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Supplementary Materials for

Balancing economic and epidemiological interventions in the early stages of pathogen emergence

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Supplementary Material

Optimization and Numerical Analysis

Here we describe the main features of our optimization procedure, from both the theoretical and the numerical approximation viewpoints. Although our optimization problem falls into the class of the so-called Optimal Control Problems (OCP) in continuous time, it is not standard in some aspects. In the next subsections, we describe the problem, its main features, and the methods we use to find solutions and to approximate them. We pay particular attention to the points which are non-trivial and technically involved.

Basic ingredients of the optimal control problem. In an OCP one identifies the key variables which describe the state of the system, or the "state" variables, and the ones which are under the control of the optimizing agent, the "control" variables. The state variables belong to a set Z, the "state space." This is typically a subset of $\mathbb{R}^n$, where n is the number of state variables. The control variables belong to a set K, the "control space." This is typically a subset of $\mathbb{R}^m$, where m is the number of control variables. Both state and control variables may change with time, t, where $t \in [0, T]$. We refer to 0 as the initial time, while T is the final time, also called the "horizon" of the problem. We term a "control strategy" a function

$$k(\cdot) : [0, T] \to K$$

and a "state trajectory" a function

$$z(\cdot) : [0, T] \to Z.$$

Clearly not all control strategies are suitable for a given problem. Hence, later, we will introduce the set of admissible control strategies. Given the above, the main ingredients of an OCP are:

- the state equations which describe the time evolution of the state variables; i.e., the state trajectory $z(\cdot)$, depending on the choice of the control strategies $k(\cdot)$;
- the objective functional to be optimized on the time span $[0, T]$ over all admissible control strategies.

The state/control variables and the state equations. The control variables in our model are

- the lockdown intensity $p \in [0, p_{max}]$, where p_{max} is its maximum value;
- the testing intensity $\tau \in [0, \tau_{max}]$, where τ_{max} is the maximum rate of testing (or testing capacity).

Hence in our setting $k := (p, \tau)$ and $K = [0, p_{max}] \times [0, \tau_{max}]$. We will usually set $p_{max} = 1$, assuming that complete lockdown is possible, and $\tau_{max} = 365/2$ in days^{-1}, which is a reasonable value for the maximum testing capacity of a developed country. Clearly such parameters can be chosen differently, as we do for example in Section 3 where we consider the case where the maximum testing capacity is reduced by a factor 1/3.

The state variables are the eight epidemiological variables as discussed in the main text: $S, C, E_1, E_2, I_1, I_2, R, D_c$ which, for brevity, will be denoted by the vector $z \in \mathbb{R}^8$, with the components in the above order. All state variables are required to be positive and smaller than the initial size of the population N_0. Hence, the state space, Z, is the compact subset of $\mathbb{R}^8$ given by $Z := [0, N_0]^8$. For brevity we will often denote the control strategies at time t by $k(t)$ and the state trajectories at time t by $z(t)$.

The state equations form a system of eight Ordinary Differential Equations (ODEs) which we rewrite here emphasizing the time dependent variables:

$$\dot{S}(t) = \mu N(t) - \mu S(t) - (1-p(t))^2 \beta \frac{S(t)\left(E_2(t)(1-\gamma) + \gamma I_1(t)\right)(1+c)}{N(t)}$$

$$+ \phi R(t) + \tau(t)\left(r + \frac{1-r}{1+c}\right) C(t) s_p + \delta C(t) - \tau(t) r S(t)(1-s_p) \tag{1}$$

$$\dot{C}(t) = (1-p(t))^2 \beta (cS(t) - (1-\epsilon_C)C(t)) \frac{E_2(t)(1-\gamma) + \gamma I_1(t)}{N(t)}$$

$$+ \tau(t) r S(t)(1-s_p) - \mu C(t) - \tau(t)\left(r + \frac{1-r}{1+c}\right) C(t) s_p - \delta C(t) \tag{2}$$

$$\dot{E}_1(t) = (1-p(t))^2 \beta (S(t) + (1-\epsilon_C)C(t)) \frac{E_2(t)(1-\gamma) + \gamma I_1(t)}{N(t)}$$

$$- (\mu + \varphi_1) E_1(t) - \tau(t)\left(r + \frac{1-r}{1+c}\right) s_e E_1(t) \tag{3}$$

$$\dot{E}_2(t) = \varphi_1 E_1(t) - (\mu + \varphi_2) E_2(t) - \tau(t)\left(r + \frac{1-r}{1+c}\right) s_e E_2(t) \tag{4}$$

$$\dot{I}_1(t) = \varphi_2 E_2(t) + \tau(t) s_e \left(r + \frac{1-r}{1+c}\right)(E_1(t) + E_2(t))$$

$$- (\mu + \mu_{I_1} + (1-\eta)\delta_1 + \eta\delta_1) I_1(t) \tag{5}$$

$$\dot{I}_2(t) = \eta\delta_1 I_1(t) - (\mu + p_M\delta_2 + (1-p_M)\delta_2) I_2(t) \tag{6}$$

$$\dot{R}(t) = (1-\eta)\delta_1 I_1(t) + (1-p_M)\delta_2 I_2(t) - (\mu + \mu_R + \phi) R(t) \tag{7}$$

$$\dot{D}_c(t) = \mu_{I_1} I_1(t) + p_M \delta_2 I_2(t) + \mu_E E_2(t) + \mu_R R(t) \tag{8}$$

where

$$N(t) := S(t) + C(t) + E_1(t) + E_2(t) + I_1(t) + I_2(t) + R(t), \qquad \forall t \in [0, T] \tag{9}$$

with $N(0) = N_0$. It is useful to observe that in the state equations above we have, for all $t \in [0, T]$,

$$N'(t) + D_c'(t) = 0 \quad \Longleftrightarrow \quad N(t) + D_c(t) = N_0.$$

Since $D_c(\cdot)$ is always positive and increasing, it follows that $N(t)$ is decreasing.

Remark 1 If the mortality rates due to the epidemic are low, as is the case for COVID-19 (which is our driving example), then $N(t)$ remains very close to N_0. In that case, (see e.g. (40) and (41)) , we can substitute $N(t)$ with N_0 in the above equations (and also below in the objective functional). While not necessary for our numerical procedure, this step simplifies the numerical approximations.

For later use, we introduce the notation for the state equations. We call $F : Z \times K \to \mathbb{R}^8$ the function defined by the right hand side of the state equations; i.e.,

$$F(S, C, E_1, E_2, I_1, I_2, R, D_c; p, \tau) = \tag{10}$$

$$\begin{pmatrix} \mu N - \mu S - (1-p)^2 \beta \frac{S(E_2(1-\gamma)+\gamma I_1)(1+c)}{N} + \phi R + \tau \left(r + \frac{1-r}{1+c}\right) C s_p + \delta C - \tau r S(1-s_p) \\ (1-p)^2 \beta (cS - (1-\epsilon_C)C) \frac{E_2(1-\gamma)+\gamma I_1}{N} + \tau r S(1-s_p) - \mu C - \tau \left(r + \frac{1-r}{1+c}\right) C s_p - \delta C \\ (1-p)^2 \beta (S + (1-\epsilon_C)C) \frac{E_2(1-\gamma)+\gamma I_1}{N} - (\mu + \varphi_1) E_1 - \tau \left(r + \frac{1-r}{1+c}\right) s_e E_1 \\ \varphi_1 E_1 - (\mu + \varphi_2) E_2 - \tau \left(r + \frac{1-r}{1+c}\right) s_e E_2 \\ \varphi_2 E_2 + \tau s_e \left(r + \frac{1-r}{1+c}\right)(E_1 + E_2) - (\mu + \mu_{I_1} + (1-\eta)\delta_1 + \eta\delta_1) I_1 \\ \eta \delta_1 I_1 - (\mu + p_M \delta_2 + (1-p_M)\delta_2) I_2 \\ (1-\eta)\delta_1 I_1 + (1-p_M)\delta_2 I_2 - (\mu + \mu_R + \phi) R \\ \mu_{I_1} I_1 + p_M \delta_2 I_2 + \mu_E E_2 + \mu_R R \end{pmatrix} \tag{11}$$

With this notation, the state equations can be rewritten more concretely as

$$z'(t) = F(z(t); k(t)). \tag{12}$$

Remark 2 The model can be studied for any choice of the initial conditions, z_0, for the 8 state variables. Since our focus is the study of an emerging pathogen, our initial conditions will be those typical of the beginning of an epidemic; i.e.,

$$S(0) \sim N_0; \qquad E_1 > 0 \text{ small}; \qquad E_2, C, I_1, I_2, R, D_C = 0.$$

See e.g. (40,41,48).

Choosing the admissible control strategies. The set of admissible control strategies, $\mathcal{K}_{ad}$, must be chosen in a way that the above system of state equations admits a unique solution for any initial conditions $z(0) \in Z$ and remains in Z at all times. Moreover such a set of admissible strategies should be realistically implementable by the policymakers. The typical choice (see Section III.5 of (50)) is to consider control strategies which are simply Lebesgue integrable (this class of functions is usually chosen in similar contexts, since it contains the piecewise constant policies commonly employed by decision makers in practice, note that piecewise constant functions are not continuous, hence the class of continuous control strategies would not be an appropriate choice in our context) i.e.,

$$\mathcal{K}_{ad} := \{k(\cdot) : [0, T] \to K, \text{ Lebesgue integrable}\}.$$

For any such strategy, $k(\cdot) \in \mathcal{K}_{ad}$, and any initial condition $z_0 \in Z$ the state equation has a unique solution. We refer to this as the state trajectory corresponding to $k(\cdot)$ and we denote it by $z^{z_0, k(\cdot)}(t)$.

The above set $\mathcal{K}_{ad}$ consists of admissible policies that vary continuously with time. However, since in practice our control policies cannot be changed continuously, for our simulations in the main text we employ control policies that assume only a finite number of values. We call an *n-phase policy* for $n > 1$ any control function that is piecewise constant and assumes only n different values on contiguous time intervals, excluding the value 0. Moreover, we also impose that our control policy is null (inactive) before a fixed time, which we call *delay* in the main text, and which we denote hereafter by t_0. Therefore we define the set of admissible controls with $n > 1$ phases (assuming only one between p and τ is considered)

$$\begin{aligned}\mathcal{K}^n_{ad} = \{&k(\cdot) : [0,T] \to K, \exists (t_1,\dots,t_n) \in (t_0,T]^n, (k_1,\dots,k_n) \in K^n s.t.\\ &k(t) \equiv 0 \text{ for } t \le t_0, k(t) \equiv k_i \text{ for } t \in (t_{i-1}, t_i] \text{ for } i = 1,\dots,N, k(t) \equiv 0 \text{ for } t > t_n\},\end{aligned}$$

where we implicitly assumed the vector of times $(t_1,\dots,t_n)$ to be monotonically increasing. It is clear from the previous definition that any function in $\mathcal{K}^n_{ad}$ can be represented with $2n$ parameters, n for the vector of times when the policy changes, and n for the vector of intensity of the control policy in each phase. For $n = 0$ we define $\mathcal{K}^0_{ad} = \{k : [0,T] \to K, k(t) \equiv 0\}$, the space consisting only of the zero function; that is, in this case the control does not affect the system.

Even if the function space $\mathcal{K}^n_{ad}$ is not finite-dimensional as a vector space, the parameterization adopted here allows us to represent each element of $\mathcal{K}^n_{ad}$ using a finite number of parameters ($2n$). However, notice that this representation is not unique, since two different combinations of the $2n$ parameters may describe the same function (in particular in the case where two consecutive i_i are equal).

In the case where both (p,τ) are present, we introduce the sets $\mathcal{K}^{n,p}_{ad}$ and $\mathcal{K}^{M,\tau}_{ad}$ for each of the controls and we define

$$\mathcal{K}^{n,M}_{ad} = \mathcal{K}^{n,p}_{ad} \times \mathcal{K}^{M,\tau}_{ad}.$$

Notice in particular that the number of lockdown phases, n, can be different from the number of testing interventions, M.

In what follows, the above sets of strategies will be denoted generically by $\mathcal{K}_{ad}$ or, in case when the specification of the number of phases is needed, by $\mathcal{K}^{n,M}_{ad}$.

The objective functional to optimize: the starting problem (P_0). To define the objective functional we set

$$Y(t) = A_0(1-p(t))^{\Delta+\alpha} L(t)^\alpha - \Phi(x(t), N(t)) \tag{13}$$

with $N(t)$ as in (9) and

$$\begin{aligned}&L(t) = S(t) + \epsilon_C\,[C(t) + E_1(t) + E_2(t)] + R(t)\\ &x(t) = \tau(t)\,[C(t) + E_1(t) + E_2(t) + rS(t)], \qquad \text{the total number of people tested,}\end{aligned}$$

where the function Φ for the cost of testing is given by

$$\Phi(x,N) = \rho_0 x + \exp\left(\frac{\rho_1}{N-x}\right) - \exp\left(\frac{\rho_1}{N}\right)$$

The objective functional to maximize is the Total Social Welfare (TSW) function specified in equation (10) in the main paper text over all admissible control strategies, i.e. over all $k(\cdot) \in \mathcal{K}_{ad}$. Here we denote it by $J_0(k(\cdot))$ to underline its dependence on the control strategies chosen by the policy-maker.

$$J_0(k(\cdot)) = \int_0^T U(Y(t)) - \theta V(D_c(t))\, \mathrm{d}t \tag{14}$$

with:

$$\begin{aligned} U(Y) &= \frac{Y^{1-\sigma}}{1-\sigma} & \sigma \in (0,1) \\ V(D_c) &= \frac{D_c^{\omega}}{\omega} & \omega > 1; \end{aligned}$$

Note that the utility from production is strictly concave while the disutility from deaths is strictly convex. Our framework allows us to consider different types of utility/disutility functions.

In what follows, we refer to "Problem (P_0)" as the problem of maximizing the functional $J_0(k(\cdot))$ over all $k(\cdot) \in \mathcal{K}_{ad}$, under the state equations (12). Note that, using the above setup, we can write the so-called running cost (i.e. the function appearing inside the integral in J_0) as a function $f_0(z(t); k(t))$ where $f_0 : \mathbb{R}^8 \times \mathbb{R}^2 \to \mathbb{R}$, more precisely (for brevity we write, as usual, N for $S + C + E_1 + E_2 + I_1 + I_2 + R$)

$$\begin{aligned} &f_0(S, C, E_1, E_2, I_1, I_2, R, D_c; p, \tau) = \qquad (15) \\ &U\left(A_0(1-p)^{\Delta+\alpha}(S + R + \epsilon_C[C + E_1 + E_2])^{\alpha} - \Phi\left(\tau[C + E_1 + E_2 + rS], N\right)\right) - \theta V\left(D_c\right) \end{aligned}$$

The objective functional to optimize: the real exit time problem (P). As formulated above, problem (P_0) has an important drawback due to the fact that, even if all the infected compartments (E_1, E_2, I_1, I_2) are close to zero, the epidemic may restart. To avoid this artifact of continuous numbers of individuals, we introduce the following modification.

We first define for any given admissible state trajectory $z^{z_0,k(\cdot)}(\cdot)$ (associated to an initial datum $z_0 \in Z$ and a control strategy $k(\cdot) \in \mathcal{K}_{ad}$), and any closed region $\mathcal{O} \subset Z$, the first entry time $T_{\mathcal{O}}$ as

$$T_{\mathcal{O}}(z^{z_0,k(\cdot)}(\cdot)) := \inf \{t \geq 0 : \; z(t) \in \mathcal{O}\}$$

For brevity we will write $T_{\mathcal{O}}(z_0, k(\cdot))$, or simply $T_{\mathcal{O}}$ when no confusion is possible.

In our problem, as explained in the main text (see Section 3.2), we set

$$\mathcal{O} := \left\{z \in \mathbb{R}^8 : \; 0 \leq z_3, z_4, z_5, z_6 \leq 1\right\}$$

The idea is that after time $T_{\mathcal{O}}$ the epidemic is no longer present and so both control variables and all the state variables E_1, E_2, I_1, I_2 are set equal to 0. The system still moves in the components S, R, D_c following equations

$$\begin{aligned} \dot{S}(t) &= \mu N(t) - \mu S(t) + \phi R(t) \\ \dot{R}(t) &= -(\mu + \mu_R + \phi) R(t) \\ \dot{D}_c(t) &= \mu_R R(t) \end{aligned} \tag{16}$$

with initial conditions given by their values at $T_{\mathcal{O}}$).

If $T_{\mathcal{O}} < T$, these components still have an influence on the utility/disutility functions so we have to take them into account. Hence, we consider the following modification of the above functional J_0:

$$J_1(k(\cdot)) := \int_0^{T\wedge T_{\mathcal{O}}} U(Y(t)) - \theta V(D_c(t))\,\mathrm{d}t + \int_{T\wedge T_{\mathcal{O}}}^{T} U(Y(t)) - \theta V(D_c(t))\,\mathrm{d}t \tag{17}$$

where in the second integral, $Y(t) = A_0(S(t) + R(t))^{\alpha}$ and where the variables $S(t), R(t), D_c(t)$ solve (16) with initial conditions given by their values at $T_{\mathcal{O}}$.

Note that to apply the standard optimality conditions for exit time problems (see Section 8.4 of (51)), the target set $\mathcal{O}$ should not have corners like the one we use here. However, this does not present a problem here, as we use a discrete set of policies.

Note that the second integral, when $T_{\mathcal{O}} < T$ can be expressed as a given function Ψ (which can be explicitly computed) of $T_{\mathcal{O}}$ and $z\big(T_{\mathcal{O}}\big)$ so we can rewrite the functional J_1 as follows

$$J_1(k(\cdot)) := \int_0^{T\wedge T_{\mathcal{O}}} U(Y(t)) - \theta V(D_c(t))\,\mathrm{d}t + \Psi\left(T_{\mathcal{O}}, z\left(T_{\mathcal{O}}\right)\right)\mathbf{1}_{[0,T)}(T_{\mathcal{O}}) \tag{18}$$

The notation $\mathbf{1}_A$ stands for the indicator function of a set A; i.e., a function which is 1 on the set A and 0 elsewhere.

Our Problem (P) is then maximizing J_1 with the above change of state equations.

This is a standard optimal control problem with exit time (see Chapter 8 in (51)).We have the following:

Definition 1 *Admissible policy $k^*(\cdot)$ is an optimal control policy if it maximizes J_1; i.e.,*

$$J_1(k^*(\cdot)) \geq J_1(k(\cdot)) \qquad \forall k(\cdot) \in \mathcal{K}_{ad}.$$

We define an optimal state trajectory $z^*(\cdot)$ as the unique solution of the state equation associated to an optimal control strategy.

We define the optimal $(k^*(\cdot), z^*(\cdot))$ as an ordered pair of functions where the first is an optimal control strategy and the second is its associated state trajectory.

In some cases it is useful to emphasize in the notation the fact that the objective functional depends, through the state equations, also on the initial condition $z(0) = z_0$, by writing $J_1(k(\cdot); z_0)$ in place of $J_1(k(\cdot))$. We then define the value function as the function

$$W(z_0) := \sup_{k(\cdot)\in\mathcal{K}_{ad}} J_1(k(\cdot); z_0)$$

defined for all $z_0 \in Z$. In economic terms this will be called the welfare function.

Optimization in a finite dimensional setting (fixed number of phases). Under a fixed number of control phases corresponding to the values taken by the piecewise constant controls (p, τ), see subsection , the problem can be reduced to a finite-dimensional optimization problem with inequality constraints.

Fix N to be the number of phases of the control p, and assume $M = 0$; i.e., the control τ representing the testing policy is inactive at all times. The general case where $N, M > 0$ can be treated in the same manner by using a slightly more cumbersome notation. For every control policy $p \in \mathcal{K}_{ad}^{N,p}$, we identify by I_p and t_p as the vectors of $\mathbb{R}^N$ that represent intensities and times of phase change associated with the function p. We use the notation p_{I_p,t_p} to denote such a policy.

Therefore, our finite dimensional optimization problem becomes

$$\begin{aligned}
\max_{I_p,t_p} \quad & J_{0,P'}(p_{I_p,t_p}(\cdot)) & \\
\text{s.t.} \quad & 0 \le i_j \le 1 & \forall j = 1,\dots,N, \\
& t_j \le t_{j-1} & \forall j = 1,\dots,N, \\
& t_N \le T. &
\end{aligned}$$

This is a standard finite dimensional optimization problem in $\mathbb{R}^{2N}$ under $2N+1$ inequality constraints. The problem here is that the objective function cannot be computed explicitly given the parameters of the controls. Hence we have to rely on a numerical approximation to compute the objective function, as well as to perform the numerical optimization. To compute the objective function we rely on classical numerical methods for ODEs. However, due to the lack of regularity of the controls, we employ a numerical method which is suitable for stiff problems, such as implicit Runge-Kutta methods of high order. To approximate the continuous time integration we employed a Gauss-Kronrod quadrature rule.

The process of numerical optimization is challenging due to the lack of convexity in the objective function. In the absence of such convexity, there is no guarantee that a local optimum will also be a global one. Therefore, during the numerical optimization, we have to rely on a global optimization algorithm, which is numerically more demanding than a local numerical optimizer. In figure S1, we analyze the case of a single lockdown phase, showing the shape of the objective function for many combinations of the parameter describing the control policy using a very fine grid. In this case we see that concavity is indeed a feature of the objective with respect to this parameterization. A general result concerning the shape of the objective function is however still lacking.

It is well known that global optimization is one of the many problems suffering from the *curse of dimensionality*, that is the number of function evaluations required for a thorough search in the state space grows exponentially with the dimension of the problem. For the optimization procedure we employed both the DIRECT and DIRECT-L algorithm proposed in (43,47). Both algorithms are deterministic procedures based on a subdivision of the domain in iteratively smaller rectangles until convergence is reached. To ensure robustness of our results, we also performed some additional tests based on a combination of a brute-force approach on a very fine grid with a local refinement based on a local optimizer. An additional confirmation of the accuracy of the optimization procedure is that the maximum value of the objective function shifts in the expected direction when changing some of the parameters (e.g. when increasing the delay in the control policies the objective function decreases).

The added benefits of wearing masks

Arguably the most economical control strategy during the Covid pandemic has been the wearing of surgical masks at work and in public places. We have not incorporated the use of masks in our analysis, partly because we see it as an important 'safety play' that is unlikely to stop the pandemic; masks may provide some protection to those that wear them, while also significantly reducing transmission from asymptomatic infected hosts. An important limitation here is that we do not have good estimates of the efficacy of masks (neither for transmission blocking, nor for protection). The best available methods suggest an average efficiency of around 45 percent (averaged across both transmission blocking and susceptible

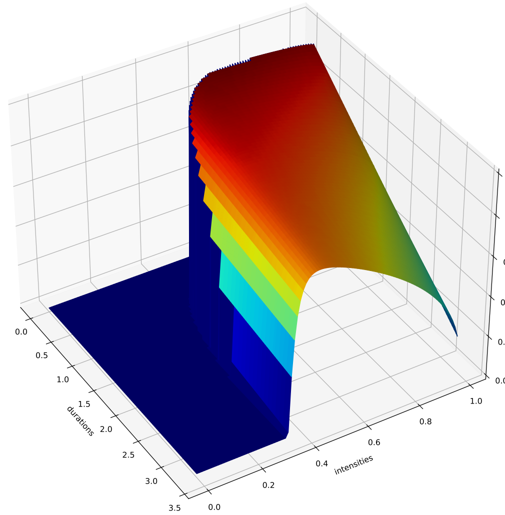

FIGURE S1. Normalized objective function $J_{0,P'}$ for the control policy consisting of a single phase lockdown (values below zero have been cut for graphical purposes). The x-axis "durations" refers to the length of the single lockdown phase, while the y-axis "intensities" refers to the strength of the policy. In this case the objective function is convex with respect to the parameterization adopted.

protection). Similarly, the proportion of people wearing masks varies widely and may follow current levels of infection and perceived risk in the population.

We have not explicitly examined the dynamic consequences of wearing face masks (see (39)). Instead we present a simple static analysis that compares the efficacy of face masks with that of a vaccine (Appendix 2). We do this as there has been resistance to wearing face masks and wearing them is subject to asymmetric and poorly quantified levels of efficacy. Specifically, when an infected individual wears a mask it may trap a high proportion of the viral infection stages they produce, e_i, but not all of them. Similarly, uninfected hosts who wear masks reduce their level of exposure but are unlikely to completely exclude viral particles, their masks have a protective efficiency of e_p. The efficacy of both processes will vary with the type of mask and how it is worn. We can examine the static implications of this by modifying the standard equation for the proportion of people who need to be vaccinated, p_c, to reduce the Basic Reproductive Number of a pathogen, R_0, below unity (19, 20, 40),

$$p_c = (1 - 1/R_0) \tag{20}$$

The proportion of people who have to wear masks at different levels of R_0 is given by

$$p_c = (1 - \sqrt{e_p e_i}/R_0) \tag{21}$$

We coarsely quantify mask efficacy as the geometric mean of the two types of mask efficacy, this is plainly a conservative calculation as there are always more uninfected than infected hosts. Figure S2 illustrates the relationship between the Basic Reproductive Number, R_0, of the pathogen and the proportion of people who have to wear masks at different average levels of mask efficacy as protective devices. A key point to notice from the earlier

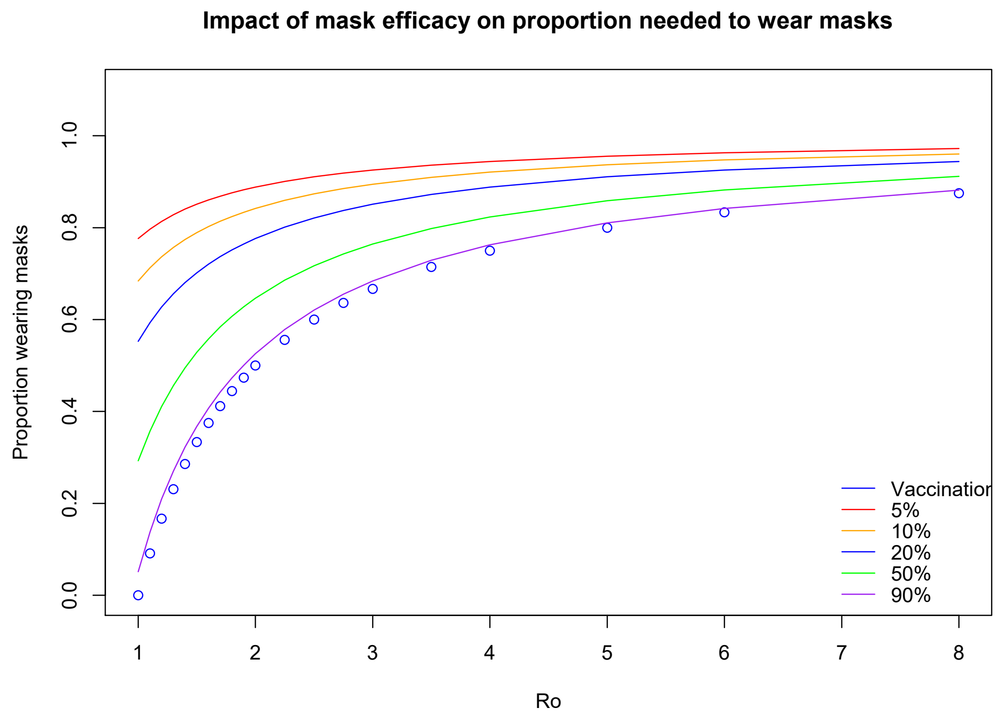

Figure S2. The relationship between the Basic Reproductive Number of a pathogen, R_0 and the proportion of people who need to wear masks so that R_0 is reduced below unity and the pathogen contained. The dotted lines illustrate the "classical' curve for a totally efficient vaccine that generates lifetime immunity. The colored lines illustrate the cases for different levels of protective efficiency for mask-wearers; this efficiency is assumed to be the geometric mean efficacy of protecting the wearer from infection and preventing transmission from an infectious person. The purple line signifies 90 percent mean efficacy, green 50 percent, etc.

figure is that even when masks are 90 percent efficient, they are always less efficient than vaccination and the proportion of people wearing them has to increase rapidly as their efficacy declines. Nonetheless, the figure serves to underline our main point, a high proportion of people have to wear very efficient masks if they are to be effective in reducing R_0 below unity. They will still serve the useful function of slowing the epidemic and reducing the pressure on health care services. Ultimately, we see them as a useful adjunct to the other forms of NPI discussed in more detail below. If more people wear masks, then lock-downs can be shorter and impact a smaller proportion of the economy.

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