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Incorporating Preference Shocks and Liquidity Shocks into the Bond Pricing New Keynesian Model

By

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Abstract

In standard bond pricing macroeconomic models, the weights of expected inflation errors are too high in the nominal yields forecast error variance decomposition. This paper tries to solve the problem by incorporating preference shocks and liquidity shocks into a bond pricing New Keynesian model. This paper documents that the incorporation of preference shocks offers negligible improvements, and the incorporation of liquidity shocks could decrease the weights of expected inflation forecast errors at short maturities, but still offer no improvements at long maturities. Furthermore, this paper provides an optional method to model a liquidity provider and financial intermediaries in a bond pricing New Keynesian model.

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1 Introduction

Economists keep trying to rationalize the characteristics of nominal yield curves using standard bond pricing macroeconomic models. There are some common empirical facts in bond markets that are not well explained by standard models. First, a few standard models generate a downward-sloping nominal yield curve, such as the textbook model with price rigidity in Galí (2015). Second, a number of production economies have relatively small volatility of nominal yields, especially at long maturities, e.g. den Haan (1995), Kung (2015). Third, certain standard models cannot generate significant average positive term premiums, or the term premiums implied in the models are too stable to reject the expectation hypothesis, e.g. Backus *et al.* (1989), Donaldson *et al.* (1990). This contradicts with empirical literature, e.g. Fama (1984a), Fama (1984b).

With the introduction of Epstein-Zin recursive preferences, as well as negative correlation settings between inflation and consumption growth, the endowment economies including Piazzesi & Schneider (2006), Bansal & Shaliastovich (2013), and the production economies including Rudebusch & Swanson (2012) and Kung (2015) all produce sizable average positive term premiums with upward-sloping nominal yield curves. This is because the low consumption growth is accompanied by persistently high inflation in these models.[1] Persistent high inflation is bad news for bond prices, especially for long-term bonds. Moreover, representative households with Epstein-Zin recursive preferences are strongly adverse to low expected consumption growth, so they need higher compensations for such potential risks in the future. The above settings in standard models are successful in matching the stylized facts in bond markets.

However, as critiqued by Duffee (2018), although above standard models are successful in rationalizing the stylized facts, expected inflation innovations in these models drive too much variation of nominal yields innovations. In other words, the weights of expected inflation errors in these models are too high in the forecast error variance decomposition of nominal yields. This puts forward a serious question about the effectiveness of these standard models, because his empirical results show that the inflation variance ratio – which reflects the weights of expected inflation errors in the nominal yields forecast error variance decomposition – are usually less than 20% at monthly frequency, while the counterparts in above standard bond pricing models exceed 100%. Different parameterizations of standard models have been tried in Duffee (2018), while little improvements are achieved.

To solve Duffee (2018)'s critique, Gomez-Cram & Yaron (2021) introduce preference shocks into households' preference settings with an endowment economy. In their model, preference shocks play a key role in decreasing the inflation variance ratio – from around 80% to less than 20% measured by the posterior median at the ten-year maturity. Most importantly, they document that the smoothed preference shock series have strong negative correlations with liquidity measures like the AFNC index. [2] Thus, as they suggested, it is

[1] In a production economy with price rigidity, a positive productivity shock usually causes marginal costs to decrease, then the inflation would decrease correspondingly, and vice versa. The settings for the bond pricing New Keynesian models proposed in Rudebusch & Swanson (2012) and Kung (2015) both imply that technology shocks would have persistent effects on inflation.

[2] The Adjusted National Financial Conditions Index. Positive values of the ANFCI have been historically associated with financial conditions that are tighter than what would be typically suggested by prevailing

meaningful to model a liquidity provider and financial intermediaries to examine the effects of liquidity shocks on nominal yields, which may be an alternative method to solve Duffee (2018)'s critique without reference to preference shocks.

From the empirical side, the effect of liquidity shocks on nominal bond prices and yields has been well documented in a lot of literature. The increasing bond demands and market liquidity caused by the large-scale purchasing of central banks have significant and persistent effects on the nominal yield curve, e.g. Greenwood & Vayanos (2014), Swanson (2021).[3] While this influence channel is not well captured in above standard bond pricing macroeconomic models. Thus, the lack of liquidity shocks could be an important source for the high inflation variance ratio.

This paper tries to solve Duffee (2018)'s critique using both potential solutions based on the bond pricing New Keynesian model proposed in Kung (2015). Preference shocks are incorporated in section 2, however, negligible improvements are achieved. Therefore, the central bank is modeled as a liquidity provider referring to Sims & Wu (2021) and Sims *et al.* (2021) in section 3. The paper documents that by incorporating liquidity shocks, the contribution of expected inflation innovations at short maturities decreases a lot. However, no improvements could be found at long maturities. Section 3 also tries to incorporate both shocks at the same time, and the results are similar with the situation only having liquidity shocks. Section 4 concludes.

2 Incorporating Preference Shocks

In this section, I incorporate preference shocks into the bond pricing New Keynesian model proposed in Kung (2015). I keep the settings of the production sector, the central bank, bond pricing schemes, and exogenous processes the same as Kung's original settings. The only difference are the representative households' preferences.

2.1 Households

As shown in equation (2.1), representative households maximize recursive utilities over a modified variable $C_t^\star \equiv C_t(\overline{L}-L_t)^\tau$, which is a combination of consumption C_t and working time L_t. $\overline{L}$ represents the time endowment, so $\overline{L} - L_t$ represents leisure. τ measures the weight of leisure in households' utility function compared with consumption.

$$U_t = \max_{C_t^\star} \quad \{(1-\beta)\lambda_t (C_t^\star)^{\frac{1-\gamma}{\theta}} + \beta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\}^{\frac{\theta}{1-\gamma}} \tag{2.1}$$

macroeconomic conditions. https://www.chicagofed.org/research/data/nfci/background

[3]This could also be checked with a simple reduced-form VAR model. Following the method in Diebold *et al.* (2006), I incorporate macro factors including the quantity of treasury notes issued by the Fed, as well as the yield curve level, slope, and, curvature factors in the DNS Model into one VAR. The results are similar. The DNS model is proposed in Diebold & Li (2006), which well captured the shape and movements of the nominal yield curves.

In equation (2.1), $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ is a parameter defined for convenience, where γ measures the risk aversion of representative households, and ψ measures the elasticity of intertemporal substitution.[4] The preference shock $\varepsilon_{\lambda,t}$ has effects on households' preferences through the preference coefficient λ_t. The log of the parameter coefficient is assumed to follow the AR1 process shown in equation (2.2) with time-varying volatility shown in equation (2.3).

$$\ln \lambda_t = \rho_\lambda \ln \lambda_{t-1} + \sigma_t \varepsilon_{\lambda,t}, \qquad \varepsilon_{\lambda,t} \sim N(0,1) \tag{2.2}$$

$$\sigma_t^2 - \overline{\sigma}^2 = \lambda(\sigma_{t-1}^2 - \overline{\sigma}^2) + \sigma_e e_t, \qquad e_t \sim N(0,1) \tag{2.3}$$

The households are subject to the budget constraint shown in equation (2.4), where P_t represents the aggregate price level, B_t represents the total nominal value of bonds held by the households, R_{t+1} represents the interest rates set by the central bank in period t, D_t stands for the dividends of firms, W_t stands for the nominal wage.

$$P_t C_t + \frac{B_{t+1}}{R_{t+1}} = D_t + W_t L_t + B_t \tag{2.4}$$

The intertemporal condition of the households is shown in equation (2.5) and equation (2.6).

$$1 = E_t[M_{t+1} \frac{P_t}{P_{t+1}}] R_{t+1} \tag{2.5}$$

$$1 = E_t[M_{t+1}^{\$}] R_{t+1} \tag{2.6}$$

where the real pricing kernel or the real stochastic discount factor, M_{t+1}, and the nominal pricing kernel or the nominal stochastic discount factor, $M_{t+1}^{\$}$ are shown in equation (2.7) and (2.8).

$$M_{t+1} = \beta(\frac{\lambda_{t+1}}{\lambda_t})(\frac{C_{t+1}^{\star}}{C_t^{\star}})^{\frac{1-\gamma}{\theta}}(\frac{C_{t+1}}{C_t})^{-1}(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]})^{\frac{\theta-1}{\theta}} \tag{2.7}$$

$$M_{t+1}^{\$} = \beta(\frac{\lambda_{t+1}}{\lambda_t})(\frac{C_{t+1}^{\star}}{C_t^{\star}})^{\frac{1-\gamma}{\theta}}(\frac{C_{t+1}}{C_t})^{-1}(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]})^{\frac{\theta-1}{\theta}} \frac{P_t}{P_{t+1}} \tag{2.8}$$

2.2 Bond Pricing

According to the standard consumption-based asset pricing scheme, the real price Q_t and nominal price $Q_t^{\$}$ of any asset without dividend payments in $t+1$ follow equation (2.9) and (2.10).

$$Q_t = E_t[M_{t+1} Q_{t+1}] \tag{2.9}$$

$$Q_t^{\$} = E_t[M_{t+1}^{\$} Q_{t+1}^{\$}] \tag{2.10}$$

[4]The higher γ is, households will be more adverse to uncertainty; the higher ψ is, households will shift more today's consumption to future when they are faced with the same change in relative marginal utility; the higher τ is, households will be more concerned with leisure.

$P_t^{(n)}$ represents the real price of an n-maturity bond at time t, and $P_t^{(n)\$}$ represents the nominal price of an n-maturity bond at time t. According to equation (2.9) and (2.10) we have

$$P_t^{(n)} = E_t[M_{t+1}P_{t+1}^{(n-1)}] \tag{2.11}$$

$$P_t^{(n)\$} = E_t[M_{t+1}^{\$}P_{t+1}^{(n-1)\$}] \tag{2.12}$$

Normalization implies $P_t^{(0)\$} = 1$ and $P_t^{(1)\$} = \frac{1}{R_{t+1}}$.

Assuming the log normal distribution for the pricing kernel, we can get the expression for the nominal yields of an n-maturity bond at time t,[5]

$$y_t^{(n)\$} = -\frac{1}{n}E_t(\sum_{j=1}^{n} m_{t+j}^{\$}) - \frac{1}{2n}\text{Var}_t[\sum_{j=1}^{n} m_{t+j}^{\$}] \tag{2.13}$$

Similarly, for the real yields of an n-maturity bond at time t, we have

$$y_t^{(n)} = -\frac{1}{n}E_t(\sum_{j=1}^{n} m_{t+j}) - \frac{1}{2n}\text{Var}_t[\sum_{j=1}^{n} m_{t+j}] \tag{2.14}$$

m_{t+j} and $m_{t+j}^{\$}$ represent the log of the real and nominal pricing kernel respectively.

2.3 Nominal Bond Yields Forecast Error Variance Decomposition

The nominal bond yields forecast error variance decomposition follows the methods in Campbell & Ammer (1993), Duffee (2018), and Gomez-Cram & Yaron (2021). This decomposition is purely from the perspective of accounting.

The nominal excess bond returns of an n-maturity bond are defined as the one-period nominal returns of that bond, $p_{t+1}^{(n-1)\$} - p_t^{(n)\$}$, minus the one-period nominal yields of a short-term 1-maturity bond, $y_t^{(1)\$}$, as shown in equation (2.15).

$$rx_{t+1}^{(n)\$} \equiv p_{t+1}^{(n-1)\$} - p_t^{(n)\$} - y_t^{(1)\$} \tag{2.15}$$

The one-period real risk-free rates are defined as the one-period real yields.

$$r_{f,t} \equiv y_t^{(1)} \tag{2.16}$$

We can decompose the nominal bond yields into three parts using above definitions as shown in equation (2.17), which are related to the real risk-free rates, expected inflation,

[5] Here the definition $P_t^{(n)\$} \equiv (1 + y_t^{(n)\$})^{-n}$ is used. Approximately, this definition can be written as $y_t^{(n)\$} \equiv -\frac{1}{n}p_t^{(n)\$}$, where $p_t^{(n)\$} = \ln P_t^{(n)\$}$. Derivations in detail could be found in the section 2.2, Appendix.

and nominal excess returns respectively.

$$y_t^{(n)\$} = \frac{1}{n}\sum_{i=1}^{n} E_t(r_{f,t+i-1}) + \frac{1}{n}\sum_{i=1}^{n} E_t(\Pi_{t+i}) + \frac{1}{n}\sum_{i=1}^{n} E_t(rx_{t+i}^{(n-i+1)\$}) \tag{2.17}$$

Usually, the term premium is defined as the average one-period excess return of bonds with maturities across 1 to n,

$$tp_{t,n}^{\$} \equiv \frac{1}{n}\sum_{i=1}^{n} E_t(rx_{t+1}^{(n-i+1)\$}) \tag{2.18}$$

Since the ex-ante version of equation (2.17) also holds, we can then decompose the innovation of nominal bond yields, or the forecast error of nominal bond yields into three parts as shown in equation (2.19) and (2.20)

$$\varepsilon_{y^{\$},t}^{(n)} = \varepsilon_{r,t}^{(n)} + \varepsilon_{\pi,t}^{(n)} + \varepsilon_{tp^{\$},t}^{(n)} \tag{2.19}$$

$$\begin{aligned}
\varepsilon_{y^{\$},t}^{(n)} &= E_t[y_t^{(n)\$}] - E_{t-1}[y_t^{(n)\$}] \\
\varepsilon_{r,t}^{(n)} &= \frac{1}{n}\sum_{i=1}^{n} E_t(r_{f,t+i-1}) - \frac{1}{n}\sum_{i=1}^{n} E_{t-1}(r_{f,t+i-1}) \\
\varepsilon_{\pi,t}^{(n)} &= \frac{1}{n}\sum_{i=1}^{n} E_t(\Pi_{t+i}) - \frac{1}{n}\sum_{i=1}^{n} E_{t-1}(\Pi_{t+i}) \\
\varepsilon_{tp^{\$},t}^{(n)} &= \frac{1}{n}\sum_{i=1}^{n} E_t(rx_{t+i}^{(n-i+1)\$}) - \frac{1}{n}\sum_{i=1}^{n} E_{t-1}(rx_{t+i}^{(n-i+1)\$})
\end{aligned} \tag{2.20}$$

Finally, we can decompose the forecast error variance of nominal bond yields as shown in equation (2.21).

$$Var(\varepsilon_{y^{\$},t}^{(n)}) = Var(\varepsilon_{r,t}^{(n)}) + Var(\varepsilon_{\pi,t}^{(n)}) + Var(\varepsilon_{tp^{\$},t}^{(n)}) + 2Cov(\varepsilon_{r,t}^{(n)}, \varepsilon_{\pi,t}^{(n)}) + 2Cov(\varepsilon_{r,t}^{(n)}, \varepsilon_{tp^{\$},t}^{(n)}) + 2Cov(\varepsilon_{\pi,t}^{(n)}, \varepsilon_{tp^{\$},t}^{(n)}) \tag{2.21}$$

The inflation variance ratio, VR_π, proposed in Duffee (2018), is defined as equation (2.22).

$$VR_\pi \equiv \frac{Var(\varepsilon_{\pi,t}^{(n)})}{Var(\varepsilon_{y^{\$},t}^{(n)})} \tag{2.22}$$

2.4 Simulation Results

After the incorporation of preference shocks, only the persistence of preference shocks, ρ_λ, needs to be calibrated – I set the value as 0.981[6]. The rest parameters are the same as reported in Kung (2015). [7] Thus, the model still has the same good features as the original

[6] I also try smaller values for ρ_λ, and the results are similar. The simulation results for alternative calibration are reported in Section 7, Appendix

[7] Except in the Taylor rule, the sensitivity of interest rate to inflation, ρ_π, is adjusted to 0.45, and the sensitivity of interest rate to output, ρ_y, is adjusted to 0.3. Calibration details are reported in Section 6,

model proposed in Kung (2015), such as nice matches with the moments of macroeconomic variables and many characteristics of nominal yields.

Table 1 and 2 report the nominal yields forecast error variance decomposition results for situations without and with preference shocks respectively. The model is solved by Dynare with a second-order approximation at quarterly frequency. I stochastically simulate the model for 10,000 periods.

It could be seen that in both situations, the variance of expected inflation innovations contributes to more than 100% of the total variance of nominal yield innovations. At the long maturity, the inflation variance ratio decreases less than 10% after the introduction of preference shocks, which is negligible compared with its magnitude. In addition, at the short maturity, the inflation variance ratio even increases compared with the situation without preference shocks.

The results suggest that simply incorporating preference shocks may not well solve Duffee (2018)'s critique. Thus, I model the central bank as a liquidity provider and incorporate liquidity shocks in section 3.

Maturity	$\mathrm{Var}[\varepsilon^{(n)}_{r\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r,t}, \varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{\pi,t}, \varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r\$,t}, \varepsilon^{(n)}_{tp\$,t}]$
1	255.58%	106.51%	0.00%	-262.09%	0.00%	-0.00%
2	212.34%	137.38%	0.00%	-249.72%	-0.00%	-0.00%
3	163.30%	159.75%	0.00%	-223.05%	-0.00%	-0.00%
4	122.44%	171.96%	0.00%	-194.39%	-0.00%	-0.00%
5	92.94%	177.30%	0.00%	-170.23%	-0.00%	0.00%
6	72.57%	179.02%	0.00%	-151.59%	-0.00%	0.00%
7	58.46%	179.04%	0.00%	-137.50%	-0.00%	0.00%
8	48.48%	178.32%	0.00%	-126.80%	-0.00%	0.00%
9	41.21%	177.32%	0.00%	-118.54%	-0.00%	0.00%
10	35.79%	176.25%	0.00%	-112.04%	-0.00%	0.00%
11	31.64%	175.21%	0.00%	-106.85%	-0.00%	0.00%
12	28.39%	174.23%	0.00%	-102.62%	-0.00%	0.00%
13	25.80%	173.34%	0.00%	-99.13%	-0.01%	0.00%
14	23.70%	172.53%	0.00%	-96.22%	-0.01%	0.00%
15	21.97%	171.79%	0.00%	-93.76%	-0.01%	0.00%
16	20.53%	171.13%	0.00%	-91.65%	-0.01%	0.00%
17	19.32%	170.53%	0.00%	-89.84%	-0.01%	0.00%
18	18.29%	169.99%	0.00%	-88.27%	-0.01%	0.00%
19	17.40%	169.50%	0.00%	-86.90%	-0.01%	0.00%
20	16.64%	169.06%	0.00%	-85.70%	-0.01%	0.00%

Table 1: Nominal yields forecast error variance decomposition without preference shocks

Appendix.

The percentage in the table represents the weights of each item in the nominal yields forecast error variance decomposition shown in equation (2.21). The second column reports the inflation variance ratios at different maturities.

Maturity	$\mathrm{Var}[\varepsilon^{(n)}_{r\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r,t}, \varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{\pi,t}, \varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r\$,t}, \varepsilon^{(n)}_{tp\$,t}]$
1	249.01%	113.86%	0.00%	-262.88%	-0.00%	0.00%
2	199.50%	144.03%	0.00%	-243.53%	-0.00%	-0.00%
3	147.92%	163.10%	0.00%	-211.02%	-0.00%	-0.00%
4	107.87%	171.70%	0.00%	-179.57%	-0.00%	-0.00%
5	80.37%	174.32%	0.00%	-154.69%	-0.00%	0.00%
6	62.01%	174.21%	0.00%	-136.22%	-0.00%	0.00%
7	49.56%	173.05%	0.00%	-122.61%	-0.00%	0.00%
8	40.89%	171.55%	0.00%	-112.44%	-0.00%	0.00%
9	34.64%	170.03%	0.00%	-104.68%	-0.00%	0.00%
10	30.01%	168.61%	0.00%	-98.62%	-0.00%	0.00%
11	26.49%	167.31%	0.00%	-93.80%	-0.00%	0.00%
12	23.74%	166.16%	0.00%	-89.89%	-0.00%	0.00%
13	21.56%	165.13%	0.00%	-86.68%	-0.01%	0.00%
14	19.79%	164.22%	0.00%	-84.00%	-0.01%	0.00%
15	18.34%	163.41%	0.00%	-81.75%	-0.01%	0.00%
16	17.14%	162.69%	0.00%	-79.82%	-0.01%	0.00%
17	16.12%	162.05%	0.00%	-78.16%	-0.01%	0.00%
18	15.26%	161.47%	0.00%	-76.73%	-0.01%	0.00%
19	14.52%	160.96%	0.00%	-75.48%	-0.01%	0.00%
20	13.88%	160.50%	0.00%	-74.38%	-0.01%	0.00%

Table 2: Nominal yields forecast error variance decomposition with preference shocks

The percentage in the table represents the weights of each item in the nominal yields forecast error variance decomposition shown in equation (2.21). The second column reports the inflation variance ratios at different maturities.

3 Incorporating Liquidity Shocks

This section modifies the bond pricing New Keynesian model in Kung (2015) to incorporate liquidity shocks with an enlarged financial sector referring to Sims *et al.* (2021) and Sims & Wu (2021). The modified model has four distinct features compared with Kung's original model.

First, the modified model has two different kinds of households, parent households, and child households. There are various income sources for parent households. Parent households work, receive wages and have savings. Besides, parent households own production firms, financial intermediaries, and the central bank, so parent households also receive dividends or operating surplus. While child households finance their consumption by issuing private bonds.

Second, financial intermediaries are added into the model. Financial intermediaries hold savings from the parent households as debts and buy the private bonds issued by the child households as assets. In addition, financial intermediaries need to put reserves at the central bank.

Third, financial intermediaries are owned by the parent households and financial intermediaries' operation is supported by the capital from the parent households. A credit constraint is imposed in the modified model, which constrains financial intermediaries' leverage for their private bonds holding according to the capital transferred from the parent households.

Fourth, the central bank can directly hold the private bonds issued by the child households. Modeled as the liquidity provider, the central bank injects or absorbs liquidity by buying and selling private bonds. As a result, a positive liquidity shock would stimulate child households' consumption. Moreover, with increasing bond demands, nominal bond prices tend to increase, and nominal yields tend to decrease, generating liquidity effects.

3.1 Households and Bond Pricing

Parent households have the same recursive preferences as Kung (2015). They maximize the variable $C_t^\star$, which is combined with consumption C_t, and leisure $\overline{L} - L_t$.

$$U_t = \max_{C_t^\star} \quad \{(1-\beta)(C_t^\star)^{\frac{1-\gamma}{\theta}} + \beta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\}^{\frac{\theta}{1-\gamma}} \tag{3.1}$$

The budget constraint is different. In equation (3.2), Sa_t represents the savings. W_t represents the nominal wage. D_t represents the dividends from intermediate goods production firms. D_t^{FI} represents the dividends from financial intermediaries. While T_t represents the operating surplus from the central bank. Besides, parent households pay X_t^{FI} to financial intermediaries as capital. X_t represents the nominal value they transfer to child households. P_t is the aggregate price level of the final goods. R_{t-1}^s is the gross nominal interest rate for the savings set at period $t-1$.

$$P_t C_t + Sa_t = W_t L_t + R_{t-1}^s Sa_{t-1} + D_t + D_t^{FI} + T_t - X_t - X_t^{FI} \tag{3.2}$$

The intertemporal condition for the parent households is

$$1 = E_t[M_{t+1}\Pi_{t+1}^{-1}]R_t^s \tag{3.3}$$

where M_{t+1} is the real pricing kernel or the real stochastic discount factor.

$$M_{t+1} = \beta\left(\frac{C_{t+1}^\star}{C_t^\star}\right)^{\frac{1-\gamma}{\theta}}\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]}\right)^{\frac{\theta-1}{\theta}} \tag{3.4}$$

and

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{3.5}$$

The bond pricing scheme is the same as section 2.2, which uses parent households' stochastic discount factor to price the bonds. Thus, the bond prices are determined by the consumption growth of the parent households as well as expected inflation.

The intratemporal condition for the parent households is

$$\frac{W_t}{P_t} = \frac{\tau C_t}{\overline{L} - L_t} \tag{3.6}$$

Child households receive the transfer, X_t, from parent households. Child households issue private bonds, B_t, to support their consumption. R^b_{t-1} is the gross nominal interest rate for bonds at $t-1$. Child households' budget constraint is

$$P_t C_{b,t} + B_{t-1} R^b_{t-1} = B_t + X_t \tag{3.7}$$

3.2 Production

The settings of the production sector are the same with Kung (2015), because its production sector settings allow productivity shocks to have persistent effects on inflation.[8] Final goods Y_t are produced using intermediate goods $X_{i,t}$ with a constant elasticity of substitution technology. The inverse demand schedule is

$$P_{i,t} = P_t Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}} \tag{3.8}$$

where ν is elasticity of substitution, and $P_{i,t}$ is the price for goods i.

The intermediate goods are produced using firm-specific physical capital $K_{i,t}$ and labor $L_{i,t}$, with α as capital share,

$$X_{i,t} = K^{\alpha}_{i,t}(Z_{i,t} L_{i,t})^{1-\alpha} \tag{3.9}$$

where $Z_{i,t}$ represents firm-specific total factor productivity, defined by equation (3.10).

$$Z_{i,t} \equiv A_t N^{\eta}_{i,t} N_t^{1-\eta} \tag{3.10}$$

A_t represents aggregate productivity. $N_{i,t}$ represents firm-specific R&D capital. N_t represents aggregate R&D capital defined by equation (3.11).

$$N_t \equiv \int_0^1 N_{j,t} dj \tag{3.11}$$

The intermediate goods production firms take aggregate price level P_t, aggregate physical capital K_t, aggregate R&D capital N_t, final goods demand Y_t, and aggregate productivity A_t as given. They need to decide physical investments $I_{i,t}$, R&D investments $S_{i,t}$, labor demand $L_{i,t}$, as well as optimal price $P_{i,t}$ every period to maximize their dividends. The evolution of physical capital $K_{i,t}$ is determined by physical investments as shown in

[8] A positive productivity shock tends to stimulate firms' investments, which in turn increases the expected productivity growth continuously and keeps marginal costs at a low level, generating persistently depressing effects on expected inflation.

equation (3.12).

$$K_{i,t+1} = (1-\delta_k)K_{i,t} + \Phi_k(\frac{I_{i,t}}{K_{i,t}})K_{i,t} \tag{3.12}$$

δ_k is the depreciation rate for physical capital. Physical capital adjustment cost, $\Phi_k(\frac{I_{i,t}}{K_{i,t}})$, is set as

$$\Phi_k(\frac{I_{i,t}}{K_{i,t}}) = \frac{\alpha_{1,k}}{1-\frac{1}{\xi_k}}(\frac{I_{i,t}}{K_{i,t}})^{1-\frac{1}{\xi_k}} + \alpha_{2,k} \tag{3.13}$$

Similarly, the evolution of R&D capital $N_{i,t}$ is shown in equation (3.14),

$$N_{i,t+1} = (1-\delta_n)N_{i,t} + \Phi_n(\frac{S_{i,t}}{N_{i,t}})N_{i,t} \tag{3.14}$$

δ_n is the depreciation rate for R&D capital. R&D capital adjustment cost, $\Phi_n(\frac{S_{i,t}}{N_{i,t}})$, is set as

$$\Phi_n(\frac{S_{i,t}}{N_{i,t}}) = \frac{\alpha_{1,n}}{1-\frac{1}{\xi_n}}(\frac{S_{i,t}}{N_{i,t}})^{1-\frac{1}{\xi_n}} + \alpha_{2,n} \tag{3.15}$$

The standard Rotemberg pricing scheme is adopted in the intermediate goods production settings, with price adjustment costs set as

$$G(P_{i,t}, P_{i,t+1}; P_t, Y_t) = \frac{\phi_R}{2}(\frac{P_{i,t}}{\Pi_{ss}P_{i,t-1}} - 1)^2 Y_t \tag{3.16}$$

ϕ_R represents the degree measure of price adjustment costs. Π_{ss} represents steady-state inflation.

Therefore, the dividends from intermediate firms can be written as

$$D_{i,t} = P_t Y_t^{\frac{1}{\nu}}[K_{i,t}^{\alpha}(A_t N_{i,t}^{\eta} N_t^{1-\eta} L_{i,t})^{1-\alpha}]^{1-\frac{1}{\nu}} - W_{i,t}L_{i,t} - P_t I_{i,t} - P_t S_{i,t} - P_t G(P_{i,t}, P_{i,t+1}; P_t, Y_t) \tag{3.17}$$

Intermediate goods production firms' constraints for their dividends maximization include the evolution process of physical capital and R&D capital, as well as the inverse demand schedule of final goods. First-order conditions and envelop conditions could be found in section 4.3, Appendix.

3.3 Financial Intermediaries

The balance sheet of financial intermediaries is

$$B_t^{FI} + RE_t = Sa_t + N_t \tag{3.18}$$

The left-hand side of equation (3.18) is the asset of financial intermediaries. They hold the bonds, B_t^{FI}, issued by the child households. They also put reserves, RE_t, into the central bank. The right-hand side of equation (3.18) is the debt and equity of financial intermediaries. They receive the savings, Sa_t, as well as the capital, X_t^{FI}, from the parent households.

The dividends of financial intermediaries are

$$D_{t+1}^{FI} = (R_t^b - R_t^s)B_t^{FI} + (R_t^{re} - R_t^s)RE_t + R_t^s X_t^{FI} \tag{3.19}$$

Financial intermediaries need to pay back savings and their interests in the next period, namely, $R_t^s S_t = R_t^s(B_t^{FI} + RE_t - X_t^{FI})$. They also receive the bonds and interests paid back by the child households, $R_t^b B_t^{FI}$, in the next period, as well as the reserves and interests transferred back by the central bank, $R_t^{re} RE_t$.

Financial intermediaries maximize the next period expected dividends according to the nominal stochastic discount factor of the parent households by choosing the amount of reserves and private bonds they hold.

$$\max_{B_t^{FI}, RE_t} \quad E_t[M_{t+1}\Pi_{t+1}^{-1} D_{t+1}^{FI}] \tag{3.20}$$

$$\text{subject to} \quad B_t^{FI} = \Theta_t X_t^{FI} \tag{3.21}$$

I assume a credit constraint on the bonds held by the financial intermediaries as in previous literature, which constrains that financial intermediaries' leverage for their private bonds holding cannot exceed Θ_t. The Θ_t follows the process shown in equation (3.22).

$$\ln \Theta_t = (1 - \rho_\theta) \ln \Theta + \rho_\theta \ln \Theta_{t-1} + s_\theta \varepsilon_{\theta,t} \tag{3.22}$$

The first-order conditions are shown in equation (3.23) and (3.24), where Ω_t is the Lagrangian multiplier.

$$E_t M_{t+1} \Pi_{t+1}^{-1} (R_t^b - R_t^s) = \Omega_t \tag{3.23}$$

$$E_t M_{t+1} \Pi_{t+1}^{-1} (R_t^{re} - R_t^s) = 0 \tag{3.24}$$

3.4 Central Bank

The balance sheet of central bank is

$$B_t^{cb} = RE_t \tag{3.25}$$

The left-hand side of equation (3.25) is the asset of the central bank, and the right-hand side of equation (3.25) is the debt of the central bank. The central bank directly holds the bond, B_t^{cb}, issued by the child households. While the financial intermediaries put their reserves, RE_t, at the central bank.

I assume the quantitative easing of the central bank QE_t follows the process shown in equation (3.26).

$$\ln QE_t = (1 - \rho_q) \ln QE + \rho_q \ln QE_{t-1} + s_q \varepsilon_{q,t} \tag{3.26}$$

The variable $\varepsilon_{q,t}$ represents the liquidity shock. The liquidity injection and absorption are achieved by buying and selling private bonds issued by the child households. Thus, I assume

$$QE_t = b_t^{cb} \tag{3.27}$$

where $b_t^{cb} = \frac{B_t^{cb}}{P_t}$.

Besides, the central bank follows a Taylor rule to control the gross saving interest rate R_t^s.

$$\ln(\frac{R_t^s}{R_{ss}}) = \rho_r \ln(\frac{R_{t-1}^s}{R_{ss}}) + (1-\rho_r)[\rho_\pi \ln(\frac{\Pi_t}{\Pi_{ss}}) + \rho_y \ln(\frac{\hat{Y}_t}{\hat{Y}_{ss}})] + \sigma_\xi \xi_t \tag{3.28}$$

where the detrended output is defined as $\hat{Y}_t \equiv \frac{Y_t}{N_t}$, and $\hat{Y}_{ss}$ is the steady-state value for the detrended output.

The operating surplus of the central bank, T_{t+1}, would be transferred to the parent households.

$$T_{t+1} = R_t^b B_t^{cb} - R_t^{re} RE_t \tag{3.29}$$

3.5 Market Clearing Conditions and Other Exogenous Processes

The goods market clearing condition is

$$Y_t = C_t + C_{b,t} + S_t + I_t + \frac{\Phi_R}{2}(\frac{\Pi_t}{\Pi_{ss}} - 1)^2 Y_t \tag{3.30}$$

The bond market clearing condition is

$$B_t = B_t^{FI} + B_t^{cb} \tag{3.31}$$

For simplicity, I assume[9]

$$R_t^b = R_t^{re} = R_t^s \tag{3.32}$$

as well as the constant capital of financial intermediaries.

$$X_t^{FI} = X^{FI} \tag{3.33}$$

I also assume the consumption of the child households is equal to b_t, where $b_t = \frac{B_t}{P_t}$. This implies the child households' consumption is totally covered by issuing private bonds.

$$C_{b,t} = b_t \tag{3.34}$$

Besides, I keep the original settings for the productivity shock process and the volatility shock process, as shown in equation (3.35) and (3.36).

$$\ln A_t = (1-\rho)a^\star + \rho \ln A_{t-1} + \sigma_{t-1}\varepsilon_t, \qquad \varepsilon_t \sim N(0,1) \tag{3.35}$$

$$\sigma_t^2 = \overline{\sigma}^2 + \lambda(\sigma_{t-1}^2 - \overline{\sigma}^2) + \sigma_e e_t, \qquad e_t \sim N(0,1) \tag{3.36}$$

The full set of equilibrium conditions are shown in section 5, Appendix.

[9] This implies the first-order conditions for the financial intermediaries always hold with $\Omega_t = 0$.

3.6 Simulation Results for the Model with Liquidity Shocks

Following Sims *et al.* (2021), the steady-state leverage Θ is calibrated at 5, and the steady-state QE is calibrated as 10 percent of the detrended output. Besides, the persistence of credit shocks, ρ_θ, the persistence of liquidity shocks, ρ_q, are both calibrated at 0.8. Moreover, the standard deviations of credit shocks and liquidity shocks are calibrated at 0.01. Furthermore, in order to generate appropriate liquidity effects, the steady-state child consumption is set at half of the steady-state total consumption. This would in turn determine the steady-state amount of transfer from the parent households to the child households, X, and to the financial intermediaries, X^{FI}.

Figure 1 plots the impulsive response functions of macroeconomic variables and nominal yields when there is a positive liquidity shock[10], which would generate positive stimulating effects for output and both physical and R&D investments. Since I assume the child consumption is totally supported by private bonds, a positive liquidity shock will increase private bond demands and in turn stimulate the consumption for child households. Moreover, a positive liquidity shock would also generate depressing effects on inflation and interest rates. With liquidity effects from the increasing bond demands and depressing effects for inflation and interest rates, the nominal yields are also depressed for all maturities. Short-term bonds like 1-year maturity are affected more than long-term bonds like 5-year maturity.

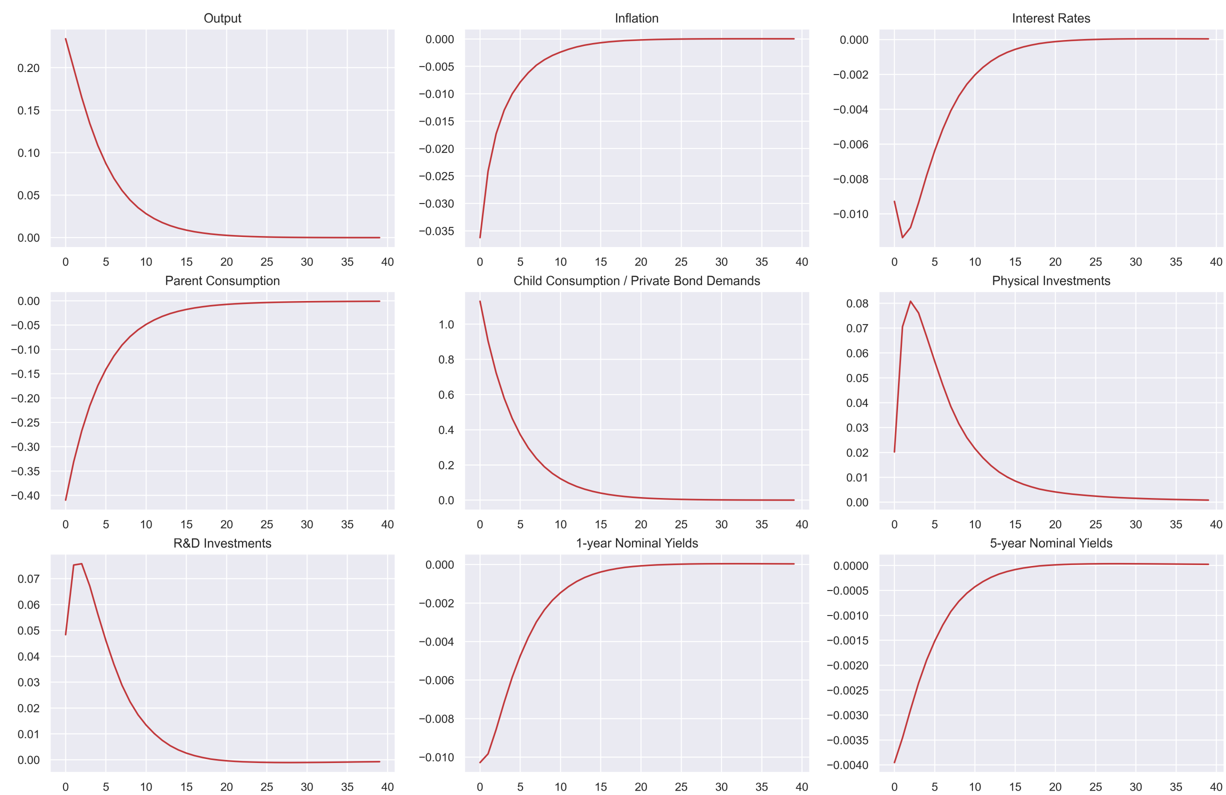

Figure 1:Impulsive Response for a Positive Liquidity Shock

[10]It is intuitive to consider it as a large-scale purchasing project conducted by the central bank.

Figure 2 plots the impulsive response functions of macroeconomic variables and nominal yields when there is a positive credit shock. Similarly, a positive credit shock would generate positive stimulating effects for output, investments, child consumption, as well as depressing effects on inflation, interest rates, and nominal yields. Besides, the stimulating effects and liquidity effects generated by a credit shock are stronger than the counterpart of a liquidity shock.

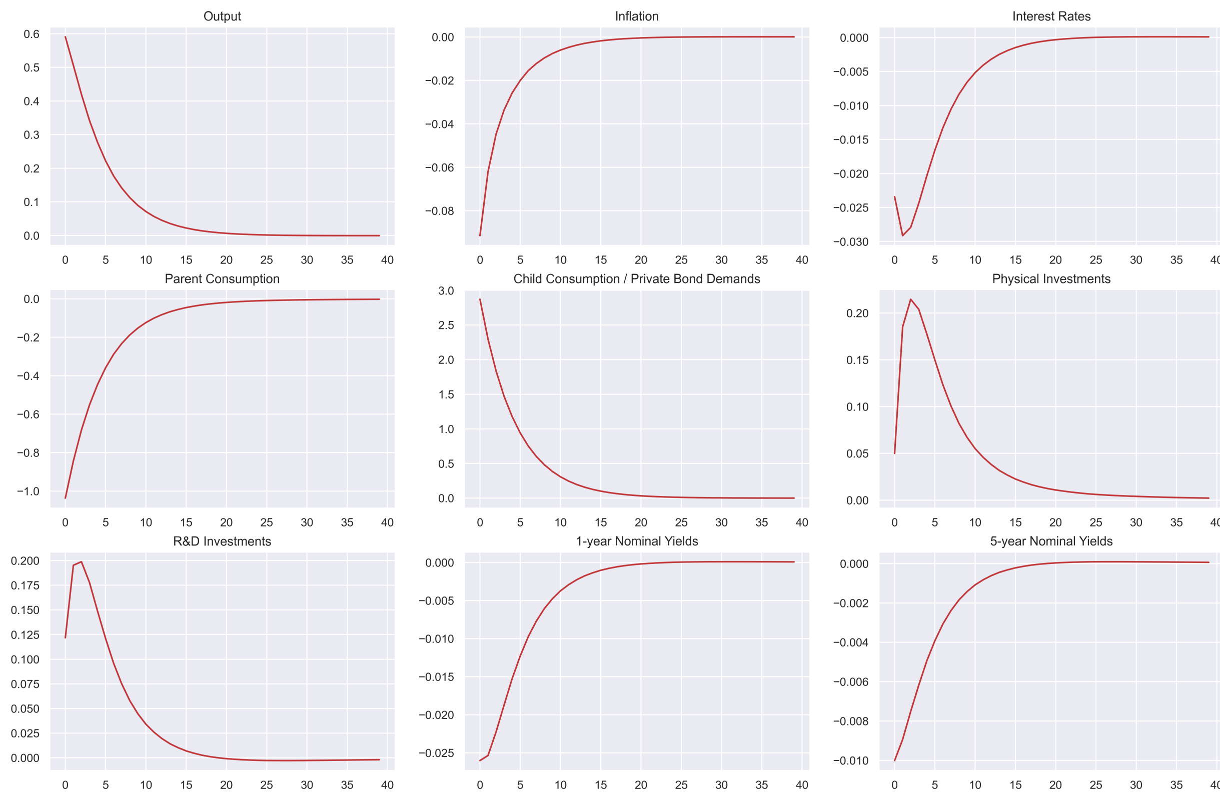

Figure 2:Impulsive Response for a Positive Credit Shock

Table 3 reports the nominal yields forecast error variance decomposition results for situations with liquidity shocks, and table 4 reports the results with both preference shocks and liquidity shocks. The models are solved by Dynare with a second-order approximation and a stochastic simulation for 10,000 periods.

It could be seen that at the short maturity, the inflation variance ratio decreases by about 50% compared with Kung's original model, which is an optimistic signal.[11] However, the improvements decrease as the maturity becomes longer. In addition, the magnitude is still too large compared with Duffee (2018)'s empirical results – the inflation variance ratio is larger than 100% for the bonds with the maturity longer than 1 year. Furthermore, after incorporating preference shocks further, the improvements only become slightly better and the conclusions do not change.

[11]In the meanwhile, the weights of the real risk-free rate innovations also decrease at the short maturity. But the weights for the covariance between the real risk-free rate innovations and the expected inflation innovations increase at the short maturity.

Maturity	$\mathrm{Var}[\epsilon^{(n)}_{r\$,t}]$	$\mathrm{Var}[\epsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Var}[\epsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\epsilon^{(n)}_{r,t}, \epsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Cov}[\epsilon^{(n)}_{\pi,t}, \epsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\epsilon^{(n)}_{r\$,t}, \epsilon^{(n)}_{tp\$,t}]$
1	230.66%	51.06%	0.00%	-181.71%	0.00%	0.00%
2	221.02%	66.99%	0.00%	-188.01%	0.00%	-0.00%
3	204.62%	85.87%	0.00%	-190.49%	0.00%	-0.00%
4	183.72%	105.08%	0.00%	-188.80%	-0.00%	-0.00%
5	161.63%	122.36%	0.00%	-183.99%	-0.00%	-0.00%
6	140.87%	136.64%	0.00%	-177.51%	-0.00%	-0.00%
7	122.69%	147.87%	0.00%	-170.56%	-0.00%	-0.00%
8	107.37%	156.48%	0.00%	-163.85%	-0.00%	-0.00%
9	94.68%	163.01%	0.00%	-157.70%	-0.00%	0.00%
10	84.24%	167.99%	0.00%	-152.22%	-0.00%	0.00%
11	75.63%	171.79%	0.00%	-147.42%	-0.00%	0.00%
12	68.50%	174.73%	0.00%	-143.22%	-0.00%	0.00%
13	62.55%	177.02%	0.00%	-139.57%	-0.00%	0.00%
14	57.56%	178.83%	0.00%	-136.39%	-0.00%	0.00%
15	53.33%	180.27%	0.00%	-133.60%	-0.00%	0.00%
16	49.73%	181.44%	0.00%	-131.17%	-0.00%	0.00%
17	46.64%	182.39%	0.00%	-129.03%	-0.00%	0.00%
18	43.97%	183.17%	0.00%	-127.14%	-0.00%	0.00%
19	41.65%	183.82%	0.00%	-125.47%	-0.00%	0.00%
20	39.61%	184.37%	0.00%	-123.98%	-0.00%	0.00%

Table 3: Nominal yields forecast error variance decomposition with liquidity shocks

The percentage in the table represents the weights of each item in the nominal yields forecast error variance decomposition shown in equation (2.21). The second column reports the inflation variance ratios at different maturities.

	$\mathrm{Var}[\varepsilon^{(n)}_{r\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Var}[\varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r,t}, \varepsilon^{(n)}_{\pi\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{\pi,t}, \varepsilon^{(n)}_{tp\$,t}]$	$\mathrm{Cov}[\varepsilon^{(n)}_{r\$,t}, \varepsilon^{(n)}_{tp\$,t}]$
1	229.15%	50.37%	0.00%	-179.53%	0.00%	0.00%
2	218.83%	65.83%	0.00%	-184.66%	0.00%	-0.00%
3	201.84%	84.10%	0.00%	-185.94%	0.00%	-0.00%
4	180.61%	102.64%	0.00%	-183.25%	-0.00%	-0.00%
5	158.42%	119.29%	0.00%	-177.71%	-0.00%	-0.00%
6	137.76%	133.04%	0.00%	-170.79%	-0.00%	-0.00%
7	119.76%	143.86%	0.00%	-163.62%	-0.00%	-0.00%
8	104.66%	152.17%	0.00%	-156.83%	-0.00%	-0.00%
9	92.19%	158.50%	0.00%	-150.70%	-0.00%	0.00%
10	81.95%	163.34%	0.00%	-145.29%	-0.00%	0.00%
11	73.52%	167.06%	0.00%	-140.58%	-0.00%	0.00%
12	66.54%	169.95%	0.00%	-136.49%	-0.00%	0.00%
13	60.73%	172.22%	0.00%	-132.95%	-0.00%	0.00%
14	55.85%	174.02%	0.00%	-129.87%	-0.00%	0.00%
15	51.73%	175.47%	0.00%	-127.20%	-0.00%	0.00%
16	48.22%	176.65%	0.00%	-124.86%	-0.00%	0.00%
17	45.20%	177.62%	0.00%	-122.82%	-0.00%	0.00%
18	42.60%	178.42%	0.00%	-121.02%	-0.00%	0.00%
19	40.33%	179.10%	0.00%	-119.43%	-0.00%	0.00%
20	38.35%	179.68%	0.00%	-118.03%	-0.00%	0.00%

Table 4: Nominal yields forecast error variance decomposition with both preference shocks and liquidity shocks

The percentage in the table represents the weights of each item in the nominal yields forecast error variance decomposition shown in equation (2.21). The second column reports the inflation variance ratios at different maturities.

4 Conclusion

In general, incorporating preference shocks into the bond pricing New Keynesian model only offers negligible improvements. However, incorporating liquidity shocks seems more successful and promising. In order to decrease the inflation variance ratio of the New Keynesian model to an appropriate level, two problems need to be solved in the future. The first problem is how to make liquidity effects stronger, because the lowest inflation variance ratio at the short maturity is around 50%, still higher than the 20% shown in Duffee (2018). The second problem is how to make the effects of liquidity shocks more persistent. After the incorporation of liquidity shocks, the inflation variance ratios are only improved at the short maturity and exceed 100% beyond the 1-year maturity. In my modeling of the liquidity provider and financial intermediaries, different calibration for parameters has been tried and fewer improvements are found. Thus, a more complete and effective model for the liquidity provider and financial intermediaries should be considered in future researches.

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