariffs, with and without fixed inputs



Notes: The three panels in this figure show the distribution of the changes in firm-level variables due to a uniform 5 percent increase in foreign tariffs on Belgian exports. Panel (a) shows the distribution of firm-level output changes, $\hat{q}_k$, panel (b) shows the distribution of firm-level marginal cost changes, $\hat{c}_k$, and panel (c) shows the distribution of firm-level nominal wage changes, $\hat{w}_k$. In all panels, the white bars represent the distributions when one allows for fixed inputs, and the grey bars represent the distributions when one does not allow for fixed inputs. In this figure, we allow for imperfect competition in the labor market, $\varepsilon = 3.5$.

2.6.4 How fixed costs affect the impacts of foreign demand shocks

We now shift attention to how fixed overhead costs affect the propagation and implications of foreign demand shocks. To do so, Figure 2.5 compares the impacts of these shocks in our representation of the actual Belgian economy (white bars) to those we obtain in the counterfactual economy where firms face upward-sloping labor supply curves but no fixed costs in labor or intermediate goods (grey bars).

The results suggest that fixed overhead costs lead to foreign demand shocks having larger and more dispersed impacts on both output and marginal cost. In the economy without fixed overhead costs, the median firm reduces output by 0.5 percent and marginal costs by 3.5 percent. By comparison, output and marginal costs decline by 4.7 and 4.8 percent if one incorporates the fixed overhead costs. These differences are mirrored in the changes in nominal wages. Shutting down fixed costs attenuates the decline in the nominal wage of the median firms by almost 4 percentage points (from 8.8 percent in the economy with fixed costs to 5.1 percent in the economy with no fixed costs).

Fixed overhead costs amplify the consequences of foreign demand shocks because they make foreign inputs (labor) relatively more (less) important for total variable costs. To understand why, recall that the Lerner Symmetry Theorem establishes the equivalence between any intervention that increases the cost of import and export by the same amount. Thus, the five percent increase in foreign tariffs is equivalent to a five percent increase in tax on imports. An increase in the cost of imports has a larger impact on the firms' total variable costs, and, in turn, on output and real wages, if the economy has sizable fixed overhead costs in labor while imported inputs are predominately variable costs. Empirically, we show, in Figure B.4 of Appendix B.4.2, that (direct and indirect) imports make up a considerably larger share of the total variable costs when we take our estimates of fixed overhead costs into account.

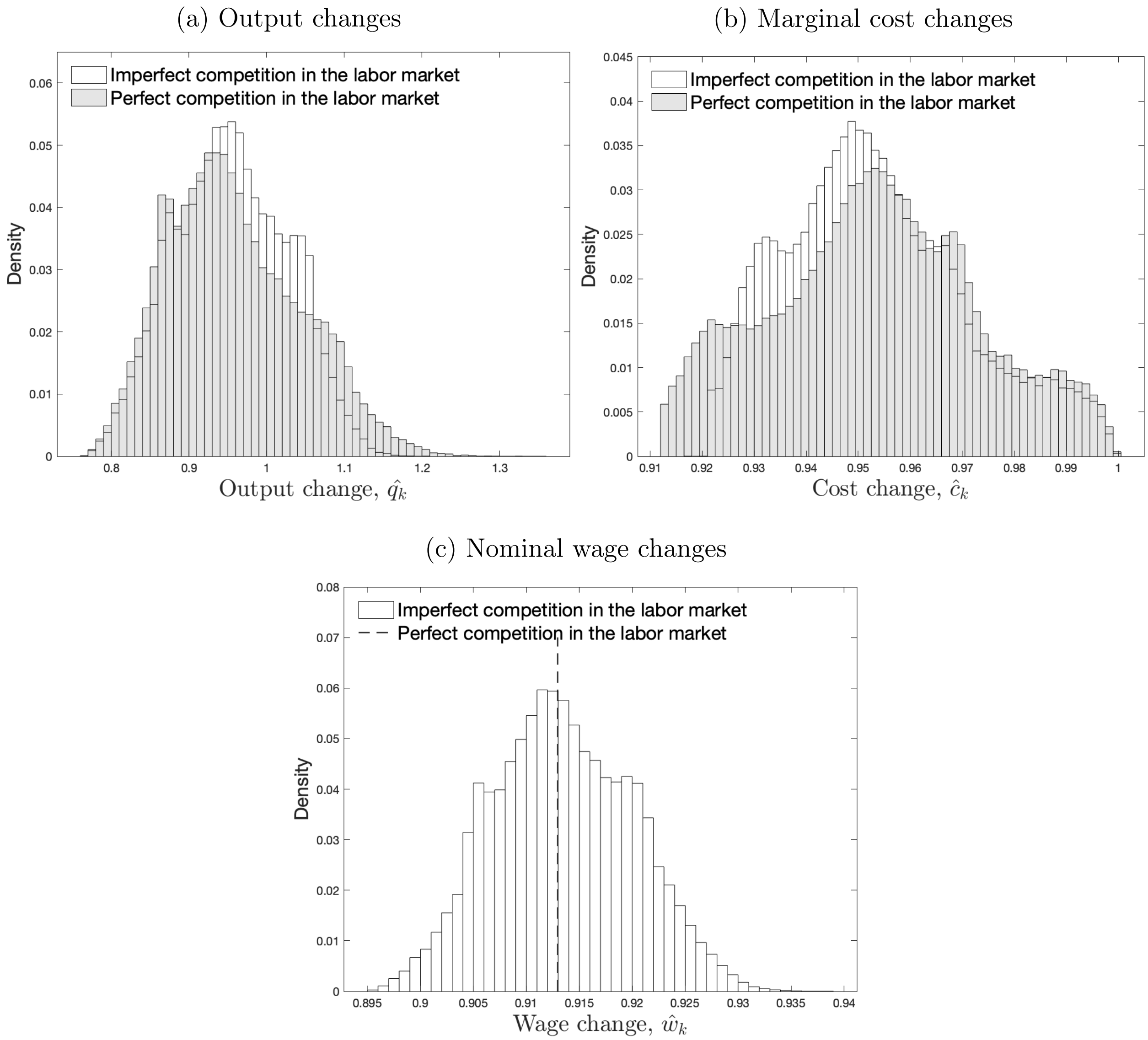
2.6.5 How imperfect competition in the labor market affects the impacts of foreign demand shocks

We now turn to an analysis of how imperfect competition in the labor market affects the propagation and implications of foreign demand shocks. To do so, Figure 2.6 compares the impacts of these shocks in our representation of the actual Belgian economy (white bars) to those we obtain in the counterfactual economy where firms face fixed costs in labor and intermediate goods and perfectly elastic labor supply curves (grey bars). This figure shows the firm-level distributions of changes in output (panel (a)), marginal costs (panel (b)), and nominal wages (panel (c)).

The results suggest that imperfect competition in the labor market does not greatly change the impacts of the foreign demand shocks on output or marginal costs. These findings are mirrored in the changes in nominal wages. In the economy without imperfect competition in the labor market, the five percent increase in foreign tariffs on Belgian exports produces an 8.7 percent decline in the nominal wages of all firms. By comparison, nominal wages decline by 8.8 percent for the median firm if one incorporates that firms face upward-sloping labor supply curves. Taken together, these findings suggest that the essential feature to accurately predict the impacts of the foreign tariff is fixed overhead costs, not imperfect competition in the labor market.

It is important to observe that this conclusion is an empirical result and does not follow by assumption. In our model, imperfect competition in the labor market should amplify the impacts of the foreign demand shocks on output, marginal costs, and (nominal and real) wages. To see this, recall our discussion of equation (2.16) in Section 2.5.2. This equation shows that the elasticity of labor cost with respect to a demand-driven change in the firm's output depends on the share of labor inputs that is variable and the labor supply curve it faces. All else equal, the steeper the labor supply curve, the larger the elasticity of labor costs. However, the sensitivity of the elasticity of labor cost to the labor supply elasticity

Figure 2.6: Firm-level distribution of changes in output, marginal costs, and wages in response to a 5 percent increase in foreign tariffs, with and without imperfect competition in the labor market



Notes: The three panels in this figure show the distribution of the changes in firm-level variables due to a uniform 5 percent increase in foreign tariffs on Belgian exports. Panel (a) shows the distribution of firm-level output changes, $\hat{q}_k$, panel (b) shows the distribution of firm-level marginal cost changes, $\hat{c}_k$, and panel (c) shows the distribution of firm-level wage changes, $\hat{w}_k$. In all panels, the white bars represent the distributions when one allows for imperfect competition in the labor market, and the grey bars represent the distributions when one assumes perfect competition in the labor market. Because all firms have common wages under perfect competition in the labor market, the corresponding wage change is depicted as a vertical line in panel (c). In the figure, we allow for fixed input costs in both labor inputs and input purchases.

depends on the variable labor shares. Given our estimates of the variable labor shares, the estimated value of the labor supply elasticity of 3.5 has only a modest impact on the firm responses to the foreign demand shocks.

2.6.6 Implications for real wages

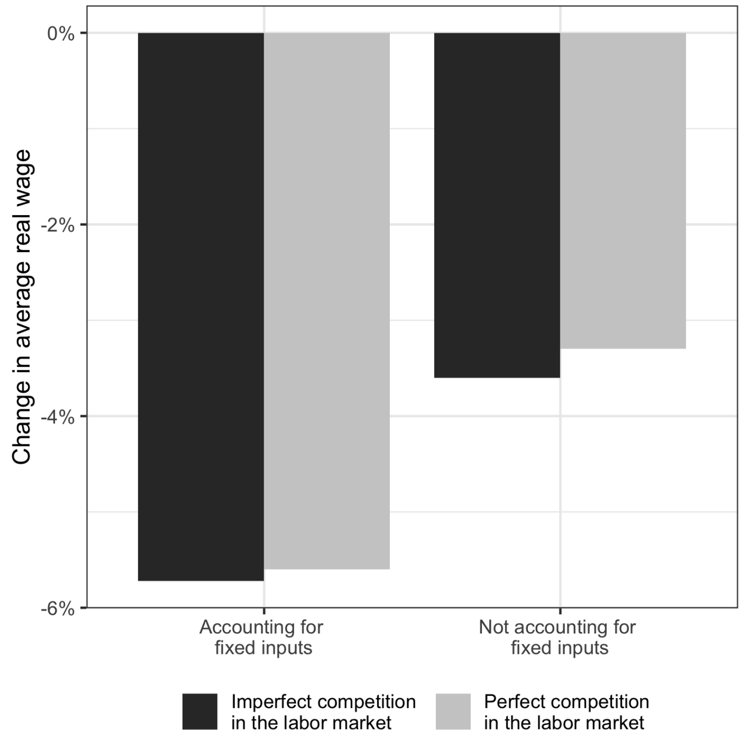
We conclude the analysis of foreign demand shocks by studying how the implications for *real* wages vary across the counterfactual economics. The results are reported in Figure 2.7. For each counterfactual economy, this figure presents our estimate of how the increase in foreign tariffs on Belgian exports would affect the average real wage, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$.[12]

The reduction in real wages is largest in our representation of the actual Belgian economy with firms that face fixed overhead costs and upward sloping labor supply curves. In this economy, the five percent increase in foreign tariffs on Belgian exports produces a 5.7 percent fall in the average real wage. In the counterfactual economy with fixed costs and perfectly elastic labor supply, this increase in foreign tariffs results in a 5.6 percent fall in the average real wage. By comparison, the average real wage declines only by 3.6 percent in the counterfactual economy with no fixed costs and upward-sloping labor supply curves. The smallest effect on the average real wage, a 3.3 percent decline, is found in the counterfactual economy with no fixed costs and perfectly elastic labor supply.

Taken together, these findings show that the essential feature to accurately predict the impacts of foreign tariffs on the real wages of Belgian workers is fixed overhead costs, not imperfect competition in the labor market. Furthermore, our results suggest that the way the labor market is typically modeled in existing research on foreign demand shocks—with no fixed costs and perfectly elastic labor supply—may grossly understate the decline in real wages due to an increase in foreign tariffs.

12. In Appendix B.4.3, we report the same set of results for the changes in real income, $\hat{E}/\hat{P}$, which captures not only the impacts on wages and consumer prices but also the effects on profits. The results are qualitatively the same. However, the quantitative impacts are larger, reflecting the negative effects of foreign tariffs on firms' profits.

Figure 2.7: Changes in average real wage in response to a 5 percent increase in foreign tariffs



Notes: In this figure, we report the changes in average real wage, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$, due to a uniform 5 percent increase in foreign tariffs on Belgian exports. Each bar represents the responses under different parameterizations of the model presented in Section 2.4. We use our estimated labor supply elasticity $\varepsilon = 3.5$ in the counterfactual Belgian economies with upward-sloping labor supply curves. Wages are common across all firms under the parameterization in which we assume $\varepsilon = \infty$, hence $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P} = \hat{w} / \hat{P}$. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 2.5.

2.6.7 Shocks to variables other than foreign demand

While our paper is centered around foreign demand shocks, the presence of fixed costs and imperfect competition could also have important implications for the analyses of other types of shocks. In Appendix B.4.4, we explore this by considering the propagation and implications of domestic productivity shocks, $\left\{\hat{\phi}_k\right\}$.

The analyses in this appendix closely follow the analyses in Subsections 2.6.3-2.6.6, except that we now replace the foreign demand shocks with a uniform 5 percent reduction in the productivity of all manufacturing firms. The results echo the key insights from our analyses of the foreign demand shocks. Fixed overhead costs and, to a lesser extent, imperfect competition in the labor market matter considerably for the predicted impacts of domestic productivity shocks.

2.7 Conclusion

The goal of our paper was to quantify and explain the firm responses and worker impacts of foreign demand shocks to domestic production networks. To capture that firms can be indirectly exposed to such shocks by buying from or selling to domestic firms that import or export, we used Belgian data with information on both domestic firm-to-firm sales and foreign trade transactions. Our estimates of firm responses suggest that Belgian firms pass on a large share of a foreign demand shock to their domestic suppliers, face upward-sloping labor supply curves, and have sizable fixed overhead costs in labor.

Motivated and guided by these findings, we developed and estimated an equilibrium model that allows us study how idiosyncratic and aggregate changes in foreign demand propagate through a small open economy and affect firms and workers. Our results suggest that the way in which the labor market is typically modeled in existing research on foreign demand shocks—with no fixed costs and perfectly elastic labor supply—would grossly understate the decline in real wages due to an increase in foreign tariffs. When interpreting these results, it is useful to observe that our model is parsimonious and restrictive in several ways. For example, we take all buyer-supplier relationships as given, in terms of both the domestic firm-to-firm links and the firms' direct export and import participation. Furthermore, we do not model firm entry and exit. Such adjustments could be especially important if one studies larger foreign demand shocks.

APPENDIX A

APPENDIX TO JOB LADDER OVER PRODUCTION NETWORKS

A.1 Data appendix

A.1.1 Aggregation of VAT identifiers into firms

As we describe in Section 1.2.1, all the information from the B2B datasets is reported at the level of VAT identifiers. Because some firms may have several VAT identifiers for accounting or tax purposes, the VAT identifiers do not necessarily correspond to the notion of firms or establishments. In order to focus on the firm-level analyses, we follow the same procedure as in Dhyne et al. (2021) to aggregate multiple VAT identifiers into the firm identifiers.

In order to collect multiple VAT identifiers that belong to the same firm, we proceed as follows. First, we determine whether a pair of VAT identifiers can be aggregated into the same firm based on their ownership structure. We use the information from ownership filings in the annual accounts as well as the Balance of Payments survey and collect multiple VAT identifiers that are linked with at least 50 percent of ownership. We also aggregate multiple VAT identifiers into the same firm if at least 50 percent of their shares are held by the same foreign parent firm. The foreign parent firms are recorded by their names, so we apply a “fuzzy string matching" method to compare all possible pairs of foreign firms’ names and determine the foreign parent firm of a given VAT identifier. Lastly, we also link a pair of VAT identifiers if they are linked one year before and one year after, so that we avoid potential misreporting.

After following the aggregation procedure above, we select the “most representative" VAT, or “head" VAT, identifiers among each collection of VAT identifiers. See Appendix C.4 of Dhyne et al. (2021) for the selection criteria. We use these head VAT identifiers as the identifiers of the firms and sum up all the variables across VAT identifiers to the firm level. For some variables that cannot be added up, such as firms' primary industry and the location of their main economic activities, we take those of the head VAT identifiers. For variables such as total sales and inputs, we further adjust them by the amount of B2B sales among the pairs of VAT identifiers that belong to the same head VAT identifier, so that we correct for the double counting of transactions within the same firm.

A.1.2 Merging procedures for the NBB and CBSS datasets

The information on the employers from the matched employer-employee data is recorded at the level of Banque Carrefour des Entreprises (Crossroads Bank for Enterprises, BCE) identifiers. All businesses in Belgium are assigned the unique identifiers upon their registration with the BCE. Because these businesses are required to register with the BCE when they pay VAT, their BCE identifiers can be easily converted to VAT identifiers. When we merge the matched employer-employee data from the CBSS datasets with the NBB datasets, we first convert all BCE identifiers into VAT identifiers. We then follow the same aggregation procedure explained in Appendix A.1.1 and aggregate multiple VAT identifiers into firms.

A.1.3 Descriptive statistics on the merged sample

In Section 1.2.3, we construct our main analysis sample of firms and workers. Table A.1 reports the descriptive statistics of firm characteristics and worker characteristics in the main analysis sample in 2012. For firm characteristics, we report both firm-level and worker-level averages. For instance, the average firm in 2012 buys from and sells to around 51 firms, whereas the employment-weighted averages reveal that the employer of the average worker is connected to thousands of firms.

Table A.1: Descriptive statistics

	Firm-level average	Employment-weighted average
Firm characteristics in 2012		
Value added (in million euro)	1.68	412
Labor cost (in million euro)	1.02	259
Employment (FTE)	18.6	4,827
Number of buyers	51.4	3,648
Number of suppliers	51.4	1,190

	All	Blue-collar	White-collar	Male	Female
Worker characteristics in 2012Q1					
Share of all workers	1.00	0.51	0.49	0.66	0.34
Average quarterly earnings (FTE)	8,000	6,026	10,010	8,477	7,044

Notes: This table shows the descriptive statistics of firm characteristics and worker characteristics. The table is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

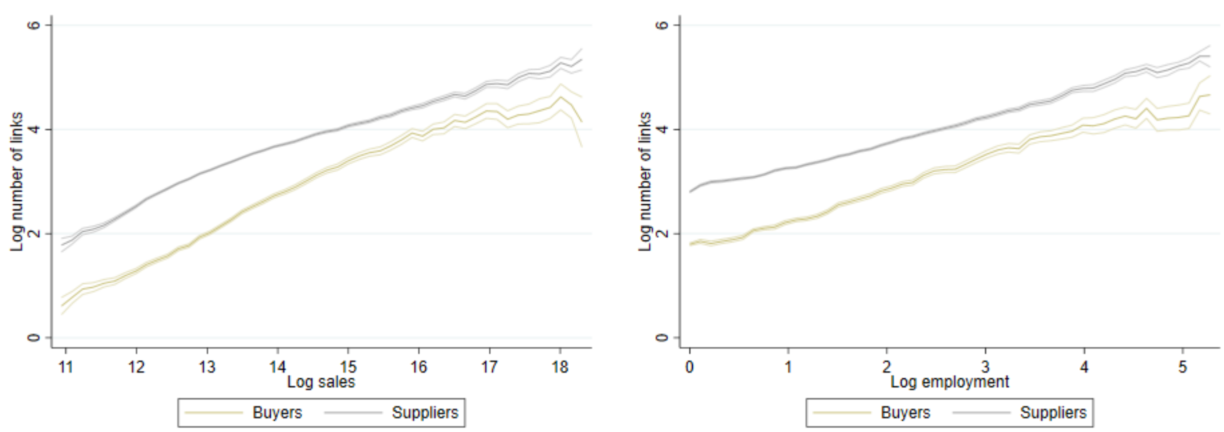
A.2 Additional empirical results

A.2.1 Firm size and number of buyers and suppliers

In Section 1.3.1, we show that the worker-level average of labor market connectedness is higher than the firm-level average. In this section, we show that the Belgian firms with a greater number of buyers and suppliers tend to be larger on average. To do so, we use local polynomial regressions to non-parametrically estimate the relationships between log firm size and log number of links. In Figure A.1, we show that larger firms, measured in terms of both log sales and log employment, have a greater number of buyers and suppliers. These relationships further corroborate the finding in Table A.1 of Appendix A.1.3 that the employer of the average worker in Belgium has more connections than the average firm.

Figure A.1: Relationship between firm size and number of buyers and suppliers

(a) Sales and number of links (b) Employment and number of links



Notes: The figures display the relationship between firm-level sales, employment, and number of links, using the smoothed values of kernel-weighted local polynomial regression estimates with 95 percent confidence intervals. We use the Epanechnikov kernel function with kernel bandwidth of 0.05. Log sales are trimmed at the top and bottom 1 percentiles. The figures are based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

A.2.2 *Alternative measures of labor market connectedness*

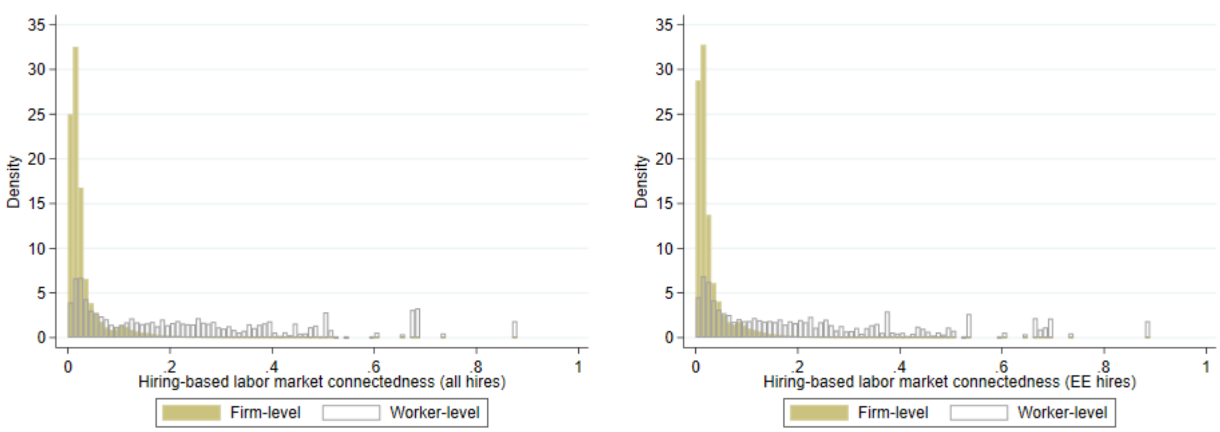
In Section 1.3.1, we define the employment-based labor market connectedness and present its distribution in the Belgian economy. In this section, we consider several alternative measures of labor market connectedness. First, we define the *hiring-based labor market connectedness*, which captures the share of total gross hires accounted for by the firms that firm j is directly connected to in production networks. Firm j's hiring-based labor market connectedness at time t, denoted by $\mathbb{C}^h_{j,t}$, can be computed as follows:

$$\mathbb{C}^h_{j,t} = \frac{\sum_{i\in\Omega^B_{j,t}\cup\Omega^S_{j,t}} (\text{gross hires})_{i,t}}{\sum_{i\in\Omega\setminus\{j\}} (\text{gross hires})_{i,t}}, \tag{A.1}$$

where the term $(\text{gross hires})_{i,t}$ is computed from the worker-level data, which captures the number of workers employed by firm i at time t but not at $t-1$.

Figure A.2: Distribution of hiring-based labor market connectedness

(a) Based on all hires (b) Based on EE hires



Notes: These figures show the distributions of hiring-based labor market connectedness. The hiring-based labor market connectedness of firm j, denoted by $\mathbb{C}^h_j$ and defined in equation (A.1), is the share of total gross hires accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. Panel (a) displays the hiring-based labor market connectedness based on all hires. In Panel (b), we only consider the employment-to-employment (EE) hires, in which the new hire is required to be employed at another firm in the previous period. The figures are based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

In Figure A.2, we present the distributions of hiring-based labor market connectedness. In 2012, an average Belgian firm is connected to around 3 percent of all hires, whereas an average worker is connected to around 24 percent of all hires through the direct links of their employer.

We summarize different measures of labor market connectedness in Table A.2. In addition to the firm-level and mover-level averages of labor market connectedness in 2012, we also report the mover-level average of each measure as well as the average over the entire sample period 2003-2014. We also report the average labor market connectedness based on the employment-to-employment hires next period. It is worthwhile to note that the mover-level average of labor market connectedness based on the employment-to-employment hires next period, which we report in the last row of Table A.2, corresponds to the statistical random benchmark discussed in Section 1.3.2 without any additional controls.

Table A.2: Average labor market connectedness

	Average		
	Firm-level	Worker-level	Mover-level
Labor market connectedness in 2012			
Employment-based	0.04	0.23	0.22
Hiring-based (all hires)	0.03	0.24	0.23
Hiring-based (EE hires)	0.03	0.23	0.23
Hiring-based (EE hires next period)	0.03	0.23	0.23
Labor market connectedness in 2003-2014			
Employment-based	0.05	0.23	0.22
Hiring-based (all hires)	0.03	0.23	0.21
Hiring-based (EE hires)	0.03	0.22	0.20
Hiring-based (EE hires next period)	0.03	0.22	0.20

Notes: This table reports the averages of different measures of labor market connectedness. The employment-based labor market connectedness is defined in equation (1.1) and measures the share of total employment accounted for by the firms directly connected in production networks. The hiring-based labor market connectedness is defined in equation (A.1) and measures the share of total gross hires accounted for by the firms directly connected in production networks. In the third and fourth rows, we only consider the employment-to-employment (EE) hires, in which the new hire at time t or $t+1$ is required to be employed at another firm in the previous period (at time $t-1$ or t, respectively). The table is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

A.2.3 Labor market connectedness without links from retailers, wholesalers, and utility companies

In Section 1.3.1, we compute the employment-based labor market connectedness using all the firm-to-firm linkages observed in the B2B datasets. One potential concern is that our results might be driven by a certain set of firms that have small transactions with the majority of firms. In this section, we show the robustness of labor market connectedness when only considering the subset of firm-to-firm linkages. In Figure A.3, we show the distribution of labor market connectedness, computed after dropping the links from retailers, wholesalers, and utility companies to their buyers. This procedure drops around 45 percent of firm-to-firm linkages and 36 percent of transaction volume. In this alternative measure, an average Belgian worker is still connected to around 20 percent of total employment through the other links.

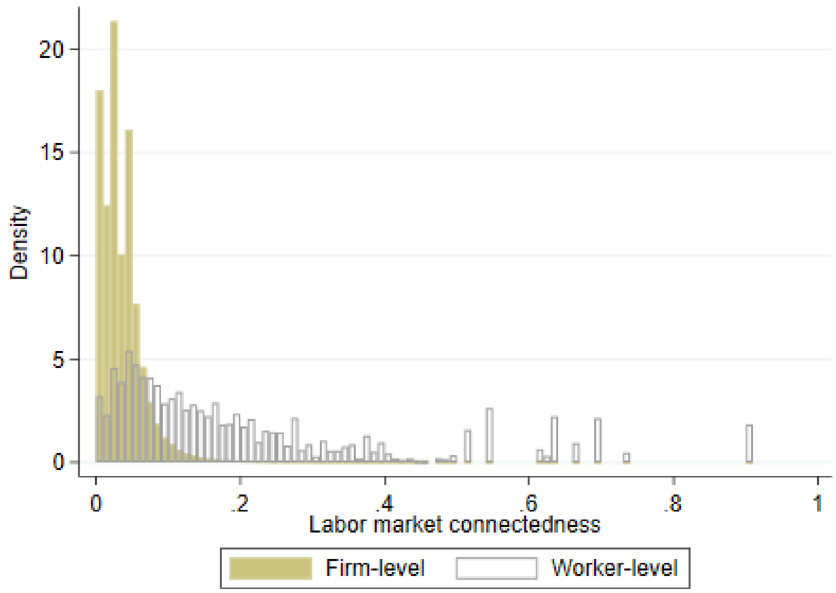
A.2.4 Additional results for the share of B2B moves

In Section 1.3.2, we show that workers move disproportionately more into the buyers and suppliers of their current employers. In this section, we provide several additional evidence for the shares of B2B moves, reported in Table A.3.

In the first panel of Table A.3, we consider the locations of establishments. Importantly, we do not observe the exact workplace of each worker if firms have multiple establishments. Therefore, the movers whose current and next employers report different geographic regions may not necessarily moved across regions. To alleviate this concern, we consider whether two firms have any establishments that operate in the same regions. Our results are robust for the movers between two firms that have no overlapping business coverage.

In the second panel, we split the movers based on the industries of their current employers and report the B2B shares excluding movers from consulting firms and temporary employment agencies. One might be concerned that our results for the industry switchers

Figure A.3: Distribution of employment-based labor market connectedness, excluding the links from retailers, wholesalers, and utility companies



Notes: This figure shows the distribution of employment-based labor market connectedness, in which we exclude the firm-to-firm linkages with retailers, wholesalers, and utility companies as the suppliers. The employment-based labor market connectedness of firm j, denoted by $\mathbb{C}_j^e$ and defined in equation (1.1), is the share of total employment accounted for by the firms that firm j is directly connected to in production networks. The white bars represent the distribution of the firm-level measure of labor market connectedness in which one weights the firms by the number of workers at each firm. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

are driven by selected industries, such as consultants at the consulting firms moving to their former client firms or workers at temporary employment agencies moving to their customers. While the movers from those firms have a higher share of movements to buyers, our result among the other movers is still robust in comparison to the random benchmark.

Table A.3: Share of B2B moves: additional results

	Share of movers	Share of B2B moves	
		Data	Random benchmark
Locations of establishments			
movers between firms with overlaps	0.85	0.46	0.25
movers between firms with no overlaps	0.15	0.16	0.05

	Share of movers	Share of moves to buyers	
		Data	Random benchmark
Consulting firms and temp agencies			
movers from consulting firms	0.03	0.34	0.12
movers from temp agencies	0.35	0.52	0.25
movers from other firms	0.62	0.21	0.07
movers from any firm	1.00	0.32	0.14

Notes: This table reports the shares of B2B moves among different sets of job-to-job movers. B2B moves refer to the job-to-job transitions where the origin and destination firms are connected in production networks, and the share of B2B moves is defined in equation (1.2). In the first panel, we split the movers into two groups based on whether their origin and destination firms have any establishments that operate in the same NUTS two-digit regions. We report the share of movers for each group in the first column. In the second and third columns, we report the observed shares in the data and the results from simulation exercises to compute the statistical random benchmarks, respectively. The random benchmarks compute the share of B2B moves if movers were to be randomly matched with hiring firms (see Appendix A.4.1 for details). In the second panel, we split the movers based on the industries of their current employers and report the B2B shares excluding movers from consulting firms and temporary employment agencies. This table is based on the main analysis sample of 467,194 movers in Belgium over the period 2003-2014 (see Section 1.2.3 for details).

A.3 Model appendix

A.3.1 Derivations of optimal revenue net of intermediate input costs

In this section, we provide a formal argument to the derivations of firms' optimal revenue net of intermediate input costs discussed in Section 1.4.1. Formally, the static problem of firm j with n workers to maximize its revenue net of intermediate input costs can be written as follows:

$$R_j(n) = \max_{\substack{\{p_{jk}\}_{k \in \Omega_j^B \cup \{H\}} \\ \{q_{jk}\}_{k \in \Omega_j^B \cup \{H\}} \\ \{q_{ij}\}_{i \in \Omega_j^S}}} \left(\int_{\Omega_j^B \cup \{H\}} p_{jk} q_{jk} dk - \int_{\Omega_j^S} p_{ij} q_{ij} di \right),$$

subject to the following constraints:

$$\begin{aligned}
q_j &= \int_{\Omega_j^B \cup \{H\}} q_{jk} dk \\
q_j &= \phi_j n^{\alpha} m_j^{1-\alpha} \\
m_j &= \left(\int_{\Omega_j^S} \left(\gamma_{ij} q_{ij}\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
q_{jk} &= \gamma_{jk}^{\sigma-1} \frac{p_{jk}^{-\sigma}}{z_k^{-\sigma}} m_k \quad \text{for all } k \in \Omega_j^B \\
q_{jH} &= \beta_{jH}^{\sigma-1} \frac{p_{jH}^{-\sigma}}{P^{1-\sigma}} E.
\end{aligned}$$

As discussed in the main texts, the cost minimization problem of firm j yields equations (1.8) and (1.9), implying that the following relationship holds true for the optimizing firm:

$$z_j m_j = \int_{\Omega_j^S} p_{ij} q_{ij} di.$$

Plugging this in, we rewrite firm j's problem, suppressing the constraints, as follows:

$$\begin{aligned}
R_j(n) &= \max_{\{p_{jk}\}_{k\in\Omega_j^B\cup\{H\}},m} \left(\int_{\Omega_j^B\cup\{H\}} p_{jk}q_{jk}(p_{jk})dk - z_j m_j \right) \\
&= \max_{\{p_{jk}\}_{k\in\Omega_j^B\cup\{H\}}} \left[\int_{\Omega_j^B\cup\{H\}} p_{jk}q_{jk}(p_{jk})dk - z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} \left(\int_{\Omega_j^B\cup\{H\}} q_{jk}(p_{jk})dk \right)^{\frac{1}{1-\alpha}} \right].
\end{aligned}$$

The first order condition with respect to p_{jk} yields:

$$p_{jk}\frac{dq_{jk}}{dp_{jk}} + q_{jk} - \frac{1}{1-\alpha} z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} q_j^{\frac{\alpha}{1-\alpha}} \frac{dq_{jk}}{dp_{jk}} = 0.$$

Notice that for all customers $k \in \Omega_j^B \cup \{H\}$, the shape of demand curve implies that $dq_{jk}/dp_{jk} = -\sigma q_{jk}/p_{jk}$. Therefore, the optimal price p_{jk}^* satisfies:

$$p_{jk}^* = \frac{\sigma}{\sigma-1}\frac{1}{1-\alpha} z_j \phi_j^{-\frac{1}{1-\alpha}} n^{-\frac{\alpha}{1-\alpha}} q_j^{\frac{\alpha}{1-\alpha}}. \tag{A.2}$$

This expression holds true for all $k \in \Omega_j^B \cup \{H\}$, and thus, firms optimally charge the same price to all of their customers.[1]

Now we derive the exact expressions for equations (1.14) and (1.15). Using equations (1.6) and (1.11), we rewrite the static problem of firms as follows:

$$R_j(n) = \max_m \left(\phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{\frac{(1-\alpha)(\sigma-1)}{\sigma}} - z_j m \right).$$

The first order condition with respect to m yields:

$$\frac{(1-\alpha)(\sigma-1)}{\sigma} \phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{*\frac{-1-\alpha(\sigma-1)}{\sigma}} = z_j$$

1. This result is not unique to our setting and holds in a wide range of production network models that assume a common and constant demand elasticity across all customers. See, for example, Huneeus et al. (2022).

Solving this equation for m^*, we obtain:

$$m^* = \left[\frac{(1-\alpha)(\sigma-1)}{\sigma} z_j^{-1} \phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}}\right]^{\frac{\sigma}{1+\alpha(\sigma-1)}} \tag{A.3}$$

Using this expression, we have:

$$\begin{aligned} p^* &= \phi_j^{-\frac{1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{-\frac{\alpha}{\sigma}} m^{-\frac{1-\alpha}{\sigma}} \\ &= \left[\left(\frac{\sigma}{(1-\alpha)(\sigma-1)}\right)^{1-\alpha} z_j^{1-\alpha} \phi_j^{-1} \chi_j^{\alpha} n^{-\alpha}\right]^{\frac{1}{1+\alpha(\sigma-1)}}, \end{aligned} \tag{A.4}$$

and:

$$\begin{aligned} R_j(n) &= \left(\phi_j^{\frac{\sigma-1}{\sigma}} \chi_j^{\frac{1}{\sigma}} n^{\frac{\alpha(\sigma-1)}{\sigma}} m^{*\frac{-1-\alpha(\sigma-1)}{\sigma}} - z_j\right) m^* \\ &= \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_j m^* \\ &= \Phi_j n^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}, \end{aligned} \tag{A.5}$$

where

$$\Phi_j = \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} \left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{1+\alpha(\sigma-1)}} z_j^{-\frac{(1-\alpha)(\sigma-1)}{1+\alpha(\sigma-1)}} \phi_j^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} \chi_j^{\frac{1}{1+\alpha(\sigma-1)}}. \tag{A.6}$$

A.3.2 Derivations of product market equilibrium in Claim 1

In this section, we provide the derivation of equations (1.32)-(1.34) which characterize the product market equilibrium in Claim 1. First, the derivation of equation (1.32) immediately follows from equation (1.9). Second, in order to derive equation (1.33), we use the expression for the optimal revenue net of intermediate input costs in equation (A.5), which yield

$$p_j q_j - z_j m_j = \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_j m_j.$$

Rearranging this equation and using equation (1.11), we obtain the following:

$$
\begin{aligned}
z_j m_j &= \frac{(1-\alpha)(\sigma-1)}{\sigma} p_j q_j \\
&= \frac{(1-\alpha)(\sigma-1)}{\sigma} p_j^{1-\sigma} \chi_j,
\end{aligned}
$$

which implies

$$
\tilde{m}_j = \frac{(1-\alpha)(\sigma-1)}{\sigma} \chi_j. \tag{A.7}
$$

Now we can use equations (1.31) and (A.5) to rewrite the expression for χ_j in equation (1.12) as follows:

$$
\begin{aligned}
\chi_j &= \beta_{jH}^{\sigma-1} \int_{\Omega} R_k(n_k) dk + \int_{\Omega_j^B} (\gamma_{jk} z_k)^{\sigma-1} z_k m_k dk \\
&= \beta_{jH}^{\sigma-1} \int_{\Omega} \frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} z_k m_k dk + \int_{\Omega} \mathbb{1}_{\{k \in \Omega_j^B\}} \gamma_{jk}^{\sigma-1} z_k^{\sigma-1} z_k m_k dk \\
&= \int_{\Omega} \left(\frac{1+\alpha(\sigma-1)}{(1-\alpha)(\sigma-1)} \beta_{jH}^{\sigma-1} + \mathbb{1}_{\{k \in \Omega_j^B\}} \gamma_{jk}^{\sigma-1} \tilde{z}_k^{-1} \right) \tilde{p}_k \tilde{m}_k dk.
\end{aligned} \tag{A.8}
$$

Substituting equation (A.8) into equation (A.7) yields equation (1.33).

Lastly, we derive equation (1.34) using the optimal pricing equation (A.2). Substituting equation (1.6) into equation (A.2), we obtain the following:

$$
\begin{aligned}
p_j &= \frac{\sigma}{(1-\alpha)(\sigma-1)} z_j \phi_j^{-\frac{1}{1-\alpha}} n_j^{-\frac{\alpha}{1-\alpha}} \left(\phi_j n_j^{\alpha} m_j^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} \\
&= \frac{\sigma}{(1-\alpha)(\sigma-1)} \phi_j^{-1} n_j^{-\alpha} z_j^{1-\alpha} \tilde{m}_j^{\alpha} p_j^{\alpha(1-\sigma)}.
\end{aligned} \tag{A.9}
$$

Solving equation (A.9) for $\tilde{p}_j \equiv p_j^{1-\sigma}$ yields equation (1.34).

It is useful to observe that $\tilde{p}_j$ in equation (1.34) only depends on variables at firm j. Therefore, we can further reduce the number of equations by substituting equation (1.34) into equations (1.32) and (1.33). We can rewrite the system of equations in Claim 1 as follows:

$$
\begin{aligned}
x_{j1} &= \sum_{k\in\Omega} f_{jk2}(x_{k1}, x_{k2}) \\
&= \sum_{k\in\Omega} \omega_{kj}\gamma_{kj}^{\sigma-1} \left(\frac{(1-\alpha)(\sigma-1)}{\sigma}\phi_k n_k^{\alpha}\right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} x_{k1}^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} x_{k2}^{-\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} \\
x_{j2} &= \sum_{k\in\Omega} f_{jk1}(x_{k1}, x_{k2}) \\
&= \sum_{k\in\Omega} \left(\frac{1+\alpha(\sigma-1)}{\sigma}\beta_{jH}^{\sigma-1} + \frac{(1-\alpha)(\sigma-1)}{\sigma}\omega_{jk}\gamma_{jk}^{\sigma-1}x_{k1}^{-1}\right) \\
&\quad \times \left(\frac{(1-\alpha)(\sigma-1)}{\sigma}\phi_k n_k^{\alpha}\right)^{\frac{(\sigma-1)}{1+\alpha(\sigma-1)}} x_{k1}^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} x_{k2}^{\frac{1}{1+\alpha(\sigma-1)}}.
\end{aligned}
$$

A.3.3 Derivations of employment distribution in Claim 2

In this section, we provide the derivation of equation (1.35) which characterizes the steady-state employment in Claim 2. In the steady state where the distributions of labor productivity Φ_j and workers n_j remain unchanged, firm j's value Π_j in equation (1.24) has a closed-form solution. Using equations (1.14) and (1.23), we obtain the following expression for Π_j:

$$
\Pi_j(n) = \frac{\Phi_j}{\rho+\delta_j\tilde{\alpha}}\left[1 - \frac{\eta}{1-\eta(1-\tilde{\alpha})}\tilde{\alpha}\right] n^{\tilde{\alpha}} - \frac{\bar{w}}{\rho+\delta_j}n.
$$

Taking its derivative with respect to employment, we find that the following expression holds to satisfy the optimal vacancy-posting decision in equation (1.26):

$$
\frac{c}{\mu_j} = \frac{\Phi_j\tilde{\alpha}}{\rho+\delta_j\tilde{\alpha}}\left[1 - \frac{\eta}{1-\eta(1-\tilde{\alpha})}\tilde{\alpha}\right] n_j^{\tilde{\alpha}-1} - \frac{\bar{w}}{\rho+\delta_j} \tag{A.10}
$$

Rearranging equation (A.10) for n_j yields equation (1.35).

It is useful to observe that the vacancy-filling rate μ_j and separation rate δ_j in equation (1.35) are determined by firms' vacancy-posting decisions $\{v_j\}_{j\in\Omega}$, which are then characterized by the set of employment $\{n_j\}_{j\in\Omega}$ and labor productivity $\{\Phi_j\}_{j\in\Omega}$. First notice that the law of motion for employment implies that the following relationship holds in the steady state:

$$v_j = \frac{\delta_j}{\mu_j} n_j. \tag{A.11}$$

Next, we can rewrite the value for an unemployed worker in equation (1.27) as follows in the steady state:

$$U = \frac{b}{\rho + \lambda^m} + \frac{\lambda^m}{\rho + \lambda^m} \int_\Omega \tilde{v}_k W_k dk. \tag{A.12}$$

The value for a worker employed at firm j in equation (1.27) yields

$$(\rho + \delta_0) W_j = w_j + \int_\Omega \lambda_{jk} \tilde{v}_k \left(W_k - W_j\right)^+ dk + \delta_0 U. \tag{A.13}$$

We can then characterize μ_j and δ_j using equations (1.30) and (1.29), respectively.

A.4 Estimation and computation details

A.4.1 Computing the random benchmark for B2B moves

In this section. we describe the procedures to compute the statistical random benchmarks for B2B moves presented in Section 1.3.2. The goal of this exercise is to compute the share of B2B moves if movers were to be matched with hiring firms randomly. We compute the random benchmark for the share of B2B moves as follows:

1. Randomly draw a mover from the set of all movers. Denote the quarter and employer before move as time $\hat{t}$ and firm $\hat{j}$, respectively.
2. Create a list of firms that satisfies the following conditions:
   (a) firms exist at time $\hat{t}+1$ and are different from firm $\hat{j}$: $k \in \Omega_{\hat{t}+1} \setminus \{\hat{j}\}$.
   (b) we observe at least one new hire at time $\hat{t}+1$: there exists worker i such that $k = j(i, \hat{t}+1)$ and $k \neq j(i, \hat{t})$.
3. Randomly draw a destination firm from the list. Call it firm $\hat{k}$.
4. Check if the destination firm $\hat{k}$ is connected to firm $\hat{j}$ in production network at time $\hat{t}$ and record the value of $\mathbf{1}_{\left\{\hat{k} \in \Omega^S_{\hat{j},t} \cup \Omega^B_{\hat{j},t}\right\}}$.
5. Repeat Steps (1)-(4) 100,000 times and report the average value of $\mathbf{1}_{\left\{\hat{k} \in \Omega^S_{\hat{j},t} \cup \Omega^B_{\hat{j},t}\right\}}$.

When computing the random benchmarks with additional controls reported in the second panel of Table 1.1 as well as Table A.3, we introduce additional conditions to Steps (1) and (2). For instance, we only draw from the set of movers who are blue-collar workers at the origin firms when computing the random benchmark for blue-collar movers. Similarly, when we compute the random benchmark for movers within industries, we require the destination firm $\hat{k}$ to be in the same NACE two-digit industry as firm $\hat{j}$.

A.4.2 Estimating labor market parameters

In this section, we describe the procedures to estimate the labor market parameters discussed in Section 1.5.2. The goal is to estimate the vector of six labor market parameters $\Theta = \{\bar{\lambda}, \zeta, \delta_0, c, \bar{w}, \eta\}$ by the method of simulated moments. Recall that we estimate the labor market parameters by minimizing the following objective function:

$$\hat{\Theta} = \arg\min_{\Theta}[\hat{y} - y(\Theta^*)]'[\hat{y} - y(\Theta^*)].$$

where $\hat{y}$ and $y(\Theta)$ denote the vectors of targeted moments in the data and model-implied moments at the parameter value Θ, respectively. In what follows, we explain the algorithm to compute $y(\Theta)$ given the choice of parameter values at Θ.

In the estimation step, we take as given the parameters we externally set in Section 1.5.1, namely $\{\rho, \alpha, \sigma, L, \xi\}$. We use the distribution of data-implied labor productivity $\{\Phi_j\}$ presented in Section A.5.1 as well as the network structures $\{\omega_{IJ}\}$ as inputs. We then compute the vector of moments $y(\Theta)$ for a given choice of parameters Θ as follows:

1. guess $\{n_j\}$, $\{\delta_j\}$, and $\{\mu_j\}$
2. compute $\{w_j\}$ using equation (1.23) and guess $\{W_j\}$ and U
3. compute $\{v_j\}$, $\{s_j\}$, and $\{\lambda_{jk}\}$ using equations (1.35), (A.10), (1.25), and (1.20)
4. update $\{\delta_j\}$ and $\{\mu_j\}$ using equations (1.29) and (1.30)
5. update $\{W_j\}$ and U using equations (A.13) and (A.12)
6. update $\{n_j\}$ using the law of motion $n_j^{new} = n_j + (\mu_j^{new} v_j - s_j - \delta_j^{new} n_j)$
7. repeat Steps 3-6 until $\{n_j\}$ converges
8. compute the moments $y(\Theta)$ given the steady-state distribution of employment $\{n_j\}$ and the mobility decisions of workers

A.4.3 Estimating product market parameters

In this section, we describe the procedures to estimate the product market parameters discussed in Section 1.5.3. The goal is to estimate the product market parameters for each firm group while concurrently solving for the product market equilibrium.

In the estimation step, we take as given the parameters we externally set in Section 1.5.1, namely $\{\rho, \alpha, \sigma, L, \xi\}$. We use the distribution of data-implied labor productivity $\{\Phi_j\}$ presented in Section A.5.1, the network structures $\{\omega^B_{IJ}, \omega^S_{IJ}\}$, network sales shares $\{r^{net}_j\}$, and the estimated supplier fixed effect, buyer fixed effect, and buyer-supplier residual $\{\log \Gamma^S_j, \log \Gamma^B_k, \log \tilde{\Gamma}_{jk}\}$ as inputs. We also take as given the employment $\{n_j\}$ and the labor market parameters estimated in Section 1.5.2. We then estimate the product market parameters and solve for the product market equilibrium as follows:

1. guess $\{z_j\}$ and $\{\chi_j\}$
2. estimate $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ using Propositions 1 and 2
3. compute $\{\phi_j\}$ using equation (A.6)
4. compute $\{m_j\}$ and $\{p_j\}$ using equations (A.3) and (A.4)
5. update $\{z_j\}$ and $\{\chi_j\}$ using equations (1.41), (1.9), and (1.12)
6. repeat Steps 3-5 until $\{z_j\}$ converges
7. estimate $\{\gamma_{jk}\}$ and $\{\beta_{jH}\}$ using Propositions 1 and 2
8. go back to Step 3 and repeat until $\{\beta_{jH}\}$ converges

A.4.4 Solving for the steady state

In this section, we describe the procedures to solve for the steady state distributions of labor productivity $\{\Phi_j\}$ and employment $\{n_j\}$. In doing so, we take advantage of Claim 3 and solve for the product market equilibrium and the stationary distribution of workers in a sequential manner. Within each step, the procedures to solve for the product market and labor market resemble the estimation procedures described in Appendices A.4.2 and A.4.3.

Given the initial guess for the distributions of labor productivity $\{\Phi_j\}$ and employment $\{n_j\}$, we proceed as follows. We first solve for the product market equilibrium and update the labor productivity given the distribution of employment:

1. guess $\{z_j\}$ and $\{\chi_j\}$
2. compute $\{\phi_j\}$ using equation (A.6)
3. compute $\{m_j\}$ and $\{p_j\}$ using equations (A.3) and (A.4)
4. update $\{z_j\}$ and $\{\chi_j\}$ using equations (1.41), (1.9), and (1.12)
5. repeat Steps 2-4 until $\{z_j\}$ converges
6. update $\{\Phi_j\}$ using equation (A.6)

Using the updated distribution of labor productivity $\{\Phi_j\}$, we then solve for the new distribution of employment $\{n_j\}$:

1. guess $\{n_j\}$, $\{\delta_j\}$, and $\{\mu_j\}$
2. compute $\{w_j\}$ using equation (1.23) and guess $\{W_j\}$ and U
3. compute $\{v_j\}$, $\{s_j\}$, and $\{\lambda_{jk}\}$ using equations (1.35), (A.10), (1.25), and (1.20)
4. update $\{\delta_j\}$ and $\{\mu_j\}$ using equations (1.29) and (1.30)
5. update $\{W_j\}$ and U using equations (A.13) and (A.12)
6. update $\{n_j\}$ using the law of motion $n_j^{new} = n_j + (\mu_j^{new} v_j - s_j - \delta_j^{new} n_j)$
7. repeat Steps 3-6 until $\{n_j\}$ converges

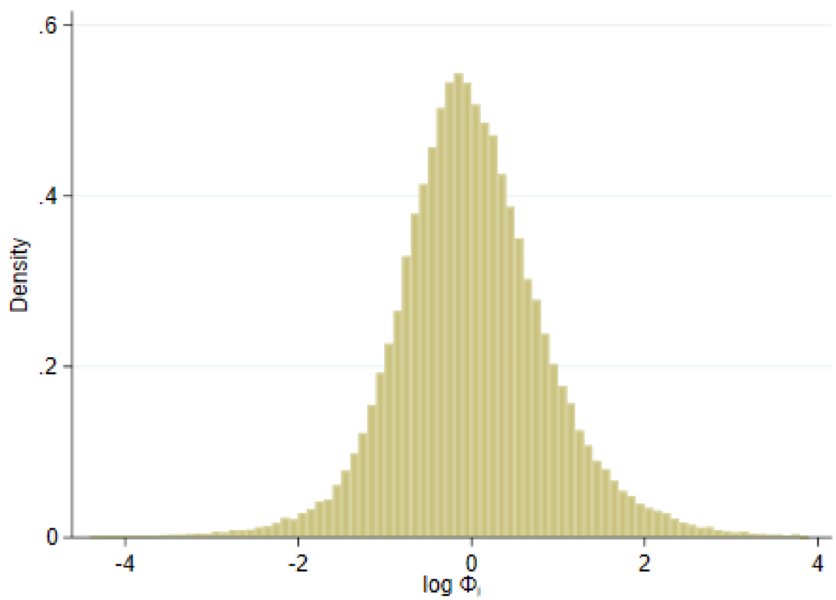
If the new distribution of employment is not close enough to the initial distribution, we construct the new distribution of employment as a linear combination of the previous guess and the computed values and go back to the first step. We iterate between these two steps until $\{n_j\}$ converges.

A.5 Estimation and computation results

A.5.1 Steady-state distribution of labor productivity Φ_j

In Figure A.4, we present the distribution of log labor productivity Φ_j. For each firm, we compute its labor productivity Φ_j based on equation (1.14), using the observed levels of value added and employment. The return to scale parameter for labor α and substitutability parameter for goods σ in equation (1.14) are set externally at 0.37 and 4, respectively, following the discussions in Section 1.5.1. We also normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero.

Figure A.4: Distribution of log labor productivity Φ_j



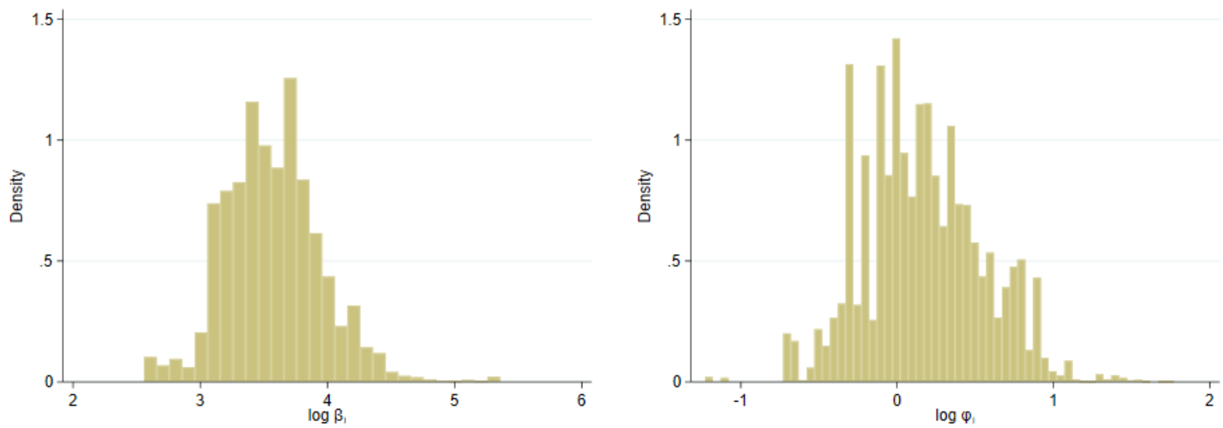
Notes: This figure shows the distribution of log labor productivity Φ_j. For each firm, we compute its labor productivity Φ_j based on equation (1.14), using the observed levels of value added and employment. The return to scale parameter for labor α and substitutability parameter for goods σ in equation (1.14) are set externally at 0.37 and 4, respectively, following the discussions in Section 1.5.1. We normalize labor productivity such that the average of $\log \Phi_j$ is set to be zero. Top and bottom 1 percent of the estimated parameters are trimmed in the figures for illustrative purposes. The figure is based on the main analysis sample of 98,599 private-sector firms in Belgium in 2012 (see Section 1.2.3 for details).

A.5.2 Estimated product market parameters

In Section 1.5.3, we provide the identification arguments for estimating the product market parameters in our model. In Figure A.5, we present the distributions of the estimated saliency parameters in households' preference $\{\beta_{jH}\}$ as well as firm's own productivity $\{\phi_j\}$. Following the procedure explained in Section 1.5.2, we first cluster firms into firms groups and estimate the parameter values for each firm group. In Figure A.5, we then present the firm-level distributions by weighting the firm groups by the number of firms within each firm group.

Figure A.5: Distribution of estimated product market parameters

(a) $\log \beta_{jH}$ (b) $\log \phi_j$



Notes: These figures show the distributions of the estimated product market parameters. Panel (a) displays the distribution of the estimated saliency parameters in households' preference $\{\beta_{jH}\}$, and Panel (b) shows the distribution of the estimated firm's own productivity $\{\phi_j\}$. The identification strategies for estimating these parameters are discussed in Section 1.5.3. For both parameters, we estimate their values for each firm group, which is constructed by following the procedure explained in Section 1.5.2. In each panel, we present the firm-level distribution by weighting the firm groups by the number of firms within each firm group. Top and bottom 1 percent of the estimated parameters are trimmed in the figures for illustrative purposes.

A.5.3 Long-run response to productivity shocks

In Section 1.6.2, we presented the instantaneous responses of labor productivity and wages to a 5 percent reduction in manufacturing productivity. In this section, we consider the long-run response of firms by comparing the initial steady state to the new steady state after the shock. Compared to the analysis in the main text, where we focus on the instantaneous impacts on workers, we now consider the firms' responses after taking into account the endogenous reallocation of employment.

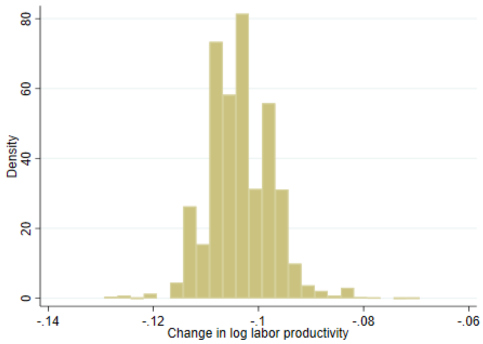
Panel (a) of Figure A.6 shows the long-run responses in log labor productivity Φ_j. As in Figure 1.5, both manufacturing firms and non-manufacturing firms are affected by the decline in manufacturing productivity. Nonetheless, a small fraction of non-manufacturing firms now experience positive changes in their labor productivity relative to the initial steady state. This is possible in the long run because some non-manufacturing firms experience gains in their employment, which is also displayed in Figure A.7.

In Panel (b) of Figure A.6, we show the relationship between firms' own labor cost changes and the average labor cost changes of the matched firms. Because we now compare different workers at two steady states, we present the changes in firm-level labor costs instead of the changes in average wage. The overall patterns are similar to the findings in 1.5. While the average decline in labor cost among the firms that workers meet through the market search does not depend on the current employers, the firms that are hit harder by the productivity shocks are more likely to be connected to other firms with larger labor cost declines through the network search channel.

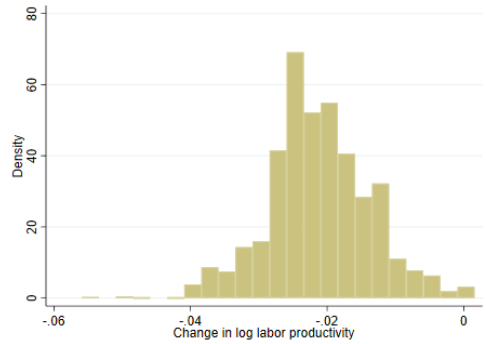
Figure A.6: Long-run response to 5 percent reduction in manufacturing productivity

(a) Changes in log labor productivity

Manufacturing firms



Non-manufacturing firms



(b) Changes in log labor cost: own vs matched firms

Manufacturing firms

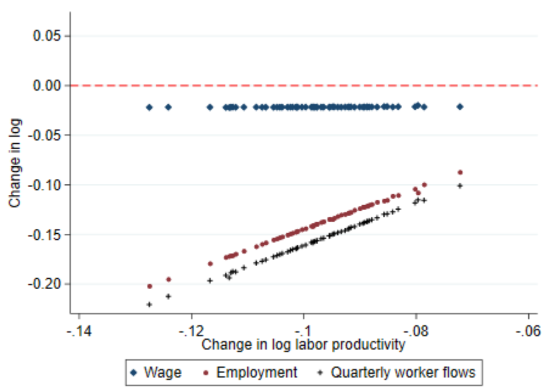
Non-manufacturing firms

Notes: In this figure, we report the long-run changes in log labor productivity and log labor cost due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. In Panel (a), we show the distributions of long-run changes in log labor productivity $\{\Phi_j\}$ for both manufacturing firms and non-manufacturing firms. We compute the long-run changes by solving for the new steady state after the shock, following the procedure in Appendix A.4.4. In Panel (b), we present the relationship between firms' own labor cost changes and the average labor cost changes of the firms with which workers are matched through the market search and network search channels. For each bin of firm group, sorted by the percentiles of the own labor cost changes, we compute the average labor cost changes of the matched firms, weighted by the likelihood of the matching. The blue diamonds represent the average labor cost changes of the firms matched through the market search channel, whereas the red markers represent the average labor cost changes of the firms matched through the network search channel. The dashed red lines represent the employment-weighted average of the labor cost changes in the entire economy.

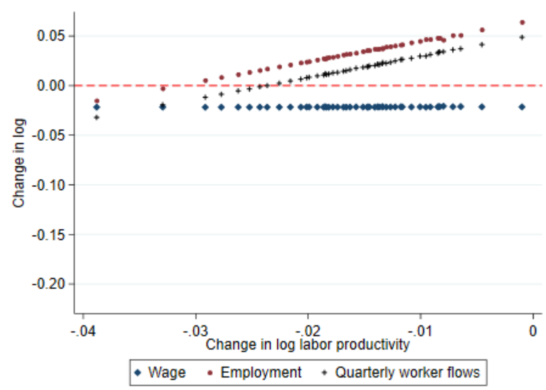
Figure A.7: Long-run response to 5 percent reduction in manufacturing productivity

(a) Changes in log wage, employment, and quarterly worker flows

Manufacturing firms



Non-manufacturing firms



Notes: In this figure, we report the long-run changes in log wage, employment, and quarterly worker flows due to a 5 percent reduction in manufacturing firms' own productivity $\{\phi_j\}$. We compute the long-run changes by solving for the new steady state after the shock, following the procedure in Appendix A.4.4. For each bin of firm group, sorted by the percentiles of the changes in their log labor productivity $\{\Phi_j\}$, we compute the average change in each variable.

APPENDIX B

APPENDIX TO FOREIGN DEMAND SHOCKS TO PRODUCTION NETWORKS: FIRM RESPONSES AND WORKER IMPACTS

B.1 Data appendix

B.1.1 Aggregating VAT identifiers into firms

As discussed in the main text, the NBB datasets are available at the level of VAT identifiers. In order to conduct our analyses at the firm level, we aggregate multiple VAT identifiers into the firm identifiers, using the same procedure as in Dhyne et al. (2021). We leverage the information from ownership filings in the annual accounts as well as the Balance of Payments survey to determine if a pair of VAT identifiers belongs to the same firm. We aggregate multiple VAT identifiers to the same firm if they are linked with at least 50 percent of ownership or if they share the same foreign parent firm that holds at least 50 percent of their shares. In order to determine the foreign parent firm of a given VAT identifier, we apply a "fuzzy string matching" method, comparing all possible pairs of foreign firms' names. Lastly, we correct for potential misreporting by linking the pair of VAT identifiers that are linked one year before and one year after.

After collecting multiple VAT identifiers that belong to the same firm, we then assign their firm identifier using the "most representative" VAT identifiers among them. The criteria for selecting such "head VAT" identifiers are explained in detail in Appendix C.4 of Dhyne et al. (2021). Once we determine the head VAT identifiers for all firms that have multiple VAT identifiers, we then sum up all the variables across VAT identifiers to the firm level. In order to avoid the double counting of transactions within the firm, we further adjust total sales and inputs by the amount of B2B sales between the pair of VAT identifiers that belong to the same head VAT identifier. For other variables such as firms' age, their primary industry, and location of their main economic activities, we take those of head VAT identifiers.

B.1.2 Merging NBB datasets with BCSS datasets

The BCSS datasets are available at the level of Banque Carrefour des Entreprises (Crossroads Bank for Enterprises, BCE) identifiers. All businesses in Belgium are required to register with the BCE, which assigns them the unique identifiers upon registration. Registration with the BCE is required for firms to pay VAT, so the BCE identifiers can be easily converted to VAT identifiers. In order to match the BCSS datasets with the NBB datasets, we first convert all BCE identifiers to VAT identifiers and then aggregate multiple VAT identifiers into firms, as explained in Appendix B.1.1.

B.1.3 Coverage and summary statistics on the merged sample

Table B.1 reports the coverage of our main estimation sample (NBB sample) in 2012 and compares it to the official aggregate statistics obtained from Eurostat. Our sample covers a large majority of the aggregate value added, gross output, labor costs, exports, and imports in the Belgian economy. We also report the coverage of the subsample of firms for which we have additional information from the worker data (merged NBB-BCSS sample for the firms with 10 or more FTE employees at least once from 2002 to 2014), which still makes up most of the total sales, inputs, and trade in the Belgian economy. In Table B.2, we also present summary statistics on the workers and the firms by firms' export status and worker types, obtained from the merged NBB-BCSS dataset.

Table B.1: Coverage of NBB and NBB-BCSS datasets in 2012

	Eurostat
GDP (excl. Gov.&Fin.)	248
Output (excl. Gov.&Fin.)	672
Import	310
Export	311

	NBB sample	NBB-BCSS sample
Count:		
Total	98,745	26,470
Direct exporters	11,892	7,024
Indirect exporters	74,529	18,043
Value added	164	145
Total sales	796	704
Network sales	225	190
Import	292	279
Export	292	281
Labor cost	100	90
Employment (FTE)	1,824,066	1,578,505

Notes: All numbers except for count and employment are denominated in billion euro in current prices. Belgian GDP and output are for all sectors excluding the public and financial sectors. Data for Belgian GDP, output, imports, and exports are from Eurostat. Firms' value added is from the reported values from the annual accounts. Firms' sales consist of their sales to other firms in the NBB sample (network sales), sales to households at home, and direct exports to foreign markets.

Table B.2: Summary statistics by firms' export status and worker types

	(1) All	(2) Blue-collar	(3) White-collar	(4) Male	(5) Female
Number of stayers					
All firms	49.41	24.58	24.83	33.02	16.39
Exporter	114.27	50.06	64.22	79.57	34.70
Non-exporter	25.57	15.21	10.36	15.92	9.66
Average stayer wage					
All firms	32,246	24,006	40,403	34,180	28,352
Exporter	35,467	26,032	42,823	37,270	31,335
Non-exporter	26,958	21,557	34,890	28,502	24,413

Notes: All summary statistics are computed on the merged NBB-BCSS sample in 2012. Workers are considered as stayers at firm j in 2012 if they work for firm j throughout 2012 as well as in the last quarter of 2011 and the first quarter of 2013.

B.2 Model appendix

B.2.1 General equilibrium of the model in Section 2.4.1

We characterize the firm-level outcomes implied by the firms' profit maximization and cost minimization problem. First, the sum of the variable and fixed costs of firm k can be written as

$$\begin{aligned} TC_k =&\phi_k^{-1} \frac{\frac{1+\varepsilon}{\varepsilon}\left(1-\alpha_{\ell k}\right)+\alpha_{\ell k}}{\left(\frac{1+\varepsilon}{\varepsilon}\left(1-\alpha_{\ell k}\right)\right)^{1-\alpha_{\ell k}} \alpha_{\ell k}^{\alpha_{\ell k}}}\left(\sum_{j \in Z_k}\left(\omega_{jk}^v\right)^\sigma p_j^{1-\sigma}+\left(\omega_{Fk}^v\right)^\sigma p_{Fk}^{1-\sigma}\right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} w_k^{\alpha_{\ell k}} q_k \\ &+\left(\sum_{j \in Z_k}\left(\omega_{jk}^f\right)^\sigma p_j^{1-\sigma}+\left(\omega_{Fk}^f\right)^\sigma p_{Fk}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \bar{q}_k^f \\ &+w_k \bar{\ell}_k^f, \end{aligned} \tag{B.1}$$

where the first term represents the variable costs, the second term represents the fixed input purchases, and the last term represents the fixed labor costs. Note that firms face a common demand elasticity of σ regardless of whom they sell to; hence,

$$p_k=\mu_k c_k=\frac{\sigma}{\sigma-1} c_k. \tag{B.2}$$

Taking the total derivative of the total cost with respect to output quantity, one can derive the firm's marginal cost,

$$c_k=\frac{1}{\phi_k \alpha_{\ell k}^{\alpha_{\ell k}}\left(1-\alpha_{\ell k}\right)^{1-\alpha_{\ell k}}}\left(\sum_{j \in Z_k}\left(\omega_{jk}^v\right)^\sigma p_j^{1-\sigma}+\left(\omega_{Fk}^v\right)^\sigma p_{Fk}^{1-\sigma}\right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}}\left(\frac{1+\varepsilon}{\varepsilon} w_k\right)^{\alpha_{\ell k}}. \tag{B.3}$$

The marginal cost follows the standard structure except that the firm's wage enters the cost with a wedge of $\frac{1+\varepsilon}{\varepsilon}$. One can then derive the total variable input cost of the firm—the first term in equation (B.1)—in terms of its sales $p_k q_k$, by substituting in equations (B.2) and (B.3):

$$w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v = \left(\frac{\varepsilon_m}{\varepsilon_m + 1} \alpha_{\ell k} + 1 - \alpha_{\ell k} \right) \frac{\sigma - 1}{\sigma} p_k q_k. \tag{B.4}$$

The firm's variable labor input share out of its variable cost, $s_{\ell k}^v$, is a constant but lower than the Cobb-Douglas parameter $\alpha_{\ell k}$ as a result of the upward-sloping labor supply curve:

$$s_{\ell k}^v = \frac{\frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}}{1 - \alpha_{\ell k} + \frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}}. \tag{B.5}$$

The share of variable inputs from firm j out of firm k's variable cost can be expressed as the share of variable input purchases times the share of firm j's goods out of the variable input purchases:

$$s_{jk}^v = \frac{1 - \alpha_{\ell k}}{1 - \alpha_{\ell k} + \frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}} \frac{\left(\omega_{jk}^v \right)^\sigma p_j^{1-\sigma}}{\sum_{j \in Z_k} \left(\omega_{jk}^v \right)^\sigma p_j^{1-\sigma} + \left(\omega_{Fk}^v \right)^\sigma p_{Fk}^{1-\sigma}}. \tag{B.6}$$

Analogously, the share of variable imports in variable cost is expressed as

$$s_{Fk}^v = \frac{1 - \alpha_{\ell k}}{1 - \alpha_{\ell k} + \frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}} \frac{\left(\omega_{Fk}^v \right)^\sigma p_{Fk}^{1-\sigma}}{\sum_{j \in Z_k} \left(\omega_{jk}^v \right)^\sigma p_j^{1-\sigma} + \left(\omega_{Fk}^v \right)^\sigma p_{Fk}^{1-\sigma}}. \tag{B.7}$$

Similar to the variable input purchases, one can write firm j's share and import share in firm k's total purchases of fixed intermediate inputs as follows:

$$s_{jk}^{f} = \frac{\left(\omega_{jk}^{f}\right)^{\sigma} p_j^{1-\sigma}}{\sum_{j\in Z_k} \left(\omega_{jk}^{f}\right)^{\sigma} p_j^{1-\sigma} + \left(\omega_{Fk}^{f}\right)^{\sigma} p_{Fk}^{1-\sigma}} \tag{B.8}$$

$$s_{Fk}^{f} = \frac{\left(\omega_{Fk}^{f}\right)^{\sigma} p_{Fk}^{1-\sigma}}{\sum_{j\in Z_k} \left(\omega_{jk}^{f}\right)^{\sigma} p_j^{1-\sigma} + \left(\omega_{Fk}^{f}\right)^{\sigma} p_{Fk}^{1-\sigma}}. \tag{B.9}$$

Firm-level sales consist of the sum of domestic sales to other firms as either variable or fixed inputs, domestic sales to domestic final demand, and exports. Therefore, we have the following equation for firm k's sales:

$$p_k q_k = \sum_{i\in W_k} s_{ki}^{v} \frac{p_i q_i}{\mu_i} + \sum_{i\in W_k} s_{ki}^{f} c_i^{f} \bar{q}_i^{f} + s_{kH} E_H + p_k^{1-\sigma} D_{kF}, \tag{B.10}$$

where W_k is the set of firm k's domestic buyers and

$$s_{kH} = \frac{\beta_{kH}^{\sigma-1} p_k^{1-\sigma}}{\sum_j \beta_{jH}^{\sigma-1} p_j^{1-\sigma}} \tag{B.11}$$

is firm k's share in household expenditure.

We close the model by assuming that all variable profits generated by firms are transferred back to households. We obtain the following expression for aggregate household income:

$$E_H = \sum_k w_k \ell_k^{v} + \sum_k \frac{\mu_k - 1}{\mu_k} p_k q_k - \sum_j \sum_k p_j q_{jk}^{f} - \sum_k p_{Fk} q_{Fk}^{f} - TB, \tag{B.12}$$

where TB is the aggregate trade balance. Labor market clearing implies that firms' labor demand equals the total labor supply in each labor market:

$$L_m = \sum_k \frac{1}{w_k} s_{\ell k}^{v} \frac{p_k q_k}{\mu_k} + \sum_k \bar{\ell}_k^{f}. \tag{B.13}$$

Definition 2 (Equilibrium)**.** *Given the set of price of imports p_{Fk}, foreign demand shifters D_{kF}, aggregate trade balance TB, aggregate labor supply L, firms' domestic supplier sets Z_k and their importing and exporting decisions, and firms' fixed overhead input requirements $\bar{q}_k^f$ and $\bar{\ell}_k^f$, an equilibrium is the firms' wages, $\{w_k\}$, and the aggregate expenditure, E_H, such that equations* (2.6)–(2.8), (2.12)–(2.14), *and* (B.2)–(B.13) *hold.*

B.2.2 Derivations of equations (2.16) *and* (2.17)

To obtain equation (2.16), we take the total derivative of equation (2.11) while holding supply-side technology parameters fixed. From equation (2.11), the right-hand side of which is constant, we have

$$d\log \ell_k^v w_k\left(\ell_k\right) = d\log p_k q_k\left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right).$$

Further rearranging using equation (2.9), we obtain

$$\begin{aligned} d\log \ell_k w_k\left(\ell_k\right) + d\log\left(1 - \frac{\bar{\ell}_k^f}{\ell_k}\right) &= d\log p_k q_k\left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right) \\ d\log \ell_k w_k\left(\ell_k\right) + \frac{\bar{\ell}_k^f}{\ell_k^v} d\log \ell_k &= d\log p_k q_k\left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right). \end{aligned}$$

We know from the labor supply curve of equation (2.6) that

$$\begin{aligned} d\log \ell_k &= \varepsilon d\log w_k \\ &= \frac{\varepsilon}{1+\varepsilon} d\log w_k \ell_k. \end{aligned}$$

Plugging this in, we have

$$\left(1 + \frac{\bar{\ell}_k^f}{\ell_k^v}\frac{\varepsilon}{1+\varepsilon}\right) d\log w_k \ell_k = d\log p_k q_k\left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right),$$

and hence

$$\frac{d\log w_k\ell_k}{d\log p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}=\frac{\ell_k^v}{\ell_k}\frac{1+\varepsilon}{\frac{\ell_k^v}{\ell_k}+\varepsilon}.$$

We take a similar approach in deriving equation (2.17). The output elasticity in equation (2.15) can be written as

$$\frac{\partial\log q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}{\partial\log q_{jk}^v}=\left(1-\alpha_{\ell k}\right)\omega_{jk}^v\left(\frac{q_{jk}^v}{q_k^v}\right)^{\frac{\sigma-1}{\sigma}},$$

where q_k^v is the CES bundle of variable intermediate inputs. The term $\frac{q_{jk}^v}{q_k^v}$ depends only on the relative prices of firm k's suppliers, which we assume to be constant. Then one can write the total derivative of equation (2.15) as

$$d\log p_j q_{jk}^v=d\log p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right).$$

Further rearranging using equation (2.9), we obtain

$$d\log p_j q_{jk}+d\log\left(1-\frac{q_{jk}^f}{q_{jk}}\right)=d\log p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right).$$

The fixed input purchases from firm j are given by equation (2.13) and only depend on the prices of firm j and the prices of other suppliers of firm k, which are all taken as fixed. Hence, one can further rearrange and obtain the following:

$$\begin{aligned}d\log p_j q_{jk}+\frac{q_{jk}^f}{q_{jk}^v}d\log q_{jk}&=d\log p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)\\ \frac{d\log p_j q_{jk}}{d\log p_k q_k\left(\ell_k^v,\left\{q_{jk}^v\right\},q_{Fk}^v\right)}&=\frac{q_{jk}^v}{q_{jk}}.\end{aligned}$$

B.2.3 System of counterfactual changes in variables

Counterfactual changes in response to import price and foreign demand shocks

As outlined in Appendix B.4.1, we assume that firms charge a common markup of $\frac{\sigma}{\sigma-1}$, as in Section 2.4, and in the baseline assume that firms have monopsony power in labor markets, $\varepsilon = 3.5$. We introduce the term adj_k, which represents the discrepancy between a firm's theory-implied variable input cost, $\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k$, and its observed variable input cost, $varinput_k = w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v$:

$$adj_k = \underbrace{\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k}_{\text{theory implied input}} - \underbrace{varinput_k}_{\text{observed input}}.$$

In this counterfactual exercise, we assume that the ratio of adj_k relative to the firm's variable inputs is fixed, leading to the following relationship:

$$\widehat{varinput}_k = \frac{\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k}{varinput_k}\widehat{p_k q_k} - \frac{adj_k}{varinput_k}\widehat{adj}_k,$$

where we have $\widehat{varinput}_k = \widehat{p_k q_k} = \widehat{adj}_k$.

Therefore, the steps to solve for the counterfactual outcomes are as follows:

1. Guess the changes in firm-level wages $\hat{w}_k$. If $\varepsilon = \infty$, then the guess is common across all firms.

2. Compute the firm-level changes in total and variable labor inputs, $\hat{\ell}_k$ and $\hat{\ell}_k^v$, using equations (2.6) and (2.9):

$$\hat{\ell}_k = \frac{\hat{w}_k^{\varepsilon}}{\sum_j \ell_j \hat{w}_j^{\varepsilon}} L$$
$$\hat{\ell}_k^v = \frac{\ell_k}{\ell_k^v}\hat{\ell}_k - \frac{\bar{\ell}_k^f}{\ell_k^v}.$$

Skip this step if $\varepsilon = \infty$.

3. Solve for $\hat{c}_k$ using equation (B.3):

$$\hat{c}_k = \left(\sum_{j \in Z_k} \frac{s^v_{jk}}{1 - s^v_{\ell k}} \hat{c}_j^{1-\sigma} + \frac{s^v_{Fk}}{1 - s^v_{\ell k}} \hat{p}_{Fk}^{1-\sigma} \right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} \hat{w}_k^{\alpha_{\ell k}}.$$

4. Compute the change in shares and prices using equations (B.6), (B.7), (B.8), (B.9), and (B.11):

$$\begin{aligned}
\hat{s}^v_{jk} &= \frac{\hat{c}_j^{1-\sigma}}{\sum_{j \in Z_k} \frac{s^v_{jk}}{1-s^v_{\ell k}} \hat{c}_j^{1-\sigma} + \frac{s^v_{Fk}}{1-s^v_{\ell k}} \hat{p}_{Fk}^{1-\sigma}} \\
\hat{s}^v_{Fk} &= \frac{\hat{p}_{Fk}^{1-\sigma}}{\sum_{j \in Z_k} \frac{s^v_{jk}}{1-s^v_{\ell k}} \hat{c}_j^{1-\sigma} + \frac{s^v_{Fk}}{1-s^v_{\ell k}} \hat{p}_{Fk}^{1-\sigma}} \\
\hat{s}^f_{jk} &= \frac{\hat{c}_j^{1-\sigma}}{\left(\hat{c}^f_k\right)^{1-\sigma}} \\
\hat{s}^f_{Fk} &= \frac{\hat{p}_{Fk}^{1-\sigma}}{\left(\hat{c}^f_k\right)^{1-\sigma}} \\
\left(\hat{c}^f_k\right)^{1-\sigma} &= \sum_{j \in Z_k} s^f_{jk} \hat{c}_j^{1-\sigma} + s^f_{Fk} \hat{p}_{Fk}^{1-\sigma} \\
\hat{s}_{kH} &= \frac{\hat{c}_k^{1-\sigma}}{\sum_j s_{jH} \hat{c}_j^{1-\sigma}}.
\end{aligned}$$

5. Solve for $\widehat{p_k q_k}$ using equation (B.10):

$$\widehat{p_k q_k} = \sum_{i \in W_k} r^v_{ki} \hat{s}^v_{ki} \widehat{p_i q_i} + \sum_{i \in W_k} r^f_{ki} \hat{s}^f_{ki} \hat{c}^f_i + r_{kH} \hat{s}_{kH} \hat{E}_H + r_{kF} \hat{c}_k^{1-\sigma} \hat{D}_{kF}$$

where we have the following revenue shares of firm k:

$$r^v_{ki} = \frac{s^v_{ki} p_i q_i}{p_k q_k \mu_i}, \; r^f_{ki} = \frac{p_k q^f_{ki}}{p_k q_k}, \; r_{kH} = \frac{s_{kH} E_H}{p_k q_k}, \; r_{kF} = \frac{p_k^{1-\sigma} D_{kF}}{p_k q_k}.$$

The change in aggregate household expenditure is written as

$$\begin{aligned}\hat{E}_H =& \sum_k \frac{w_k \ell_k}{E_H} \hat{w}_k + \sum_k \frac{\pi^v_k}{E_H} \widehat{p_k q_k} - \sum_j \sum_k \frac{p_j q^f_{jk}}{E_H} \hat{s}^f_{jk} \hat{c}^f_k - \sum_k \frac{p^f_{Fk} q^f_{Fk}}{E_H} \hat{s}^f_{Fk} \hat{c}^f_k \\ & - \sum_k \frac{w_k \bar{\ell}^f_k}{E_H} \hat{w}_k - \frac{TB}{E_H} + \sum_k \frac{adj_k}{E_H} \widehat{adj}_k, \end{aligned} \tag{B.14}$$

where $\widehat{adj}_k = \widehat{p_k q_k}$ and $\pi^v_k = \left(1 - \left(\frac{\varepsilon_m}{\varepsilon_m + 1} \alpha_{\ell k} + 1 - \alpha_{\ell k}\right) \frac{\sigma - 1}{\sigma}\right) p_k q_k$.

6. Update $\hat{w}_k$ with the following and iterate from Step 2 until $\hat{w}_k$ converges:

$$\begin{aligned}\hat{w}^{new}_k &= \frac{\widehat{p_k q_k}}{\hat{\ell}^v_k} \\ \hat{w}_k &= d \hat{w}^{new}_k + (1 - d) \hat{w}_k.\end{aligned}$$

If $\varepsilon = \infty$, then use the following to update the common guess of wage change:

$$\hat{w}^{new}_k = \frac{\sum_k w_k \ell^v_k \widehat{p_k q_k} + \sum_k w_k \bar{\ell}^f_k \hat{w}_k}{\sum_k w_k \ell_k}.$$

7. Finally, check that the trade balance holds (i.e., the exogenous TB is unchanged).

Counterfactual changes in response to firm productivity shocks

The system of counterfactual changes in variables when one considers changes in firms' productivities is similar to that presented in Appendix B.2.3. Instead of the changes in import price $\hat{p}_{Fk}$, we consider changes in productivities, $\hat{\phi}_k$. Hence, we replace Step 3 in

Appendix B.2.3 with the following equation that solves for $\hat{c}_k$ given the shocks $\hat{\phi}_k$ and guess of $\hat{w}_k$:

$$\hat{c}_k = \hat{\phi}_k^{-1} \left(\sum_{j \in Z_k} \frac{s_{jk}^v}{1 - s_{\ell k}^v} \hat{c}_j^{1-\sigma} \right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} \hat{w}_k^{\alpha_{\ell k}}.$$

B.2.4 Total import shares

Consider a change in the price of imported goods when the labor market is competitive ($\varepsilon = \infty$). The first-order approximated change in the aggregate price index upon small changes in prices of imports $\frac{dp_{Fk}}{p_{Fk}}$, foreign demand shifters $\frac{dD_{kF}}{D_{kF}}$, and the changes in wages $\frac{dw}{w}$ can be computed as follows. First, the changes in firm k's marginal costs $\frac{dc_k}{c_k}$ can be written as

$$\frac{dc_k}{c_k} = \sum_{j \in Z_k} s_{jk}^v \frac{dc_j}{c_j} + s_{Fk}^v \frac{dp_{Fk}}{p_{Fk}} + \alpha_{\ell k} \frac{dw}{w}. \tag{B.15}$$

Changing to vector notation, this can be further arranged to

$$d\boldsymbol{c} = \left(I - S^{'} \right)^{-1} \left(d\boldsymbol{c}_F + d\boldsymbol{c}_L \right), \tag{B.16}$$

where $d\boldsymbol{c}$ is a $I \times 1$ vector whose k's element is the percentage change in k's marginal cost, $\frac{dc_k}{c_k}$. The $I \times I$ matrix S records the variable input cost shares from the domestic production network—the (j, k) element of matrix S is s_{jk}^v. The cost-based Leontief inverse matrix $\left(I - S^{'} \right)^{-1}$ captures firms' overall exposure to all other firms as buyers of their goods. The $I \times 1$ vectors $d\boldsymbol{c}_F$ and $d\boldsymbol{c}_L$ record the direct variable cost effect of import price and labor cost changes: the k'th element of $d\boldsymbol{c}_F$ is $s_{Fk}^v \frac{dp_{Fk}}{p_{Fk}}$, and the k'th element of $d\boldsymbol{c}_L$ is $\alpha_{\ell k} \frac{dw}{w}$. The aggregate price change is a weighted average of the firm-level cost changes, using the household expenditure share on firm k, s_{kH}, as the weight:

$$\frac{dP}{P} = \sum_k s_{kH} \frac{dc_k}{c_k}. \tag{B.17}$$

If one assumes a uniform price change across all imports, then the above equation for firms' cost changes becomes

$$\frac{dc_k}{c_k} = s_{Fk}^{v,Total} \frac{dp_F}{p_F} + \left(1 - s_{Fk}^{v,Total}\right) \frac{dw}{w}, \tag{B.18}$$

where

$$s_{Fk}^{v,Total} = s_{Fk}^{v} + \sum_{j \in Z_k} s_{jk}^{v} s_{Fj}^{v,Total} \tag{B.19}$$

is the total import share of variable inputs.

B.3 Additional empirical results

B.3.1 Additional results on exporter premium on wages

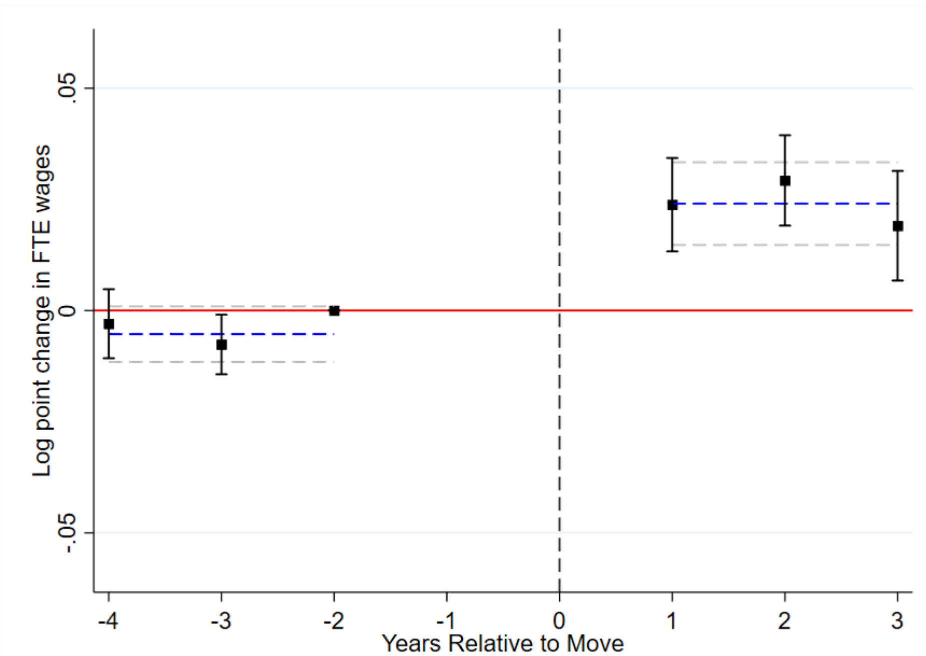
In Section 2.3.3, we run a set of wage regressions on a sample of movers to estimate wage premium for firms that directly export. Identification is achieved from a common trend assumption in the workers' wages in the absence of moving to direct exporters. In this section, we empirically assess this assumption by performing the following movers analysis. We consider a sample of workers who switch their main jobs between $t-1$ and t and have tenures of no fewer than four years at both origin and destination firms. We then use the balanced panel of movers from $t-4$ to $t+3$ and estimate the effects of moving from non-exporters to exporters by running the following regression:

$$\log w_{n,s} = \sum_{\kappa=-4}^{3} \eta_\kappa \mathbf{1}[s=\kappa] + \sum_{k=-4}^{3} \tau_\kappa \mathbf{1}[s=\kappa, T(n)=1] + \zeta_n + \xi_{n,s}, \tag{B.20}$$

where $\log w_{n,s}$ denotes mover n's log wage in year s (relative to the year of move), $T(n)$ is an indicator for the move from non-exporters to exporters, and ζ_n is a worker fixed effect. In order to ensure that we only use full-year employment spells in a given firm, we drop the observations in years $t-1$ and t. We also pool all movers in the regression and assume that the effects of moving from exporters to non-exporters are symmetric.

Figure B.1 presents a graphical representation of the exporter wage premium. In this figure, we report the estimated coefficients τ_κ in equation (B.20) for κ from -4 to 3 and normalize the estimates by setting $\tau_{-2}=0$. As in Table 2.2, we additionally control for calendar year effects, observable time-varying worker characteristics, and sector fixed effects. Our findings support common trends prior to the move, suggesting that the wage growth of workers moving to a firm that does not directly export can be a valid counterfactual for those moving to a firm that directly exports.

Figure B.1: Graphical representation of exporter wage premium from movers analysis



Notes: This figure uses the subsample of firms for which we have additional information from the worker data (see Section 2.2.3 for details). We run a worker-level regression based on equation (B.20) and report the estimated coefficients $\{\tau_\kappa\}_{\kappa=-4}^{3}$. We define movers in year t as workers who are employed by the origin firms at no later than $t-4$, switch their jobs between $t-1$ and t, and stay at their destination firms at least until $t+3$. The sample of movers is balanced from $t-4$ to $t+3$. Observations in years $t-1$ and t are dropped from the regression to ensure that we only use full-year employment spells in a given firm. The estimates are normalized by setting $\tau_{-2}=0$. The assignment to the exporter or non-exporter category is made based on firms' export participation status at $t-2$ for both origin and destination firms. Industry fixed effects are included at the NACE four-digit level.

B.3.2 Fixed labor input shares by firm categories

In Section 2.5.2, we assume that the fixed share of labor inputs is homogeneous across all firms in the Belgian economy. In this section, we allow the fixed shares of labor inputs to vary across firm categories. Table B.3 reports our estimates when we distinguish between exporters and non-exporters or between manufacturing firms and non-manufacturing firms. In doing so, we first estimate the cumulative elasticities of labor cost and employment for each firm category by interacting our IV model in equation (2.23) with firm categories. We then use equation (2.18) and equation (2.16) to solve for the labor supply elasticity (ε) and fixed share of labor inputs ($1 - \ell_k^v/\ell_k$), respectively. In the third column, we also report the weighted averages of fixed labor input shares, weighted by the shares of aggregate sales by firm categories. We find that these weighted averages are not substantially different from our main estimate in the first row, in which the fixed shares of labor inputs are assumed to be homogeneous across all firms.

Table B.3: Labor supply elasticities and fixed shares of labor inputs by firm categories

	Labor supply elasticity (ε)	Fixed share of labor inputs ($1 - \ell_k^v/\ell_k$)	
		by category	weighted average
All firms	3.48	0.52	
Exporters	3.23	0.52	0.52
Non-exporters	3.83	0.52	
Manufacturing firms	3.41	0.63	0.55
Non-manufacturing firms	3.73	0.45	

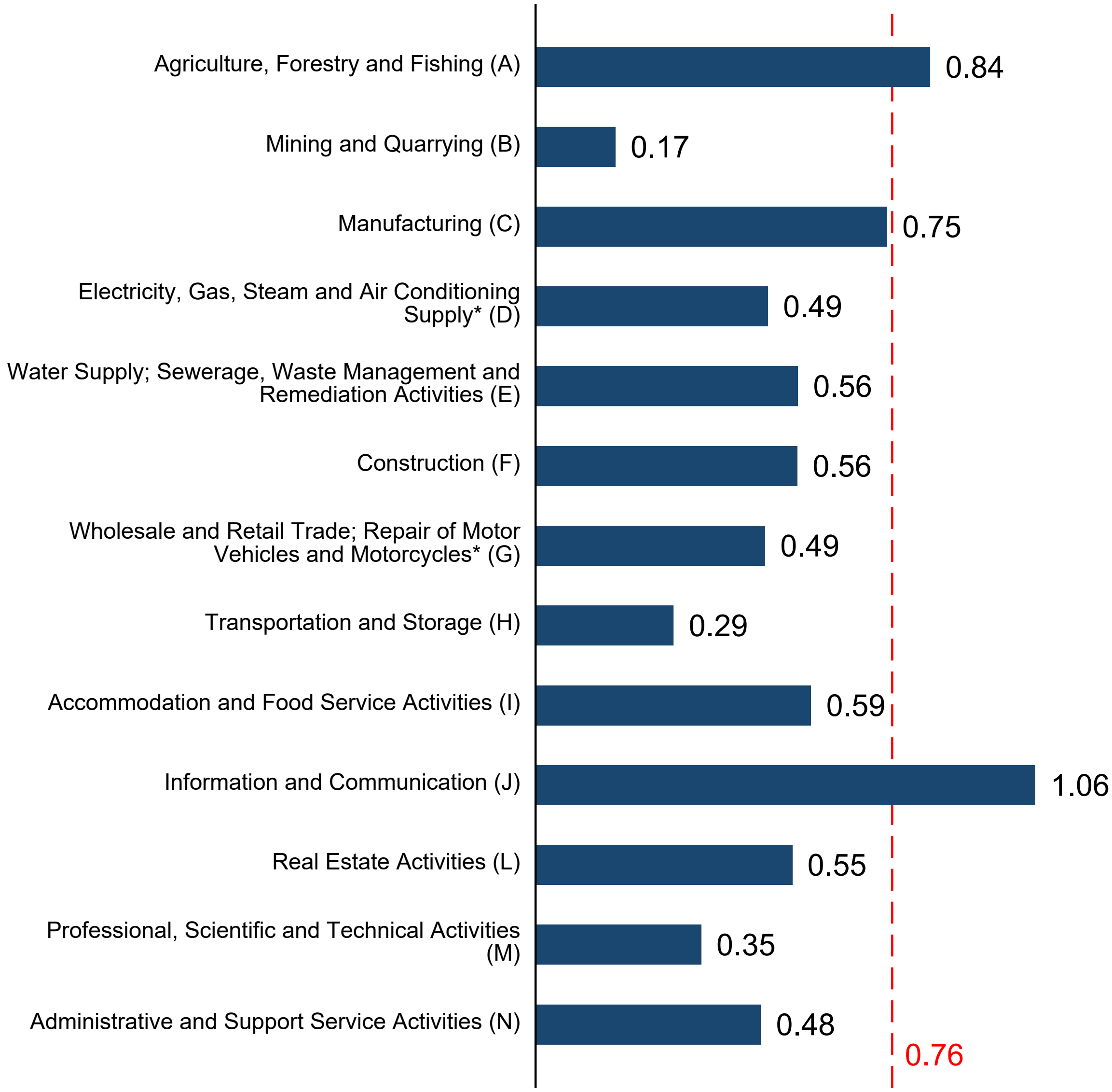
Notes: This table uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). For each row, we report the labor supply elasticity (ε) and fixed share of labor inputs ($1 - \ell_k^v/\ell_k$) by firm category. To obtain the labor supply elasticity, we first estimate the cumulative elasticities ($\sum_{\kappa=0}^{3} \gamma^\kappa$) of labor cost and employment based on equation (2.23) and use equation (2.18) to solve for the labor supply elasticity. We then use equation (2.16) to solve for the variable share of labor inputs (ℓ_k^v/ℓ_k). We use the shares of aggregate sales by firm categories as weights when computing the weighted average of fixed labor input shares.

B.3.3 *Elasticities of input purchases by suppliers' industries*

In this section, we allow the elasticity of input purchases, and, thus, the fraction of an input that is used as a fixed factor, to vary across the types of inputs. In order to estimate the cumulative elasticities of input purchases by different types of inputs, we first categorize the purchases of inputs by the industry of supplier for domestic purchases and by the HS product code for import transactions. We then use an HS to NACE concordance to map the product-level import transactions to the industry level, so that we classify both domestic and foreign input purchases by supplying industries.

Figure B.2 shows the cumulative elasticities of (domestic and foreign) input purchases at the NACE one-digit level. We report those elasticities relative to the cumulative increase in total sales of 0.76, as referenced by the dotted red line. For instance, we find that purchases from the manufacturing industry, which account for around half of all input purchases in the Belgian economy, increase by 7.5 percent when firms receive foreign demand shocks to increase their sales by 7.6 percent. On the other hand, input purchases from most of the service industry (NACE G to N one-digit sectors) do not increase as much, implying that service inputs have higher fixed input cost shares.

Figure B.2: Elasticities of input purchases by suppliers' NACE one-digit industries



Notes: This figure uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). For each bar, we report the cumulative elasticities of input purchases from suppliers' respective industries. To compute the cumulative elasticities, we run four firm-level regressions based on equation (2.23) for κ from 0 to 3 and compute the sum of four coefficients $\{\gamma^{\kappa}\}_{\kappa=0}^{3}$. Variables are winsorized at the top and bottom 0.5 percentiles. The dotted red line corresponds to the cumulative response of sales. We report the cumulative elasticities at NACE one-digit sections. We exclude the public and financial sectors from our sample, and we drop NACE S (Other Service Activities) because of the small sample size. (*) We include the input purchases from NACE 46.71 (Wholesale of solid, liquid and gaseous fuels and related products) and NACE 47.3 (Retail sale of automotive fuel in specialised stores) in NACE D (Electricity, Gas, Steam and Air Conditioning Supply) instead of NACE G (Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles).

B.3.4 Specification checks

In this section, we consider several alternative specifications to our main results presented in Table 2.3. For each alternative specification, we also report the cumulative increase in sales ($\sum_{\kappa=0}^{3} \beta^{\kappa}$ in equation (2.22)) relative to its instantaneous response to the foreign demand shock (β^0). Table B.4 shows the sensitivity of our results to additionally controlling for location-year fixed effects. We use level 2 of the Eurostat NUTS classification as a measure of location. In Table B.5, we restrict our estimation sample to a balanced panel of firms that are observed for at least seven consecutive years (from κ equal to -3 to 3). Table B.6 reports the results in which we weight each firm by its lagged employment. In these specifications, our IV estimates relative to the cumulative increase in sales are not substantially affected.

Table B.4: IV estimates of the impact of changes in firm sales that are induced by the foreign demand shocks: including location-year fixed effects

	(1) Sales	(2) Average wage	(3) FTE Employment	(4) Labor cost	(5) Input purchases	(6) Domestic input purchases
First stage						
Instantaneous response	0.312***					
(β^0)	(0.0261)					
Cumulative response	0.235***					
($\sum_{\kappa=0}^{3} \beta^{\kappa}$)	(0.0291)					
Ratio	0.754					
($\sum_{\kappa=0}^{3} \beta^{\kappa} / \beta^0$)						
Second stage						
Instantaneous response		0.0897***	0.0667**	0.155***	0.942***	0.760***
(γ^0)		(0.0299)	(0.0321)	(0.0369)	(0.0765)	(0.0670)
Cumulative response		0.108**	0.323***	0.432***	0.779***	0.597***
($\sum_{\kappa=0}^{3} \gamma^{\kappa}$)		(0.0456)	(0.0571)	(0.0705)	(0.145)	(0.126)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes	
Location-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). In Column (1), we estimate the responses of sales to the total export shock defined in Section 2.4.3. We run four firm-level regressions based on equation (2.22) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (β^0) as well as the cumulative response (the sum of four coefficients $\{\beta^{\kappa}\}_{\kappa=0}^{3}$) and compute their ratio. For each outcome variable in Columns (2)-(6), we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (2.23) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (γ^0) as well as the cumulative response (the sum of four coefficients $\{\gamma^{\kappa}\}_{\kappa=0}^{3}$). The first-stage F-statistics for excluded instruments is 142.8. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B.5: IV estimates of the impact of changes in firm sales that are induced by the foreign demand shocks: using a balanced panel of firms

	(1) Sales	(2) Average wage	(3) FTE Employment	(4) Labor cost	(5) Input purchases	(6) Domestic input purchases
First stage						
Instantaneous response	0.423***					
(β^0)	(0.0327)					
Cumulative response	0.223***					
($\sum_{\kappa=0}^{3} \beta^\kappa$)	(0.0363)					
Ratio	0.530					
($\sum_{\kappa=0}^{3} \beta^\kappa / \beta^0$)						
Second stage						
Instantaneous response		0.0999***	0.0400	0.139***	0.863***	0.730***
(γ^0)		(0.0316)	(0.0363)	(0.0345)	(0.0975)	(0.0921)
Cumulative response		0.104***	0.199***	0.307***	0.413***	0.318***
($\sum_{\kappa=0}^{3} \gamma^\kappa$)		(0.0332)	(0.0405)	(0.0567)	(0.0978)	(0.104)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample (see Section 2.2.3 for details). The analysis is based on 307,435 firm-year observations of private-sector firms in Belgium from 2002 to 2014 that are observed for at least seven consecutive years (from κ equal to -3 to 3). In Column (1), we estimate the responses of sales to the total export shock defined in Section 2.4.3. We run four firm-level regressions based on equation (2.22) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (β^0) as well as the cumulative response (the sum of four coefficients $\{\beta^\kappa\}_{\kappa=0}^{3}$) and compute their ratio. For each outcome variable in Columns (2)-(6), we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (2.23) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (γ^0) as well as the cumulative response (the sum of four coefficients $\{\gamma^\kappa\}_{\kappa=0}^{3}$). The first-stage F-statistics for excluded instruments is 160.9. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B.6: IV estimates of the impact of changes in firm sales that are induced by the foreign demand shocks: weighted by employment at $t-1$

	(1) Sales	(2) Average wage	(3) FTE Employment	(4) Labor cost	(5) Input purchases	(6) Domestic input purchases
First stage						
Instantaneous response	0.308***					
(β^0)	(0.0456)					
Cumulative response	0.140**					
($\sum_{\kappa=0}^{3} \beta^\kappa$)	(0.0591)					
Ratio	0.454					
($\sum_{\kappa=0}^{3} \beta^\kappa / \beta^0$)						
Second stage						
Instantaneous response		0.0807*	0.0869	0.195***	1.081***	0.764***
(γ^0)		(0.0471)	(0.0582)	(0.0715)	(0.144)	(0.115)
Cumulative response		0.0484	0.208*	0.268**	0.511***	0.383***
($\sum_{\kappa=0}^{3} \gamma^\kappa$)		(0.0868)	(0.119)	(0.125)	(0.117)	(0.121)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.2.3 for details). In Column (1), we estimate the responses of sales to the total export shock defined in Section 2.4.3. We run four firm-level regressions based on equation (2.22) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (β^0) as well as the cumulative response (the sum of four coefficients $\{\beta^\kappa\}_{\kappa=0}^{3}$) and compute their ratio. For each outcome variable in Columns (2)-(6), we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (2.23) for $\kappa \in \{0, 1, 2, 3\}$ and report the instantaneous response (γ^0) as well as the cumulative response (the sum of four coefficients $\{\gamma^\kappa\}_{\kappa=0}^{3}$). The first-stage F-statistics for excluded instruments is 45.53. Variables are winsorized at the top and bottom 0.5 percentiles. In all regressions, we weight each firm by its employment at $t-1$. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

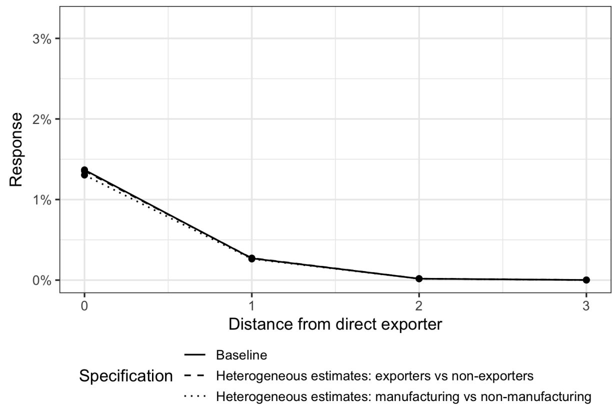
B.3.5 Direct and indirect effects of foreign demand shocks to production networks using heterogeneous estimates

In our main simulation in Section 2.5.4, we assume that all firms in the Belgian economy face the same labor supply curve and have the same fixed labor input share. In this section, we relax this assumption and allow the labor supply elasticities and fixed shares of labor inputs to vary across firm categories. Using the heterogeneous estimates by firm categories reported in Table B.3 of Appendix B.3.2, we redo our simulation of foreign demand shock transmissions along the supply chain.

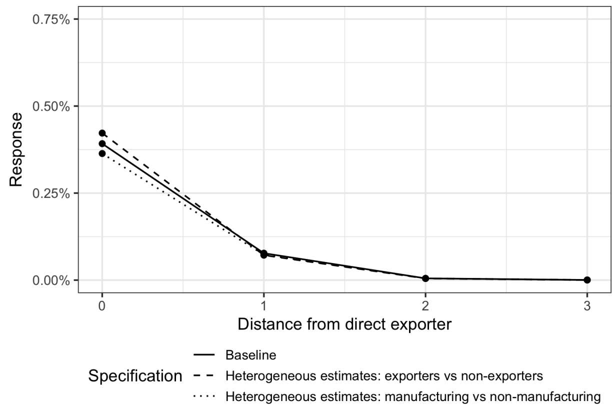
Figure B.3 reports the simulated responses of FTE employment and average wage as well as additional worker rents generated by a foreign demand shock when we allow the labor supply curve elasticities and fixed shares of labor inputs to vary across firm categories. The results from the simulation exercise using separate estimates for the direct exporters and for the firms that do not directly export are reported by the dashed lines, and the dotted lines show the simulation results in which we use separate estimates for manufacturing and non-manufacturing firms. We find that our baseline results in Figure 2.4, also reported in Figure B.3 as the solid lines, are robust to allowing for heterogeneity in the labor supply curve elasticities and fixed shares of labor inputs.

Figure B.3: Simulation results of foreign demand shock transmission along the supply chain: using heterogeneous fixed labor input shares

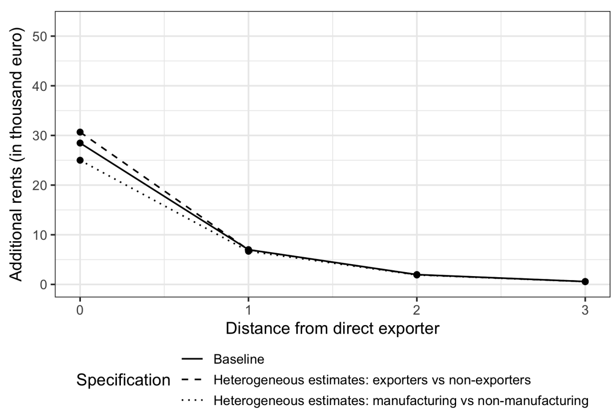
(a) Response of FTE employment



(b) Response of average wage



(c) Additional worker rents



Notes: For each panel, we report the simulation results of the transmission of foreign demand shocks along the supply chain (see the discussion in Section 2.5.4 for how the simulation is done). The first two panels present the employment and wage response at the direct exporter, the direct exporter's key supplier, the key supplier of the exporter's key supplier, and so on. The bottom panel aggregates the rents to the workers in firms that direct export, to workers in their direct suppliers, to workers in their suppliers' suppliers, and so on (up to three links). In each line of every figure, we make different assumptions regarding the fixed shares of labor inputs. For the solid lines, we use our estimated labor supply elasticity $\varepsilon = 3.5$ as well as the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level); and for the stippled and dotted lines, we use heterogeneous labor supply curve elasticities and fixed labor input shares by firm categories reported in Appendix B.3.2.

B.4 Additional counterfactual results

B.4.1 Setup of the counterfactual exercise

In our main counterfactual exercise in Section 2.6, we assume that firms charge a common markup of $\frac{\sigma}{\sigma-1}$ as in Section 2.4 and that firms have monopsony power in labor markets by setting $\varepsilon = 3.5$ using estimates from Section 2.5. We lay out the detailed steps to solve for the counterfactual outcomes in Appendix B.2.3.

By having firms set a common markup of $\frac{\sigma}{\sigma-1}$, we have a discrepancy between a firm's theory implied variable input cost, $\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k$, and its observed variable input cost, $varinput_k = w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v$. We denote these firm-level discrepancies by adj_k:

$$adj_k = \underbrace{\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k}_{\text{theory-implied input}} - \underbrace{varinput_k}_{\text{observed input}} .$$

One interpretation of this term adj_k is the usage of firm k's inventories. If $adj_k > 0$, then the firm is purchasing fewer variable inputs than what is implied from the theory and hence is using the past inventory of inputs to produce. If $adj_k < 0$, then the firm is purchasing more variable inputs than what is implied from the theory and hence is accumulating inventory for future use.

In the counterfactual exercise, we follow Dhyne et al. (2022b) and assume that the ratio of adj_k relative to the firm's variable inputs—both the theory-implied inputs and the observed inputs—is fixed. This is consistent with an interpretation in which the fraction of how much inventory the firm uses (or accumulates) relative to its inputs and sales does not change in response to foreign shocks. With this assumption, we have the following relationship:

$$\widehat{varinput}_k = \frac{\frac{\sigma-1}{\sigma}p_k q_k}{varinput_k}\widehat{p_k q_k} - \frac{adj_k}{varinput_k}\widehat{adj}_k,$$

where we have $\widehat{varinput}_k = \widehat{p_k q_k} = \widehat{adj}_k$.

This treatment of the differences in variable input costs is isomorphic to assuming that firms charge firm-specific markups of $\mu_k = \frac{p_k q_k}{w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v}$, which can be read from the data. To see this, we refer to equation (B.14) in Appendix B.2.3, which illustrates how the change in aggregate income is affected by changes in firms' variable profits $\left(\frac{\pi_k^v p_k q_k}{E_H}\widehat{p_k q_k}\right)$ and the changes in the discrepancy terms $\left(\frac{adj_k}{E_H}\widehat{adj}_k\right)$. If one assumes that firms charge markups of μ_k, then the effect of the changes in their variable profits on aggregate income can be summarized by $\frac{\frac{\mu_k - 1}{\mu_k} p_k q_k}{E_H}\widehat{p_k q_k}$. With the assumption that firm sales and the discrepancy terms move in tandem $\left(\widehat{adj}_k = \widehat{p_k q_k}\right)$, the effects on aggregate income are isomorphic to each other.

B.4.2 Total import shares

To gain intuition on how accounting for firms' fixed inputs affects firms' and aggregate responses to foreign demand shocks, we focus on the firm-level measure of total import share, defined in Dhyne et al. (2021). Because we impose the trade balance condition, the uniform foreign demand shock that we consider in the exercises can also be seen as a shock where the prices of imports uniformly increase.[1] A firm's total import share, which measures how much of the firm's variable inputs originate directly or indirectly from abroad, is a useful statistic that captures the degree of the firm's exposure to the foreign shock.

Firm k's total import share, $s_{Fk}^{v,Total}$, is defined in a recursive manner as follows:

$$s_{Fk}^{v,Total} = s_{Fk}^v + \sum_{j \in Z_k} s_{jk}^v s_{Fj}^{v,Total}, \tag{B.21}$$

where s_{Fk}^v and s_{jk}^v are the shares of foreign imports and inputs from firm j in the firm's

1. This symmetry is called Lerner's symmetry. It implies that the outcomes from this uniform change in foreign demand can be mapped into an equivalent set of outcomes from a uniform change in import prices. In this case, the 5 percent increase in foreign tariffs on Belgian exports is equivalent to a 5 percent uniform increase in the price of Belgian imports.

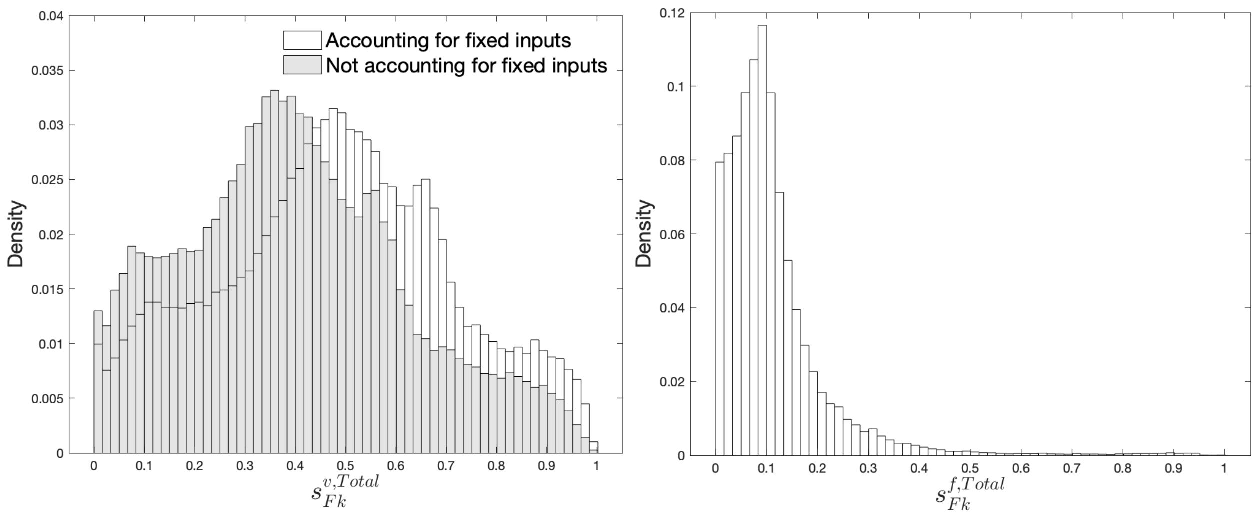
variable costs. As shown in Appendix B.2.4, firms' total import shares become relevant statistics in predicting firm-level outcomes at a first-order approximation: when the labor market is competitive, the costs of firms with higher total import shares increase more than those of firms with lower total import shares in response to a uniform increase in the price of imports.

Through the measure of firms' total import shares, one can see the two main effects of fixed inputs. On the one hand, if for example a large fraction of labor costs is a fixed input, the variable cost shares of s^v_{Fk} and s^v_{jk} become larger. This will magnify any direct cost shock from an import price change and indirect cost shocks from domestic suppliers. On the other hand, some of the foreign inputs are fixed as well, which, all else equal, lowers the direct cost shock through lower values of s^v_{Fk}. Quantitatively, however, more than 80 percent of imports are calculated as variable inputs (based on the estimated elasticities for the NACE one-digit level classification), and since around 50 percent of labor costs are fixed, the direct foreign input share tends to be larger under fixed inputs as well.

Panel (a) of Figure B.4 plots the distributions of the total import shares, $s^{v,Total}_{Fk}$, one accounting for and another not accounting for fixed inputs. When one accounts for fixed inputs, the total import shares of firms in variable costs are larger (with the median firm having a share of 48 percent) than the total import shares of firms when not accounting for fixed inputs (with the median firm having a share of 39 percent). Relatedly, we compute and plot the share of how much of a firm's fixed inputs originate directly or indirectly from abroad in panel (b) of the figure. We find that these shares are generally much lower than the total import share of variable inputs: 9 percent of the median firm's fixed inputs originates from abroad.

Figure B.4: Total import shares

(a) Total import share in variable input costs (b) Total import share in fixed input costs

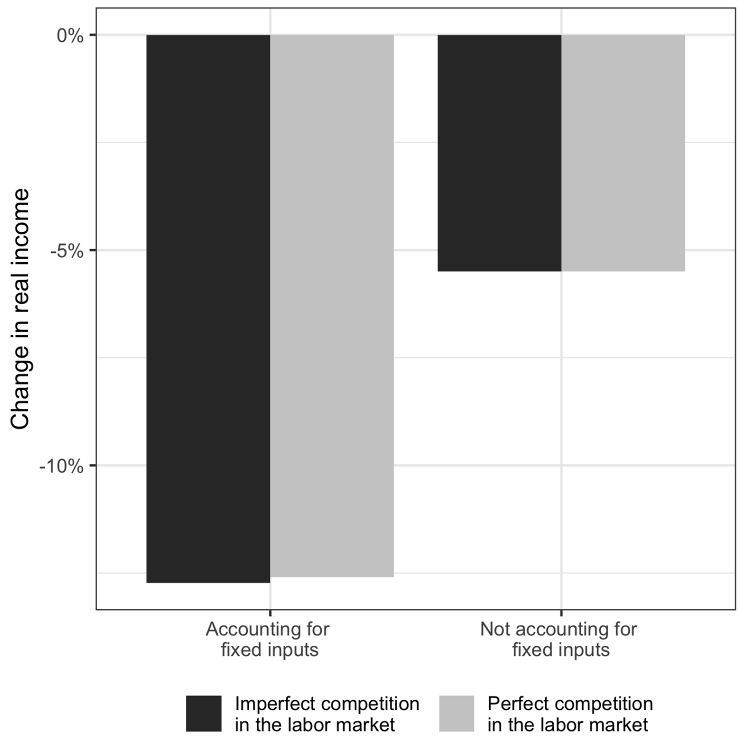


Notes: The left panel shows the distribution of the firm-level total import shares in variable input costs, $s_{Fk}^{v,Total}$, defined in equation (B.21). The white bars show the distribution of the shares when one accounts for fixed inputs, and the grey bars show the distribution of the shares when one does not account for fixed inputs. The right panel shows the distribution of the firm-level total import shares in fixed input costs, $s_{Fk}^{f,Total}$. Firms' total import shares in fixed input costs are defined recursively as in $s_{Fk}^{f,Total} = s_{Fk}^{f} + \sum_{j \in Z_k} s_{jk}^{f} s_{Fj}^{f,Total}$.

B.4.3 Change in real income

Figure B.5 reports the changes in real income, $\hat{E}/\hat{P}$, in response to a 5 percent increase in foreign tariffs.

Figure B.5: Changes in real income in response to a 5 percent increase in foreign tariffs

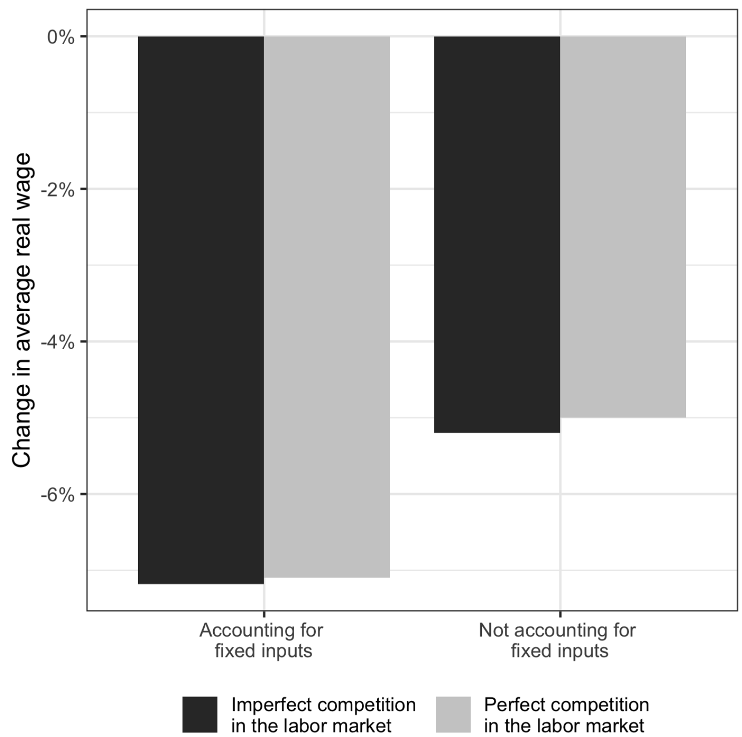


Notes: In this figure, we report the changes in real income, $\hat{E}/\hat{P}$, due to a uniform 5 percent increase in foreign tariffs on Belgian exports. Each bar represents the response under different parameterizations of the model presented in Section 2.4. We use our estimated labor supply elasticity $\varepsilon = 3.5$ in the counterfactual Belgian economies with upward-sloping labor supply curves. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 2.5.

B.4.4 Domestic productivity shocks

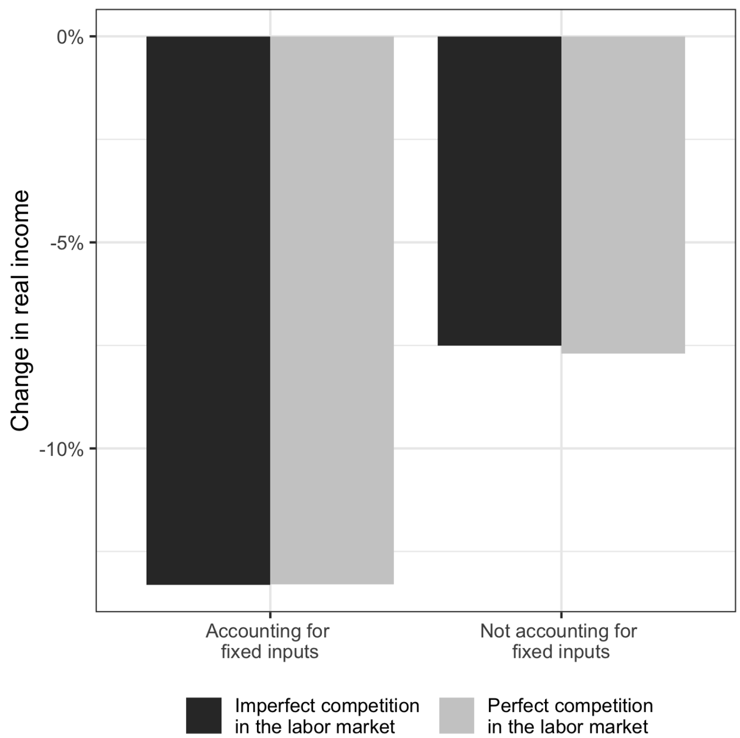
Figures B.6 and B.7 report the changes in average real wage and real income in response to a 5 percent reduction in productivity ϕ_k for all manufacturing firms. We outline the steps to solve for the counterfactual outcomes in Appendix B.2.3.

Figure B.6: Changes in average real wage in response to a 5 percent reduction in manufacturing firms' productivity



Notes: In this figure, we report the changes in average real wage, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$, due to a 5 percent reduction in manufacturing firms' productivity. Each bar represents the response under different parameterizations of the model presented in Section 2.4. We use our estimated labor supply elasticity $\varepsilon = 3.5$ in the counterfactual Belgian economies with upward-sloping labor supply curves. Wages are common across all firms under the parameterization in which we assume $\varepsilon = \infty$, hence $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P} = \hat{w} / \hat{P}$. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 2.5.

Figure B.7: Changes in average real income in response to a 5 percent reduction in manufacturing firms' productivity



Notes: In this figure, we report the changes in real income, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$, due to a 5 percent reduction in manufacturing firms' productivity. Each bar represents the response under different parameterization of the model presented in Section 2.4. We use our estimated labor supply elasticity $\varepsilon = 3.5$ in the counterfactual Belgian economies with upward-sloping labor supply curves. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 2.5.

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