Supplemental materials for correlated frequency noise in a multimode acoustic resonator

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I. EXPERIMENTAL SETUP

The SAW resonator is mounted in a dilution refrigerator and cooled down to around 10 mK. We measure the reflection coefficient of the device using a multi-frequency lock-in amplifier (MLA), which can measure signals up to 32 tones simultaneously. We employ frequency modulation circuits for converting RF signals to the detector operating range. A bandpass filter is inserted after the frequency upconversion to eliminate unwanted lower sidebands. There are 14 modes of SAW available within the 2.37 - 2.40 GHz range. A schematic representation of our experimental setup is given in Fig. S1.

II. PHASE FLUCTUATION TIME TRACES

We extract the resonance frequencies from the SAW resonator profile shown in Fig. 2(a). Additionally, we define a control tone at 2.41 GHz, which is outside the SAW bandwidth and located on the positive side relative to all SAW modes. The drive power of the control tone is -47 dBm, equal to SAW drive tones. We measure the SAW resonator profile shown in Fig.1(b) under the presence of the control tone so that it reflects the same condition as in the frequency fluctuation measurement.

We measure reflection coefficients S_{11} of 14 SAW resonance modes and the control tone simultaneously for 7 hours 15 minutes. The control tone serves as a reference for comparing with the SAW probing tones so that we can identify fluctuations from other noise sources in the environment. The SAW probing tones exhibit continuous fluctuations while the control tone shows insubstantial frequency drift, as shown in Fig. S2. The amplitude and phase fluctuations of the control tone show no correlation with any other SAW modes. Note that the amplitude and phase of the control tone cannot be converted to frequency as this is a reference tone that does not correspond to any resonance mode.

III. TIME-DOMAIN ANALYSIS

We perform frequency domain analysis by estimating the noise power spectral density using Welch's method. However, power spectral density can create more artifacts or uncertainty. Thus, we compute the Allan deviation (ADEV) σ_y, which is a time-domain analysis tool for analyzing a slow noise process and addressing the source of fluctuation [1]. The mathematical expression of σ_y of the $1/f$ and the white noise is written as [2]:

$$\sigma_y(\tau) = \sqrt{2\ln(2)h_{-1}} + \sqrt{\frac{h_0}{2\tau}}, \tag{S1}$$

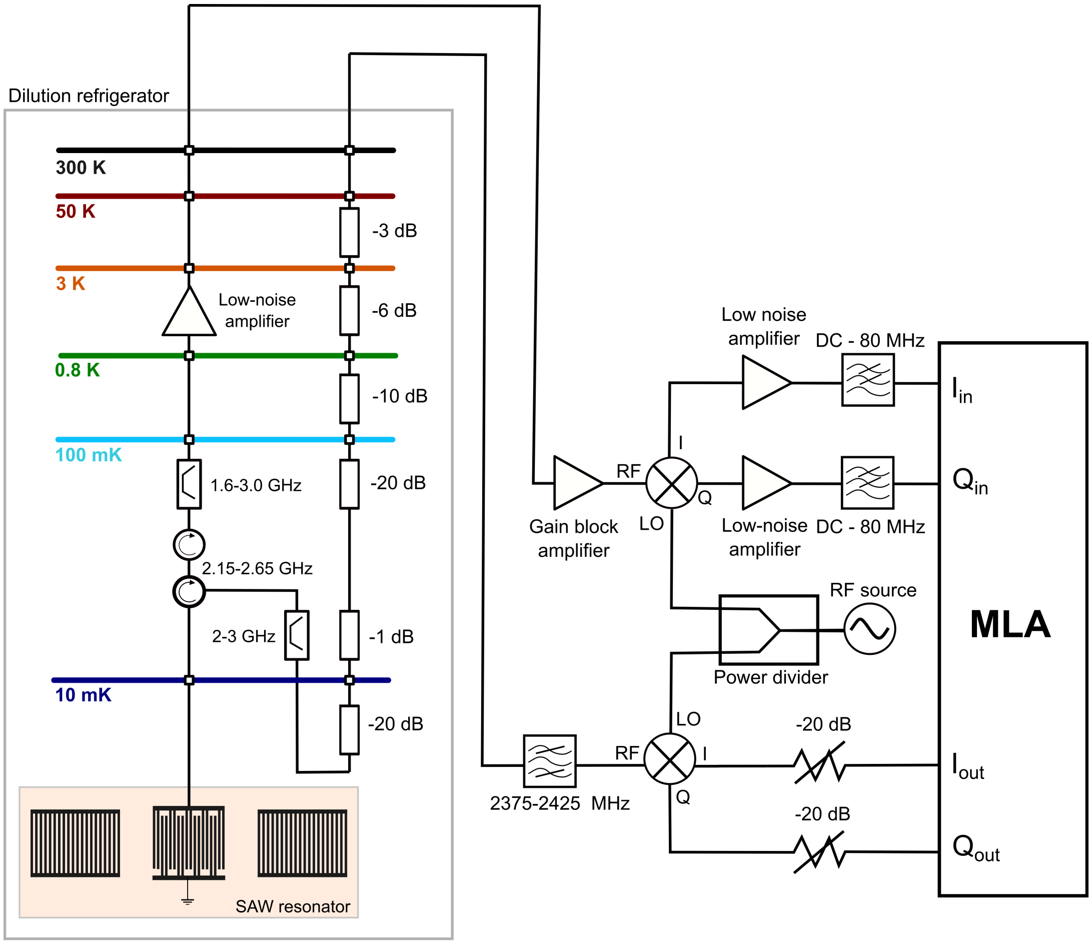

FIG. S1. A schematic of our experimental setup shows a SAW resonator installed in a dilution refrigerator and a circuit diagram for measuring the reflection coefficient of the SAW resonator.

while ADEV of the Lorentzian noise process is expressed by [3]:

$$\sigma_{y_L}(\tau) = \frac{A\tau_0}{\tau}\left(4e^{-\tau/\tau_0} - e^{-2\tau/\tau_0} + 2\frac{\tau}{\tau_0} - 3\right)^{1/2}. \tag{S2}$$

Figure S3 presents the comparison between the pure $1/f$ characteristics and the combination of the $1/f$ with the Lorentzian noise type. The ADEV of the resonance frequency of the full time trace has a local maximum, which is a characteristic of the Lorentzian noise process, whereas the noise in the quiet window shows a relatively flat ADEV. The parameters obtained from fitting are $A = 53.12$ Hz, $\tau_0 = 1.22$ s, $h_{-1} = 145.0$ Hz^2, and $h_0 = 0$.

IV. POWER DEPENDENCE ANALYSIS

We also study the frequency fluctuations at different probe powers. The noise power spectral density (PSD) at varied drive powers is shown in Fig. S4. The plot shows that the noise level reduces as the drive power increases which is in agreement with the prediction of the STM [4] and previous findings [5, 6].

Deviations from the $1/f$ trend are most pronounced for the -47 dBm probe power in the 10^{-4} to 1 Hz frequency range, where the Lorentzian noise characteristic is present (See Fig.4 in the main article). At lower power these fluctuations are not visible and PSD follow the theoretical prediction of TLS behavior.

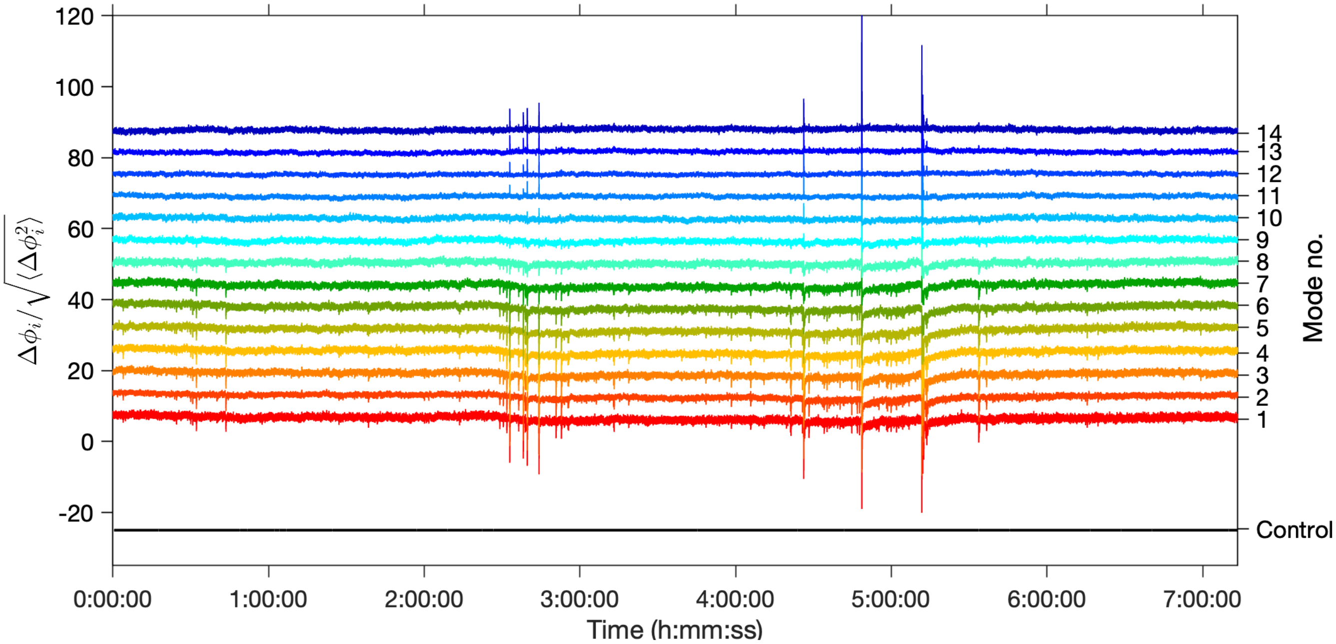

FIG. S2. Normalized time traces of phase fluctuations of the control tone and 14 modes of the SAW resonator. All traces are vertically shifted for better visualization. The resonator probing tones show fluctuations, whereas the control tone is rather steady for the entire measurement.

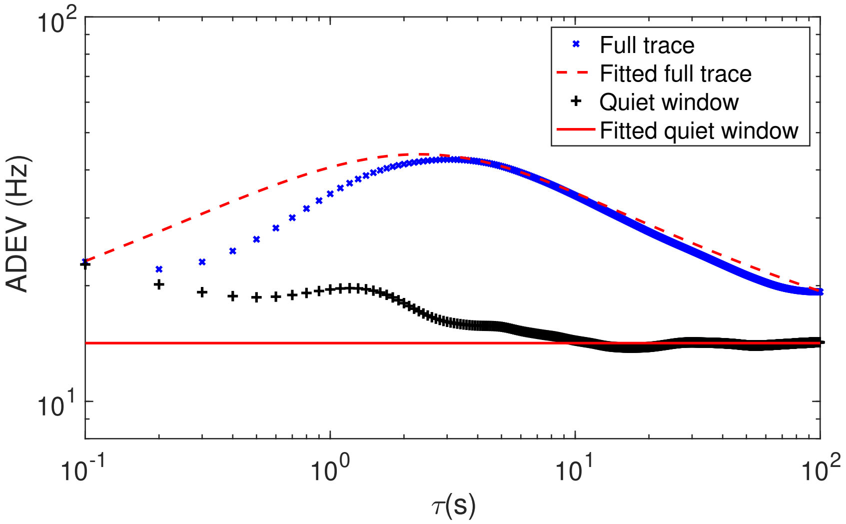

FIG. S3. Allan deviation of the resonance frequency mode no.8 calculated from the entire trace and the quiet time window. The lines represent the fitted power law noise model (Eq. S1) and the sum of the power law noise and the Lorentzian noise model (Eq. S1 + S2).

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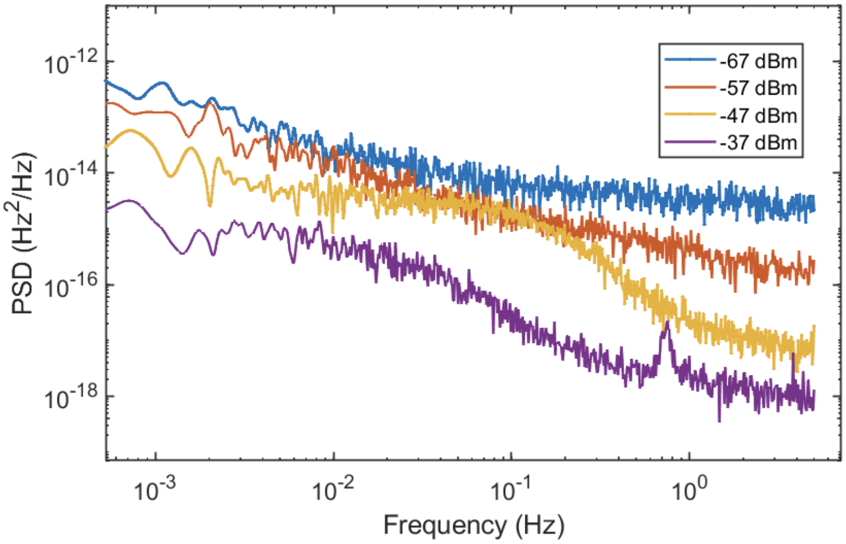

FIG. S4. Power spectral density of noise from SAW resonator mode no. 8 obtained using varied drive powers. Generally, the noise level is suppressed with increased SAW driving power, except for drive power of -47 dBm. At this power, we observe anomalous frequency fluctuations exhibiting non-TLS behaviors.

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