Supplemental material

More on the breaking of shift symmetry. In the Letter, we have employed the Langrangian in Eq. (5) as a concrete example to demonstrate the effect of the time-dependent mass. Here we briefly comment on the validity of this model and effects of higher order corrections.

It is well known that the inflaton dynamics has an approximate shift symmetry $\phi \to \phi + c$ as indicated by the near scale invariance of the primordial power spectrum. For this reason, we have chosen the trilinear coupling $(\partial_\mu \phi)^2 \sigma$ to be shift symmetric and of lowest dimension.

At the same time, also well known is that the shift symmetry is broken by quantum gravity effects. In this Letter, we have exactly considered such an effect by including the SDC-motivated mass term $-\frac{1}{2} e^{-2\alpha\phi/M_{\rm Pl}} m^2 \sigma^2$ in (5). One might worry that this term would introduce other shift-symmetry-breaking interactions between σ and ϕ, but it is easy to show that such terms must be small, and the σ-mass term is the leading shift-symmetry breaking term.

To see this, we note that the broken shift symmetry in the σ-mass term can be restored by promoting m^2 to a spurion $m^2 \to \chi$, which transforms under the inflaton shift as $\chi \to e^{2\alpha c/M_{\rm Pl}} \chi$. The σ-mass $m^2 \sim \mathcal{O}(H^2)$ is recovered when χ takes a vev $\chi_0 \sim \mathcal{O}(H^2)$.

After restoring the shift symmetry, the spectator σ should couple to ϕ via the shift-symmetric combination $\chi e^{-2\alpha\phi/M_{\rm Pl}}$. It is then obvious that the lowest dimension coupling is exactly the σ-mass term, given that the tadpole of σ is properly canceled. To see the impact of the shift symmetry breaking on other operators in (17), we take the next-order term $\lambda_3 \sigma^3$ as an example. The spurion procedure shows that the shift-symmetry-breaking correction to this term would be of order $\chi_0 \sigma^3/\Lambda$, where the cutoff $\Lambda \sim \mathcal{O}(M_{\rm Pl})$ as expected for a quantum gravity effect. Then the coefficient of this coupling would be $\chi_0/\Lambda \sim \mathcal{O}(H^2/M_{\rm Pl})$, and this is much less than $\lambda_3 \sim \mathcal{O}(H)$, showing that the shift-symmetry-breaking effect on the σ^3 term is negligible. Similar analyses can be done to other operators in (5) which all lead to the same conclusion.

The scalar tilt. In the main text we have showed the correction to the scalar tilt Δn_s due to the time-dependent mass, in Fig. 2. To get this figure, we have numerically evolved the mode equations (12). The Bunch-Davies initial condition is imposed as usual, at an initial time when all relevant modes are well inside the horizon. Then, we collect the values of the resulting mode function at a final time where all relevant modes are well outside the horizon. With these mode functions, we get a numerical result for the power spectrum P_ζ in (14) for a range of comoving momentum k. We then fit the numerical result of $\log P_\zeta$ by a linear function $X \log k + Y$ in $\log k$. The scalar tilt n_s can thus be extracted from the slope of this fitted linear function, $n_s - 1 = X$. We have also checked numerically that the running of the scalar tilt is around or below $\mathcal{O}(10^{-2})$. Therefore, the linear function $X \log k + Y$ is a good fit of the numerical power spectrum for the parameter space of interest.

We emphasize again that Fig. 2 shows the corrections to the n_s from the time-dependent mass $m(\tau)$ alone. The complete result of n_s should also include the slow-roll corrections. The slow-roll correction can be included by rewriting the mode equations (12) with a general slow-roll background $a(\tau)$ instead of the slow-roll limit $a(\tau) = -1/(H\tau)$. Numerically evolving this slow-roll corrected mode equations can give the full result of n_s. In this Letter, we chose not to include the slow-roll correction in the mode equation, in order to isolate and highlight the effect of the time-dependent mass.

To clarify the relation between our result and the scalar tilt from a constant-mass spectator computed in [32], we write the power spectrum P_ζ in the following way:

$$P_\zeta = P_\zeta(m/H, \mu/H), \tag{20}$$

where m is the spectator's mass and μ is its two-point coupling with the inflaton. The scale (k) dependence in P_ζ is obtained from the time (τ) dependence of m/H and μ/H, by evaluating m/H and μ/H at the time of horizon exit, $\tau \simeq -1/k$. In [32], the scalar tilt is obtained by including the time dependence in H:

$$P_\zeta = P_\zeta\Big(\frac{m}{H(\tau)}, \frac{\mu}{H(\tau)}\Big), \tag{21}$$

while in this Letter, we consider the time dependence in m, and switched off the time dependence in H:

$$P_\zeta = P_\zeta\Big(\frac{m(\tau)}{H}, \frac{\mu}{H}\Big). \tag{22}$$

Numerical implementation. In the main text we presented the numerical result of the in-in integral (17) which computes the Feynman diagram in (16). Here we spell out the procedure to carry out this numerical calculation, essentially following the treatment introduced in [32].

The computation begins with the usual canonical quantization of the Fourier modes $\varphi_{\mathbf{k}}(\tau)$ and $\sigma_{\mathbf{k}}(\tau)$, which can be expanded in terms of a set of creation and annihilation operators $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$, $b_{\mathbf{k}}$, $b_{\mathbf{k}}^\dagger$, as follows.

$$\varphi_{\mathbf{k}}(\tau) = \big[\varphi_k^{(1)} a_{\mathbf{k}}(\tau) + \varphi_k^{(2)} b_{\mathbf{k}}(\tau)\big] e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}, \tag{23}$$

$$\sigma_{\mathbf{k}}(\tau) = \big[\sigma_k^{(1)} a_{\mathbf{k}}(\tau) + \sigma_k^{(2)} b_{\mathbf{k}}(\tau)\big] e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}, \tag{24}$$

where the variables $\varphi_k^{(n)}$ and $\sigma_k^{(n)}$ ($n = 1, 2$) with italic subscript k are mode functions, satisfying the same set of equations (12) as the field operators. Here we have included $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ in both field operators, allowing for the mixing between the two fields. To solve the mode

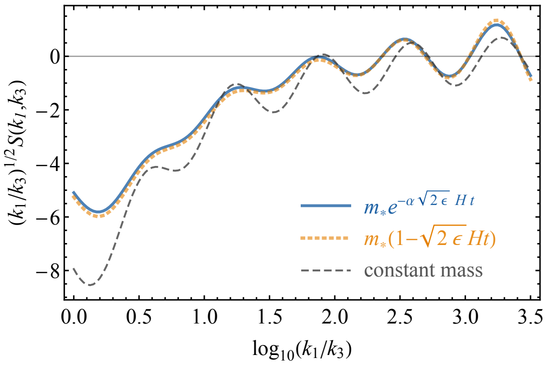

FIG. 4. The shape function (18) with two parameterizations of the mass $m_{\rm eff}(t)$, plotted with the constant-mass shape function. The blue curve corresponds to (6) and the orange curve corresponds to (29). For this plot we choose $\alpha = 1$ and $16\epsilon = 0.005$, and other parameters are chosen to be the same with in Fig. 3.

equations (12), we impose the usual Bunch-Davies initial condition in the early-time limit $\tau \to -\infty$ with proper normalizations:

$$\varphi_k^{(n)} = \frac{H}{2k^{3/2}}(-k\tau)^{1-(-1)^n \mathrm{i}\mu/(2H)}e^{-\mathrm{i}k\tau}, \quad (25)$$

$$\sigma_k^{(n)} = (-1)^{n-1}\mathrm{i}\varphi_k^{(n)}. \quad (26)$$

The unusual power dependence on τ is due to the mixing of the two modes. Then, in principle, we can numerically solve (12) with initial conditions spelled out above. However, one can anticipate that the integrand of (17) becomes highly oscillatory in the early-time limit $\tau \to -\infty$, making the numerical computation difficult. Therefore, we Wick-rotate the time variable $\tau \to \pm\mathrm{i}\tau$ such that the mode function is always exponentially damped when $\tau \to -\infty$. Note that this specification of the contour rotation is consistent with the standard iϵ-prescription, which is ultimately from selecting the correct vacuum initial state.

There is one more complication in the numerical computation: After the Wick rotation, the two oscillatory branches of the mode function become exponential functions of the imaginary time iτ, either growing or dying exponentially fast as $\tau \to 0$. In the numerical calculation, we need to specify the initial condition at some large $-\tau_i$, and then evolve the equation numerically towards $\tau = 0$. The existence of an exponentially growing branch means that we have to specify the initial condition exponentially precisely in order to get a result of any reasonable precision. This is virtually impossible. However, there is a simple way out: Since we know that the mode functions would be essentially exponential functions in the early-time limit, we can simply factor this exponential out before the numerical computation. This motivates us to rewrite

$$\varphi_k(\tau) = A_k(\tau)e^{-\mathrm{i}k\tau}, \qquad \sigma_k(\tau) = B_k(\tau)e^{-\mathrm{i}k\tau}. \quad (27)$$

Then the functions A_k and B_k would have no dangerous exponential dependence on τ, making the numerical computation safe. It is simple to get a pair of equations for A_k and B_k, written with a dimensionless variable $z = -\mathrm{i}k\tau$:

$$A_k'' + \frac{2(z-1)}{z}A_k' - \frac{2}{z}A_k - \frac{\mu}{z}B_k' - \frac{\mu(z-3)}{z^2}B_k = 0,$$

$$B_k'' + \frac{2(z-1)}{z}B_k' + \frac{m^2(z)-2z}{z^2}B_k + \frac{\mu}{z}(A_k' + A_k) = 0,$$

where $m^2(z) = m_*^2(z^2/k^2)^{\alpha\sqrt{2\epsilon}}e^{\mathrm{i}\pi\alpha\sqrt{2\epsilon}}$ is the time-dependent mass after the Wick rotation, and the prime denotes the derivative with respect to z.

With the above numerical procedure, the in-in integral (17) can be carried out directly in terms of the mode functions:

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle' = 2\lambda_3\,\mathrm{Im}\int_{-\infty}^0 \mathrm{d}\tau\, a^4 \times \prod_{i=1}^3\Big[\sigma_{k_i}^{(1)}(\tau)\varphi_{k_i}^{(1)}(0) + \sigma_{k_i}^{(2)}(\tau)\varphi_{k_i}^{(2)}(0)\Big], \quad (28)$$

where we have taken $\tau_f = 0$.

Alternative mass ansatz. In the main text we focus on an exponential ansatz for the time-dependent mass of the spectator field. Here, instead of (6), we consider an alternative parametrization of the time dependence in the mass:

$$m_{\rm eff}(t) = m_*(1 - \sqrt{2\epsilon_*}H_*t), \quad (29)$$

which could come from a direct coupling $\lambda\phi^2\sigma^2$. Due to the smallness of ϵ_*, we can expect that the signal from this direct coupling would be similar to the exponential ansatz (6) for at least a range of k. In Fig. 4 we show both parameterizations of the mass and it is clear that the two shape functions with time-dependent mass are very close to each other, and both show clear deviations from the constant-mass signal. Therefore, the main conclusion of our paper is insensitive to the particular type of direct coupling between the spectator field and the inflaton.