FOR ONLINE PUBLICATION APPENDIX

A Robustness analysis

In this section, we present several robustness analysis of our main results. First, we investigate the case in which liquidity services are delayed by one period with respect to when money is held in the agents' portfolio. Second, we sketch out the implications of imperfect substitutability between currencies. Finally, we provide a detailed model involving credit cards.

A.1 Delayed liquidity services

An important assumption of our framework is liquidity immediacy, i.e. that the liquidity services provided by a currency occur at the same date t that money is added to the agent's portfolio. However, some models, such as the third example in Section 7, postulate instead that liquidity premia are to be received a period after portfolio choices are made, i.e. with delay in $t+1$:

Assumption A.1 (Liquidity delay)**.** *The purchase of the global currency and currency h in country h at t yields delayed liquidity premia L_{t+1} receivable in $t+1$. Analogously, the time t purchase of global currency and currency f in country f at t yields delayed liquidity premia L^*_{t+1} receivable in $t+1$.*

In this case, equations (9), (10) and (11) need to be replaced with

$$
\begin{aligned}
1 &\geq E_t[\mathcal{M}_{t+1}(1+L_{t+1})], & \text{(A.1)}\\
Q_t &\geq E_t[\mathcal{M}_{t+1}(1+L_{t+1})Q_{t+1}], & \text{(A.2)}\\
\frac{i_t}{1+i_t} &\geq E_t[\mathcal{M}_{t+1}L_{t+1}]. & \text{(A.3)}
\end{aligned}
$$

The liquidity premia are appropriately discounted by the stochastic discount factor. Since we focus on equilibria in which all currencies are used, we set (A.1), (A.2), (A.3) with an equality sign.

In country f, one must likewise replace (12), (13) and (14) with

$$\begin{aligned} 1 &\geq E_t[\mathcal{M}^*_{t+1}(1+L^*_{t+1})], && \text{(A.4)} \\ Q^*_t &\geq E_t[\mathcal{M}^*_{t+1}(1+L^*_{t+1})Q^*_{t+1}], && \text{(A.5)} \\ \frac{i^*_t}{1+i^*_t} &\geq E_t[\mathcal{M}^*_{t+1}L^*_{t+1}]. && \text{(A.6)} \end{aligned}$$

Again, in what follows, we will assume that the above equations hold with an equality sign. Define the conditional covariance under the home country risk-adjusted measure as

$$\widetilde{cov}_t(X,Y) \equiv \widetilde{E}_t[XY] - \widetilde{E}_t[X]\,\widetilde{E}_t[Y] \tag{A.7}$$

For a random variable X, define the risk-adjusted expectation in country f as the equivalent to $\tilde{E}_t[\cdot]$ via

$$\tilde{E}^*_t[X] \equiv \frac{E_t[\mathcal{M}^*_{t+1}X]}{E_t[\mathcal{M}^*_{t+1}]} \tag{A.8}$$

Let

$$\Delta_t \equiv i_t - i^*_t$$

be the differences between the nominal interest rates. Maintaining all other assumptions, we next turn to deriving implications for the exchange rate. The next results apply independently of whether liquidity premia are delayed, and they need as input solely the interest rate differential, like in (A.11).

Proposition A.1 (Delayed Liquidity Services and Exchange Rates)
In a stochastic economy, assuming liquidity delay, complete markets, and all currencies being used: the expected liquidity services differences and exchange rates then satisfy

$$\Delta_t = \tilde{E}_t[L_{t+1}] - \tilde{E}^*_t\left[L^*_{t+1}\right] \tag{A.9}$$

and

$$\frac{\tilde{E}_t\left[S_{t+1}\right]}{S_t} = 1 + \frac{\Delta_t}{1+i^*_t} \tag{A.10}$$

This corollary is a strict consequence of the given interest differential: the presence of the global currency is not necessary to establish these consequences. Note how the results here are adjusted relative to the expressions in our benchmark result. The (expected) liquidity services now differ by the interest rate differential. If the rate is zero, as in the main result, so is the (expected) liquidity service difference. The exchange rate is no longer a risk-adjusted martingale: instead, there is an adjustment term that depends on the interest rate differential. If that interest rate differential is zero, as in the main result, we are back to the risk-adjusted martingale.

Proof. [Proposition A.1] Note that (A.1) and (A.3) can be written as

$$i_t = \tilde{E}_t[L_{t+1}].$$

Likewise, (A.4) and (A.6) can be written as

$$i_t^* = \tilde{E}_t^*[L_{t+1}^*].$$

The combination yields (A.9). Finally, consider the uncovered-interest-parity relationship (18) to obtain (A.10). □

Corollary A.1 (Stochastic Economy under Delayed Liquidity Premia)
In a stochastic economy, assuming liquidity delay, complete markets, and all currencies being used, the nominal interest rate differential satisfies

$$i_t^* - i_t = \frac{\widetilde{cov}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t[Q_{t+1}]} + \frac{\widetilde{cov}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]} \tag{A.11}$$

Note that the benchmark result of interest rate equality in case of liquidity immediacy is a direct consequence of (A.11), since the conditional covariance terms must be zero, if L_{t+1} and L_{t+1}^* are known in t. In the general case, nonzero covariance terms arise and equation (A.11) informs us, in which direction one needs to adjust the interest differential.

Proof. [Corollary A.1.] Since all currencies are used, (A.3) and (A.6) hold with equality. With (7) and (8), rewrite (A.3) and (A.6) using the risk-adjusted measures as

$$i_t = \tilde{E}_t \left[L_{t+1}\right] \tag{A.12}$$

and

$$i_t^* = \tilde{E}_t^* \left[L_{t+1}\right] = \frac{\tilde{E}_t \left[L_{t+1}^* S_{t+1}\right]}{\tilde{E}_t \left[S_{t+1}\right]} \tag{A.13}$$

where in the latter we have also used the assumption of complete markets. Combining the two equations above, we can write the interest-rate differential as

$$i_t^* - i_t = \tilde{E}_t \left[L_{t+1}^*\right] - \tilde{E}_t \left[L_{t+1}\right] + \frac{\widetilde{\mathrm{cov}}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]}, \tag{A.14}$$

Note that this equation holds, regardless of whether there is a global currency or not. The presence of the global currency, however, delivers a restriction on the difference between the expected liquidity services. Use (A.5) together with the assumption of complete markets and the equivalence $Q_t = S_t Q_t^*$ to obtain

$$Q_t = E_t[\mathcal{M}_{t+1}(1 + L_{t+1}^*)Q_{t+1}] \tag{A.15}$$

This can be written under the risk-adjusted measure as

$$(1 + i_t)Q_t = \tilde{E}_t \left[(1 + L_{t+1}^*)Q_{t+1}\right]. \tag{A.16}$$

Writing (A.2) using the risk-adjusted measure

$$(1 + i_t)Q_t = \tilde{E}_t \left[(1 + L_{t+1})Q_{t+1}\right], \tag{A.17}$$

and compare it with the equation above to obtain that

$$0 = \tilde{E}_t \left[(L_{t+1}^* - L_{t+1})Q_{t+1}\right] \tag{A.18}$$

and thus

$$\tilde{E}_t \left[L_{t+1}^*\right] - \tilde{E}_t \left[L_{t+1}\right] = \frac{\widetilde{\mathrm{cov}}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t[Q_{t+1}]}, \tag{A.19}$$

Plugging (A.19) into (A.14) delivers (A.11). □

Note that equation (A.19) determines the expected difference in the liquidity premia, by which we can retrieve the result of the benchmark case of equal liquidity premia when L_{t+1} and L^*_{t+1} are known at time t.

B Imperfect Substitutability of currencies: Linear Scaling

In this section, we model the imperfect substitutabiliy of currencies per assuming that the liquidity services provided by the global currency are a linear multiple of the liquidity services provided by official domestic currency. In order to frame this approach and and as a general starting point, suppose that liquidity services, L_t^G and $L_t^{G,*}$ are paid on global currency at home respectively abroad. Let L_t^H and L_t^F the liquidity services on home and foreign currency. By imperfect substitutability, we can generically have $L_t^H \neq L_T^G$ and $L_t^F \neq L_t^{G,*}$. The pricing equations at home become

$$\begin{aligned} 1 &\geq L_t^H + E_t[\mathcal{M}_{t+1}] && \text{(B.20)} \\ 1 &\geq L_t^G + E_t[\mathcal{M}_{t+1}\frac{Q_{t+1}}{Q_t}], && \text{(B.21)} \end{aligned}$$

with equality when the according currency is used. At foreign, we have

$$\begin{aligned} 1 &\geq L_t^F + E_t[\mathcal{M}^*_{t+1}] && \text{(B.22)} \\ 1 &\geq L_t^{G,*} + E_t[\mathcal{M}^*_{t+1}\frac{Q^*_{t+1}}{Q^*_t}]. && \text{(B.23)} \end{aligned}$$

again with equality if the currencies are used. We maintain the assumption that in each country at least one currency is used and we focus on the case where the global currency is held in at least one country, i.e. where the global currency has value. We retain the bond pricing equations

$$E_t[\mathcal{M}_{t+1}] = \frac{1}{1+i_t} \tag{B.24}$$

$$E_t[\mathcal{M}^*_{t+1}] = \frac{1}{1+i^*_t} \tag{B.25}$$

Lemma B.1. *Without loss of generality, assume the global currency is used abroad. Then* $L^G_t \leq L^{G,*}_t$*, and it holds* $L^G_t = L^{G,*}_t$ *if and only if the global currency is used in both countries.*

Lemma B.1. Without loss of generality, assume the global currency is used abroad. Then $1 = L^{G,*}_t + E_t[\mathcal{M}^*_{t+1}\frac{Q^*_{t+1}}{Q^*_t}] = L^{G,*}_t + E_t[\mathcal{M}_{t+1}\frac{Q_{t+1}}{Q_t}] \leq 1 - L^G_t + L^{G,*}_t$, where the second step uses that the global currency is traded arbitrage-free in international capital markets. If the global currency is not traded at home, the last step holds with strict inequality, implying, $L^G_t < L^{G,*}_t$. If the global currency is traded at home, the last step holds with equality, implying $L^G_t = L^{G,*}_t$. Vice versa, $L^G_t = L^{G,*}_t$ requires the global currency to be traded at home. □

Proposition B.1 (Imperfect Substitutability)
Assume $L^H_t = \xi L^G_t$ *and* $L^F_t = \xi^* L^{G,*}_t$ *where* $\xi, \xi^* > 0$*. Assume that the home currency is used at home and that the foreign currency is or is not used at the foreign country. Assume the global currency is used at the foreign country. Consider* (i^*, ξ, ξ^*) *with* $i^*_t > 0$*.*
a) If $\xi < \xi^*$ *or* $\xi > \xi^*$ *and* $i^*_t \in (0, \frac{1}{\frac{\xi}{\xi^*}-1})$ *then the global currency is not adopted at home if the home central bank sets* $i_t > 0$ *which satisfies*

$$i_t < \frac{1}{\frac{\xi^*}{\xi}\left(1 + \frac{1}{i^*_t}\right) - 1} \tag{B.26}$$

b) If $\xi > \xi^*$ *and* $i^*_t \in (\frac{1}{\frac{\xi}{\xi^*}-1}, \infty)$ *then for every* $i_t > 0$*, the global currency is not adopted at home.*
c) If $\xi = \xi^*$*, then the global currency is not adopted at home if the home central bank sets* $i_t < i^*_t$*.*

Proof. Assume, the global currency is used at the foreign country. The previous Lemma B.1 jointly with the bond pricing equations yield that the global currency is not used at home if

$$0 < L_t^{G,*} - L_t^G = \frac{1}{\xi^*} L_t^F - \frac{1}{\xi} L_t^H \tag{B.27}$$

$$\leq \frac{1}{\xi^*}\left(1 - \frac{1}{1+i_t^*}\right) - \frac{1}{\xi}\left(1 - \frac{1}{1+i_t}\right) \tag{B.28}$$

$$= \frac{1}{\xi^*}\left(\frac{1}{1+1/i_t^*}\right) - \frac{1}{\xi}\left(\frac{1}{1+1/i_t}\right) \tag{B.29}$$

which is equivalent to requiring

$$1/i_t > \frac{\xi^*}{\xi}\left(1 + 1/i_t^*\right) - 1 \tag{B.30}$$

Case 1: Assume $\xi < \xi^*$. Then for all $i_t^* > 0$: $\frac{1}{i_t^*} > 0 > \frac{\xi}{\xi^*} - 1$ and thus $\frac{\xi^*}{\xi}(1 + \frac{1}{i_t^*}) - 1 > 0$. Thus, the right hand side of (B.30) is strictly positive and the inequality (B.30) can only hold for i_t satisfying (B.26). In that case, the global currency is not adopted at home.

Case 2: Assume $\xi > \xi^*$.

Case 2a: Assume in addition $i_t^* \in (0, \frac{1}{\frac{\xi}{\xi^*}-1})$. Then, again, the right hand side of (B.30) is strictly positive and the inequality (B.30) can only hold for i_t satisfying (B.26). In that case, the global currency is not adopted at home.

Case 2b: Assume $\xi > \xi^*$ and $i_t^* \in (\frac{1}{\frac{\xi}{\xi^*}-1}, \infty)$. Then, the right hand side of (B.30) is negative. Therefore, every $i_t > 0$, satisfies B.30. Thus, for every $i > 0$ the global currency is not adopted at home.

Case 3: For $\xi = \xi^*$, then inequality (B.30) simplifies to

$$1/i_t > 1/i_t^* \tag{B.31}$$

So that the global currency is not adopted at home for every $i_t < i_t^*$.

□