

Optimal Bank Regulation in the Presence of Credit and Run Risk

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We modify the 1983 Diamond and Dybvig model so that banks offer liquidity services to depositors, raise equity funding, make risky loans, and invest in safe, liquid assets. Banks monitor borrowers to ensure that they repay loans and they are susceptible to depositor runs. We model the run decision by solving a novel global game. Relative to a social planner, banks opt for a more deposit-intensive capital structure, their assets may be more or less lending intensive, and the level of lending may be higher or lower. Correcting these three distortions requires a package of three regulations.

I. Introduction

It is well understood that financial intermediaries (hereafter, banks) provide useful services through both the lending they undertake and the

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deposits they offer. Banks use both sides of their balance sheet to perform these services, which exposes them to credit risk and funding (run) risk. The efficiency of the resulting allocations from a social point of view depends jointly on the overall level of lending and the mix of bank assets and liabilities. Yet most banking models tend to focus on only one service or type of risk, often neglecting how bank fragility endogenously affects the structure of bank balance sheets and vice versa. We propose a new model where banks create economic surplus for both borrowers and depositors but choose a balance sheet that results in excessive run risk and an inefficient level of lending. We use the model to study alternative regulations that have been proposed since the Global Financial Crisis and derive the optimal regulatory mix.

We start with the setup developed by Diamond and Dybvig (1983), which explains how a bank can provide liquidity services to depositors (savers) with uncertain consumption/funding needs. We modify the Diamond-Dybvig model so that the bank also raises equity, makes loans that are risky, and can invest in safe, liquid assets. We also assume that banks and borrowers are subject to limited liability so that each of them has incentives to take risks that they do not fully bear the costs of. We suppose, similar in spirit to Diamond (1984), that banks monitor the loans to guarantee that they are repaid. Monitoring is costly and hence must be profitable for banks to do so. Even if savers were able to monitor borrowers as efficiently as the bank, delegating this function to banks is optimal, because banks can combine monitoring and deposit granting, which is itself valuable. However, because the loans are risky and have an uncertain liquidation value, depositors may opt to run on the bank, depending on the conjectured ability of the bank to serve early withdrawals and the bank's incentives to continue monitoring borrowers. Finally, the bank is managed by a banker who maximizes the profits accruing to her rather than worrying about the utility of depositors or borrowers. The banker internalizes how her choices affect run risk, deposit supply, and loan demand. Depositors and borrowers rationally expect the equilibrium level of run risk when making these decisions but are atomistic and do not internalize their marginal impact on the bank's portfolio and run risk.

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We characterize the run decision by introducing a new specification of the type of global game developed by Goldstein and Pauzner (2005) to analyze Diamond-Dybvig style models. In our setup, depositors get signals about the (liquidation) value of loans that are called early to help pay depositors. Depositors use a threshold rule in deciding to run, such that if their signal about the liquidation value is below the threshold, they decide to withdraw even if they do not need to consume immediately. Conversely, if the signal is above the threshold, only depositors with urgent consumption need withdraw, while the rest keep their deposits in the bank. The threshold depends on the deposit contract, the balance sheet, and the profitability of the bank, because the deposit contract specifies the return from waiting instead of withdrawing early, the balance sheet determines the capacity of the bank to serve early withdrawals, and the profitability of the bank governs its incentives to monitor borrowers.

The proof to establish the uniqueness of the threshold is novel and may be of independent interest for other applications of global games. As in Goldstein and Pauzner (2005), there are no global strategic complementarities for depositors, but in our setup, the property of state monotonicity is also absent. This property relates to whether the incentive to take an action is monotonic with respect to the underlying fundamentals (state) of the economy, typically required in canonical global games (see Morris and Shin 2003). In our model, state monotonicity would obtain if the relative incentive to run becomes stronger as the liquidation value of loans becomes smaller. If the bank has liquid resources to serve all depositors queuing to withdraw, then state monotonicity holds. But it fails to hold once the bank runs out of liquid resources, and thus only a percentage of withdrawals can be met. Intuitively, once liquid assets are exhausted, the payoff from withdrawing is higher when the liquidation value of the bank's assets is higher, because the probability of being at a lucky spot in the queue is higher. Hence, state monotonicity is absent.

We employ a new argument to prove uniqueness, which does not require state monotonicity everywhere but only at threshold points. This property is easily satisfied in our framework, and we provide intuition about generalizing it to other environments where such perverse state monotonicity obtains. Hence, the proof we propose could be adapted to other applications in which improvements in fundamentals reduce the incentives to act in some regions and increase them in others.

The modifications to the Diamond-Dybvig model that we propose are all necessary to assess the differences between the private and social optima. First, microfounding the probability of a run is crucial to understand the determinants of run risk. In partial equilibrium, one expects that raising capital requirements or liquidity requirements will reduce the probability of a run. We show that this is true in our model. But in general equilibrium, agents will adjust their behavior in response to regulation, and

their new choices will feed back to alter run risk via equilibrium interest rates.

Second, it is important to explicitly model the optimization behavior of bankers, depositors, and borrowers in order to understand how run risk affects their choices and welfare. Instead of assuming ad hoc welfare criteria, such as the minimization of run risk, we model the effect of the choices on agents' utilities. This allows us to compare privately and socially optimal choices. In general, they differ, though we also describe a special case where the private equilibrium is constrained efficient.

Third, introducing costly monitoring highlights the importance of the bank's profitability for its stability (over and above the relative liquidity of its assets and the runability of its liabilities). Monitoring is unobservable and costly, so it occurs only if the banker finds it ex post profitable. Policy interventions that crush bank profits can backfire if the banker loses the incentive to monitor. In this case, depositors can opt to run because they realize bank credit risk has endogenously increased because of the lack of monitoring. Hence, we highlight a novel connection between moral hazard and run risk, which is important for studying regulations that compress banking profits.¹

Fourth, by modeling the utility impact of agents' choices, it becomes clear that borrowers and depositors care about the amount of lending and the level of deposits that are available. Thus, the overall size of the banking sector matters beyond just the composition of assets and liabilities. The Modigliani-Miller theorem is violated in various ways in our model, but allowing for endogenous equity issuance helps us understand the implications of these violations for the size of the banking sector and how the banking sector converts deposits into loans.

We focus on the normative properties of the model and derive four main findings. First, there are three separate distortions in private banking choices. Both the private and the social planner's equilibria can be characterized by the mix of loans versus liquid assets, the mix of deposits versus equity, and the level of lending that is funded by deposits (summarized by the loan-deposit spread). In the private equilibrium, the banker wants to maximize her profits from collecting deposits to extend loans. Because she understands how run risk and how the profit margin between the loan and deposits rates are each determined, she chooses levels of liquidity and equity that are profit maximizing for her. Compared with the banker, the social planner cares about how these choices affect not only bank profits but also the utility of depositors and borrowers. We show that the other two agents are disadvantaged in the private equilibrium

¹ This possibility may be of independent interest to researchers, as it highlights that runs can occur even for financial institutions that are very liquid if they lack the incentives to monitor.

and explain how the asset mix, liability mix, and loan-deposit spread can be adjusted to improve the welfare of the savers or borrowers.

Second, each distorted choice by the banker has a component tied to the run and another component that reflects the surplus created by the bank that accrues to either the borrowers or savers. The run component arises because bankers are prone to take risks that raise the probability of a run without internalizing how run risk directly matters for borrowers' and savers' welfare. This is because the banker is a monopolist and maximizes her own profits subject to limited liability, while borrowers and savers are atomistic, so that they do not internalize how their choices affect run risk.

Third, how the planner corrects the distortions depends on the weights placed on borrowers versus savers, that is, on the implicit preference between credit extension and liquidity provision. Run risk arises from choices on both sides of the bank's balance sheet, and the planner would opt for both a less deposit-intensive capital structure and a less lending-intensive asset allocation to reduce run risk. But doing so may restrict credit extension and reduce the surplus to borrowers. Thus, the desire to reduce run risk is balanced against the benefits from offering deposits and extending loans.

When savers are favored, the planner chooses a more equity-intensive capital structure, a more liquidity-intensive asset allocation, and a lower level of lending. Although more deposit taking requires higher deposit rates that compress bank profit margins and discourage equity issuance, cutting lending allows the banker to raise loan rates. The higher loan rates offset the higher deposit costs, and equity issuance is still possible. The deposits that are not channeled to lending are invested in the liquid assets, further reducing run risk and offsetting some of the increase in deposit rates.

By contrast, when borrowers are favored, the planner shifts to a relatively more lending-intensive asset allocation and a higher level of lending while choosing a more equity-intensive capital structure. Increasing lending by raising more deposits would compress profits and reduce the incentives to provide equity. So, instead, the planner reduces the liquid asset holdings and shifts funding away from deposits to release resources for loan extension and maintain the incentives for equity issuance.

Fourth, individual regulations can be used to move the private choices closer to those that a planner would choose. But no single regulation can deliver the planning outcomes. Capital and liquidity regulations reduce run risk but inefficiently restrict credit creation. Moreover, as banks take excessive risk on both sides of their balance sheet, controlling risk on one side may result in risk materializing on the other side. To replicate the planner's preferred allocations, a combination of three regulations are needed to correct the three distortions associated with the bank's asset

allocation, capital structure, and level of lending that is deposit funded. The asset and liability composition distortions can be corrected using a capital and a liquidity requirement. Capital and liquidity requirements are jointly helpful and should be treated as complementary, since they operate on different sides of the balance sheet. Neither is very effective at boosting the level of lending, which the planner may favor in order to expand lending and help borrowers. To boost (or shrink) lending, other regulations, such as deposit or lending subsidies/taxes, would be needed.

Related literature.—Our analysis of runs can be contrasted to several other approaches in the literature. For instance, a bank run in our setup can occur because the information about fundamentals is very bad. Thus, our analysis can be compared with many prominent papers that analyze information-based runs, such as Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), and Uhlig (2010). According to this fundamental view of runs, policy interventions are needed to correct pecuniary externalities arising from missing markets; otherwise, the private equilibrium is constrained efficient (Allen and Gale 2004). Although we have abstracted from pecuniary externalities, the fundamental run probability in our model depends on the bank's liquidity and capital. Thus, regulation could be used to address not only the adverse consequences of any pecuniary externalities but also the probability that they arise.

However, a bank run can also occur because of a coordination problem among depositors even if the bank is solvent. This type of run can be interpreted as being panic based; hence, it is important to know what determines the panic. In the Diamond-Dybvig model, panics are a multiple equilibrium outcome. There are several ways to tackle the multiplicity of equilibria such that the model is suitable for policy analysis. Cooper and Ross (1998), Peck and Shell (2003), and Diamond and Kashyap (2016) consider that the probability of a bank run is driven by sunspots. Ennis and Keister (2005) take an axiomatic approach to equilibrium selection and link the probability of a particular equilibrium being played to the appropriately defined incentives of agents. Despite the fact that these approaches may be, to varying degrees, adequate for certain types of positive analysis, the derivation of optimal policy requires microfoundations for the incentives to run on the bank.

In order to address such concerns, we follow the global games approach—developed by Carlsson and van Damme (1993) and Morris and Shin (1998) and applied to bank runs by Goldstein and Pauzner (2005)—to derive a unique probability of run. This approach ties run incentives to the fundamentals of the economy, the balance sheet structure of the bank, and the equilibrium payoffs to depositors withdrawing early and late. Rochet and Vives (2004) and Vives (2014) also take a global game approach but assume simpler payoffs for depositors that wait and depositors

that withdraw, which are exogenous and do not depend on equilibrium allocations and interest rates. Despite the microfoundation for run dynamics, these approaches are arguably not adequate to study optimal regulation, because deposit rates, which are important for welfare in general equilibrium, do not directly influence the incentives of depositors to run.

Goldstein and Pauzner (2005) consider the impact of deposit contracts on run incentives but assume a fixed liquidation value for loans and an exogenously set upper dominance region for the global game. We propose a new way to endogenize the liquidation value and the upper dominance region. Apart from the new type of global game we consider, we also differ from the aforementioned papers because we endogenize the supply of deposits, equity issuance, and loan demand. Hence, we allow the deposit and loan rates to be determined in equilibrium and respond to regulations. As we show, this is important in order to capture the welfare effects on different agents, assess the consequences of banks' private choices, and determine how a planner would alter these choices.

We should note that in both our model and the literature we have surveyed so far, deposit rates cannot be indexed by the realization of the state of the economy. Instead, if contracts could incorporate suspension clauses, as in Ennis and Keister (2009) and Keister (2015), then run dynamics would differ. As noted in these papers, suspension of convertibility may not be ex post efficient, but the interaction between run dynamics, ex post intervention, and ex ante convertible deposit contracts is an interesting topic for future research.

Our analysis abstracts from policy interventions such as deposit insurance and bank resolution. Government guarantees may be useful policy interventions to reduce run risk, but it is unclear whether they would unambiguously improve outcomes, because removing the disciplining role of deposits can encourage risk-taking (see Cooper and Ross 2002). Allen et al. (2018) study the implications of deposit insurance in a global games framework, with a simpler banking sector than ours, while Davila and Goldstein (2023) compute the optimal deposit insurance under a sufficient statistics approach. Schilling (2022) studies the optimal bank resolution using a global games framework and finds that aggressive intervention may be counterproductive.

Finally, there is a growing literature studying the complementarity of capital and liquidity regulations, such as Walther (2016) and Kara and Ozsoy (2019) in the presence of fire sale externalities and Van den Heuvel (2022) in a real business cycle model with risk-taking. Our conclusion, relevant to this literature, is that the number of optimal regulatory interventions is equal to the number of distorted margins rather than the number of externalities in the model. In other words, it does not make much difference if the same externality or multiple externalities distort

multiple margins in distinct ways. What is important is whether different margins are distorted.

II. Model

The model consists of three periods, $t = \{1, 2, 3\}$, features a single consumption good, and includes three types of agents. In particular, there is a continuum of entrepreneurs, a continuum of savers, and one banker. We will be referring to individual entrepreneurs and savers as the entrepreneur (E) and the saver (S), and we will be also using the singular or plural for these agents depending on the context. But it should be clear that each of these agents is atomistic and representative of her type, taking equilibrium variables as given. By contrast, the banker (B) internalizes how her choices affect the equilibrium variables that matter for her profitability.

Each entrepreneur has access to a productive—but illiquid—risky technology and chooses how much to borrow to invest in it. Funds invested at $t = 1$ yield A per unit of investment at $t = 3$ with probability ω (which we call the “good” state of the world) and zero otherwise (the “bad” state). The project delivers no output at $t = 2$, but it can be liquidated for an uncertain amount $\xi \sim U[\underline{\xi}, \bar{\xi}]$ with $0 < \underline{\xi} < 1 < \bar{\xi}$, which is realized at $t = 2$. Define $\Delta_\xi \equiv \bar{\xi} - \underline{\xi}$. All projects of individual entrepreneurs yield the same payoff at $t = 3$ and have the same liquidation value at $t = 2$. Hence, we are always referring to the representative entrepreneur and the representative project.

The banker manages an institution (bank) that stands between the entrepreneurs and savers. The bank is funded with equity from the banker and deposits from savers. The funds raised at $t = 1$ are invested into either a liquid storage asset or a loan to entrepreneurs. The loan contract with the entrepreneur specifies an uncontingent loan rate, and it is backed by the entrepreneur’s project and the cash flows it generates. If the entrepreneur fails to repay the loan, the bank can seize the project and any cash flows that have been generated. Loans are also callable, that is, the bank does not need to wait for the project to mature but can request full repayment at any point in time.

Moreover, the banker decides whether to monitor the entrepreneur’s project at $t = 3$. Monitoring is essential because the payoff realization is private information to the entrepreneur. Without monitoring, the entrepreneur would report the bad state of the world and default. Because individual projects’ payoffs are perfectly correlated, the banker needs to incur the monitoring cost once for the whole loan portfolio, which we assume is independent of how many loans are extended.

Savers are identical *ex ante*, and each of them has a large endowment at $t = 1$ that is used to fund initial consumption and savings. Each saver

receives an idiosyncratic preference shock at $t = 2$ to consume early or late, as in the Diamond-Dybvig model. Given that the preference shocks are independent and identically distributed, some fraction of savers will need to consume at $t = 2$, and the rest wait to consume at $t = 3$. Thus, savers are *ex ante* identical but *ex post* heterogeneous, which we will refer to as impatient and patient, respectively. The (*ex ante*) representative saver invests in bank deposits or holds a liquid storage asset at $t = 1$. Deposits are demandable, which is important to provide incentives to the banker to monitor, as we shall explain later.

A loan can be recalled to pay deposits. Upon being recalled, it yields an immediate gross amount ξ . When a loan is called, the entrepreneur forfeits the portion of the project that is funded by the loan. Depending on the value of ξ , the bank may not have enough resources to fully pay any level of withdrawals, while monitoring incentives may be weak or strong. Aside from extremely high or low realizations of ξ , the bank is at risk for self-fulfilling runs: a patient saver will demand her deposits early if she believes that other patient savers will do the same. To address the coordination problem and obtain a unique equilibrium, we assume that savers receive noisy signals about the true realization of ξ at $t = 2$. These signals provide information not only about the fundamental ξ but also about the beliefs of other savers and so serve to coordinate the patient savers' decisions. We show that there is a threshold strategy that patient savers will follow such that when the signal is above a threshold value, they do not withdraw early, and when it is below the threshold, they run.

Moreover, on the basis of the signal, the savers form posterior beliefs about ξ , and we show that there is a unique threshold ξ^* , such that all patient savers withdraw their deposits when they believe the true realization is below that value and they keep their deposits in the bank otherwise. We will refer to ξ^* as the run threshold and denote the probability of a run by $q = (\xi^* - \xi)/\Delta_\xi$.

The liquidation value may be understood in several ways. For instance, the incomplete project could have a secondary use in the interim period because it can be used in conjunction with an alternative short-term technology. Alternatively, we could assume that it can be sold to some outside investors, as in Shleifer and Vishny (1992). In other words, ξ does not strictly represent the salvage value but rather the liquidation/resale value of long-term investment. The upper bound $\bar{\xi}$ has to be high enough that the bank can always withstand a panic for some realizations, while the lower bound ξ has to be low enough that the bank may run out of liquidity even if a panic does not occur. We describe the importance of these bounds in section II.D.

We simplify the model in three ways by assuming that savers have quasilinear preferences, entrepreneurs have linear preferences and no initial

endowment, and the production technology is linear. The main cost of linearity is that it may leave agents at a point where they break even from their savings and borrowing choices. Hence, the bank creates no value for them, which does not distort the basic properties of the private equilibrium but has powerful implications for regulation. We reintroduce curvature into the savers' and entrepreneurs' problems in other ways that make the results less extreme while keeping the model tractable. Hereafter, we explain the implications of each of these assumptions, but none of them are crucial for our main results.

Figure 1 presents the timeline of the model. Sections II.A–II.D describe the agents' optimization problems and the derivation of the run threshold. Section II.E characterizes the private equilibrium.

A. *Savers*

Savers are endowed with e_s at $t = 1$ and decide how much to invest in bank deposits, D , and how much to hold in the liquid asset, L_s . At $t = 2$, a portion of savers, δ , receive a preference shock to consume immediately, while the rest, $1 - \delta$, want to consume at $t = 3$. These shocks are private information, are independent and identically distributed, and are not contractible ex ante.

Deposits are demandable, early withdrawals are serviced sequentially, and the $t = 2$ and $t = 3$ uncontingent gross interest rates are r_D and R_D , respectively. We restrict all gross rates to be higher than 1, an assumption that we relax in our extensions. This contract structure creates the possibility of a run, since patient savers may choose to demand their deposits early depending on their own information and their expectations about the actions of other patient savers.

If there is no run, that is, $\xi \in [\xi^*, \bar{\xi}]$, only impatient depositors withdraw, and they receive Dr_D . Patient depositors' repayments will depend on the state realization at $t = 3$; they receive their promised payment, DR_D , with probability ω and zero otherwise, since B defaults. In a run, all depositors attempt to withdraw, and there is probability $\theta(\xi)$, defined later, that any depositor gets Dr_D .

$t = 1$	$t = 2$	$t = 3$
E borrow and invest in risky projects S invest in demandable bank deposits and potentially in the liquid asset B raises equity and deposits and invests in loans and liquid assets	S learn their type S receive noisy signals about ξ and decide whether to withdraw B recalls loans and pays withdrawals If $\xi < \xi^*$, a run occurs	Investment payoff is determined E privately learn the payoff realization B decides whether to monitor Loans and deposits are repaid if the payoff is A and default otherwise

FIG. 1.—Timeline.

Savers have quasilinear preferences for consumption, such that they value consumption linearly at $t = 2$ and $t = 3$. This is our first assumption that greatly simplifies the patient savers' decision to join a run, because it means that all that must be computed is the expected payoff from deposits at $t = 2$ versus $t = 3$. By contrast, if savers were risk averse, finding the threshold that determines whether to run is much more complicated. The complication arises because a saver needs to compute her expected utility differential between waiting and withdrawing, accounting for all possible out-of-equilibrium beliefs about the actions of other savers. Computing the expected deposit payoffs is simpler than computing the expectation of a nonlinear function of the deposit payoffs. Moreover, quasilinearity yields a very tractable deposit supply schedule, which substantially simplifies the normative analysis. The disadvantage of linearity is that it reduces the usefulness of a deposit to a saver. Essentially, the deposit becomes a pure financial instrument whose only value is that it pays more interest than the liquid asset. Put differently, the deposit would be equivalent to a bond.

To make deposits more useful, we assume that impatient savers enjoy transaction benefits from having deposits when there is no run. To do this, we suppose that this advantage is described by a concave function V that is increasing in the amount promised and repaid, Dr_d , and it accrues only for $r_d \geq 1$, providing some justification for the assumption on gross returns. Peck and Shell (2010) also have a similar assumption about the utility from the transaction services of deposits. This additional benefit of deposits partially offsets the stark implications of quasilinear preferences. Modeling things this way leads to no qualitative changes relative to a model with concave utility and no transactions services but makes it substantially easier and faster to solve the model.

The expected utility of an individual saver is given by

$$\begin{aligned}
 \mathbb{U}_S = & \overbrace{U(e_s - D - L_s)}^{\text{Utility at } t=1} + \overbrace{\int_{\xi}^{\xi^*} [\beta\delta + \beta^2(1 - \delta)][\theta(\xi)Dr_d + L_s] \frac{d\xi}{\Delta_\xi}}^{\text{run}} \\
 & + \underbrace{\int_{\xi^*}^{\bar{\xi}} \beta\delta[Dr_d + L_s] \frac{d\xi}{\Delta_\xi}}_{\text{no run, impatient}} + \underbrace{\int_{\xi^*}^{\bar{\xi}} \beta^2(1 - \delta)[\omega DR_d + L_s] \frac{d\xi}{\Delta_\xi}}_{\text{no run, patient}} + \underbrace{\int_{\xi^*}^{\bar{\xi}} V(Dr_d) \frac{d\xi}{\Delta_\xi}}_{\text{transaction services}}.
 \end{aligned}
 \tag{1}$$

U is the utility function for $t = 0$ consumption with $U' > 0$ and $U'' < 0$; V captures the transaction services of deposits with $V(0) = 0$, $V' > 0$, and $V'' < 0$; $\beta \leq 1$ is the time discount factor.²

² The transaction services accrue to only the impatient depositors, so δ is subsumed inside V .

Savers choose the level of deposits and their holdings of the liquid asset to maximize (1). An individual saver takes the run threshold, ξ^* , and the probability of being repaid in a run, $\theta(\xi)$, as given. These objects depend on the aggregate bank portfolio, and we suppose that the individual saver is atomistic so as to not account for her impact on them. In contrast, a social planner would internalize the effect of the choices. Nevertheless, individual savers have rational expectations and correctly anticipate the equilibrium level of run risk when making their decisions. Finally, short selling of deposits and the liquid asset is not allowed, that is, $D \geq 0$ and $L_s \geq 0$.

The optimal choice of deposits by S yields the following deposit supply schedule:

$$U'(\epsilon_s - D - L_s) \geq [\beta\delta + \beta^2(1 - \delta)]r_D \int_{\underline{\xi}}^{\xi^*} \theta(\xi) \frac{d\xi}{\Delta\xi} \quad (2)$$

$$+ [\beta\delta r_D + \beta^2(1 - \delta)\omega R_D + V'(Dr_D)r_D](1 - q),$$

which holds with strict equality if savers choose to hold deposits in equilibrium, that is, $D > 0$. Condition (2) says that savers equate the marginal utility of forgone consumption at $t = 1$ to the expected marginal utility gain from holding deposits in the future. In a run, all savers withdraw; an individual saver will receive r_D per unit of deposits with probability $\theta(\xi)$ for each realization of $\xi < \xi^*$. Otherwise, with probability $1 - q$, a run does not occur, and only impatient savers withdraw. In this case, an individual saver either is impatient with probability δ and receives the period 2 deposit rate r_D or is patient with probability $1 - \delta$ and receives the period 3 deposit rate R_D with probability ω . Absent a run, S also enjoys the marginal benefit of transaction services, $V'(Dr_D)r_D$.

Savers may also self-insure by holding the liquid asset. The optimal L_s is given by

$$U'(\epsilon_s - D - L_s) \geq \beta\delta + \beta^2(1 - \delta). \quad (3)$$

This condition compares the marginal loss in consumption with the expected value of storage and holds with strict equality if $L_s > 0$. Without loss of generality, we consider equilibria in which $L_s = 0$ but $D > 0$, that is, (2) holds with equality, while (3) is slack. Savers will not use the storage technology whenever (i) their endowments are not excessive, (ii) the bank offers high enough deposit rates, or (iii) transaction services are sufficiently valuable.

Substituting the deposit schedule (2) into (1), we get the indirect utility function,

$$\mathbb{U}_S^* = U(\epsilon_s - D) + U'(\epsilon_s - D)D + (1 - q)[V(Dr_D) - V'(Dr_D)Dr_D], \quad (4)$$

for the benchmark case in which $L_s = 0$.³ Given our assumptions about $V(\cdot)$, it is easy to show that the third term in (4) is strictly positive.⁴ Moreover, the first two terms in (4) must be higher than \mathbb{U}_s^a (the utility level from saving only using liquid assets), otherwise savers could choose $L_s > 0$ and attain that level of utility. Hence, savers are always better off using the bank compared with autarky.

Even with our simplifying assumptions, this framework produces an equilibrium and a set of intuitive decision rules for savers. In particular, savers use the bank because it offers a better way to save for the future and facilitates transactions. The former is captured by the benefit in terms of period 1 consumption (the first two terms in [4]). The latter is captured by the benefit from the transaction services (the last term in [4]).

B. *Entrepreneurs*

Entrepreneurs have the rights to operate real projects that are in perfectly elastic supply, require a unit of funding at $t = 1$, are infinitely divisible when liquidated, and mature at $t = 3$. For simplicity, E does not have an endowment but borrows I from the bank at gross rate R_t to invest in the risky technology. Moreover, E is risk neutral and derives utility only from consumption at $t = 3$. Finally, E is protected by limited liability when projects mature and loans are due.

The risk neutrality of entrepreneurs together with the absence of initial endowment is our second technical assumption. Risk neutrality means that the entrepreneur cares only about the expected profits from operating the technology. The absence of an endowment means that the decision to default will depend only on the productivity of the technology, not on entrepreneurs' leverage. Otherwise, the savers' run decision becomes more intertwined with how much the entrepreneur wants to borrow, and the relative risk aversion of the two parties will matter for how risks are shared.

The linearity of the production function is our third technical assumption. It simplifies the derivation of the loan demand but comes at the cost of also implying that R_t depends only on A . If the production function had curvature, it would substantially complicate the calculations. The

³ For completeness, if $L_s > 0$ then the savers' indirect utility is $\mathbb{U}_s^a + (1 - q)[V(Dr_D) - V'(Dr_D)Dr_D]$, where \mathbb{U}_s^a is the utility in autarky given by $\mathbb{U}_s^a = U(U'^{-1}(\beta\delta + \beta^2(1 - \delta))) + [\beta\delta + \beta^2(1 - \delta)][e_s - U'^{-1}(\beta\delta + \beta^2(1 - \delta))]$.

⁴ Since $D > 0$, the term can be written as $(1 - q)D[V(D)/D - V'(D)]$. Because $V(0) = 0$, $V(D)/D$ is the slope of the straight line emanating from zero and passing through D . Since V is strictly concave, its image is always above the image of the straight line for any point $x \in (0, D)$. Because V' is strictly decreasing, V will necessarily cross the straight line connecting zero and D from above. Hence, the derivative of V at D is strictly smaller than the slope of the straight line.

complication arises because the realized production depends on the bank's decision to recall loans, which occurs after the realization of ξ is known. The loan demand trades off the expected revenue from borrowing to produce against the cost of repaying the loan. And it is easier to compute the expected profits to entrepreneurs over $\xi \geq \xi^*$ when production is linear rather than when it is a nonlinear function of the number of projects that are not recalled.

In order to make the determination of R_f less mechanical, while maintaining analytical tractability, we introduce a cost of effort that the entrepreneur incurs upon investing. We suppose that this cost is incurred before the payoff from the investment is known; we could also call this cost an adjustment cost. We assume that this cost is convex and it pertains to total investment.

The expected utility of an individual entrepreneur can be written as

$$\mathbb{U}_E = \int_{\xi^*}^{\xi} \left\{ \underbrace{\omega[A(1 - y(\xi, \delta))I]}_{\text{realized output}} - \underbrace{(1 - y(\xi, \delta))IR_f}_{\text{loan obligation}} - \underbrace{c(I)}_{\text{effort}} \right\} \frac{d\xi}{\Delta_\xi}, \quad (5)$$

where $y(\xi, \delta)$ is the portion of loans recalled to serve withdrawals from impatient savers (and will be derived below). If a run does not occur, E repays the outstanding loans in the good state, $(1 - y(\xi, \delta))I$, as long as the per unit payoff, A , is higher than the promised gross loan rate R_f . Naturally, E defaults in the bad state when the project pays zero. In a run, all projects funded by bank loans are liquidated and no production takes place. Finally, $c(I)$ is the effort/adjustment cost with $c(0) = 0$, $c' > 0$, and $c'' > 0$. E needs to incur this cost in order to produce before she learns the realization of the state of the world and after run uncertainty has been resolved. Hence, E will choose not to exert effort and thereby avoid the cost in a run. Absent a run, given that the net payoff to E is increasing in ξ , she will choose to produce if the following incentive compatibility constraint holds:

$$\omega[A - R_f](1 - y(\xi^*, \delta))I - c(I) > 0. \quad (6)$$

Entrepreneurs choose the level of investment and thus borrow I to maximize (5). An individual entrepreneur takes the run threshold, ξ^* , and the aggregate portion of loans recalled, $y(\xi, \delta)$, as given. These objects depend on the aggregate bank portfolio, and we suppose that individual entrepreneurs are atomistic so that they do not account for their impact on them. A social planner would internalize the effect of the choices. However, individual entrepreneurs have rational expectations and correctly anticipate the equilibrium level of run risk when making their decisions.

The optimal choice of I by E yields the following loan demand schedule:

$$\int_{\xi^*}^{\bar{\xi}} \left\{ \omega[A - R_l](1 - y(\xi, \delta)) - c'(I) \right\} \frac{d\xi}{\Delta_\xi} = 0. \quad (7)$$

Condition (7) says that E equates the expected profit margin on the remaining projects—given by the difference between the marginal product of investment and the gross loan rate—to the marginal effort cost over the realizations of ξ , where a run does not materialize.

Substituting the loan demand schedule (7) into (5), we get the following indirect utility function:

$$\mathbb{U}_E^* = (1 - q)[c'(I)I - c(I)]. \quad (8)$$

Given our assumptions about c , it is easy to show that (8) is always strictly positive, that is, E is strictly better off than in autarky, where E 's utility is zero.⁵ If effort is costless, then $\mathbb{U}_E^* = 0$, and the loan rate is equated to the marginal product of the project, that is, $R_l = A$ from (7).

Despite our simplifying assumptions about entrepreneurs' preferences and shape of the production function, entrepreneurs' choices are very conventional. Their welfare in equilibrium, given by (8), is increasing in I and decreasing in q . Our setup captures these relationships in a tractable way and succinctly identifies the distortions that any policies will aim to correct.

C. Banker

The banker (B) makes all investment and funding decisions to maximize her own utility. At $t = 1$, she is endowed with e_B and decides how much equity, E , to put into the bank. In addition, she decides how many deposits to raise, D , and how much to invest in the liquid assets, L , and illiquid loans, I , subject to the following balance sheet constraint:

$$I + L = D + E. \quad (9)$$

If a bank run occurs at $t = 2$, that is, $\xi < \xi^*$, the bank is liquidated and the proceeds are distributed on a first-come, first-served basis. The maximum level of withdrawals that can be served in full is

⁵ To show that (8) is strictly positive, it suffices to show that $c'(I) - c(I)/I > 0$, given that $I > 0$; otherwise, E is in autarky (short selling of projects is not allowed). Because $c(0) = 0$, $c(I)/I$ is the slope of the straight line starting at zero and passing through I . Given that c is strictly convex, its image is always below the image of the straight line for any point $x \in (0, I)$. Because c' is strictly increasing, c will necessarily cross the straight line connecting zero and I from below, and hence the derivative of c at I is strictly higher than the slope of the straight line.

$$\theta(\xi) = \frac{\xi I + L}{Dr_D}, \tag{10}$$

which is also equal to the probability of being served if all depositors withdraw.

If there is not a run, that is, $\xi \geq \xi^*$, the balance sheet and profits after $t = 2$ depend on the realization of ξ and the level of withdrawals, λ . The bank will recall and liquidate a portion $y(\xi, \lambda)$ of its loan portfolio to serve the early withdrawals.⁶ The amount recalled is given by

$$y(\xi, \lambda) = \max \left[\frac{\lambda Dr_D - L}{\xi I}, 0 \right]. \tag{11}$$

The profits that B receives in the good state at $t = 3$ are equal to the repayment on the remaining loans minus the payment on the remaining deposits. In the bad state, B defaults, so profits are zero. B needs to incur a monitoring cost, X , to learn the true state of the world. Alternatively, B can forgo the monitoring, in which case the entrepreneur will report that productivity was zero and default, independent of state realization. Hence, there is a moral hazard problem, akin to Holmström and Tirole (1997), such that B will choose to monitor only if expected profits exceed X .

Given a level for the liquidation value ξ and the level of withdrawals λ , B will choose to monitor if the following incentive compatibility constraint is satisfied:

$$\omega[(1 - y(\xi, \lambda))IR_l - (1 - \lambda)DR_D] - X \geq 0. \tag{12}$$

The monitoring decision is taken before B learns the true state of the world, so incentive compatibility requires that the expected payoff to B is higher than X . In an equilibrium where the bank is taking deposits to make loans, (12) must be satisfied when a run does not occur, that is, for $\xi \geq \xi^*$ (and $\lambda = \delta$); otherwise, monitoring would not take place, depositors would always run, and there is no gain to operating the bank. In section II.D, we prove that (12) is always slack for $\xi \geq \xi^*$.

The banker’s expected utility is given by

$$\begin{aligned} U_B = & W(e_B - E) + \int_{\xi^*}^{\bar{\xi}} \left\{ \underbrace{\omega[(1 - y(\xi, \delta))I]}_{\text{outstanding loans}} \underbrace{R_l}_{\text{loan rate}} - \underbrace{(1 - \delta)D}_{\text{patient deposits}} \underbrace{R_D}_{\text{deposit rate}} \right\} \\ & - \underbrace{X}_{\text{monitoring cost}} \left. \right\} \frac{d\xi}{\Delta_\xi}, \end{aligned} \tag{13}$$

⁶ The bank might also want to liquidate loans beyond the need to serve early withdrawals and carry the proceeds forward using the storage technology. But this would occur only if the realization of ξ is higher than the expected return from holding the loan to maturity, which we rule out by assumption.

where $W(\cdot)$ is B's utility function for $t = 1$, with $W'(\cdot) > 0$ and $W''(\cdot) < 0$. The banker has also quasilinear preferences but, unlike the saver, never needs to consume in the interim period.

D. Global Game and Bank Run Threshold

Having described the choices of all the agents, we can now determine when individual patient savers will want to withdraw their deposits at $t = 2$. The source of fundamental uncertainty in our model is the liquidation value ξ for which individual patient savers receive a private noisy signal. The decision depends not only on each saver's belief about the bank's financial health but also on their beliefs about how other savers will behave given their individual signals. We show that a unique threshold signal exists such that the individual savers withdraw only if their own signal is lower than that threshold. As the noise in private signals goes to zero, this threshold converges to a unique threshold for fundamentals ξ^* , which we have used in the analysis so far.

We prove this result in six steps using a series of lemmas and propositions. First, we derive the regions of ξ where savers act unilaterally, that is, without considering others actions. Lemmas 1 and 2 establish these regions, known as the lower and upper dominance regions. In the former, an individual patient saver decides to withdraw even if no other patient savers withdraw, while in the latter, she does not withdraw even if all savers withdraw. Second, we show that for intermediate values of ξ , individual actions depend on beliefs about aggregate withdrawals, as they govern the bank's incentive to monitor and whether the bank has adequate liquidity to meet withdrawals (lemma 3). Third, we show that if a threshold strategy is followed by all savers, then individuals can form beliefs about aggregate withdrawals and compute unique cutoffs for the regions of ξ where the bank is expected to monitor and have enough liquidity to meet withdrawal requests (lemma 4). Equipped with these beliefs and cutoffs, individuals can compute the expected payoff from withdrawing given their private signals and posteriors about ξ . Fourth, we show that following a strategy that supposes a common threshold signal is an equilibrium. Fifth, we establish the existence and uniqueness of the threshold signal and present the logic and mathematical characterization for our novel proof, leading to proposition 1, which establishes the uniqueness of ξ^* . Finally, proposition 2 summarizes the results about the existence and uniqueness of a threshold strategy equilibrium.

Step 1.—A bank is immune to runs if it both has enough liquidity to serve all potential withdrawals and maintains incentives to monitor irrespective of the level of withdrawals. The maximum level of withdrawals that the bank can sustain for given ξ is equal to $\theta(\xi)$ in (10), which is increasing in ξ . Thus, there is a cutoff $\hat{\xi}$ such that the bank does not have

enough liquidity to even serve early withdrawals for $\xi < \hat{\xi}$ and another value $\hat{\xi}$ such that it has enough liquidity to serve all potential withdrawals for $\xi \geq \hat{\xi}$. These cutoffs are the solutions to $\theta(\hat{\xi}) = \delta$ and $\theta(\hat{\xi}) = 1$, which yields

$$\hat{\xi} = \frac{\delta Dr_D - L}{I} \text{ and } \hat{\xi} = \frac{Dr_D - L}{I}. \tag{14}$$

Turning to monitoring incentives, note that (12) is increasing in ξ and, for now, suppose that it is decreasing in λ ; we establish this below. Then, the banker will not monitor for $\xi < \xi_{ld}$ even if only impatient savers withdraw, while she will monitor for $\xi \geq \xi_{ud}$ even if all savers withdraw. These cutoffs are obtained by setting (12) to zero and solving for ξ when $\lambda = \delta$ and $\lambda = 1$, which yields

$$\xi_{ld} = \frac{\delta Dr_D - L}{I - (1 - \delta)DR_D/R_l - X/(\omega R_l)} \text{ and } \xi_{ud} = \frac{Dr_D - L}{I - X/(\omega R_l)}. \tag{15}$$

Note that $\xi_{ld} > \hat{\xi}$ and $\xi_{ud} \geq \hat{\xi}$, with strict inequality for $X > 0$. This implies that the cutoffs related to monitoring ξ_{ld} and ξ_{ud} rather than $\hat{\xi}$ and $\hat{\xi}$ are the candidate ones to define the lower and upper dominance regions. Consider, for example, a $\xi \in (\hat{\xi}, \xi_{ld})$ and $\lambda = \delta$. The banker would not monitor even though the bank has enough liquidity to serve withdrawals, because expected profits are not high enough to cover the monitoring cost. Similarly, for $\xi \in (\hat{\xi}, \xi_{ud})$ and $\lambda = 1$, the banker would not monitor even though the bank has enough liquidity to serve withdrawals from all savers, because the bank has to liquidate so many loans that the expected profits from the remaining ones are not enough to cover the monitoring cost. To establish the lower and upper dominance regions, we need to show that $\xi_{ld}, \xi_{ud} \in (\hat{\xi}, \bar{\xi})$ and that (12) is indeed decreasing in λ . The next two lemmas provide the conditions for these properties to hold.

LEMMA 1 (Existence of the lower dominance region). For sufficiently high $\hat{\xi}$ and A , the bank has incentives to choose $\delta D > L$.

Proof. Define $z \equiv \delta D - L \in [0, \delta D]$. Using (9), we obtain $I(z) = D + E - L = z + (1 - \delta)D + E$, that is, z are the loans not funded by long-term deposits and equity. We can also solve for R_D and R_l as functions of z , denoted by $R_D(z)$ and $R_l(z)$, by substituting $I(z)$ into (2) and (7). Substituting all expressions in (13) and taking the first-order condition with respect to z yields

$$\begin{aligned} & \omega R_l(z) \int_{\xi^*}^{\hat{\xi}} \left(1 - \frac{1}{\xi}\right) d\xi + \omega R_l'(z) \int_{\xi^*}^{\hat{\xi}} (1 - \gamma(\xi, \delta)) I(z) d\xi - \omega R_D'(z) \int_{\xi^*}^{\hat{\xi}} (1 - \delta) D d\xi \\ & + \frac{d\xi^*}{dz} \left(-\Pi(\xi^*) + \omega \frac{dR_l}{d\xi^*} \int_{\xi^*}^{\hat{\xi}} (1 - \gamma(\xi, \delta)) I(z) d\xi - \omega \frac{dR_D}{d\xi^*} \int_{\xi^*}^{\hat{\xi}} (1 - \delta) D d\xi \right) + \psi_z \Delta_\xi = 0, \end{aligned} \tag{16}$$

where ψ_z is the Lagrange multiplier on $z \geq 0$ and $\Pi(\xi^*) = \omega(1 - y(\xi^*, \delta))I(z)R_l(z) - (1 - \delta)DR_D(z) - X$ are bank's expected profits for $\xi = \xi^*$. The Kuhn-Tucker condition requires that $\psi_z \geq 0$ for $z = 0$ to be a solution, that is, the sum of the first four terms must be negative or zero. We prove by contradiction that $z = 0$ cannot be an equilibrium. To do this, we sign each of the four individual terms. We then explain how by invoking the conditions in the lemma, we can make their sum positive at the prevailing equilibrium allocations, ruling out the $z = 0$ equilibrium. Finally, we guess that there is a perturbation so that there is an equilibrium with $z > 0$ and verify that in our subsequent analysis.

The first term in (16) captures the direct payoff from increasing z and is positive if $\int_{\xi^*}^{\bar{\xi}} (1 - 1/\xi) d\xi > 0$, or $\bar{\xi} - \log \bar{\xi} > \xi^* - \log \xi^*$. Since $\bar{\xi} > 1$, the left-hand side is increasing in the level of $\bar{\xi}$, and thus given a ξ^* , the first term can be made positive for sufficiently high $\bar{\xi}$. The second and third terms capture the direct effect of higher z on R_l and R_D and are both negative. From (7), we can obtain $R_l'(z) = -[\alpha_1 c'(z + (1 - \delta)D + E) + \alpha_2 c''(z + (1 - \delta)D + E)] < 0$, because α_1 and α_2 are positive. From (2), we get $R_D'(z) = (\beta\delta + \beta^2(1 - \delta))/((1 - q)\omega\beta^2(1 - \delta)) \int_{\xi^*}^{\bar{\xi}} (1 - \xi)/D d\xi/\Delta_\xi > 0$.

The fourth term captures the effect of higher z on the run threshold ξ^* and operates through the three subterms in parentheses multiplying $d\xi^*/dz$: for a marginal increase in ξ^* , (i) the bank forgoes profits $\Pi(\xi^*)$ (first subterm); (ii) $dR_l/d\xi^* > 0$ from (7), meaning that loan rates increase, which increases profitability (second subterm); and (iii) $dR_D/d\xi^* > 0$ from (2), implying that deposit rates rise, which also reduces profitability (third subterm). In any (reasonable) equilibrium where z is higher, run risk is higher, meaning that $d\xi^*/dz > 0$, so the whole fourth term is negative for sufficiently low c' , resulting in sufficiently low $dR_l/d\xi^*$.

To find an equilibrium with $z > 0$, begin with the $z = 0$ solution to (16) and start increasing A , allowing R_l to adjust while keeping all other variables at the prevailing equilibrium values. Then, the first term in (16) can be increased enough so that the sum of all four terms is strictly positive. We now guess we can perturb the $z = 0$ allocations by making the second, third, and fourth terms more negative and the first one less positive so that (16) holds with equality for $\psi_z = 0$ and $z > 0$. While performing this calculation, we kept other variables at their $z = 0$ values. Thus, we need to verify that when all the other variables are optimally chosen and $z > 0$, the lemma is still true. For now, we guess that this is the case and verify it in section III.B. QED

The intuition underlying the lemma is simple. $\delta D - L$ are the loans that are not funded by long-term deposits and equity, so that they represent loans that are liquidated to serve early withdrawals. If lending is profitable enough and loans can be potentially liquidated at high enough values, then planning to liquidate some loans to meet early withdrawals is desirable. Given that $r_D \geq 1$, the lemma implies that $\xi_{id} > 0$ and thus establishes

the existence of a lower dominance region because we can choose a $\underline{\xi}$ sufficiently close to zero such that $\underline{\xi} < \xi_{ld}$.⁷

LEMMA 2 (Existence of the upper dominance region). For sufficiently low V' and high e_s , there exists $\xi_{ud} < R_l r_D / R_D$ and (12) is decreasing in λ .

Proof. With the definition of ξ_{ud} from (15), the claim we want to establish becomes

$$IR_l r_D - DR_D r_D + LR_D - \frac{Xr_D}{\omega} > 0. \quad (17)$$

For $r_D = 1$, when we use the balance sheet constraint (9) and lemma 1, (17) becomes $(D - L)(R_l - R_D) + ER_l - X/\omega > (1 - \delta)D(R_l - R_D) + ER_l - X/\omega > 0$, provided $R_l - R_D > 0$. This condition is intuitive: it says that B's profits from offering late deposits plus investing her own equity are positive after monitoring costs; otherwise, the bank would not operate. If $IR_l - DR_D - X/\omega > 0$, (17) is also increasing in r_D , so no further assumptions are needed. If this is not true, (17) would, by continuity, hold for $r_D > 1$ for r_D sufficiently close to 1. The latter will hold if the marginal benefits from payments services, captured by V' in the deposit supply schedule (2), are sufficiently small with e_s sufficiently high. We verify in the equilibria we study that these conditions hold. QED

Lemma 2 implies that there is $\bar{\xi}$ higher than ξ_{ud} , such that the upper dominance cutoff exists, but with $\bar{\xi}$ less than $R_l r_D / R_D$. The upper bound on $\bar{\xi}$ ensures that liquidating a loan and storing the proceeds is less attractive than letting the loan mature. Hence, the bank's profits given by (12) are decreasing in λ for all ξ (because the derivative of (12) with respect to λ is $-R_l r_D / \xi + R_D < 0$).

Taken together, lemmas 1 and 2 guarantee that ξ_{ld} and ξ_{ud} are indeed the appropriate lower and upper dominance cutoffs. Figure 2 shows graphically the lower and upper dominance regions.

Step 2.—For $\xi \in [\xi_{ld}, \xi_{ud}]$, the incentives to monitor depend on the level of withdrawals, and a patient saver may decide to withdraw early if she believes that other patient savers will withdraw as well. The strategic complementarity in the withdrawal decision creates multiple equilibria. The withdrawal decision depends on the perceived payoff differential between waiting until $t = 3$ and withdrawing at $t = 2$. We denote this differential

⁷ The liquidity regulations that have been implemented since the global financial crisis, such as the liquidity coverage ratio (LCR), still allow for banks to choose $\delta D > L$, so that lemma 1 can continue to hold. This is because these regulations pertain to banks' expected net liquidity needs. In particular, regulatory liquidity is calculated by summing the liquid assets that banks hold (L in our model) plus some less liquid assets (such as corporate bonds or payments coming from all other banking activities) and subtracting expected fund outflows (δD in our model). If the realized level of inflows is unexpectedly small, then L will be far below the level of expected deposit outflows.

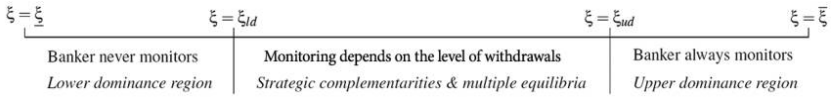


FIG. 2.—Lower and upper dominance regions.

by $v(\xi, \lambda)$ and report in (18) the value it takes for different beliefs about the level of withdrawals λ when the realization of fundamentals is equal to ξ :

$$v(\xi, \lambda) = \begin{cases} \omega DR_D - Dr_D & \text{if } \hat{\lambda}(\xi) \geq \lambda \geq \delta, \\ -Dr_D & \text{if } \min(\theta(\xi), 1) \geq \lambda > \hat{\lambda}(\xi), \\ -(\xi I + L)/\lambda & \text{if } 1 \geq \lambda > \min(\theta(\xi), 1). \end{cases} \quad (18)$$

We discuss the three regions in (18) in turn. The first case concerns beliefs for λ under which the banker has an incentive to monitor, so that full repayment at $t = 3$ is possible. The maximum level for which this is true is given by

$$\hat{\lambda}(\xi) = \frac{(\xi \cdot I + L)R_I - \xi(DR_D + X/\omega)}{D(R_I r_D - \xi R_D)}, \quad (19)$$

with $d\hat{\lambda}(\xi)/d\xi > 0$, using the chain rule and lemma 2. The payoff differential in this region is $\omega DR_D - Dr_D$, which is strictly positive; otherwise, no patient savers would ever wish to wait. The remaining regions have no monitoring. The second one is when $\lambda > \hat{\lambda}(\xi)$ but less than $\min(\theta(\xi), 1)$. Here the banker has enough liquidity to serve all λ withdrawals. Recall that $\theta(\xi) = (L + \xi I)/(Dr_D)$ from (10). Lemma 3 shows that $\hat{\lambda}(\xi) < \theta(\xi)$. So a patient saver's payoff differential is $-Dr_D$. Finally, if $\min(\theta(\xi), 1) < \lambda \leq 1$, the bank cannot fully repay all savers that withdraw and is liquidated. A saver gets zero if she waits, while she is repaid with probability $\theta(\xi)/\lambda$ if she withdraws and the expected payoff is $\theta(\xi)/\lambda Dr_D = (L + \xi I)/\lambda$. Hence, the expected payoff differential is equal to $-(L + \xi I)/\lambda$. Below we refer to this region of λ 's as the *full-run* region.⁸

The next lemma says that an individual saver may still want to withdraw, even if the bank has enough liquidity to serve all withdrawals, when she believes that the banker will not monitor.

LEMMA 3 (Strategic complementarities and monitoring). $\delta \leq \hat{\lambda}(\xi) < \theta(\xi)$ for $\xi \in [\xi_{ld}, \xi_{ud}]$.

⁸ For completeness, note that this region will not exist if $\xi \in [\hat{\xi}, \xi_{ud}]$.

Proof. $\theta(\xi)$ and $\hat{\lambda}(\xi)$ are increasing in ξ , the latter from lemma 2. Thus, $\hat{\lambda}(\xi) \geq \delta$ with strict inequality for $\xi > \xi_{ld}$. Moreover, from the definitions of $\theta(\xi)$ and $\hat{\lambda}(\xi)$ in (10) and (19), $\theta(\xi) \geq 1$ and $\hat{\lambda}(\xi) < 1$ for $\xi \in [\hat{\xi}, \xi_{ud})$. For $\xi \in [\xi_{ld}, \hat{\xi})$, $\hat{\lambda}(\xi) < \theta(\xi)$ can be rewritten as

$$\theta(\xi) \frac{\xi R_D}{R_I r_D - \xi R_D} < \frac{\xi(R_D + X/(\omega D))}{R_I r_D - \xi R_D}, \tag{20}$$

which is always true because $\theta(\xi) < 1$ for $\xi \in [\xi_{ld}, \hat{\xi})$, while $R_I r_D - \xi R_D > 0$ from lemma 2. QED

Step 3.—Because the sign of $\nu(\xi, \lambda)$ in (18) depends on beliefs about λ , a coordination problem among patient savers arises, leading to multiple equilibria. To resolve the multiplicity, we model a global game whereby each patient saver i receives a private noisy signal $x_i = \xi + \varepsilon_i$ at $t = 2$; ε_i are small error terms that are independent and uniformly distributed over $[-\varepsilon, \varepsilon]$. These signals provide information not only about ξ but also about other savers' signals, so that inference about their actions is possible. We seek a symmetric equilibrium characterized by a threshold signal x^* such that an individual patient saver will withdraw her deposits if $x_i < x^*$. Under such a threshold strategy, the number of savers that withdraw at a given level of fundamentals ξ is

$$\lambda(\xi, x^*) = \begin{cases} 1 & \text{if } \xi < x^* - \varepsilon, \\ \delta + (1 - \delta)\text{Prob}(x_i \leq x^*) & \text{if } x^* - \varepsilon \leq \xi \leq x^* + \varepsilon, \\ \delta & \text{if } \xi > x^* + \varepsilon. \end{cases} \tag{21}$$

If ξ is lower than $x^* - \varepsilon$, then all patient savers receive signals $x_i < x^*$ and withdraw, that is, $\lambda(\xi, x^*) = 1$. The opposite is true for $\xi > x^* + \varepsilon$. In this case, all patient savers receive signals $x_i > x^*$ and keep their deposits in the bank, while only the impatient ones withdraw, that is, $\lambda(\xi, x^*) = \delta$. Finally, if $\xi \in [x^* - \varepsilon, x^* + \varepsilon]$, some patient savers will receive signals that are lower than x^* and will withdraw their deposits; others will receive a signal higher than x^* and will keep their deposits in the bank. Because ε_i is independently and identically distributed, the law of large numbers obtains and the number of savers withdrawing for a given level of ξ is $\lambda(\xi, x^*) = \delta + (1 - \delta)\text{Prob}(x_i \leq x^*) = \delta + (1 - \delta)(x^* - \xi + \varepsilon)/2\varepsilon$, which is increasing in x^* and decreasing in ξ .

Given x^* , there are two cutoffs for fundamentals $\xi_\theta(x^*)$ and $\xi_\lambda(x^*)$ such that the bank can serve all withdrawals for realizations $\xi \geq \xi_\theta(x^*)$ and the banker has incentives to monitor for realizations $\xi \geq \xi_\lambda(x^*)$. These two cutoffs are, respectively, the solutions to $\lambda(\xi_\theta(x^*), x^*) = \theta(\xi_\theta(x^*))$

and $\lambda(\xi_{\hat{\lambda}}(x^*), x^*) = \hat{\lambda}(\xi_{\hat{\lambda}}(x^*))$, the properties of which are established in lemma 4.

LEMMA 4 (Regions of ξ with illiquidity and no monitoring). Consider signal threshold x^* . Then, there exist unique cutoffs $\xi_{\theta}(x^*)$ and $\xi_{\hat{\lambda}}(x^*)$, with $\xi_{\theta}(x^*) \leq \xi_{\hat{\lambda}}(x^*)$.

Proof. First, for $x^* \in [\xi_{ld} + \varepsilon, \xi_{ud} - \varepsilon]$, all ξ lie in $[\xi_{ld}, \xi_{ud}]$. If $\xi \in [\xi_{ld}, \hat{\xi})$, then with (10) and (21), $\lambda(\xi_{\theta}(x^*), x^*) = \theta(\xi_{\theta}(x^*))$ yields

$$\xi_{\theta}(x^*) = \frac{\varepsilon[(1 + \delta)Dr_D - 2 \cdot L] + x^*(1 - \delta)Dr_D}{2\varepsilon I + (1 - \delta)Dr_D}, \tag{22}$$

which lies in $(x^* - \varepsilon, x^* + \varepsilon)$ because (i) $\lambda(\xi, x^*)$ is continuous and decreasing in ξ , (ii) $\theta(\xi)$ is continuous and increasing in ξ , (iii) $\lambda(x^* - \varepsilon, x^*) = 1 > \theta(x^* - \varepsilon)$ for $x^* < \hat{\xi} - \varepsilon$, and (iv) $\lambda(x^* + \varepsilon, x^*) = \delta < \theta(x^* + \varepsilon)$ for $x^* > \xi_{ld} + \varepsilon$. For $\xi \in [\hat{\xi}, \xi_{ud})$, $\theta(\xi) \geq 1$, so we set $\xi_{\theta}(x^*) = x^* - \varepsilon$, that is, the full-run region is the null set. Similarly, $\xi_{\hat{\lambda}}(x^*)$ is given by $\lambda(\xi_{\hat{\lambda}}(x^*), x^*) = \hat{\lambda}(\xi_{\hat{\lambda}}(x^*))$, which yields

$$\delta + (1 - \delta) \frac{x^* + \varepsilon - \xi_{\hat{\lambda}}(x^*)}{2\varepsilon} = \frac{(\xi_{\hat{\lambda}}(x^*)I + L)R_l - \xi_{\hat{\lambda}}(x^*)(DR_D + X/\omega)}{D(R_l r_D - \xi_{\hat{\lambda}}(x^*)R_D)}. \tag{23}$$

Though (23) is quadratic, there exists a unique $\xi_{\hat{\lambda}}(x^*)$ lying in $(x^* - \varepsilon, x^* + \varepsilon)$. First, $\lambda(\xi, x^*)$ is continuous and decreasing in ξ , while $\hat{\lambda}(\xi)$ is continuous and increasing in ξ because of lemma 2. Second, $\lambda(x^* - \varepsilon, x^*) = 1 > \hat{\lambda}(x^* - \varepsilon)$ for $x^* < \xi_{ud} - \varepsilon$, and third, $\lambda(x^* + \varepsilon, x^*) = \delta < \hat{\lambda}(x^* + \varepsilon)$ for $x^* > \xi_{ld} + \varepsilon$. So, the two functions cross once in $(x^* - \varepsilon, x^* + \varepsilon)$ and there exists a unique $\xi_{\hat{\lambda}}(x^*)$. Because $\hat{\lambda}(\xi) < \theta(\xi)$ for any $\xi \in [\xi_{ld}, \xi_{ud}]$ from lemma 3, we get that $\xi_{\theta}(x^*) < \xi_{\hat{\lambda}}(x^*)$.

To conclude, for x^* such that $\xi < \xi_{ld}$, we have $\lambda(\xi, x^*) = 1$ and thus set $\xi_{\theta}(x^*) = \xi_{\hat{\lambda}}(x^*) = x^* + \varepsilon$. For x^* such that $\xi > \xi_{ud}$, we have $\lambda(\xi, x^*) = \delta$ and set $\xi_{\theta}(x^*) = \xi_{\hat{\lambda}}(x^*) = x^* - \varepsilon$. QED

Under threshold strategy x^* , the beliefs of an individual saver about aggregate withdrawals are well defined, allowing her to compute the expected payoff differential between waiting and withdrawing given her private signal. The saver uses the signal to update her beliefs about the realization of ξ . Given that both ξ and ε_i are uniformly distributed, the posterior distribution of ξ given x_i is $\xi|x_i \sim U[x_i - \varepsilon, x_i + \varepsilon]$. Thus, the expected payoff differential is

$$\Delta(x_i, x^*) = \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} \nu(\xi, \lambda(\xi, x^*)) d\xi. \tag{24}$$

A patient saver prefers to withdraw if $\Delta(x_i, x^*) < 0$ for all $x_i < x^*$ and prefers to wait if $\Delta(x_i, x^*) > 0$ for all $x_i > x^*$. $\Delta(x_i, x^*)$ is continuous in x_i , because a change in the signal changes only the limits of integration $[x_i - \varepsilon, x_i + \varepsilon]$ and the integrand is bounded. Hence, a patient saver

who receives signal $x_i = x^*$ is indifferent between waiting and withdrawing if

$$\bar{\Delta}(x^*) \equiv \Delta(x^*, x^*) = \frac{1}{2\varepsilon} \int_{x^*-\varepsilon}^{x^*+\varepsilon} \nu(\xi, \lambda(\xi, x^*)) d\xi = 0. \tag{25}$$

We need to establish two results: first, that a threshold strategy is an equilibrium, that is, $\Delta(x_i, x^*) < 0$ for $x_i < x^*$ and $\Delta(x_i, x^*) > 0$ for $x_i > x^*$; second, and more importantly, that there exists a unique x^* satisfying $\bar{\Delta}(x^*) = 0$. The proof of the first result is as follows.

Step 4.—For signal $x_i < x^*$, decompose the intervals $[x_i - \varepsilon, x_i + \varepsilon]$ and $[x^* - \varepsilon, x^* + \varepsilon]$ into a common part $c = [x_i - \varepsilon, x_i + \varepsilon] \cap [x^* - \varepsilon, x^* + \varepsilon]$ and two disjoint parts $d^l = [x_i - \varepsilon, x_i + \varepsilon] \setminus c$ and $d^* = [x^* - \varepsilon, x^* + \varepsilon] \setminus c$. Under a common threshold strategy, all savers form beliefs according to (21). Individual signals matter for savers’ posterior beliefs about ξ and the possible realizations of λ . Hence, (24) and (25) can be also decomposed to common and disjoint parts, respectively, as $\Delta(x_i, x^*) = \Delta_{\xi \in c} + \Delta_{\xi \in d^l}$ and $\bar{\Delta}(x^*) = \Delta_{\xi \in c} + \Delta_{\xi \in d^*}$. From (21), λ is always 1 over d^l , thus $\Delta_{\xi \in d^l} = \int_{\xi \in d^l} \nu(\xi, 1) d\xi = - \int_{\xi \in d^l} (\xi I + L) d\xi < 0$ from (18). Hence, it suffices to show that $\Delta_{\xi \in c} < 0$. We will use the facts that $\bar{\Delta}(x^*) = 0$ and that ν changes sign (crosses zero) only once while being positive and negative for higher and lower values of $\xi \in [x^* - \varepsilon, x^* + \varepsilon]$. Hence, $\Delta_{\xi \in d^*} > 0$ and $\Delta_{\xi \in c} < 0$, since the fundamentals are higher over d^* than c . The fact that ν may be increasing in ξ in the lower segment of c does not matter, because ν is still negative in that segment. Essentially, observing a signal x_i below x^* shifts probability from positive values of ν to negative values of ν —recall that noise is uniformly distributed—and $\Delta(x_i, x^*) < \bar{\Delta}(x^*) = 0$. The argument holds trivially if the interval c is empty. The proof for $x_i > x^*$ is similar. As in Goldstein and Pauzner (2005), single crossing of ν at zero suffices for a threshold strategy, should it exist, to be an equilibrium.

Step 5.—To prove existence, note that $\bar{\Delta}(x^*) < 0$ for $x^* \leq \xi_{ld} - \varepsilon$, because this level of signal x^* implies that $\xi \leq \xi_{ld}$ and thus $\nu(\xi, \lambda(\xi, x^*)) < 0$ always. Similarly, $\bar{\Delta}(x^*) > 0$ for $x^* \geq \xi_{ud} + \varepsilon$. Moreover, all the integrands and the limits of integration in $\bar{\Delta}(x^*)$ are bounded and continuous in x^* . Hence, $\bar{\Delta}(x^*)$ is continuous in x^* , which implies that there are solutions to $\bar{\Delta}(x^*) = 0$ given the existence of ξ_{ld} and ξ_{ud} for lemmas 1 and 2.

To prove that x^* is unique, it is typical in the global games literature to consider environments in which state monotonicity holds (Morris and Shin 2003; Rochet and Vives 2004; Goldstein and Pauzner 2005). This property requires that $\bar{\Delta}(x^*)$ is always increasing in x^* , which implies that the solution to $\bar{\Delta}(x^*) = 0$ is unique since $\bar{\Delta}$ can cross zero only once. However, this property does not necessarily hold in our model because, as we show in detail below, $\nu(\xi, \lambda)$ is decreasing in ξ in the full-run region. This means that we cannot ensure that $\bar{\Delta}$ is always increasing.

The ambiguity arises because savers have higher incentives to run on a stronger bank than a weaker bank, conditional on a run occurring, because their chances of getting paid are higher. We call this property *perverse state monotonicity*. To see the issue, use lemma (4) to write $\bar{\Delta}(x^*)$ as

$$\bar{\Delta}(x^*) = - \int_{x^*-\varepsilon}^{\xi_\theta(x^*)} \frac{\xi I + L}{\lambda(\xi, x^*)} \frac{d\xi}{2\varepsilon} - \int_{\xi_\theta(x^*)}^{\xi_\lambda(x^*)} Dr_D \frac{d\xi}{2\varepsilon} + \int_{\xi_\lambda(x^*)}^{x^*+\varepsilon} (\omega DR_D - Dr_D) \frac{d\xi}{2\varepsilon}. \quad (26)$$

Varying x^* changes both the threshold strategy and the signal that the threshold saver receives, changing both the limits of integration and $\lambda(\xi, x^*)$ in (26). It is convenient to set $x^* = \tilde{x} + \Delta x$ and vary Δx while keeping \tilde{x} fixed. Defining $\tilde{\xi} = \xi - \Delta x$, we can express $\bar{\Delta}(x^*)$ as $\bar{\Delta}(\tilde{x} + \Delta x)$ equal to

$$\begin{aligned} & - \int_{\tilde{x}-\varepsilon}^{\xi_\theta(\tilde{x}+\Delta x)-\Delta x} \frac{(\tilde{\xi} + \Delta x)I + L}{\lambda(\tilde{\xi}, \tilde{x})} \frac{d\tilde{\xi}}{2\varepsilon} \\ & - \int_{\xi_\theta(\tilde{x}+\Delta x)-\Delta x}^{\xi_\lambda(\tilde{x}+\Delta x)-\Delta x} Dr_D \frac{d\tilde{\xi}}{2\varepsilon} + \int_{\xi_\lambda(\tilde{x}+\Delta x)-\Delta x}^{\tilde{x}+\varepsilon} (\omega DR_D - Dr_D) \frac{d\tilde{\xi}}{2\varepsilon}, \end{aligned} \quad (27)$$

where we have used the fact that $\lambda(\tilde{\xi} + \Delta x, \tilde{x} + \Delta x) = \lambda(\tilde{\xi}, \tilde{x})$, that is, perceived withdrawals are unaffected by varying Δx , as a higher threshold means that the threshold saver also has higher posterior over ξ . The transformation in (27) shows that increasing Δx is equivalent to having the threshold saver think that the liquidation value is higher by an amount Δx for all values ξ in the interval $[\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$. This enables the easy comparison of $\bar{\Delta}(\tilde{x} + \Delta x)$ for liquidation values in the same interval $[\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$ but for different values of Δx given by

$$\frac{d\bar{\Delta}(\tilde{x} + \Delta x)}{d\Delta x} = - \int_{\tilde{x}-\varepsilon}^{\xi_\theta(\tilde{x}+\Delta x)-\Delta x} \frac{I}{\lambda(\tilde{\xi}, \tilde{x})} \frac{d\tilde{\xi}}{2\varepsilon} + \frac{1}{2\varepsilon} \left[1 - \frac{d\xi_\lambda(\tilde{x} + \Delta x)}{d\Delta x} \right] \omega DR_D. \quad (28)$$

Note that the limit of integration, $\xi_\theta(\tilde{x} + \Delta x) - \Delta x$, also changes with Δx , but it does not affect $\bar{\Delta}(\tilde{x} + \Delta x)$ because $((\tilde{\xi} + \Delta x)I + L)/\lambda(\tilde{\xi}, \tilde{x})$ evaluated at $\tilde{\xi} = \xi_\theta(\tilde{x} + \Delta x) - \Delta x$ is equal to Dr_D using $\lambda(\xi_\theta(\tilde{x} + \Delta x) - \Delta x, \tilde{x}) = \lambda(\xi_\theta(\tilde{x} + \Delta x), \tilde{x} + \Delta x) = \theta(\xi_\theta(\tilde{x} + \Delta x)) = (\xi_\theta(\tilde{x} + \Delta x)I + L)/(Dr_D)$.

Thus, the overall effect on $\bar{\Delta}$ is ambiguous. A higher threshold signal pushes down the payoff differential in the full-run region (first term in [28]), but it increases the region where monitoring occurs (second term in [28]). To see that the latter term is positive, totally differentiate $\lambda(\xi_\lambda(\tilde{x} + \Delta x), \tilde{x} + \Delta x) = \hat{\lambda}(\xi_\lambda(\tilde{x} + \Delta x))$, which is the condition that gives $\xi_\lambda(\tilde{x} + \Delta x)$. Doing so yields $d\xi_\lambda(\tilde{x} + \Delta x)/d\Delta x = \partial\lambda/\partial\Delta x \cdot (d\hat{\lambda}/d\xi - \partial\lambda/\partial d\xi)^{-1}$, which is less than 1 because $\partial\lambda/\partial\Delta x = \partial\lambda/\partial x^* = -\partial\lambda/\partial d\xi > 0$ from (21) and $d\hat{\lambda}/d\xi > 0$ from lemma 2.

Because of the perverse state monotonicity, we need to use a new proof that covers three possible cases that are depicted in figure 3. A first

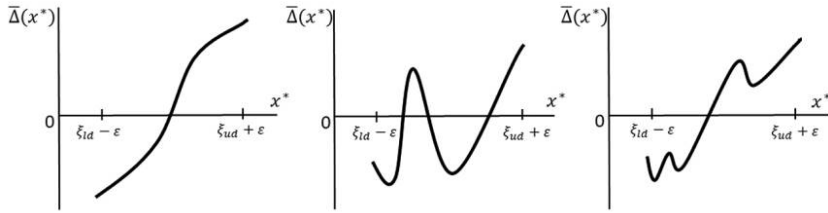


FIG. 3.—Perverse state monotonicity.

possibility is similar to what is typically studied in the literature, where $\nu(\xi, \lambda)$ is always increasing in the state. If this happens to be true, the threshold solving $\bar{\Delta}(x^*) = 0$ will be unique, as depicted in the left panel in figure 3. However, $\nu(\xi, \lambda)$ may not be increasing in the state. Instead, it could be a situation (as in the middle panel) where there are multiple solutions or a situation (as in the right panel) where there is a unique solution, despite the fact that $\bar{\Delta}(x^*)$ is not increasing in x^* everywhere in the domain.

To establish uniqueness under perverse state monotonicity, notice that to rule out the problematic middle case, $\bar{\Delta}(x^*)$ does not need to be increasing for any x^* (as is typically required) but instead only needs to be strictly increasing at candidate solutions that solve $\bar{\Delta}(x^*) = 0$, which we know exist. This weaker requirement is graphically depicted in figure 4. In the left panel, where there are multiple solutions, $\bar{\Delta}$ will necessarily cross the x -axis both from below and from above. This means that the derivative of $\bar{\Delta}$ at the candidate solutions (circles on the x -axis) can be either positive or negative. On the contrary, in the unique solution in the right panel, the derivative at the candidate solution is strictly positive. Hence, the strategy to establish uniqueness requires showing that the derivative of $\bar{\Delta}$ at a solution is always strictly positive.

We analyze the equilibrium, as the noise shrinks to zero, $\varepsilon \rightarrow 0$. Note that if we take the limit $\varepsilon \rightarrow 0$ and use the expressions in lemma 4, the

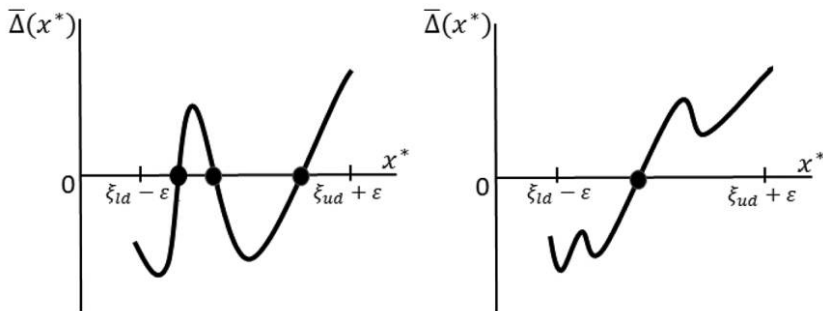


FIG. 4.—Graphical representation of uniqueness argument.

thresholds for fundamentals $\xi_\theta(x^*)$ and $\xi_{\hat{\lambda}}(x^*)$ converge to the same threshold ξ^* , which equals x^* . When we work with $\varepsilon \rightarrow 0$, it is convenient to change the variable of integration in $\bar{\Delta}(x^*)$ from ξ to λ . Recall that a saver who receives signal x^* believes that λ is given by $\lambda(\xi, x^*) = \delta + (1 - \delta)(x^* - \xi + \varepsilon)/2\varepsilon$ and that $\xi \sim U[x^* - \varepsilon, x^* + \varepsilon]$. Thus, as ξ increases uniformly from $x^* - \varepsilon$ to $x^* + \varepsilon$, λ decreases uniformly from 1 to δ . Given ξ , define

$$\Delta_{wt}(\xi) = \int_{\delta}^{\hat{\lambda}(\xi)} \omega DR_D \frac{d\lambda}{1 - \delta},$$

$$\Delta_{wd}(\xi) = \int_{\delta}^{\theta(\xi)} DR_D \frac{d\lambda}{1 - \delta} + \int_{\theta(\xi)}^1 \frac{\xi I + L}{\lambda} \frac{d\lambda}{1 - \delta},$$

where for completeness, we recall equations (10) and (19),

$$\theta(\xi) = \frac{\xi I + L}{DR_D} \text{ and } \hat{\lambda}(\xi) = \frac{(\xi I + L)R_I - \xi(DR_D + X/\omega)}{D(R_I r_D - \xi R_D)}.$$

Rewrite $\xi = x^* + \varepsilon - 2\varepsilon(\lambda - \delta)/(1 - \delta)$ as a function of $\lambda \sim U[\delta, 1]$. Per change of variable of integration in (25) from ξ to λ and examination of $v(\xi, \lambda)$ in (18), one can see that $\Delta_{wt}(\xi^*)$ is the payoff from waiting, while $\Delta_{wd}(\xi^*)$ is the payoff from withdrawing for a patient saver when $\xi = \xi^*$. The indifference condition (25) at the equilibrium ξ^* can be written, for $\varepsilon \rightarrow 0$, as

$$\Delta_{wt}(\xi^*) = \Delta_{wd}(\xi^*). \tag{29}$$

The following proposition establishes the key result.

PROPOSITION 1 (Threshold uniqueness). The solution ξ^* to (29) is unique.

To prove the proposition, we compare the elasticities of Δ_{wt} and Δ_{wd} at $\xi = \xi^*$. Define and calculate the elasticities as

$$\eta_{wt}(\xi) = \Delta'_{wt}(\xi) \frac{\xi}{\Delta_{wt}(\xi)} = \hat{\lambda}'(\xi) \frac{\xi}{\hat{\lambda}(\xi) - \delta},$$

$$\eta_{wd}(\xi) = \Delta'_{wd}(\xi) \frac{\xi}{\Delta_{wd}(\xi)} = \frac{\int_{\theta(\xi)}^1 (\xi I/\lambda)[d\lambda/(1 - \delta)]}{\Delta_{wd}(\xi)},$$

where we have used that $DR_D = (\xi I + L)/\theta(\xi)$ from the definition of $\theta(\xi)$ and therefore the derivatives with respect to the integral bounds cancel. We shall show that

$$\eta_{wt}(\xi^*) > 1 > \eta_{wd}(\xi^*). \tag{30}$$

With that and (29), $\Delta'_{wt}(\xi^*) > \Delta'_{wd}(\xi^*)$, that is, the difference $\Delta_{wt}(\xi) - \Delta_{wd}(\xi)$ has a positive slope at $\xi = \xi^*$ and can cross zero only from below, establishing the proposition.

Equation (30) will be established using lemmas 1 and 2 as well as the next two lemmas.

LEMMA 5 (Elasticity of payoff from waiting). Given lemmas 1 and 2, $\hat{\lambda}'(\xi) > (\hat{\lambda}(\xi) - \delta)/\xi$.

Proof. Using the definition of $\hat{\lambda}(\xi)$, taking the derivative, and recombining terms, we get

$$\hat{\lambda}'(\xi) = \frac{\hat{\lambda}(\xi) - \delta}{\xi} \left(1 + \frac{\xi R_D}{R_I r_D - \xi R_D} \right) + \frac{(\delta D r_D - L) R_I}{\xi D (R_I r_D - \xi R_D)} > \frac{\hat{\lambda}(\xi) - \delta}{\xi}, \tag{31}$$

because $\delta D r_D - L > 0$ from lemma 1 and $R_I r_D - \xi R_D > 0$ for $\xi \leq \bar{\xi}$ from lemma 2. QED

The lemma establishes that $\eta_{wt}(\xi^*) > 1$, the left part of (30). This is an intuitive property, because it implies that a saver believes that the probability of monitoring, given by $(\hat{\lambda}(\xi) - \delta)/(1 - \delta)$, would increase by more than 1% if she received a 1% higher signal about ξ . This is because a better ξ reduces the need to recall loans to pay impatient savers—the second term of $\hat{\lambda}'(\xi)$ in (31)—and the loans that are left in place can serve even more withdrawals by patient savers if needed—the first term of $\hat{\lambda}'(\xi)$ in (31).

LEMMA 6 (Elasticity of payoff from withdrawing).

$$\int_{\theta(\xi)}^1 \frac{\xi I}{\lambda} \frac{d\lambda}{1 - \delta} < \Delta_{wd}(\xi).$$

Proof. This follows, since the term on the left is one of the summands defining $\Delta_{wd}(\xi)$ and since all the summands are nonnegative, with at least one strictly positive. QED

This lemma establishes that $1 > \eta_{wd}(\xi^*)$, the right part of (30). This is intuitive, because a part of $\Delta_{wd}(\xi)$ is linearly increasing in ξ (in the full run region) and another part is positive but independent of ξ (in the region where deposit withdrawals are paid in full). So raising ξ increases the payoff from withdrawing by less than one for one. Hence, proposition 1 is now established.

Step 6.—Consequently, we have the following proposition.

PROPOSITION 2 (Threshold strategy equilibrium). Assume that the model parameters are such that lemmas 1 and 2 obtain. For $\varepsilon \rightarrow 0$, there exists unique ξ^* such that a patient depositor does not withdraw at $t = 2$ if $x_t > \xi^*$ and withdraws if $x_t < \xi^*$. Therefore, a run does not occur if $\xi \geq \xi^*$, and the bank is fully liquidated if $\xi < \xi^*$.

Proof. Given lower and upper dominance regions from lemmas 1 and 2, proposition 1 shows that there exists a unique ξ^* that the threshold signal x^* and threshold for fundamentals, $\xi_\theta(x^*)$ and $\xi_\lambda(x^*)$, converge to for

$\varepsilon \rightarrow 0$. Moreover, we showed earlier that a threshold strategy—whereby an individual patient saver withdraws only if x_i is below a common x^* —is an equilibrium. QED

In the rest of paper, we use the following notation for the condition that yields ξ^* as a function of all other predetermined endogenous variables:

$$\text{GG} \equiv \lim_{\varepsilon \rightarrow 0} \bar{\Delta}(x^*) = \Delta_{\text{wt}}(\xi^*) - \Delta_{\text{wd}}(\xi^*) = 0. \quad (32)$$

The appendix, available online, shows that proposition 2 is also true for positive but sufficiently small ε .

E. *Private Equilibrium*

The banker will choose both sides of the balance sheet to maximize her own profits, as shown in (13). The choices of loans, liquidity, deposits, and equity need to satisfy the global game constraint (32), which determines the run threshold; the deposit supply schedule (2), which determines the deposit rates; and the loan demand schedule (7), which determines the loan rate. The balance sheet identity also holds so only three of the four variables can be freely chosen. The banker will internalize the effect of her actions on the run threshold, on deposit supply, and on loan demand. As a result, conditions (32), (2), and (7) not only need to be satisfied in equilibrium but also will be explicit constraints in the banker's optimization problem, the solution to which defines the *private equilibrium* (PE). Essentially, this means that the banker also chooses directly the level of the run threshold as well as the deposit and loan rates rather than letting them be determined competitively in equilibrium. Finally, the banker will never choose allocations that result in lower welfare than her outside option. The bank can either invest in the storage technology or lend to entrepreneurs without accepting savers' deposits. Thus, the banker's outside option is $\bar{\mathbb{U}}_{\text{B}} = \max[\mathbb{U}_{\text{B}}^{\alpha}, \mathbb{U}_{\text{B}}^n]$, where $\mathbb{U}_{\text{B}}^{\alpha}$ is the utility in autarky (where the bank holds only liquid assets) and \mathbb{U}_{B}^n is the utility when the banker does not take deposits and lends to E using only her own capital.⁹

DEFINITION 1. The private equilibrium is defined as the set of bank assets, $\{I, L\}$, bank liabilities, $\{D, E\}$, the run threshold, ξ^* , deposit rates, $\{r_D, R_D\}$, and the loan rate, R_L , that maximize banker's utility, \mathbb{U}_{B} , defined in (13) subject to the balance sheet (BS) constraint (9), the global game

⁹ The utility in autarky is $\mathbb{U}_{\text{B}}^{\alpha} = W(e_{\text{B}} - \alpha) + \alpha$, where $\alpha > 0$ is the storage investment satisfying $W'(e_{\text{B}} - \alpha) = 1$. Note that if $W'(e_{\text{B}}) > 1$, then $\alpha = 0$. If the banker lends to entrepreneurs using only her own capital, her utility is $\mathbb{U}_{\text{B}}^n = W(e_{\text{B}} - n) + \omega(1 + r_n)n - X$, where r_n is the interest rate determined by the loan demand $1 + r_n = A - c'(n)/\omega$. Because B internalizes how the choice of n affects the loan rate that entrepreneurs are willing to accept, the optimal $n > 0$ is the solution to $-W'(e_{\text{B}} - n) + \omega A - c'(n) - c''(n)n = 0$.

(GG) constraint (32), the deposit supply (DS) schedule (2), and the loan demand (LD) schedule (7).

Each first-order condition in the private equilibrium takes the form

$$\frac{\partial \mathbb{U}_B}{\partial \mathcal{C}} + \sum_y \psi_y \frac{\partial \mathcal{Y}}{\partial \mathcal{C}} = 0, \quad (33)$$

where $\mathcal{C} \in \{I, L, D, E, \xi^*, r_D, R_D, R_I\}$, ψ_y are the shadow values on constraints $\mathcal{Y} \in \{\text{BS, GG, DS, LD}\}$ and $\partial \mathcal{Y} / \partial \mathcal{C}$ the partial derivatives capturing the effect of choice \mathcal{C} on these constraints. Take, for example, the lending choice, that is, $\mathcal{C} = I$. Then (33) says that the optimal level of lending is determined by having the banker trade off the marginal return accruing to her, $\partial \mathbb{U}_B / \partial I$, against the shadow cost of funds, ψ_{BS} , and the way it affects the run threshold determination, the deposit supply, and the loan demand. The optimality condition for the other variables can be similarly interpreted, and we report the detailed expressions for the partial derivatives in the appendix.

The optimality condition with respect to E , which is $\psi_{\text{BS}} = W'(\epsilon_B - E)$, deserves special attention because it highlights another way in which our model differs from others in the literature. This condition says that injecting more equity requires the banker to give up consumption in the initial period in exchange for increasing the funds of the bank.¹⁰ Thus, the shadow cost of funds, ψ_{BS} , is inversely related to the amount of equity the banker puts in the bank. In banking models without endogenous credit and run risk, the higher funding costs of injecting more equity would feed in higher loan rates and lower investment. In our model, this is not necessarily true: higher equity can improve the safety of deposits, which can be compatible with lower loan rates and more investment.

While the banker is free to choose any three of the quantities on her balance sheet, to explain how the model works, it is helpful to summarize the choices in terms of three different combinations. In particular, we will summarize the private equilibrium in terms of the asset allocation choice, the capital structure choice, and the level of lending (that is deposit funded). The three margins can be easily interpreted (see the appendix for detailed expressions).

The *asset allocation margin* is

$$\text{AAM}_{\text{PE}} = \left(\frac{d\mathbb{U}_B}{dL} - \frac{d\mathbb{U}_B}{dI} \right) + \frac{d\mathbb{U}_B}{d\xi^*} \left(\frac{d\xi^*}{dL} - \frac{d\xi^*}{dI} \right). \quad (34)$$

¹⁰ Note that the condition does not include the term for the effect of additional equity on constraints GG, deposit supply, and loan demand. This is true because E does not appear directly in (32), (2), or (7), but this does not mean that equity is irrelevant for their determination. On the contrary, equity issuance can affect the run probability, the deposit supply, and the loan demand through its joint determination with other equilibrium variables.

AAM_{PE} captures the decision to shift a unit of loans into liquid assets, consisting of the effect on B's utility via bank profitability (the first term) and the effect via the run probability (the second term).

Similarly, the *capital structure margin* is

$$CSM_{PE} = \left(\frac{dU_B}{dE} - \frac{dU_B}{dD} \right) - \frac{dU_B}{d\xi^*} \frac{d\xi^*}{dD}, \quad (35)$$

and it captures the decision to replace a unit of deposits with equity.

Last, the *deposit lending margin* is

$$DIM_{PE} = \left(\frac{dU_B}{dI} + \frac{dU_B}{dD} \right) + \frac{dU_B}{d\xi^*} \left(\frac{d\xi^*}{dI} + \frac{d\xi^*}{dD} \right), \quad (36)$$

and it captures the decision to raise a unit of deposits in order to expand lending. This decision will raise run risk because both sides of the balance sheet become riskier, and the only reason to do it will be to make more credit available to the entrepreneur. So this margin is one way to describe how the banker's private choices affect the entrepreneur relative to the saver.

To understand how these margins determine the structure of the bank's balance sheet, start in reverse order. Given a level of D and L , DIM_{PE} determines the level of lending by fixing the spread between loans and deposits; higher I requires a lower spread, all else being equal, and vice versa. Then, AAM_{PE} and CSM_{PE} simultaneously fix L and D , or the liquidity ratio $\ell = L/(I + L)$ and the leverage ratio $k = E/(E + D)$, given that $E = I + L - D$ from the balance sheet identity.

COROLLARY 1. The liquidity ratio, ℓ , the leverage ratio, k , and the loan-deposit spread, $R_l - R_d$, characterize the banker's optimal choices.¹¹

Before we compare the private equilibrium and the social one, we briefly discuss the assumptions about the banker's behavior. First, the possibility of a run disciplines the banker, who internalizes how her lending and funding choices affect ξ^* and hence the probability that she will make profits (see also Calomiris and Kahn 1991; Diamond and Rajan 2000). These considerations are captured by the terms multiplied by the shadow value on the global game constraint, ψ_{CG} , in (33).

Second, the banker internalizes how her choices affect r_b and R_b . This is possible because the debt is demandable and savers can fully observe the bank's balance sheet. At the moment of entering into a deposit contract,

¹¹ The statistics in corollary 1 are not exclusive. For example, one could combine (33) with respect to L and D , on the one hand, and with respect to E and I , on the other hand, to obtain alternative margins to AAM and CSM . These margins would be proxied by a reserve ratio, L/D , and a risk-weighted capital ratio, $CR = E/I$, respectively.

savers observe the balance sheet and can rationally compute the underlying risks. Hence, if the banker alters the balance sheet, savers can request new contract terms, so that their optimal supply of deposits reflects the new risks. The banker anticipates this behavior and internalizes the impact of all of her choices on deposit supply. These considerations are captured by the terms multiplied by the shadow value on the deposit supply schedule, ψ_{DS} , in (33).

Third, the banker internalizes how all of her choices affect R_t . These considerations are captured by the terms multiplied by the shadow value on the loan demand schedule, ψ_{LD} , in (33).

We relax these assumptions in the extensions in section V.

III. Efficient Allocations

Our bank offers socially useful services but faces the risk of a run. Run risk is harmful for all agents in the economy, but the effects on their respective welfare differ. B would benefit from low run risk, but reducing the risk lowers profits. S would prefer lower run risk and more deposit services, while E would benefit from higher lending accompanied by lower run risk.

The private banking choices are socially inefficient. B fully internalizes how her lending and capital decisions change the probability of a run, the deposit rates, and the loan rate. Yet B maximizes her own utility, disregarding the direct effect of her actions on the welfare of S and E. In turn, S and E are atomistic and do not internalize how their own decisions affect the bank (and the probability of a run). To examine how private choices distort outcomes, we consider a social planner who internalizes the effects of banking choices on all agents but is constrained by the market structure of the economy. We show that all three margins derived in section II.E are distorted from the planner's perspective. Section III.A sets up the planner's problem and identifies the sources of differences between the private and social optimization margins. Section III.B presents a numerical solution to the model and describes how the private and social planner's allocations differ.

A. Social Planner

The social planner maximizes the following social welfare function:

$$U_{SP} = U_B + w_S U_S + w_E U_E, \quad (37)$$

where $w_S \geq 0$, $w_E \geq 0$ are the weights assigned to S and E, while we have normalized the weight on B to 1 to facilitate the comparison with the private optimality conditions (other normalizations are also possible). Agents' utilities are given by (1), (5), and (13). The planner is constrained

by the market structure, that is, she cannot use lump-sum transfers to allocate resources across agents or complete any missing markets.¹² Moreover, the planner respects the run determination given by (32) as well as the deposit supply and loan demand schedules (2) and (7). Given that the latter two hold with equality in the planner's problem, we can substitute the indirect utilities (4) and (8) into (37) to get

$$\mathbb{U}_{\text{SP}}^* = \mathbb{U}_{\text{B}} + w_{\text{S}}\mathbb{U}_{\text{S}}^* + w_{\text{E}}\mathbb{U}_{\text{E}}^*. \quad (38)$$

Using (38) instead of (37) adds a lot of tractability in the optimizing conditions and, as we show below, allows us to pinpoint the sources of distortions in the private equilibrium that a planner would like to address. In particular, \mathbb{U}_{S}^* depends on only D and ξ^* , with

$$\frac{\partial \mathbb{U}_{\text{S}}^*}{\partial D} = -U''(e_{\text{S}} - D) - (1 - q)V''(Dr_{\text{D}})Dr_{\text{D}}^2 > 0, \quad (39)$$

$$\frac{\partial \mathbb{U}_{\text{S}}^*}{\partial \xi^*} = -[V(Dr_{\text{D}}) - V'(Dr_{\text{D}})Dr_{\text{D}}] \frac{1}{\Delta_{\xi}} < 0. \quad (40)$$

Therefore, a planner who cares primarily about S would like to increase D and reduce ξ^* .

Similarly, \mathbb{U}_{E}^* depends on only I and ξ^* , with

$$\frac{\partial \mathbb{U}_{\text{E}}^*}{\partial I} = (1 - q)c''(I)I > 0, \quad (41)$$

$$\frac{\partial \mathbb{U}_{\text{E}}^*}{\partial \xi^*} = -[c'(I)I - c(I)] \frac{1}{\Delta_{\xi}} < 0. \quad (42)$$

Hence, a planner who cares primarily about E would like to increase I and reduce ξ^* .

DEFINITION 2. The social planner's equilibrium is defined as the set of bank assets, $\{I, L\}$, bank liabilities, $\{D, E\}$, the run threshold, ξ^* , deposit rates, $\{r_{\text{D}}, R_{\text{D}}\}$, and the loan rate, R_{L} , that maximize social welfare, \mathbb{U}_{SP}^* , defined in (38) subject to the balance sheet constraint (9), the global game constraint (32), the deposit supply schedule (2), and the loan demand schedule (7).

Each first-order condition in the planner's equilibrium takes the form

$$\frac{\partial \mathbb{U}_{\text{B}}}{\partial \mathcal{C}} + w_{\text{S}} \frac{\partial \mathbb{U}_{\text{S}}^*}{\partial \mathcal{C}} + w_{\text{E}} \frac{\partial \mathbb{U}_{\text{E}}^*}{\partial \mathcal{C}} + \sum_{\text{y}} \zeta_{\text{y}} \frac{\partial \mathcal{Y}}{\partial \mathcal{C}} = 0, \quad (43)$$

¹² Because of market incompleteness, we cannot unambiguously obtain a welfare criterion to maximize total surplus. Thus, we assign weights for different agents in a social welfare function and study different constellations of these weights.

where ζ_y are the shadow values on the constraints $\mathcal{Y} \in \{\text{BS, GG, DS, LD}\}$ (analogous to ψ_y in the private equilibrium). The planner’s optimality conditions differ from the private ones, because the planner accounts for how her choices affect S and E and thus assigns different shadow values on the same constraints \mathcal{Y} . Following the same steps as for the private equilibrium, we can derive the same three margins for the planner (see the appendix for detailed expressions).

The asset allocation margin for the planner can be written as $\text{AAM}_{\text{SP}} = \text{AAM}_{\text{PE}} + \text{AAM}_{\text{WD}}$, where AAM_{PE} is given by (34) and AAM_{WD} is a wedge, which captures the additional distortions that the planner takes into account and will try to correct, given by

$$\text{AAM}_{\text{WD}} = \underbrace{\left(w_S \frac{\partial \mathcal{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathcal{U}_E^*}{\partial \xi^*} \right) \left(\frac{\partial \xi^*}{\partial L} - \frac{\partial \xi^*}{\partial I} \right)}_{\text{Run consequences from asset allocation}} - \underbrace{w_E(1 - q)c''(I)I}_{\text{Surplus to E from additional } I}. \quad (44)$$

The first term in (44) captures the impact on run risk due to the choice of the asset allocation between liquid assets and loans. If shifting a unit of loans to liquid assets reduces the run probability—that is, $d\xi^*/dL - d\xi^*/dI$ is negative—then because the planner cares about S and E, she would want a more liquid asset mix than in the private equilibrium. The last term in (44) captures the surplus created for the entrepreneur from an additional unit of investment, and this consideration leads the planner to increase the level of investment to favor E. Note that in principle, a more liquid asset mix and more investment can be consistent if they are accompanied by a bigger balance sheet.

Similarly, the capital structure margin for the planner is $\text{CSM}_{\text{SP}} = \text{CSM}_{\text{PE}} + \text{CSM}_{\text{WD}}$, where CSM_{PE} is given by (35) and the wedge CSM_{WD} is given by

$$\text{CSM}_{\text{WD}} = - \underbrace{\left(w_S \frac{\partial \mathcal{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathcal{U}_E^*}{\partial \xi^*} \right) \frac{\partial \xi^*}{\partial D}}_{\text{Run consequences from liability mix}} + \underbrace{w_S[U''(\epsilon_S - D)D + (1 - q)V''(Dr_D)Dr_D^2]}_{\text{Surplus to S from additional } D}. \quad (45)$$

Shifting funding from deposits to equity reduces run risk and helps S and E but reduces the surplus to S, because the last term is negative (because of concavity of U and V).

Finally, the deposit loan margin for the planner is $\text{DIM}_{\text{SP}} = \text{DIM}_{\text{PE}} + \text{DIM}_{\text{WD}}$, where DIM_{PE} is given by (36) and the wedge DIM_{WD} is given by

$$\text{DIM}_{\text{WD}} = \underbrace{\left(w_S \frac{\partial \mathcal{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathcal{U}_E^*}{\partial \xi^*} \right) \left(\frac{\partial \xi^*}{\partial I} + \frac{\partial \xi^*}{\partial D} \right)}_{\text{Run consequences of boosting loans and deposits}} - \underbrace{w_S[U''(\epsilon_S - D)D + (1 - q)V''(Dr_D)Dr_D^2] + w_E(1 - q)c''(I)I}_{\text{Surplus to S and E from boosting loans and deposits}}. \quad (46)$$

If an additional unit of deposits to fund loans increases the probability of a run—that is, $\partial\xi^*/\partial I + \partial\xi^*/\partial D > 0$ —the planner would like to do less of that. Yet the run considerations may be offset by the second term, because taking more deposits to make more loans increases the surplus to S and E.

Having described the various wedges, we have established three of our four main results. First, we have seen that there are three distorted margins that differ between the private and social allocations. Second, all three wedges feature a component linking the structure of the balance sheet to run risk and a component that captures the surplus created for S and/or E. The planner trades off reducing run risk against improving the surplus accruing to S and/or E when the run does not occur. In doing so, she chooses a different asset allocation, capital structure, and level of lending funded by deposits, which have a direct impact on the surplus and an indirect impact on run risk.

Our third main result is that correcting these distortions involves trade-offs, as it skews allocations toward either favoring E over S or vice versa. For instance, holding more safe assets and fewer loans helps S because it makes their deposits safer. Conversely, holding more loans and fewer safe assets creates more opportunities for E but reduces the buffer that helps mitigate the riskiness of deposits.

In section IV, we show how to implement (decentralize) the planner's solution with regulation. This will take us to our fourth main result that the three distortions are distinct, and three independent tools are generally needed to replicate the planner's allocations.

For completeness, the following corollary shows the conditions for constrained efficiency.

COROLLARY 2. The private and social planner's equilibria coincide if all of the following conditions hold: (i) $c''() = 0$, (ii) $V''() = 0$, and (iii) $U''() = 0$ or $e_s > \bar{e}_s$ such that $L_s > 0$.

Proof. Set $c(x) = a_c x$ and $V(x) = a_v x$, with $a_c, a_v \geq 0$ (recall that $c(0) = V(0) = 0$). Then, $\partial\mathbb{U}_E^*/\partial\xi^* = -[c'(I)I - c(I)]/\Delta_\xi = 0$ and $\partial\mathbb{U}_S^*/\partial\xi^* = -[V(Dr_D) - V'(Dr_D)Dr_D]/\Delta_\xi = 0$. Moreover, $(1 - q)c''(I)I = 0$ and $(1 - q)V''(Dr_D)Dr_D^2 = 0$. Given $U'' = 0$, all wedges $AAM_{WD} = CSM_{WD} = DIM_{WD} = 0$. Note that $U'' = 0$ occurs endogenously if $L_s > 0$, since $U' = \beta\delta + \beta^2(1 - \delta)$ from (3). QED

Corollary 2 says that if the bank is creating no welfare gains for S and E, there is nothing a planner can do to improve outcomes under the assumed market structure. This is not surprising because the private and social allocations differ only because B disregards the direct effects of her choices on S and E. If the bank cannot create any value for S and E, there is no scope for an intervention by the planner to improve outcomes. In section V, we explore extensions of the model under which there are additional reasons why private and social allocations differ. In those cases,

a planner might make different choices than private agents even under the conditions in corollary 2.

Finally, it is also possible that changing the nature of competition in the deposit or loan markets could overturn corollary 2. For instance, if there were multiple banks competing to raise deposits, it is possible that they would be led to offering contracts that implicitly maximize saver's welfare subject to the banks' participation constraints. In that case, the private choices might match the ones made by the planner when the weights are $w_s = 1$ and $w_E = 0$, while banks break even. A similar argument could be made for the case of severe competition in the loan market. More work investigating the role of competition would be a promising direction for future research.

B. Numerical Example

This section presents our benchmark numerical example and compares the private equilibrium with the planner's solution under different w_E and w_s , focusing on the distortions in the three margins.

1. Parameterization

The full set of parameters we used to solve the model is shown in table 1. The parametrization should be taken more as an illustrative example to highlight the mechanisms in the model rather than as a realistic calibration of the economy that would be suitable for making quantitative statements about the absolute optimal level of banking regulations. We have experimented with various other parameter choices, and the findings that we emphasize are quite robust.

TABLE 1
PARAMETERIZATION

Parameter	Value
e_s	2.50
e_b	.10
A	4.50
ω	.80
c_D	.27
c_I	.1
ϕ_I	3
X	.10
γ	.15
δ	.50
β	.70
ξ	2.20
$\underline{\xi}$.01

Our model would require further modifications to use it for quantitative policy analysis. For example, all liabilities are unsecured, while in practice, certain types of deposits are insured. Deposit insurance, even if partial, would reduce the reliance on market discipline exerted by depositors. Hence, deposit rates would be much lower, as we see in advanced economies. It is also not clear whether the various regulations in practice (Basel requirements, stress tests) are binding and whether one should be calibrating to match a regulated economy rather than an unregulated private equilibrium. Another potential concern is that our model consists of three periods, so one would need to decide how to interpret the length of each period. Finally, the quasilinearity of preferences as well as the finite horizon of the model make depositors willing to accept a higher probability of a run than if they were risk averse or if there was a continuation value for the bank. One could add convex bankruptcy costs to mimic a higher degree of risk aversion as well as model the continuation value, but doing so is not important to make our fundamental analytic points.

With these caveats in mind, let us call attention to some of the considerations we took into account while choosing the model parameters. Our objective in picking them was to obtain a private equilibrium that is rich enough to describe the mechanisms we want to highlight rather than to match observable variables. First, the bank is profitable enough, and the initial equity of the banker and her preference for current consumption are such that she voluntarily uses some of her endowment to buy more equity in the bank. So the banker finds operating the bank to be profitable. Second, the deposit services provided by the bank lead savers to forgo self-insuring by directly holding the liquid asset. If savers were opting to self-insure, then the banking sector is underperforming as a provider of liquidity, and hence regulations that make the bank more stable would have an additional positive effect. Third, we have chosen the parameters so that the bank makes loans and invests in liquid assets but also plans to liquidate some loans to serve early withdrawals. The key parameters responsible for this outcome are the size of the liquidity preference shock, the distribution of the liquidation values for recalled loans, and the riskiness of investment. Fourth, we have chosen log utility for $t = 1$ consumption for both the savers and the banker, while the banker values future consumption more than the savers do. In particular, $U(x) = \log(x)$ and $W(x) = \gamma \log(x)$, where $\gamma < 1$.¹³ Finally, we set $V(x) = c_D \log(1 + x)$

¹³ Assigning to the banker the same utility function requires high enough e_b or low enough γ such that she invests enough of her wealth in equity to provide benefits to savers. We do the second because we want the banker endowment to represent only a small part of the total endowment in the economy, with the vast majority accruing to the savers. For $\gamma = 1/\beta^2$, such that savers and the banker discount the future the same way, and for log utility, we can obtain the same equilibrium for banker's wealth $\bar{e}^B = E + (e^B - E)/(\beta^2\gamma)$, where E is the equilibrium value of contributed equity.

and $c(x) = c_I x^{\phi_I}$, with $\phi_I > 1$, which satisfy the general properties that were assumed for the transaction services' and effort cost functions.

2. Comparing the Private and Social Equilibria

Table 2 shows the PE and SP outcomes for small perturbation of the welfare weights away from the private equilibrium.¹⁴ We focus the analysis around the three margins derived in section III.A.

Before turning to the details, it is helpful to recall three things about the nature of the distortions that the planner is trying to correct. First, the banker is already internalizing everything that matters for her own welfare. The problem is that she is ignoring the consequences of her choices on the saver and the borrower. Therefore, anything the planner does to take this into account will make the banker worse off. So the planner will be constrained on this front by the need to make sure that the banker will still find it profitable enough to monitor loans. The saver generally wants safer deposits. This can be accomplished by reducing the riskiness of the asset mix or by raising more equity from the banker. The banker will contribute more equity only if the expected profit yield is high enough. Finally, the borrower would like to get more loans, but since the demand curve is downward sloping, more lending requires lower interest rates.

Initially, consider the case where the planner favors E and S equally, for example, $w_E = w_S = 0.1$ in table 2. To address run risk, the planner would like to increase liquid holdings in the asset mix, because $\partial \xi^* / \partial L - \partial \xi^* / \partial I < 0$ in (44), and increase the amount of equity in the liabilities mix, because $\partial \xi^* / \partial D > 0$ in (45). However, to benefit E and S, the planner needs to pay attention to the level of loans and deposits such that they do not drop. To achieve these various goals, the overall size of the bank's balance sheet should grow while making assets more liquidity intensive and liabilities more equity intensive. The reduction in run risk mitigates the upward pressure on the deposit rate, and the volume of lending has to be high enough so that, even though the loan rate will be lower, expected profits grow by enough to induce the banker to supply more equity.

One way to think about what is happening in this experiment is to recognize that in the private equilibrium, the banker is restricting lending to prop up loan rates and limiting deposits to suppress the cost of deposits. The planning allocations correct these problems. In doing so, the banker is made slightly worse off, but the other two agents are much better off. Overall, social welfare rises.

Next, consider the case that the planner wants to favor S but cares little about E, for example, $w_E = 0$ and $w_S = 0.2$. Similar to the first case, the

¹⁴ Given $t = 1$ allocations, ξ^* is unique, but there may be other private equilibria, each with a different unique ξ^* .

TABLE 2
PRIVATELY VERSUS SOCIALLY OPTIMAL SOLUTIONS

	PE	SP FOR WEIGHTS (w_E, w_S)				
		(.00, .20)	(.05, .15)	(.10, .10)	(.15, .05)	(.20, .00)
I	.862	.785	.841	.873	.899	.906
L	.052	.221	.119	.060	.012	.000
D	.875	.962	.919	.894	.873	.867
E	.038	.044	.041	.039	.038	.038
R_l	4.097	4.198	4.131	4.089	4.051	4.042
R_D	1.717	1.804	1.778	1.767	1.761	1.758
q	.407	.386	.398	.403	.407	.408
ℓ	.057	.219	.124	.065	.013	.000
k	.042	.044	.043	.042	.042	.042
$R_l - (1 - \delta) R_D$	3.239	3.296	3.242	3.205	3.170	3.163
$I + L$.914	1.006	.960	.933	.911	.906
$I - E$.824	.741	.800	.834	.861	.867
$E(\text{Div})$.745	.755	.750	.747	.743	.743
$\Delta U_{E(\%)}$...	-1.66	-.44	.33	1.02	1.19
$\Delta U_{S(\%)}$...	3.63	1.74	.71	-.10	-.30
$\Delta U_{B(\%)}$...	-.44	-.13	-.05	-.08	-.09
$\Delta U_{SP(\%)}$29	.11	.05	.07	.14
$\Delta S_{SP(\%)}$...	1.53	1.18	.99	.84	.79

NOTE.—The welfare changes (last five rows) are computed relative to the level of welfare in the private equilibrium, which is normalized to 1 for each agent.

planner would like to improve the liquidity of the asset mix to address run risk. But the planner is now less concerned about the surplus accruing to E and can more easily shift some of the investment toward the liquid asset. At the same time, the planner would like to increase equity in the liabilities mix in order to address the run risk but without cutting deposits, which is what matters for the surplus accruing to S. The planner can increase both liquidity and deposits and, at the same time, guarantee that B will inject more equity in the bank by cutting the level of lending. This set of changes is enough to make equity funding attractive for the banker because the loan-deposit spread rises boosting bank profitability. The overall size of the balance sheet goes up, but the amount of lending that is deposit funded falls.

Finally, consider the case where the planner wants to favor E but cares little about S, for example, $w_E = 0.2$ and $w_S = 0$. The planner would like to increase lending and the surplus accruing to E. Thus, the planner shifts liquidity to loans, which makes deposits more expensive. The planner could then substitute away from deposits to equity to fund the higher investment, but this would be possible only if such a shift is accomplished by maintaining enough profitability so that equity injection by the banker remains attractive. Given the optimality of private allocations from B's perspective, such a shift from deposits to equity—with a shift from liquidity to loans at the same time—would not be profitable for her. However, because the planner is not concerned with helping S, it is possible to reduce

deposit taking, which lowers the deposit rate and also the necessary amount of liquidity that is needed to be carried for early withdrawals. The planner shifts allocations so that more deposits are being used to support lending, even though the bank operates with a smaller balance sheet.

3. Lending, Run Risk, and Welfare in Private and Social Equilibria

The private equilibrium may exhibit over- or underinvestment (or lending) compared with the planner's outcomes depending on w_E and w_S . Lending falls and liquid asset holdings rise with higher w_S . Both factors contribute to a higher liquidity ratio $\ell \equiv L/(L + I)$. Lower lending is accompanied by a higher loan rate, and this induces the banker to provide more equity. The leverage ratio $k \equiv E/(E + D)$ improves as a result. Even though this also helps deposits increase somewhat, the amount of deposits that support lending is falling, so that the loan-deposit spread rises. The opposite is true when w_E is higher. In this case, investment jumps, but deposits fall.

Naturally, the run probability is lower when more weight is placed on S. But lower run risk accompanied by higher ℓ and k can still be consistent with higher lending. This can happen when the bank has a bigger balance sheet and uses more deposit funding to support lending. Notice that this is what happens when the planner cares equally about entrepreneurs and savers.

The enhanced stability of both the asset portfolio and the capital structure of the bank is beneficial to S especially because, as discussed above, it can be accompanied by a higher level of deposit services. Lower run risk is also beneficial to E but may come at the cost of lower investment and hence lower surplus from production. Indeed, when investment falls below its level in the private equilibrium, the entrepreneur will be worse off. As more weight is placed on E, the level of investment increases, pushing up the run probability and reducing the surplus from deposit services. S's welfare goes down, and after a point, she is worse off compared with the private equilibrium. In this example, both S and E can be made better off when the planner cares about them equally. B is always worse off, because she already internalized what mattered for her and any deviation from the private equilibrium reduces her welfare. Note that the planner not only increases social welfare $U_{SP} = U_B + w_S U_S^* + w_E U_E^*$, which depends on the weights, but also the overall surplus in the economy, $S_{SP} = U_B + U_S^* + U_E^*$. Thus, the planner could improve the welfare of all agents if she were able to make lump-sum transfers across agents.

IV. Regulation

We now explore how the planner's solution can be decentralized via various regulatory interventions. We group tools into two categories. The first

category includes tools that target the capital structure in order to get more equity in the bank. The second category includes tools that aim to make the asset mix more liquid. Sections IV.A and IV.B discuss the effects when the tools are used in isolation. Section IV.C discusses how the regulations can be optimally combined.

Recall that the leverage and liquidity ratios are $k \equiv E/(I + L)$ and $\ell \equiv L/(I + L)$. Also, denote the risk-weighted capital ratio by $CR \equiv E/I$ (with zero weight on L and weight of 1 on I), the liquidity coverage ratio by $LCR \equiv (L + \xi I)/(D r_D)$, and the net stable funding ratio by $NSFR \equiv E + (1 - \delta)D/I$. On top of these regulatory ratios, consider a tax (subsidy) τ_D on D , a tax (subsidy) τ_I on I , a tax (subsidy) τ_E on E , and a tax (subsidy) τ_L on L .

A. Capital Regulation

We first examine higher requirements for k and CR. The direct effect of both tools would be to increase the level of equity in the bank's liabilities. Both k and CR affect the probability of a run directly by changing allocations in (32) and indirectly by influencing loan and deposit rates. Recall that the banker will invest in additional equity only if the bank profits rise. Profitability depends on the spread between the R_l and R_d , so rising profitability requires either higher loan rates or lower deposit rates. Hence, either lending will have to fall so that the loan rate can rise or deposits have to be made safer so that the deposit rate can fall. We first show that the direct effect (i.e., keeping rates and other allocations constant) of higher k or CR is to reduce run risk.

PROPOSITION 3. For X low enough, the partial equilibrium effect of higher k or CR on ξ^* is negative.

Proof. Set $X = 0$ to make the determination of ξ^* via (32) scale invariant assuming constant lending and deposit rates. Dividing by $I + L$, we can express (32) in terms of k and ℓ , yielding the following derivative with respect to k , keeping ℓ and lending/deposit rates constant:

$$\frac{\partial GG}{\partial k} = \left(\frac{\xi^* R_d}{R_l r_D - \xi^* R_d} + \delta \right) \omega R_d + (\theta(\xi^*) - \delta) r_D > 0.$$

Thus, $\partial \xi^* / \partial k = -(\partial GG / \partial k) / (\partial GG / \partial \xi|_{\xi=\xi^*}) < 0$, since $\partial GG / \partial \xi|_{\xi=\xi^*} > 0$ from proposition 1. Also, $CR = k(1 - \ell)$. So higher k is equivalent to higher CR, all else being equal, and thus higher CR also reduces ξ^* . By continuity, the proposition holds for positive but low enough X . QED

Hence, in partial equilibrium, tightening k or CR is the same as long as $\Delta k = \Delta CR(1 - \ell)$, where ℓ is the liquidity ratio in the private equilibrium.

However, the general equilibrium effects of the two regulations on the incentive of bankers to hold liquidity and on the deposit supply and loan demand will differ. These differences stem from the way that k and CR

affect the three margins. Both affect CSM and DIM in the same way by requiring more equity relative to deposits. Increasing k does not affect the AAM because the bank can costlessly switch from liquid assets to loans. This substitution is possible only if lending rates fall, implying that deposit rates must also fall to boost profitability. By contrast, increasing CR directly affects the AAM by lowering lending, which will lead to a higher loan rate. The higher loan rate means that the bank can take on more deposits than if lending had risen.

Table 3 reports the results for individually tightening k and CR, with $\Delta k = \Delta \text{CR}(1 - \ell)$. We show cases in which both requirements increase by a little and by a lot. The beneficial effect on run risk continues to hold with general equilibrium effects under both regulations. However, the magnitude of the decrease differs, and so do the effects on other components of the bank's balance sheet.

First, consider the case where k and CR are marginally tightened. To compare the effects, notice how the AAM is differentially impacted. Loans decrease much more under CR than k , which improves bank's asset liquidity. Thus, run risk goes down even further than what we would expect by just increasing capital. The lower run risk makes deposits cheaper, and the bank increases its deposit taking. S gains, while both E and B lose. The results are similar for k , but the effects are less pronounced because

TABLE 3
SINGLE CAPITAL REGULATIONS

	PE	MILDER INCREASE IN		BIGGER INCREASE IN	
		CR	k	CR	k
I	.862	.856	.862	.770	.880
L	.052	.064	.055	.225	.019
D	.875	.880	.876	.941	.841
E	.038	.040	.040	.053	.058
R_f	4.097	4.106	4.099	4.214	4.077
R_v	1.717	1.713	1.711	1.704	1.605
q	.407	.405	.406	.387	.405
ℓ	.057	.069	.060	.226	.021
k	.042	.044	.044	.053	.065
CR	.045	.047	.047	.069	.066
$R_f - (1 - \delta)R_v$	3.239	3.249	3.243	3.362	3.274
$I + L$.914	.920	.916	.994	.899
$I - E$.824	.816	.821	.716	.821
$E(\text{Div})$.745	.749	.749	.779	.791
$\Delta U_{E(\%)}$...	-.13	-.01	-2.03	.51
$\Delta U_{S(\%)}$17	.02	2.70	-1.23
$\Delta U_{B(\%)}$...	-.01	-.01	-.61	-1.24
$\Delta S_{SP(\%)}$03	.01	.06	-1.96

NOTE.—The welfare changes (last four rows) are computed relative to the level of welfare in the private equilibrium, which is normalized to 1 for each agent.

the bank can shift some liquidity to lending and still satisfy the regulation. Because of the additional lending, the bank does not boost liquidity by as much, so the scope to increase deposits is reduced. The gains for S are smaller, and so are the losses for E.

Considering a much larger increase in capital, we obtain the same but stronger effects for CR. However, for k , the substitution toward lending becomes much more pronounced. The bank actually increases lending relative to the private equilibrium and substantially reduces its holding of liquid assets.¹⁵ Given those changes, the cost of deposits must fall in order to make the bank profitable enough to support the higher level of capital. This drop occurs because the total deposits fall, and S are made worse off from this change. The higher level of lending make E better off.

B. Liquidity Regulation

We next examine higher regulatory requirements on ℓ , LCR, and NSFR.

PROPOSITION 4. For X low enough and $\delta > \exp^{-1}$ or $\ell > \hat{\ell} \in R$, the partial equilibrium effect of higher ℓ , LCR, or NSFR on ξ^* is negative.

Proof. When we set $X = 0$ and follow the same steps as in the proof of proposition 3, the derivative of GG with respect to ℓ , keeping k (and lending/deposit rates) constant, is

$$\frac{\partial \text{GG}}{\partial \ell} = (1 - \xi^*) \left[\frac{\omega R_D R_I}{R_I r_D - \xi^* R_D} + \log \theta(\xi^*) \right], \quad (47)$$

which is positive if $\log \theta(\xi^*) > -1$, given that $\omega R_D > r_D$ and lemma 2. In turn, this is satisfied under sufficient conditions $\delta > \exp^{-1}$ since $\theta(\xi^*) > \delta$ or $\ell > \hat{\ell} \equiv [\exp^{-1}(1 - k)r_D - \xi^*]/(1 - \xi^*)$, which is true for high enough ξ^* . Also, $\text{LCR} = ((1 - \xi)\ell + \xi)/(kr_D)$ and $\text{NSFR} = (k + (1 - \delta)(1 - k))/(1 - \ell)$. So increasing ℓ is equivalent to increasing LCR or NSFR, all else being equal, and thus higher LCR or NSFR reduce ξ^* . By continuity, the proposition holds for positive but low enough X . QED

This proposition establishes that if one fixes the leverage ratio, k , as well as interest rates, then tightening ℓ , LCR, or NSFR reduces run risk in exactly the same way, because $\text{LCR} = ((1 - \xi)\ell + \xi)/((1 - k)r_D)$ and $\text{NSFR} = (k + (1 - \delta)(1 - k))/(1 - \ell)$. Hence, in a partial equilibrium setting, one would conclude that these three regulations are equivalent.

These regulations have the unintended consequence that extra liquidity raises patient savers' incentives to join the full run. We show that these

¹⁵ This is in contrast to models where the bank cannot raise additional equity, where stricter capital/leverage requirements (mechanically) result in a drop in credit extension (see, e.g., Clerc et al. 2015 and references therein).

perverse incentives do not dominate when the fraction of patient depositors is small enough or when the liquidity ratio in the private equilibrium is above a cutoff. In the private equilibrium we examine, $\hat{\ell} < 0$ because the run risk is big enough to limit the strength of this channel, so the partial equilibrium effect of higher ℓ on ξ^* is negative.

As with capital regulations, the general equilibrium effects of liquidity regulations may differ because they alter the three critical margins differently. Table 4 reports the results for individually tightening ℓ , LCR, or NSFR, with $\Delta\text{LCR} = \Delta\ell(1 - \xi)/((1 - k)r_D)$ and $\Delta\text{NSFR} = \Delta\ell/(1 - \ell)\text{NSFR}$, where k , NSFR, and ℓ are the leverage, net stable funding, and liquidity ratios in PE.

First, focus on ℓ and LCR, which are very similar. Mandating that the bank must hold more liquidity changes the trade-off between investing in risky loans and liquid assets. The higher liquid asset holdings allow the bank to raise more deposits without increasing run risk. Although deposits increase, the portion that is channeled to loans falls. At the same time, equity goes up.

Why does requiring the bank to hold more liquidity induce it to raise both more deposits and more equity? The bank can raise more deposits and invest them in the liquid asset to satisfy the regulation. This is preferable to raising equity in order to invest in the liquid asset, because equity is more expensive. Despite the fact that run risk decreases, the increased

TABLE 4
SINGLE LIQUIDITY REGULATIONS

	PE	ℓ	LCR	NSFR
I	.862	.746	.747	.770
L	.052	.258	.257	.217
D	.875	.959	.959	.942
E	.038	.045	.045	.044
R_I	4.097	4.236	4.236	4.211
R_D	1.717	1.746	1.745	1.736
q	.407	.385	.385	.390
ℓ	.057	.257	.256	.220
k	.042	.045	.045	.045
LCR	.069	.277	.276	.239
NSFR	.552	.703	.702	.670
$R_I - (1 - \delta)R_D$	3.239	3.363	3.363	3.343
$I + L$.914	1.004	1.004	.987
$I - E$.824	.701	.701	.726
$E(\text{Div})$.745	.756	.757	.756
$\Delta U_{E(\%)}$...	-2.50	-2.49	-2.05
$\Delta U_{S(\%)}$...	3.50	3.48	2.76
$\Delta U_{B(\%)}$...	-.51	-.51	-.35
$\Delta S_{SP(\%)}$49	.49	.36

NOTE.—The welfare changes (last four rows) are computed relative to the level of welfare in the private equilibrium, which is normalized to 1 for each agent.

demand for deposits pushes up the deposit rate and thus makes loans less profitable. The bank will reduce lending to secure higher loan rates to raise profitability. For a large enough fall in lending, the loan-deposit spread widens so much that it becomes desirable to increase the amount of equity.

The NSFR regulation operates via the same channels, but the effects are less pronounced. The reason is that this regulation also partially resembles a risk-weighted capital requirement. Notice that the NSFR can be rewritten as $1 - \delta + \delta CR + (1 - \delta)\ell/(1 - \ell)$ so that it operates through affecting both capital and liquidity. Relative to the other liquidity regulations, the NSFR has a milder effect on deposits. S is better off, while E and B are worse off under all liquidity regulations.

C. The Optimal Regulatory Mix

We examine whether and how regulation can be combined to implement the planner's solution as a private equilibrium. We consider the following set of tools $\mathbb{T} = \{k, CR, \ell, LCR, NSFR, \tau_D, \tau_I, \tau_E, \tau_L\}$. We expand the set of tools beyond the capital and liquidity regulations for two reasons. First, for managing the loan-deposit spread, these tools are inadequate. Second, it is simpler to describe how the distorted wedges are connected using the tax/subsidy tools. In part, this is because using the liquidity and capital tools requires checking whether the regulations are binding.¹⁶

The following proposition establishes our fourth main result.

PROPOSITION 5. For nonzero and linearly independent wedges (44)–(46), three tools are needed to replicate the planner's preferred allocations.

- i. Any combination of three tools among $\tau_D, \tau_I, \tau_E,$ and τ_L can be used to implement the planner's solution.
- ii. If $CR < AAM_{WD}/(AAM_{WD} + CSM_{WD} + DIM_{WD}) < 1 - \ell$ and $\delta > \exp^{-1}$ or $\ell > \hat{\ell}$, the planner's solution can be implemented with a combination of CR, ℓ , and τ_D .
- iii. If $CR < AAM_{WD}/(AAM_{WD} + CSM_{WD} + DIM_{WD})$ and $\delta > \exp^{-1}$ or $\ell > \hat{\ell}$, the planner's solution can be implemented with a combination of CR, τ_I , and τ_D .

Proof. In order for us to replicate the SP solution as a PE, the tools must alter the margins to equate the wedges in (44)–(46); see the appendix for why this is a necessary and sufficient condition. Implementation is simple using taxes/subsidies chosen independently. Given that there are

¹⁶ Without loss of generality, the easiest way to model the additional tools is to tax (reward) the banker $\tau_D, \tau_I, \tau_E,$ and τ_L per unit of $D, I, E,$ and L and then return to (extract from) her a lump-sum amount equal to $\tau_D D, \tau_I I, \tau_E E,$ and $\tau_L L$.

three independent wedges, a combination of three taxes is needed, establishing part i.

The situation is less straightforward with capital and liquidity regulations, as the Lagrange multipliers on them need to be positive so that regulations are binding. Take, for example, an implementation with CR, ℓ , and τ_D . Implementation requires that $\psi_{CR}CR + \psi_\ell = AAM_{WD}$, $\psi_{CR} + \tau_D = CSM_{WD}$, and $-\psi_{CR}CR - \psi_\ell\ell - \tau_D = DIM_{WD}$, where ψ_{CR} and ψ_ℓ are the multipliers on capital and liquidity requirements $E \geq I \cdot CR$ and $(1 - \ell)L \geq \ell I$. Solving for multipliers and the τ_D , we get $\psi_{CR} = (AAM_{WD}\ell + CSM_{WD} + DIM_{WD})/(1 - CR(1 - \ell))$, $\psi_\ell = (AAM_{WD} - (AAM_{WD} + CSM_{WD} + DIM_{WD})CR)/(1 - CR(1 - \ell))$, and $\tau_D = (-AAM_{WD}\ell - CSM_{WD}CR(1 - \ell) - DIM_{WD})/(1 - CR(1 - \ell))$. Note that $AAM_{WD} + CSM_{WD} + DIM_{WD} = (w_S\partial U_S^*/\partial \xi^* + w_E\partial U_E^*/\partial \xi^*)\partial \xi^*/\partial L > 0$ from proposition 4 for $\delta > \exp^{-1}$ or $\ell > \bar{\ell}$. Thus, $\psi_{CR}, \psi_\ell > 0$ if the conditions in point ii hold.

The implementation with CR, τ_I , and τ_D requires $\psi_{CR} = AAM_{WD} + CSM_{WD} + DIM_{WD} > 0$, $\tau_I = AAM_{WD} - (AAM_{WD} + CSM_{WD} + DIM_{WD})CR$, and $\tau_D = -(AAM_{WD} + DIM_{WD})$; τ_I is positive under point ii and negative under point iii, also verifying that ℓ can be part of the optimal mix only in point ii. QED

Proposition 5 shows that typically capital requirements and deposit-taking subsidies are useful ingredients of the optimal regulatory mix. The third tool with which they would be combined varies and depends on the sign and size of $AAM_{WD} = (w_S\partial U_S^*/\partial \xi^* + w_E\partial U_E^*/\partial \xi^*)(\partial \xi^*/\partial L - \partial \xi^*/\partial I) - w_E(1 - q)c''(I)I$, which depends on the weights w_S and w_E . Setting $w_E = 0$, the wedge captures the increase in savers' utility from lower run risk that follows from requiring higher liquid asset holdings. The higher these holdings are, the easier it is to satisfy the condition in part ii of proposition 5. However, for large enough w_E , AAM_{WD} may become negative, so that case iii of proposition 5 could obtain, justifying a lending subsidy. This is intuitive because the cost to entrepreneurs from lower lending is more important when w_E is higher. In general, combining capital regulation and deposit-taking subsidies benefits savers and entrepreneurs, as they enhance banking stability, but the use of tools to restrict or encourage lending depends on which agents are favored most.

Having established the general properties of the optimal regulatory mix, examining it in the context of our numerical example, can provide additional insights. Table 5 reports the outcomes of combining regulatory tools and compares them with the planner's solution for a case where the planner favors the saver. We combine one capital regulation and one liquidity regulation with a tool that can push the bank toward the socially optimal level of deposit funded loans. As already mentioned, there are several potential candidates for the third type of tool, but we will consider a simple tax (or subsidy) on deposit taking. This tool can indirectly control the spread between loans and deposits, which, as discussed in corollary 1,

TABLE 5
IMPLEMENTATION OF PLANNER'S SOLUTION FOR $w_E = 0.00$ AND $w_S = 0.20$

	PE	CR	CR and ℓ	CR, ℓ , and τ_D
I	.862	.829	.776	.785
L	.052	.116	.206	.221
D	.875	.899	.938	.962
E	.038	.047	.044	.044
R_I	4.097	4.143	4.204	4.198
R_D	1.717	1.700	1.734	1.804
q	.407	.399	.392	.386
ℓ	.057	.123	.210	.219
k	.042	.049	.045	.044
$R_I - (1 - \delta)R_D$	3.239	3.292	3.337	3.296
$I + L$.914	.946	.982	1.006
$I - E$.824	.783	.732	.741
$E(\text{Div})$.745	.764	.756	.755
$\Delta U_{E(\%)}$...	-.75	-1.92	-1.66
$\Delta U_{S(\%)}$94	2.56	3.63
$\Delta U_{B(\%)}$...	-.15	-.31	-.44
$\Delta S_{SP(\%)}$04	.33	1.53

NOTE.—The welfare changes (last four rows) are computed relative to the level of welfare in the private equilibrium, which is normalized to 1 for each agent.

is a sufficient statistic for the DIM (provided that the AAM and CSM have been fixed by a capital and a liquidity tool).¹⁷

As discussed, tightening CR to target the CSM increases the equity in the bank, reduces run risk, and results in lower loan extension. If, in addition, we impose a stricter ℓ , we get closer to the planner in terms of the asset and capital structure margins. But the loan-deposit rate spread (or the amount of deposits funding investment) moves in the opposite direction of what the planner seeks. A subsidy of $\tau_D = -9.17\%$ can be levied to induce the bank to move the spread to where the planner prefers (this subsidy is equivalent to a subsidy of $\tilde{\tau}_D = -7.40\%$ on deposit interest expenses). The combination of the three tools can implement the planner's solution as a private equilibrium. More generally, using two tools that are not redundant would also typically improve welfare relative to any single regulation. However, to mimic the planner, three regulations are required.

The planner puts more emphasis on reducing run risk and increasing the surplus from deposit services for this set of weights. Hence, capital and liquidity regulations are useful, because they reduce run risk without impeding deposit taking (see secs. IV.A, IV.B). Moreover, in view of corollary 1 and the fact that the asset allocation and capital structure wedges

¹⁷ The planner cannot use two liquidity or capital tools at the same time because they will not be jointly binding. This result is consistent with the analysis in Checchetti and Kashyap (2016), who show that LCR and NSFR regulations almost surely will never bind at the same time. However, the collinearity of the CR and k regulations may be specific to our model and may not even hold for high k (see the discussion in sec. IV.A).

are independent, both regulations are needed. The complementarity of the capital and liquidity regulations arises in this case because agents' welfare is affected by both the bank's asset and liability mix. If, instead, the regulator had an ad hoc objective to enhance stability by reducing run risk, then either capital or liquidity regulation could be used to accomplish this, and the tools would be substitutes.

Finally, one might expect that capital and liquidity requirements are not useful when the planner would like to increase the lending in the economy (as would be the case when the planner puts high weights on E). Table 6 shows partial and full implementation of the planner's solution for case. To mimic the planner's outcome, we need to use deposit and lending subsidies. The complication arises because, as we see from table 2, the planner wants higher lending but much less liquidity for this set of weights. So raising ℓ will mean that the planner's allocations cannot be achieved. A CR can be helpful, but it will have to be combined with lending or deposit subsidies.

Column 2 in table 6 shows that two subsidies, $\tau_D = 2.20\%$ and $\tau_I = -4.33\%$, together can boost lending all the way to where the planner prefers but do not replicate the planner's solution. Column 3 in table 6 shows that these two subsidies can be combined with a capital requirement, $CR = 4.25\%$, to replicate the planner's allocation. In this case, capital regulation is needed because the bank is tempted to maximize its leverage to reap the benefits of the subsidies. This helps a bit with respect to run risk compared with the case where only the two subsidies are

TABLE 6
IMPLEMENTATION OF PLANNER'S SOLUTION FOR $w_E = 0.15$ AND $w_S = 0.05$

	PE (1)	τ_D and τ_I (2)	CR, τ_D , and τ_I (3)
I	.862	.900	.899
L	.052	.010	.012
D	.875	.872	.873
E	.038	.038	.038
R_I	4.097	4.050	4.051
R_D	1.717	1.762	1.761
q	.407	.408	.407
ℓ	.057	.011	.013
k	.042	.042	.042
$R_I - (1 - \delta)R_D$	3.239	3.169	3.170
$I + L$.914	.910	.911
$I - E$.824	.862	.861
$E(\text{Div})$.745	.743	.743
$\Delta U_E(\%)$...	1.03	1.02
$\Delta U_S(\%)$...	-.12	-.10
$\Delta U_B(\%)$...	-.08	-.08
$\Delta S_{SP}(\%)$84	.84

NOTE.—The welfare changes (last four rows) are computed relative to the level of welfare in the private equilibrium, which is normalized to 1 for each agent.

in place, but unlike a liquidity regulation, capital regulation does not restrict lending materially, which is the primary objective of the planner for this set of weights.

Our findings suggest that capital is useful as part of the optimal regulatory mix irrespective of which agent the planner favors, while liquidity requirements are more useful for savers.

V. Extensions

To check the generality of our results, we extend the model in several dimensions. Our main results continue to hold. The details are presented in the appendix.

Section D.1 considers the case when the banker does not internalize the effect of her actions on ξ^* , resulting in higher risk in the private equilibrium. Pareto improvements via regulation are possible in this case. Section D.2 considers incomplete deposit contracts and a lack of commitment by the banker such that the banker has an incentive to deviate toward lending more, holding fewer liquid assets, and raising less equity after the deposit contract has been signed. Again, we show that the need for regulation is stronger and Pareto improvements are possible. Section D.3 considers a banker that takes the loan rate R_l as given. Lending is higher in the private equilibrium, while the need for regulation, especially liquidity regulation, remains. Section D.4 considers a perfectly elastic loan demand curve such that E drops out of the problem. Results are consistent with those described earlier when the planner puts higher weight on S. Section D.5 derives the equilibrium under direct lending from S to E and shows that S would prefer to use a bank because of the payment services and the more efficient monitoring it provides. Section D.6 allows for negative deposit rates for early withdrawals. Our results continue to hold when run risk is present in equilibrium. We also consider negative deposit rates that implement the run-proof equilibrium and show that it is dominated by the one with positive run risk that we study. Section D.7 examines when banks can also raise outside equity, which does not affect our results, while section D.8 considers a planner that can use tools to distort the deposit supply and loan demand schedules of savers and entrepreneurs.

VI. Conclusions

Banks perform important services using both sides of their balance sheet. Yet private choices made by banks may not be socially optimal, and regulating banks' activities can improve social welfare. We examined how many of the regulations that are often mentioned in policy discussions perform in a relatively familiar model of banks. We started from

the Diamond-Dybvig benchmark precisely because it is so thoroughly studied. The modifications we study balance tractability—to keep the model relatively simple—against our preference for additional realistic forces that the baseline model excludes.

Our modifications generate endogenous credit risk in banks' portfolios as well as the risk of an endogenous funding run. This simple pair of features interact in interesting and unexpected ways. We draw several lessons from the model that we believe will carry over to many other models.

First, we identify three general margins that are distorted: the relative amounts of liquid and illiquid assets, the mix of deposits and equity, and the spread between loan and deposit rates. Second, the way that a bank privately sets these margins diverges from what a social planner would choose because of the failure to fully account for the risks created by runs.

Third, the way the planner corrects the distortions in the private equilibrium depends on the relative importance placed on savers versus borrowers. If the social planner cares sufficiently about savers, the planner chooses relatively more liquidity and equity than the banker and would reduce bank profits by boosting deposit rates and lowering loan rates. Thus, the planner reduces run risk, improves the provision of liquidity, guarantees a more stable extension of credit and real production, and delivers more loans funded by deposits compared with the private equilibrium. Capital regulation is still desirable even when the planner cares little about savers, but liquidity regulation needs to be replaced with tools that encourage credit extension and hence actually create more risk.

Fourth, more than one regulatory tool is needed to implement the socially optimal allocations. Optimal policy in models without all of these distortions can be misleading. For example, if the liability structure is constrained—say, because deposit levels are exogenously determined and equity is fixed—studying asset allocations and distortions becomes much easier. But regulation, if any is needed, will amount to fixing liquidity ratios. Similarly, shutting down the liquidity demand and liquidity risk makes it easier to focus on the optimal capital structure and level of investment. But regulation, if again any is needed, would amount to fixing capital ratios. Instead, when both sides of the bank's balance sheet are endogenously determined, the distortions from each side interact, and a combination of both capital and liquidity requirements emerge in the optimal regulatory mix.

Moreover, our analysis highlights some political economy aspects of regulation that deserve attention. The banker internalizes how her decisions matter for run risk, funding structure, and the level of lending to maximize her own welfare. Her choices from a social point of view have real macroeconomic consequences. Regulation improves aggregate welfare but reduces the rents accruing to bankers. Therefore, incentives to

engage in regulatory arbitrage could be strong. The lack of regulatory arbitrage in the model we have studied is one of its main shortcomings.

There are other interesting avenues to extend our model, some of which we have already mentioned and are analyzed in the appendix. One further direction would be to allow the issuance of long-term debt together with demandable deposits and equity. Including loss-absorbing debt instruments in the regulatory mix could introduce additional ways to tackle with run risk and credit risk. But it would not constitute a full remedy by itself because of the disciplinary role that demandable liabilities play. Moreover, our model is flexible enough to incorporate fire sale dynamics by endogenizing the liquidation value of long-term investment. Although this would introduce pecuniary externalities as an additional reason why private allocations are inefficient, it would not qualitatively overturn our main conclusions. Finally, one could enrich the set of risky investments from which a banker could choose and thus increase the scope for asset substitution. Setting the (relative) risk weights in capital requirements to capture social risks would then be highly important.

Data Availability

Code replicating the tables in this article can be found in Kashyap, Tsomocos, and Vardoulakis (2024) in the Harvard Dataverse, <https://doi.org/10.7910/DVN/YSUMQK>.

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