THE UNIVERSITY OF CHICAGO

FROM UT RE MI TO FOURTEEN-TONE TEMPERAMENT: THE GLOBAL ACOUSTEMOLOGIES OF AN EARLY MODERN CHINESE TUNING REFORM

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To the loving memory of

My grandfather, Wang Dingxin 王鼎新 *(December 24, 1928-April 16, 1999),*

Meteorologist, revolutionary,

for his ever-inspiring passion for knowledge and music.

برای عزیزم ,شادی و خوشبختی من از تو است.

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From *Ut Re Mi* to Fourteen-Tone Temperament:

The Global Acoustemologies of an Early Modern Reform to Chinese Musical Tuning

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Abstract

This dissertation examines what is commonly known as the Kangxi Emperor (r. 1661-1722)'s fourteen-tone temperament, a 1714 reform to Chinese musical tuning that effectively uses the familiar Pythagorean proportions to divide the octave into fourteen parts. Besides examining the ideological and cultural contexts of the tuning reform and correcting many long-held misconceptions, I argue that the reform largely resulted from an epistemological shift that rearticulated the empirical process of sounding and listening vis-à-vis the historicist studies of texts and records in producing musical knowledge. Besides examining it in the context of traditional Chinese scholarship, I shed particular light on the transregional and even global scale of this shift. I argue that the series of experiments and studies on which the fourteen-tone temperament was based took place within the specific political structures of the Qing Empire (1636-1912) as a conquest regime that subjugated China under its minority Manchu ruling class. I also show that the shift was itself inspired by a global exchange of musical knowledge, in which the concept of octave equivalence in Western music theory was misunderstood yet appropriated to advocate an empirical term in music theory and a reform to Chinese opera, both in turn harnessed for Qing-imperial ideological purposes. What is more, by comparing the fourteen-tone temperament to roughly contemporary discourses on texts vs. sounds, writing vs.

speech, and historicism vs. empiricism, both within the Qing Empire and beyond, I argue that the Qing's reform to musical tuning, despite its apparent parochialism, potentially reflected a much broader transformation that took place on a global scale, or what I call the "Phonological Revolution." In concluding this dissertation, I make a case for further examining how seemingly discrete rearticulations of the relation between historicism and empiricism across different discourses and praxes of language, music, writing, and songs may reveal a coeval and co-constitutive epistemological shift on a global scale in the early modern world.

Keywords: tuning, history of music theory, Qing Empire, global music history, acoustemology, Phonological Revolution

Introduction

Imagine you are Jean-Joseph Marie Amiot (1718-1793, known as Qian Deming 錢德明 in Chinese), a Jesuit from Toulon in southern France who has arrived in China in 1751 at the age of thirty-three.[1] One day, in 1752 or 1753, as the Eight Banners army of the Qing Empire (1636-1912) marches into Central Asia in hopes of finishing off the Dzungar Khanate (1634-1758), you find yourself sheltered in the residence of the royally endowed French Jesuit mission at the imperially commissioned Church of the Savior,[2] located less than half a mile from the Forbidden City in the Manchu section of Beijing.[3] Far from the war cries that have defined the northwestern frontiers of the empire for more than a century,[4] and segregated from the bustling noises of street

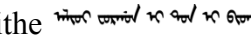
1 Amiot arrived in Beijing at midday on August 22, 1751. See his letter from Beijing dated on October 20, 1752, in Jean-Baptiste Du Halde ed., *Lettres édifiantes et curieuses*, vol. 23, 2nd edition (Paris: J. G. Merigot le Jeune, 1781), 154-180; 161.

2 The Church of the Savior, also referred to as the Western Church (*xitang* 西堂) or the Xishiku Church (*xishiku jiaotang* 西什庫教堂), is located right outside the imperial palace complex, which consisted of the Forbidden City and the several parks to its north and west. The location was granted by the Qing to the French Jesuits—who were endowed by the French monarch—in 1703. See Joseph Dehergne, "Les biens de la Maison Française de Pékin en 1776-1778," in *Monumenta Serica: Journal of Oriental Studies* 20 (1961): 246-265.

3 After the Qing conquered Beijing, it ordered that Manchus and bannermen (which also included the Mongols and most Han Chinese ruled by the Qing before its invasion of China) had to live in the northern Inner City, whereas non-bannermen Chinese could only live in the southern Outer City. The Qing maintained such ethnic apartheid in many major cities throughout China. See Mark Elliot, *The Manchu Way: The Eight Banners and Ethnic Identity in Late Imperial China* (Stanford, California: Stanford University Press, 2001), Chapter Two "Manchu Cities: Tigers on the Mountain," 89-132.

4 For the wars between the Qing, the Dzungar Khanate, and the Romanov Russian Empire, see Peter Perdue, *China Marches West: The Qing Conquest of Central Eurasia* (Cambridge, Massachusetts and London, England: The Belknap Press of Harvard University Press, 2005), Part Two "Contending for Power," 131-299.

fairs, opera theaters, and tea houses that enliven the Chinese section of the capital,[5] you are working quietly on a manuscript for Paris, entitled *De la musique moderne des chinois* (1754).[6]

For the organology section of this text, you are not spending any time plucking, blowing, or beating actual instruments but have opted instead to copy off the Qing Empire's official music theory treatise, *Orthodox Meaning of Pitch Pipes* (lülü zhengyi 律呂正義, 1714, alioi forgon i ton i bithe  in Manchu[7]).[8] Remarkably, this *yuzhi* 御製 "imperially written" treatise on musical tuning in the Chinese tradition dedicates its fifth and last volume, labelled the

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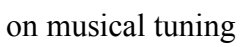
5 For an overview of the musical, especially operatic, ecologies of Beijing during Amiot's time, see Andrea S. Goldman, *Opera and the City: The Politics of Culture in Beijing, 1770-1900* (Stanford, California: Stanford University Press, 2012), Chapter Two "Metropolitan Opera, Border Crossings, and the State," 63-114.

6 Paris, Bibliothèque nationale de France, Département de Musique, Rés. Vmb. Ms. 14. I thank François Picard for providing me with his entire transcription of this source. Jean-Philippe Rameau attests to the date of this manuscript being 1754; see "Nouvelles réflexions sur le principe sonore," in *Code de musique pratique* (Paris: Imprimerie royale, 1760), 189. It is this date that makes me speculate that Amiot, based on his language skills, must have started working on this text around 1752 or 1753.

7 The Manchu script is written vertically from top to bottom, with the resulting columns proceeding horizontally from left to right. Due to typesetting limits, I have rotated all Manchu scripts in this document ninety degrees counterclockwise.

8 Though itself undated, *Veritable Records of the Kangxi Reign* (kangxichao shilu 康熙朝實錄), Vol. 260, indicates that *Orthodox Meaning of Pitch Pipes* was completed and submitted to the Kangxi Emperor December 23, 1714 of the Gregorian Calendar, or the 6th day of the 12th month of the 53rd year of the Kangxi reign in the Chinese Calendar. Such a date in the Chinese Calendar would be notated as KX 53.12.6 in the rest of this dissertation. To my knowledge, no Manchu version of *Orthodox* exists. Yet this Manchu title for the treatise, *alioi forgon i ton i bithe* , which literally means "the book of pitch pipes and calendar science," is attested to in National Library of China, Shanben Guji I22 H221.6 0650, entitled *The Imperially Approbated Manchu Language* (qinding qingyu 欽定清語, hesei toktobuha manju gisun ). While this latter source is undated, it was almost certainly compiled at the Qing court (given the exquisite paper that was used) under the reign of the Qianlong Emperor (r. 1735-1796), who instigated a "purification" campaign to purge the Manchu language of loan words, publishing various vocabularies, lexicons, and dictionaries containing the "imperially approbated" Manchu renditions of Chinese names, titles, and idioms as well as words. See Elliot, *The Manchu Way*, 294-299. See also Pamela Kyle Crossley and Evelyn S. Rawski, "A Profile of the Manchu Language in Ch'ing History," *Harvard Journal of Asiatic Studies* 53, No. 1 (1993): 63-102; see particularly 80-87.

"appended tract" (*xubian* 續編), to what is readily recognizable as Western *musica practica*: the Guidonian solfège syllables *ut re mi fa sol la*,[9] mensuration, and staff notation.[10] Indeed, the volume's preface openly attributes its content to two imperial subjects of the Qing who came

9 The solfège syllables originated as a system of mnemonics for training singers to identify the different types of intervals they would typically encounter in the repertory of Gregorian chants. It was developed by Guido of Arezzo (c. 991-1033), a Benedictine monk in Central Italy, and the earliest textual evidence of this invention is found in Guido's *Micrologus*, a treatise on musical and counterpoint for two voices Guido derived the syllables, *ut*, *re*, *mi*, *fa*, *sol*, and *la* from the first syllable of each line of the Latin hymn *Ut queant laxis*, which reads:

> **Ut** queant laxis **re**sonare fibris
> **Mi**ra gestorum **fa**muli tuorum
> **Sol**ve polluti **la**bii reatum
> **S**ancte **I**ohannes.

In the melodic setting of this hymn, each line begins on the next scale degree from the previous line. Thus, the six initial syllables are sung to notes that constitute precisely the major scale *ut re mi fa sol la*. Notice that Guido's original system only spells six syllables, as their combinations were more or less sufficient for "solmizing" all the different kinds of intervals in the chant repertory." The seventh syllable, *si* (or *ti* in North America), combining the first letters of "Sancte Iohannes" in the last line of the same hymn, was added only during the 17th century. See Stefano Mengozzi, *The Renaissance Reform of Medieval Music Theory: Guido of Arezzo between Myth and History* (Cambridge: Cambridge University Press, 2010), 1-16, 110-114.

10 In this dissertation, I use the term *musica practica* as a general term referring to the tradition of treatises and pedagogical systems of music theory in Western Europe whose primary goal was training practitioners of music, namely performers and composers. In this way, the content of *musica practica* from solfège, intervals, staff notation, rhythms and rhythmic notation, counterpoint, and (to a certain extent) the modes was contrasted during the Middle Ages to the notion of *musica speculativa* and, starting in the late 15th century, *musica theorica*. Both *musica speculativa* and *theorica* referred to studies of music from theological, cosmological, mathematical, and, increasingly during the early modern era, physical and mechanical perspectives. Of course, any neat and absolute distinction between the "practical" on the one hand and the "theoretical" or "speculative" is impossible, and this dissertation examines many mutual entanglements between these differently-aimed studies on music. Still, the extent to which a treatise on music was geared towards pedagogues and practitioners of music rather than scholars or philosophers was a crucial distinction in early modern productions of musical knowledge in Western Europe. And it would be a critical part of the overall argument of this dissertation that the type of "music theory" that defined the musical exchanges between the Qing Empire and Europe through Christian missionaries was predominantly on the *musica practica* side of the spectrum. See Thomas Christensen, "Introduction," in *The Cambridge History of Western Music Theory*, Christensen ed. (Cambridge and New York: Cambridge University Press, 2002), 1-23; see particularly 2-13 for a brief history of the notion of "music theory."

from the Far West: Tomás Pereira (1645-1708, known as Xu Risheng 徐日昇 in Chinese),[11] a Portuguese Jesuit, and Teodorico Pedrini (1671-1746, known as De Lige 德理格 in Chinese),[12] an Italian Lazarist. Later historians will even identify the prototype of this appended tract, a undated treatise entitled *Elements of Pitch Pipes* (lülü zuanyao 律呂纂要, lioi lioi z'uwan yoo bithe ᠯᡳᠣᡳ ᠯᡳᠣᡳ ᡯᡠᠸᠠᠨ ᠶᠣᠣ ᠪᡳᡨᡥᡝ in Manchu).[13]

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11 Many books and articles have been written about Pereira on his musical, clockmaking, and evangelical activities at the Qing court. See, for example, Artur K. Wardega and Antonio Vasconcelos de Saldanha ed., *In the Light and Shadow of an Emperor: Tomas Pereira, the Kangxi Emperor and the Jesuit Mission in China* (Newcastle upon Tyne, UK: Cambridge Scholars Publishing, 2012), and Catherine Jami, "Tomé Pereira (1645-1708), Clockmaker, Musician and Interpreter at the Kangxi Court: Portuguese Interests and the Transmission of Science," in Luís Saraiva and Catherine Jami eds., *The Jesuits, the Padroado and East Asian Science (1552-1773)* (Singapore and Hackensack, NJ: World Scientific Publishing, 2008).

12 Pedrini was an infinitely fascinating figure not only for the musical activities he engaged in but also for all the backstabbing palace intrigues he witnessed and went through during the 1710s and 1720s amid the so-called Chinese Rites Controversy that pitted him, a representative of the Holy See, against the Jesuits. See Fernand Combaluzier, "Théodoric Pedrini. Le missionnaire. Le musician à la Cour impériale de Pékin," *Neue Zeitschrift für Missionswissenschaft/Nouvelle revue de science missionaire* 8 (1952): 270-287, for an unabashedly biased yet still captivating account of those intrigues. See Peter C. Allsop and Joyce Lindorff, "Teodorico Pedrini: The Music and Letters of an 18th-century Missionary in China," *Vincentian Heritage Journal* 27 (2007): 43-59, for an account—also somewhat biased—of his musical activities.

13 An enormous amount of studies have been generated with respect to the "appended tract" of *Orthodox Meaning of Pitch Pipes* and *Elements of Pitch Pipes*, albeit only the Chinese version of the latter, as far as I know. In European languages, see Gerlinde Gild-Bohne, *Das Lü Lü Zheng Yi Xubian: ein Jesuitentraktat über die europäische Notation in China 1713* (Göttingen: Edition Re, 1991); François Picard and Fañch Thoraval, "*Musica mecanica, practica & speculative*: de Pereira à Pedrini, la musique européenne à la cour de Kangxi," in Luciane Beduschi, Anne-Emmanuelle Ceulemans, Alice Tacaille ed., *Musica, sive liber amicorum Nicolas Meeùs, Mélanges offerts au professeur Nicolas Meeùs* (Paris: Presses Universitaires de Paris-Sorbonne, 2014), 453-492; and Shubing Jia, *The Dissemination of Western Music through Catholic Missions in High Qing China (1662-1795)*, Ph.D. dissertation (University of Bristol, 2012). In the Chinese language, see Wang Bing 王冰, "《律呂纂要》之研究" ("A study on *Elements of Music Theory*"), *Gugong bowuyuan yuankan* 故宮博物院院刊 102, No. 2 (2002): 68-81; and Wang Bing, "徐日昇和西方音樂知識在中國的傳播" ("Tomás Pereira and the dissemination of knowledge of Western music in China"), *Wenhua zazhi* 文化雜誌 47 (2003): 71-91.

Unlike music historians writing centuries later, however, you show little interest in examining how the Chinese digested the "*ut re mi*" solfège syllables you have mastered since your choirboy days. After all, your senior Jesuit Antoine Gaubil (1689-1759) specifically asked you to gather information on *musique moderne des chinois* in order to complement the translation of a text on ancient Chinese music you have already been undertaking, also upon his recommendation: *Commentaries on the Ancient Confucian Canon of Music* (gu yuejing zhuan 古樂經傳, published in 1726) by the renowned Chinese scholar Li Guangdi 李光地 (1642-1718).[14] Thus, after acknowledging the deed of musical evangelism of your missionary forefathers immortalized in the fifth volume of *Orthodox Meaning*,[15] for the rest of your manuscript you focus only on the first four volumes of the treatise whose content, for your purpose, appears to be exclusively *chinois*.

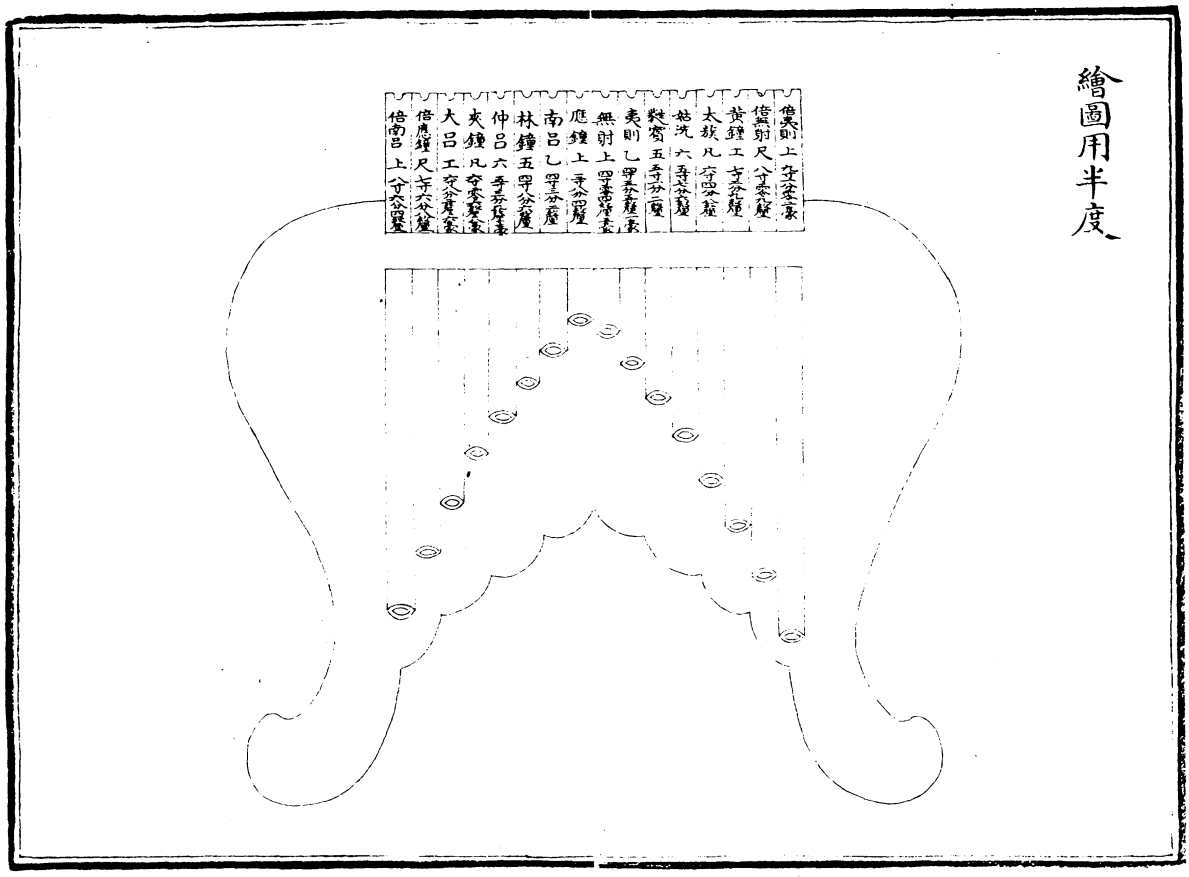
You turn the page, then, to Volume 3, folio 12 of the Qing treatise, setting your eyes on the first instrument in its organology (See Illustration I). Upon seeing its illustration, you immediately liken the *paixiao* 排簫 made of sixteen holeless even-bodied bamboo pipes to a pan-flute. While noting that the late Jesuit mathematician Marin Mersenne (1588-1648) also began his organology of *instrument à vent* with a *syringa panos* in *Harmonie universelle* (1636),[16] you know well that the Qing empire's official music treatise puts a pan-flute at the beginning of its organology not because of any Arcadian or *bergerie* associations but because of the foundational role bamboo pipes play in Chinese music theory. According to the chapter

14 Amiot describes the circumstances of his translation of Li Guangdi's treatise in his later *Mémoire de la Musique des Chinois tant anciens que modernes* (Paris, 1779), 4-5. The translation was sent to Paris in the same dispatch that contained *De la musique moderne des chinois*, yet by all indication it has been lost.

15 Amiot, *De la musique moderne des chinois*, 1-2.

16 Marin Mersenne, *Harmonie universelle* (1637), 2nde partie, "Livre premier des instruments," 227-229.

Illustration I The Pan-Flute (*paixiao *排簫) in Chapter 2, Volume 3 of* Orthodox Meaning of Pitch Pipes *(1714), f. 11bis.



"Ancient Music" (*guyue* 古樂) in *Chronicles of Master Lü* (lüshi chunqiu 呂氏春秋, c. 239 BCE)—the earliest extant and securely dated source in Chinese to discuss musical tuning—it was the Yellow Emperor (*huang di* 黃帝) himself (? 2698-2598 BCE), the legendary progenitor of Chinese civilization, who first commissioned a set of twelve tuning pitch pipes or *lülü* 律呂 made from slashing bamboo stems harvested from a valley in Mount Kunlun.[17] Upon matching their pitches to the sounds of the cosmos—specifically the calls of six male phoenixes and six female ones—the Yellow Emperor ordained the twelve tuning pitch pipes as the standard to

17 Lü Buwei 呂不韋, *Chronicles of Master Lü* (lüshi chunqiu 呂氏春秋), Chapter 25 "Ancient Music" (*guyue* 古樂).

which all instruments and performances of music ought to be tuned, including even bells, which typically rank first in the solemn ensemble of *yayue* 雅樂 "elegant music" at sacrificial rites and courtly ceremonies. Subsequent sources from the Han era (206 BCE-220 CE) even record that the Yellow Emperor and later ancient sage kings matched the twelve tuning pitch pipes to the twelve months and adopted the largest pitch pipe, *huangzhong* 黃鐘 (which literally means "yellow bell"), as the standard of measurement for length, volume, and weight.[18] Whether these broadly quoted narratives are truth or legend, almost all later texts made the twelve tuning pitch pipes their primary objects and tools for studying musical tuning both in order to guide musical practices and in order to rectify the system of metrology and even of the calendar. Even the name of these pitch pipes, *lülü* 律呂, has long become a metonym for the study of musical tuning and music theory in general—ergo the title of the Qing treatise, *Orthodox Meaning of Pitch Pipes*.[19]

Indeed, the music-theoretical significance of this sixteen-pipe pan-flute becomes all the more evident when you, Amiot, realize that the illustration scrupulously specifies the length, pitch name, and solmization syllable for every single one of its constitutive pipes (Table I). Upon inspecting their lengths, furthermore, you discover their perfect Pythagorean proportions—a fact you find utterly unremarkable. You know too well that, for at least two millennia, the Chinese have been deriving the lengths of the twelve tuning pitch pipes through the method of *sanfen sunyi* 三分損益 "triple division with one part subtracted or added," whose earliest record is also

18 See Liu An 劉安, *Writings of Master Huainan* (huainan zi 淮南子 c. 139 BCE), Chapter 3 "Patterns of Heaven" (*tianwen xun* 天文訓); Ban Gu, *Book of the Han* (han shu 漢書), "Chronicle of Pitch Pipes and Calendar" (*lüli zhi* 律曆志).

19 The phrase *lülü* "tuning pitch pipes" conjoins two characters: *lü* 律 and *lü* 呂. The former character refers to odd-numbered pitch pipes in a sequence of twelve tuning pitch pipes arranged from the longest to the shorter; the latter refers to the even-numbered pitch pipes.

Pipe No. from the left	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Pitch Name	倍南呂 Double *Nanlü*	倍應鐘 Double *Yingzhong*	大呂 *Dalü*	夾鍾 *Jiazhong*	仲呂 *Zhonglü*	林鐘 *Linzhong*	南呂 *Nanlü*	應鐘 *Yingzhong*	無射 *Wuyi*	夷則 *Yize*	蕤賓 *Ruibin*	姑洗 *Guxian*	太簇 *Taicu*	黃鐘 *Huangzhong*	倍無射 Double *Wuyi*	倍夷則 Double *Yize*
Solmization Syllable	上 *shang* [*do*]	尺 *che* [*re*]	工 *gong* [*mi*]	凡 *fan* [*fi*]	六 *liu* [*sol*]	五 *wu* [*la*]	乙 *yi* [*ti*]	上 *shang* [*do*]	上 *shang* [*do*]	乙 *yi* [*ti*]	五 *wu* [*la*]	六 *liu* [*sol*]	凡 *fan* [*fi*]	工 *gong* [*mi*]	尺 *che* [*re*]	上 *shang* [*do*]
Length in Chinese inches (*cun*)	8.64	7.68	6.826	6.068	5.393	4.86	4.32	3.84	4.045	4.551	5.12	5.76	6.48	7.29	8.09	9.102
Length ratio to pipe No. 14	32 : 27	256 : 243	2048 : 2187	16384 : 19683	131072 : 177147	2 : 3	16 : 27	128 : 243	32768 : 59049	4096 : 6561	512 : 729	64 : 81	8 : 9	1 : 1	65536 : 59049	8192 : 6561
Length ratio to adjacent pipe on the left	/	8 : 9	8 : 9	8 : 9	8 : 9	59049 : 65536	8 : 9	8 : 9	256 : 243	9 : 8	9 : 8	9 : 8	9 : 8	9 : 8	65536 : 59049	9 : 8

Table I Lengths, Pitch Names, and Proportions of Pipes of the Pan-flute from Illustration I[20]

20 In Table I, the numbering of the sixteen pipes, their pitch names, their solmization syllables, and their lengths in Chinese inches (*cun*) are taken from *Orthodox Meaning of Pitch Pipes*, Vol. 3, f. 11*bis* (i.e. Illustration I). The ratios of the length of each pipe to that of pipe No. 14 (the *huangzhong* pipe, comparable to the role of the pitch class of C in Western harmonics) and the ratios of the length of each pipe to that of the one on its left are calculated by me.

found in *Chronicles of Master Lü*.[21] This time-worn method begins by first determining the proper length of the pitch pipe of *huangzhong*, whose role is akin to that of fundamental C in common-practice Western harmonics. According to *Writings of Prince Huainan* (huainan zi 淮南子, c. 139 BCE), the second-earliest extant source to discuss musical tuning in detail, the ancients set the length of the *huangzhong* pipe to be 9 *cun* 寸 or 9 Chinese inches.[22] Though the number nine's being the largest single-digit number and thus the epitome of the *yang* 陽 or masculine principle is a rudimentary doctrine of Chinese numerology, subsequent scholars would scramble to reconstruct exactly how long this specific *cun* was, in relation to later measuring standards also bearing this particular name.

After determining the length of the *huangzhong* pipe, however, the method of "triple division with one part subtracted or added" is as clear and universal a consensus as it gets. From the length of *huangzhong*, the method produces the lengths of the remaining eleven pitch pipes one after another by alternating the ratios of 2:3, i.e. "triple division with one part subtracted," and 4:3, i.e. "triple division with one part added." As a result, these combinations of 2:1 and 1:3 generate the same set of Pythagorean ratios that had served as a backbone of musical tuning for as long a time among the Europeans as they had among the Chinese (except, of course, that the Chinese method proceeds only in the "sharp direction" of the circle of fifths). Indeed, the only notable difference between the Chinese Pythagorean method and its Greco-Roman and Western counterpart is that the latter applies these ratios not to fashioning a set of pitch pipes but to dividing the string of a monochord. Thus, all too familiar with Pythagorean tuning, you simply jot down the vapid numbers of the lengths of those pipes in the pan-flute, translate their Chinese

21 Lü Buwei, *Chronicles of Master Lü*, Chapter 27 "Pitch Pipes" (*yinlü* 音律).
22 Liu An, *Writings of Master Huainan*, Chapter 3 "Patterns of Heaven."

pitch names as specified in the illustration into French equivalents, and move on to the next instrument, never to return.

If only you had dwelled on it for but a few more seconds! For you would have seen that the sixteen pipes in the illustration are not arranged in a single series from the longest to the shortest, as in most pan-flutes and *paixiao*'s, but grouped into two series of eight: the first eight pipes on the left are decreasing in length, whereas the remaining eight are increasing in length (Table I, pipes no. 1-8 cf. pipes no. 9-16). What is more, by calculating their length proportions, you would also have discovered that each series of eight pipes seems to constitute not a diatonic octave, as the number "eight" might suggest, but what would appear to be a whole-tone scale spanning a major ninth. On the left, pipe No. 1 measures 8.64 *cun*—not the ancient *cun* in *Writings of Prince Huainan* but the Qing's standard *cun*, which equals roughly 3.21 centimeters—and pipe No. 8 measures 3.84 *cun* long, thus a 9:4 ratio; and each of the eight pipes is 8:9 or about 0.888 times the length of the one to its left, except for pipe No. 6, which is 59049:65536 or about 0.901 times the length of pipe No. 5. Inversely, on the right, pipe No. 9 measures 4.045 *cun* long and pipe No. 16 measures 9.012 *cun* long, thus a 4:9 ratio; and each of the eight pipes in this series is 9:8 or about 1.111 times the length of the one to its left, except for pipe No. 15, which is 65536:59049 or about 1.110 times the length of pipe No. 14.

But stay put before you rush to conclude that modern Chinese music, based on these two distinct series of pitch pipes, uses whole-tone scales, lest you commit the same error as Jean-Philippe Rameau (1683-1764) in his *Code de musique pratique* (1760) after seeing the pan-flute in your manuscript at the house of the *académicien* Jean-Pierre de Bougainville (1722-1763) in

Paris in the 1750s.[23] Something even less sensible appears when you factor in the *gongche* 工尺 or Chinese solmization syllables that Table I spells out for the sixteen pipes along with their pitch names and lengths to show the intervallic relations between their sounds. Somehow, each series of eight pipes related by whole tones as implied by their length proportions are solmized through the seven diatonically related *gongche* syllables: *shang* 上, *che* 尺, *gong* 工, *fan* 凡, *liu* 六, *wu* 五, and *yi* 乙, which are comparable to the Western solfège syllables *do*, *re*, *mi, fi* (that is, *fa*#), *sol*, *la*, and *ti*. Remarkably, though the lengths of pipes No. 1 and No. 8 are 9:4 in proportion, both of their pitches are solmized as *shang*'s or *do*'s, implying that they are an octave apart in sound, not a major ninth. And the same is true with pipes No. 9 and No. 16. Meanwhile, though the lengths of pipes No. 1 and No. 7 are actually 2:1 in proportion, their pitches are solmized as *shang* and *yi*, i.e. *do* and *ti*, implying that they are a major seventh apart in sound, not an octave. And the same is true with pipes No. 10 and No. 16. In consequence, between a pair of low and high *shang*'s or *do*'s—with an octave between the pitches they solmize—there are seven whole tones in terms of length proportions (six 8:9 major seconds and one 59049:65536 diminished third) instead of five whole tones and two semitones. And, should all sixteen pipes be rearranged in decreasing order of length, as shown in Table II, between two pipes whose pitches are an octave apart according to their solmization syllables, there would be fifteen pipes or fourteen semitones in terms of length proportions (six 2048:2187 augmented unisons or chromatic semitones and eight 243:256 minor seconds or diatonic semitones), instead of twelve.

23 Jean-Philippe Rameau, "Nouvelles réflexions sur le principe sonore," in *Code de musique pratique* (Paris: Imprimerie royale, 1760), 189

Pipe Number in original Pan-flute	**16**	**1**	**15**	**2**	**14**	**3**	**13**	**4**	**12**	**5**	**11**	**6**	**10**	**7**	**9**	**8**
Pitch Names	倍夷則Double *Yize*	倍南呂Double *Nanlü*	倍無射Double *Wuyi*	倍應鐘Double *Yingzhong*	黃鐘*Huangzhong*	大呂*Dalü*	太簇*Taicu*	夾鍾*Jiazhong*	姑洗*Guxian*	仲呂*Zhonglü*	蕤賓*Ruibin*	林鐘*Linzhong*	夷則*Yize*	南呂*Nanlü*	無射*Wuyi*	應鐘*Yingzhong*
Solmization Syllables	上 *shang* [*do*]	上 *shang* [*do*]	尺 *che* [*re*]	尺 *che* [*re*]	工 *gong* [*mi*]	工 *gong* [*mi*]	凡 *fan* [*fi*]	凡 *fan* [*fi*]	六 *liu* [*sol*]	六 *liu* [*sol*]	五 *wu* [*la*]	五 *wu* [*la*]	乙 *yi* [*ti*]	乙 *yi* [*ti*]	上 *shang* [*do*]	上 *shang* [*do*]
Length in Chinese inches (*cun*)	9.102	8.64	8.09	7.68	7.29	6.826	6.48	6.068	5.76	5.393	5.12	4.86	4.551	4.32	4.045	3.84
Length ratio to adjacent pipe on the left	/	243 : 256	2048 : 2187	243 : 256	243 : 256	2048 : 2187	243 : 256	2048 : 2187	243 : 256	2048 : 2187	243 : 256	243 : 256	2048 : 2187	243 : 256	2048 : 2187	243 : 256

Table II The Sixteen Pipes of the Pan-Flute from Illustration I Reordered from Longest to Shortest[24]

24 Table II is identical to Table I (without the ratios of each pipe to that of the *huangzhong* pipe), except the pipes are reordered from longest to shortest. Thus, pipe No. 16 (solmized as *shang*) and pipe No. 9 (solmized as *shang*) form an octave, despite their 9.102 to 4.045 or 9:4 length ratio; pipe No. 1 (solmized as *shang*) and pipe No. 8 (solmized as *shang*) form an octave, despite their 8.64 to 3.84 or 9:4 length ratio. Meanwhile, pipe No. 16 (solmized and *shang*) and pipe No. 10 (solmized as *yi*) form a major seventh, despite their 9.102 to 4.551 or 2:1 length ratio; pipe No. 1 and (solmized as *shang*) and pipe No. 7 (solmized as *yi*) form a major seventh, despite their 8.64 to 4.32 or or 2:1 length ratio.

What is this madness, whereby a whole-tone major ninth implied by the pipes' length proportions (4:9) nonetheless becomes a diatonic octave in sound per their solmization syllables (*shang-shang*, i.e. *do-do*), and whereby the octave is effectively divided into fourteen parts comprising two complementary diatonic scales? How did it come to be enshrined as the basis of the Qing's official music theory in an imperially sponsored treatise?

Fourteen-Tone Temperament

What I have just described, through the imagined perspective of our French Jesuit, is a tuning reform championed by the Kangxi Emperor (r. 1667-1722), presented in the Qing's official music theory treatise *Orthodox Meaning of Pitch Pipes*, and partially and unwittingly transmitted four decades later by the Jesuit in his 1754 manuscript. Its core research having been finished in 1708 and its text completed in 1714, *Orthodox Meaning* was formally published in 1723 as part of the 100-volume *Origins of Cosmological Sciences* (lüli yuanyuan 律曆淵源), a *magnum opus* of the Kangxi Emperor's scholarly patronage that also incorporated two other treatises, one on mathematics and the other on astronomy.[25]

That a ruler of China weighed in at least in name on a matter as seemingly recondite as musical tuning was not exceptional. Even without generalizing the significance of music (*yue* 樂) in Chinese cosmology, the widely quoted chapter "Record on Music" (yueji 樂記) in the 1st-century BCE Confucian classic *Records of Rites* (liji 禮記) puts it in the most pragmatic terms: the exactitude of the musical tuning of a regime not only reflects but also affects the physical and

25 The other two treatises in this compilation are: *Lixiang kaocheng* 曆象考成 ("Astronomical Observations Investigated and Compiled") on calendar-keeping, and *Shuli jingyun* 數理精蘊 ("Essence and Meaning of the Principles of Numbers") on mathematics.

mental health of its rulers, the state and fortune of their governance, and the well-being of their subjects.[26] And even though the Qing's Manchu elites hailed from Manchuria and ruled China as a foreign conquest regime, in consolidating their vast empire, which also encompassed modern-day Mongolia, Korea, Taiwan, Tibet, Vietnam, and East Turkestan, the Qing crafted a model of multiethnic rulership that is best described as "wearing many hats."[27] Indeed, the Kangxi Emperor and his successors styled himself as the *Sure Kundulen Han* ᠰᡠᡵᡝ ᡴᡠᠨᡩᡠᠯᡝᠨ ᡥᠠᠨ ("Wise and Venerated Khan") to the Manchus, the *Bogda Khagan* ᠪᠣᠭᠳᠠ ᠬᠠᠭᠠᠨ ("Great Khan") to the Mongols, the *Huangdi* 皇帝 ("Emperor and Sovereign") to the Chinese, and the འཇམ་དབྱངས་གོང་མ་ཆེན་པོ *vjam dbyangs gong ma chen po* ("Great Emperor, the *Manjusri* bodhisattva reincarnated") to the Tibetans. Maintaining the tuning accuracy of music used for the Confucian sacrificial rites and courtly ceremonies constituted part of the Kangxi Emperor's performance as Emperor of the Chinese, one of his many hats. In fact, by compiling official treatises on musical tuning as well as other scholarly projects, he created thousands of jobs for the Han Chinese literati with the prospect of rising in government ranks, which was arguably the Qing's most effective gesture towards this very body politic of Confucianism. In the short term, the Kangxi Emperor's scholarly patronage lured into his service many Han Chinese who had been hostile to the regime after the Manchu conquest of China. In the longer term, the stream of publication projects sponsored by the Qing would continue to mend the often-fraught relationships between the Manchu conquerors and the landed Chinese gentry who, despite being in many ways second-

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26 Liu Xiang 劉向 and Dai De 戴德 eds. *Liji* 禮記 ("Records of Rites," compiled in c. 1st century BCE), Chapter 19 "Yueji" 樂記 ("Records of Music").

27 See Pamela Kyle Crossley, *A Translucent Mirror: History and Identity in Qing Imperial Ideology* (Berkeley and Los Angeles: University of California Press, 1999), Chapter 5, "The Wheel-Turning King," 223-280.

class citizens, served as the empire's bread basket, tax base, and reserve of literati and bureaucrats.[28]

What *was* exceptional, however, was that in performing his part as guardian of Confucian music and musical scholarship, the Kangxi Emperor went far beyond simply finetuning what the Qing had inherited from previous Chinese regimes. Though traditional Chinese music was never as thoroughly pentatonic as its sanitized Orientalist portrayals might suggest, it was still a major break from traditions to divide the octave into fourteen parts comprising two complementary diatonic scales (indeed an oxymoron)—hence the common moniker of the Qing's tuning reform today as the "Kangxi Emperor's fourteen-tone temperament" (*kangxi shisilü* 康熙十四律). As far as extant written sources are concerned, since at least the 6th century CE, a system of twelve semitones in an octave, seven notes in a diatonic scale, and ergo eighty-four modes through transpositions and mutations has been the "common practice" of Chinese music, even though precisely how the twelve semitones were tuned ranged from the conventional Pythagorean "triple division with one part subtracted or added" to the more reform-minded twelve-tone equal temperament.[29] As it divides the octave into fourteen parts instead of twelve, however, the Qing's *Orthodox Meaning of Pitch Pipes* completely reorganizes this system into fourteen

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28 The best analysis I have seen of the complex power dynamics between the Qing Empire and the Han Chinese literati and landed-gentry class is in Michael Chang, *A Court on Horseback: Imperial Touring and the Construction of Qing Rule, 1680-1785* (Cambridge, MA: Harvard University Press, 2007), Chapter 6, "The Southern Tours as Cultural Encompassment: The Valorization of Verse & Accommodation of Han Learning," 260-304.

29 The earliest record of the system of eighty-four modes (seven notes of the diatonic scale multiplied by twelve tones of the octave) is found is the official dynastic history of the Sui (581-618): Wei Zheng 魏徵, Zhangsun Wuji 長孫無忌 et al., *Suishu* 隋書 ("Book of Sui'), Chapter 14 "Yuezhi" 樂志中 "Chronicle of Music, Part Two of Three." Incidentally, the narrative in *Suishu* of the provenance and adoption of this system of eighty-four modes would be copied word-for-word into the preface to the fifth and last volume or the "appended tract" of *Orthodox Meaning on Pitch Pipes*, the volume on Western *musica practica*.

semitones in an octave, seven notes in a diatonic-yet-also-whole-tone scale and ergo ninety-eight modes.

What is more, as I have shown above, the Kangxi Emperor's fourteen-tone temperament does not simply seek to divide the octave into fourteen parts instead of twelve. Rather, it reinvents how tuning proportions should be matched with sounding intervals in a way that appears to fundamentally defy the most basic acoustic common sense. For example, an octave in sound corresponds not to a 1:2 proportion in the physical lengths of pitch pipes, according to the pan-flute in *Orthodox Meaning* that Amiot copied into his 1754 manuscript, but instead to a 4:9 proportion, even though the latter ratio should normally produce a major ninth. And though the tuning proportions of the sixteen pipes of the pan-flute indicate two complementary whole-tone scales each spanning a major ninth, the pitches of each scale are nonetheless solmized as the eight notes forming a diatonic octave, thus creating two "complementary diatonic scales"—again, as oxymoronic as it sounds. Adding to the puzzle are the rhetorical glosses of historicism and restorationism that saturate the prose of the Qing treatise. Though styling oneself as a restorer of the proper institutions and practices of Confucianism was a mandatory cliché in performing the part of a sage Confucian ruler, it still takes more than a leap of faith to propose that a fourteen-tone temperament actually restores the "orthodox meaning" of proper musical tuning practiced by the ancient sage kings per Confucian historical imagination.

Because Amiot never fully understood what he was copying into his own manuscript from the Qing treatise, the fourteen-tone temperament never reached his contemporary readers in Europe outside the immediate reach of the Qing empire. Even today, the Qing's tuning reform remains a *res obscura* in non-Chinese-speaking academic circles, not only among music historians but also among historians of science focusing on China. Indeed, though scholars such

as Benjamin Elman and Catherine Jami have produced insightful monographs and articles on the Kangxi Emperor's scholarly patronage, with *Origins of Cosmological Sciences* as its crest, their discussions of music theory have been too perfunctory to even touch on the fourteen-tone temperament, and this despite the fact that *Orthodox Meaning of Pitch Pipes* is one of the three treatises that comprises *Origins*, as well as the first to be finished.[30] Such is the case even for ethnomusicological approaches to Chinese music, which might be understandable, given that the Qing's tuning reform ultimately exerted little impact on discursive or embodied practices of music theory beyond the Forbidden City and the imperial temples and resorts.

Among historians of Chinese music, on the other hand, the Kangxi Emperor's fourteen-tone temperament has been seen as a notorious imperial blunder and has lately been an object of historical revisionism. During the formative years of modern Chinese musicology in the mid twentieth century, scholars such as Chen Wannai 陳萬鼐 in Taiwan and Yang Yinliu 楊蔭瀏 in China dismissed outright the fourteen-tone temperament for being pragmatically-speaking pointless and scientifically- and acoustically-speaking senseless.[31] Notably, they considered the Qing's tuning reform a regression from the twelve-tone equal temperament that a prince of blood

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30 See Benjamin A. Elman, *On Their Own Terms: Science in China, 1550-1900* (Cambridge, MA and London: Harvard University Press, 2005), Chapter 4, "The Limits of Western Learning in the Early Eighteenth Century," 150-189, which discusses the *Origins of Cosmological Sciences* project but fails to mention anything substantial about its content on musical tuning. See also Catherine Jami, *The Emperor's New Mathematics: Western Learning and Imperial Authority During the Kangxi Reign* (1662-1722) (Oxford and New York: Oxford University Press, 2012); though Jami's monograph is thus far the most extensive study on the various historical backgrounds and processes that led to the *Origins* project, and though Jami does in fact discuss its content on "harmonics" in Chapter 16 "A new mathematical classic?" Section 2 "Harmonics and Astronomy," it appears from this latter section as though she has not thoroughly read through the *Orthodox Meaning* portion of *Origins*, since she effectively characterizes it as a squarely conventional work on Chinese musical tuning.

31 See Chen Wannai 陳萬鼐, Qingshi gao Yuezhi *yanjiu*《清史稿・樂志》研究 ("A Study on the "Chronicle of Music" of the *Draft History of Qing*"), 2nd edition (Beijing: Renmin chubanshe, 2010), 95-110.

of the previous Ming dynasty (1368-1644), Zhu Zaiyu 朱載堉 (1536-1610), had proposed more than a century before in 1584.[32] Curiously, it was also in the mid-twentieth century that the renowned British scientist and historian Joseph Needham (1900-1995), in his paradigm-setting *Science and Civilisation in China* (1954), portrayed Zhu's twelve-tone equal temperament as a crowning achievement of pre-modern Chinese science and mathematics and the epitome of their superiority to their contemporary European counterparts.

Since the 1990s, new generations of Chinese musicologists, including Dai Nianzu 戴念祖, Li Laizhang 李来璋, and Zheng Rongda 鄭榮達, to name just a few, have challenged the established verdict against the Qing's tuning reform.[33] By parsing the dense prose of *Orthodox Meaning* and piecing together some contextual evidence, these recent studies strive to understand the internal logic of the fourteen-tone temperament and to salvage what they deem "scientific" elements from the otherwise confused tuning reform that previous scholars have neglected in dismissing its merits outright. Still, many of the most basic questions remain unanswered. Particularly, how—materially, political, and epistemologically—did the Kangxi Emperor's learned researchers end up with such an unusual if not outright erroneous system of tuning and acoustics as the "orthodox meaning of pitch pipes"? And what could have inspired their peculiar ideas of matching the tuning proportion of 4:9 with the octave and constructing a system of complementary diatonic scales based upon it? Even more difficult, I observe, is to assess the

32 Zhu Zaihyu, *Lüxue xinshuo* 律學新說 ("A New Theory of Pitch Pipes," 1584).

33 See Dai Nianzu 戴念祖, "中國古代在管口矯正方面的成就" ("Ancient Chinese Achievements in End Corrections of Pipes"), in *Zhongguo keji shiliao* 中國科技史料 (1992:4):1-5; Li Laizhang 李来璋, "康熙與十四律" ("Kangxi Emperor and the Fourteen-Tone Temperament"), in *Huangzhong* 黃鐘—武漢音樂學院學報 (1996:1): 19-23; and Zheng Rongda 鄭榮達, "康熙三分損益十四律解—兼與陳萬鼐先生商榷" ("Solving the Kangxi Emperor's Fourteen-Tone Temperament of 'Triple Division with One Part Subtracted or Added': a Counterargument to Mr. Chen Wannai"), in *Zhongguo yinyue* 中國音樂 (2009:1): 33-50.

significance of the Qing's tuning reform under the shadow of Zhu Zaiyu and his system of twelve-tone equal temperament. As I will later show, compilers of the Qing treatise openly acknowledged Zhu Zaiyu's twelve-tone equal temperament and from every indication understood its mathematics and unique characteristics.[34] The fact that they ultimately rejected it, in favor of their own fourteen-tone temperament, however, only creates further challenges to the history of music and science in the early modern era, which is still so commonly narrated as a linear progression towards a Eurocentric vision of science and modernity, a vision that includes the twelve-tone equal temperament.

In this dissertation, I continue the refreshing efforts among Chinese scholars since the 1990s to take seriously the Kangxi Emperor's seemingly outlandish fourteen-tone temperament. To this end, however, I will not dive right into the worn-out debates over the merits or scientificity of the Qing's tuning reform itself. Instead, I will explore a major yet hitherto unnoticed twist to the plot that answers the many lingering questions regarding the provenance, internal logic, *raison d'être*, and actual developmental processes of the fourteen-tone temperament. This twist, furthermore, sheds critical new light on the significance of the Qing's tuning reform beyond the parochial concerns of the history of Chinese music or science. Indeed, it allows me to situate the fourteen-tone temperament in the context of the Qing's transregional empire-building and of the globally transformative processes of sounding and listening in the early modern era. And here is the promised twist: unbeknownst to Amiot, his readers, or most subsequent historians of music, the fourteen-tone temperament that the former unwittingly copied into his 1754 manuscript was, in fact, a direct product of the decades-long integration of his Jesuit forefathers and their knowledge into the multiethnic Qing court. In particular, I argue,

34 Beijing, National Library of China, *Putong Guji* No. 15251, 55r-56v.

the fourteen-tone temperament, as idiosyncratic as it might seem, owed much of its origins to the Kangxi Emperor having learned from the Jesuits how to sing—and perhaps express through gestures on the joints of his left hand—his *ut*, *re*, *mi*'s.

Hidden Cosmopolitan Acoustemologies

But how, many may ask, could a fourteen-tone temperament have possibly resulted from the introduction of Western music and music theory to the Qing Empire by the Jesuits? Of course, even without invoking the well-studied *archicembalo* of Nicola Vicentino (1511-1575), the century-long tradition of split keys on harpsichords, spinets, and organs can testify that early modern Europe was no stranger to more than twelve-fold divisions of the octave. So far, however, I have yet to find any fourteen-tone temperament or any oxymoronic system of two complementary diatonic scales that the Europeans could have imparted when serving the Qing Empire as astronomers, cartographers, instrument-makers, or diplomats, or when tutoring the Kangxi Emperor, his sons, and his eunuchs. And I can never repeat too many times that the Kangxi Emperor's so-called fourteen-tone temperament does not simply divide the octave fourteen-fold. Rather, it entails the puzzling acoustic reasoning that a whole-tone scale spanning a major ninth in terms of tuning proportions somehow produces a diatonic octave in sound. I have searched many 16th- and 17th-century Western European writings on musical tuning for any similar theories, only in vain.

What is more, many may wonder, even assuming that the Jesuits and their kinds of Western music and music theory did play a material role in the development of the fourteen-tone temperament, how could have this reinvention of musical tuning and acoustics been inspired not by some equally mathematically or conceptually oriented discourse but by something as

mundane as the Guidonian solfège syllables? In fact, the new mechanical paradigm of music theory spearheaded by Galileo Galilei (1564-1642) and adopted by Marin Mersenne (1588-1648) and the Jesuits' own Athanasius Kircher (1602-1680) did make its presence felt at the Qing court. Indeed, the Society's global network of textual circulation delivered the latter's *Musurgia universalis* (1650) right to the desk of the Kangxi Emperor on May 8, 1690.[35] Yet I specifically argue that these Jesuit-introduced ideas in *musica speculativa* or *theorica*—or indeed what could be considered precursors to modern scientific acoustics as a subdiscipline of physics—were only incidental to the Qing's tuning reform. As I will show, the Kangxi Emperor's researchers appropriated their diagrams and their mechanistic theory of vibration only to justify *ex post facto* the fourteen-tone temperament and its extraordinary acoustic claims. Meanwhile, what directly inspired two decades of imperially sponsored research in musical tuning—research that would conclude with the fourteen-tone temperament as presented in *Orthodox Meaning of Pitch Pipes*—had been sitting quietly in the latter's fifth and last volume all along, even though the treatise itself draws no connection between its appended tract on Western *musica practica* and its first four volumes on tuning. Indeed, a host of studies have been produced on this fifth and last volume of *Orthodox Meaning*, which is typically examined in tandem with its purported source, *Elements of Pitch Pipes*.[36] Yet to date none of them has ever suspected that its seemingly innocuous "foreign" content on rudiments of Western music holds the key to understanding the fourteen-tone temperament, the latter being not only a tuning reform to "native" musical

35 Paris, Bibliothèque nationale de France, MS *Français* 17240, f. 277v. "Le 8.e 9.e 10.e 11.e l'Emperor s'est fait expliquer par nous a l'ordinaire la leçon d'Euclide et il s'y est raisonablement appliqué. Il s'est arreté une fois fort long temps a feuilleter un gros volume du pere kirker qui traite des sons et de tout ce qui appartient a la musique. […] [emphases mine]."

36 See footnote 11 above.

practices but also a symptom of broader shifts in the epistemologies of sound, bodies, instruments, and the cosmos.

In answering these two questions—how a fourteen-tone temperament could have arisen from a tradition of Western music theory, and how this Western tradition could have been something as basic as *musica practica*—I argue that the seemingly impossible passage from *ut*, *re*, *mi* to the Kangxi Emperor's fourteen-tone temperament can become legible only if music historians listen attentively to what I call the "hidden cosmopolitan acoustemologies" of this tuning reform. In order to perceive the global embeddedness of a musical phenomenon that has never been analyzed outside its supposedly indigenous domain of culture, civilization, or nation-state, I argue, we as music historians must not limit our narrative to conscious historical experiences, representations, or constructions of "Others." Instead, we must unravel the global relationalities that constituted the very phenomenological and epistemological structures in which historical subjects experienced and mobilized sounds to produce knowledge in and about their globally integrated ecologies, even when they themselves did not opt to register difference or alterity through symbolic or semiotic marks. In other words, our current endeavors to globalize the history of music ought to go beyond chasing down the transfer of musical subjects, objects, and ideas outside their well-defined "native" lands. Rather, we should map the boundless relationalities of listening as a way of knowing and making worlds, so that any individual or social engagement with music and sound, no matter how inward-looking, local, or even parochial it may seem, can be productively analyzed as part of mutually transformative global processes.[37]

37 In this regard, I follow Sebastian Conrad's definition of global history as studying "global integration, or structured transformations on a global scale." See Conrad, *What is Global History?* (Princeton, NJ: Princeton University Press, 2016), 62.

By coining the term "hidden cosmopolitan acoustemologies," I join together two critical interventions that not only help examine the Kangxi Emperor's fourteen-tone temperament from the overdue transregional perspective but also push the global history of music beyond essentialist and Eurocentric confines of "cultures" or "civilizations." As reductive as it is, the "East and West" binary still looms large in studies on early modern China. In fact, art historian Jonathan Hay first used the term "hidden cosmopolitanism" in his foreword to *Qing Encounters: Artistic Exchanges between China and the West* (2015) as a subtle critique of the latter collection of essays, which for him fails to go deeper than the cultural combinatorics of East-West hybridities.[38] Granted, the essays in *Qing Encounters* exemplify decades of productive scholarship on how one end of Eurasia borrowed motifs and styles from the other, and how the early modern consumption of foreign cultures articulated the boundaries between subject and object and between self and Other. Hay, however, contends that their narrow focus on "fully declared encounters" between Europe and the Qing Empire "would leave the impression that the more explicit cosmopolitanisms of the time defined the horizon of intercultural engagement."[39] This, Hay argues, was not the case at least under the Qing. What is more, the neglect of "hidden" in favor of "explicit" cosmopolitanisms also means that works that assimilated non-native influences not on the highly visible stylistic or motivic level but on the epistemic and ontological level "to the point of invisibility" of their very assimilation end up ironically excluded from narratives of cross-cultural encounter.[40] As a result, while flocking to the fascinating objects of *chinoiserie* or *euroiserie* that fetishized Qing-Europe connections as transactions of tastes,

38 Jonathan Hay, "Foreword" to Petra Ten-Doesschate Chu, et al. ed., *Qing Encounters: Artistic Exchanges between China and the West* (Los Angeles: The Getty Research Institute, 2015), vii-xix.

39 Ibid., xiii.

40 Ibid., xv.

scholars largely ignore the deep, structural, and transformative effects of global integration that, only because they were not explicitly marked as such by relevant historical subjects, remain "hidden" to a global history that only admits conscious narrations of alterity into its archives.

The example Hay uses to stake his claim for "hidden cosmopolitanism" is particularly illuminating for music historians. Having originated during the Mongol rule of China, *wenrenhua* 文人畫 or literati paintings—that is, as opposed to those done by professional artists—were not particularly concerned with depicting the outside world. Instead, they performed a "lineage-defined relation to a Chinese past" as a cornerstone of the self-reflective Chinese literati identity.[41] This "inward-looking" genre, in Hay's description, became even more culturally fundamentalist during the Qing era, becoming a compensation for the Manchu conquest of China that trampled on Han Chinese ethnocentrism. Thus, it is even more remarkable that this nativist genre turned out to have been transformed by the deepening artistic and material integrations between the Qing Empire and Western Europe. This transformation also occurred not on the level of visual codes, which remained restrictive in literati painting, compared to the *euroiserie* of surface decorations, foundations, and mechanical clocks that embellished the Qing court. Rather, it took place on the level of the epistemology and ontology of the pictorial medium.

By analyzing literati paintings by scholar-officials such as Gao Qipei 高其佩 (1660-1734) and Li Shan 李鱓 (1686-1762), Hay shows that these works, whose themes and techniques are no different from previous literati paintings, nonetheless embodied a fundamental transformation in visual epistemic logic. In particular, a new fascination with light arose as "responses to an epistemological shock of European art" during the formative years of those

41 Ibid., xiv.

Chinese scholar-artist-officials at the Qing court, when Jesuit painters and their disciples dominated the Imperial Painting Workshop (*ruyi guan* 如意館).[42] Granted, in the same way that these literati painters eschewed European motifs, styles, techniques, and materials, they did not adopt the European conceptualization of light *in toto*, anchored as it was in the spatial logic of solid mass and mediated through preparatory *disegno* or drawing. Instead, they integrated their experiments with light into the characteristic "transmedial space" of literati painting that "accommodated equally calligraphy, painting, and text (poetry, prose)" in a single work; in brief, a literati painting typically consists of a "painting" *per se* and a few lines of text, often improvised, written down by the same artist in the same creative process. Nevertheless, whereas conventional literati paintings treated luminosity as a local and individual attribute of traces of the brushstroke as the unifying agent of the transmedial surface, Gao and others explored light as the pervasive agent in pictorial space that casts varying degrees of brightness onto material surfaces. This shift in the role of light (and, I may add, of the gaze), Hay argues, transformed the "pictorial ontology" of literati painting from the "logic of transmedial trace" into the "logic of transmedial image."[43] Previous literati paintings mediated and unified their calligraphic, pictorial, and textual elements through the transmedial actions and traces of the brushstroke on the two-dimensional surface. In contrast, by spatializing light and brightness beyond the local variances of brushstrokes, 18th-century literati painters reorganized the transmedial components of their paintings into a single coherent image of which the brushstroke became but an executor. Thus, by digging beyond the surface matters of styles, motifs, and tastes into the question of

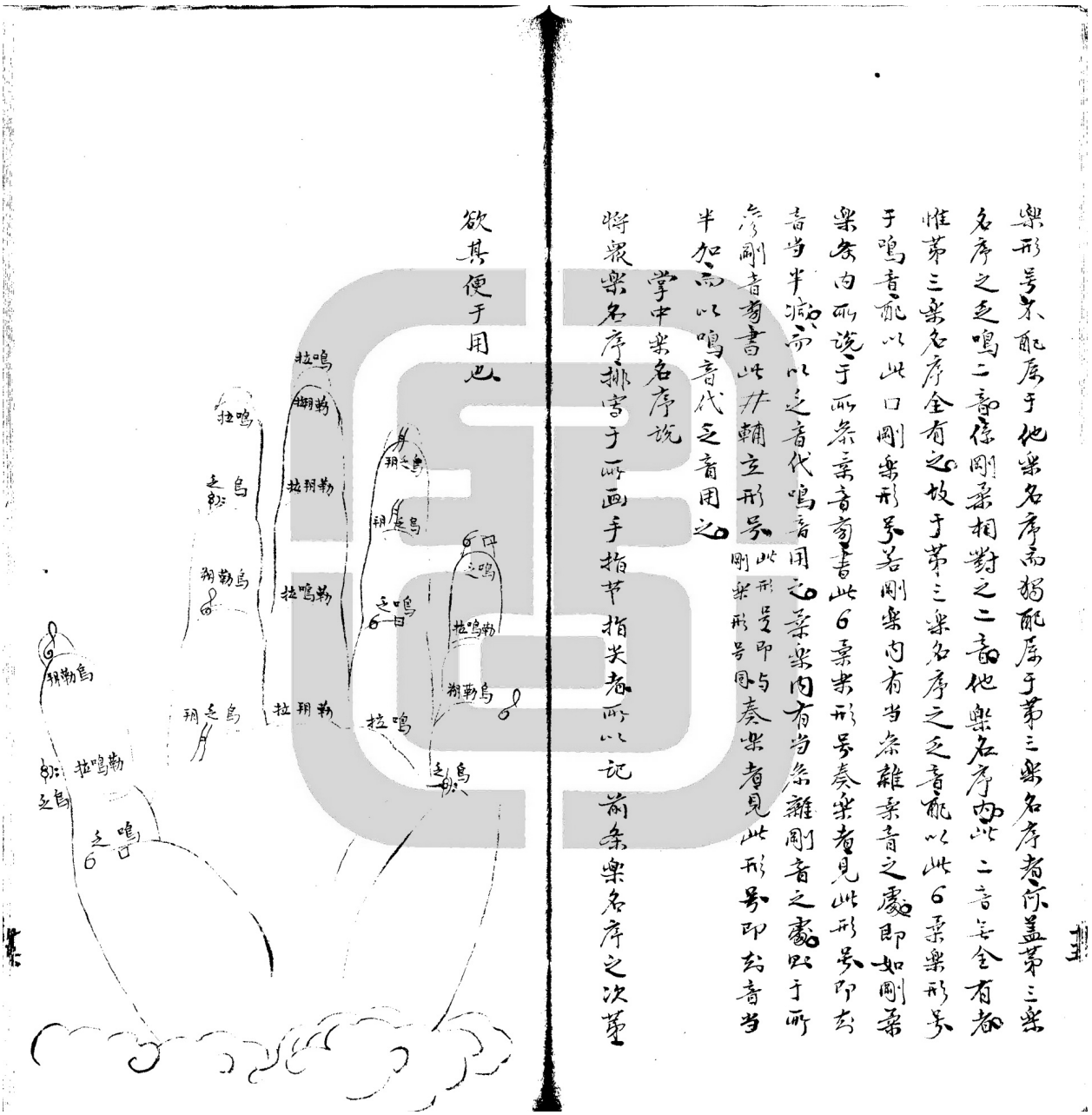
42 Ibid.
43 Ibid.

pictorial epistemes and ontologies, Hay uncovers how early modern global integration transformed the very phenomenology and epistemology of sight of literati painters.

I argue that the Kangxi Emperor's fourteen-tone temperament was a similar case of "hidden" cosmopolitanism. Just as Gao Qipei and others situated themselves in the literati painting genre whose *raison d'être* remained articulating the Chinese scholar-officials' "lineage-defined relation to [the] Chinese past," the Qing's tuning reform aligned itself with the Chinese discourse on *lülü* "tuning pitch pipes" which ultimately strove to resurrect the legendary and "orthodox" music of the ancient sage kings in the Confucian historical imagination. And just as literati painters made no attempt to construct or foreground any heterogeneity of cultures or tastes consequent of Western artists at the Qing court, *Orthodox Meaning of Pitch Pipes* draws no apparent connection between its core content of fourteen-tone temperament and its appended tract completely dedicated to Western *musica practica*, from solfège to staff notation.

Yet does analyzing such a case of hidden cosmopolitanism simply entail exposing what was once obscure, marking what was once unmarked, or bringing into the conscious knowledge of modern scholars what was once hidden in the unconscious of historical subjects? The answer, I argue, is a resolute no, because the productiveness of Hay's concept of hidden cosmopolitanism does not rest in providing a once missing label such as "Western" or "European" or in its forensic detection of influence. After all, the fact that Christian missionaries from Europe dominated the Qing's Imperial Painting Workshop in the 18th century and left important cross-cultural legacies was common knowledge among cultural historians before Hay suspected that such influences extended to the inward-looking literati paintings. Similarly, in scholarship on music in China during the 17th and early 18th centuries, there is arguably nothing more documented, studied, or celebrated than the Kangxi Emperor's fondness for the music and

Illustration II An image of the Guidonian hand in Chinese from* Elements of Pitch Pipes *(c. 1690)[44]



instruments introduced by European missionaries. From images of the Guidonian hand in Chinese and Manchu in the appended tract of *Orthodox Meaning* and its prototype *Elements of*

44 From Beijing, National Library of China (NLC) *Putong Guji* No. 15251 (c. 1707), ff. 15v-16r. The exquisite fingernails in this image were a common feature across the images of the Guidonian hand in Chinese and Manchu sources of the time.

Pitch Pipes—see Illustration II—to the many organs, bells, harpsichords, and violins whose birth at or journey to Beijing can be securely traced in historical sources,[45] from a joint performance at the court by the Kangxi Emperor on flute and Tomás Pereira on harpsichord[46] to the chastisement the former gave to his eunuchs for not learning well their *ut*, *re*, *mi* with Pedrini,[47] the Qing court's taste for music, music theory, and instruments brought or built onsite by European missionaries is as "explicit" a cosmopolitanism as it gets in current scholarship, which particularly emphasizes the roles of missionaries as agents of these exchanges. Nor am I the first to propose that the latter played a role in the development of the Kangxi Emperor's fourteen-tone temperament, whose presentation in *Orthodox Meaning of Pitch Pipes* makes no attempt to parade any cultural hybridity. Recently, in 2012, Weng Panning 翁攀峰 suggested a textual link between an important chapter in *Orthodox Meaning* and an earlier Jesuit-authored treatise, *Proportions of Pitch Pipes* (lülü jieyao 律呂節要, lioi lioi jiyei yoo bithe ᡶᡳᠣᡳ ᡶᡳᠣᡳ ᡤᡳᠶᡝᡳ ᠪᡝ ᠪᡳᡨᡥᡝ in Manchu), all of whose extant exemplars in Chinese and Manchu are paired with those of *Elements of Pitch Pipes*, the prototype text for the fifth and last volume of *Orthodox Meaning* on Western *musica practica*.[48] In fact, Weng goes as far as pronouncing at the end of his article that the fourteen-tone temperament could not have developed at all from within the native framework of traditional Chinese music theory. In the same year, Qi Mingjing 漆明镜 also showed that the

45 See, for example, Chen Wannai, Qingshi gao Yuezhi *yanjiu*, 372-387.

46 Rome, Archivum Romanum Societatis Jesu, *Jap. Sin.* 117 *Sinarum Litterae annuae 1610-1770*, "Annua. V. Provinciae Sinensis Anni 1678 & 1679," ff. 161-182; f. 167r.

47 See Fang Hao 方豪 , *Zhongxi jiaotong shi* 中西交通史 ("History of the Exchanges between China and the West") (Taipei: Zhongguo wenhua daxue, 1953), 901-902.

48 Weng Panfeng 翁攀峰, "關於'康熙十四律'思想來源的初步探討" ("A Discussion on the Origin of "The Fourteen Temperament [*sic*] of Kangxi"), *Wenhua yishu yanjiu* 文化藝術研究 6 (2013): 32-41.

seemingly innocuous gloss of a key term of musical tuning in *Orthodox Meaning* nonetheless suggests influences of the Kangxi Emperor learning *musica practica*.[49]

As Hay's analysis of the use of light in 18th-century literati paintings shows, however, "hidden cosmopolitanism" does not so much allow scholars to expose cross-cultural influences between "East and West" as to liberate them from such very frameworks of essentialist identities and alterities. Indeed, I argue, attempts to tease out native vs. foreign elements or to trace a stemma of transmissions across distinct cultures only presume and thus reinforce the inherent autarky and immutable essence of each initiating/receiving end of an "encounter" or "transfer." In defiance of this cultural essentialism, Hay's foreword shows literati paintings to be neither as inward-looking (as the tradition styled itself to be) nor as merely marking, incorporating, or exoticizing foreign elements for the *jouissance* of the Other, but instead to be relational. Rather than defining literati paintings as either "Eastern" or "Western" or as a combination of the two, Hay's analysis of the shifting use of light as part of the pictorial ontology of the transmedial genre turns the logic of such *ad hoc* labels upside down. He does so, I argue, by unfolding from inside-out the radically transformative global integrations that 18th-century consumers of foreign goods were trying to fix by constructing marked categories of civilizations in their conscious play with heterogeneous styles and tastes. It is precisely by cutting through and opening up this play that Hay reaches the once "hidden" structures of perception and knowledge in literati paintings—hidden, furthermore, not because of any innate structures of surface vs. depth but because of the false binary scholars have constructed between the inward-looking nativism of this genre and the explicit cosmopolitanism of *euroiserie*. What Hay achieves, then, is not simply

49 Qi Mingjing 漆明镜, "試從《御製律呂正義》議'康熙十四律'" ("Discussions of 'Kangxi Fourteen Temperament' from Perspectives of *Imperial Formulae for Musical Temperament and Tuning*"), *Wenhua yishu yanjiu* 文化藝術研究 5 (2012): 117-122.

adding a new entry to the list of "global things" that cultural historians who share the same taste for the hybrid and the exotic as did 18th-century elite consumers might gladly include in their archives. Instead, by positing that early modern globalization took place not only on the semiotic level of marked Othering but also on the phenomenological and epistemological level of experiencing and knowing, Hay eviscerates the presumed essentialist and self-sufficient cores of "East" and "West" or "Europe" and "China," categories that were themselves constructed as a result of globalization. As the very structures of perception are dissolved and integrated into the global processes of mutual, coeval transformations, any experience or knowledge, no matter how seemingly self-sufficient or inward-oriented, can be examined in a field of radical relationalities in which any demarcation of difference or sameness is only meaningful in relation with other demarcations parallel, perpendicular, or oblique to it.

It is precisely because of this epistemological relationality at the heart of Hay's concept of "hidden cosmopolitanism" that I conjoin to it Steven Feld's concept of "acoustemologies."[50] First developing the concept from his ethnographic work in the Bosavi rainforests of Papua New Guinea, Feld joins "acoustics" with "epistemology" to "inquire into what is knowable, and how it becomes knowable, through sounding and listening."[51] While its focus on sounds and listening as materially, socially, and ecologically situated ways of knowing may remind one of the notion of soundscapes, Feld distinguishes acoustemology from the latter. In his understanding, though the concept of soundscapes arose from R. Murray Schafer's pioneering endeavor in acoustic ecologies, its appropriation of landscape only objectifies and thus renders passive both the

50 While Feld's writings on "acoustemologies" are numerous, I have mostly relied on "Acoustemology," in David Novak and Matt Sakakeeny eds., *Keywords in Sound* (Durham, NC: Duke University Press, 2005), 12-21.

51 Ibid., 12.

ecological dimensions of sounds and their social aspects, analogized to landscape.[52] This etymological fuss is crucial for Feld's concept of acoustemology, which ultimately strives to examine "what are more typically theorized as subject-object relations" as "deeply known, experienced, imagined, enacted, and embodied […] subject-subject relations."[53] When studying Bosavi songs in relation to the rainforests, for example, Feld rejects the framework of "adaptation," which not only opposes songs to their environments but also privileges singing over and above other modes of knowing such as ornithology, mapping, and natural history.[54] By integrating "sounding-as and sounding-through-knowing" with "numerous sources of action […] that are variously human, nonhuman, living, nonliving, organic, or technological," Feld argues, studies of acoustemologies can relationalize and mobilize notions such as songs, voice, music, and environment that have too long been naturalized—only at the very expense, perhaps, of "nature" objectified and marginalized.[55]

Compared to Feld's trans- or posthumanist vision of acoustemologies, my analysis of the knowledge-producing processes of the Kangxi Emperor's fourteen-tone temperament is much more limited and conservative, at least within the framework of the first few chapters on the tuning reform itself. For one, ecological history is largely missing from my narrative. And sociologically, most of the human, textual, and material protagonists in this study circulated in a small circle of literate males that, while perhaps too diverse to be simply generalized as "elites," certainly pales in comparison to the radical subject-subject relations that Feld's concept of acoustemologies affords. Still, by joining acoustemologies with hidden cosmopolitanism, I

52 Ibid., 15.
53 Ibid., 19.
54 Ibid., 15-16.
55 Ibid., 19.

harness both concepts' emphases on structures of sounding and listening as a way of circumventing what Feld calls "symbolic and semiotic representations of modes of knowing,"[56] which, I suggest, necessarily segregate subject/self from object/Others based on ethnocentric as well as anthropocentric notions of agency. Indeed, just as the concepts of soundscapes and ecological adaptations in Feld's critique only presuppose a dichotomy between human subjects and their sociological and ecological environments, studies focusing on what Hay describes as explicit cosmopolitanisms, such as encounters with or consumptions of an Other(ed), only reinscribe essentialist notions of cultures to which any hybridization with Others is ultimately superficial. And just as Feld's acoustemologies use materially, socially, and ecologically situated processes of sounding- and listening-as-knowing in order to turn demarcations of subjects and objects into a radically expansive field of "others-in-relation," Hay's hidden cosmopolitanism analyzes the transmedial pictorial ontology of literati paintings in order to excavate the transformative effects of early modern global integration from beneath the conscious plays with heterogeneous identities as the visible symptoms of such integration.

What "hidden cosmopolitan acoustemologies" serve, then, is to examine the very processes by which historical subjects listened and sounded in relation to each other in a radical, subject-subject coevalness as embedded in their mutual transformations on a global scale. While I begin with the overlooked Western influences on the Kangxi Emperor's fourteen-tone temperament, my overall aim in this project is to use the Qing's tuning reform to illuminate those global relationalities of listening- and sounding-as-knowing without being confined by *ad hoc* demarcations of sameness or difference. By thus analyzing the varying engagements with musics and sounds under the Qing, I also aim to shed light on the very materials of those demarcations

56 Ibid., 17.

of sameness or differences in the context of the Qing's continuous building of its multiethnic empire.

Summary of Chapters

In this dissertation, I argue that the Kangxi Emperor's fourteen-tone temperament was a product of the hidden cosmopolitan acoustemologies that integrated the Qing's tuning reform in various transregional dynamics of empires, colonialism, migrations, and commerce. These acoustemologies, furthermore, put the tuning reform in relation with the coeval transformations of various "subjects" in Feld's posthuman sense, ranging from bodies, instruments, flora, and even the universe. And while what I have framed as the passage from *ut*, *re*, *mi* to fourteen-tone temperament strategically makes the European contribution to the tuning reform the most eye-catching part of these acoustemologies, fundamentally, I argue that the fourteen-tone temperament was at one and the same time a symptom of and a reaction to a much broader shift in the epistemology of sounds that unfolded in tandem with the Qing's empire-building.

During the 17th and 18th century, varying effects of the Manchu conquest of China not only reconfigured sounding and listening as ways of knowing but also transformed the broader epistemic paradigm of Confucian scholarship, which vied to construct an ideal society modelled after the legendary ancient sage kings and harmonized with the cosmos. Beginning with what I call the epistemic logic—"what is knowable and how it becomes known" in Feld's words—of the fourteen-tone temperament, I explore from inside out the hidden cosmopolitan acoustemologies that not only conditioned 17th- and 18th-century debates over how sounds and listening could produce meaningful knowledge about music and tuning but also permeated the

political and ideological transformations of bodies, instruments, subjectivities, and cosmologies, all as others-in-relation.

What follows is a narrative summary chapter by chapter. Chapter One, "From Circle of Fifths to Octave Equivalence: The Equivocation of an Imperial Mistake," begins the narrative from the very middle of the story. It examines a court gathering at the Forbidden City on February 20, 1692, which the Kangxi Emperor originally intended for dressing down his closest Han Chinese advisors for their purported ignorance in mathematics, astronomy, and musical tuning.[57] While official chronicles and private recollections of the time seized this opportunity to extol the Emperor's knowledge, particularly in musical tuning, they failed or managed not to notice that he actually committed an egregious mistake in glossing a rather basic term in Chinese musical tuning: *geba xiangsheng* 隔八相生 "mutual generation at every eighth step." While the term effectively means "circle of fifths" if translated into Western musical terminology, the Emperor mistook it for octave equivalence when demonstrating the concept on a zither and a flute.

This seemingly innocuous blunder was critical, I argue, precisely because the Emperor's gloss of the term, though technically wrong, makes perfect sense on its own. In this chapter, I analyze the Emperor's gloss both through extant sources describing the court gathering and through Chinese treatises on musical tuning that the Emperor himself commended at the gathering. I show that the Emperor's unwitting mistake uprooted the term from its established context within the twelve tuning pitch pipes—the "instrument of music theory" of Chinese

57 Sources of the court gathering include *Kangxi chao shilu* 康熙朝實錄 ("Veritable Records of the Kangxi Era") and two eyewitness accounts by Li Guangdi 李光地 (1642-1718) and two by Zhang Yushu 張玉書 (1642-1711).

musical tuning, to borrow Alexander Rehding's term[58]—and the textually transmitted mathematical data used to determine their absolute and relative lengths as originally set out by the ancients. What the reinterpretation did was to insert the term into a hitherto irrelevant context of pitch organization, system of scales and modes, and process of maneuvering and listening to sound-producing instruments in order to examine their audible patterns.

This discursive distinction between the established definition of the term meaning circle of fifths and the Kangxi Emperor's reinterpretation of it to mean octave equivalence would become an epistemic distinction leading up to *Orthodox Meaning of Pitch Pipes*. The question was how knowledge about *lülü* "tuning pitch pipes" or musical tuning, which strove both to rectify current musical practices and to resurrect ancient musical institutions, should be produced. Specifically, should scholars of *lülü* make it their priority to interpret historical sources so as to determine the physical dimensions of the tuning pitch pipes, which in turn determine the sonic patterns between their pitches? Or should they first listen to actual sounds and tones so as to determine the "orthodox" sonic patterns and then set the physical dimensions of tuning pitch pipes accordingly?

Granted, there is no direct attestation as to how the discursive distinction between the two definitions of "mutual generation at every eighth step" took on this epistemic significance between the 1692 court gathering and the 1714 *Orthodox Meaning*. In Chapter Two, "Taking an Epistemic Cue from the Emperor: Towards the Fourteen-Tone Octave," however, I analyze a treatise that Mao Qiling 毛奇齡 (1629-1716) wrote in reaction to the court gathering as a

58 Alexander Rehding, "Instruments of Music Theory," *Music Theory Online* 22 (2016). http://mtosmt.org/issues/mto.16.22.4/mto.16.22.4.rehding.html, accessed October 22, 2018.

precious piece of corroborative evidence.[59] An important Han Chinese contributor to scholarly activities at the Qing court, Mao had already retired from imperial service by the time he wrote the treatise, and he had only read about the Kangxi Emperor's music lectures at the court gathering from an issue of the imperial gazette. Yet Mao's commentary on the 1692 gathering (never before examined in detail) arguably reflects a "sonic turn" in 17th-century studies of musical tuning. Scholars like Mao and, in fact, Zhu Zaiyu, who wrote a century earlier, advocated for the epistemic reincorporation of sounds back into the study of musical tuning.[60] Whereas tuning under the ancient sages was all about patterns of sounds and their resonances with the cosmos under the ancient sages, they lamented, it had long deteriorated from this sonorous origin into mere studies of numbers and of historical sources transmitting numbers among modern scholars.

Though it might seem counterintuitive that sounds could play no role in studying musical tuning, my rather extensive and step-by-step explication of Mao and Zhu's critique of the then dominant paradigm of musical research reveals a rather absolute epistemic distinction between *sheng* 聲 "sound" and *shu* 數 "numbers." I further argue that this distinction took place in studies of *lülü* in relation to (though not in parallel with) the shift from a philosophical and speculative strand of Confucian scholarship, which became the state ideology in the 14th century, towards a more pragmatist and philological strand that began to prosper at the dawn of the Manchu conquest of China.[61] In Mao's analysis, in the Emperor's reinterpretation, the term in question

59 Mao Qiling 毛奇齡, *Shengyu yueben jieshuo* 聖諭樂本解說 ("Commentaries on the Emperor's Sage Explanation of the Essence of Music," 1692).

60 Zhu Zaiyu 朱載堉, *A New Theory of Pitch Pipes*; and *Essential Meaning of Pitch Pipes* (1596).

61 See Benjamin A. Elman, "Early Modern or Late Imperial Philology? The Crisis of Classical Learning in Eighteenth Century China," *Frontiers of History in China* 6 (2011): 3-25.

went from functioning within a research paradigm of musical tuning that relies only on extracting mathematical data from historical sources to functioning within a paradigm that relies on observing patterns and relations of actual sounds. Even though this unwitting reinterpretation is hardly defensible with respect to the established meaning of the term, for Mao, it nonetheless pointed towards the correct epistemic direction: salvaging the twelve tuning pitch pipes from their abstraction into mathematical shapes and proportions and turning them once more into sonorous, sound-producing instruments.

While it remains unclear whether compilers of *Orthodox Meaning of Pitch Pipes* ever read Mao's commentary on the 1692 gathering, Mao's treatise nevertheless predicted almost too well how the Kangxi Emperor's unwitting mis- or reinterpretation of the term ended up shaping fourteen-tone temperament. In Particular, Mao's epistemic contrast between textually transmitted mathematical data to empirically observed sonic patterns mirrored how scholars working for the Qing's tuning reform incorporated sounding and listening epistemically into the research process. In Chapter Three, "Rectifying Pitch Pipes in order to Examine Their Tones: The Imperial and Empirical Underpinnings of a Tuning Reform," I analyze the linchpin chapter in *Orthodox Meaning*, which first raises the head-splitting claim that a set of tuning pitch pipes whose lengths are proportioned in the Pythagorean manner nevertheless yield the sound of an octave divided into fourteen parts or two complementary diatonic scales.[62] As baseless as current scholars believe such a claim to be, I argue that researchers for the Qing's tuning reform took the Emperor's reinterpretation of the Chinese term as an epistemological cue that they should

62 In-c'i ᡳᠨ ᠴᡳ et al., *Lülü Zhengyi* 律呂正義 ("Orthodox Meaning of Pitch Pipes"), Vol. 1, Chapter 9.

experiment in making, sounding, and listening to bamboo pitch pipes in order to acquire meaningful knowledge regarding the patterns of their sounds.

And experiment they did. While every previous study on the fourteen-tone temperament has assumed that it lacked any empirical grounding, those studies did not take into account the manuscript *Putong Guji* No. 15251 I have discovered at the National Library of China (NLC).[63] Originating from the circle of Prince In-c'i ᠨᡳᠩᠵᡳ (Yinzhi 胤祉 in Chinese, changed in 1723 into Yūn-c'i ᠶᡡᠨᠵᡳ, or Yunzhi 允祉 in Chinese; 1677-1732), the third son of the Kangxi Emperor responsible for the entire *Origins of Cosmological Sciences* project, this manuscript contains preparatory digests and drafts for *Orthodox Meaning*, and indeed notes of the very experiment that led to the infamous observation of a fourteen-tone octave arising from Pythagorean-proportioned pitch pipes. I have also uncovered imperial documents that detail how materials used for their pipe-making and listening experiments, specifically those 7,100 bamboo pipes, were furnished through the Qing's manufacturing and spying outposts in the Yangtze River Delta, the heartland of Han Chinese culture.[64] By analyzing these sources, I reconstruct the material and social processes whereby the core research for the Qing's tuning reform took place amid the political turmoil of the 1700s. I argue that the very knowledge-producing acts of making, sounding, and listening to pitch pipes of varying shapes and sizes, which took their cue from the Emperor's reinterpretation of the term and led to the fourteen-tone temperament, unfolded within what Qing historian Michael Chang terms the "ethno-dynastic" network of

63 National Library of China, *Putong Guji* 15251.

64 Two memorials to the Kangxi Emperor from superintendents of the imperial workshops in Suzhou and Nanjing, one dated to November 5, 1713 and another dated to February 8, 1714, detail the entire process of the commission, harvesting, and shipment of these bamboos for making pitch pipes. See Items No. 189 and 197 in *Suzhou zhizao lixu zouzhe* 蘇州織造李煦奏摺 ("Memorials of Li Xu, Superintendent of the Imperial Suzhou Workshops") (Taipei: Wenhai Chubanshe, 1976), 146, 151-152.

Manchu conquest elites that superseded the traditional Han Chinese bureaucracy staffed by scholar-officials and landed gentry.[65] While it is safe to assume that most of the still anonymous bibliographers, scribes, illustrators, artisans, and musicians who worked under Prince In-c'i were Han Chinese, their labor was integrated into a system of knowledge production and patronage that resulted from and took part in the Qing's continuous efforts at empire-building decades after they conquered China.

Having shown how the Kangxi Emperor's unwitting reinterpretation of a Chinese term from meaning circle of fifths to meaning octave equivalence heralded experiments in listening to the sounds of pitch pipes that in turn led to the fourteen-tone temperament, I turn to the provenance of the Emperor's reinterpretation itself in Chapter Four, "Singing *ut*, *re*, *mi*, Singing the Empire: A Princely Vision for Qing Imperial Music Theory." Of course, I have deliberately paraded that provenance in the title of this Introduction: namely, the Jesuits, Europeans, and other Westerners who led extraordinary journeys to Qing China and committed extraordinary deeds of introducing Western music theory that ended up even influencing an apparently "native" Chinese fourteen-tone temperament in a way so extraordinary that they have thus far been hidden from the view of previous scholars. Eye-catching as all of this may be, I nonetheless argue that simply labelling some historical subjects as the "source" of all that which led to the Qing's dramatic and systematic reform of musical tuning is reductionist. To reiterate: the goal of studying "hidden cosmopolitan acoustemologies" is not to unveil previously unmarked or unverbalized foreign influences, as if in a counterintelligence operation, but to reach beyond the semiotic level of "encountering Others" and analyze how the very phenomenological and

65 Chang, *A Court on Horseback*, Chapter 2 "'Following Ancestors': The Ethno-Dynastic Imperative of Imperial Touring, 1680s & 1740s," 72-113.

epistemological structures of sounding- and listening-as-knowing were integrated into relational transformations on a global scale.

In this chapter, then, I gather and analyze a broad variety of historical sources coming from or documenting the markedly "European" or "Western" musicians, instruments, and practices in the orbits of the Qing court and its imperially expansive network of knowledge-production. By following the textual traces of the pedagogical-performative activities that involved the Jesuits, the Kangxi Emperor, his sons, and onlookers from the 1670s to the 1710s, I show that the Emperor's unwitting reinterpretation of the term resulted from these embodied interactions with sounds and sound-producing instruments. These activities, furthermore, were not merely forms of entertainment, princely education, or political-symbolic messaging, but actually participated in the collective envisionings and enactments of the Qing's empire-building, specifically in the form of Chinese opera. By analyzing court and missionary archives I show singing and singing-focused musical pedagogy to be primary concerns of the worlds from which these sources were written down. Through prefaces and in-text annotations, *Orthodox Meaning* also integrates its *musica practica* content into the broader historical imaginations and enactments of the expanding multiethnic Qing Empire through opera. Whereas scholarship on *lülü* "tuning pitch pipes" or musical tuning and aficionado studies on opera had been largely separate, scholars working under Prince In-c'i laid the research groundwork not just for the Qing's reform of musical tuning but also for a much more extensive project: refining Chinese operas and harnessing their power in service of the expanding Qing Empire.[66] Of course, it would not be until the mid 1750s that this ultimate vision laid on in NLC *Putong Guji* No. 15251

66 National Library of China, *Putong Guji* 15251, ff. 62-87.

would be complete.[67] Still, it shows that the Jesuits' *ut*, *re*, *mi* could have influenced the Kangxi Emperor's understanding of a squarely "native" term of Chinese musical tuning only because both the "initiator" and "receiver" of this influence was embedded in broader embodied activities of sounding and listening, activities that were furthermore integrated in the material and ideological building of the Qing Empire.

Though researchers for *Orthodox Meaning* derived their "fourteen tones in an octave" observation from actually listening to pitch pipes fashioned in the Chinese Pythagorean manner, this does not in itself mean that they had to fundamentally revise their systems of pitch organization, scales, and modes. Chapter Five, "An Easier Path Not Taken: End Historicism vs. Empiricism, Records vs. Sounds," supports this point. Here, I turn to the works of Zhu Zaiyu, whom both the Kangxi Emperor and the Qing researchers often drew upon (despite the animosity many current scholars tend to imagine between them),[68] as well as to Marin Mersenne, who served as a primary source for Kircher, whose *Musurgia* in turn the Kangxi Emperor had a documented record of reading.[69] Celebrated pioneers of music-theoretical empiricism who also documented their experiments, both Zhu and Mersenne obtained observations of "there being more than twelve tones in an octave in sounds" produced by pitch pipes tuned according to Pythagorean method and/or twelve-tone equal temperament similar to (if not even more

67 Namely, Wang Yiqing 王奕清 et al., *Yuding Qupu* 御定曲譜 ("The Imperially endorsed compendia of opera tunes," 1715); Wang, Yiqing et al., *Yuding Cipu* 御定詞譜 ("The Imperially endorsed compendia of lyrical tunes," 1715); In-lu ᡳᠨ ᠯᡠ et al., *Jiugong Dacheng Nanbei Ci Gongpu* 新定九宮大成南北詞宮譜 ("Grand Compendium of Northern and Southern Opera Tunes in All Modes," 1746); and Yung-iong ᠶᡠᠩ ᡳᠣᠩ et al., *Qinding Shijing Yuepu Quanshu* 欽定詩經樂譜全書 ("The imperially endorsed complete music for the Confucian *Canon of Songs*," 1788).

68 Mostly Zhu Zaiyu, *Essential Meaning of Pitch Pipes* (1596).

69 See footnote 35 above.

egregious than) the ones claimed in *Orthodox Meaning* and its drafts.[70] Current acoustics recognizes the phenomenon as the "end correction" problem, in which the length of the vibrating air column of an even-bodied pipe is necessarily longer than the length of the body of its pipe itself, therefore making pipe lengths not exactly equivalent to string lengths, so to speak, in the application of mathematically derived tuning ratios. Different from researchers under Prince In-c'i, both Zhu and Mersenne perceived the so-called end correction phenomenon as a problem that needed a solution. And their solutions were identical: instead of using, in Zhu's terms, "pipes of the same diameter" (*tongjing guan* 同徑管), where the base circumference and thickness of pipes remain constant however longer or shorter their lengths become, both opted for "pipes of the same shape" (*tongxing guan* 同形管), where the base circumferences and thicknesses of pitch pipes change proportionally, as do their lengths.[71] Though no precise mathematical formulation for solving the end correction problem exists even today, the most common approximation in current acoustics I can find is to add to the physical length of a pipe an "end correction value" equal to its diameter multiplied by a certain constant. Following on this approximation, Zhu and Mersenne's solution of using "pipes of the same shape" instead of "pipes of the same diameter" would indeed mitigate the need for making any adjustments in terms of the relative proportions between pipes.

What is immensely puzzling, however, is that the researchers for the Qing's tuning reform knew fully well Zhu and Mersenne's solution but still rejected it. The digests of previous

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70 Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, Vol. 1, f. 31r; Mersenne, *Harmonie universelle* 2nde partie, "Livre sixième de l'orgue," Proposition XIII "Determiner en quelle raison doiuent ester les longueurs des tuyaux de mesme grosseur pour faire les sons & les intervalles requis: & si l'on peut faire un Orgue, dont tous les tuyaux soient de mesme grosseur," 333-334.

71 Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, Vol. 1, ff. 31-79. Mersenne, *Harmonie universelle*, Mersenne, *Harmonie universelle* 2nde partie, "Livre sixième de l'orgue," 318-321.

treatises on musical tuning gathered in the preparatory manuscript *Putong Guji No. 15251* actually see Prince In-c'i (annotating in vermilion pigment) and his researchers praising Zhu precisely for using "pipes of the same shape" instead of "pipes of the same diameter."[72] Texts in the more "theoretical" or "speculative" strand of music theory produced and consumed with indisputable Jesuit contribution at the Qing court—including Kircher's *Musurgia*,[73] the aforementioned *Proportions of Pitch Pipes* that partially adapted Kircher's Book I and IX, and a textbook that Antoine Thomas (1644-1709) once wrote for teaching at the University of Coimbra and had later used to derive most of his lectures in mathematics for the Kangxi Emperor[74]—uniformly concurred with Mersenne's solution of making the diameters of organ pipes proportional to their lengths. In stark contrast to these concurring "native" and "foreign" sources, Prince In-c'i and his researchers unapologetically used "pipes of the same diameter" to conduct their experiments as recorded in the experiment notes of NLC *Putong Guji* No. 15251. And in turn, they used the results of these experiments to construct the fourteen-tone temperament that would later become the Qing's tuning reform in *Orthodox Meaning of Pitch Pipes*.[75]

Thus, apparently, the Qing researchers under In-c'i refused to recognize the "fourteen tones in an octave" or so-called end correction phenomenon as a problem needing any solution. They even appear to readily accept as given whatever patterns of sounds that will arise from pitch pipes whose lengths are determined through the conventional Chinese Pythagorean tuning method and whose diameter and thickness are constant. It was as though they took it as their duty

72 National Library of China *Putong Guji* No. 15251, ff. 53-54.

73 Athanasius Kircher, *Musurgia universalis* (1650), Tome I, Vol. 6 "Artis Magnae Consoni et Dissoni Liber Sextus Musica Organica, sive De Musica Instrumentali," 507-511.

74 Antoine Thomas, *Synopsis Mathematica* (Coimbra: Michael Mairesse, 1685), 1st Part, Tractatus VIII "De Musica," Articulus Quartus "De constructione fistularum ad symphonias," 457-459.

75 National Library of China, *Putong Guji* No. 15251, ff. 88-90.

to derive an entire system of pitch organization, scales, modes, transpositions, and mutations based on it, regardless of how drastic a change to musical practice it would be. As if mocking Zhu and Mersenne's solution of using "pipes of the same shape," *Orthodox Meaning of Pitch Pipes* did end up producing a set of such pipes that do not require any end correction adjustment. Nonetheless, the set of pipes of the same shape they produced featured not thirteen pipes intended to render a twelve-tone octave as they do in Zhu and Mersenne's works, but rather fifteen pipes intended to render a fourteen-tone octave, thus further inscribing the tuning reform on a much larger scale.[76]

Why, then, were the researchers for the Qing's tuning reform and compilers of *Orthodox Meaning* so adamant about following *both* "pipes of the same diameter" combined with the conventional Chinese Pythagorean tuning ratios, on the one hand, *and* the sonic pattern of "fourteen tones in an octave" they heard from these pipes, on the other hand, even to the point of unsettling the entire system of pitch organization, scales, and modes of musical practice, which had not seen such a drastic shift since at least the 6th century? It is with this question, fully articulated in Chapter Five, that I conclude this dissertation in Chapter Six, "*Shenyin* 審音 'Examining the Tones': A Phonological Revolution?" Indeed, when Zhu Zaiyu and Mersenne listened to pitch pipes or organ pipes, they used the results to amend the mathematics of their tuning systems: namely, to adjust the diameters and thicknesses of pipes proportional to their lengths, as opposed to holding them constant. This feedback loop between the numbers determining physical dimensions of pitch pipes and the sounds observed from listening to those pipes does not at all exist in *Orthodox Meaning*, however. Rather, reminiscent of the epistemic distinction that Mao Qiling identified between the conventional meaning of that Chinese term as

76 Ibid., ff. 91-95. In-c'i, *Orthodox Meaning of Pitch Pipes*, Vol. 1, Chapter 12.

circle of fifths and the Kangxi Emperor's reinterpretation of it as octave equivalence, *Orthodox Meaning* segregates its research program and textual surface into two parts, each with its distinct epistemic logic of musical tuning. The first part, which the treatise labels as *dinglü* 定律 "rectifying the pitch pipes," is a text-based process of parsing layers of historical sources in order to determine the most orthodox absolute and relative sizes of pitch pipes in compliance with the ancient institutions.[77] The second part, *shenyin* 審音 "examining the tones," is a listening process of determining the actual sonic patterns of pitch pipes that have already been fixed in the previous process.[78]

This epistemic segregation between numbers and sounds means that no sonic observations can be admitted as proper evidence for fussing with the exact physical dimensions of the tuning pitch pipes, including how many types of pitch pipes (or "pitch classes," so to speak) there should be (twelve in the earliest sources, as opposed to later scholars who expanded them varyingly to sixty, three hundred and sixty, or sixteen); how their lengths should relate to each other ("triple division with one part subtracted or added" since the earliest sources); and indeed how their diameters and thicknesses should relate to each other (the Yellow Emperor's legend stipulates that he invented the twelve tuning pitch pipes by cutting a bamboo stem, not using stems of different widths). The epistemic segregation also means, however, that no textual or historicist records can be admitted as proper evidence for modelling the patterns of sounds produced by those pitch pipes, or how these sounds should be organized into scales and modes. Thus, whereas Mao Qiling and Zhu Zaiyu in my previous analysis in Chapter Two effectively argued for an empiricist paradigm of research in musical tuning, the knowledge-production

77 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Vol. 1, Chapters 1-8.
78 Ibid., Vol. 1, Chapters 9-12.

process of the fourteen-tone temperament is neither thoroughly historicist nor thoroughly empiricist. Rather, the Qing treatise strove hard to delineate the exact "spheres of influence" of the two distinct epistemic logics so that each can be carried to its fullest extent on its own turf. As a result, having first determined the exact lengths, shapes, and proportions of pitch pipes through historical sources without any consideration for how they would end up sounding, researchers for the Qing treatise readily accepted it when the pipes thus produced yielded a fourteen-tone octave, taking this observation as the foundation for their tuning reform.

But why did *Orthodox Meaning of Pitch Pipes* opt to simultaneously segregate and conjoin a text-based historicist process of "rectifying the pitch pipes" on the one hand and a sound-based empiricist process of "examining the tones" on the other hand? To answer this question, I insert the Qing's tuning reform into yet another scholarly discourse of the time that shared many of the same dilemmas, struggles, and anxieties with sounding- and listening-and-knowing as part of a larger goal of *fugu* 復古 "restoring the ancient": historical phonology. Both studies of *lülü* "tuning pitch pipes" or musical tuning and *guyin* 古音 "ancient sounds" or ancient Chinese phonology took part in the broader *kaozheng* 考證 or evidential learning movement that blossomed in the 17th and 18th centuries.[79] Granted, in terms of its objective of "restoring the ancient," a narrative critical in the Confucian historical imagination, the evidential learning movement was by no means innovative. It stood out, however, with respect to its emphasis on philology, whose Chinese name at the time, *xiaoxue* 小學 "little learning," betrays the disregard

79 For the evidential learning movement, see Benjamin A. Elman, *From Philosophy to Philology: Intellectual and Social Aspects of Change in Late Imperial China* (Cambridge, MA and London: Council on East Asian Studies, Harvard University, 1984), and Elman, "Early Modern or Late Imperial Philology? The Crisis of Classical Learning in Eighteenth Century China," *Frontiers of History in China* 6 (2011): 3-25.

Confucian scholars had conventionally accorded to it. After the conquest of China by the Manchus, whom Han Chinese literati under the Ming (1368-1644) vehemently disparaged as barbarians, however, many Han Chinese scholars blamed the humiliation of conquest on previous generations of Confucian scholars, particularly on their penchant for ethical and speculative interpretations of ancient classics. These philosophical musings, they argued, only obscured the original intentions of the ancients. Instead, the only way to recover what the ancients meant from the texts they left behind and thus resurrect their thoughts and practices was to diligently examine their use of language—hence the particular importance of philology for the evidential learning movement. Thus, in contrast to Confucian scholars of the Song and Ming era—as well as their adherents in the Qing-era—who strove to fathom the *yili* 義理 "essence and principle" of the Confucian canons, scholars of *kaozheng* or evidential learning strove to *kao* "investigate" and *zheng* "verify with evidence" what each individual signifier and name refers to in a text while paying limited attention to fathoming its overall moral teaching. Although the latter remained an ultimate goal, evidential scholars considered it getting way ahead of themselves to claim to understand what ancient texts mean without exhausting the contextual and intertextual meanings of each and every word.

A particularly apposite example of this new approach to ancient texts was the Confucian *Canon of Songs* (shijing 詩經, varyingly translated as *Book of Songs*, *Book of Poetry*, *Book of Odes*, etc.). A collection of folk tunes (without the "Das Volk" aspect of this term being overly marked) from different realms of the Former Zhou (c. 1046-771 BCE) dynasty, the so-called *feng* 風 "regional songs" section of *Canon of Songs* was revered both by Confucius and by later Confucian scholars as the epitome of the perfect governance and mores of the ancients. For centuries, scholars have debated how best to understand the many songs in this collection, whose

lyrics can appear outright licentious (at least according to the proper patriarchal morals of conventional interpretations of Confucianism), and the many explanations ranging from political allegories, satires, and counterexamples of what would happen when the ruler fails to properly edify his people to authentic expressions of human passions. The puzzling ethics and aesthetics of this Confucian song collection became even more significant during the 16th and 17th centuries, when a "folksong mania" struck Han Chinese scholars who regarded peasant tunes, mountain songs, urban ditties, and, by extension, operas as the authentic descendants of the perfect music of the ancients, rather than all the text-based studies that have tried in vain to resurrect ancient practices from fragments of their sources.

Evidential scholars of the Qing era, however, turned the interpretation of *Canon of Songs* completely upside down. Rather than explicating the overall meanings of these songs, they used them as the very evidence to reconstruct the historical language of the ancients. In particular, they analyzed their rhyming patterns to reconstruct the ancient pronunciations of characters, whose difference from later pronunciations are not at all apparent from their texts, due to the non-phonetic nature of Chinese writing. Indeed, since at least the 13th century, scholars reciting lyrics from *Canon of Songs* noticed the conspicuous irregularities of their rhymes. And from the time of the late 16th century, an explanation gradually gained traction that these apparent "irregularities" are because the ancients pronounced the same characters differently than the moderns. As a result, whereas the same lines of poetry would rhyme perfectly when the ancients recited them, their rhyme patterns would be broken when the moderns recited them with their own pronunciations of characters. This meant that by using the rhymes of thousands of poetic lines from *Canon of Songs*, evidential scholars could reconstruct ancient phonology and identify which words were very likely homophones. Homophones contain very important pieces of

information, since many a word in ancient texts that had been extremely puzzling when taken literally at grammatological-semantic face value of its character suddenly became easily explicable when taken as a rebus stand-in for a semantically irrelevant homophone. Indeed, even today, these 17th- and 18th-century efforts to reconstruct ancient pronunciations of Chinese characters and explicate the use of individual characters in ancient Chinese texts serve as the foundation for historical Chinese phonology and philology.

I argue that this "discovery" of an ancient phonology distinct from the modern one and of the sonic rather than visual-grammatological aspect of explicating ancient Chinese texts should not be understood as a teleological progress, as it tends to be narrated in scholarship on 17th- and 18th-century Chinese linguistics. Indeed, almost a millennium passed from Wu Yu 吳棫 (c. 1100-1154)[80] who first attempted to address systematically the apparent rhyming irregularities in *Canon of Songs* to Duan Yucai 段玉裁 (1735-1815) whose categories of ancient vowel types and application of them to philology are broadly considered the epitome of Qing-era research on ancient linguistics.[81] And though Chen Di 陳第 (1541-1617) had conjectured in as early as the late 16th century that ancient pronunciations of the same words and characters were expressively different from those of the moderns,[82] it was only in the mid-18th century that this "ancient phonology" hypothesis became the generally accepted explanation for the apparent rhyming irregularities in *Canon of Songs* and was gradually adopted for studying ancient philology and explicating ancient texts. The reason, I show, is that up to the early to mid 18th century, debates

80 Wu Yu 吳棫 (c. 1100-1154) in his *Yunbu* 韻補 ("Supplementing the Rhymes," c. 12th century).

81 Duan Yucai 段玉裁, *Shuowen jiezi zhu* 說文解字註 ("Annotations of *Explicating Glyphs and Analyzing Characters*," 1815).

82 Chen Di 陳第, *Maoshi guyin kao* 毛詩古音攷 ("An Investigation on the Ancient Pronunciations of *Canon of Songs* as edited by Mao Heng," 1606).

over whether ancient phonology was systematically different from its modern counterpart *in toto* only drew upon textually transmitted rhyming data, namely the various rhyming relations between thousands of characters extracted from the lyrics of *Canon of Songs*. As a result, the hypothesis that the ancients pronounced the same written characters differently from the moderns was but one method for making sense of the rhyming data. A competing hypothesis, for example, was that the ancients did not at all pronounce the characters themselves differently from the moderns, but were simply much looser in their pronunciations and their use of rhymes, than later rules of poetry demanded.[83]

So what eventually allowed the "ancient phonology" hypothesis win out at the end of the day? The answer, I argue, lies in exactly the same term with which *Orthodox Meaning of Pitch Pipes* frames its experiments of listening to pitch pipes as the sole empirical basis for reforming the systems of pitch organization around the fourteen-tone octave: *shenyin* 審音 "examining the tones." A major figure in the evidential learning movement with a special proclivity for phonology and musical tuning, Jiang Yong 江永 (1681-1762) lamented in his paradigm-shifting *Standards of Ancient Rhymes* (guyun biaozhun 古韻標準, 1771) that previous scholars who had embraced the "ancient phonology" explanation of the apparent rhyming irregularities in *Canon of Songs* accomplished much in *gaogu* 考古 "investigating ancient things" but ignored *shenyin* "examining the tones."[84] What he proposed, then, was to introduce the modern study of *dengyun* 等韻 ("classified rhymes") or phonetics to the analyses of the rhyming relations in *Canon of*

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83 Zhang Minquan 張民權, *Qingdai qianqi guyinxue yanjiu* 清代前期古音學研究 ("The Study of the Archaic Chinese Phonology in Early Qing Dynasty") (Beijing: Beijing guangbo xueyuan shubanshe, 2002), Vol. 1, 42-88; Vol. 2, 135-153.

84 Jiang Yong 江永, *Guyun biaozhun* 古韻標準 ("Standards of Ancient Rhymes," 1771), a facsimile version available on available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5452, accessed October 22, 2018), Part 1, 14.

Songs. Granted, the study of *dengyun* or phonetics originated only in the 9th century and used as their primary materials what is now known as Middle Chinese, which was considered part of the "modern" in the Qing era. Jiang, however, argued that the rhyming data extracted from the texts of *Canon of Poetry* are ultimately meaningless in reconstructing the ancient language unless scholars borrow the methods of modeling tone shifts, rhyme transformations, prenuclear glides, places of articulation, and so on from the study of modern phonetics—or, in his words, unless they start "examining the tones." He even suggested that proving the differences between ancient and modern pronunciations should ultimately rely not on extracting rhyming patterns from characters featured in *Canon of Poetry* but on examining these patterns through phonetic principles derived from studying modern pronunciations.

It was by using modern phonetics to guide the analyses of ancient rhyming data and the reconstructions of ancient pronunciations that Jiang and his student Dai Zhen 戴震 (1727-1777) put a decisive end to the centuries-long debate over whether the ancients pronounced the same written characters differently (the answer is yes).[85] Granted, previous proponents of the "ancient phonology hypothesis" managed to distill various vowel groups that comprise characters pronounced with different vowels in modern speech. Nonetheless, every attempt before Jiang to rationalize the apparently irregular rhyming patterns in *Canon of Songs* by organizing ancient pronunciations of characters into different vowel groups only ended up creating more irregularities, as characters from different ones of these newly constructed ancient vowel groups could still rhyme on occasion. These newly created irregularities actually became a major weakness of the "ancient phonology hypothesis," exploited by those who believed that the

85 See Wang Li 王力, *Qingdai guyin xue* 清代古音學 ("Ancient Phonology in the Qing Era") (Beijing: Zhonghua shuju, 1990/2012), 140-141.

ancients did not pronounce any characters differently but were simply much freer in rhymes and pronunciations in poetry. Rather than inventing even more patterns to give order to ancient rhyming data, however, Jiang and Dai focused on examining the principles whereby different vowels—ancient or modern—could transform into one another on the ahistorical phonetic level. In so doing, they constructed an intricate system detailing under what conditions characters from different reconstructed ancient vowel groups can still be taken to rhyme through shifts in tones, glides, and allophones. Thus, their endeavor in "examining the tones" cleared a major hurdle that had hitherto prevented the "ancient phonology hypothesis" from becoming the dominant narrative of Qing-era phonology.

But what exactly did "examining the tones" mean for Jiang Yong as a way of knowing-in-sound and knowing-about-sound? Jiang was certainly not at all the first person to heed the importance of sounds in studying *Canon of Songs* with respect to ancient pronunciations. For the centuries before and after him, entering students of Chinese literature have been reciting those lyrics and casually changing the pronunciation of individual words when they apparently do not rhyme as they should in whatever dialect they speak. And it could only have been from reciting those lyrics—that is, treating the texts of *Canon of Songs* as teleologically or potentially sonorous traces of poetry-as-enunciated-sounds—that scholars at the turn of the 17th century first hypothesized that the ancient pronunciations of the same characters were decidedly different from the modern ones. What changed in Jiang's advocacy for "examining the tones" *id est* using modern models and principles of phonetics to give structure and sense to ancient rhyming data, I argue, was how sounding and listening functioned as a way of knowing—indeed as Feld's acoustemology. Before, when patterns of vowel groups were established solely from making sense of the textual rhyming data of *Canon of Songs* themselves and ultimately fed back to the

sonorous recitation of those texts, sounds were integrated and embedded in written texts, characters, and graphemes. The 18th century, however, saw sounds disconnected from texts and becoming a prior epistemological entity. This disembodiment of sounds from written texts is what I call a "Phonological Revolution." Rather than using textual rhyming data to demarcate sonic patterns of ancient pronunciations, studies of ahistorical phonetic principles became the agent that inversely acted upon and regulated those rhyming data. And rather than having sonic patterns return to the recitation of the texts from which they were first derived, scholars such as Duan Yucai began appropriating them to fundamentally reform the field of philology. Equipped with reconstructed ancient vowel groups and ancient pronunciations of characters, scholars discovered more and more characters in ancient texts as rebus-homophone stand-ins: that is, instances when a written character serves not to directly signify a unit of meaning but to signify a particular sound that in turn signifies a unit of meaning. Thus, by the early 19th century, Duan Yucai pronounced—in stark contrast to the Chinese historical linguistics tradition that used to emphasize grammatology over phonology—that sounds were not only independent of written characters but were even historically and logically prior to them.[86]

Of course, the experiments in "examining the tones" of pitch pipes in *Orthodox Meaning* preceded the studies of "examining the tones" of phonetic principles in Jiang Yong's work on ancient phonology and philology by several decades. I argue, however, that both processes of "examining the tones" pointed towards an epistemological shift in which sounding and listening went from being part of a correlative and perhaps relational process integrated with bodies, pasts,

86 Duan Yucai, "Preface" to Wang Niansun 王念孫, *Guangya shusheng* 廣雅疏証 ("Commentaries and Proofs for '"Towards Elegance" Extended,'" 1795), a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=1490, accessed October 22, 2018), 3.

cosmos, and texts to being one link in a chain of causations and deductions operated and observed from a disembodied vantage point. Whereas Zhu Zaiyu, for example, created a process of musical tuning consisting of multiple feedback loops wherein knowledge is not produced by any single instrument, set of numbers, or humans but by all of these subjects in webs of correlations,[87] *Orthodox Meaning* spells out a deductive chain of commands: first, read historical texts and figure out the proper proportions and sizes of pitch pipes according to ancient practice; next, make a set of tuning pitch pipes according to these textually transmitted data; then, listen to the sounds of these pipes and discern their patterns; afterwards, build an entire system of pitch organization, scales, and modes based on these sonically derived patterns; and finally, apply this system to all other instruments. Within this chain, numbers, sounds, and instruments each have a proper circumscribed place to occupy and in which to function, rather than constantly shifting, morphing, or relating from one to the other, while the detached and disembodied knowing subject maintains the overall order, as though operating a machine or flow chart. A similar procedure, I argue, took place during the Phonological Revolution. As Duan Yucai wrote: "For studying the ancient classics, there is nothing more important than acquiring the meanings of characters, and for acquiring their meaning, there is nothing more critical than acquiring their pronunciations."[88] Indeed, as "examining the tones" became the standard method for not only making sense of textual rhyming data in reconstructing ancient pronunciations but also for identifying uses of rebus-homophones in explicating characters in ancient texts, sounds were transformed from participating in the embodied experience of reciting or even singing lyrics to being distinct links in the deductive chain of resurrecting the institutions of the ancients—the

87 Zhu Zaiyu, *A New Theory of Pitch Pipes*, Vol. 1, Chapters 7-9.
88 Duan Yucai, "Preface," 3.

chain that moves from characters to sounds, sounds to meanings, meanings to interpretation, and finally interpretation to resurrection of the ancients.

Chapter One

From the Circle of Fifths to Octave Equivalence:

The Equivocation of an Imperial Mistake

On February 20, 1692, the Kangxi Emperor almost made a fool of himself. I say almost, because hardly any contemporary witness or subsequent historians ever cared (or dared) to notice it, even though all that is needed to detect this fountainhead of the Qing's notorious fourteen-tone temperament is duly recorded in the official *Veritable Records of the Kangxi Reign* (kangxi chao shilu 康熙朝實錄, 1732).[1] According to the latter, on that morning, before dawn, the Emperor summoned some of his most trusted Han Chinese advisors to the Gate of Heavenly Purity (*qianqing men* 乾清門) at the heart of the Forbidden City, where he had made it a custom to deliberate on the most pressing matters of the ever-expanding empire in the open air.[2] The subject of the day, however, concerned nothing of political urgency. Amid lingering threats of a

1 *Shengzu ren huangdi shilu* 聖祖仁皇帝實錄 ("Veritable Records of the Shengzu Emperor Ren"), also known as *Kangxi chao shilu* 康熙朝實錄 "Veritable Records of the Kangxi Reign" (1732), Vol. 153. The relevant entry is dated as 康熙三十一年正月甲寅 "the day of *jiayin* of the 1st month of the 31st year of the Kangxi Reign," or 康熙三十一年正月初四 "the 4th day of the 1st month of the 31st year of the Kangxi Reign" (KX 31.1.4), which is February 20, 1692 in the Gregorian Calendar. For this chapter, I have relied on the digitized text on *Veritable Records of the Kangxi Reign* on the Chinese Text Project. See *Qingshilu kangxi chao shilu* 康熙朝實錄 ("Veritable Records of the Kangxi Reign"), part 50, segment 375 (https://ctext.org/wiki.pl?if=gb&chapter=259761, accessed March 7, 2019). The only historian to my knowledge who has given a somewhat substantial account or analysis of this day's event is Catherine Jami—see *The Emperor's New Mathematics: Western Learning and Imperial Authority during the Kangxi Reign, 1662-1722* (Oxford, UK: Oxford University Press, 2012—although Jami does not seem to capture the music-theoretical significance of it, which is my focus in this chapter. See also footnote no. X below.

2 See Zhang Naiwei 章乃煒 et al., *Qinggong shuwen* 清宮述聞 ("Accounts and Knowledge of the Qing Palace," 1941), Chen Xiaodong 陳曉東 ed. (Beijing: Gugong chubanshe, 2009), vol. 1, 323.

Dzungar assault on Beijing and reports of crop failures in northwestern China,[3] the only item on the Emperor's agenda four days after the Chinese New Year was to ambush this highly select gathering of scholar-officials with what amounted to an oral exam on celestial navigation, geometry, hydraulics, and, especially for our purpose, musical tuning.[4]

Unprepared though they were, the Han Chinese veterans of the Emperor's innermost retinue knew exactly what was awaiting them. By then, it was an established pattern of behavior of the Manchu ruler to personally dress down his Han Chinese confidants with pop quizzes on astronomy, mathematics, mechanics, and music—subjects over which he knew these otherwise learned scholars to have a tenuous command.[5] Granted, since the 16th century, scholars had been

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3 For much of the 17th and 18th centuries, the Dzungar Khanate (1634-1758) was the defining national security threat of the Qing Empire, as it threatened the Qing Emperor's nominal and material control of all the Mongol tribes as the *bogda khagan* or the Great Khan, a title through which they claimed political descent from none other than Chinggis himself. And since the First Dzungar-Qing War (1687-1697) fought over control for the Khalkhas, who lived in the southeastern part of the Mongolian plateau, many of the military conflicts unrolled in close proximity to Beijing, so much so that any substantial defeat of the Qing's army would have posed a grave threat to the Empire's capital. In 1690 and 1696, the Kangxi Emperor even led personal military expeditions. See Peter Perdue, *China Marches West: The Qing Conquest of Central Eurasia*, 152-161, and 175-190. As for crop failures, they were discussed on KX 31.1.2, or February 18, 1692, two days before the court gathering taken up in this chapter. See *Veritable Records of the Kangxi Regin*, Part 50, Segments 373-374. (https://ctext.org/wiki.pl?if=gb&chapter=259761, accessed March 7, 2019)

4 While several Manchu and bannermen-Chinese officials were also present at the court gathering, according to *Veritable Records*, the Emperor only appeared to have interacted with a few Han Chinese scholar-officials. For a full roster of the twenty people present at the court gathering, see National Palace Museum (國立故宮博物院), *Qingdai qijuzhu ce kangxi chao* 清代起居注冊康熙朝 ("Imperial Diaries of the Qing Era, Kangxi Reign"), Vol. 3 (Taipei: Yingyin faxingzhe lianjing chuban shiye gongsi, 2009), facsimiles no. T01183-T01184. See also footnote No. 9 below.

5 In fact, just three years beforehand, on a visit to the Nanjing Observatory on March 18, 1689, the Kangxi Emperor had such a dress-down session with Li Guangdi, who, as luck would have it, turned out to be one of the several scholar-officials summoned for the pop quiz and dress-down of February 20, 1692. For the 1689 visit to the Nanjing Observatory, the Emperor's interaction with Li Guangdi there, and the ethnic motivations of his schooling of his Han Chinese scholar-officials in general, see Jami, *The Emperor's New Mathematics*, 120-131.

paying more attention to such subjects thanks to the rise of the *shixue* 實學 or "concrete learning" movement, which preached a pragmatist, statecraft-oriented approach to Confucian scholarship.[6] Still, most Han Chinese literati in the 17th century dedicated themselves predominantly to the more metaphysical and moralist Neo-Confucianism epitomized by Zhu Xi 朱熹 (1130-1200) whose teachings and exegeses of classics they had to know inside out in order to pass the civil examinations and join government ranks.[7] By hitting the Empire's most respected Han Chinese scholar-officials right in their the blind spot, the Kangxi Emperor not only got to expose how these bookish guardians of Confucian learning were clueless when it came to any practical know-how about the world—particularly when it required a bit of *suanfa* 算法 "algorithm" or "methods of calculation"—but also to flaunt how he, the supposed barbarian according to their Sinocentric worldview, had actually mastered these skills.

Indeed, everything on that morning unraveled according to this script of ethnically charged humiliation, both for the aggressor and for his victims. As usual, none of the victims produced a satisfactory solution to any one of the four rather random problems the Emperor raised:

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6 See Willard Peterson, "Confucian Learning in Late Ming Thought," in Denis C. Twitchett and Frederick W. Mote eds., *The Cambridge History of China*, Vol. 8, *The Ming Dynasty 1368-1644*, Part 2 (Cambridge, UK: Cambridge University Press, 1998), 708-788, particularly 772-788.

7 Of course, that the Neo-Confucian or *daoxue* 道學 "learning of the way" interpretation of Confucianism and curation of its classics reached orthodox status did not mean they had complete ideological control or could foreclose the possibilities of debate, conflict, or non-orthodox influences, such as those from the "concrete learning" or the later *kaozheng* "evidential learning" movements, which emphasized evidence and processes of proof and verification. See Benjamin Elman, *Cultural History of Civil Examinations in Late Imperial China* (Berkeley and Los Angeles, California: University of California Press, 2000), 421-459.

(1) Determine whether Cai Yuanding 蔡元定 (1135-1198), a disciple of the revered Zhu Xi, was correct in applying the rule of thumb "the circumference of a circle of three, when the diameter is one" (*jingyi weisan* 徑一圍三) to calculating the circumferences of the base of tuning pitch pipes in his *New Treatise on Pitch Pipes* (lülü xinshu 律呂新書 c. 1180s);

(2) Explicate the broadly quoted principle in musical tuning, "mutual generation at every eighth step" (*geba xiangsheng* 隔八相生);

(3) Determine the algorithm for calculating the flow rate of water through a sluice gate;

(4) Predict the noontime altitude of the sun at Beijing on that day, and verify this prediction with a sundial.[8]

Upon laying bare, yet another time, the ignorance of these supposed scholar-officials, the Emperor smirked: "You Han Chinese, you know absolutely nothing about methods of calculation [*suanfa*]" (你們漢人全然不曉得算法)! As it made plain the ethnic undertone of the entire event, the imperial diarists (*qiju zhu guan* 起居註官) in attendance probably deemed the Emperor's denigration of his Han Chinese scholar-officials too politically sensitive to record. The remark did not end up in *Veritable Records*, and it was never even transcribed into the imperial diaries (*qiju zhu* 起居注), which, as classified court documents for internal use, served

8 A full picture of the series of questions the Emperor posed to his scholar-officials can be pieced together by collating the entry in *Veritable Records of the Kangxi Reign* with an account by Li Guangdi (see footnotes No. 15 and 16 below), both of the court gathering itself and of a meeting between the Kangxi Emperor and Xiong Cilü 熊賜履 (1635-1709) that touched on the first two of the four questions. Though Li Guangdi and Xiong Cilü were highly critical of each other's work, both were instrumental for establishing Neo-Confucianism as the Qing's state orthodoxy, and both played instrumental roles in recruiting Han Chinese scholars for literary and political service of the Empire.

as the main primary source for the published *Veritable Records*.[9] Still, the transcripts of the imperial diarists rendered an exceptionally detailed account of how the Emperor schooled his Han Chinese scholars in a *tour de force*. First, the Emperor exhibited a diagram of a circle inscribed with a right hexagon and explained that multiplying the diameter threefold only begets the circumference of the hexagon, whereas that of the circle is slightly over 3.141 times the diameter. Next he had someone fetch a traverse flute or *di* 笛 and a twenty-five-string plucked zither or *se* 瑟 from the Altar of Heaven located three miles to the south,[10] asked Daoist musicians to play the scale on each,[11] and invited his scholar-officials to listen with their own ears to how the same tone returned at every eighth step of the scale, thus producing "mutual generation at every eighth step." As for measuring flow rate, explained the Emperor, all one needs is to measure the area of the sluice gate and the linear distance the water flows every second and multiply the former by the latter. And finally, regarding the solar zenith angle, *Veritable Records* is thin on the Emperor's words but certainly not on his actions:

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9 The complete entry of the court gathering in the imperial diaries can be found at National Palace Museum, *Imperial Diaries of the Qing Era, Kangxi Reign*, Vol. 3, facsimiles no. T01183-T01193; the entry is almost identical to the later account of the court gathering in *Veritable Records*.

10 The provenance of the very flute and zither that the Emperor used for his demonstration is not specified in *Veritable Records*; instead, it was recorded by Li Guangdi in his account of the court gathering; see footnote No. 16 below. I cannot quite make out why the Emperor specifically asked for instruments from the Altar of Heaven when many must have been available inside the Forbidden City.

11 Following precedents set by the Ming courts, all musical services provided by the Office of Imperial Sacrifices (*taichang si* 太常寺) for sacrificial rites and stately ceremonies were performed by Daoist priests in the 17th and 18th centuries until an overhaul involving ceremonial music was made to the official and courtly institutions in the 1740s. Meanwhile, inner court entertainments, including operas, were provided by eunuchs, who since the time of the Qing conquest of Beijing replaced those women musicians who worked within the Ming's inner court music establishment, the Royal Music Academy (*jiaofang si* 教坊司). See Ye Xiaoqing, "Imperial Institutions and Drama in the Qing Court," in *European Journal of East Asian Studies* 2, no. 2 (2003): 329-364, particularly 331-340.

> […] 又命取測日晷表，以御筆畫示，曰：“此正午日影所至之處。”遂置乾清門正中，令諸臣候視。至午正，日影與御筆畫處恰合，毫髮不爽。諸臣等奏曰：“臣等今日仰承聖訓，得聞所未聞，見所未見，不勝歡慶之至！”[12]
>
> […] Then, [the Emperor] ordered that an observational sundial be fetched and dashed off a stroke [on the dial] with his imperial brush, saying: “this is where the shadow of the sun will reach at noon.” He then had it placed at the center of the Gate of Heavenly Purity and asked all officials to wait and see. When noontime struck, the shadow of the sun coincided exactly with the dash of the imperial brush without deviating by as much as a single hair’s width. All the officials submitted: “Today, we your subjects have been enlightened by the Sagely Instruction and got to hear about what we have never before heard about and see what we have never before seen — how overwhelming is our joy and jubilation!”[13]

The Subtle Art of Flattery

Now, anyone who had stood for hours in the wintry cold of early-February Beijing only to wait for noontime would have probably greeted its arrival with “joy and jubilation.” Yet the awe and amazement that conclude the entry for February 20, 1692 in *Veritable Records* is corroborated in unofficial accounts written by those Han Chinese advisors present at the court gathering. By “unofficial,” I do not mean “private”: all of them were eventually printed and circulated when their authors were still alive, and one was actually addressed to the throne formally as a petition or *shu* 疏. This “public” nature makes their paeans to the Emperor’s knowledge all the more unremarkable: after all, what else were these scholar-officials supposed to write when the Emperor so clearly wanted to assert his intellectual dominance over them?

12 See *Veritable Records of the Kangxi Reign*, Part 50, Segment 375 (https://ctext.org/wiki.pl?if=gb&chapter=259761, accessed March 7, 2019).

13 All translations in this dissertation are mine unless otherwise noted.

What is remarkable is that in their reminiscences of the court gathering, the scholar-officials differ significantly from the imperial diarists over what exactly awed and amazed them. The Emperor's celestial forecast conclude the court gathering with a admittedly garish spectacle, and the action-packed description in *Veritable Records* and its marked secrecy on his actual methods for calculating the solar zenith angle also make it a captivating intrigue. However, all of the Han Chinese scholar-officials who watched the Emperor accurately predict the noontime solar altitude unanimously ignored this portion of the court gathering in their accounts of the day. This omission is baffling. Given the mathematical skills and special apparatus it demanded—only a small number of people in the Qing Empire or around the world knew spherical trigonometry or had a portable sundial at their disposal—the Emperor's astronomical feat would have been a natural target of flattery. It was as though none of his Han Chinese advisors had any wonder or blandishment left for the celestial forecast, since they had showered them all onto another part of the Emperor's lecture despite how unworthy it may seem: his explanation of the music theory term *geba xiangsheng* "mutual generation at every eighth step."

To be clear, the concept of octave equivalence, which the Emperor effectively demonstrated without using the term as such (an English one, after all), is by no means timeless or universal. Even for Western music, it remains an unsettled matter for some musicologists whether a concept of octave equivalence existed in the Middle Ages at a time when music pedagogy was based, at least nominally, on Guido's hexachords.[14] Nevertheless, while the

14 The most captivating account of (the rather risible yet still somewhat widespread notion) of whether the diatonic pitch system of Medieval and Renaissance music in Western music was fundamentally based on major sixths instead of octaves—on account of there being only six instead of seven syllables in Guido's hexachord *ut*, *re*, *mi*, *fa*, *sol*, and *la*—is found in Mengozzi, *The Renaissance Reform of Medieval Music Theory*, particularly the "Interlude: All hexachords are 'soft,'" 110-114.

Table 1-1 Guido's solfège syllables,* gongche *syllables, and "the five proper notes and two altered notes"

	1	**2**	**3**	**4**	**5**	**6**	**7**
Guido's syllables	*do*/*ut*	*re*	*mi*	*fi* (*fa#*)	*sol*	*la*	*[ti]*[15]
Gongche syllables[16]	*shang* 上	*che* 尺	*gong* 工	*fan* 凡	*liu* 六 *he* 合[17]	*wu* 五 *si* 四	*yi* 乙 *yi* 一
The five proper notes and two altered (*bian*) notes	*gong* 宮	*shang* 商	*jue* 角	*bianzhi* 變徵	*zhi* 徵	*yu* 羽	*biangong* 變宮

received solmization system in Western Europe added a seventh syllable *ti* to Guido's original *ut re mi fa sol la* only during the 17th century, Chinese solmization systems had been thoroughly heptatonic, as shown in Table 1-1 above. The *gongche* 工尺 syllables mentioned in the Introduction (*shang*, *che*, *gong*, *fan*, *liu*/*he*, *wu* /*si*, and *yi*) often served as a performance shorthand, though scholarly treatises and musical scores of high production value—particularly

15 The original solfège system of Guido d'Arezzo had only six syllables before the seventh one, *si* (*ti* as used in North America), was added in the 17th century. See footnote no. 9 in the Introduction.

16 It should be noted that the *gongche* notation syllables in and of themselves do not prescribe any intervallic relations. In other words, they are more like the concept of "scale degrees" rather than solfège syllables in Western music theory. In contrast, the "five proper notes and two altered notes" are almost always taken as indices of intervallic relations, and are indeed comparable to solfège syllables. As a result, different historical periods have different preferences of fitting the *gongche* notational syllables to the "five proper …" solfège syllables. For example, in the pan-flute from Volume 3 of *Orthodox Meaning of Pitch Pipes* puts the *shang* 上 syllable of the *gongche* notable on the *yu* 羽 (*la*) note of "five proper …" solfège syllables. In contrast, in this table, which represents the "consensus" so to speak of music theory and practice before *Orthodox*, locates the same *shang* syllable on the *gong* 宮 (*do*) note. For the rather complicated history of the *gongche* syllables both in books and in practice, see Wu Xiaoping 吳曉萍, *Zhongguo gongchepu yanjiu* 中國工尺譜研究 ("A Study of the Chinese *Gongche* Notation") (Shanghai: Shanghai yinyue xueyuan chubanshe, 2005).

17 The three syllables *he* 合, *si* 四, and *yi* 一 are typically used to indicate the notes that sound an octave below *liu* 六, *wu* 五, and *yi* 乙 respectively. As a result, if I were not comparing the *gongche* syllables to both Guido's solfège syllables and the "five proper notes and two altered notes," it would have been more appropriate to really speak of ten *gongche* syllables from *he* 合 to *yi* 一 and then from *shang* 上 to *yi* 乙.

those documenting the tunes for sacrificial rites and court ceremonies—favored the more classical solmization scheme, which also spells seven syllables: *gong* 宮, *shang* 商, *jue* 角, *bianzhi* 變徵, *zhi* 徵, *yu* 羽, and *biangong* 變宮, roughly equivalent, as are the *gongche* syllables, to *do*/*ut*, *re*, *mi*, *fi* (that is, *fa#*), *sol*, *la*, and *ti*.

Granted, of these seven syllables, five—*gong*, *shang*, *jue*, *zhi*, and *yu*, which render a pentatonic scale (*do/ut*, *re, mi*, *sol*, and *la*)—are referred to as *wu zhengsheng* 五正聲 "the five proper notes," often shortened as *wusheng* 五聲 "the five notes." Meanwhile, *bianzhi* (*fi* or *fa#*) and *biangong* (*ti*) are separately referred to as *er biansheng* 二變聲 "the two altered notes," often shortened as *erbian* 二變 "the two altered," since they were considered the results of the *bian* 變 "altering" (specifically lowering) of two of the five proper notes, *zhi* (*sol*) and *gong* (*do/ut*), respectively. Still, the phrases *wusheng erbian* 五聲二變 "five proper notes and two altered notes" and *geba xiangsheng* "mutual generation at every eighth step" were ubiquitous both in generalist works of scholarship and in specialist treatises on music. One can only be so credulous on encountering Li Guangdi 李光地 (1642-1718) recorded in his posthumous *Quotations of Sir Rongcun: A Sequel* (rongcun yulu xu 榕村語錄續, 1894) as having said the following to his disciples right after the court gathering:

> […] 皇上真大聰明，我輩平日只知道隔八相生，不曉得隔八相生之根。上因不解，命道士攜琴瑟彈聽之，猜云："或是宮、商、角、徵、羽及變宮、變徵，去七聲，八位便生次弦。"卻得隔八相生之根子。余因遍試九位、七位、六位、五位，惟除去二變聲，六位合，添二變聲，八位合，餘俱不能合到次位。[18]

18 Li Guangdi 李光地, *Rongcun yülu xu* 榕村語錄續 ("Quotations of Sir Rongcun: A Sequel"), Huang Jiading 黃家鼎 ed. (1894), vol. 17, f. 25v. Rongcun 榕村 was Li Guangdi's pen name or *hao* 號.

> […] The Emperor is such a great genius! People like us only know the words "mutual generation at every eighth step" but do not know the origin of "mutual generation at every eighth step." Because the Emperor didn't quite understand it, he ordered Daoists to play the zithers and listened, speculating: "it is probably that, passing through the seven notes of *gong* [*do*], *shang* [*re*], *jue* [*mi*], *zhi* [*sol*], *yu* [*la*], and *biangong* [*ti*] and *bianzhi* [*fa#*], the eighth position yields the string in the next series." In so doing, however, he discovered the origin of "mutual generation at every eighth step." Thereafter, I thoroughly tried the ninth, seventh, sixth, and fifth positions, and the correspondence exists at the sixth position when not counting *biangong* [*ti*] or *bianzhi* [*fa#*], and at the eighth position when counting *biangong* [*ti*] and *bianzhi* [*fa#*]; meanwhile, none of the other positions works.

A much shorter entry from Li Guangdi's *Quotations* conveys a similar account of the Emperor's demonstration of "mutual generation at every eighth step," here quoted in full:

> 上問隔八相生之義，命取天壇二十五弦之瑟，指之曰："莫非以一絃至第七絃，至第八絃則又以宮起麼？"某因讚之曰："此卻能指出隔八相生源頭，古未有及此者。"蓋若以五六之數生之，卻不能。[19]

> When asking about the meaning of "mutual generation at every eighth step," the Emperor fetched a twenty-five-string *se* zither from the Altar of Heaven. He pointed to it, saying: "could it be that, having progressed from the first string to the seventh string, the eighth string then starts again at *gong* [*do*]?" Someone thus praised it, saying: "From this, the Emperor was actually able to pinpoint the origin of 'mutual generation at every eighth step,' and no one since the ancient times has ever achieved this." Indeed, if one tries to generate it with [step] number five or number six, it will not work.

Notably, while the official narrative in *Veritable Records* portrays the Emperor as a triumphant and all-knowing lecturer, both quotations above portray the Emperor as more of a curious learner who himself did not quite understand the much-repeated phrase in music treatises *geba xiangsheng* "mutual generation at every eighth step" at the start of the court gathering. In

19 Ibid., vol. 18, f. 18v.

addition, the two quotations (which likely originated from the same story Li told his confidants after the court gathering, even if later retold and written down by different people) disagree significantly over exactly how the Emperor demonstrated his understanding of "mutual generation at every eighth step" and how Li replicated his demonstration. In the first quotation, the description of Li's replication of the Emperor's demonstration pinpoints the different finger positions on a single string yielding the seven-note diatonic scale. Meanwhile, in the second quotation, the description pinpoints different open-position strings on a twenty-five-string *se* zither, which *Veritable Records* confirms as the instrument used during the Emperor's demonstration.

Neither *Veritable Records* nor its primary source material, the imperial diaries, clarifies which of the two descriptions better matches the specific mechanisms of the Kangxi Emperor's demonstration on the *se* zither, yet I suggest that the second quotation is more accurate. Compared to using different finger positions on the same string, plucking different open strings is not only a more idiomatic way for playing the *se* zither but was probably the whole point of the Emperor using a twenty-five-string *se* zither in the very first place. Indeed, it might seem odd that the Emperor chose this particular type of zither instead of the much more common seven-string *qin* 琴 zither. The two zithers were often mentioned in a pair, *qinse* 琴瑟, to denote zithers and plucked-string instruments in general—this turn of phrase is used in the first quotation of Li Guangdi above—and they even became a metaphor for husband and wife in Classical Chinese.[20] Yet the seven-string *qin* zither had the special cultural significance as the emblematic instrument of the Chinese literati, who displayed their refinements and sensibilities not only by playing but also by writing about *qin* zithers and procuring rare instruments. Eager to convince the Han

20 For example, 琴瑟和鳴 "a *qin* zither and a *se* zither sounding in harmony."

Chinese scholar-officials that the Manchu Emperors were really "one of them," the Qing court became arguably the most successful collector of *qin* zithers of the 17th and 18th centuries. Had the Kangxi Emperor wished to show off the Chinese cultural capital he had accumulated besides schooling his scholar-officials on the meaning of octave equivalence, he could have simply sent for any one of the *qin* zithers located in his studies throughout the Imperial City complex, many of which date back to as early as the Tang (618-907) and Song (960-1276) eras.

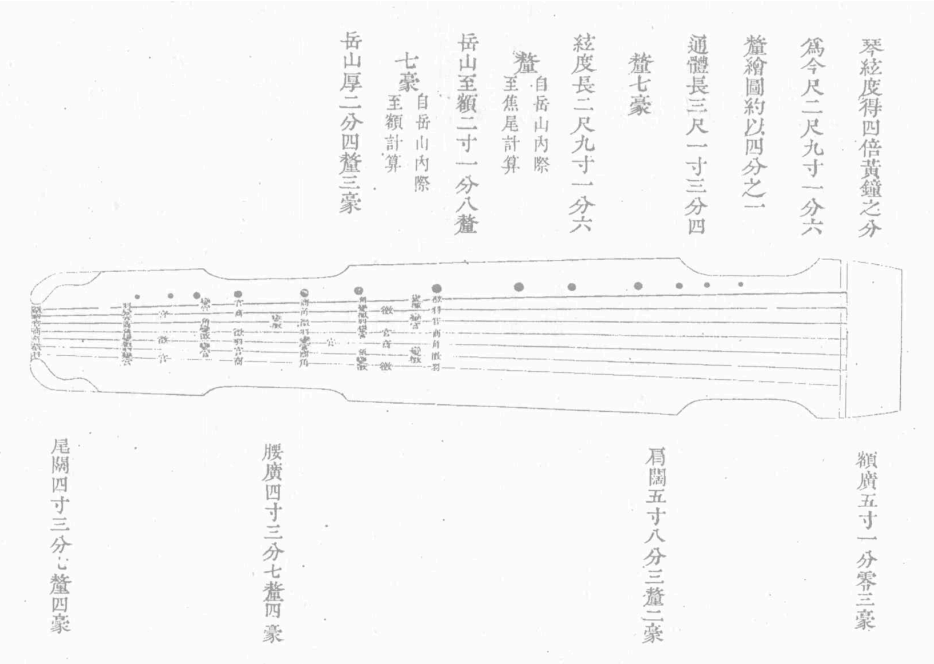
The twenty-five-string *se* zither lacked any comparable cultural or collectible value. Kept largely for their functionality, the only *se* zithers the Kangxi Emperor likely had at his disposal would have come from the various imperial ritual venues where *se* and *qin* zithers were both featured in the *yayue* 雅樂 "elegant music" ensembles that regularly accompanied sacrificial rites and courtly ceremonies in accordance with Confucian traditions.[21] Collating the two quotations of Li Guangdi cited above suggests that both the twenty-five-string *se* zither and the Daoist musician(s) whom the Kangxi Emperor asked to play the zither came from the Daoist Temple of Sacred Music (*shenyue guan* 神樂觀; later reformed into *shenyue shu* 神樂署 or the Office of Sacred Music).[22] Located by the western wall of the Altar of Heaven, which is mentioned in Li's second quotation above, the Temple had been providing ritual music for the imperial court since

21 While *yayue* "elegant music" was a generic term for music performed at rites and ceremonies following Confucian traditions, the repertory of music that accompanied the most solemn occasions at the Qing court is referred to as *zhonghe shaoyue* 中和韶樂 "Centering and Harmonizing Music of Brightness." See Wan Yi 萬依 and Huang Haitao 黃海濤, *Qingdai gongting yinyue* 清代宮廷音樂 ("Court Music of the Qing Era") (Hong Kong: Zhonghua shuju Xianggang fenju, 1985), 12-19.

22 See Yu Minzhong 于敏中 et al., *Rixia jiuwen kao* 日下舊聞考 ("Verifying Zhu Yizun's *Descriptions of the Imperial Capital*, "1782), Vol. 58, 城市外城南城二 ("The City-Outer City-Southern Part").

the early Ming (1368-1644) era, in addition to gaining immense popularity among the Beijing citizenry for its herbal medicines and temple fairs.

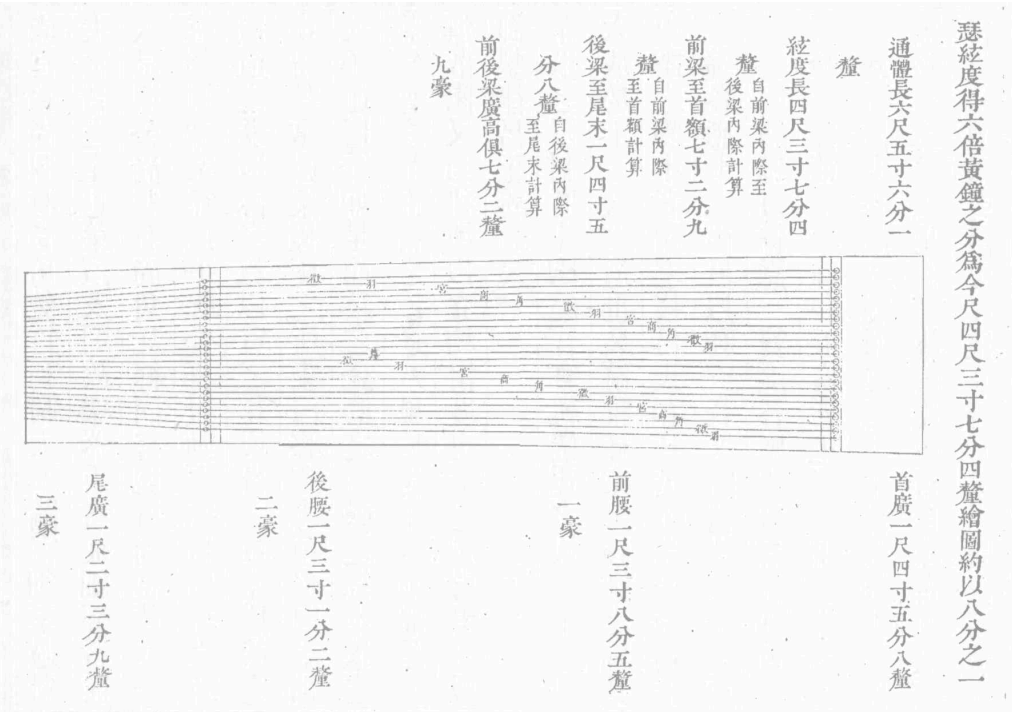
***Illustration 1-1 The seven-string* qin *zither in* Orthodox Meaning of Pitch Pipes *(1714), Chapter 1, Volume 4f. 23*bis[23]**



So why did the Kangxi Emperor send for the twenty-five-string *se* zither six miles away, even though several seven-string *qin* zithers were readily available nearby? I argue that the most logical explanation is the audio-visual affordances of the twenty-five-string *se* zither. As Illustration 1-1 shows, because the *qin* zither features only seven strings, playing a seven-note diatonic scale spanning an entire octave in order to demonstrate "mutual generation at every eighth step" or octave equivalence would require using different finger positions on one or

23 This illustration shows a standard seven-string *qin* zither. The thirteen black dots arranged in a row right above the upper most string in the illustration indicate the different finger positions. The various Chinese characters written onto the various junctures of the seven strings—most of them aligned rather well with one of the thirteen black dots qua finger positions—indicate the different notes the corresponding finger position produces on the respective string.

***Illustration 1-2 The twenty-five-string* se *zither in* Orthodox, *Chapter 2, Volume 4, f. 31*bis**[24]

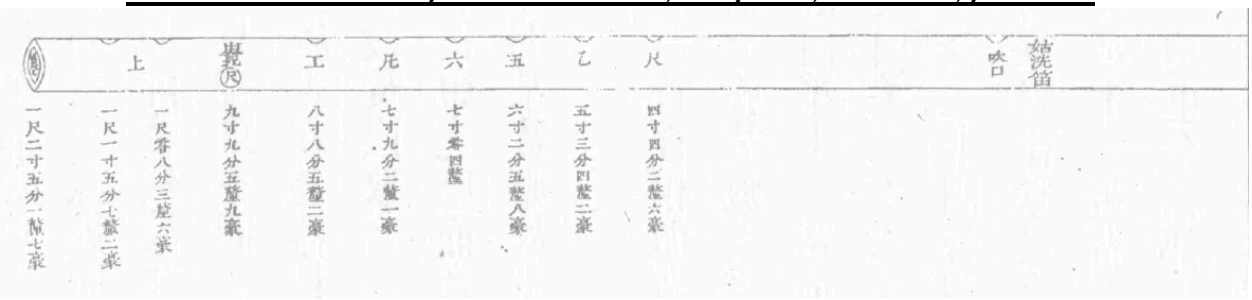


several strings; using these various finger positions is indeed an integral part of the *qin* zither's play style. Meanwhile, as Illustration 1-2 shows, the *se* zither features as many of twenty-five strings. Typically, *se* zithers were tuned to produce only "the five proper notes" of the Chinese pentatonic scale (*gong*, *shang*, *jue*, *zhi*, and *yu*, which are comparable to *do*, *re*, *mi*, *sol*, and *la* in Western solfège), yet some were tuned to produce all the "the five proper notes and the two altered notes" of the Chinese heptatonic scale (adding *bianzhi* and *biangong*, or *fa*# and *ti*). Based on the description in Li Guangdi's second quotation above, the special *se* zither brought from the Altar of Heaven was indeed tuned to the heptatonic scale. Under such a configuration, demonstrating "mutual generation at every eighth step" as octave equivalence would simply require plucking one open string after another, which, as I mentioned above, is the idiomatic way

24 This illustration shows a standard twenty-five-string *qin* zither tuned to the pentatonic scale. The Chinese character on each string indicates the name of the note it produces (that is, when in open-position).

of playing the instrument. Thus, while the two plucked-string instruments could produce the same sonic pattern, the successive open strings on the twenty-five-string *se* zither would render more conspicuous visual signposts for the seven-note diatonic octave than would the subtle back-and-forth movements between different finger positions on one or several strings on the seven-string *se* zither.

Illustration 1-3 The* di *flute in* Orthodox, *Chapter 3, Volume 3, f. 36*bis-*1[25]



In addition to picking the relatively scarce twenty-five-string *se* zither over the practically ubiquitous seven-string *qin* zither, the Kangxi Emperor's resolve to use the most intuitive instruments to demonstrate the principle of octave equivalence also brought in front of his Han Chinese scholar-officials the traverse *di* flute. Though neither one of Li Guangdi's two quotations above mentions the flute, its presence at the court gathering is confirmed by *Veritable Records*. And the wind instrument would have made an optimal tool for showing "mutual generation at every eighth step" in exactly the same way as did the twenty-five-string *se* zither. As Illustration 1-3 shows, a *di* flute is typically carved with six finger holes on the upper side of its body, which is the same side of the embouchure hole. Analogous to the successive plucking of open-position strings on the twenty-five-string *se* zither, the *di* flute ascends one note along

25 This illustration shows a standard six-hole *di* flute. Notice that the first hole on the upper edge of the body of the flute in the diagram is the embouchure hole, and the second to the seventh holes are finger holes to be opened or covered, while the last two holes are for the passage of air out of the flute.

the seven-note diatonic scale with the successive opening of each finger hole, from the farthest to the nearest in relation to the embouchure hole. Even more useful for the purpose of the Kangxi Emperor, when the *di* flute reaches the eighth note after passing through the seven-note diatonic scale, not only is the pitch sonically an octave above the first note, but the fingering is also visually identical to that of the first note, albeit producing a different harmonic partial, with all finger holes on the side of the embouchure hole closed.

Together, Li Guangdi's two quotations indicate that the Emperor's demonstration of *geba xiangsheng* "mutual generation at every eighth step" with the zither and the flute was as carefully planned a spectacle as was his prediction of noontime solar altitude. Verifying the Emperor's prediction of solar altitude required that a portable sundial be brought to the middle of the courtyard in front of the Gate of Heavenly Purity. Similarly, explaining the phrase "mutual generation at every eighth step" as octave equivalence required that a twenty-five-string *se* zither tuned specifically to the heptatonic as opposed to the pentatonic scale be brought from the Temple of Sacred Music at the Altar of Heaven. In addition, the Emperor asked the Han Chinese scholars to watch with their own eyes the shadow of the sun meet his brushstroke on the sundial at the exact moment of noontime. Similarly, he asked them to listen with their own ears to the pattern of sounds on the two instruments against the visual reference of the successive plucking of strings on the zither and the successive lifting of the fingers off the finger holes on the flute.

Yet there was a major difference between the Kangxi Emperor's two spectacles of knowledge-demonstration: one of them clearly impressed his Han Chinese spectators much more than did the other. It is here that Li Guangdi's two quotations is somewhat at odds with *Veritable Records*. As mentioned in the previous section of this chapter, the official chronicle effectively paints the Emperor's prediction of noontime solar altitude as the highlight of the day. Li's

recollection of the court gathering completely ignores the Emperor's astronomical feat and casts his demonstration of "mutual generation at every eighth step" as the primary if not sole cause of the scholar-officials' glowing praise. What is more, even though hyperbolic praise is a genre-defining rhetorical device for *Veritable Records*—the formal commemoration of the deeds of an emperor—Li's paean to the Kangxi Emperor's demonstration, uttered in front of his friends and disciples, is rather incongruous. How could he have claimed to know only the name of a concept as banal as octave equivalence and only a phrase as ubiquitous as *geba xiangsheng* "mutual generation at every eighth step," and never understood what they stand for?

The truth is, Li was one of the most learned scholar-officials in the Qing Empire, having achieved the highest degree possible from the civil examinations, *jinshi* 進士, at the age of twenty-eight. What is more, he also penned the very *Commentaries on the Ancient Confucian Canon of Music* (gu yujing zhuan 古樂經傳, 1726) that Amiot would translate into French in the 1750s upon recommendation by Antoine Gaubil (1689-1759), who was arguably the most learned European sinologist of the 18th century.[26] Even if Li simply wanted to ingratiate himself with the Emperor by concocting these self-deprecating flatteries, why didn't he direct them to the Emperor's solar forecast, an area of knowledge where Li would not have needed to feign any ignorance, at least if his scholarly output is any indication? If Li had very few opportunities to lay his fingers on a portable sundial to practice his solar altitude calculations and observations, he had no problem procuring a zither right after the court gathering. He also knew perfectly well how to maneuver around its various strings and finger positions in order to duplicate the Emperor's demonstration of *geba xiangsheng* "mutual generation at every eighth step." Given

26 See Jean-Joseph Marie Amiot, *Mémoire sur la musique des chinois tant anciens que modernes* (Paris: Nyon l'aîné, 1779), 4-5.

such adroitness at playing the zither, it is unimaginable that Li never noticed that the pitch produced by plucking the eighth string (or by plucking the same string while pressing on the finger position of the eighth note, which is the string's bisecting point) is almost identical to that produced by plucking the first string (or plucking the same string open).

In fact, even Li himself attested to the authority in music and music theory that his colleagues accorded him at the time. Those colleagues included the "someone" Li quotes in the second quotation above praising the Emperor's demonstration of "mutual generation at every eighth step" as a historic discovery: Zhang Yushu 張玉書 (1642-1711). Another Han Chinese scholar-official present at the gathering, Zhang stopped Li as they were exiting the palace gate and begged him to explain, of all things, *geba xiangsheng* "mutual generation at every eighth step." According to Li's first quotation (a part not quoted above), Zhang said to Li: "at least you actually know something about this stuff [你還知道些]!"[27] Reportedly, Li explained the term to Zhang by making a *huotu* 活圖 a "movable diagram" consisting three concentric paper discs. Though *Quotations* does not replicate that diagram, its description recalls similar illustrations contemporary music theory treatises employ to illustrate *xuangong* 旋宮 "rotating the *gong* [*do*] note" and *zhuandiao* 轉調 "turning the mode," or the transpositions of scales and mutations of modes.[28] This further attests to Li's expertise in music theory and the absurdity of his claim of not knowing what *geba xiangsheng* "mutual generation at every eighth step" means.

Meanwhile, our new friend Zhang Yushu went home right after his make-up lesson with Li Guangdi and wrote an even more glaringly incredible account of the court gathering: a

27 Li Guangdi, *Quotations of Sir Rongcun: A Sequel*, vol. 17, f. 25r.

28 See, for a few examples, In-c'i et al., *Orthodox Meaning of Pitch Pipes*, vol. 2, f 37v, and Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, vol. 7, f. 4r.

Petition that Treatises on Musical Tuning and Mathematics be Compiled (qing bianci yuelü suanshu shu shu 請編次樂律算術書疏, 1692), which he then submitted to the throne. As historians of science have shown, Zhang's petition triggered a pivotal shift in Qing policies and politics in the direction of "the natural sciences." While filled with the compulsory expressions of praise, the petition concludes by subtly provoking the Kangxi Emperor to share his knowledge in these matters with his subjects. Implicitly, this was a plea not to hold it back only to whip it out to humiliate his poor courtiers. And though the Kangxi Emperor formally rebuked Zhang's *Petition*, saying that he had better things to do with his precious time, he eventually accepted Zhang's requests. Besides opening up his once guarded library of scientific instruments and manuscripts, he also recruited Han Chinese scholars to compile the *Origins of Cosmological Sciences* (lüli yuanyuan 律歷淵源, 1723) whose volumes 43-47, *Orthodox Meaning of Pitch Pipes* (lülü zhengyi 律呂正義), is of primary concern in this dissertation.[29] Notably, while *Origins* ended up allocating only five volumes to *Orthodox*, compared to the forty-two volumes for a treatise on astronomy and calendar-keeping and fifty-three volumes for a treatise on mathematics, Zhang's *Petition* shared Li's complete silence on the astronomy and mathematics portion of the court gathering. In fact, Zhang went even farther than Li in eulogizing the Emperor's otherwise mundane demonstration of the mundane phenomenon of octave equivalence in Chinese music theory and practice.

Of course, in contrast to Li Guangdi's specialist knowledge in music theory, Zhang Yushu never authored any treatise on music or touted any specialist knowledge of musical tuning. Yet he was no mere pedant of Neo-Confucian philosophical musings either. A *jinshi* at

29 For the reception of Zhang's *Petition* by the Kangxi Emperor, particularly regarding scholarships on astronomy and mathematics, see Jami, *The Emperor's New Mathematics*, 229-235.

the even more impressive age of nineteen, Zhang was an experienced overseer of the Qing's river regulations, a favorite subject of inquiry for the *shixue* or concrete-learning scholars, given the strategic importance of canals and the frequency of floods (indeed, the significance of water management for practically oriented Confucian scholars was reflected in the Emperor's question about calculating the water flow rate at sluice gates). Zhang was also one of the two *zongzhuan* 總撰,"authors in chief," of the imperially sponsored *History of the Ming* (ming shi 明史, 1739), the official dynastic history of the Qing's predecessor regime in China. He thus had under his command some of the most learned Han Chinese scholars on music as well as astronomy, geomancy, calendar-keeping, geography, rites, imperial processions, garments, education, civil service, economics, river regulations, penal code, and bibliography: in accordance with the established paradigm of dynastic historiography, *History of the Ming* features many *zhi* 志 "treatises," each dedicated to one of these highly specialized and technical aspects of Ming statecraft.

All this means that it was one thing for Zhang to concede his own ignorance over the meaning of *geba xiangsheng* "mutual generation at every eighth step" and quite another to describe the Emperor's demonstration of the broadly used term as follows:

> […] 至于十二律隔八相生，宋儒載其圖、具其說，而其自然之理與所以然之故，未有能實指之者。皇上命樂工以篴和瑟審其聲音。七音高下次第相生，至第八聲復還其始。所謂隔八相生之法，其本原實在乎是，從來論樂者，皆未之及。蓋千古未發之祕！自我皇上今日發之非聖心神悟本于天授何由得此？[30]
>
> […] As for "mutual generation at every eighth step" for the twelve tuning pitch pipes, Song-era [960-1279] Confucian scholars drew diagrams of it and detailed explanations of it, yet there has not

30 Zhang Yushu, *Zhang Wenzhen gong ji* 張文貞公集 ("Anthology of Sir Zhang Wenzhen," c. 1710), a facsimile version available on available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5042, accessed April 14, 2019), Part 1, 88-89.

> been anyone who has concretely pinpointed its natural principle or its cause. The Emperor asked musicians to play a *di* flute in harmony with a *se* zither in order to examine their sounds and tones. The seven notes of varying heights followed one another in succession, and they returned to their beginning at every eighth step. This is really the essence and origin of the so-called principle of "mutual generation at every eighth step," yet from the past until the present, those who discuss music have never understood this. Indeed, an insight that had not been uncovered for all eternity has been discovered on this day by our Emperor: how could it be, except that his Sagely Mind and Godly Intellect are granted by Heaven? […]

Even stripped of its conventional hyperbole about the Emperor's mind and intellect, Zhang's description makes stunning claim: the Emperor did not simply demonstrate the principle of octave equivalence as the meaning of *geba xiangsheng* "mutual generation at every eighth step," he discovered it. While Li Guangdi presumed to speak only for "people like us," Zhang wrote as if no one had ever revealed this patently rudimentary feature of Chinese music and organology as the essence of and reason for a phrase as hackneyed as "mutual generation at every eighth step." Zhang wrote as if neither Zhu Xi, nor Cai Yuanding, nor Zhu Zaiyu 朱載堉 (1536-1610)—who, like Cai, was actually mentioned by name at the court gathering (more on this later)—nor any of the scholars under his command working on the "Treatise on Music" (*yuezhi* 樂志) of *History of the Ming* had ever been enlightened enough to figure this out: when proceeding by the order of *gong* (*do*), *shang* (*re*), *jue* (*mi*), *bianzhi* (*fa*#0), *zhi* (*sol*), *yu* (*la*), and *biangong* (*ti*)—or *shang*, *che*, *gong*, *fan*, *liu/he*, *wu*/*si*, and *yi*, if one prefers the less solemn *gongche* syllables—on either a zither or a flute, one necessarily returns to *gong* (*do*; or *shang*) at every eighth step and begins a new cycle of the seven-note diatonic scale, i.e. "the five proper notes and two altered notes."

In reality, it is no surprise that Li Guangdi and Zhang Yushu characterized the Emperor's musical lecture as a revolutionary discovery. Both Li and Zhang, like the imperial diarists,

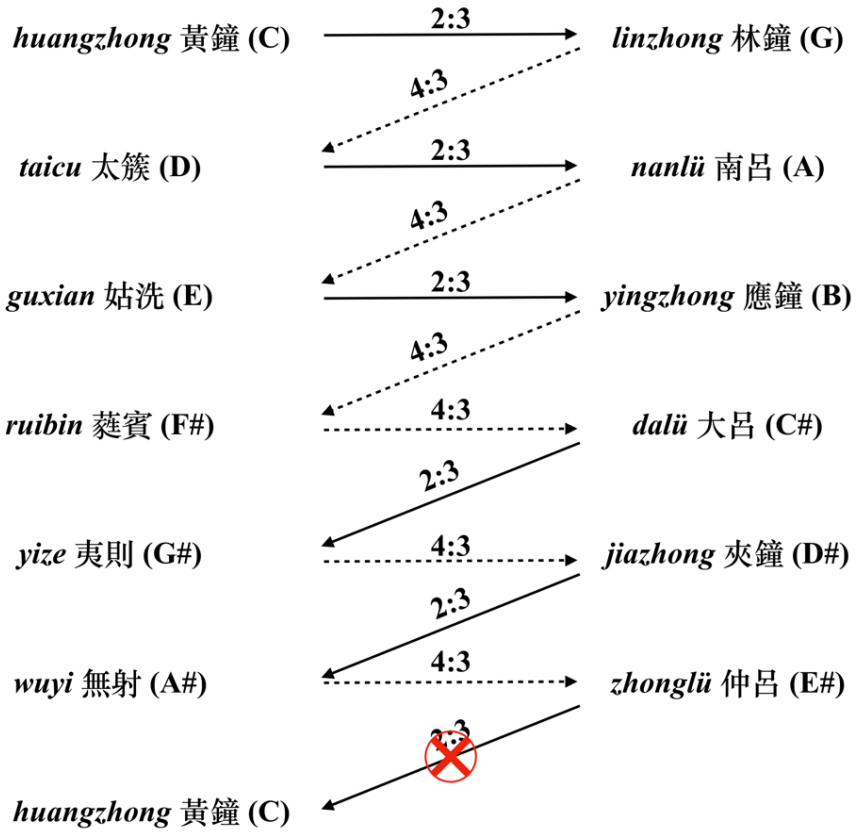
skipped over a detail in their adulations of the Emperor's demonstration of *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence: he actually got it completely wrong.

Putting the Emperor on Trial

In its every documented usage, the phrase *geba xiangsheng* 隔八相生 "mutual generation at every eighth step" has had nothing to do with octave equivalence, notwithstanding what a fundamental a principle of traditional Chinese music it has been. Rather, before the Emperor's groundbreaking demonstration of February 20, 1692, the phrase had referred to a specific relationship among the twelve tuning pitch pipes that sufficiently and necessarily results from the Chinese Pythagorean tuning method of *sanfen sunyi* 三分損益 "triple division with one part subtracted or added."[31] In fact, if there were to be a term from Western music theory deemed more or less equivalent to "mutual generation at every eighth step," it would not be octave equivalence but rather circle of fifths, which I will use as a translation of *geba xiangsheng* for convenience's sake from time to time. Indeed, the two phrases *sanfen sunyi* "triple division with one part subtracted or added" and *geba xiangsheng* "mutual generation at every eighth step" were frequently found together in music theory prose because they are really two sides of the same coin. The former prescribes how the lengths of a whole sequence of twelve tuning pitch pipes can be generated from the single predetermined length of the *huangzhong* pipe, namely by alternating between ratio 2:3 and its double, 4:3. The latter phrase, *geba xiangsheng* "mutual

31 See also Qi Mingjing, "試從《御製律呂正義》議'康熙十四律'" ("Discussions of 'Kangxi Fourteen Temperament' from Perspectives of *Imperial Formulae for Musical Temperament and Tuning*"), *Wenhua yishu yanjiu* 文化藝術研究 5 (2012): 122, in which the author also notices the Kangxi Emperor's mistake, albeit in a somewhat cursory remark.

Illustration 1-4 The "mutual generation" of the lengths of the twelve tuning pitch pipes through "triple division with one part subtracted or added"[32]



generation at every eighth step," in turn, describes where this generative ratio of 2:3 is to be found after such a sequence of twelve tuning pitch pipes is rearranged in decreasing order of

32 In this illustration, the ratio of 2:3 is represented by a solid line and the ratio of 4:3 is represented by a dashed line; in both cases, the arrow originates from the pipe to whose length the respective ratio is applied and pointing towards the pipe that is being generated. Notice a break of the alternation between the two ratios at *ruibin* 蕤賓 (F#). While *ruibin* is itself generated from the shorter *yingzhong* pipe through the 4:3 ratio, were one to apply the alternate 2:3 ratio to *ruibin*, the resulting pipe would be shorter than even the half-length *huangzhong* 黃鐘 (C) pipe; therefore, the 4:3 ratio is applied again to *ruibin*, so that the resulting pipe will fall between *huangzhong* and *taicu* 太簇 (D) in length. Notice also that, while the alternation between 2:3 and 4:3 would suggest that the same 2:3 ratio should be applied to the *zhonglü* 仲呂 (E#) pipe. Nonetheless, as I will explain more in detail later in this chapter, the length ratio that would get one from *zhonglü* (E#) back to *huangzhong* (C)—technically the half-length *huangzhong* pipe—would not be 2:3 (approximately 0.6667) but 177147:262144 (approximately 0.6758), which is known as the Pythagorean wolf in Western music theory.

lengths: specifically between any two pipes that are eight pipes apart (with one recurrent exception) when both pipes are counted.

To explain from the beginning: *xiangsheng* 相生 "mutual generation" describes the situation where, in a series of tuning pitch pipes, the length of any one pipe is *sheng* 生 "generated" from the length of another pipe by applying a specific predetermined proportion. Under the Chinese Pythagorean tuning method of "triple division with one part subtracted or added," this proportion can only be either 2:3 or its double 4:3. For example, after the standard length of the *huangzhong* 黃鐘 pipe (1st of the twelve tuning pitch pipes), is established, the length of the *linzhong* 林鐘 pipe (8th) is computed by applying the 2:3 ratio to the length of the *huangzhong* pipe. Thus, the *huangzhong* pipe is said to have *sheng* "generated" the *linzhong* pipe. In turn, the length of the *taicu* 太簇 pipe (2nd) is computed by applying the 4:3 ratio to the length of the pipe of *linzhong*, and thus the *linzhong* pipe—itself having been generated by *huangzhong*—is in turn said to have generated the *taicu* pipe. Illustration 1-4 shows the "mutual generation" of all the twelve tuning pitch pipes fashioned according to "triple division with one part subtracted or added": the length of all twelve tuning pitch pipes are derived one after another from the single preestablished length of the *huangzhong* pipe through the successive alternation of two predetermined ratios, 2:3 and 4:3.

What about *geba* 隔八 "at every eighth step?" One can be easily led astray by the luring affinity between "eight" and "octave": in any diatonic scale, every two notes that are eight notes apart, with themselves counted, yield an octave. Nonetheless, if one were to borrow Western terminologies again, the "eighth step" in "mutual generation at every eighth step" refers not to the eighth diatonic step but actually to the eighth chromatic step. Illustration 1-5 rearranges in decreasing orders of lengths all the twelve tuning pitch pipes that have been "generated" in

Illustration 1-5 The "mutual generation at every eighth step" as the pattern of 2:3 ratios among the twelve tuning pitch pipes[33]

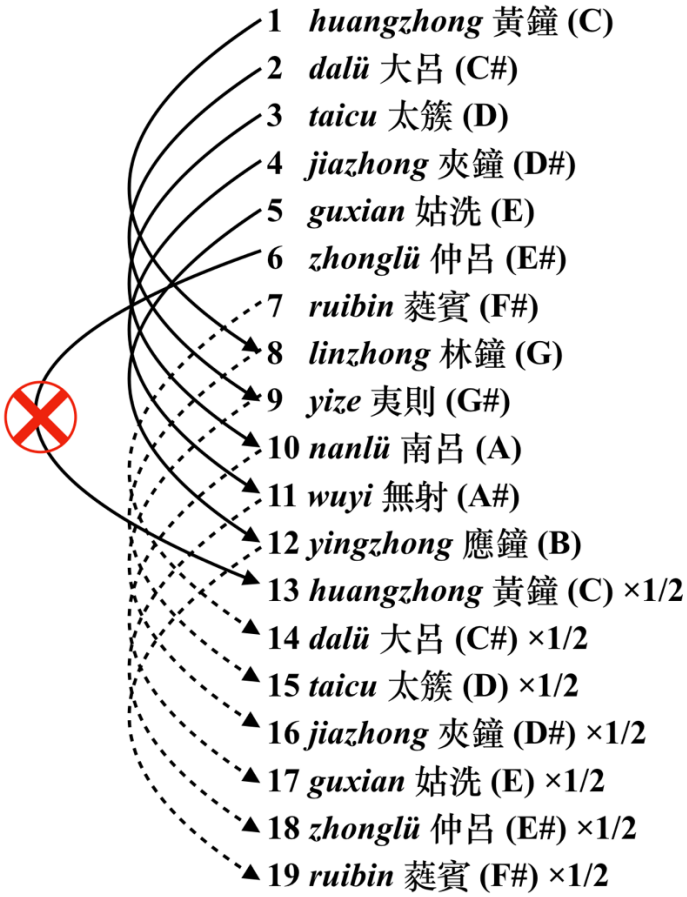


Illustration 1-4 according to the Chinese Pythagorean tuning method. Seven additional pipes, whose lengths are half as long as those from *huangzhong* (C) to *ruibin* 蕤賓 (F#), are added. In Western musical terminology, such a sequence of pitch pipes originally generated through the Pythagorean proportions of 2:3 and 4:3 create a chromatic scale, wherein the notes of any two adjacent pitch pipes are a semitone apart. In such a sequence in Illustration 1-5, furthermore, for

33 Notice that in this illustration, both the solid lines and the dashed lines represent a length ratio of 2:3. Nevertheless, whereas the solid lines connect two pipes whose lengths bear a 2:3 ratio in the previous Illustration 1-4, the dashed lines connect two pipes whose lengths bear a 4:3 ratio in the said previous illustration; the difference for the latter pipes is that the pipes whose lengths are generated through a 4:3 proportion in Illustration 1-4 are here replaced with pipes that are half as long, thus rendering those 4:3 ratios into 2:3.

any two pitch pipes eight pitches apart, always with themselves counted, it must be the case that one of them has "generated" the other pipe through a 2:3 or 4:3 ratio back in Illustration 1-4. For example, in Illustration 1-4, the pipe of *huangzhong* (1st pipe) has generated the pipe of *linzhong* (8th), whose length is defined as 2/3 of *huangzhong*; accordingly, in Illustration1-5, there are exactly eight pipes from the pipe of *huanghzong* to the pipe of *linzhong*: *huangzhong*, *dalü* (2nd), *taicu* (3rd), *jiazhong* (4th), *guxian* (5th), *zhonglü* (6th), *ruibin* (7th), and *linzhong*. Similarly, in Illustration 1-4, the pipe of *linzhong* has generated the pipe of *taicu* (2nd), whose length is computed as 4/3 of *linzhong* (or 2/3 of *linzhong* doubled); accordingly, in Illustration 1-5, there are exactly eight pipes from the original pipe of *linzhong* to the pipe of half-length *taicu* (2/3 of *linzhong*): *linzhong*, *yize* (9th), *nanlü* (10th), *wuyi* (11th), *yingzhong* (12th), half-length *huangzhong* (13th), half-length *dalü* (14th), and half-length *taicu* (15th). In fact, as Illustration 1-5 shows, except for between *zhonglü* (6th) and any one of the two *huangzhong* pipes—the length proportion between the original *zhonglü* and the half-length *huangzhong* pipes is 177147:262144, identical to the so-called "wolf interval" in Pythagorean tuning, for example the augmented third between E# and C—this *xiangheng* "mutual generation" relationship corresponding to a length proportion of 2:3 necessarily occurs between any two pipes eight pipes apart with themselves counted. Ergo *geba xiangsheng* "mutual generation at every eighth step."

Logically, in order to prove that my above explanation of "mutual generation at every eighth step" was its sole universally received meaning prior to February 20, 1692, I ought to enumerate and analyze all known usages of the phrase up to that point. In truth, however, only one author needs to be called to the witness stand: Zhu Zaiyu. Invoking a Ming's *prince du sang* to disprove a Qing emperor's music-theory lecture might seem utterly unfair, especially since music historians today often compare the Kangxi Emperor's tuning reforms unfavorably to Zhu

Zaiyu's celebrated twelve-tone equal temperament. Yet the attitude of the Kangxi Emperor towards Zhu Zaiyu and his theories was much more amicable than most modern scholars have imagined. Besides, using Zhu's discussion of "mutual generation at every eight step" in his magnum opus *Essential Meaning of Pitch Pipes* (lülü jingyi 律呂精義, 1596) as an example for the phrase's received meaning also offers a convenient springboard for examining his broader proposal of twelve-tone equal temperament, which, though tangential at this moment, will become a critical point of reference for unpacking the sources and knowledge-production processes of the Qing's tuning reform. Most importantly, Zhu Zaiyu's systematic repudiations of the received orthodoxies of Chinese musical tuning in favor of his twelve-tone equal temperament actually make him the best witness not only to how most people at the time understood "mutual generation at every eighth step" as an individual phrase, but also to where this concept belonged and how it functioned in the context of the larger epistemic structure of music. This latter significance holds the key to understanding contextually the role of the Kangxi Emperor's otherwise innocuous misinterpretation of "mutual generation at every eighth step" within the development of the fourteen-tone temperament itself and as part of the broader epistemological shift in which the tuning reform participated.

Admittedly, it does not require too far a reach reading to get a sense of dynastic rivalry out of the reception of Zhu Zaiyu, the Ming prince's monumental writings on music theory during the Qing era. The rivalry reached its peak towards the end of the 18th century when, under the Qianlong Emperor (r. 1736-1793), Qing imperial treatises on music would become overtly hostile towards Zhu Zaiyu's works, including his proposal of twelve-tone equal temperament,

among other writings on *lülü* "tuning pitch pipes" or musical tuning.[34] Nonetheless, no evidence allows us to trace such hard feelings back to the Kangxi Emperor. On the contrary, accounts from the court gathering on February 20, 1692 show that the Emperor appeared to consider Zhu Zaiyu an underappreciated maverick to the point of defending him in front of his Han Chinese advisors. According to Li Guangdi, when the Emperor quizzed his scholar-officials on how to calculate the circumference of a circle, he specifically demanded that they choose sides between Cai Yuanding's *New Treatise on Pitch Pipes* (c. 1180s) and Zhu Zaiyu's *Essential Meaning of Pitch Pipes* (1596). Cai, as mentioned earlier, followed the old rule of thumb *jingyi weisan* "the circumference is three when the diameter is one," which Zhu Zaiyu repudiated by proposing a new approximation wherein, visualized in modern algebraic notation, the circumference c is computed in relation to the diameter d as

$$c = \frac{40}{9} \times \sqrt{\frac{d^2}{2}}$$

or roughly 3.1426968 times diameter.[35] In Li Guangdi's description, the Emperor became *da buping* 大不平 "greatly indignant" when his scholar-officials sided with Cai only on account of his reputation as a disciple of the revered Zhu Xi, the aforementioned progenitor of Neo-Confucianism, who also happened to have penned the very preface to Cai's *New Treatise*.[36] After showing how Zhu Zaiyu's method of calculating the circumference was much more accurate, the Emperor lamented the state of scholarship, as Zhang Yushu recounted in another recollection of

34 See Weng Panfeng 翁攀峰, "皇權與樂律—乾隆時期對十二平均律的批判" ("Imperial Power and Musical Tuning: Criticism of Twelve-Tone Equal Temperament during the Qianlong Era"), *Ziran bianzhengfa tongxun* 自然辯證法通訊, 2016, No. 1: 86-91.

35 Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, Vol. 1, f. 33r.

36 Li Guangdi, *Quotations of Sir Rongcun: A Sequel*, vol. 17, f. 24v.

the event, published in his posthumous *Anthology of Sir Zhang Wenzhen* (zhang wenzhen gong ji 張文貞公集, c. 1710s; Wenzhen 文貞 was Zhang Yushu's posthumous courtesy name):

> […] 今人所講算法 […] 皆踵襲宋人舊說，不自知其非是。且人縱知徑一圍三之誤，若以此語人，必羣起而非之，以為宋人既主此論，不可不從。究竟試諸實用，一無所驗。爾等第依其法，試之，當自了然也。[…][37]
>
> […] The methods of calculation preached by scholars today […] all eagerly follow the old tenets of those from the Song era without knowing for themselves what is correct or what is mistaken. And even if there were to be someone who knew "the circumference is three when the diameter is one" to be wrong, were they to openly profess it to someone else, there would end up being a mob rising up to attack them, arguing: "given that scholars from the Song era have affirmed these theories, they are not to be disobeyed." Yet in the end, none of those theories can be substantiated when put to the test through concrete applications. You will realize this yourself once you follow [these different methods of calculation] one after another and try them out. […]

Admittedly, the Emperor's "great indignance" was more epistemic than music-theoretical. Rather than anointing Zhu Zaiyu as the new authority in musical tuning or *suanfa* "methods of calculation" or "algorithm," he criticized the very credulity among his learned Han Chinese scholar-officials towards established authorities, at the expense of evidence-based verification. In fact, in the same harangue, the Emperor remarked that "not even could Zhu Zaiyu's relentless censure of 'triple division with one part subtracted or added' could be impeccable"; this referred to Zhu Zaiyu's proposal of a twelve-tone equal temperament, which substituted irrational ratios for the simple Pythagorean proportions of 2:3 and 4:3.[38] Similarly, by documenting the Emperor's disapproval of Cai Yuanding and "people from the Song era," Li

37 Zhang Yushu, *Anthology of Sir Zhang Wenzhen*, a facsimile version available on available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5042, accessed April 14, 2019), Part 4, 101-102.

38 Ibid, 99-100.

Guangdi and Zhang Yushu aimed not at elevating Zhu Zaiyu personally but rather at joining their voices to the growing chorus of literati who favored concrete learning as against Zhu Xi's Neo-Confucianism, which was often referred to, somewhat derogatorily, as *songxue* 宋學 "Song learning," after the dynasty under which Zhu Xi, Cai Yuanding, and many of his disciples lived.[39] Still, the anecdote proved that the Kangxi Emperor was perfectly receptive to the Ming prince even though he appeared to know perfectly well Zhu Zaiyu's characteristic iconoclasms, including especially his rejection of *sanfen sunyi* "triple division with one part subtracted or added."

Where "Mutual Generation" Belongs

Ironically, it is precisely in rejecting this Chinese Pythagorean tuning method as the age-old cornerstone of *lülü* or musical tuning that Zhu Zaiyu left some of the most reliable testaments to the "common practice" of Chinese musical tuning—including the received meanings of the very principles, such as *geba xiangsheng* "mutual generation at every eighth step," that his twelve-tone equal temperament sought to supplant. Having first formulated his *xinfa* 新法 "New Method" in *A New Theory of the Study of Pitch Pipes* (lüxue xinshuo 律學新說, 1584),[40] Zhu Zaiyu authored the much more expansive *Essential Meaning of Pitch Pipes* twelve years later to systematically take down the *jiufa* 舊法 "Old Method," with the Chinese Pythagorean tuning method at its heart. Granted, Zhu Zaiyu never explains in plain words what "mutual generation at

39 See Benjamin Elman, *From Philosophy to Philology: Intellectual and Social Aspects of Change in Late Imperial China* (Cambridge, MA: Harvard University Asia Center, 1984), translation in Chinese by Zhao Gang 趙剛 (Nanjing: Jiangsu People's Publishing Ltd., 2012), 45-47.

40 Zhu Zaiyu, *A New Theory of the Study of Pitch Pipes*, vol. 1, ff. 5r-7r.

every eighth step" actually means, probably deeming it too rudimentary to be worth the effort. Nevertheless, analyzing his discussion of the phrase in *Essential Meaning* not only shows his understanding of it to be identical to my explanation above but also sheds light on the position of this concept in the established epistemic structure of Chinese musical tuning.

Indeed, the organization of the opening chapters of Zhu Zaiyu's *Essential Meaning* shows that the Kangxi Emperor's gloss of *geba xiangsheng* "mutual generation at every eighth step" was not simply "wrong" but also completely off the mark:

> 總論造律得失第一
> 不宗黃鐘九寸第二
> 不用三分損益第三
> 不拘隔八相生第四
> 不取圍徑皆同第五
> 新舊法參校第六
> 新舊律實驗第七[41]
>
> One A Summary on Previous Successes and Failures of Making Pitch Pipes
> Two "The Length of the *Huangzhong* Pipe should be 9 *cun*" is Not to be Followed
> Three "Triple Division with One Part subtracted or added" is Not to be Used
> Four "Mutual Generation" is Not to be Restricted to "at Every Eighth Step"
> Five The Same Circumference or Diameter is Not to be Applied to All Tuning Pitch Pipes
> Six A Mutual Comparison of the Old and New Methods
> Seven Concretely Verifying the Old and New Pitch Pipes

As shown earlier, by explaining *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence, the Kangxi Emperor associated it with the seven-note diatonic scale system, i.e. *wusheng erbian* "the five proper notes and two altered notes." But in Zhu Zaiyu's treatise not even the names of the seven notes of the scale are mentioned until later in Chapter Nine, which

41 Extracted from Zhu Zaiyu, *Essential Meaning of Pitch Pipes* (1596), vols. 1-5.

introduces the concepts of modes and transpositions using, not unlike the Emperor, a multi-string zither.[42] Meanwhile, Zhu Zaiyu places his chapter on "mutual generation at every eighth step" squarely in a different realm of musical knowledge involving a different type of musical instrument: the measuring and proportioning of the twelve *lülü* "tuning pitch pipes."

The specific affinity between *geba xiangsheng* "mutual generation at every eighth step" and the twelve tuning pitch pipes in Zhu Zaiyu's *Essential Meaning* is not a matter of organological preference. Instead, it is an instantiation of the role of pitch pipes as the material foundation of musical tuning—and, accordingly, an instantiation of the epistemic association of "mutual generation at every eighth step" with the proportioning of the twelve tuning pitch pipes as a foundation of Chinese musical tuning. Indeed, Zhu Zaiyu's New Method was not merely an abstract mathematical exercise in calculating the various irrational ratios that constitute the twelve-tone equal temperament. Rather, much like the Old Method he was railing against, his New Method was a "user guide" that explained as well as justified how a set of twelve tuning pitch pipes can be fashioned as tangible objects, so that they could in turn serve as the standard for tuning all other instruments.

Thus, though it is Zhu Zaiyu's replacement of the rational ratios of 2:3 and 4:3 with the irrational ratios based on various integer exponentiations of $1:\sqrt[12]{2}$ that would make him the famous inventor of twelve-tone equal temperament, his New Method altered almost every other aspect of the twelve tuning pitch pipes in equally iconoclastic fashion. This iconoclasm is evident even from the titles of Chapters Two to Five of *Essential Meaning* that present the New Method, each comprising a negation—*buzong* 不宗 "not following," *buyong*, 不用 "not using," *buju* 不拘 "not being restricted to," and *buqu* 不取 "not applying"—of long-established tenets

42 Ibid., vol. 9, ff. 1-10.

for making the twelve tuning pitch pipes. Chapter Two dismisses widely esteemed sources on historical metrology and on the exact length of the *huangzhong* pipe as heretical fabrications (more on this in the next chapter of this dissertation).[43] Chapter Three, the crux of Zhu Zaiyu's twelve-tone equal temperament, formally parts ways with the two-millennium-old ratios of 2:3 and 4:3.[44] Instead, it first obtains $\sqrt[12]{2}$ through consecutive extractions of two square and one cube roots and then applies the reciprocal ratio, $1:\sqrt[12]{2}$, to the length of *hunagzhong* eleven times, thus yielding the remaining eleven pitch pipes. Chapter Five, to which I will return in the fourth chapter of this dissertation, rejects the conventional design of *tongjing guan* 同徑管 "pipes of the same diameter," which applies the base diameter of the *huangzhong* pipe to all the other tuning pitch pipes, regardless of their length. Instead, Zhu Zaiyu opts for *tongxing guan* 同形管 "pipes of the same shape," a design that applies the length-to-diameter ratio of the *huangzhong* pipe to all the other pipes, so that the same Pythagorean proportions generating the various lengths of all pitch pipes also generate their diameters.[45] And finally, Chapter Seven concludes this manual on matching pitch pipes by discussing where to harvest the best bamboos, what types of files and rasps are needed for bamboo-working, how to carve open an embouchure hole, and how to properly blow the pipes when using them either to compare different tuning systems or to tune other instruments.[46]

Within this progressive series of guidelines on pitch-pipe measuring and making, Chapter Four discussing—or, to be more precise, refuting—*geba xiangsheng* "mutual generation at every eighth step" follows right after Chapter Three on *sanfen sunyi* "triple division with one part

43 Ibid., vol. 1, ff. 7v-9r.
44 Ibid., vol. 1, ff. 9r-13v.
45 Ibid., vol. 2, ff. 1-36 and vol. 3, ff. 37-79.
46 Ibid., vol. 5, ff. 116r-116v.

subtracted or added." In further testimony to the close affinity between the two concepts, the opening lines of Chapter Four immediately make clear that "mutual generation at every eighth step," as I have earlier explained, refers to a logically sufficient and necessary outcome of "triple division with one part subtracted or added," that is, the Chinese Pythagorean tuning method, so that Zhu Zaiyu's rejection of the latter in favor of twelve-tone equal temperament inevitably demands that "mutual generation" be emended:

> 新法不拘隔八相生，而相生有四法。或左旋、或右旋，皆循環無端也，以證三分損益往而不返之誤。[…][47]
>
> Under the New Method, "mutual generation" is not restricted to "at every eighth step"; instead, there are four patterns of mutual generation. Some of them rotate to the left, some of them rotate to the right, and all of them are limitless cycles, proving that "triple division with one part subtracted or added" errs in not being able to return to where it has begun.

Crucially, Zhu Zaiyu does not reject "mutual generation at every eighth step" *in toto* but only takes issue with the part about "at every eighth step." Even to this latter part, Zhu Zaiyu does not reject it per se it so much as he seeks to perfect and expand it. This attempt to perfect "mutual generation at every eighth step" is evident in the first of the four patterns of "mutual generation" Zhu Zaiyu enumerates, which should be quite familiar by now:

> […]其一：黃鐘生林鐘、林鐘生太簇、太簇生南呂、南呂生姑洗、姑洗生應鐘、應鐘生蕤賓、蕤賓生大呂、大呂生夷則、夷則生夾鐘、夾鐘生無射、無射生仲呂、仲呂生黃鐘。長生短，五億乘之；短生長，十億乘之，皆以七億四千九百一十五萬三千五百三十八除之。[…][48]
>
> […] The first pattern: *huangzhong* [rough equivalent of C] generates *linzhong* [G], *linzhong* generates *taicu* [D], *taicu* generates *nanlü* [A], *nanlü* generates *guxian* [E], *guxian* generates *yingzhong* [B], *yingzhong* generates *ruibin* [F#], *ruibin* generates

47 Ibid., vol. 1, f. 13v.
48 Ibid.

> *dalü* [C#], *dalü* generates *yize* [G#], *yize* generates *jiazhong* [D#], *jiazhong* generates *wuyi* [A#], *wuyi* generates *zhonglü* [E#], and *zhonglü* generates *huangzhong*. When a longer pipe generates a shorter pipe, the ratio is 500,000,000:749,153,538 [= 0.6674199 …]; when a shorter pipe generates a longer pipe, the ratio is 10,000,000:749,153,538 [=1.33483985…].

Indeed, even though Zhu Zaiyu never takes pains to explain that once these tuning pitch pipes are arranged in a decreasing order of their lengths, there are exactly eight pipes from *huangzhong* (C) to *linzhong* (G), from *linzhong* to half-length *taicu* (D; specifically, the pipe whose length is half that of the *taicu* mentioned in the quote above), from *taicu* to *nanlü* 南呂 (A), from *nanlü* to half *guxian* 姑洗 (E), … and from *wuyi* 無射 (A#) to *zhonglü* 仲呂 (E#), there can be hardly any doubt that he understands this generative cycle of pitch pipes as being exactly what "mutual generation at every eighth step" means. Not only does he refer to it as one of the four patterns of "mutual generation," he even lists it as the first pattern in a chapter that seeks to introduce new patterns of "mutual generation" in addition to "at every eighth step."

What is different about Zhu Zaiyu's *geba xiangsheng* "mutual generation at every eighth step," however, is that he transforms it from a definite chain into an indefinite circle. As Illustrations 1-4 and 1-5 above have shown, under Chinese Pythagorean tuning, *zhonglü* (E#), the last one of the twelve tuning pitch pipes to have its length generated through successive alternations of 2:3 and 4:3, cannot proceed likewise to "generate" the half *huangzhong* pipe (C), which would be the "eighth step" if one begins with *zhonglü* and counts towards the shorter pipes. This is because the length ratio between them is not 2:3 (=0.66666…) but rather the notably larger 177147:262144 (=0.67576…). As a result, *geba xiangsheng* "mutual generation at every eighth step" under the Chinese Pythagorean *sanfen sunyi* "triple division with one part subtracted or added" necessarily requires an exception whenever it comes to a *zhonglü-huanghzong* (E#-C) pair. In Zhu Zaiyu's words, this chain of mutual generations *wang er bufan*

往而不返 "fails to return to where it has begun," that is, is unable to use the same ratios of 2:3 or 4:3 to return to *huangzhong* from whose predetermined length all other pitch pipes have been generated through those ratios. Once Zhu Zaiyu substitutes a tempered $1:\sqrt[12]{2^7}$ (which he calculates as 500,000,000:749,153,538) for 2:3 and a tempered $2:\sqrt[12]{2^7}$ (which he calculates as 1,000,000,000:749,153,538) for 4:3, however, the length ratio between *zhonglü* (E#) and half *huangzhong* also become $1:\sqrt[12]{2^7}$, hence no different from the ratio between any other two pipes eight ones apart in the decreasing order of length when both pipes are counted. By thus closing the "mutual generation" gap between *zhonglü* and half *huangzhong*, Zhu Zaiyu's twelve-tone equal temperament transforms—or, in his opinion, improves—*geba xiangsheng* "mutual generation at every eighth step" into *xunhuan wuduan* 循環無端 "a cycle without end" or, in the term of Western music theory, a true circle of fifths without the Wolf-tone problem.

Tempering the two Chinese Pythagorean ratios of 2:3 and 4:3 also allows for Zhu Zaiyu's three other patterns of "mutual generation." By applying a $1:\sqrt[12]{2^5}$ ratio (which Zhu Zaiyu calculates as 500,000,000:667,419,927) when a longer pipe generates a shorter pipe and a $2:\sqrt[12]{2^5}$ ratio (which Zhu Zaiyu calculates as 1,000,000,000:667,419,927) when a shorter pipe generates a longer pipe, Zhu Zaiyu produces a second pattern that might be called "mutual generation at every sixth step," indeed also a circle of fifths identical to his first "mutual generation" pattern "at every eighth step," except in the reverse direction: from *huangzhong* (C) to *zhonglü* (E#), to *wuyi* (A#), to *jiazhong* (D#), to *yize* (G#) … to *taicu* (D), to *linzhong* (G), and finally back in full circle to *huangzhong* (C).[49] By applying the $1:\sqrt[12]{2}$ ratio (which Zhu Zaiyu calculates as 500,000,000:529,731,547) in succession, Zhu Zaiyu produces a third pattern that

49 Ibid, vol. 1, f. 14r.

might be called "mutual generation at every second step," or a cycle of semitones in Western terminology: from *huangzhong* (C) to *dalü* (C#), to *taicu* (D), to *jiazhong* (D#), to *guxian* (E) … and finally to *yingzhong* (B) before reaching half *huangzhong* (C).[50] And lastly, by applying the $1:\sqrt[12]{2^{23}}$ ratio (which Zhu Zaiyu calculates as 1,000,000,000:943,874,312) in succession, Zhu Zaiyu produces a fourth pattern identical to the third, except in the reverse direction: from half *huangzhong* (C) to *yingzhong* (B), to *wuyi* (A#), to *nanlü* (A), to *yize* (G#) … and finally to *dalü* (C#) before reaching *huangzhong* (C).[51] None of these closed cycles would have been possible under "triple division with one part subtracted or added," which not only engenders a "Wolf" ratio of 177147:262144 between *huangzhong* and *zhonglü* but also stipulates among different pairs of adjacent pipes two different ratios of 243:256 and 204:2187, which are numerically identical to the diatonic and chromatic semitones in Western Pythagorean tuning. After exhausting these perfected and expanded patterns of "mutual generations," Zhu Zaiyu proudly summarizes his achievement in a postscript to Chapter Four:

> 謹按：[…] 新法不用三分損益、不拘隔八相生。然而相生有序，循環無端，十二律呂一以貫之。此蓋二千餘年之所未有，自我聖朝始也。[…][52]
>
> We respectfully note: […] The New Method does not use "triple division with one part subtracted or added" and does not restrict "mutual generation" to "at every eighth step." Rather, the cycles of mutual generations are orderly and limitless, and they are implemented throughout all the twelve tuning pitch pipes. This has indeed not been discovered over the past two millennia, yet it now starts with Our Sacred Dynasty [referring to the Ming dynasty].

To conclude, the iconoclast who seeks to replace *sanfen sunyi* "triple division with one part subtracted or added" and *geba xiangsheng* "mutual generation at every eighth step," the

50 Ibid.
51 Ibid.
52 Ibid, vol. 1, f. 30v.

Pythagorean cornerstone of Chinese musical tuning, only further attests to the kinship between the two concepts in centuries of scholarship, until it was ruptured by the Kangxi Emperor on February 20, 1692.

An Innocuous—but not Insignificant—Mistake

No wonder Li Guangdi and Zhang Yushu reacted to the Emperor's explanation of *geba xiangsheng* "mutual generation at every eighth step" as though nobody had ever explained a concept as simple as octave equivalence: nobody had ever committed such a basic error on record. From a technical standpoint (and taking a liberty yet another time with Western terminology), the Emperor mistook an octave-based concept (every eighth diatonic step, or every octave) for a fifth-based one (every eighth half step, or every perfect fifth). As my analysis of Zhu Zaiyu's discussion of "mutual generations" has shown, however, the imperial blunder ran much deeper than simply messing up the type of interval or scale. Not only did the Emperor jettison the practically universal consensus over the meaning of the phrase in its thousands of years of usage, he even failed to identify the proper epistemic context in which the phrase functions in music-theoretical discourse. By misconstruing *geba xiangsheng* "mutual generation at very eighth step" as octave equivalence, the Emperor uprooted the phrase from its established signifying associations with the measuring and proportioning of the twelve tuning pitch pipes and misplaced it instead in the hitherto unrelated realm of the Chinese diatonic scale system. Thus, whereas virtually every author beforehand had situated "mutual generation at every eighth step" in their discussion on *sanfen sunyi* "triple division with one part subtracted or added" regardless of their opinion on the latter tuning method *per se*, the Emperor mismatched the phrase with another concept, *wusheng erbian* "the five proper notes and two altered notes," ot the diatonic scale system. This latter realm of knowledge is scarcely tangential to the Chinese

Pythagorean tuning method and to the twelve tuning pitch pipes as the "instruments of music theory."

But am I being a pedant here, spending page after page only to fuss over jargon? Even if the phrase "mutual generation at every eighth step" does not *technically* mean what the Emperor demonstrated using a flute and a zither, so what? Doesn't his explanation of the phrase make perfect sense on its own terms, within the new realm of musical knowledge in which he misplaced the phrase? Indeed, barring the received meaning of these signifiers, isn't it perfectly plausible—if not arguably more intuitive—to read "at every eighth step" as referring to a diatonic octave comprising "the five proper notes and the two altered notes" rather than a perfect fifth encompassing eight pitch pipes? Isn't it perfectly plausible to read "mutual generation" as referring to the replication of the same intervallic pattern and the same series of solmization syllables at every octave, rather than the presence of a specific tuning proportion between every eighth pitch pipe?

Moreover, even though there weren't any historical sources to support him, didn't the Emperor make his case with equally if not even more convincing evidence? Just as the conventional discourse on *geba xiangsheng* "mutual generation at every eighth step" employs the twelve tuning pitch pipes as proportionally generated and related, the Emperor demonstrated his understanding of the phrase with a zither and a flute. Just as the established explanation relies on observing the mathematical patterns of the lengths of the twelve tuning pitch pipes, the Emperor asked his courtiers to listen to the sonic patterns of the various pitches produced by the two instruments as configured through their varying finger positions. Thus, even if the Emperor ought to stand corrected for deviating from the agreed-upon definition of *geba xiangsheng*

"mutual generation at every eighth step" in the scholarly discourse of *lülü* "tuning pitch pipes" or "music theory," at most his was just an innocuous if not creative mix-up.

Perhaps, to acknowledge that the Emperor's explanation of "mutual generation at every eighth step" was indeed not wrong *per se* but only discursively inattentive, I should characterize it as a "reinterpretation" of the phrase, as opposed to a judgmental "misinterpretation." Such a change in diction would imply, however, that the Emperor consciously perceived himself to be revising an extant definition, even though all available sources of the court gathering on February 20, 1692 suggest that he did not appear in the least cognizant of its canonical definition. This unwittingness is crucial: it reveals the fact that the Emperor, as well as several of his sons, had acquired training in how to listen to and conceptualize about music from non-canonical sources, as I will show in detail in the fourth chapter of this dissertation.

Also unbeknownst to the Emperor or to any other attendant at the court gathering on February 20, 1692 were the consequences of his musical lecture-demonstration. Though never amounting to an unprecedented discovery as Li Guangdi's and Zhang Yushu's adulation would suggest, the Emperor's explanation of *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence instead of the circle of fifths effected a seismic epistemological reinvention of how musical knowledge was to be produced in the subsequent two decades of imperially sponsored research. It was this reinvention, furthermore, that culminated in the notorious fourteen-tone temperament in *Orthodox Meaning of Pitch Pipes*, commissioned in July 1713 and completed in December 1714.[53] Specifically, although "mutual generation at every

53 *Veritable Records of the Kangxi Reign* (1732), Vol. 254, see part 82, segment 122 of https://ctext.org/wiki.pl?if=gb&chapter=422212, accessed April 15, 2019. The relevant entry was dated on KX 52.6.23 or July 23, 1713; and Vol. 260, see part 83, segment 126 of https://ctext.org/wiki.pl?if=gb&chapter=422212, accessed April 15, 2019. The relevant entry was dated on KX 53.11.17 or December 23, 1714.

eighth step" would never come close to meaning octave equivalence in its established definition or usage, the Emperor's demonstration of his novel interpretation on a *se* zither and a *di* flute ended up elevating the acts of sounding and listening as indispensable sources of musical knowledge alongside historical and mathematical data. In so doing, it reinvented the field of *lülü* or musical tuning and music theory. It redrew and thereby problematized the epistemological boundaries between textual sources and listening experiments, between numbers and sounds, and between chordophones and aerophones in the study of musical tuning. This reinvention, furthermore, took place at a time when Qing-era researchers already faced new demands to simultaneously differentiate and reconcile historicist and empiricist types of evidence amid the rise of the so-called *kaozheng* 考證 or evidential learning movement, which emphasized proof and verification as the epistemic yardsticks of scholarship. As I will show in the following chapters, it was precisely this profound epistemic reinvention of music theory, inadvertently precipitated by the court gathering on February 20, 1692, that engendered the fourteen-tone temperament, a fact that offers crucial clues to the internal logic and raison d'être of the Qing's tuning reform that have evaded generations of music historians and music theorists.

Chapter Two

Taking an Epistemic Cue from the Emperor:

Towards the Fourteen-Tone Octave

Attributing a 1714 tuning reform seeking to divide the octave fourteen-fold back to a 1692 demonstration of a phenomenon as mundane as octave equivalence is far-fetched. But the keyword here is "epistemic."[1] The legacy of the court gathering, at least as far as the Qing's notorious tuning reform was concerned, was not redefining any technical term or redividing the octave into fourteen or some other unconventional number of portions. Rather, in transforming *geba xiangsheng* "mutual generation at every eighth step" from a notion related to proportioning the twelve tuning pitch pipes (circle of fifths) into one related to the diatonic scale system (octave equivalence), the Kangxi Emperor raised the unwitting question of how knowledge about tuning, music, and sound in general could or should be produced. Particularly, how could such knowledge be produced through the interactive and embodied processes of sounding, listening, and playing instruments, the very methods whereby the Emperor demonstrated his understanding of "mutual generation at every eighth step"?

If it seems like a foregone conclusion that empirical activities such as listening for octave relationships and mapping recurrent pitch-height patterns play a primary role in studying tuning and acoustics, let us consider the following. Though his gloss of "mutual generation at every eighth step" as octave equivalence was unprecedented, the Kangxi Emperor was neither the first

1 While I am aware of the Foucauldian connotations of this term (as well as the potential Foucauldian underpinnings of my use of it), I use the word "epistemic" simply to describe something that is related to the structure of knowledge or the process in which knowledge is produced and/or articulated. In contrast, I use the word "epistemological" specifically to describe something that is related to an explicit or implicit philosophy of how knowledge works.

nor the last to lambast Confucian scholars for their willingness to trust written wisdoms and reluctance to verify the latter through actual observations or experiments. This critique was particularly potent in musical tuning, for which practical knowledge was available through not only *yuegong* 樂工 "professional musicians," who typically came from the lowest social strata, but also through many literati themselves, such as Li Guangdi, who played the seven-string *qin* 琴 zither, a patent symbol of a scholar's artistic cultivation and expressive sensibility.[2]

Yet the epistemic challenge the Emperor's lecture-demonstration posed was not about how to incorporate empirical experiences *per se* but rather how to balance empirical processes and the data they generate against the study of ancient texts and records. Arguably, the modern disciplines of music theory and acoustics arguably tend less to acknowledge their cultural, ideological, or even at times aesthetic underpinnings. Yet the scholarly paradigm of *lülü* or musical tuning rested on its ideological promise of restoring the proper music of the ancient sages and, with it, their harmonious political institutions and social mores. This promise of restoration became all the more important amid the rising *shixue* "concrete leaning" and *kaozheng* "evidential learning" movements of the 16th to 18th centuries.[3] In fact, even the name *lülü*, which literally means "tuning pitch pipes," attests to the centrality of historicism and

2 A potent example is Zhu Zaiyu's *New Theory of Pitch Pipes* (lüxue xinshuo 律學新說, 1584), in which he first develops his renowned twelve-tone equal temperament. He justifies his rejection of "triple division with one part subtracted or added" through two arguments. First, he makes a philological argument that those ratios of 2:3 and 4:3 transmitted in historical sources must not be taken as precise numbers but only as simple approximations. Second, he observes that musical practitioners have already been using some form of twelve-tone equal temperament, which, he infers, they must have learned through generations of oral instead of textual transmissions since the time of the ancients themselves. He thus elevates embodied and hands-on musical practice as a source of musical knowledge that is as legitimate if not more as historical texts. See Zhu Zaiyu, *A New Theory of Pitch Pipes*, ff. 5r-5v.

3 For a cogent analysis of the restorationist underpinnings of the evidential learning movement, see Angela Zito, *Of Body and Brush: Grand Sacrifice as Text/Performance in Eighteenth-Century China* (Chicago: University of Chicago Press, 1997), 110-113.

restorationism in musical tuning. As explained in the introduction to this dissertation, the function of pitch pipes as the instruments *de rigueur* for musical tuning discursively harked back to the myth of the Yellow Emperor. According to sources from as early as the 3rd century BCE, this legendary progenitor of Chinese music and civilization writ large commissioned a set of twelve bamboo pipes whose sounds resonated with the cosmos. Later studies of musical tuning became known as *lülü* "tuning pitch pipes" because they largely strove to recreate those original, cosmically resonant pipes, which the Yellow Emperor supposedly made not only the foundation of all music (or at least the *yayue* "elegant music" Confucian scholars deemed worthy of studying) but also the basis of measuring units. Thus, since at least the 2nd century BCE when the earliest datable versions of the Confucian canons were written down, a rich tradition of scholarship had developed to sort through the many fragmentary and often contradictory records on exactly how long those original pipes were, both in and of themselves and in relation to each other. During the 17th and 18th centuries, this line of textual-historicist inquiry became all the more promising. As the *kaozheng* "evidential learning" movement developed increasingly nuanced theories of language and systematic methods of philology, it appeared increasingly possible that the textual traces of the ancient sages and their music would finally be rendered transparent and self-evident once and for all.[4]

Thus, the epistemic question the court gathering posed may be formulated to ask how knowledge about music and musical tuning should be produced, given that both empirical observations and textual criticism were deemed potentially legitimate sources of such knowledge, each in their own right. As I will show in a later part of this dissertation, the raison

4 See ibid., for the "optimism" regarding the restorationist project, which evidential learning largely derived from new theories of language and philology.

d'être of the fourteen-tone temperament in *Orthodox Meaning of Pitch Pipes* was in many ways to offer a solution to this conundrum. In particular, the decision to divide the octave fourteen-fold and thereby complicate various conventions of Chinese musical tuning and practice arose from attempts to clarify the epistemic boundaries between experience-based empiricism and text-based historicism. Before analyzing *Orthodox* as an attempt to solve this epistemic question, this chapter examines how the Emperor's mis- or reinterpretation of such a lone, commonplace piece of jargon as *geba xiangsheng* "mutual generation at every eighth step" could have possibly opened up a deeply epistemic question in the very first place. Specifically, I will analyze a short treatise that has thus far received little if any scholarly attention: *Explication of the Sage Instruction on the Essence of Music* (shenyu yueben jieshuo 聖諭樂本解說, 1692), a series of commentaries on Zhang Yushu's *Petition that Treatises on Musical Tuning and Mathematics Be Compiled*, the *Explication* was penned less than three months after the court gathering by Mao Qiling 毛奇齡 (1629-1716). Arguably the most highly regarded Chinese scholar of his generation, Mao was as prolific as he was idiosyncratic, as much a polymath as an iconoclast.[5] Unlike either Li Guangdi's or Zhang Yushu's accounts of the court gathering, Mao's exceptionally detailed and erudite *Explication* did not refrain from spelling out the actual, received definition of "mutual generation at every eighth step" as well as its established connotations with the twelve tuning pitch pipes; to my knowledge, Mao's was the only text from the time that dared to contradict the Emperor's novel gloss of the term, if tacitly to be sure.

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5 See Zhang Minquan 張民權, *Qingdai qianqi guyinxue yanjiu* 清代前期古音學研 "The Study of the Archaic Chinese Phonology in Early Qing Dynasty"), Vol. 2, 135-153. Mao Qiling also had the honor of having sixty-eight titles included in the "Summary Catalogue and Digest" (*zongmu tiyao* 總目提要) of *The Emperor's Complete Library in Four Sections* (siku quanshu 四庫全書, 1782); see a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5932, accessed April 15, 2019).

In spite of this, not only does Mao end up defending the validity of the Emperor's new definition of the term, he also effectively dedicates his entire *Explication* to proving that the Emperor's definition, which he himself shows to be defying all its established usages and connotations, is actually superior. Critical to the "epistemic question" central to the present chapter is that Mao makes his case by explicitly characterizing the Emperor's gloss of the term as octave equivalence to be an epistemological intervention. Above all, Mao takes the Emperor's lecture-demonstration as a directive to reinvent the entire scholarly discourse of music theory, specifically by substituting *sheng* 聲 "sounds" for *shu* 數 "numbers" as the core of the knowledge-producing process. Unlike scholars of *lülü* "tuning pitch pipes" who used historical records to reconstruct the precise measurements of the absolute and relative sizes of the ancients' original twelve tuning pitch pipes, Mao argues that no meaningful knowledge about music, ancient or not, could possibly be produced except through embodied experiences of sounding and listening. Thus, the lesson Mao claims to have learned from the court gathering is to abandon pursuit of the consecrated pitch pipes of the ancients and reinvent the study of music around the multivalent, embodied notion of *sheng*, which can mean both "sound" and "musical notes."

Even from its title, it is clear that *Orthodox Meaning of Pitch Pipes* (1714) did not implement Mao's radical proposal. Nonetheless, with a copy admitted to the imperial library in 1696, Mao's *Explication* was likely a critical source for the compilers of the imperial music theory treatise, even though the project's archival records and early-stage draft materials are silent on the matter. As I show towards the conclusion of this chapter, Mao's interpretation of the epistemological underpinnings of the Emperor's novel gloss of "mutual generation at every eighth step" as octave equivalence is entirely replicated in *Orthodox*. Indeed, the treatise's overall organization embodies the epistemic contrast between *shu* "numbers" and *sheng*

"sounds" that Mao draws in his *Explication* between the received definition and the Emperor's new definition of the term. Moreover, just like Mao, compilers of *Orthodox* suggest that the Emperor's (unwitting) reinterpretation of "mutual generation at every eighth step" as octave equivalence was a rallying cry for empirical experiments with *sheng*, which, in this particular context, was taken to mean both "sound" and "musical notes." It was precisely these experiments in *sheng* "sounds"/"notes" inspired by Emperor's lecture-demonstration (argue the authors of *Orthodox*) that led them down the road towards a fourteen-fold division of the octave.

Everybody Gets a Trophy

When the court gathering took place in Beijing inside the Forbidden City on February 20, 1692, Mao Qiling was in his hometown near Hangzhou, having retired after a mere nine years of service at the Qing court. Less than two months afterwards, when a doctor's appointment brought him to that prosperous urban center on the southern terminus of the Grand Canal, Mao chanced upon an issue of the *dibao* 邸報 "palace bulletin" from Beijing.[6] A type of proto-newspaper issued by the Qing government, palace bulletins circulated across the empire both through liaison offices for local governments at the capital and through privately run copying and printing houses.[7] Many issues even reached the Parisian *philosophes* during the 18th century, thanks to the French Jesuits, particularly Amiot, who joined their regular readership across the

6 Mao Qiling, *Shengyu yueben jieshuo* 聖諭樂本解說 ("Explication of the Sage Instruction on the Essence of Music," 1692), vol. 1, f. 2v.

7 See Kai-wing Chow, *Publishing, Culture, and Power in Early Modern China* (Stanford: Stanford University Press, 2004), 16-17.

Empire's official and bureaucratic ranks.[8] Typically, a palace bulletin would report the emperor's latest edicts, major appointments and judicial decisions, events of war and natural disasters, and select memorials from high-ranking officials, often with the emperor's responses appended.

Though I have yet to locate a copy of the exact bulletin Mao read in Hangzhou, it certainly had to have included the aforementioned *Petition that Treatises in Musical Tuning and Mathematics Be Compiled*, which Zhang Yushu drafted and submitted to the throne in response to the court gathering immediately thereafter. Per its title, Mao's *Explication of the Sage Instruction on the Essence of Music* takes the Emperor's lecture-demonstration itself as its object of analysis. Yet because Mao could not claim a first-hand account of the court gathering, his treatise takes the form of a *zhushu* 註疏 "notes and comments" to Zhang's *Petition*, a familiar genre of Confucian scholarship, which often embedded new ideas in annotating and interpreting classics. Mao's *Explication* divides the 736 characters of Zhang's *Petition* and the 154 characters of the Emperor's formal response into nine segments. When commenting on each segment, Mao first quotes Zhang's original text verbatim and then follows with his own remarks, whose lengths typically exceed the original text by many times over. Bracketing these commentaries are a preface explaining what led him to read and compose a commentary on Zhang's *Petition*—which reportedly included the apparition of his deceased brother's ghost in a dream—and how he envisioned dedicating it to the Kangxi Emperor one day,[9] and a postface

8 See René Poupardin, *Catalogue des manuscrits des collections Duchesne et Bréquigny* (Paris: Ernest Leroux, 1905); Louis-Georges-Oudard de Bréquigny (1714-1795) was a key contact person in Paris for Amiot during the 1760s-1790s, and thus the collection of his papers at the Bibliothèque nationale de France contains the majority of materials Amiot sent back to France from China.

9 Mao, *Explication of the Sage Instruction on the Essence of Music*, vol. 1, ff. 1-6r.

describing how Mao after several tries eventually succeeded in presenting this work to the Emperor. [10]

Table 2-1 The Structure of Mao Qiling's* Explication *(1692) as a commentary on Zhang Yushu's* Petition *(1692)

/	Zhang Yushu's *Petition*, summarized	Mao Qiling's *Explication*
		[Preface] Volume 1, ff. 1r-6r.
Segment 1	Summary of the court gathering event	Volume 1, ff. 6r-7v.
Segment 2	Previous scholars were ignorant of how to calculate the circumference of a circle.	Volume 1, ff. 7v-12r.
Segment 3	The Emperor demonstrated that when the diameter is 1 the circumference is 3.14.	Volume 1, ff. 12r-14r.
Segment 4	The Emperor demonstrated that "the circumference is 3 when the diameter is 1" applies only to hexagons, not circles.	Volume 1, ff. 14r-15v.
Segment 5	Previous scholars were ignorant of the true meaning of "mutual generation …"	Volume 2, ff. 1r-6r.
Segment 6	The Emperor demonstrated the phrase's meaning as octave equivalence on a *se* zither.	Volume 2, ff. 6v-13r.
Segment 7	The Emperor opined on historical metrology as a basis for making tuning pitch pipes.	Volume 2, ff. 13v-16v.
Segment 8	The cosmic significance of studying music theory, mathematics, and astronomy.	Volume 2, ff. 16v-19r.
Segment 9	The Emperor should consider publishing books on mathematics and music theory, a suggestion rebuked by the Emperor in his response.	Volume 2, ff. 19r-23v.
		[Postface] Volume 2, ff. 23v-25v.

Table 2-1 outlines Mao's segmentation of Zhang's *Petition* and summarizes the contents of each resulting segment of Zhang's text. As Zhang's *Petition* omitted the Emperor's discussion of water flowrate and his forecast of noontime solar altitude, Mao, who seemed to have no other source of information on the court gathering, focuses exclusively on the Emperor's two questions

10 Ibid., vol. 2, ff. 23v-25v.

concerning *yue* 樂 "music," as indicated in the title of his *Explication*, *yueben* 樂本 "the essence of music": how to calculate the circumference of a circle from its diameter, which is applicable to calculating the base circumference of *lülü* or tuning pitch pipes, and how to understand the phrase *geba xiangsheng* "mutual generation at every eighth step." Both of the two questions' respective segments in Zhang's original *Petition* follow the same prose structure. Zhang begins by describing—or rather exaggerating—how the question has completely eluded generations of scholars. He then reports the Emperor's lecture-demonstration of his answer to the question, before finally concurring with and praising the Emperor's wisdom. And while Mao divides the 179-character-long passage in Zhang's original *Petition* pertaining to the first question on calculating the circumference of a circle into three segments (Segments 2 to 4), he accordingly splits the shorter 117-character-long passage pertaining to the second question on *geba xiangsheng* "mutual generation at every eighth step" into two segments (Segments 5 and 6).

It is Mao Qiling's remarks on Segment 5 that sets him apart from all other contemporary commentators on the court gathering. To recall, in their respective recollections of the Emperor's lecture-demonstration, neither Li Guangdi nor Zhang Yushu mentions the canonic definition of *geba xiangsheng* "mutual generation at every eighth step," a notion comparable (as we have seen) to the "circle of fifths" in Western musical terminology. Indeed, in Segment 5 of his original *Petition*, Zhang misleadingly describes the phrase as a millennia-old scholarly conundrum, here quoted again in full:

> 原疏：
> 至于十二律隔八相生，宋儒載其圖、具其說，而其自然之理與所以然之故，未有能實指之者。[11]

Zhang Yushu's original *Petition* writes:

11 Ibid., vol. 2, ff. 1r.

> As for "mutual generation at every eighth step" for the twelve tuning pitch pipes, Song-era [960-1279] Confucian scholars drew diagrams and detailed explanations of it, yet there has not been anyone who has concretely pinpointed its natural principle or its cause reason.

Yet Mao's comment on this segment does not equivocate at all on the actual meaning of "mutual generation at every eighth step." After laying out the basic information regarding the twelve tuning pitch pipes themselves, Mao proceeds straight ahead (pointed brackets indicate subscript notes in Mao's original text):

> […] 相生之法 […] 以十二律相間排列。自黃鐘陽律起。越大吕〈陰〉、太簇〈陽〉、夾鐘〈陰〉、姑洗〈陽〉、中吕〈陰〉、蕤賓〈陽〉七位，至第八位陰律而生林鐘。林鐘陰律起，越夷則〈陽〉、南吕〈陰〉、無射〈陽〉、應鐘〈陰〉、黃鐘〈陽〉、大吕〈陰〉七位，至第八位陽律而生太簇。推之十二律相生次第皆然，謂之隔八相生。[…][12]
>
> […] The method of mutual generation is as follows: […] in a sequence of twelve tuning pitch pipes that alternates between the masculine [*yang*, odd numbered] pipes and the feminine [*yin*, even numbered] pipes, from the masculine *huangzhong* pipe [roughly equivalent of C], one skips over seven pipes—the feminine *dalü* [C#], the masculine *taicu* [D], the feminine *jiazhong* [D#], the masculine *guxian* [E], the feminine *zhonglü* [E#], and the masculine *ruibin* [F#] pipes—and generates *linzhong* [G] at the position of the eighth, feminine pipe. From the feminine *linzhong* pipe, one skips over seven positions—the masculine *yize* [G#], the feminine *nanlü* [A], the masculine *wuyi* [A#], the feminine *yingzhong* [B], the masculine *huangzhong* [C], and the feminine *dalü* [C#] pipes—and generates *taicu* [D] at the position of the eighth, masculine pipe. This pattern is the same when applied successively to the mutual generation of all the twelve tuning pitch pipes, and it is thus called "mutual generation at every eighth step." […]

Mao's explanation of the standard definition of *geba xiangsheng* "mutual generation at every eighth step" could hardly be more meticulous. He seems so committed to making his

12 Ibid., vol. 2, ff. 2r-2v.

readers understand exactly what the number eight refers to in the phrase, in fact, that he literally counts each series of the "eight steps" one by one, first from *huangzhong* (C) to *linzhong* (G), and then from *linzhong* (G) to *taicu* (D; this is really the half-length *taicu* pipe, i.e. the pipe whose length is half of that of the *taicu* pipe between *huanghzong* and *linzhong*). In thus disclosing the actual meaning of this rather rudimentary phrase in the discourse of music theory, Mao risks exposing the Emperor as having made an egregious mistake in his lecture-demonstration. Indeed, the passage immediately following the one quoted above further implies that the Emperor was not only mistaken about the definition of the term itself but also misunderstood the discursive context in which it operates. As I have shown in the previous chapter, the concept of *geba xiangsheng* "mutual generation at every eighth step" specifically pertains to making the twelve *lülü* or tuning pitch pipes as the foundation of any tuning system. To be still more precise, the phrase describes a pattern of length proportions among the twelve tuning pitch pipes that necessarily arises from the Chinese Pythagorean *sanfen sunyi* "triple division with one part subtracted or added." As Mao explains:

> 第其法，則在《漢志》爲三分損益，在《史記》為三法倍四，而總以黃鐘九寸為始事。如黃鐘九寸之管。三分而損其一分。則生林鐘，為六寸。林鐘六寸之管，三分而益其一分，則生太簇，為八寸。此損益法也。如黃鐘九寸，倍其實為十八寸，而以三乘之，三六一十八，則為林鐘六寸。林鐘六寸四倍之為二十四寸，而以三乘之，三八二十四，則為太簇八寸。此倍四法也。大抵下生用損、用倍法，上生用益、用四倍法。其三分、三法，則雖損益倍四各有參變，而其用三則並同。[13]
>
> However, as for the method [of mutual generation at every eighth step], it is recorded as "triple division with one part subtracted or added" in the treatise ["Treatise on Music"] in *Book of the Han*, and as "triple division of the doubled or the quadrupled" in *Records of the Grand Historian*. Both methods begin with the nine-*cun*-long *huangzhong* [roughly equivalent to C] pipe. Take

13 Ibid., vol. 2, ff. 2v-3r.

> the nine-*cun*-long *huangzhong* pipe, divide it threefold and subtract one part thereof, and thus the *linzhong* [G] pipe is generated; take this six-*cun*-long *linzhong* pipe, divide it threefold and add one part thereof, and thus the *taicu* [D] pipe is generated, which is eight *cun* long. This is the method of "one part subtracted or added." Take the nine-*cun*-long *huanghzong* pipe, double its length into eighteen *cun*, and divide it by three: three times six equals eighteen, and thus *linzhong* pipe is six-*cun*-long; take this six-*cun*-long *linzhong* pipe, quadruple it into twenty-four *cun*, and divide it by three: three times eight equals twenty-four, and thus *taicu* is eight-*cun*-long. This is the method of "doubled or quadrupled." To sum it up, generating a shorter pipe uses subtraction or doubling, whereas generating a longer pipe takes addition or quadrupling. Therefore, even though "subtracted or added" and "doubled or quadrupled" each has its own variances, the two methods are identical in their using "triple division."

This explanation of the Chinese Pythagorean tuning method as the discursive context of the exact meaning of "mutual generation at every eighth step" reeks of a desire to show off bibliographic as well as musical erudition.Indeed, Mao could have simply explained the tuning method as it was almost unanimously described in music theory treatises since at least Cai Yuanding's paradigm-setting *New Treatise on Pitch Pipes* (c. 1180s): that is, as *sanfen sunyi* "triple division with one part subtracted or added." Nonetheless, even though Mao does give a standard, step-by-step account of "triple division with one part subtracted or added," he appears unable to resist the pedantic urge to point out that, in reality, two of the earliest extant sources on how to proportion the lengths of the twelve tuning pitch pipes spell out two nominally different methods: the well-known *sanfen sunyi* "triple division with one part subtracted or added," and the rather obscure *fasan beisi* 法三倍四 "triple division of the doubled or the quadrupled." Per Mao's description above of these two methods, of course, the difference pertains not at all to their results but to the nitty-gritty of the computational process they prescribe. According to "triple division with one part subtracted or added," a length is first divided into three equal

portions before one such portion is either subtracted therefrom or added thereto; thus, the lengths (l) of *linzhong* and *taicu* are ultimately derived from the length of *huangzhong* as such:

$$l_{linzhong\ [G]} = l_{huangzhong\ [C]} - \frac{l_{huangzhong\ [C]}}{3}$$

$$l_{taicu\ [D]} = l_{linzhong\ [G]} + \frac{l_{linzhong\ [G]}}{3}$$

Meanwhile, according to "triple division of the doubled or the quadrupled," a length is first doubled or quadrupled before it is, in both cases, divided threefold; thus, the lengths of *linzhong* and *taicu* are ultimately derived from the length of *huanghzong* as such:

$$l_{linzhong\ [G]} = \frac{2 \times l_{huangzhong\ [C]}}{3}$$

$$l_{taicu\ [D]} = \frac{4 \times l_{linzhong\ [D]}}{3}$$

Of course, simplifying both sets of expressions makes their algebraic identity explicit:

$$l_{linzhong\ [G]} = l_{huangzhong\ [C]} - \frac{l_{huangzhong\ [C]}}{3} = \frac{2 \times l_{huangzhong\ [C]}}{3} = \frac{2}{3} l_{huangzhong\ [C]}$$

$$l_{linzhong\ [D]} = l_{linzhong\ [G]} + \frac{l_{linzhong[G]}}{3} = \frac{4 \times l_{linzhong[G]}}{3} = \frac{4}{3} l_{linzhong\ [G]}$$

And indeed, Mao himself demonstrates the identity of these two nominally different computational methods through his actual calculations of the lengths of *linzhong* and *taicu* pipes proportional to the length of the *huangzhong* pipe, which is given as nine *cun* 寸 (more on this later).

The only point in Mao even bringing up the obscure *fasan beisi* "triple division of the doubled or the quadrupled" besides *sanfen sunyi* "triple division with one part subtracted or added," then, was bibliographic technicality. Yet that is not a minor point in explaining what the term "mutual generation at every eighth step" conventionally refers to, particularly in light of the

Emperor's confusion of it with octave equivalence. Granted, neither of the two titles evoked in the quote above, Sima Qian 司馬遷 (c. 145 or 135-86 BCE)'s *Records of the Grand Historian* (taishigong shu 太史公書, also known as shiji 史記, 94 BCE) and Ban Gu 班固 (32-92 CE)'s *Book of the Han* (hanshu 漢書, 111 CE), was the earliest securely dated source on the twelve tuning pitch pipes available to scholars of the 16th to 18th centuries. That honor would go to the *Chronicles of Master Lü* (c. 239 BCE),[14] followed by *Writings of Prince Huainan* (c. 139 BCE),[15] both mentioned in the introduction to this dissertation. Nonetheless, both *Records* and *Book* were critical early sources for the study of *lülü* "tuning pitch pipes" qua a metonym for music theory, because they collectively set the paradigm for *zhengshi* 正史 "orthodox histories." Officially sanctioned or, since the 7th century, officially compiled, these dynastic histories were typically organized in the so-called *jizhuan ti* 紀傳體 "deeds-and-biographies format," consisting of a series of *ji* 紀 "deeds [of emperors]" and *zhuan* 傳 "biographies [of notables]," thus the name "deeds-and-biographies format," as well as *biao* 表 "tables [of aristocrats and officials]," and, for our purpose, *zhi* 志 "treatises" (which are called *shu* 書 "books" in Sima's *Records*). As discussed in the previous chapter, these thematic treatises cover a variety of technical subjects relating to statecraft, including the twelve *lülü* or tuning pitch pipes as the foundation to musical tuning. Indeed, Sima's *Records* features a "Book on Pitch Pipes" (*lü shu* 律書), which contains an account of the *fasan beisi* "triple division of the doubled or the quadrupled" method,[16]

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14 Lü Buwei, *Lüshi chunqiu* 呂氏春秋 ("Chronicles of Master Lü," c. 239 BCE), Chapter 27 "Music and Pitch Pipes" (*yinlü* 音律).

15 Liu An, *Huainan zi* 淮南子 ("Writings of Prince Huainan," c. 139 BCE), Chapter 3 "Patterns of Heaven" (*tianwen xun* 天文訓).

16 Sima Qian 司馬遷, *Taishigong shu* 太史公書 ("Records of the Grand Historian"), also known as *Shiji* 史記 (94 BCE), Chapter 25 "Book on Pitch Pipes" (*lü shu* 律書).

whereas Ban's *Book* features a "Treatise on Pitch Pipes and Calendar" (*lüli zhi* 律曆志), which accordingly contains an account of the *sanfen sunyi* "triple division with one part subtracted or added" method.[17]

Together, Sima's "Book on Pitch Pipes" and Ban's "Treatise on Pitch Pipes and Calendar" set the example for treatises on pitch pipes in subsequent orthodox histories modelled after their *Records* and *Book*. For scholars of the 16th to 18th centuries, these official dynastic histories were virtually the only sources available that document historical studies and uses of pitch pipes under different regimes before the 12th century, when Cai Yuanding's *New Treatise on Pitch Pipes* made stand-alone monographs the default genre for scholarship on *lülü* "tuning pitch pipes" or music theory. Therefore, although the fine details of Sima and Ban's dynastic histories might seem superfluous, by evoking these two early and canonical sources on the Chinese Pythagorean tuning method, Mao embeds the concept of "mutual generation at every eighth step" in the specific bibliographic context of the twelve tuning pitch pipes in a two-millennia-long genealogy of records. In so doing, Mao tacitly points out that not only did the Emperor's gloss of the term itself deviate from its received definition but it also missed the mark entirely by overlooking its discursive associations with the twelve tuning pitch pipes.

So, having presented a dense accumulation of sources showing the Emperor's definition of "mutual generation at every eighth step" to be incorrect and the Emperor himself to be clueless about the established discourse of *lülü* "tuning pitch pipes" or musical tuning, did Mao formally declare that on February 20, 1692 the Emperor made a fool of himself in front of his Han Chinese scholar-officials--the very people he had wanted to shame for their ignorance?

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17 Ban Gu 班固, *Hanshu* 漢書 ("Book of the Han," 111 CE), Chapter 21 "Treatise on Pitch Pipes and Calendar" (*lüli zhi* 律曆志).

Well, not quite. For, as meticulously as Mao unpacked the received definition and discursive context of the term when commenting on Segment 5 of Zhang's *Petition*, he just as eagerly concurred with the Emperor's novel gloss of it when commenting on Segment 6. In fact, much of his remarks on Segment 6 appear as though their main objective is to save face on behalf of the Emperor. Here is Segment 6 of Zhang's original *Petition* in full:

> 原疏：
> 皇上命樂工以篴和瑟審其聲音。七音高下次第相生，至第八聲復還其始。所謂隔八相生之法，其本原實在乎是，從來論樂者，皆未之及。蓋千古未發之祕！自我皇上今日發之非聖心神悟本于天授何由得此？[18]
>
> Zhang Yushu's original *Petition* states:
> The Emperor asked musicians to play a *di* flute in harmony with a *se* zither in order to examine their sounds and tones. The seven notes of varying heights followed one another in succession, and they returned to their beginning at every eighth step. This is really the essence and origin of the so-called principle of "mutual generation at every eighth step," yet from the past until the present, those who discuss music have never understood this. Indeed, a secret that had not been uncovered for thousands of years since antiquity has been discovered on this day by our Emperor: how could it be, except that his Sage Mind and Godly Intellect are granted by Heaven?

To recall, when commenting on the previous Segment 5, Mao has specifically revealed that the number eight in "mutual generation at every eighth step" refers to the number of pitch pipes that the 2:3 proportion spans in a sequence of the twelve tuning pitch pipes arranged in the decreasing order of their lengths. Indeed, Mao even holds his readers' hands through the rather vague "at every eighth step" formulation by literally counting the eight steps from *huangzhong* (roughly C) to *linzhong* (G) and from *linzhong* to half-length *taicu* (D). In stark contrast to this highly specific meaning of number eight in his commentary on Segment 5, however, Mao begins

18 Mao, *Explication*, vol. 2, f. 6v.

his commentary on Segment 6 by spewing forth a list of numerological significances of *ba* 八 "eight." As if trying to excuse the Emperor for mistaking "at every eighth step" for something other than eight pitch pipes, Mao suggests that "eight" could refer to so many different things even when limited only to music. The "eight timbres" (*bayin* 八音) refer to the number of categories in the conventional classification of instruments. The "eight rows and eight files" (*bayi* 八佾) refer to the number of dancers in each dimension of the formation at an imperial ceremony. The "eight winds" (*bafeng* 八風) refer to the number of cardinal directions to which different pitch pipes correspond. And the "two rows of eight bells and eight chimes" (*zhongqing erba* 鍾磬二八) refer to the number of bells and chimes in each *bianzhong* 編鐘 "arranged bells set" and *bianqing* 編磬 "arranged stone-chimes set," two prominent instruments in the traditional court music ensemble. Mao even digresses into the significance of "eight" in geomancy, an area of knowledge with which the twelve tuning pitch pipes bear some correlative correspondences, indeed as do pretty much any social or cultural practice.[19] No wonder the Emperor was confused.

When Mao finally steers himself back on course, he continues to shy away from confronting directly the Emperor's gloss of "mutual generation at every eighth step," which his remarks on the previous segment of Zhang's *Petition* have just shown to be misguided. Instead, it appears as though Mao recognizes that the Emperor's misunderstanding of the phrase stemmed from a broader ignorance of the discursive context of its usage, that is, the proportioning of the twelve tuning pitch pipes, particularly through the Chinese Pythagorean method of "triple division with one part subtracted or added." Thus, Mao sets the stage for unpacking the

19 Ibid., ff. 7r.

Emperor's gloss by explaining that new discursive context into which he unwittingly recast the phrase at the court gathering. Mao writes:

> […]乃以旋律之方定生聲之法。娶妻生子，連環遞代，隔九既有餘，而隔七復不足，若是者何也？則以生必隔八。此非他為之，而聲為之也。夫樂也者，聲也。律也者，和聲者也。人聲不齊，而以五聲為之準。五聲，宮、商、角、徵、羽也。然而聲有和者，即有繆者。[…] 五聲為和，二變聲為繆。二變聲者，變宮與變徵也。[…] 而要之和為五聲，繆為七聲，如是止矣。[…][20]
>
> […] Yet the Emperor used the pattern of rotating the pitch pipes [*lü*] to determine the generation of notes [*sheng*]. When "a gentleman takes a wife to generate an heir" [*quqi shengzi*], when chains of cycles continuously succeed one another, it is superfluous at every ninth step, yet conversely insufficient at every seventh step—why is it so? Indeed, generation must occur at every eighth step. This is not effected by anything but sound [*sheng*]. Indeed, music [*yue*] is sound. Pitch pipes [*lü*] are what accord with sounds. Human voices [*sheng*] vary, and thus the five notes [*sheng*] are used as yardsticks. The five notes are *gong* [*do/ut*], *shang* [*re*], *jue* [*mi*], *zhi* [*sol*], and *yu* [*la*]. Nonetheless, so long as there are harmonious notes, there are discordant ones. […] The five notes are harmonious, whereas the two altered notes are discordant. The two altered notes are *biangong* [*ti*] and *bianzhi* [*fa#*]. […] If we limit the notes to only the harmonious ones, there are five notes; with the discordant ones, there are seven notes, and there are no more.

From the very beginning of this passage, Mao acknowledges that the Emperor's lecture-demonstration transferred the term "mutual generation at every eighth step" from its established signifying context of the twelve tuning pitch pipes and the Chinese Pythagorean tuning method to a new context of the seven-note diatonic scale and the consequent phenomenon of octave equivalence. Put in Mao's own terms in his first sentence above, the Emperor recast the phrase from having something to do with *lü* 律 "pitch pipes" (a metonymic synonym for *lülü*) to having

20 Ibid., ff. 7r-8r.

something to do with *sheng* 聲, a character that means "musical notes" (notes on a scale) in this context. This transition from pitch pipes to is embodied in Mao's usage of the patriarchal metaphor *quqi shengzi* 娶妻生子 "a gentleman takes a wife to generate an heir." A somewhat established turn of phrase in writings on the tuning pitch pipes, "a gentleman takes a wife to *generate* an heir" analogizes the patterns of mutual *generations* (the 2:3 ratios among the twelve tuning pitch pipes) to the patterns of the Five Phases (*wuxing* 五行) among the Sexagenary Cycle of Stems and Branches (*ganzhi* 干支); a fuller explanation of the Sexagenary Cycle and of the mechanism of this analogy can be found in the footnote.[21] Here, suffice it to say that typically the metaphor refers to "mutual generation at every eighth step" in the received sense of the successive proportioning of the twelve tuning pitch pipes through the 2:3 ratio (as well as its double, 4:3), as in "*huangzhong* [C] takes *linzhong* [G] as a wife in order to generate *taicu* [D]."[22]

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21 The Sexagenary Cycle multiplies the ten *tiangan* 天干 "Heavenly Stems" with the twelve *dizhi* 地支 "Earthy Branches" into sixty *ganzhi* 干支 "Stems and Branches": the first of them, *jiazi* 甲子, combines the first heavenly stem *jia* 甲 with the first earthly branch *zi* 子, the second of them, *yichou* 乙丑, combines the second heavenly stem *yi* 乙 with the second earthly branch *chou* 丑, and so on, until the sixty possible combinations are exhausted in this order. The so-called *nayin* 納音 "assigning the tones" assigns one of the Five Phases (*wuxing* 五行)—*jin* 金 "metal," *mu* 木 "wood," *shui* 水 "water," *huo* 火 "fire," and *tu* 土 "earth"—to all of these sixty "Stems and Branches" combinations. The rule, as outlined by tbe Ming-era scholar Wan Minying 萬民英 in his paradigm-setting treatise *Sanming tonghui* 三命通會 ("Complete Compendium on Fortune-Telling," 16th century), vol. 1, is that the same phase is to be assigned to every 1st, 2nd, and 9th ones of the stem-and-branch combinations. For example, *jiazi*, being the first, takes *jin* "gold" as its phase; it then "takes" the next and thus second, *yimao*, "as spouse," which also gets assigned *jin* as phase. Together, *jiazi* and *yimao* "produce an heir," the 9th stem-and-branch combination, *renshen* 壬申, which also gets assigned the *jin* as phase. Wan uses "mutual generation at every eighth step" as a metaphor for this pattern of the Five Phases among the Sexagenary Cycle.

22 See, for example, Zhu Zaiyu, *Lülü jingyi waibian* 律呂精義外編 ("Peripheral Series of *Essential Meaning of Pitch Pipes*," 1596), vol. 3, ff. 41r-42r.

As the Emperor's definition of "mutual generation at every eighth step" severs it from the *lü* 律 "pitch pipes" and affixes it newly with the *sheng* 聲 "notes," what is being *sheng* 生 "generated"—which, in keeping with the patriarchal metaphor, can literally mean "given birth to"—are no longer the proportional lengths of the twelve tuning pitch pipes but rather the seven notes of the diatonic scale. Indeed, using the metaphor as a pivot, Mao continues in the passage above to give a rather classic exposition of *wusheng erbian* 五聲二變 "the five proper notes and the two altered notes," the Chinese diatonic scale system. Granted, neither the adjective *he* 和 "concordant" that Mao uses to describe the five proper notes nor the adjective *miu* 繆 "discordant" that he uses to describe the two altered notes is a received technical term. In most writings on music, the two altered notes, *biangong* (roughly *ti*) and *bianzhi* (roughly *fi* or *fa#*), were in no way considered "dissonant" in terms either of their syntactical functions or of their affective associations. Still, from the Confucian *yayue* "elegant music" performed at the imperial court to the various traditions of opera burgeoning in urban centers and village fairs across southern China, many melodies in the early modern Chinese soundscape did typically eschew the two altered notes, thus embodying the pentatonicism for which Chinese music would become stereotyped to Western-trained ears.[23] And because glossing "mutual generation at every *eighth* step" as octave equivalence would only make intuitive sense within a heptatonic scale rather than a pentatonic one—otherwise, it would have been "at every sixth step"— Mao understandably found it necessary to explain away the apparent difference between the five-note system that

23 Typically, during the 16th to 18th centuries, *beiqu* 北曲 "northern opera" employed a heptatonic scale, whereas *nanqu* 南曲 "southern opera," which flourished in the Yangtze River Delta area where Mao lived, employed the stereotypically "Chinese" pentatonic scale. See Liang Mingyue, *Music of the Billion: An Introduction to Chinese Musical Culture* (New York: Heinrichshofen, 1985), 238-243.

pervaded many musical practices of the time and the seven-note system that hovered in the theoretical background.

Thus, just like Li Guangdi in his longer account of the court gathering quoted in the previous chapter of this dissertation, Mao describes what is effectively the principle of octave equivalence—what the Emperor takes "mutual generation at every eighth step" to mean—in both pentatonic and heptatonic contexts. In the same fastidious fashion as he unpacks the canonic definition of the phrase when commenting on Segment 5, Mao hand-holds his readers through every detail of the Emperor's new gloss. Whereas Zhang Yushu's original text spends a mere patch of sixteen characters describing the Emperor's lecture-demonstration, Mao again indulges in literally counting every one of the eight steps in the regenerating cycles of the seven-note diatonic scale, lest anyone be confused about what "eight" refers to under the new definition:

> […] 乃凡歌曲者。累聲成曲。不拘和與繆。而高下抗墜。每不止五七而清聲生焉。聲止有五。五之外無聲。而高于五聲者則仍以五聲。周之第六聲即第一聲。第七聲即第二聲。第八聲即第三聲。第九聲即第四聲。自一至五所謂宮、商、角、徵、羽者。而自六至九則所謂宮清、商清、角清、徵清。 […] 乃若七聲，則仍是五聲。而二變不閟，則加二聲。[…] 七聲以閟變而減為五，五聲以啟閟而增為七。則五聲之以第六聲為第一聲者，實即第八聲為第一聲也。第七聲為第二聲者，實即第九聲為第二聲也。[…] 是五聲除五而生聲，實即七聲除七而生聲。所謂隔八相生者，葢以七調之除五聲二變而聲又生，故隔八也。[24]

> […] Whenever one sings an opera tune [*qu*], one forms a tune by accumulating notes [*sheng*], no matter whether the notes are concordant or discordant. Yet as the tune ascends [*gao*] and descends [*xia*], loudens and softens, whenever the five and seven notes are not enough, the clear [*qing*]notes are generated. When only the five notes are used, there are no notes other than these five, whereas those higher than the five notes are still the five notes. In continuation, the sixth note is the first note, the seventh note is the second note, eighth note is the third note, the ninth note

24 Mao, *Explication*, vol. 2, ff. 8r-9r.

> is the fourth note. Those that are called *gong* [roughly *do/ut*], *shang* [*re*], *jue* [*jiao*], *zhi* [*sol*], and *yu* [*la*] from the first note to the fifth, are those that are called the clear *gong*, the clear *shang*, the clear *jue*, and the clear *zhi* from the sixth note to the ninth. […] As for the seven notes, they are the same as the five notes except the two altered notes are no longer silenced. […] And thus, the sixth note's being the first note for the five notes is really the same as the eighth note's being the first note of the seven notes, and the seventh note's being the second note is really the same as the ninth note's being the second note. […] Thus, when the five notes are exhausted, the notes are generated anew, and this is really the same as when the seven notes are exhausted, the notes are also generated anew.

Continuing on the matter of "the five proper notes and the two altered notes," Mao first introduces the concept of the *qing* 清 and the *xia* 下 notes. Meaning "clear," *qing* is often used in opposition to *zhuo* 濁 "muddy." This binary concerns not the timbre of sounds, which their literal meanings might seem to suggest, but rather the notion of pitch height in Western music—except that the two adjectives most commonly describe only the minute upward or downward fine-tuning of what are categorically still the same notes in a scale or pitch system. Describing categorical differences in pitch heights between different notes and the shapes of melodic contours often involves another binary of adjectives: *gao* 高 "high" and *xia* 下 "low," a pair directly translatable to the "high-low" binary in Western musical parlance.

What might be confusing, however, is that the standard nomenclature in the Chinese scale system mixes-and-matches these two otherwise clearly defined binaries of pitch height wherein one pair, *qing* "clear" vs. *zhuo* "muddy," refers to minute fine-tuning and the other, *gao* "high" vs. *xia* "low," refers to categorical pitch-class distinctions. Indeed, when the adjective *qing* "clear" is specifically attached as an attributive to a note name—"the clear *gong*, the clear *shang*, the clear *jue*," and so forth, as in Mao's quote above— it no longer refers to fine-tuning. Instead, adding *qing* "clear" to a note name raises it by an octave, just as adding *xia* "low" to a

note name lowers it by an octave. Thus, what Mao refers to as "the clear notes" (*qingsheng* 清聲) is not the result of slightly raising the pitches of the notes of the diatonic scale but rather the next series of these notes, thus raising them by an octave above. Following his explanation, in a pentatonic system, proceeding from *gong* (roughly *do/ut*) to *shang* (*re*), *jue* (*mi*), *zhi* (*sol*), and *yu* (*la*), the next note will be *qinggong* 清宮 "clear *gong*" whose name indicates it to be the same note as *gong* except higher (by an octave). In the reverse direction, proceeding from *yu*, *zhi*, *jue*, *shang*, and *gong*, the next note will be *xiayu* 下羽 "low *yu*" whose name indicates it to be the same note as *yu* except lower (by an octave).

It is only after having fully examined the seven-note diatonic scale system—the "five proper notes and the two altered notes" and the "clear notes" an octave above them—that Mao finally unveils, for the very first time in his *Explication*, what the Emperor explained "mutual generation at every eighth step" to be at the court gathering:

> […] 然則隔八者，以生聲而言也。以正聲之生清聲，而清復還正，故為言也。[25]
>
> […] The so-called "mutual generation at every eighth step" is that notes are generated anew whenever a seven-note scale has passed through all the five proper notes and the two altered notes, ergo "at every eighth step." Yet this "at every eighth step" refers to the generation of notes. It refers to the original notes generating the clear notes, and the clear notes in turn returning to the original notes, and hence the name.

All that is left is applying this explanation to the specific organological contexts of the Emperor's demonstration, namely the six-hole *di* flute and the twenty-five-string *se* zither. Here, Mao largely repeats his analysis of the diatonic scale system and of the octave equivalences the system necessarily engenders, except now in detailed reference to the finger holes of the *di* flute

25 Ibid., 9r-9v.

and the specific strings of the *se* zither. Analyzing the Emperor's gloss of "mutual generation at every eighth step" specifically as a series of demonstrations on two instruments allows Mao to conclude his commentary on Segment 6 by concurring with Zhang Yushu regarding the stakes of the Emperor's new definition of the term:

> […] 以第八聲為圜轉生聲之準。此千秋秘旨，雖帝舜、后夔亦罕言及，而皇上一指破之。謂非聖人篤生，實為周秦以來，開二千餘年，古樂之亡，不可得也。[26]
>
> […] Using the eighth note as the yardstick for the generation of notes in cycles: even Emperor Shun [legendary, c. 2294-2184 BCE] and his court musician Hou Kui scarcely spoke of this insight, which had remained recondite for millennia, yet Our Emperor unlocked it with one finger. If the Emperor had not been born a Sage, then the extinction of ancient music, two millennia after the Zhou [c. 1046-249 BCE] and the Qin [221-206 BCE] eras, would really have been irreversible.

While echoing Zhang's original text by lauding the "Sagacity" of the Emperor, Mao nonetheless concludes his commentary on Segment 6 (by far the longest of all the commentaries in his *Explication*) by adding yet another layer of historiographic context. To recall, Zhang's *Petition* simply glorifies the Emperor's gloss of the term as "an insight that had not been uncovered for all eternity." Mao's paean in the quote above, however, describes the Emperor's gloss not so much as a discovery as a *re*discovery of some long-lost ancient wisdom. Indeed, Mao even identifies a specific moment in the Confucian historical imagination when knowledge about "mutual generation at every eighth step," meaning octave equivalence, supposedly still existed: the reign of Emperor Shun 舜, whom conventional historiography typically counted as the last of the legendary ancient sage kings, the so-called Three Sovereigns and Five Emperors (*sanhuang wudi* 三皇五帝). While the Yellow Emperor and his musician Ling Lun mentioned in

26 Ibid., 13r.

the introduction to this dissertation had already fashioned the original twelve tuning pitch pipes and used them to lay the foundation for all music a few generations beforehand, it was Emperor Shun who ordered his musician Hou Kui 后夔 to establish an entire institution of music at his court. The Confucian *Canon of History* (shang shu 尚書, earliest extant version c. 3rd century BCE), a collection of reputedly ancient documents, famously records:

> 帝曰：“夔！命汝典樂，教胄子，直而溫，寬而栗，剛而無虐，簡而無傲。詩言志，歌永言，聲依永，律和聲。八音克諧，無相奪倫，神人以和。”夔曰：“於！予擊石拊石，百獸率舞。”[27]
>
> Emperor Shun said: “Oh Kui! I order you to establish the institution [*dian*] of music, to teach our descendants to be frank yet gentle, lenient yet solemn, steadfast yet not cruel, majestic yet not vain. May poetry express the will and songs chant such expression; may notes [*sheng*] follow such chanting and pitch pipes [*lü*] accord with such notes. May the eight timbres agree and never disrupt their order, and thus may deities and humankind be in harmony.” Kui responded: “Aye! I hit and tap the stone [chimes], and all beasts obediently dance along.”

According to the standard narrative repeated in almost every dynastic history starting with Sima’s *Records* and Ban’s *Book*, generations and generations of Chinese rulers after Emperor Shun upheld the *dian* 典 “institution” of music he had established by Hou Kui. This so-called Three Dynasties period (*sandai* 三代, partly legendary, beginning c. 2070 BCE) that immediately followed the ancient sage kings culminated in the Former Zhou 周 (c. 1046-771 BCE), an era that Confucius (551-479 BCE) himself revered as the epitome of rites, mores, and governance. Beginning with the tumultuous Later Zhou period (c. 771-249 BCE) that saw constant warfare and seismic socio-economic transformations, however, the ancient institution of

27 Anon., *Shangshu* 尚書 (“Canon of History,” also referred to as *Shujing* 書經, c. 3rd century BCE), “Book of Shun” (*shun dian* 舜典).

music as well as the political and ritual order as a whole deteriorated—or as was lamented by Confucius, who had already started to yearn for the better past. And when the Qin 秦 Dynasty (221-206 BCE) put an end to the Warring States Period (476-221 BCE) and subjugated much of China-proper under its centralized control, it did not herald what Confucian scholars would consider a return to the ways of the ancient sage kings. On the contrary, the Qin, the arch-villains in the Confucian historical imagination, not only replaced the supposedly benevolent governance and harmonizing rituals of the ancients with exceptionally harsh laws, but also went out of their way to persecute Confucianism (among many other non-state-sanctioned schools of thought) during the "burning of the books and burying of the scholars" (*fenshu kengru* 焚書坑儒) campaign. While recent historians have questioned the extent of this infamous intellectual genocide, standard Confucian historiography identified the Qin era as the final completion of the perennial, cataclysmic loss of the *gu* 古 "ancient." Not only did "rites collapse and music crumble" (*libeng yuehuai* 禮崩樂壞), but even the very transmission of knowledge about such rites and music was interrupted, be it oral or written.[28] In consequence, as Mao writes when concluding his commentary on Segment 6, for the "two millennia after the Zhou and Qin eras," Confucian scholars have been scrambling to reconcile the often-contradictory historical records and the different versions of Confucian canons that had varyingly survived the Qin persecutions, in hopes of "reversing" the "extinction of ancient music."

28 This narrative of the "fall" of the perfect ancient music through the centuries after the Zhou era is ubiquitous in official dynastic histories, studies on music and tuning pitch pipes, and even casual writings on the subjects. See, for examples, Sima Qian, *Records of the Grand Historian*, "Book on Music" (*yueshu* 樂書) and Wei Zheng 魏徵, Zhangsun Wuji 長孫無忌 et al., *Suishu* 隋書 ("Book of the Sui," 636/656 CE), vol. 13, "Treatise on Music, Part One of Three" (*yuezhi shang* 樂志上).

Mao thus praises the Kangxi Emperor not just as any generic "Sage Mind [or] Godly Intellect," as Zhang Yushu did in his *Petition*, but as a pioneering restorer of lost knowledge of and about the ancients. Accordingly, his gloss of "mutual generation at every eighth step" as octave equivalence was not just any unprecedented discovery, but a specifically philological recuperation of the supposedly long-lost meaning of a phrase that permeated through the dense records and studies on music. In so doing, Mao seems to be portraying the Kangxi Emperor as a Confucian scholar: one who submits himself to the intellectual and political genealogy that traces all the way back to the ancient sage kings and dedicates himself to restoring and practicing the rites, mores, ethics, and ways of the ancients. Moreover, the fact that the Emperor's lecture-demonstration of octave equivalence took the form of the gloss of a single term aligned him rather neatly with the trend of Confucian scholarship of the late 17th and 18th centuries. As the rising *kaozheng* "evidential learning" movement emphasized philology over hermeneutics in textual analyses, the most acclaimed 18th-century works on the Confucian canons took the form not of an exegesis or meditation on their cosmic or moral principles but of a collection of *kao* 考 "investigations," *xun* 訓 "philological explications," or *zheng* 証 "proofs" glossing particular words and names found in them.[29] A precise understanding of each and every one of these signifiers could in turn, it was hoped, help reconstruct the exact ancient institutions whose preservation hung on in the Confucian texts by a thread.

Embedding the Kangxi Emperor and his novel gloss of *geba xiangsheng* "mutual generation at every eighth step" in the historiography and genealogy of Confucian scholarship was by no means an exceptional narrative strategy. For much of the century after the Qing invaded China in the 1640s, portraying the Qing emperors as producers of Confucian scholarship

29 See Elman, *From Philosophy to Philology*, Zhao trans., 25-38.

defined the relationship between the Qing Empire that ruled China as a foreign conquest regime and the landed and lettered Chinese gentry who served as the empire's tax base, bread basket, and supply for literary and bureaucratic personnel. For the Manchu conquest elites, subjecting themselves, however nominally, to transmitting texts and knowledge in the Confucian tradition allowed them to claim a place in the lineage of legitimate rulers of China despite their "barbaric" provenance. For the lettered and landed Chinese gentry qua the very body politic of Confucianism, recognizing the Manchus as the rightful heirs of the Confucian rulership allowed them to preserve their social, economic, and political privileges.[30] Under the Kangxi Emperor, especially, large-scale scholarly projects such as *History of Ming* and *Origins of Cosmological Sciences* supplemented the civil examinations based on the Confucian corpus in opening up some of the most expedient channels for Chinese scholars to rise in government ranks and join the inner circle of Qing imperial politics.[31] Even without invoking the delicate ethnic politics of the Qing rule of China, praising a ruler or regime for finally reversing the perennial loss of ancient music was as established a trope as the very narrative of such a loss itself. In fact, even Zhu Zaiyu touted his own twelve-tone equal temperament as a more precise reconstruction of the ancients' twelve tuning pitch pipes than the conventional Chinese Pythagorean method, despite literally calling his own system the *xinfa* "New Method."[32] Last but not least, because the Emperor called the court gathering in order to criticize the Chinese scholar-officials for "knowing nothing about methods of calculation," the act of showing how his lecture-

30 Chang, *A Court on Horseback*, Chapter 6 "The Southern Tours as Cultural Encompassment: The Valorization of Verse & Accommodation of Han Learning," 260-304.

31 For an example of how *History of the Ming* brought to Beijing in serving the Qing various types of scholarly talents whose specialties fell outside the normal curriculum of the civil examinations, see Jami, *The Emperor's New Mathematics*, 214-218.

32 See footnote No. 2 above.

demonstration of "mutual generation at every eighth step" as octave equivalence, however novel, still belonged to the restorationist discourse of Confucian scholarship allows Mao to defend his fellow scholars against the Emperor's dress-down.

Innocuous as it is, however, claiming that the Emperor's unprecedented gloss did belong to the discourse of Confucian scholarship after all is particularly problematic for Mao, who when commenting on the previous Segment 5 has just detailed the actual, received definition and discursive associations of the phrase. The problem is two-fold. First, as I have shown in the previous chapter, any survey of scholarly discourse in the late 17th century would have revealed that there was simply no dispute or confusion over what the phrase refers to: namely, the patterns of 2:3 proportions among the twelve tuning pitch pipes arranged in the decreasing order of their length, after they have been successively generated through the Chinese Pythagorean "triple division with one part subtracted or added" method. It is disingenuous, then, for Mao to say that the Kangxi Emperor made any valuable contribution within the discourse of Confucian scholarship by recuperating some lost knowledge of the ancients, because he himself has already shown that, when it came to "mutual generation at every eighth step," there wasn't really anything to recuperate. Here I should note that Mao is categorically different from both Li Guangdi and Zhang Yushu. The latter two technically didn't commit perjury in front of the Emperor by praising his lecture-demonstration of "mutual generation at every eighth step" as unprecedented: not only was misinterpreting the well-known and well-understood phrase as octave equivalence an unprecedented mistake as far as written records were concerned, but the very concept of "octave equivalence" was in itself an unmarked notion in Chinese music, as logical a consequence of any seven-note diatonic scale system as it may be. Mao, in contrast, puts his disingenuousness on full display by explaining what the term has always unambiguously

referred to, only to contradict himself later by casting its meaning as a "profound insight" that had been lost for the two thousand years after the ancient sages until the Kangxi Emperor finally revealed it.

Second, by praising his lecture-demonstration as the resurrection of the long-lost meaning of "mutual generation at every eighth step," Mao celebrates the Emperor for producing meaningful knowledge within the established discourse of music theory, a discourse, to reiterate, aimed at bringing back the ancients' institution of music. Nonetheless, even Mao showed that the Emperor's gloss of the term was not only mistaken in and of itself but also ignorant of its broader discursive position. As Mao puts it in his commentary on Segment 6, the Emperor recast a phrase from being associated with *lü* "pitch pipes" to being associated with *sheng* "notes," that is, from proportioning the twelve tuning pitch pipes to regenerating the seven-note diatonic scale in cycles. Now, it would be perfectly fair for Mao to claim that the phrase was somehow polysemic in its ancient usage, and that the Emperor discovered its long-lost second meaning while scholars had only known its first meaning for the past two millennia. But Mao's paean to the Emperor's lecture-demonstration at the end of his commentary on Segment 6 leaves no room for the coexistence of two meanings or two sets of discursive associations in this one term.

Thus, in his two adjacent commentaries on Segments 5 and 6 of Zhang's *Petition*, Mao puts forth two mutually exclusive arguments on "mutual generation at every eighth step." On the one hand, in order to show his erudition in the rich scholarly discourse on *lülü* "tuning pitch pipes" or music theory by citing some of its earliest extant sources, Mao presents the standard definition of the ubiquitous term meaning the pattern of 2:3 proportions among the twelve tuning pitch pipes when commenting on Segment 5. On the other hand, in order to join the laudatory chorus that overflows from the very *Petition* he comments on, Mao acclaims the Emperor's

lecture-demonstration of octave equivalence for rediscovering the meaning of the phrase that had been a conundrum for Confucian scholars over the past two millennia when commenting on Segment 6. But how could Mao spell out the "correction answer" on the question of "mutual generation at every eighth step" yet simultaneously award the Kangxi Emperor for his misguided lecture-demonstration of it? How could he reconcile the Emperor's disregard if not outright ignorance of the established scholarly discourse on music with his claim that the Emperor made profound contributions within a discourse aimed at restoring the ancient intuitions of music?

It turns out that the answer to Mao's apparent self-contradiction between his own erudition in the discourse of musical scholarship and the Emperor's resolution of a two-millennia old conundrum in this very discourse lies in the age-old question: what is "music"?

Sheng 聲 "Sound" vs. *Shu* 數 "Number"

Before diving into the problem of ontology—which, as I will argue shortly, is not what is at stake—let me recalibrate the analyses made thus far of Mao's *Explication* as a series of commentaries on the February 20, 1692 court gathering through the medium of Zhang Yushu's *Petition*.[33] As I showed in the previous chapter, the Kangxi Emperor's interpretation of "mutual generation at every eighth step" is not wrong *per se*, except insofar as it deviates completely from its received definition and associations in scholarly discourse. Under the Emperor's definition of it as octave equivalence, the phrase signifies in reference to "the five proper notes and the two altered notes" of the seven-note diatonic scale system. Meanwhile, under the

33 For the question of ontology, which I will unpack with much more detail regarding the concept of *yue* 樂 "music" soon later in this section, see Philip V. Bohlman, "Ontologies of Music," in Nicholas Cook and Mark Everist ed., *Rethinking Music* (Oxford, UK: Oxford University Press, 1999), 17-34.

received definition of it as (to abuse Western terminology again) the circle of fifths, the phrase signifies in reference to the Chinese Pythagorean "triple division with one part subtracted or added" method for proportioning the lengths of the twelve tuning pitch pipes.

In the previous section of this chapter, I showed that Mao's commentaries on Segments 5 and 6 of Zhang's *Petition* appear to embody this discursive distinction. When commenting on Segment 5, Mao explains the received definition of "mutual generation at every eighth step" by unpacking the proportional patterns between the twelve tuning pitch pipes without ever mentioning "the five proper notes and the two altered notes." Meanwhile, when commenting on Segment 6, Mao explains the Kangxi Emperor's lecture-demonstration on his understanding of the phrase by enumerating the sonic patterns in pentatonic and heptatonic scale systems, again without ever implicating the twelve tuning pitch pipes, their measurements, or their proportional relations. Under this bipartite organization of Mao's remarks on the question of "mutual generation at every eighth step," not only do both the received definition and the Kangxi Emperor's new definition of the phrase each appear to be capable of mobilizing relevant signifiers to justify itself, but their respective signification processes are also mutually exclusive. Thus, because the two definitions operate in completely distinct discursive realms without mutual entanglement, they can be considered equally and simultaneously valid, so long as such discursive distinction is clarified, as is the case in Mao's neatly separated commentaries on Segments 5 and 6 of Zhang's *Petition*.

Yet Mao does not see things this way. Despite arguing that the received definition embeds the phrase in the discourse of *lü* "pitch pipes" and that the Emperor's new definition embeds it in the different discourse of *sheng* "notes," Mao ultimately does not wall off the Emperor from the established scholarly discourse on music. This established discourse clearly

falsifies the Emperor's gloss of "mutual generation at every eighth step" as octave equivalence instead of circle of fifth. Yet still, Mao does not create a segregated symbolic order for the Emperor for the sole purpose of allowing his definition of the phrase to hold up as what's nowadays referred to in public life as an "alternative fact." On the very contrary, Mao concludes his commentary on the Emperor's lecture-demonstration by praising it for contributing to the established scholarship on music in furthering its two-millennia-long goal of recuperating the lost knowledge of and about the ancient institution of music. Moreover, this praise evoking the restorationist agenda of Confucian scholarship implies that Mao actually finds the Emperor's definition of "mutual generation at every eighth step" superior to its received definition—even though it was Mao himself who, unlike either Li Guangdi or Zhang Yushu, failed to demur on the latter definition and thus put the Emperor's lecture-demonstration in jeopardy.

If Mao does not draw a fundamental discursive distinction between the received definition of "mutual generation at every eighth step" related to *lü* "pitch pipes" and the Emperor's new definition related to *sheng* "notes"—even though such a distinction could allow for a happy polysemic coexistence and save the Emperor the embarrassment of appearing to be wrong—how does he differentiate the two definitions? Specifically, on what grounds does Mao distinguish the Emperor's definition from its received one, having tacitly falsified it by explaining the established discursive connotations of the phrase? The answer, I argue, lies in the realm of episteme, in the way knowledge is produced. Indeed, the primary lens through which Mao distinguishes the two definitions is not their varying discursive associations—*lü* "pitch pipes" vs. *sheng* "notes"—but their distinct processes of knowledge-production. It is only because Mao reads such an epistemic distinction out of the two definitions that he is able to assert the superiority of one definition over the other. As it turns out, this epistemic cue as the

distinction between how *lü* "pitch pipes" and *sheng* "notes" each produce knowledge was Mao's most significant takeaway from the court gathering on February 20, 1692—and he wasn't the only one either.

To explain how Mao reads an epistemic contrast out of the two definitions of "mutual generation at every eighth step," I turn to the very beginning of his commentary on Segment 5 of Zhang's *Petition*. Before unpacking the received definition of the phrase in reference to *lü* "pitch pipes," Mao introduces the fundamentals of the latter. Though seemingly generic, the introduction actually holds the key to understanding why Mao elected to write *Explication* in the very first place. Here is the first paragraph of Mao's commentary on Segment 5 quoted in full:

> 十二律，始于十二管。六陽、六陰，合成十二，以為調和五聲之用。此固一齊俱見，並無一律生一律之理。茍截管定寸，或長或短。凡其減長而為短，增短以為長，必有一定矩法，使短長可準。于是，先立一黃鐘九寸之管以為之質，而後減之而短與增之而長，皆有成法以推之。此不過假此度數長短以示律之有高下，其于實實聲音升降圜轉之節，全不相合。故曰：此作律之本，亦造歷審度同量衡權之本，而于樂本無與焉。且其諸說亦各有不同。[34]
>
> The twelve *lülü* originated from twelve pipes. Six of them *yang* [masculine and odd number], six of them *yin* [feminine and even number], they amount to twelve in total and are used to bring the five notes [*sheng*] into harmony. Originally, these pipes were seen together at the same time, and there wasn't such a principle whereby one pitch pipe generated [*sheng*, as in *geba xiangsheng* "mutual generation at every eighth step"] another. However, when it comes to slashing [bamboo stems to make] the pipes and determining their lengths, some of them are long and some of them are short. Whenever a long pipe is reduced to make a short one, or a short pipe is stretched out to make a long one, there needs to be a certain ruler or standard, in order that their varying lengths can be measured. Therefore, a 9-*cun*-long *huangzhong* pipe is first established as the foundation, and then, whether to make a shorter pipe via reduction or to make a longer pipe via elongation, there are established methods to deduce the results. This, however, is

34 Mao, *Explication*, vol. 2, ff. 1r-1v.

> nothing more than borrowing these measurements and numbers [*shu*] of varying magnitudes to show that the tuning pitch pipes can be either high [*gao*] or low [*xia*], yet it does not at all match the patterns of ascent, descent, transpositions, or mutations of the actual, concrete sounds [*sheng*] and tones [*yin*]. Therefore, we say: this is the essence of making pitch pipes and the essence of establishing the calendar and examining the standards of length, volume, and weight, yet it is irrelevant to the essence of music. And its theories also vary from each other.

Pronouncing something to be "irrelevant to the essence of music" is the severest dismissal possible in a treatise entitled *Explication of the Sage Instruction on the Essence of Music*—and Mao has done just that. Before even starting to explain the received definition of "mutual generation at every eighth step" in relation to the twelve tuning pitch pipes, Mao warns that the ensuing discussion on the *ben* 本 "essence" of *lü* "pitch pipes" bears in no way on the *shengyu* 聖諭 "Sage Instruction" the Emperor imparted to his Han Chinese scholar-officials at the court gathering: the essence of *yue* "music." Instead of the latter, Mao relegates the essence of *lü* "pitch pipes" to those of *li* 曆 "calendar," *du* 度 "length," *liang* 量 "volume," and *hengquan* 衡權 "weight and balance"—in other words, to maths.

It is shocking that Mao dismisses the endeavor of determining the absolute and relative lengths of the twelve tuning pitch pipes as irrelevant to the essence of music. To begin with, the concept of *yue* 樂, notwithstanding my translation, did not suggest the same kind of totalizing universality in early modern Chinese scholarship as does the word "music" in modern English.[35] Rather, the center of the field of connotations of *yue* was specifically trained on the solemn and decorous music that befits the sacrificial rites and courtly ceremonies performed by a Confucian

35 As my following discussion focuses exclusively on the concept of *yue* "music" in the early modern era, for that in ancient China—an era that defined the historical imagination of early modern Confucian scholars–, see Erica Fox Brindley, *Music, Cosmology, and the Politics of Harmony in Early China* (New York: SUNY Press, 2012), 5-6 and 25-42.

ruler, in emulation of the ancients. Put bluntly, *yue* "music" can be understood as a shorthand for *guyue* 古樂 "ancient music" and *yayue* 雅樂 "elegant music": that is, music that is worthy of preservation and, given the narrative of loss in the Confucian historical imagination, worthy of resurrection. And the twelve tuning pitch pipes conjured precisely with such *gu* 古 "ancient" and *ya* 雅 "elegant" connotations regarding the restorationist agenda of Confucian scholarship. As previously explained, the first set of twelve *lülü* or tuning pitch pipes were commonly attributed to the Yellow Emperor, who also laid down many of the Sage mores, institutions, and laws Confucian scholars would later canonize.

Even without evoking legends, the established structure of knowledge as reflected in *mulu xue* 目錄學 "bibliographic science" of the 17th and 18th centuries unambiguously deemed the twelve tuning pitch pipes not just relevant but also essential for *yue* "music." Initiated in 1700 and completed with imperial approbation in 1726, the ten-thousand-volume *leishu* 類書 "encyclopedia by category" *Imperial Encyclopedia of Illustrations and Writings from the Earliest to Current Times* (gujin tushu jicheng 古今圖書集成) features a *yuelü dian* 樂律典 "Division on Music and Pitch Pipes" as one of its thirty-two constitutive divisions (*dian* 典).[36] Here, the mutual relevance of *yue* "music" and *lü* "pitch pipes" is apparent not only in the title of this division but also in the latter's organization. Comprising forty-six sections (*bu* 部), "Division on Music and Pitch Pipes" features, after an extensive introductory first section, a second section on *lülü* 律呂 "tuning pitch pipes," a third section on *shengyin* 聲音 "notes and

36 Chen Menglei 陳夢雷 et al., *Gujin tushu jicheng. mulu* 古今圖書集成.目錄 ("Table of Content of *Imperial Encyclopedia of Illustrations and Writings from the Earliest to Current Times*"), vol. 38; a facsimile version is available at https://ctext.org/library.pl?if=gb&file=91392&page=1, accessed April 15, 2019, 62-70.

tones" (i.e. the scale system), a fourth section on *xiao* 嘯 "transcendental whistling," a fifth section on *ge* 歌 "singing," and a sixth section on *wu* 舞 "choreography," while the remaining forty sections all focus on different types of instruments, both historical and current. These materials cover the broad variety of subjects pertinent to resurrecting the music performed for the ancient sage kings. And the prominence it accords to the twelve tuning pitch pipes at its start embodies the much-repeated maxim from Sima Qian's aforementioned *Records of the Grand Historian*: "the *huangzhong* pipe is the root and essence of all things" (*huangzhong wei wanshi genben* 黃鐘為萬事根本). Just as the *huangzhong* pipe (roughly equivalent to C) is the progenitor of all the other eleven tuning pitch pipes through various proportions, the twelve tuning pitch pipes as a whole lay the foundation for the system of notes, scales, and modes, which in turn govern the melodic and tuning specifics of songs and various classes of instruments.

This organizational scheme of the "Division on Music and Pitch Pipes" in *Imperial Encyclopedia*, in which pitch pipes perform their role as the "root and essence" by spearheading a sequential presentation of musical knowledge, was the standard format of monograph treatises on music from Cai Yuanding's *New Treatise on Pitch Pipes* in the 12th century to Zhu Zaiyu's *Essential Meaning of Pitch Pipes* in the 16th century and in fact the Qing's official music theory treatise *Orthodox Meaning of Pitch Pipes*. These three treatises as well as many others on tuning bear only *lülü* "tuning pitch pipes" and not *yue* "music" in their titles, yet they would all be classified in the category (*lei* 類) of *yue* 樂 "music" in *The Emperor's Complete Library in Four Sections* (siku quanshu 四庫全書, 1782). Compared to the *Imperial Encyclopedia* that predated it by half a century, *The Emperor's Complete Library* is even more adamant that the twelve *lülü*

or tuning pitch pipes are the essence and, arguably, the only essence for the study of *yue* "music."

Commissioned in 1773 by the Qianlong Emperor (r. 1735-1795), grandson of the Kangxi Emperor, *The Emperor's Complete Library* is an enormous anthology of more than three thousand books is organized into four sections (thus the title): *jing* 經 "[Confucian] canons," *shi* 史 "histories" (including geographies and *zhengshu* 政書 "compendia of administrative statutes and precedents," as well as dynastic histories and other chronicles), *zi* 子 "philosophies and arts" (including schools of thoughts other than Confucianism and studies on miscellaneous subjects such as mathematics, medicine, military tactics), and *ji* 集 "literary anthologies (including both literary works and criticisms). Within this traditional four-section bibliographic organization, the category of *yue* "music" containing treatises on *lülü* or tuning pitch pipes falls under the first section on *jing* or Confucian canons.[37] This standard bibliographic categorization of *yue* "music" follows the notion that there once existed a core Confucian corpus of *liujing* 六經 or The Six Canons—*yi* 易 "change," *shu* 書 "history" (or "Documents"), *shi* 詩 "songs," *li* 禮 "rites," *yue* 樂 "music," and *chunqiu* 春秋 "chronicles" (or "Spring and Autumn"). While the texts of the other five canons had all survived, albeit in various versions requiring philological investigations, Confucian scholars since at least the first century in the common era had been trying to explain why no such text of a *Canon of Music* was ever available. A commonly (and typically casually) evoked narrative laments that the *Canon of Music* was lost in the fires of the

37 See Ji Yun 紀昀 et al., *Sikuquanshu zongmu tiyao* 四庫全書總目提要 ("Summary Catalogue and Digest of *The Emperor's Complete Library in Four Sections*," 1798), vols. 38 and 39. See also R. Kent Guy, *The Emperor's Four Treasuries: Scholars and the State in the Late Ch'ien-lung Era* (Cambridge and London: Council on East Asian Studies Harvard University, 1987), 69-79.

Qin's aforementioned campaign of "burning of the books and burying of the scholars." While this explanation suits very well the "loss" narrative that defined the Confucian historical imagination regarding *yue* "music," some scholars doubted whether such a *Canon of Music* had ever been written down, or whether it existed separately from the other five canons that did survive and contain certain chapters on the institutions of music at the courts of the ancient sage kings.

Here, it is helpful to cite the short preface to the category of *yue* "music" in *The Emperor's Complete Library*. Along with many other such prefaces, the text is not found in the book collection proper but in the "Summary Catalogue and Digest" (*zongmu tiyao* 總目提要), which lists and summarizes both the more than three thousand titles included in the final collection and the more than six thousand other titles that did not make the cut, all in the order of the four sections and the various categories under them. Besides unpacking the subtle nuances surrounding the notion of a lost *yuejing* 樂經 "Confucian canon of music," the preface reveals just how narrow the operating concept of *yue* "music" is in this Qing-imperial bibliographic project that articulated the entire episteme of early modern Chinese scholarship through cataloguing books:

> 沈約稱《樂經》亡於秦。考諸古籍，惟《禮記經解》有"《樂》教"之文，伏生《尚書大傳》引"辟雝舟張"四語，亦謂之《樂》。然他書均不雲有《樂經》[…]。大抵樂之綱目具於《禮》，其歌詞具於《詩》，其鏗鏘鼓舞，則傳在伶官。漢初制氏所記，蓋其遺譜，非別有一經為聖人手定也。特以宣豫導和，感神人而通天地，厥用至大，厥義至精，故尊其教，得配於經。而後代鐘律之書，亦遂得著錄於《經部》，不與《藝術》同科。顧自漢氏以來，兼陳雅俗，豔歌側調，並隸《雲》《韶》。於是諸史所登，雖細至箏琶，亦附於經末。循是以往，將小說稗官，未嘗不記言記事，亦附之《書》與《春秋》乎？悖理傷教，於斯為甚。今區別諸書，惟以辨律呂、明雅樂者，仍列於經。其謳歌末技，弦管繁聲，均退列《雜藝》、

《詞曲》兩類中。用以見大樂母音，道侔天地，非鄭聲所得而奸也。[38]

According to Shen Yue [441-513 CE], the Confucian *Canon of Music* perished under the Qin [221-206 BCE]. From a survey of ancient books, only the chapter "Explicating the Canons" from the *Records of Rites* [edited c. 1st century BCE, which forms a part of the Confucian *Canon of Rites* corpus] mentions: "The [*Canon of*] *Music* teaches …," and Fu Sheng [fl. 3rd century BCE, who was famously responsible for preserving many Confucian texts through the Qin's persecution] in the chapter "Commentary on the Reign of Emperor Shun and on the Xia Era" in his *Grand Commentary on the Confucian Canon of History* also mentions a [*Canon of*] *Music*. Yet no other [ancient] book speaks of any *Canon of Music*. […] This is probably because the outline of [the institution of] music [*yue*] was articulated in the [*Canon of*] *Rites* [e.g. the *Records of Rites* that contains a chapter "Records of Music"], its lyrics recorded in the [*Canon of*] *Songs*, and its melodies, rhythms, percussions, and dances transmitted through the musical practitioners at the courts. Thus, what Zhi Shi [fl. late 3rd century and early 2nd century BCE] wrote down during the early years of the Former Han [202 BCE-8 CE] was what remained from those transmitted melodies and rhythms, not that there was a *Canon* [*of Music*] that was personally formulated by Confucius himself. Nonetheless, on special account of the fact that music spreads happiness and heralds peace, moves gods and humans alike, and communicates with Heaven and Earth and that its usefulness is extremely grand and its essence extremely subtle, [the ancients] venerated the edifying power of music and counted it alongside the Confucian canons. As a result, later treatises on bells and pitch pipes [*lü*] got to be catalogued under the section of Confucian canons [*jing*], separate from the section of arts and trades [i.e. *zi* "philosophies and arts"]. If we survey [the categorization of knowledge and books] since the Han era, we see that [many treatises and sections under the heading of *yue* "music"] contain both the elegant [*ya*] and the vulgar [*su*], and that they put licentious ditties and zither songs alongside the *Yumen* music played at the court of the Yellow Emperor and the *Shao* music played at the court of Emperor Shun during the ancient time. As a result, in the records of the various histories, even matters as trivial as the *zheng* zither and the pipa are included at the end of [the heading on music] under the section of Confucian canons [*jing*]. Were we to follow this line of reasoning, wouldn't have at to include under the headings of the Confucian *Canon of History* and

38 Ji Yun et al., "Summary Catalogue and Digest," Vol. 38, ff. 1r-2r.

> the Confucian *Canon of Chronicles* illicit histories and hearsays, which also record the deeds of individuals and the course of events? Here the violation of ethical principles and the damage of moral edification reach such an extreme. Therefore, here, we distinguish between the various kinds of books, and we only take books that study the tuning pitch pipes [*lülü*] and elucidate the "elegant music" [*yayue*] and continue to list them under the section of Confucian canons [*jing*]. As for ditties and songs, theatrics, accompanied songs, and lascivious tunes, those are all excluded [from the category of *yue* "music" under the section of Confucian canons] and included instead in the category of "arts and skills" [*yishu*] [in the section of *zi* "philosophies and arts"] and the category of "lyrics and opera tunes" [*ciqu*] [in the section of *ji* "literary anthologies"]. We use this organization to make clear that the Way of the Grand Music and *Ur*-Sound of the cosmos resonate with Heaven and Earth and shall not be violated by licentious sounds such as those from the depraved Zheng region [according to Confucius].

When presented with a performance of *ou* 謳 "ditties," *ge* 歌 "songs," *moji* 末技 "theatrics," *guanxian* 弦管 "accompanied arias [with strings and winds]," or *fansheng* 繁聲 "lascivious tunes" enumerated towards the end of the above-quoted passage, hardly anyone vested in the early modern Western European and trans-Atlantic Republic or Letters or in the current globalized music industry or the European-North American academia would think twice before labelling these activities as forms of music (or musique, musica, música, Musik, Музыка, and so forth). Here, however, the editors of *The Emperor's Complete Library in Four Sections* contend that all these music-making activities do not belong to the category of *yue* 樂 "music" but rather to the category of *ciqu* 詞曲 "lyrics and opera tunes" as well as the category of *yishu* 藝術 "arts and skills." In other words, opera, music theater, and accompanied arias whose myriad genres and ubiquitous performances from the Forbidden City to village temple fairs defined much of the soundscape of the 17th- and 18th-century Qing Empire did not even qualify as "music" according to what is effectively the Empire's official episteme.

The preface explains their deliberate decision to exclude opera and other traditions of songs from the category of music by articulating a concept of *yue* "music" in ethical as well as historicist terms, which are ultimately conterminous in this case. Weighing in on the scholarly debate on the Confucian *Canon of Music* (or the lack of existence thereof), the preface doubts that Confucius or the early followers of his school of thought had ever composed a corpus on *yue* "music" in the same way they had compiled the other *jing* or canons on change, history, songs, and so forth whose texts had survived. Instead of a single text, the preface argues, different types of information were recorded in different sources. *Rites of Zhou* (zhouli 周禮, c. 5th to 3rd century BCE) describes the supposed institutions of music at the court of the Former Zhou (c. 1046-771 BCE). *Records of Rites* (liji 禮記, c. 5th to 3rd century BCE, edited in c. 1st century BCE) contains teachings on the ethics and decorum of proper music. *Canon of Songs* (shijing 詩經, c. 11th century BCE to 6th century BCE) comprises lyrics of songs performed at the courts of the various ancient sage kings and those collected from among the people. And lastly, generations of *lingguan* 伶官 or professional musicians had been passing down the melodies, rhythms, instrumentations, and choreographies of the ancient *yayue* "elegant" music through oral and embodied transmissions, at least up until the early Former Han era. Thus, whereas all the other categories under the *jing* "Confucian canons" section of *The Emperor's Complete Library in Four Sections* each consist solely of hermeneutics, exegeses, commentaries, and philological studies on a single one of the several canonic Confucian texts, because there is no such a text of *Canon of Music* to bespeak, the category of *yue* "music" under the same "Confucian canons" section contains treatises on music in general.

This "in general," however, does not at all translate into a comprehensive approach to selecting books for the *yue* "music" category. On the exact opposite, the editors of the

bibliographic collection articulate in the preface a very specific standard of what may count towards a book on *yue* "music" qua part of the core Confucian knowledge. Though unlike the other five of the *liujing* or Six Confucian Canons there had never existed a *yuejing* 樂經 "canon of music," an "honorary canon status" (so to speak) is granted to *yue* "music" in Confucian scholarship, the preface explains, because the ancient sage kings valued *yue* both for its resonances with the cosmos and for its power in edifying their subjects and harmonizing the governance of their realm. As a result, *yue* "music" is a proper constituent of the Confucian canon despite the lack of any consistent body of text(s) inasmuch as it conforms to the political ethics underpinning its use by the ancient sage kings. Conversely, then, any discourse or practice that does not strive to emulate the ancients in their ethical concerns or, even worse, dares to violate the moral propriety and decorum that that ancient kings put front and center in their use of music should not be deemed a proper instantiation of *yue* "music" at all. Put differently, there is no such thing as *yue* "music" that isn't already *guyue* "ancient music" or *yayue* "elegant music," the former being the perfect music of the past that awaits reconstruction, the latter being a current practice that strives precisely for such reconstruction. The *gu* "ancient" and the *ya* "elegant" are synonymous, because what drives Confucian scholars towards reviving the music of the ancient sage kings is its *ya* "elegance": the power of its moral edification, social harmonization, and cosmic resonances, or indeed the very quality that has made the study of *yue* "music" part of the study of the *jing* or Confucian canons despite the lack of any text of a *Canon of Music*.

As editors of *The Emperor's Complete Library* reveal, this symbiosis of the *gu* "ancient" and the *ya* "elegant" effects the bifurcation between books on tuning pitch pipes on the one hand and those on "ditties, songs, theatrics, accompanied songs" and the likes on the other. "Books on

bells and pitch pipes"—bells (*zhong* 鐘) being literally and figuratively the weightiest instrument in an actual ensemble of *yayue* "elegant music" performed for imperial courts, and pitch pipes being the foundation of musical tuning, though rarely if ever used as actual instruments for performances—are admitted under the section of *jing* or Confucian canons because they ultimately contribute to the resurrection of the *ya* "elegant" *guyue* "ancient music." Meanwhile, the editors inveigh against some previous categorization of books and knowledge for confusing the *ya* "elegant" with the *su* "vulgar" (or popular). Some of them put "licentious ditties and zither songs" under the same category of *yue* "music" as they do the perfect music played at the courts of the Yellow Emperor and other ancient sage kings. An example the editors have in mind here might be *Comprehensive History of Institutions* (tong dian 通典, c. late 8th century) by Du You 杜佑 (735-812). A 200-volume *zhengshu* or history of administrative institutions covering regimes ruling China from the legendary ancients to the contemporary Tang era (618-907), *Comprehensive History* set the paradigm for the *tong* 通 "comprehensive" genre of administrative histories. Organized according to subject matter, it features an entire section on "Institution of Music" (*yuedian* 樂典). In addition to discussing the twelve tuning pitch pipes or practices of *yayue* "elegant music" across different generations of rulers, Du's "Institution of Music" spends two out of its seven volumes on different traditions of songs, dances, and theatrics. These include those *wangguo zhiyin* 亡國之音 "tunes of a dying nation" such as the infamous *Flower of the Fairy Tree in the Backyard* (yushu houtinghua 玉樹後庭花)[39] and the

39 Du You 杜佑, *Tong Dian* 通典 ("Comprehensive History of Institutions," c. late 8th century), Vol. 145, "Fifth Part of Institution of Music: Songs, Miscellaneous Song Tunes, Dances, Miscellaneous Dance Tunes" (樂典五歌雜歌曲舞雜舞曲).

many non-Chinese "ethnic" traditions from Central and Northern Asia that flourished at the Tang court as a result of its expansion into Inner Asia.[40]

For editors of *The Emperor's Complete Library*, most of these above contents to which Du granted the status of *yue* "music" would only qualify as "licentious ditties" (*yange* 豔歌), since they at best disregard and at worst deviate from the search for the perfect and morally edifying music of the ancients, the very thread that makes *yue* "music" part of the study of the Confucian canons. Distancing themselves from the supposed moral latitude of previous bibliographers when it comes to music, the editors purge all the books on what may be best characterized as "popular singing traditions" from the *yue* "music" category in the consecrated *jing* "Confucian canons" section and move them instead into the *ji* "literary anthologies" section, specifically under the *ciqu* "lyrics and opera tunes" category under the *ji* "literary anthologies" section. Notably, although contemporary opera, which, as mentioned above, dominated the soundscape of early modern China, is granted its own *shu* 屬 or subcategory *nanbei qu* 南北曲 "northern and southern operas" under the "lyrics and opera tunes" category, only two titles under this subcategory were even deemed worthy enough to be included in the actual bibliographic collection, with eight more titles featured in the "Summary Catalogue and Digest." What is more, although the early 18th-century *Imperial Encyclopedia* also excludes from its section on music operas and song traditions outside the institution of *yayue* "elegant music" or the revered Confucian *Canon of Songs*, *The Emperor's Complete Library* goes even further. As discussed above, the "Division on Music [*yue*] and Pitch Pipes [*lü*]" in the *Imperial Encyclopedia* begins

40 Ibid., Vol. 146, "Six Part of Institution of Music: Han-era Court Music, Tang-era Upright and Seated Ensembles, Foreign Music from the Four Directions, Theatrics and Acrobatics, Miscellaneous Music of the Previous Dynasties" (樂典六清樂坐立部伎四方樂散樂前代雜樂).

with the twelve tuning pitch pipes and concludes with scores of chapters dedicated to various stypes of instruments, from the bells and the zithers (*qin* and *se*) to the more obscure ones. Though never mentioning it by name, the editors of *The Emperor's Complete Library* complain precisely about this kind of organization in which "even matters as trivial as the *zheng* zither and the pipa are included at the end of [the heading on music] under the section of Confucian canons [*jing*]." Accordingly, in their organization of knowledge, the editors move all the books that focus primarily on musical instruments from the category of *yue* "music" under *jing* "Confucian canons" to that of *yishu* "arts and skills" under *zi* "philosophies and arts." Notably, despite its significance for Chinese literati culture I have explained in the previous chapter, even treatises, manuals, and anthologies of musical scores for the seven-string *qin* zither fail to qualify for *yue* "music" in *The Emperor's Complete Library*. Instead, the editors create for them a dedicated subcategory entitled *qin* "seven-string zither" under the category of *yishu* "arts and skills," with only four titles included in the actual collection and twelve more mentioned in the "Summary Catalogue." Two other titles, one on the different ensembles of instruments playing the various types of songs and dances at the multicultural Tang court, the other on the *xiegu* 羯鼓, an hourglass-shaped drum from Central Asia that also flourished at the Tang court, are included under another subcategory *zaji* 雜技 "miscellaneous tricks" under the "arts and skills" category.

What results, then, is a highly specific and admittedly narrow definition of *yue* "music" in *The Emperor's Complete Library*. Effectively, only books that take *lülü* "tuning pitch pipes" as it primary and foundational object of study qualify as studies on *yue* "music" as part of the scholarship on *jing* or the Confucian canons. And though a few titles do focus on the *qin* zither, when summarizing their contents in the "Summary Catalogue and Digest," the editors focus heavily on their contribution to the study of *lülü* "tuning pitch pipes" as a metonym for musical

tuning. In fact, at the end of the category of *yue* "music" in the "Summary Catalogue," the editors plain say: "here, when selecting and including books, we focus predominantly on those that make discoveries or give explanations of the tuning pitch pipes" (今所採錄多以發明律呂者為主). Thus, the structure of *The Emperor's Complete Library* not only attests to the thorough integration of the twelve tuning pitch pipes to the study of music, but also shows that *lü* "pitch pipes" were relevant to *yue* "music" precisely as the latter constituted part of the *jing* or Confucian canons that hark back to the ancient time. Arguably, within the context of Confucian scholarship whose primary concern with *yue* "music" was the emulating the ancient sages as not just an exercise in historical reconstruction but also an aspiration for the moral and ethical perfection of their governance, the study of *lü* "pitch pipes" whose primary concern was reconstructing the original twelve tuning pitch pipes that the ancient sage kings ordained as the foundation of all music functioned as a metonym not only for musical tuning but also for *yue* "music" itself.

❦

Now, it is important to reiterate that Mao's *Explication* predated the completion of the *Imperial Encyclopedia* by more than a decade and that of *The Emperor's Complete Library* by almost a century. As I will also discuss in later chapters, the rather purist view the editors of *The Emperor's Complete Library* espoused in excluding opera and popular song traditions from the category of *yue* "music" was more reflective of emerging debates on whether opera of the modern times might in fact contribute to the reconstructionist project on the perfect music of the ancients, rather than being representative of any universal scholarly consensus on the matter. Still, compared to the editors of *The Emperor's Complete Library* who considered opera and popular songs irrelevant to *yue* "music" as part of the study of the Confucian canons, Mao's

analogous argument positing of *lü* "pitch pipes" as irrelevant to the essence of *yue* "music" was a much more radical proposition. As suggested by *radix* "root," the Latin origin of the word "radical," Mao sought to fundamentally overhaul the study of music that had always posited the tuning pitch pipes as "the root and essence of all things," even in texts such as Du You's *Comprehensive History of Institutions* that do consider opera and songs a legitimate part of musical knowledge.

So how could Mao go against the established knowledge structure of Confucian scholarship and dismiss *lü* "pitch pipes" as irrelevant to *yue* "music?" To answer this question, it is necessary first to ask the reverse question: if not the twelve tuning pitch pipes, what does Mao consider essential to the *yueben* "essence of music" on which he felt enlightened by the Emperor's "Sage Instruction"? The answer lies in a passage I have already quoted verbatim in the previous section, but the pertinent part is worth repeating here:

> […]隔九既有餘，而隔七復不足，若是者何也？則以生必隔八。此非他為之，而聲為之也。夫樂也者，聲也。律也者，和聲者也。人聲不齊，而以五聲為之準。五聲，宮、商、角、徵、羽也。 […][41]

> […] [mutual generation] is superfluous at every ninth step, yet conversely insufficient at every seventh step—why is it so? Indeed, generation must occur at every eighth step. This is not effected [*sic*] by anything but sound [*sheng*]. Indeed, music is sound. Pitch pipes [*lü*] are what accord with sounds. Human voices [*sheng*] vary, and thus the five notes [*sheng*] are used as yardsticks. The five notes are *gong* [*do/ut*], *shang* [*re*], *jue* [*mi*], *zhi* [*sol*], and *yu* [*la*]. […]

To recall, this passage from Mao's commentary on Segment 6 of Zhang's *Petition* immediately precedes his analysis of "the five proper notes and the two altered notes" qua the discursive context of the Emperor's gloss of "mutual generation at every eighth step" as octave

41 Mao, *Explication*, vol. 2, .

equivalence. Though exhaustive, Mao's presentation of the seven-note diatonic scale system is unremarkable—except, however, for a wordplay on the Chinese character *sheng* 聲. On the one hand, *sheng* in Classical Chinese refers to "sounds" of all kinds and in general, whether they come from human, animate, or inanimate sources. In this regard, *sheng* can be contrasted with *yin* 音, which tends to refer to more musically or phonetically "organized" sounds. These include the timbre of musical instruments, as in the *bayin* 八音 "eighth timbres" mentioned in the previous section, and the sounds of human speech, such as *guyin* 古音 "ancient pronunciation" and *xiangyin* 鄉音 "native dialect." One exception is that the "human voice," inasmuch as there is a directly equivalent category in Classical Chinese, is rarely called "*renyin* 人音," be it singing or speaking, but almost always *rensheng* 人聲, as Mao called it in the passage above. On the other hand, *sheng* in a more specific sense is a technical term for the "note" of the "five proper notes and the two altered notes."[42] To a certain extent, *yin*, given its "organized sound" connotation, can be substituted for this specific meaning of *sheng* as notes; for example, both *wusheng buquan* 五聲不全 "lacking the five notes [*sheng*]" and *wuyin buquan* 五音不全 "lacking the five notes [*yin*]" describe those who have amusia. Still, *sheng* is far and away the preferred term in the context of the diatonic scale system.

This double-meaning of *sheng* sheds critical light on the two brisk pronouncements in the passage above: "Music [*yue*] is sounds/notes [*sheng*]. Pitch pipes [*lü*] are what accord with sounds/notes [*sheng*]" (夫樂也者，聲也。律也者，和聲者也。). The emphatic construction in

42 For the intricate connections between the concepts of *sheng* and *yin* to the study of speech, language, and phonetics, see Jiang Yong, "Foreword" to *Guyun biaozhun* 古韻標準 ("A Standard for Ancient Rhymes," c. 1759), a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5452, accessed January 22, 2019), Preface Volume (*juanshou* 卷首), 7-28.

this pair of sentences, *A* 者 *B* 也, literally "that which is *A* is *B*," highlights their centrality to Mao's overall argument regarding the "essence of music" in his *Explication*: the essence of music is *sheng*, whereas *lü* "pitch pipes" are not only secondary to but also ought to accord with *sheng*. Here, I argue, *sheng* should be understood in both of its specific and general meanings. In the specific sense, the Kangxi Emperor's interpretation of "mutual generation at every eighth step" as octave equivalence places the phrase squarely in relation to the *sheng* "notes" of *wusheng erbian* "the five proper notes and the two altered notes." This, of course, is the discursive distinction I analyzed in the previous section whereby, in Mao's words, the Kangxi Emperor recast the phrase from a signifier having to do with *lü* "pitch pipes" to one having to do with *sheng* "notes."

Because *sheng* means not only "notes" in the specific sense but also "sounds" in the general sense, however, it follows that the Emperor's gloss of "mutual generation at every eighth step" differs from its received definition not only discursively but also epistemically. Put differently, compared to the received definition of the phrase, the Emperor's new definition not only produces knowledge *about sheng* "notes"—that is, the same note returns at every eighth step in a seven-note diatonic scale system—but also produces knowledge *through sheng* "sounds." Indeed, whereas *sheng* "notes" define the discursive positionality of the Emperor's definition of "mutual generation at every eighth step" itself, *sheng* "sounds" played a pivotal role in how the Kangxi Emperor arrived at such a definition. Rather than referencing any canonical Confucian texts or renowned treatises (which, after all, would have made his mistake more than evident), the Emperor went out of his way to fetch a *di* flute and a *se* zither from the Altar of Heaven three miles to the south of the Forbidden City. And rather than simply matching the number "eight" in "mutual generation at every eighth step" with the various fingering positions

on the two instruments, the Emperor invited his scholar-officials to listen with their own ears to the patterns of different *sheng* "sounds"/"notes" the two instruments produced through these various fingerings. In fact, even though their recollections of the court gathering are significantly shorter than Mao's verbose commentaries in *Explication*, both Zhang Yushu and Li Guangdi highlight sounding and listening as the very processes whereby the Emperor produced knowledge regarding "mutual generation at every eighth step." According to Zhang, the Emperor fetched the two instruments specifically "in order to examine their sounds [*sheng*] and tones [*yin*]" (*shen qi shengyin* 審其聲音). According to Li, the Emperor himself did not quite know what "mutual generation at every eighth step" meant before the court gathering, and he only shared his understanding of it as octave equivalence with his courtiers after they had *ting* 聽 "listened" to Daoist musicians play the zither.

Thus I argue that by pronouncing *sheng* "notes"/"sound" as the *ben* "essence" of *yue* "music," Mao is not making an ontological claim but rather an epistemological one. According to Mao, what distinguished the Emperor's definition of "mutual generation at every eighth step" from its received definition and what therefore makes it closer to the "essence of music" is not just a discursive distinction, whereby the former bespeaks *sheng* "notes" and the latter bespeaks *lü* "pipes." Indeed, were this discursive distinction the primary difference between the two definitions, it would be very difficult for Mao to defend let alone aggrandize the Emperor's understanding of the phrase, which completely disregards scholarly consensus on its meaning and discursive associations. Rather, for Mao, what allowed the Emperor's lecture-demonstration of "mutual generation at every eighth step" to get at *sheng* "notes"/"sound" qua the essence of music was the process through which it produced knowledge: the embodied, integrated production and reception of sounds. In other words, because *sheng* "notes"/"sound" is the

essence of *yue* "music," producing any knowledge meaningful and pertinent to *yue* "music" requires sounding and listening to play a pivotal role in the process.

It might seem that the above epistemology whereby sounding and listening is a prerequisite for knowing anything about the "essence of music" comes more from my interpretation of the court gathering itself than from Mao's *Explication*. Admittedly, Mao spills little ink on epistemology in his commentary on Segment 6 in which he explicitly identifies *sheng* "notes"/"sound" as the "essence of music." Nevertheless, the argument that no meaningful knowledge about music can be produced without sound playing a critical, epistemic role permeates his entire treatise. Particularly, just as it was the epistemic involvement of *sheng* "sound"/"note" in the form of sounding and listening that allowed the Emperor's lecture-demonstration to get at the essence of music, for Mao, it was also due to the lack of such sonic epistemic involvement that the received definition of the phrase in reference to pitch pipes falls short of such a goal. To recapitulate the passage from the very beginning of Segment 5 where Mao explicitly dismisses the proportioning of pitch pipes as irrelevant to the essence of music:

> […] 于是，先立一黃鐘九寸之管以為之質，而後減之而短與增之而長，皆有成法以推之。此不過假此度數長短以示律之有高下，其于實實聲音升降圜轉之節，全不相合。故曰：此作律之本，亦造歷審度同量衡權之本，而于樂本無與焉。[…][43]
>
> […] Therefore, a 9-cun-long *huangzhong* pipe is first established as the foundation, and then, whether to make a shorter pipe via reduction or to make a longer pipe via elongation, there are established methods to deduce the results. This, however, is nothing more than borrowing [*jia*] these measurements and numbers [*shu*] of varying magnitudes to show that the tuning pitch pipes can be either high [*gao*] or low [*xia*], yet it does not at all match the patterns of ascent, descent, transpositions, or mutations of the actual, concrete [*shishi*] sounds [*sheng*] and tones [*yin*]. Therefore, we say: this is the essence of making pitch pipes and the essence of establishing the calendar and examining the standards

43 Mao, *Explication*, vol. 2, f. 1v.

> of length, volume, and weight, yet it is irrelevant to the essence of music. […]

Notice the contrast between *jia* 假, which means "to borrow" or "to substitute" here but can also mean "fake," and *shishi* 實實 "actual, concrete," which is formed through the emphatic reduplication of one adjective, *shi* 實. Here, Mao puts critical pressure on the link between the absolute and relative lengths of the twelve tuning pitch pipes and the sounds these pitch pipes produce in varying patterns of *gao* "high," *xia* "low," *shengjiang* 升降 "ascent and descent," and *huanzhuan* 圜轉 "transposition and mutation" (an abbreviation of *xuangong zhuandiao* 旋宮轉調 "rotating the *gong* [i.e. transposition to a different key, in Western terms] and shifting the mode [i.e. switching from one mode to another without changing the key, for example from C-Ionian to D-Dorian]). While conceding that the patterns of *du* 度 "measurements" and *shu* 數 "numbers" of the lengths of the twelve tuning pitch pipes can represent the patterns of their *sheng* "notes"/"sounds" to a certain extent, Mao argues that, fundamentally, there is no direct, immediate relation among or homology between them. Rather, measurements and numbers can only be considered supplements for the "actual, concrete" sounds—that is, signifiers that are *jia* "borrowed" to *shi* 示 "show" the variance between sounds without ever being able to fully re-present or index their specific differences. Indeed, Mao summarizes this epistemological lesson when concluding his commentary on Segment 5:

> […] 是"隔八相生"四字，尚未能定。而欲其知自然之理與其所以然之故，必不得之數也。[44]

> […] The meaning of these four characters, *geba xiangsheng* "mutual generation at every eighth step," cannot yet be determined. Nonetheless, if one wants to know its natural principle

44 Ibid., vol. 2, f. 6r.

or cause, one certainly cannot obtain it through numbers [*shu*].

In other words, in order to produce meaningful knowledge about *yue* "music," one must not focus on *shu* 數 "numbers" but on *sheng* "sounds"/"notes," which he will soon anoint as the essence of music in the remarks that follow on Segment 6.

Re-Sounding the Pitch Pipes

To summarize, at the heart of Mao's reflection on the court gathering in *Explication* lies an epistemological claim. By declaring *sheng* "notes"/"sound" to be the essence of *yue* "music," Mao posits that music is ultimately knowable only through *sheng* in the form of listening and sounding. Meanwhile, because *shu* "numbers" are not fully substitutable for these sonic phenomena, studying the mathematical patterns of measurements, proportions, and numbers cannot, according to Mao, bring one to this *sheng* "sound"/"notes" essence of music. Notice that in both the beginning and the end of his remarks on Segment 5, Mao draws an epistemological divide not just between *shu* "numbers" and *sheng* "sounds" in general. Instead, he specifically uses this epistemic split to argue that the "essence of making pitch pipes" is irrelevant to the essence of music qua *sheng* "sounds"/"notes." Logically, Mao could make such an argument only by maintaining that *sheng* "sounds"/"notes" in the form of listening and sounding is inherently absent from the way knowledge is produced through the twelve tuning pitch pipes and that the analyses of *dushu* "numbers and measurements" are all that the twelve tuning pitch pipes afford in producing knowledge. Put differently, Mao could use the epistemic distinction between *shu* "numbers" and *sheng* "sounds"/"notes" to justify the irrelevance of *lü* "pitch pipes" to *sheng* as the essence of music only by simultaneously arguing that the twelve tuning pitch pipes are, in

so far as they are concerned in the established discourse on *lülü* "tuning pitch pipes" or music theory, epistemically silent.

As it turns out, the argument that *lülü*, both as the twelve "tuning pitch pipes" and as the scholarly discourse of musical tuning named after them, produce knowledge only through *shu* "numbers" and not through *sheng* "sounds" would become the very basis of Mao's radical proposal in *Explication*, namely to abandon the twelve tuning pitch pipes as the *sine qua non* foundation of the restorationist project of *yue* "music," or at least reinvent *lülü* as instruments and as a field of inquiry into a subset of the study of *sheng*. To understand how Mao perceived the twelve tuning pitch pipes as knowledge-affording objects in relation to his contrary epistemes of *shu* "numbers" and *sheng* "sound," I will first examine his commentaries on Segments 3 and 4 of Zhang's *Petition*, which describe the Kangxi Emperor's lecture on how to calculate the circumference of a circle based on its diameter. Recall that the Emperor specifically disabused his scholar-officials of the maxim *jingyi weisan* "the circumference of a circle is three, when the diameter is one," which Cai Yuanding uses in his 12th-century paradigm-setting *New Treatise on Pitch Pipes*. For the study of *lülü* "tuning pitch pipes" as a metonym for musical tuning, knowing the precise mathematical relation between a circle's circumference and diameter (now defined as π) is crucial for determining the shapes and sizes of the twelve tuning pitch pipes. Indeed, the "Treatise on Pitch Pipes and Calendar" from Ban Gu's 1st-century CE *Book of the Han* provides 12th-18th-century scholars with the earliest extant record on the physical dimension of the *huangzhong* pipe: "the *huangzhong* pipe is 9 *cun*, and its hollow enclosure is 9 *fen*" (黃鐘九寸空圍九分). In order to make a *huanghzong* pipe based on this record, then, one has to compute the diameter of the *huangzhong* pipe from its stipulated 9-*fen wei* 圍 "enclosure," although scholars

disagreed over whether "enclosure" in this case means the circumference or the surface area of the inside of the pipe.

In keeping with his usual attitude, Mao's commentaries on Segments 3 and 4 praise the Emperor for his skills, specifically here in mathematics. Yet both of his Segments conclude by dismissing what he has just praised as ultimately irrelevant to *yue* "music":

> [Segment 3]
> […] 然則數卒非聲，但以製管。管本非樂，徒以造器。而製管、造器之本，尚粗疎蔑裂，無往而不見其訛舛。如此，善求樂本者，不于聲是求，而徒求之器數之末，亦獨何哉！[45]
>
> […] However, numbers [*shu*] are, in essence [*ben*], not sounds [*sheng*], but they are only used to make pipes. Pipes are, in essence, not music [*yue*], but are only used to manufacture instruments. Still, the essence of making pipes and manufacturing instruments is still poorly understood and broken down to a state of disrepair, and there is nowhere one can go without seeing errors and misunderstandings in this regard. As such, those who are skillful in the pursuit of the essence of music [*yueben*] do not pursue instruments in sounds [*sheng*] yet only pursue it in the minor details of the measurements [*shu*] of instruments: what can you do about it?
>
> [Segment 4]
> […] 彼徒知備數製器為實用，而不知與聲音之道。相隔萬里，實空言也。夫空言雖精猶無用，况訛舛耶？[46]
>
> […] These people [referring to Cai Yuanding] only know how to produce numbers [*shu*] in order to make instruments [*qi*] for use, yet they do not know the way of sounds [*sheng*] or tones [*yin*]. They are far away from the latter, and what they say are just empty words. And empty words remain useless even when they are right—not to mention when they are wrong!

One cannot help but marvel at Mao's mastery at sneering, particularly at the end of his commentary on Segment 4. The wreckage comes at two levels. To begin with, Mao concurs with

45 Ibid., vol. 1, ff. 13v-14r.
46 Ibid., vol. 1, f. 15v.

the Emperor that Cai's calculation of the circumference of a circle as three times its diameter is utterly imprecise. What is more, in both conclusions, Mao summarily rejects the making of the twelve tuning pitch pipes as involving only *shu* "numbers" and not *sheng* "sounds" and therefore having nothing to do with *yue* "music" at all. A scholar would get nowhere close to the essence of music even if they had figured out the precise measurements for making the twelve tuning pitch pipes—and Cai Yuanding could not even get that right!

Besides confirming Mao's dismissal of *dushu* "measurements and numbers" of the twelve tuning pitch pipes, the two conclusions quoted above also specify that *shu* "numbers" promise false hope in illuminating the essence of music specifically by contributing to the making of *qi* 器 "instruments." As Mao states rather clearly in his conclusion to the commentary on Segment 3: "numbers are in essence not sounds, but are only used to make pipes. Pipes are, in essence, not music, but are only used to manufacture instruments." Crucially, Mao does not at all question the standard scholarly agenda of *lülü* "tuning pitch pipes" as musical tuning: securing the most exact *shu* "numbers" for measuring and proportioning the twelve tuning pitch pipes, making pitch pipes according to these numbers, and using pitch pipes thus made to tune and manufacture all other musical instruments. Indeed, Mao's disenchantment with *shu* "numbers" and *qi* "instruments" (including *lü* "pitch pipes") is not a historicist disenchantment: that is, Mao does not at all appear to believe that reconstructing the exact pitch pipes the ancient sage kings used to determine the *dian* "institution" of music is ultimately futile. For Mao, the epistemic wedge, so to speak, between the modern scholars of *lülü* "tuning pitch pipes" and the perfect *yue* "music" of the ancients is found not between the *shu* "numbers" transmitted in historical sources and the ancient pitch pipes themselves. Instead, it is found between the ancient pitch pipes and ancient music. Thus even if there were to be a set of ancient pitch pipes perfectly restored, or

even if a set of ancient pitch pipes had somehow survived millennia of wars, regime changes, and physical deterioration, these *qi* "instruments" would still offer little to no meaningful knowledge regarding what the *yue* "music" of the ancients was like.

Mao makes exactly such an argument in his commentary on Segment 9, which is not part of Zhang Yushu's original *Petition* but rather the Emperor's official response to it. As mentioned in Chapter 1 above, Zhang's petition that "treatises on musical tuning [*yuelü*] and mathematics [*suanshu*] be compiled" did not meet with an enthusiastic imperial audience. In fact, the reasons and, arguably, excuses the Emperor cited in rejecting Zhang's suggestion were far more mortifying than the rejection itself:

> 旨:
> 朕于萬幾餘暇，常覽《性理》諸書。見"隔八相生"、"圍三徑一"之說載在《性理》之首。以為，如果無用，昔之儒臣何以置諸篇首？反覆披閱，因未明晰，時向儒臣講問；此蓋欲究明書義。而然今儒臣亦不知此，或竟屬不可用，或雖辨明亦有無益之處。故云爾耶。其編輯書籍，乃儒臣專責。朕三十餘年，過于焦勞，僅能夙夜兢兢，料理國政。豈可效狂妄輕浮之文人議論往哲？這所奏無益。知道了。該部知道。[47]
>
> Decreed:
> In Our leisure time among the tens of thousands of Our engagements, We often browse books such as *Grand Compendium of Natures and Principles*. We see that the theories of "mutual generation at every eighth step" and "the circumference of a circle is three, when the diameter is one" recorded in the first volumes of *Grand Compendium.* We thought, if these theories are useless, how come Confucian scholars of previous eras put these matters in its first volumes? We read and annotated these volumes again and again, and because We could not clearly illuminate [the two theories], from time to time We asked Our Confucian scholar-officials to explain; the purpose of this was to thoroughly clarify the meaning of those books. However, even today's Confucian scholar-officials do not know [the truth about the two theories], and it even turns out that some of [these theories] are either unusable or pointless, even if they were clearly understood. And

47 Ibid., ff. 19r-19v.

> therefore, We say: editing and compiling books is the special duty of Confucian scholar-officials. For thirty years, We have been overly occupied and exhausted, and We could only live in anguish day and night in order to take care of matters of the state. How could we imitate those arrogant and shallow men of letters and subject thinkers of the past to criticism? This memorial is pointless. We have noted this, and let the relevant Board be informed.

Not even such a glowing account of his lecture-demonstration as Zhang's *Petition* could make the Kangxi Emperor stop insulting his scholar-officials. This time, the Emperor not only berates the Han Chinese scholar-officials for their ignorance concerning how to calculate the circumference of a circle and in what "mutual generation at every eighth step" means, but also for the "arrogance and shallowness" of their pointless treatise-writing and book-compiling. Yet rather than commiserating with his fellow scholar-officials, who must not have had a very good time reading such a harsh rebuke, Mao takes great delight in a particular line from the Emperor's response to Zhang's *Petition*, a line that ends up becoming the only direct quote from the original *Petition* in the commentary sections of Mao's *Explication*:

> […] 惟《律吕新書》，世未考驗，遂有指是書為通古樂者。今伏讀聖旨有云："或竟屬不可用，或雖辨明亦有無益之處。"大矣哉！聖人之言也。[…][48]

> […] However, the scholarly community has not examined or verified [Cai Yuanding's] *New Treatise on Pitch Pipes*, and some suggest that this book thoroughly transmits ancient music [*guyue*]. Yet now, prostrating, I read the Emperor's Sage Decree that says: "some of [these theories] are either unusable or pointless even if they were clearly understood." What great words! These are the words of a Sage. […]

Now, there are partisan reasons why Mao would have thoroughly reveled in the Emperor's dismissal of Cai Yuanding's *New Treatise on Pitch Pipes* as "unusable" and

48 Ibid., f. 20r.

"pointless," as he was a notorious opponent to Zhu Xi's philosophically and metaphysically oriented school of Neo-Confucianism. Often referred to as *lixue* 理學 "the learning of principle," Zhu Xi's interpretation of Confucianism posits as its final objective *qiongli* 窮理 "the fathoming of principle": by meditating on the words of the ancient sages, a scholar would bring their *xing* 性 "nature" in accordance with the cosmic *li* 理 "principle" from which all things have originated. Notably, Zhu Xi and his disciples of the Song and early Ming (1368-1644) eras emphasized *gewu* 格物 "investigating things" or studying texts and phenomena of the world as the pathway towards cosmic enlightenment. Then, beginning in the 15th century, a different school of Neo-Confucian thought, most often referred to as *xinxue* 心學 "learning of the mind" and identified with the Ming-era philosopher Wang Yangming 王陽明 (1472-1529), argued that the most important revelation of the philosophical and metaphysical principles of the cosmos resided right in one's own *xin* 心 "heart-mind." Rather than studying the external world, "learning of the heart-mind" thus identified experiencing the authentic inner self as the truest way to the principle.[49]

Living in the late 17th century, however, Mao subscribed neither to Zhu Xi's *lixue* "learning of the principle," which flourished during the 13th to 15th centuries, nor to Wang Yangming's *xinxue* "learning of the mind," which gained traction in the 15th to 16th centuries. Indeed, the Qing conquest of China in the mid 17th-century left Confucian scholars asking how Ming China as the center of the civilized world could have been conquered by the Manchu "barbarians." Many accused the *xu* 虛, "vapid" philosophizing of the Ming-era "learning of the

49 See Peterson, "Confucian Learning in Late Ming Thought," 708-788, particularly 716-728.

mind," of ignoring the *shi* "concrete" matters of statecraft. Amid this consensual backlash against the radical subjectivism of "learning of the mind," many early-Qing scholars championed a return to Zhu Xi's "learning of the principle" strand of Neo-Confucianism. These included Li Guangdi, arguably the most influential Han Chinese scholar-official of the time, who convinced the Kangxi Emperor to follow the precedent of the Ming regime and enshrine Zhu Xi's school of thought as the state ideology, at least as far as civil examinations were concerned. Nonetheless, an increasing number of scholars also extended the critique of empty metaphysical musings to include not only the Ming-era "learning of the mind" but also the Song-era "learning of the principle" epitomized by Zhu Xi, which they began to call, somewhat derogatorily, *songxue* 宋學 "Song-era learning." It was these scholars, among whom the most renowned were Huang Zongxi 黃宗羲 (1610-1695) and 顧炎武 (1613-1682), who championed the statecraft-oriented *shixue* "concrete learning" and paved the way for the rise of textual criticism and philology as the heart of the *kaozheng* "evidential learning" paradigm in the 18th century.[50]

Although Mao's contemporary and immediate subsequent generations often considered his output too idiosyncratic to be fully identified with one school of thought or another, he was broadly renowned (or loathed) for his vocal criticism of Zhu Xi.[51] One of his most famous or notorious works was *Rectifying the Mistakes of Zhu Xi's Collected Annotations of Passages and Sentences from the Four Books* (sishu gaicuo 四書改錯, 18th cent.), which consists of 451 mistakes in Zhu Xi's *Collected Annotations of Passages and Sentences from the Four Books* (sishu zhangju jizhu 四書章句集註, c. 12th cent.). The latter was not only the most critical

50 See Elman, *From Philosophy to Philology*, Zhao trans., 18-27, as well as Guy, *The Emperor's Four Treasuries*, 121-156.

51 See footnote No. 5 above.

corpus of Zhu Xi's Confucian thought, it was also an official "textbook," so to speak, for any Ming- and Qing-era candidate for the civil examinations, which demanded in-depth knowledge in the "Four Books" selected and curated by Zhu Xi himself as the epitome of Confucian thought: *Mencius* (mengzi 孟子), *Analects* (lunyu 論語), *Great Learning* (daxue 大學), and *Doctrine of the Mean* (zhongyong 中庸). Thus, given that the *Grand Compendium of Natures and Principles* (xingli daquanshu 性理大全書, 1415) was an encyclopedia of Zhu Xi's school of thought and that Cai Yuanding's *New Treatise on Pitch Pipes* is one of the nine treatises featured in its entirety at the beginning of the *Grand Compendium*, it is hardly surprising that Mao would acclaim the Emperor for branding Cai's *New Treatise* as "unusable" and "pointless."

Nonetheless, although the Kangxi Emperor's snide remark against Cai Yuanding fits very well into Mao's scholarly politics, his two characterizations of Cai's theories as *bukeyong* 不可用 "unusable" and *wuyi* 無益 "pointless" match exactly Mao's arguments about the state of affairs in the study of *lülü* "tuning pitch pipes" or musical tuning. Indeed, in the conclusions to his commentaries on Segments 3 and 4 of Zhang's *Petition* quoted above, Mao concurred with the Emperor that Cai's methods for calculating the base circumference of pitch pipes is too imprecise. And in the last quote above, from his commentary on the Emperor's rebuke of Zhang's *Petition*, Mao further argues that Cai's theories of the twelve tuning pitch pipes in his *New Treatise on Pitch Pipes* have not been verified by the scholarly community, criticizing scholars for mindlessly canonizing the treatise simply out of its author's stature as a student of Zhu Xi's. To use the Emperor's diction, then, Cai's theories on how to set the exact measurements and proportions of pitch pipes are "unusable," in that they are too imprecise to be used to reconstruct the exact twelve tuning pitch pipes set by the ancient sage kings

But the word *wuyi* "pointless," literally "there would be no benefit," delegitimizes the entire endeavor of trying to reconstruct the exact *dushu* "measurements and numbers" of the twelve tuning pitch pipes, regardless of whether such reconstructions are correct. The Emperor's comment on Cai's theories, "some of [these theories] are […] pointless even if they were clearly understood," echoes Mao's earlier conclusions in his commentaries on Segments 3 and 4 that studies on the measurements and proportions needed to reconstruct the twelve tuning pitch pipes are just *kongyan* 空言 "empty words." Even though *shu* "numbers" are important for making *qi* "instruments" such as pitch pipes, Mao argues that these instruments would make no difference in creating any meaningful knowledge of music, particularly *guyue* "ancient music." Indeed, Mao asserts this ultimate futility of studying *shu* "numbers" in order to make *lü* "pitch pipes" exceptionally clear towards the end of his commentary on the Emperor's rebuke of Zhang's *Petition*:

> […] 先儒嘗云：著樂書千卷，算億算萬，不過為脩數耳。其所以備數者，不過為製器耳。則向使有古器于此，為帝舜、后夔之所親製者，而相沿至今，将必《簫韶》再興，古樂在是矣。乃隋時何妥論樂，謂秦滅齊時，《韶》之樂器在齊也。及漢滅秦，《韶》之樂器尚在秦也。漢高遷其器別為一縣，改名曰《文始》之樂。而乃使李延年為新聲，敕公孫宏、司馬相如為詩歌。明明虞廷所製器，而其所奏者為漢人之聲，所播者為漢人之詩歌。其所為《文始》之樂，則仍非虞廷之樂，而漢人之樂。然則器之必無與于樂也審矣。况《新書》所辨，尚未製器；器且不成，何益于樂？葢聖謨煌煌，昭如星日！一言之闢，已破渾頓。必欲勒成一書，為萬古法則。[…][52]
>
> […] Confucian scholars of the past have said: "the point of writing thousands of volumes of books on music [*yue*] and of calculating in the thousands and millions is nothing other than to perfect the numbers [*shu*]. And the reason why numbers should be perfected is nothing other than to make instruments [*qi*]." In this way, suppose there were to be ancient instruments [*qi*] in existence that had been made by Emperor Shun and Hou Kui themselves and had survived

52 Mao, *Explication*, vol. 2, ff. 22r-22v.

> until today, then they would have been able to bring back the exact *Shao* music played at Emperor Shun's court, and ancient music [*guyue*] would reside in them. However, when He Tuo [fl. 592] participated in the discussion of music during the Sui [581-619] era, he said that when the Qin [221-207] toppled the Qi, the original instruments that played Emperor Shun's *Shao* music was at the Qi court. When the Han [202 BCE-220 CE] toppled the Qin, those instruments were still at the Qin court. The Gao Emperor of Han moved those instruments, established them into a separate group, and renamed *Shao* as *Wenshi*. However, [the later Wu Emperor of Han] asked Li Yannian to make new tunes [*sheng*] and ordered Gongsun Hong and Sima Xiangru to make new verses [*shi*] and songs [*ge*]. Obviously, those were still instruments from the court of Emperor Shun, yet what they were playing were the tunes [*sheng*] of the Han era and what they accompanied were verses and songs of the Han era. The *Wenshi* music they played was still not the music of the court of Emperor Shun but rather the music of the Han era. Therefore, it is certain that instruments [*qi*] necessarily have no bearing on music [*yue*]. Not to mention: what [Cai Yuanding's] *New Treatise on Pitch Pipes* has argued has never been used to make instruments; if not even its instruments can be made, how can it have any bearing on music [*yue*]? Indeed, how brilliant is the Sage Deliberation, as illustrious as the stars and the sun! With a single utterance, the Emperor cut through the chaos. Of course it should be compiled into a treatise and be taken for the perennial rule and method. […]

At the start of the passage above, Mao indulges in a thought experiment. Suppose it were true that restoring the exact measurements, proportions, and other designs of the ancient musical instruments—particularly the twelve *lülü* or tuning pitch pipes, to which all the other instruments are to be tuned—is all there is to resurrecting the music of the ancient sage kings from its perennial loss. It follows that, were there really to be a set of *qi* "instruments" or *lü* "pitch pipes" exactly identical to those of the ancients, these instruments would be sufficient to bringing ancient music back to life. Conversely, had there ever been a point when ancient *qi* "instruments" were used yet still failed to resurrect the ancient *yue* "music," by the law of logical contraposition (i.e. if P → Q is true, then not P → not Q must also be true), it would follow that *qi* "instruments" and the endeavor of reconstructing the exact *dushu* "measurements and

numbers" are ultimately and fundamentally futile for creating any meaningful knowledge towards the resurrection of ancient music.

And Mao proves this contraposition through the words of He Tuo 何妥 (fl. 592), a Sogdian musician from modern-day Uzbekistan who served as a court musician under the Sui Dynasty (581-619).[53] Scholars of the 16th to 18th centuries often looked back to the Sui era and the first century of the Tang era (618-907) as a critical juncture in the history of *yue* "music." After centuries of divisions and warfare since the Later Han (25 CE-220 CE), the Sui finally brought both the Yellow River and Yangtze River drainage areas once again under a centralized regime. Though short-lived, the Sui laid the foundation for the first half of the Tang Empire, which is still often considered the golden age of pre-modern China for its economic prosperity, military expansions into Central Asia, and cultural cosmopolitanism. The institutional stability and literary cultivation of the Tang era facilitated extensive historiographic projects. Indeed, no less than eight official dynastic histories were compiled under the Tang, as well as a joint series of *zhi* "treatises" shared among five of these dynastic histories, including a "Treatise on Pitch Pipes and Calendar" and a "Treatise on Music" (*Yuezhi* 樂志).

In addition to expanding the corpus of musical and music-theoretical historiography, the Sui and early Tang eras also saw significant reforms to the Confucian institution of music practiced by the court and the state—reforms, that is, always carried out in the name of restoration. As part of its efforts to rebuild a unified state for the first time in the late 2nd century, the Sui aspired to reconstruct and renew the proper institutions of music, and thus its founding Emperor Wen (r. 581-604), whose regnal title was Kaihuang between 581-600, summoned the

53 See Fang Xuanling et a., *Book of the Sui*, vol. 75, "Confucian Schoalrs" (*rulin* 儒林); a facsimile version is available at https://ctext.org/library.pl?if=gb&file=57366&page=1, accessed April 15, 2019, 117-133.

famous “Discourse on Music of the Kaihuang Era” (*kaihuang yueyi* 開皇樂議). Various proposals were tabled, often from competing factions of courtiers, and the ideological underpinning of the dispute focused on where the orthodox tradition of music had survived the most intact. Though the Sui rose in northern China and claimed to be the rightful heirs to the Confucian rulership, a significant part of its ruling body politic descended from the nomadic peoples who migrated in droves during the 4th century from Northern and Central Asia into the Yellow River region during the so-called “The Five Barbarians throwing China into chaos” (*wuhu luanhua* 五胡亂華). Meanwhile, even though the Sui denigrated the southern Chinese regimes it eventually conquered as *man* 蠻 “southern barbarians,” most of the political and cultural elites of southern China at the time had actually migrated from the Yellow River to the Yangtze River area during the 4th century—known as “Chaos of the Yongjia Era” (*yongjia zhiluan* 永嘉之亂; Yongjia was the regnal title of Emperor Huai of Jin, r. 307-313)—precisely when northern China was being overrun by nomadic invasions and migrations. Adding on to this dispute over northern vs. southern legitimacy, since the 6th century, a constant influx of migrants, goods, and cultural influences from Central Asia also exerted considerable influence on the soundscape of northern China, thanks to political alliances and marriages with the Turkic Khaganate that had recently unified the Central Eurasian steppe. Indeed, He Tuo was far from being the only Central Asian musician who contributed to the Wen Emperor’s “Discourse on Music,” of which extensive records are found in the *Book of Sui* (suishu 隋書, 636/656). As I will show in Chapter Four, a key conclusion of this Discourse, a system of eighty-four modes combining the twelve pitch pipes with the seven-note diatonic scale, derived largely from Central Asian influences and would exert lasting legacies on Chinese music and its historiography.

Of course, Mao evokes He Tuo not to harness his Central Asian or cosmopolitan connections but to invoke his claim that, at least by the earlier half of the Former Han (206 BCE-9 CE), the regime that succeeded the "arch-villain" Qin, the ancient institution of music established by Emperor Shun was still in a sound state. In the last passage quoted above, the quotation of He Tuo paraphrases a disquisition he reportedly made at Emperor Wen's "Discourse of Music" as recorded in his biography in the *Book of Sui*. According to the latter source, He tuo narrated an entire history of *yue* "music" from the ancient times up to his age. Unlike most historiographies of *yue*, He Tuo identified the cataclysmic moment of the loss of *guyue* "ancient music" neither with the 3rd-century BCE Qin dynasty nor with the more recent "Chaos of the Yongjia Era" that saw massive migrations throughout northern and southern China. Rather, He Tuo argued that ancient music had never been lost at all until the Houjing Rebellion (*houjing zhiluan* 侯景之亂) in 548 CE. By that time, He Tuo suggested, the music of the court of Emperor Shun, known as the *Shao* 韶 (literally "illustrious") music, had been passed down through the courts of dynasty after dynasty: from the venerated Zhou—the epitome of ancient institutions—to the Kingdom of Qi (-221 BCE), then to the Qin, to the Han, to the Wei (220-226 CE), and finally to the Former Jin (266-316). And although the "Chaos of the Yongjia Era" had forced this ancient institution of *yue* "music" to relocate to the south of the Yangtze River along with many of the Chinese aristocrats and elites, He Tuo argued that court musicians up to the Southern Liang (502-557) were still performing the very same music, albeit under a different name than "Shao." The Houjing Rebellion, however, dispersed the Southern Liang's court musicians. Some of them migrated north to the Northern Qi (550-577), but still, the latter only retained these musicians and their supposedly ancient music for the private entertainment of its rulers. Thus, He Tuo argued that the Sui should put them to good use in restoring anew the

ancient institution of music, now that the Sui had long annexed Northern Qi in its campaign to unify China.

The merits of He Tuo's arguments may be debatable, and his report that some musicians had brought the remnants of the ancient *Shao* music from southern China back to northern China seems suspiciously convenient, since much of the Discourse on Music during the early Sui based in northern China was motivated by its need to establish political and cultural legitimacy over southern China, which it had recently conquered. But for the purpose of this chapter, I note that Mao egregiously twisted He Tuo's words when paraphrasing them in the last passage quoted above. According to the *Book of Sui*, He Tuo's genealogy of the transmission of Emperor Shun's *Shao* music from dynasty to dynasty identified the object of such transmission as *yue* "music." This formulation might seem generic inasmuch as He Tuo specifically referred to music used at *zongmiao* 宗廟 "ancestral temples," yet there is little doubt that he refers to the performances of *ge* "songs" and *wu* "dances" accompanying the solemn sacrificial rites. What is more, when he did specify exactly which components of this *yue* were transmitted and subsequently dispersed, He Tuo focused on *yuesheng* 樂聲 "the sounds of music" and *yueshi* 樂師 "masters of music": in his description, it was the *sheng* "sounds" of *guyue* "ancient music" that migrated to the south of the Yangtze River in the 4th century, and it was the "masters of music" who brought it back to the north shortly before the rise of Sui.[54]

Mao, however, distorts He Tuo's words, so that what had survived since ancient times until at least the Former Han era was neither the *sheng* "sound" of ancient music nor the embodied knowledge constitutive of the duty of a *yueshi* "masters of music" performing it at courts, but instead *qi* "instruments." Indeed, in the last passage quoted above, Mao, claiming to

54 Ibid., 128-129.

quote He Tuo, describes the transmission history from Emperor Shun all the way to Emperor Wu of Han (r. 157-87 BCE) not of *sheng* "sounds" or even of *yue* "music," but specifically of the original instruments Emperor Shun's court used to play the *Shao* music. In reality, He Tuo did not even mention the word *qi* "instrument," as far as the record in *Book of Sui* is concerned. Furthermore, whereas He Tuo specifically argued that the sounds and tunes of this ancient *Shao* music were never lost before the 6th century, so that Southern Liang court musicians could still perform it, Mao warps He Tuo's words to suggest that by the 1st century BCE under Emperor Wu of Han those instruments of *Shao* music were the *only* things that had survived from Emperor Shun's court. To shore up his distorting appropriation of He Tuo's narrative, Mao evokes Li Yannian 李延年 (d. 90 BCE), who, not surprisingly, was not even mentioned by He Tuo. The brother of a consort of Emperor Wu, Li Yannian was remembered in Sima Qian's *Records* and Ban Gu's *Book* for creating *xinsheng* 新聲 "new sounds" and *biansheng* 變聲 "changed sounds" (not to be confused with the *bian* or altered notes in the scale) at the Han court. Not only are *xin* 新 "newness" and *bian* 變 "change" prone to suspicious judgements in contrast to the venerated *guyue* "ancient music," but the moral character of Li's music was thrown into further doubt owing to his intimate relationship with Emperor Wu, best described as homoerotic as far as Sima and Ban's histories are concerned.

Thus, by borrowing yet perverting He Tuo's 6th-century history of Emperor's Shun's *Shao* music, Mao argues that instruments in general are insufficient for bringing back the lost ancient music. Had pitch pipes been sufficient for the restorationist agenda of generations of scholars of *lülü* "tuning pitch pipes" or music theory, those instruments of ancient music from Emperor Shun's court would have forestalled Li Yannian's musically and morally deviant "new sounds." And because in the scholarship of musical tuning, the only purpose in studying *dushu*

"measurements and numbers" is to reconstruct the ancient instruments according to the exact lengths, sizes, and proportions prescribed by the ancient sage kings, this whole scholarly endeavor, being solely focused on *shu* "numbers" and *qi* "instruments," is ultimately *wuyi* "pointless" for illuminating the essence of *yue* "music." While proceeding from historical records of *shu* "numbers" to exact reconstructions of ancient *qi* "instruments" is achievable, continuing from the ancients' pitch pipes thus rebuilt to the ancient *sheng* "sound" and *yue* "music" would be nothing more than a pipe dream.

Lülü without Pitch Pipes

Now, Mao's claim that instruments alone are not sufficient for resurrecting any historical practice of music would likely leave many readers vested in Western ontological notions of "music" utterly unimpressed. Of course, one may argue, the twelve tuning pitch pipes in and of themselves cannot perform their function as, per Sima Qian, the "root and essence of all things" (*wanshi genben* 萬事根本); they need to be sounded, and it is the pitches they produce, not their lengths or any other tangible physical feature, that are used for tuning other instruments. In any case, doesn't the Yellow Emperor legend specify that the twelve tuning pitch pipes actually resonated with the *yuansheng* 元聲 "*ur*-sound" of the cosmos, when the Yellow Emperor played them after they were fashioned by his musician Ling Lun? Indeed, according to the 3rd-century BCE *Chronicles of Master Lü*, the earliest securely dated source of the twelve tuning pitch pipes, the cosmic sounds of the pipes resonated with the birdsongs of six male phoenixes (ergo the six *yang* 陽 "masculine" or odd-numbered pitch pipes) and six female phoenixes (ergo the six *yin* 陰

"feminine" or even-numbered pitch pipes).[55] Thus, as much a point as Mao may have had in arguing that simply using *shu* "numbers" to reconstruct the twelve tuning pitch pipes or any *qi* "instrument" of the ancients is in itself *insufficient* to shed light on the essence of *yue* "music," it does not follow that the twelve tuning pitch pipes are pointless or unnecessary for that purpose. For without there being any *qi* "instruments," how can any *sheng* "sound" or "note" be made in order to be examined and patterned?

What Mao actually inveighs against, though, is neither instruments nor organology, for which *shu* "numbers" in the form of measurements and proportions are quintessential. Instead, he opposes the centrality that the established scholarly discourse on music and musical tuning have granted to instruments, particularly to the twelve tuning pitch pipes. Because Mao considers the essence of music to be *sheng* "sounds," he argues that no meaningful knowledge about *yue* "music" or *guyue* "ancient music" can be produced except when sounding and listening play a pivotal role in the knowledge-producing process. The problem for Mao with the twelve *lülü* or tuning pitch pipes, then, is not with the pipes themselves or whether they can be played to produce sound. Instead, it is the fact that the making of pitch pipes has become the foundational knowledge-producing process in the Confucian restorationist scholarship on *yue* "music." As I have shown, using the classification and structure of knowledge in the *Imperial Encyclopedia* (1726) and the *Emperor's Complete Library* (1782) as examples, scholars of *lülü* "tuning pitch pipes" engrossed themselves in the mathematics and the historical data related to reconstructing the ancient pitch pipes. It is only by using the twelve tuning pitch pipes thus reconstructed that scholars could dabble in the pattern of *sheng* "notes" qua "the five proper notes and the two altered notes" of the diatonic scale system; and even then, discussions of the scale system were

55 Lü Buwei, *Chronicles of Master Lü*, Chapter 25 "Ancient Music" (*guyue* 古樂).

almost always perfunctory compared to the heated intertextual references, debates, and polemics that became common during the 15th and 16th centuries. As a result, embodied experiences of sound—if they serve any knowledge-producing function at all, which they do not in Cai Yuanding's *New Treatise on Pitch Pipes*, for example—and the entire discourse on music have become harnessed under the yoke of the epistemically silent endeavor of determining the exact *dushu* "measurements and numbers" of the ancient pitch pipes, to the point that even the latter's names had become a metonym not just for tuning but also for the entire terrain of knowledge associated with the restoration of *guyue* "ancient music."

What Mao proposes, then, is to reverse this established epistemic hierarchy in which *shu* "numbers" qua the "essence of making pitch pipes" take precedence over *sheng* "sounds"/"notes" qua the "essence of music." Ultimately, Mao envisions a new paradigm of musical and music-theoretical scholarship without pitch pipes functioning as either the discursive or the epistemic anchor point. Put differently, Mao aspires to a study of *lülü* "musical tuning" and "music theory" (as well as music and ancient music in general) without *lülü* "pitch pipes." To advocate for such a vision, which would have been very hard to imagine due to the immense significance accorded to the twelve tuning pitch pipes in the Confucian historical imagination of music, Mao sketches out two arguments in *Explication*. First, the history of *lülü* "tuning pitch pipes," Mao argues, is the gradual usurpation of their sonorous origins by the epistemically silent studies of *shu* "numbers," and it was this usurpation that gave rise to the established scholarly discourse of *lülü* "musical tuning." To remain true to their origins, then, would mean stripping the tuning pitch pipes of their paramount fundamentality in the study of music and re-subjugating them as subordinate to *sheng*, meaning both "notes" as well as "sound," even though it would not be the cosmic *yuansheng* "ur-sound" but rather the more mundane *rensheng* "human voice."

Second, not only should pitch pipes be regarded as no different in their subjugation to *sheng* "sounds"/"notes," Mao argues, but they are also exceptionally poor tools for studying such *sheng* through the embodied processes of sounding and listening. In fact, Mao describes a series of listening experiments he has done with pipes that show simply how unpredictable their pitches are in relation to their physical sizes. Thus, besides emphasizing that *sheng* "sounds"/"notes" unencumbered by any particular instrument should be the primary focus and method of musical scholarship, Mao hammers the final nail into the coffin for *shu* "numbers" or *lü* "pitch pipes" to be able to claim any relevance to musical knowledge production.

Mao lays out his interpretation of the history of pitch pipes as a history of the usurpation of *sheng* "sounds" by *shu* "numbers" in his commentary on Segment 2 of Zhang Yushu's *Petition*. Whereas Segment 1 of Zhang's original text describes the context of the court gathering on February 20, 1692, Segment 2, being the first to address the substance of the Emperor's lecture-demonstrations, describes the problem of *jingyi weisan* "the circle of a circumference is three when its diameter is one." Since computing the circumference of a circle is pertinent to reconstructing the measurements and proportions of tuning pitch pipes, Mao begins his commentary with the origin of the twelve tuning pitch pipes:

> 徑者，直也。圍者，圓圍也。黃帝作樂，始于製管。史所稱伶倫伐竹大夏，截管一十二，以分陰陽。陽律六，陰律六，合之稱十二律是也。第十二律。首名黃鐘。黃鐘為萬事根本，而黃鐘之管上生下生又為諸律管之本。故管律分寸從黃鐘始。[…][56]
>
> "Diameter" is straight. "Circumference" is circular. When the Yellow Emperor established music [*yue*], it began with making the pipes. Historians say that Ling Lun went to Bactria [*Daxia*] to harvest bamboo, and he cut them into twelve pipes in order to distinguish between the masculine [*yang*] and the feminine [*yin*]. The masculine pipes are six, the feminine pipes are six, and they

56 Mao, *Explication*, ff. 7v-8r.

> are combined into twelve. Nonetheless, of the twelve tuning pitch pipes, the first one is called *huanghzong*. "*Huanghzong* is the root and essence [*ben*] of all things," and the upward and downward generation [*sheng*] of pipes from *huangzhong* is in turn the essence [*ben*] of all the tuning pitch pipes. Therefore, the measurements of the tuning pitch pipes begin with *huangzhong*. […]

Notably, as though negating his later pronouncement that making pitch pipes is irrelevant to the "essence of music," here, Mao explicitly identifies "making pitch pipes" or *lü* as the very start of *yue* "music." After that, Mao even utters the phrase from Sima Qian elevating the *huangzhong* pipe to being the "root and essence of all things," as the measurements of all the other pitch pipes are generated proportionally in relation to *huangzhong*. And, indeed, following the passage quoted above, Mao invokes various canonical sources on the tuning pitch pipes, specifically in relation to calculating the circumference and diameter of the *huangzhong* pipe. Specifically, Ban Gu's "Treatise on Pitch Pipes and Calendar" from the 1st century CE records the length of the *huangzhong* pipe to be 9 *cun*, its circumference is 9 *fen* or a tenth of its length, and, by the rule of "the circumference is three when the diameter is one," the diameter is 3 *fen*, and that Cai Yuanding's *New Treatise on Pitch Pipes* follows these data to the letter.

At this point Mao neither continues to outline various other early sources on the lengths and circumferences of the *huangzhong* and other pitch pipes, which will constitute the second half of his commentary on Segment 2, nor joins the Emperor in criticizing the inaccuracy of the rule of thumb "the circumference is three when the diameter is one" (which he will do when commenting later on Segment 3). Instead, for the very first time in *Explication*, Mao criticizes the study of *lülü* "tuning pitch pipes" as irrelevant to *yueben* "essence of music," a turn of phrase he uses for the very first time in this treatise. And he does so precisely by picking up the issue of their origins. While at the start of this commentary on Segment 2 Mao follows the Yellow

Emperor legend as prescribed in the established scholarship on music and musical tuning, here, Mao argues for a slightly different account:

> […] 夫律管所設，一稟人聲，並無有一定之數生乎其間。惟西京備數之家，以為數起乎律，即借此律器遍推之，為歷、度、量、衡四事之準。乃于是不言聲而言數，而徑一圍三之說生焉。此但約略律數，全非樂理。故謂之律本，而非樂本。所謂樂本，即下律吕生聲、旋宫轉調之法。故《史》《漢》以製管備數載入《律書》、《律歷志》，而以《九歌》、《六詩》、"依永和聲"之本別為《樂書》、《禮樂志》，可驗也。今聖諭以此祛作律之本而以生聲終，始定作樂之本。辨律本之訛，立樂本之實，即數言而全樂之旨俱具焉。此實三代至今萬古長夜，而一旦豁然夢覺焉。何其神與！[…][57]
>
> […] Indeed, when the twelve tuning pitch pipes were established, they were thoroughly endowed with the human voice [*rensheng*], and there was no inalterable number that was generated between them. It was only those scholars of numbers [*shu*] at Chang'an during the Han era who thought that numbers arise from pitch pipes, and they thus borrowed these instruments to study everything and made them the yardsticks for calendar [*li*], length [*du*], volume [*liang*], and weight [*heng*]. From then on, scholars no longer spoke of sounds [*sheng*] but only spoke of numbers [*shu*], and thus arose the theory "the circumference is three when the diameter is one." This, however, is only approximating the numbers [*shu*] of pitch pipes [*lü*], and it does not pertain at all to the principle of music [*yue*]. Therefore, it is called the essence [*ben*] of pitch pipes [*lü*] but not the essence of music. What is called the essence of music [*yueben*], then, are the methods of generating notes [*sheng*] from the tuning pitch pipes and of transposing the scales and mutating the modes. Therefore, both [Sima Qian's] *Records* and [Ban Gu's] *Book* record the making of pitch pipes and their numbers in their "Book on Pitch Pipes" and "Treatise of Pitch Pipes and Calendar." Meanwhile, they create a separate "Book on Music" [*yue*] and "Treatise on Rites and Music" [*liyue*] to discuss the essence of *Nine Songs of Chu*, *Canon of Songs* and the matters of [as described by Emperor Shun in *Canon of Documents*] "following chanting according with notes." This can be verified. Today, the Sage Instruction uses this [distinction] to discard the essence of making pitch pipes that ends with generating sounds, but begins to determine the essence of making music. By eradicating the error of the essence of making pitch

57 Mao, *Explication*, ff. 8v-9v.

> pipes and laying out the concreteness of the essence of music, with just a few words the Emperor thoroughly details the quintessence of all music. Really, it has been an eternal night since the Three Dynasties era until now, when suddenly we are awakened one morning from the stupor. How marvelous! […]

What this dense passage puts forth, I argue, is a "fall" narrative of the twelve tuning pitch pipes, one that differs from the narrative of perennial loss of *guyue* "ancient music" related in the established discourse on pitch pipes and music. Whereas scholars of *lülü* or musical tuning fret about the irretrievability of the exact measurements of the pitch pipes of the ancients, Mao argues that the fall was not about the pitch pipes themselves but what kind of knowledge-producing objects they are made out to be in the study of music: namely, studies and applications of the *shu* "numbers" aspects of the pipes took over their origins from *sheng* "sounds." Notably, however, for Mao, the sonorous origin of the pitch pipes lay not with the twelve phoenixes singing and hovering over a bamboo forest on Mount Kunlun, from which Ling Lun harvested the bamboo stems he used to fashion the original twelve tuning pitch pipes. Instead, Mao argues that the ancients' tuning pitch pipes were endowed with a much more mundane kind of sound: *rensheng* 人聲 "human sounds," i.e. the human voice. While contradicting the most broadly cited sources of the Yellow Emperor legend such as *Chronicle of Master Lü*, Mao nonetheless appears to evoke another canonic source on the origin of pitch pipes, and indeed of music in general, under the ancient sage kings: "May poetry express the will, and songs chant such expression; my notes [*sheng*] follow such chanting, and pitch pipes [*lü*] accord with such notes" (詩言志, 歌永言, 聲依永, 律和聲)—this reportedly uttered by Emperor Shun when he ordered that the institution (*dian*) of music (*yue*) be established, according to the passage from the

Confucian *Canon of History* quoted in a previous section of this chapter.[58] Commonly featured in writings on literature and aesthetics, the chain of genesis from *shi* 詩 "poetry" to *ge* 歌 "song," *sheng* "sounds"/"notes," and finally to *lü* "pitch pipes" served as the *de facto* motto during the 16th to 18th centuries for scholars of the aforementioned Confucian *Canon of Songs*.[59] It was also often invoked by collectors of urban ditties and mountain songs and even linguists and philologists, who made up the core of the *kaozheng* "evidential learning" movement.[60]

That Mao substitutes the songs of humans for the songs of marvelous fowl might seem a minor discursive detail: both the Yellow Emperor legend and Mao's history of the twelve tuning pitch pipes identify certain *sheng* "sounds" as their origins. But Mao's specification of this origin as the human voice turns out to be a critical point in his historiographical analysis of early sources on the twelve tuning pitch pipes. As he describes in the second half of the last passage quoted above, both Sima Qian's *Records of the Grand Historian* and Ban Gu's *Book of the Han*, each of which were paradigm-setting official dynastic histories, feature a treatise on *yue* "music": the "Book on Music" (*yueshu* 樂書) in *Records* and the "Treatise on Rites and Music" (*liyue zhi* 禮樂志) in *Book*. Neither of these two treatises, however, discuss the twelve tuning pitch pipes. In fact, as much *huangzhong* is the "root and essence of all things," neither chapter even mentions the name *huangzhong*, let alone the other eleven tuning pitch pipes. Instead, Mao sums up their contents precisely with that quote from Emperor Shun from *Canon of History*:

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58 The *Canon of Songs* has been alternatively translated as the "Classic of Poetry" or the "Book of Odes."

59 See footnote No. 26 above.

60 See, for examples, Chen Di 陳第, *Maoshi guyin kao* 毛詩古音攷 ("An Investigation on the Ancient Pronunciations of *Canon of Songs* as edited by Mao Heng," 1606) and Feng Menglong 馮夢龍, *Shan'ge* 山歌 ("Mountain Songs," c. 1630s), both of which I will discuss in the last chapter of this dissertation.

"following the chanting" (*yiyong* 依詠) and "according with the notes" (*hesheng* 和聲). Indeed, Sima's "Book on Music" and Ban's "Treatise on Rites and Music" focus exclusively on the performative aspects of music at the courts of various rulers within the timeframe of their histories, including the uses of songs and dances for rites and ceremonies, the lyrics of songs and the setup of their instrumental accompaniments, anecdotes of rulers singing and prescribing the institution of music, and discussions of the broader political and moral significance of music. As examples, Mao mentions the *Canon of Songs*, which supposedly compiled odes accompanying various rituals at the courts of reputed ancient kings of the Three Dynasties period and even included what may be best described as folksongs that the Kings of Zhou had collected from different regions of their realm to gauge the well-being of their subjects. Meanwhile *Nine Songs of Chu* (jiuge 九歌) is an anthology from about the 3rd century BCE of ritual songs from the Chu region, an area that was culturally quite distinct from the other regions under Zhou's rule.

On the other hand, as mentioned above, it is in their respective treatises on *lü* "pitch pipes" and *lüli* "pitch pipes and calendar" that Sima and Ban discuss the other twelve tuning pitch pipes and, importantly, declare the *huangzhong* pipe as the "root and essence" of all things. As it turns out, however, "all" those things have nothing to do with matters of singing, ritual and performance, or indeed anything that requires sounding and listening. The concern of Sima's "Book on Pitch Pipes" and Ban's "Treatise on Pitch Pipes and Calendar" falls squarely on *shu* "numbers." Besides the obvious topic of the measurements and proportions of the twelve tuning pitch pipes, much of these two treatises appropriates the twelve tuning pitch pipes as the foundation of metrology and even as a set of tools for setting the calendar, ergo the confusing combination of "pitch pipes and calendar" in Ban's treatise. In fact, the latter was the earliest extant account on how these standards of *du* "length," *liang* "volume," *heng* "weight," *quan*

"balance," and *li* 曆 "calendar" were derived from the pitch pipes. According to Ban, after the twelve tuning pitch pipes were established, the ancients took the length of *huangzhong* as the standard for nine *cun* 寸, with nine being the largest masculine or odd number. This account of length concurs with the aforementioned *Writings of Prince Huainan*, which preceded Ban's *Book of the Han* by two centuries. The ancients, Ban records, also took the hollow-cylinder volume of the *huangzhong* pipe as the standard for one *yue* 龠. Seeing that the *huangzhong* pipe, reportedly, could contain at most 1,200 *shu* 黍 or grains of millet, they also took the weight of these grains of millet as the standard for twelve *zhu* 銖.

Whereas these metrological systems rely only on the *huangzhong* pipe, Ban also suggests that the ancients matched each of the twelve tuning pitch pipes with every other one of the twenty-four solar periods, or *qi* 氣 ("air," "breath," "spirit," or "vital force"), of the tropical year, an account also found in *Chronicles of Master Lü* and *Writings of Prince Huainan*: *huanghzong* (roughly equivalent to C), corresponds to Winter Solstice in the eleventh month, *jiazhong* (D#), to Spring Equinox in the second month, *ruibin* (F#) to Summer Solstice in the fifth month, *nanlü* to Autumn Equinox in the eighth month, and so on. And while these correspondences might be in the realm of the metaphysical or metaphorical in these sources, the fifth-century *Book of the Later Han* (*houhan shu* 後漢書) by Fan Ye 范曄 (398-445 or 446),[61] a sequel to Ban's *Book of the Han* covering the Later Han period (25-220 CE) during which Ban lived, renders the connections between pitch pipes and the solar periods physically observable, or at least to the extent such connections exist. In a procedure called *houqi* 候氣 "awaiting the *qi*," a calendar-

61 Fan Ye 范曄, *Houhan shu* 後漢書 ("Book of the Latter Han," 445), "First Treatise: Pitch Pipes and Calendar, Part One of Three" (*xuhanzhi diyi lüli zhi* 續漢志第一律曆志).

setter would make manifest the exact moment of a solar period—say, the Spring Equinox—by filling its corresponding pitch pipe—in this case, the pipe of *jiazhong* (D#)—with reed ashes and placing it in a special chamber. Then, they would "await" the very moment when the *qi* "solar period" under question took a physical form as *qi* "air," penetrated through the pitch pipes it corresponds to, and stirs up the ashes within it, thus leaving an observable effect. Granted, beginning in the 16th century, many scholars increasingly ridiculed as bogus the "awaiting the *qi*" method of calendar-keeping, and some, including Zhu Zaiyu, even conducted experiments in "awaiting the *qi*," hoping to falsify this idea through their consistent failure to replicate the result recorded in those early sources.[62] Still, prone to cosmic and esoteric imaginations, many prominent Song-era scholars including Cai Yuanding himself in *New Treaitse on Pitch Pipes* endorsed using the twelve tuning pitch pipes to "await" the solar periods as a valid and promising line of inquiry,[63] to the point that "awaiting the *qi*" effectively became the official calendar-keeping method of the Qing Empire, at least ideologically, between 1664 and 1668 during the early years of the Kangxi reign.[64] And although the method was imperially repudiated in the 1670s, not only did "awaiting the *qi*" remain a clichéd poetic metaphor for changes of months and seasons, but the close epistemic as well as discursive connection between *lü* "pitch pipes" and *li* "calendar" is even embodied in the title of *Origins of Cosmological Sciences*,

62 See Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, vol. 8, 117-128.

63 See Cai Yuanding, *New Treatise on Pitch Pipes*, , Volume 1, Chapter 10 "Awaiting the *qi*" (*houqi* 候氣), and Volume 2, Chapter 9 "Awaiting the *qi*" (*houqi* 候氣). See also Wang Yumin 王玉民, *Houqishu: guren guannian zhong tian di ren zhi niudai* 候氣術：古人觀念中天地人之紐帶 ("Awaiting the *qi*: a link in the ancients' conceptions of Heaven, Earth, and Human") (Zhengzhou: Zhongzhou guji chubanshe, 2016), 46-58, 106-109.

64 See Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, vol. 8, 117-128. For a dramatic debate on the merits of "awaiting the *qi*" that implicated Han Chinese scholars, Hui-Muslim astronomers, European Jesuits, and Manchu nobles under the Qing Empire during the 1660s, see Elman, *On Their Own Terms*, 133-144, and Jami, *The Emperor's New Mathematics*, 49-54.

whose title, *lüli yuanyuan* 律曆淵源, literally means "the recondite origins of pitch pipes and calendar." Admittedly, the treatise makes no attempt at "awaiting the *qi*" or even drawing any meaningful knowledge-producing relations between its volumes on music theory, *Orthodox Meaning of Pitch Pipes*, and those on calendar-keeping, *Astronomical Observations Investigated and Compiled* (lixiang kaocheng 曆象考成). Yet the title attests to the millennia-old integration between pitch pipes and calendar in the established structure of knowledge, following on the broader notion of pitch pipes being the "root and essence of all things," which also includes standards of measurements for length, volume, and weight.[65]

This sharp divide between *yue* "music" on the one hand and "pitch pipes and calendar" as well as metrology on the other hand in both Sima's *Records* and Ban's *Book* becomes a critical juncture in Mao's history of the twelve tuning pitch pipes. As Mao argues in the last passage quoted above, the ancients' tuning pitch pipes accorded completely and uniquely with *rensheng* or the human voice, without measurements or proportions playing any role. But this thorough embedment of *lü* "pitch pipes" in *sheng* "sounds" during ancient times cracked at the hands of Han-era scholars, indeed including Sima and Ban. Here, Mao's direct reference to *Xijing* 西京 "the Western Capital,"[66] a major center of scholarship after Confucianism became the Han's state ideology in the 2nd century BCE, implicates *hanxue* 漢學 "Han-era learning," 17th and 18th-century Confucian scholars characterized (and somewhat romanticized) as much more textually and philologically oriented than the increasingly maligned *songxue* "Song-era learning"

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65 See Jami, *The Emperor's New Mathematics*, 368-372, in which the author discusses the connection between pitch pipes and calendar evoked in the title, although, as I have commented in footnote No. 30 in the Introduction, Jami does not appear to know the exact content of *Orthodox Meaning of Pitch Pipes* as part of *Origins of Cosmological Sciences*.

66 The name for Chang'an 長安 under the Later Han whose main capital was Luoyang 洛陽 to its east.

identified with Zhu Xi. Perhaps indicative of his idiosyncratic take on scholarship, Mao identifies Han-era scholarship as the beginning of a "fall," whereby the tuning pitch pipes lost their sonorous origins and became epistemically silent objects in the studies of metrology and calendar. As scholars appropriated their tangible physical dimensions and calendrical correspondences to establish standards of measurements and time-keeping, Mao argues, they no longer used the sonic affordances of pitch pipes to produce any knowledge, since restoring the exact measurements and proportions of pitch pipes of the ancients and using these numbers to regulate metrology and the calendar do not involve sounding or listening in any knowledge-producing fashion. Thus, pitch pipes became irrelevant to *yueben* "essence of music," a fact explicitly attested by their treatment in Sima's "Book on Pitch Pipes" and Ban's "Treatise on Pitch Pipes" in contrast to their total exclusion from "Book on Music" and "Treatise on Rites and Music."

What Mao describes the Kangxi Emperor as having achieved through the court gathering is therefore "discarding the essence of making pitch pipes that ends with generating sounds" and instead "beginning to determine the essence of music." Notably, this formulation implies a change of epistemic priority: rather than making pitch pipes first and then using them to produce the sounds that would in turn be studied, Mao praises the Emperor for getting rid of the twelve tuning pitch pipes *in toto* and studying sounds directly as the "essence of music." Indeed, given that pitch pipes originated from *sheng* "sounds" and that subsequent scholarship uprooted them from their sonorous origins, Mao effectively argues that the twelve tuning pitch pipes are not only insufficient for shedding light on *yue* "music" but are also dispensable for that purpose. And indeed, the fact that Mao specifies the sonorous origins of pitch pipes as *rensheng* "human sounds" or the human voice further makes *qi* "instruments" only optional for producing

meaningful knowledge of music and ancient music through sounding and listening. In fact, even when instruments are to be involved in the study of *sheng* "sounds"/"notes," Mao argues that pitch pipes are far from being the optimal *qi* "instruments" or tools for this purpose owing to their organological features. This argument comes later in *Explication*, in Mao's commentary on Segment 7 of Zhang Yushu's *Petition*. In the original text, this segment comes after Zhang has already described the Emperor's lecture-demonstration of "mutual generation at every eighth step" using a *di* flute and a *se* zither. And here, Mao substantiates the inutility of pitch pipes by appealing not to historical sources but to an actual experiment of sounding and listening he claims to have conducted himself:

> 况律管長短則尤于聲律無所用者，向曾製管以驗之矣。取會稽竹篠厚薄均等者，不論古尺今尺：長則俱長，短則俱短。先作黃鐘管九寸，準以徑一圍三之說，徑三分，圍九分。含氣呼之，不能成聲。乃缺其脣而吹之，其聲弇而卑，全不類黃鐘之音。[…] 乃遍截他管，準諸寸法。而太簇八寸與夾鐘姑洗之六寸零無可辨焉，應鐘四寸零與夷則南吕之五寸零亦無可辨焉。是不特毫釐杪忽，不能剖析。而即其洪纖高下、長短清濁、大□ [illegible character] 小窕之顯然者，或竹同而聲異，或聲同而竹異。彼推此驗。百無一當。[…] 而欲以此定宫商，辨律吕，正五聲七調、四清二變、九聲十二律之數，不其寃乎！[…] 所由祛制管之訛，立生聲之法，聲中律和，由此而驗也。[67]
>
> What is more, the length patterns of pitch pipes are particularly useless to the rules of sounds, and I once verified this by making pipes. I took bamboo stems of even thickness from Mount Kuaiji, and I did not care whether I was using ancient or modern units of length: [so long as I only was using one type of unit,] the long ones would always be long, and the short ones would always be short. I first made a *huangzhong* pipe that is 9 *cun*, subject it to the theory of "the circumference is three when the diameter is one," with the diameter being 3 *fen* and the circumference 9 *fen*. I inhaled air and blew the pipe, yet it did not produce any sound. Then I retracted my [upper] lip and blew the pipe, its sound was muffled and feeble, totally unlike [what] the tone of *huangzhong* [ought to be]. […] Then, I slashed bamboos here and there to make the other

67 Mao, *Explication*, vol. 2, ff. 15r-16v.

> pipes, subjecting them all to the same standard of length. Yet the 8-*cun taicu* pipe and the more-than-6-*cun guxian* pipe were indistinguishable, and the 4-*cun yingzhong* pipe and the more-than-5-*cun yize* and *nanlü* pipes are also indistinguishable. Thus, unless one specially exerts oneself on the minute differences of length, they cannot be examined. What is more, among those whose variances in volume, pitch height, length, and physical size are evident, some pipes are physically identical yet sound different, while some pipes sound the same yet are physically different. […] Trying to use these to determine the *gong* from the *shang*, distinguish the different pitches and rectify the likes of the five notes of the scale, the seven scale-types, the four clear notes, the two altered notes, the nine pentatonic notes, and the twelve pitch pipes—what an injustice is that! […]

As Mao was living in Hangzhou when he composed *Explication*, he easily had access to bamboos from Mount Kuaiji, the burial place of King Yu 禹 (? 2123-2055 BCE), founder of the Xia Dynasty (? 2070-1559 BCE), the first of the Three Dynasties period—even though, despite the cultural and historical significance of Mount Kuaiji, I have not found any source attesting to its special connection with tuning pitch pipes. In either case, by describing his experiments with making and sounding bamboo pipes himself, Mao identifies two sonic aspects of pitch pipes that make them sub-par instruments for studying *sheng* "sounds"/"notes" as the essence of *yue* "music." First, not only does producing a consistent pitch on these pipes require various minute adjustments of the embouchure, but the timbre of their pitches is also "muffled and feeble"; this is likely because, unlike reed pipes, tuning pitch pipes produce sound solely through the vibration of their air columns, and because Mao, following the earliest sources on the matter, did not carve out a mouth hole on the blowing end of the pipes. Second, and more importantly, Mao argues that the physical dimensions of these pipes do not at all correspond to the patterns of pitches they produce: "among those whose variances in volume, pitch height, length, and physical size are evident, some pipes are physically identical yet sound differently, while some pipes sound the same yet are physically different." Indeed, this particular observation gives a

sonic, empirical dimension to Mao's earlier claim in Segment 5 that the measurements and proportions of pitch pipes "do not at all match the patterns of ascent, descent, transpositions, or mutations of actual, concrete sounds and tones." As I will show in later chapters, Mao was far from being the first or the last during the 16th to 18th centuries to remark on such a discrepancy between the *shu* "numbers" and *sheng* "sounds" of pitch pipes. It suffices to say here, though, that these experiments in making and sounding pitch pipes prove for Mao the ultimate irrelevance of them for studying *yue* "music," whose essence, for him, is sounds.

A Unique and Uncharted Legacy

Dense and erudite though it is, Mao's *Explication* can be summed up in two major arguments. On the specific level, Mao argues that the Emperor's gloss of *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence differs from its received definition as circle of fifths not just discursively but also epistemically. By interpreting a concept related to *lü* "pitch pipes" as one related to *sheng* "notes," the Emperor replaces *shu* "numbers" with *sheng* "sounds" as the linchpin of the knowledge-producing process of his gloss. Whereas the received definition draws on the measurements and proportions of the twelve tuning pitch pipes to give meaning to the phrase, the Emperor's lecture-demonstration relies solely on the experience of sounding the *di* flute and *se* zither and listening to the patterns of their varying notes. And thus, on the general level, Mao advocates that scholars of *yue* "music" whose primary concern is reviving the perfect music of the ancient sage kings must put the embodied examinations and experiments with *sheng* "sounds" front-and-center in their knowledge-producing process. As indicated in the title of his *Explication*, the most significant lesson Mao claims to have learned from the Emperor's lecture-demonstration is that *yueben* "the essence of

music" is none other than *sheng* "sounds." For that reason, no meaningful knowledge about *yue* "music" or *guyue* "ancient music" can be produced without sounding or listening playing a pivotal role in the process. While the twelve *lülü* or tuning pitch pipes originated from certain sounds, be it the cosmic *yuansheng* "ur-sound" or the more mundane *rensehng* "human sound" or the human voice, subsequent scholarship ruptured them from their sonorous origin by striving only to reconstruct their absolute and relative sizes and borrow them to regulate standards of metrology and the calendar. Since this scholarly discourse of *lülü* "tuning pitch pipes" qua musical tuning and music theory produces knowledge only through *shu* "numbers" yet does not involve sounding or listening in any knowledge-producing capacity, Mao calls on scholars to follow the lead of the Emperor to disregard the issues of pitch pipes and engage directly with sounds in order to shed true light on the "essence of music." In fact, by using his own experiments with making, playing, and listening to pitch pipes, Mao even suggests that the size proportions between different pitch pipes are exceptionally poor indicators of the sonic relations between their pitches, thus rendering them unusable *qi* "instruments" for studying *sheng* "sounds"/"notes" compared, for example, to the flute and zither the Kangxi Emperor used. Thus, according to Mao, not only should the study of *lülü* as musical tuning pivot from the epistemically silent calculations, proportions, and historical metrological data, but it should also be completely reconfigured, so that the twelve *lülü* or tuning pitch pipes are no longer be the "root and essence" of all things when it comes to the study of *yue* "music" or even *lülü* "musical tuning" yet make way for direct experiences with sounds.

I should note that a survey of early modern Chinese scholarship on music and tuning pitch pipes would find many sympathetic ears for Mao's advocacy for *sheng* "sounds" instead of *lü* "pitch pipes" or *shu* "numbers" as the essence of music. For all his mathematical prowess in

calculating the twelfth-root of two, Zhu Zaiyu, the renowned inventor of twelve-tone equal temperament, consistently emphasizes the overlooked importance of *sheng* "sounds" and *ting* 聽 "listening" in the study of not just *yue* "music" but also *lü* "pitch pipes" and musical tuning. As I will analyze in a later chapter, in his *A New Theory of the Study of Pitch Pipes* (1584) where he first laid out the basic parameters of his "New Method" or twelve-tone equal temperament, Zhu Zaiyu actually characterizes his discovery of twelve-tone equal temperament not as a mathematical feat but as revelation from the embodied knowledge of playing, maneuvering, and listening to instruments alongside musical practitioners. And before he lists the relevant data and proportions of the new temperament in Chapter 3 of *A New Theory*, Zhu Zaiyu opines in Chapter 2 on the general state of the scholarship on tuning pitch pipes as the established pathway towards reconstructing the perfect music of the ancient sages. Zhu Zaiyu's arguments in the opening of this chapter must seem rather familiar by now:

> 上古造律，其次聽律，其後筭律。《虞書》《周禮》，有聽律之官，無筭律之法。[…] 至於律同合聲，陽左旋而陰右轉，觀其次序，不以筭法論矣。筭法之起，殆因律管有長短。此筭家因律以命術，非律命於筭也。[…] 故曰：古之為鍾律者，以耳齊其聲，後人不能，始假數以正其度。雅樂之不可興，聲音之學不傳也。
>
> The ancients first made the pitch pipes, then listened to the pitch pipes, and afterwards made calculations [*suan*] of the pitch pipes. The first five chapters of the Confucian *Canon of History* and the *Rites of Zhou* all stipulate an official tasked to listen to the pitch pipes, yet they do not contain any methods for calculating the pitch pipes. […] When it comes to matching the sounds of the pitch pipes to the notes of the scale [*sheng*] and to observing the order of the pitch pipes whereby the odd-numbered ones generate the next pipe to their left and the even-numbered ones generate the next pipe to their right, these are not bespoken through methods of calculation. Methods of calculation probably originated because pitch pipes had varying lengths. This means that mathematicians used pitch pipes to invent their craft, not that pitch pipes were made from calculations. […] Therefore, we say: those who made

> bells and pitch pipes during the ancient times used their ears to tune their sounds; later people could not do so and thus began to borrow [*jia*] numbers to rectify their measurements. The failure to revive elegant music [*yayue* of the ancients] lies in the extinction of the art of sounds [*shengyin*].

Like Mao in his *Explication* examined earlier, Zhu Zaiyu uses the word *jia* 假 "to borrow" "to substitute" to describe the use of *shu* "numbers" in the study of pitch pipes. Here, the word allows Zhu Zaiyu to make the same argument as does Mao that numbers, proportions, and measurements were not the foundation of the original twelve tuning pitch pipes of the ancients but were, on the contrary, derived from the pitch pipes. After positing the sonorous origins of the tuning pitch pipes, Zhu Zaiyu then proceeds to spell out the same narrative of "The Fall," so to speak, whereby subsequent generations lost the ancients' ability to tune the pitch pipes and actual instruments such as the bells by ear. Effectively echoing what would be Mao's argument that the essence of *yue* "music" lies in *sheng* "sounds," Zhu Zaiyu concludes by equating the loss of ancient music to the loss of "the art of sounds" (*shengyin zhixue* 聲音之學), hence implying the essence of the former to be the latter.

There is no sufficient evidence determining whether Mao consulted Zhu Zaiyu's *A New Theory* when writing his own *Explication* in 1692; the fact that it took Mao no more than two months to complete his commentary on the court gathering might suggest he did not. Besides, in stark contrast to Mao's proposal to bypass the tuning pitch pipes or for that matter any *qi* "instruments" and reorganize the study of music around sound, Zhu Zaiyu remained committed to the study of *lülü*, writing several treatises bearing the title "tuning pitch pipes" where he uses them to present his theories on musical tuning. Still, the parallel between Zhu Zaiyu and Mao's identification of *sheng* "sounds" rather than *shu* "numbers" as the origins of *lü* "pitch pipes" and thereby the essence of *yue* "music" under the ancient sage kings reflects a recurrent theme in

early modern Chinese scholarship. As I will explore in more detail in Chapters 4 and 6, the rising *shixue* "concrete learning" and *kaozheng* "evidential learning" exerted critical and ethical pressure on the resurrection of *guyue* "ancient music" in all its details as a way towards retrieving an aspect of the perfect governance of the ancient sage kings, and tension between *sheng* "sounds" and *shu* "numbers" became more salient in a variety of manifestations. In fact, one could even argue that the dialectics between numbers and sounds—between textually transmitted historical data and empirically observed sonic patterns—has been a perennial and perhaps universal problem of musical tuning. The Western European tradition of music theory, for example, posits at its very Greco-Roman origin a dichotomy between the legendary Pythagoras of Samos (c. 570-495 BCE) and Aristoxenus of Tarentum (fl. 335 BCE), a disciple of Aristotle (384-322 BCE). Whereas Pythagoras famously discovered the mathematical proportions of various intervals—octave = 1:2, perfect fifth = 2:3, in particular—and enshrined numbers as the foundation of music and the entire cosmos, Aristoxenus counters in his *Elementa harmonica* that the properties of intervals, scales, and melodies must primarily be studied not by mathematical ratios but by the ear.[68] The contrast between Pythagoras's numbers and Aristoxenus's sounds in musical tuning manifested in various ways in the discourse of music theory in early modern Western Europe, that is, around the time of the late-Ming and early-Qing era under examination in this dissertation. The renewed interests in reviving *antica musica* or ancient music brought forth two waves of debates during the 16th century over the chromatic and

68 See Thomas J. Mathiesen, *Apollo's Lyre: Greek Music and Music Theory in Antiquity and the Middle Ages* (Lincoln and London: University of Nebraska Press, 1999), 321-322.

enharmonic genera.[69] According to various ancient sources including Aristoxenus's writings, chromaticism and enharmonicism contributed much to the legendary emotive powers of Ancient Greek music, yet attempts to incorporate them into the largely diatonic musical practice of 16th-century Western Europe posed significant challenges to the established Pythagorean tuning system based on rational numbers.[70] The development of the field of mechanical acoustics in the 17th century witnessed the tension between sounds and numbers in musical tuning play out against the larger backdrop of epistemological debates during the so-called Scientific Revolution between the rationalism and empiricism and between mathematical and mechanistic models of observed physical phenomena.[71]

So what is special about Mao's discussion on *sheng* "sounds" vs. *shu* "numbers" and his unqualified advocacy for the former, even though he was neither the first or the last to articulate this dyad within (let alone beyond) the Chinese tradition of musical tuning? While a complete answer would require a thorough examination of the dialectics between numbers and sounds throughout the entire discourse of *lülü* "tuning pitch pipes" or musical tuning across all times—an examination that falls far beyond the scope of the current dissertation—, here, I highlight the specific context under which Mao developed his proposal for a reinvention of musical tuning pivoting away from *lü* "pitch pipes" and towards *shu* "numbers" and *sheng* "sounds": the Kangxi

69 See Maria Rika Maniates, "Introduction," in Nicola Vicentino, *Ancient Music Adapted to Modern Practice* (1555), Maniates trans., Claude Palisca ed. (New Haven and London: Yale University Press, 1996), xi-lxiii; see in particular xxv-xxxvi, hwere Maniates gives an overview of most of the Greco-Roman sources used by scholars in the 16th century debate over chromaticism and enharmonicism.

70 Mathieson, *Apollo's Lyre*, 306-318.

71 See H. F. Cohen, *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580-1650* (Dordrecht, The Netherlands: D. Reidel Publishing Company, 1984), in particular 187-201, for a series of communications between René Descartes (1596-1650) and Marin Mersenne over these debates.

Emperor's misguided lecture-demonstration of *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence with a *se* zither and a *di* flute. As I have shown in this chapter, Mao's *Explication* was not just a general commentary on the Emperor's lecture-demonstration, furnishing the Emperor's words and deeds with rich historical references and explanations. Rather, Mao specifically interprets the Emperor's technically erroneous gloss of the term as an epistemic directive to heed *sheng* "sounds" instead of *shu* "numbers" or *lü* "pitch pipes" as the true essence of *yue* "music" and *guyue* "ancient music" and therefore the principal pathway towards the latter's reconstruction.

The fact that Mao took an epistemic cue regarding the importance of sounding and listening in producing musical knowledge from the Emperor's gloss of "mutual generation at every eighth step" is critical. As it turned out, not only did Mao's epistemological interpretation of the Emperor's lecture-demonstration eventually reach the very hands of the Kangxi Emperor himself, but it also anticipated the exact manner whereby the Qing's official music theory treatise heralded a fourteen-fold division of the octave as the "Orthodox Meaning of Pitch Pipes." According to his own account, by June 29, 1692, Mao had not only finished his *Explication* annotating Zhang Yushu's *Petition* but also composed an eight-volume *Anthology of the Imperially Prescribed Tenets of Sound* (huangyan dingsheng lu 皇言定聲錄, 1692), which claims to take the Sage Instruction of the Emperor at the court gathering to develop a novel system of musical scales and modes. Together with yet another treatise on music not related to matters discussed at the court gathering, Mao sent his *Explication* and *Anthology* to Zhang Ying 張英 (1637-1708) in Beijing, another Han Chinese scholar-official present at the court gathering, and asked him to offer the three works to the Emperor on his behalf. Zhang Ying declined, citing the Emperor's preoccupation with the Dzungar wars looming in the borderlands between China

and Mongolia, and perhaps echoing also the Emperor's rebuke to Zhang Yushu's suggestion to compile treatises on musical tuning and mathematics.[72] It was not until April 24, 1699, during the Kangxi Emperor's third tour to the Yangtze River Delta area—a crucial occasion for the Manchu conquest regime to display its military might to the Han Chinese and to cultivate an amicable rapport with them—that Mao was summoned to the Emperor's presence as an honored retiree and presented his *Explication* and the two other treatises to the Emperor. By that time, Mao had already self-published these works, and he feared that the printed copies might be too humble for the Emperor. Still, the latter gladly asked that the printed exemplars Mao offered be admitted in the imperial libraries, provided a few lines of Mao's commentaries be censored in *Explication* and that misspelled characters from Zhang's original *Petition* be corrected.[73]

From that point on, it remains unclear whether the Kangxi Emperor or for that matter any one in his immediate retinue found the time to read Mao's texts, and it remains unclear how he reacted when reading Mao's diligent efforts to honor his interpretation of "mutual generation at every eighth step" and turn it into an epistemological overhaul of the study of music and musical tuning. Like a stone dropped into the sea, the fate of the copies Mao presented to the Emperor after they entered the palace collections has no archival trace. Nor have I been able to locate them from what survives in the Qing's imperial book repositories, the bulk of whose holdings are currently dispersed throughout a number of institutions in Beijing and Taipei. In fact, though Mao's *Explication* remained in circulation through several reprints and anthologies into the nineteenth century, it's possible that no one has seen them in the imperial book collections since

72 Ibid., vol. 2, ff. 23v-24r.

73 Ibid., vol. 2, ff. 24r-25v. Three lines in f. 20r are left blank, with a single character *que* 闕 "missing" indicating that its content has been censored. Neither what precedes nor what follows this missing segment gives any clue as for what might have triggered its censorship.

Mao's initial submission, or at least since as early as 1773, when *The Emperor's Complete Library in Four Sections* was commissioned. In particular, the previously mentioned "Summary Catalogue and Digest" of this gargantuan book collection catalogues more than ten thousand titles, providing each with its author, the provenance of the principal exemplar used by the editors, and a summary and commentary of its content. When it comes to Mao's *Explication* and the two other treatises he simultaneously presented to the Emperor, which were all admitted to the final collection, the editors indicate that they relied not on any exemplar that Mao originally presented to the Emperor—in which case they would have marked the provenance of these treatises as *neifu* 內府, referring to the Imperial Household Department—but instead on copies recently furnished by the governor of Zhejiang, the province where Mao composed and first published these treatises.[74] Additionally, though the editors note in their summary of *Explication* that Mao presented his treatises to the Emperor back in 1699, all that they mention regarding the exemplars Mao presented is an educated guess that the copies provided to them by the governor ought to have come from the same print run as the ones Mao originally submitted. Probably none of the editors of *Emperor's Complete Library*, despite their extraordinary access to the imperial book repositories, ever saw any of Mao's texts there.

That said, there is a single piece of strong textual evidence that Mao's *Explication* ended up becoming a critical source for the compilers of *Orthodox Meaning of Pitch Pipes*. The clue appears in Chapter 9, Volume 1 of *Orthodox*, and the influence of Mao's analysis of the Emperor's lecture-demonstration of *geba xiangsheng* "mutual generation at every eighth step" is already evident even without contextualizing this chapter within the broader organization of the

74 See Ji Yun et al., "Summary Catalogue and Digest," Vol. 38, ff. 33r-34v.

volume and the treatise as a whole, as will do in the following chapters. The chapter begins with its title:

> 審定十二律呂五聲二變。
> 言樂者，皆知三分損益、隔八相生。然此二者義各有在，不可一槩而論。三分損益乃制律之則也。古聖人立為算術，以別十二律呂相生之度。凡金石之厚薄、絲竹之短長，皆依以定焉。隔八相生乃審音之法也。審音之法，必取首音與第八音，叶和同聲，以為之準。即首音、八音之間，區而別之，以為五聲二變。則清濁之相應，高下之相宣，皆賴以生焉。 […][75]
>
> [Translation mine] Examining and determining "the five proper notes and two altered notes" of the twelve tuning pitch pipes.
> All those who speak about music [*yue*] know about "triple division with one part subtracted or added" and "mutual generation at every eighth step." However, these two terms each has its own meaning, and they cannot be lumped together. "Triple division with one part subtracted or added" are the proportions for establishing the pitch pipes [*lü*]. The ancient sages established them as methods of calculation and used them to distinguish the measurements of the mutual generations of the twelve tuning pitch pipes. And the thicknesses of bells and chimes and lengths of strings and [performance-used] pipes are all based on [the standards of length of the twelve tuning pitch pipes]. "Mutual generation at every eighth step" is the method of examining tones [*yin*]. The method of examining tones must have the first and eighth tones accord with each other so that they become the same note [*sheng*] and use that as the yardstick. That is: to distinguish [the notes] between the first and the eighth note as the five [proper] notes and the two altered notes. And thus the mutual correspondences of the clear and muddy notes and the mutual articulation of the high and low both depend on it for their "generation." […]

To recall, the received definition of the phrase as circle of fifths describes a particular pattern of 2:3 proportions among the twelve tuning pitch pipes pursuant to the Chinese Pythagorean *sanfen sunyi* "triple division with one part subtracted or added" method, so that the two phrases are almost always broached in close proximity with each other. By comparison, the

75 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, vol. 1, Chapter 9, f. 26r.

passage above written at some point between July 1713 and January 1714 (more on this timing in later chapters) not only adopts the Kangxi Emperor's technically mistaken definition of "mutual generation at every eighth step" back in 1692, but argues for a sharp distinction between the two concepts. And this is where Mao's *Explication* might have exerted considerable influence, for the distinction the passage heralds against conflating "mutual generation at every eighth step" with "triple division with one part subtracted or added" is an epistemic one. As the passage explains, "triple division with one part subtracted or added" is about the proportions of the twelve tuning pitch pipes and how the resultant absolute and relative sizes of the pitch pipes can in turn be used as measurement standards for other instruments. Notably, while the passage may appear to champion the traditional role of the twelve tuning pitch pipes as the tuning foundation for all instruments, here, under the banner of "triple division with one part subtracted or added," it argues that the twelve tuning pitch pipes perform this function not through "tuning" *per se* in the sonic sense but precisely through their measurements: indeed, it is not the pitches or notes of the bells, chimes, strings, and (non-tuning) pipes that are to be collated with those the pitch pipes: instead, it is the length and thickness of these instruments, parameters that can be determined without any acts of sounding or listening whatsoever playing a knowledge-producing role. Thus, this entire endeavor of figuring out the proper sizes and proportions of the twelve tuning pitch pipes and using them to size-up the other instruments is, epistemically, a silent process—or, as Mao would put it, it is only about *lü* "pitch pipes" and *shu* "numbers" and is irrelevant to *sheng* "sounds."

What about "mutual generation at every eighth step?" On the most evident level, following the Kangxi Emperor's novel gloss thereof as octave equivalence, the passage evokes the seven-note diatonic scale system of *wusheng erbian* "the five proper notes and the two

altered notes" from which an octave equivalence—that is,the first and the eighth note being "in accord" and "becoming the same note"—is a logically necessary outcome. This definition about *sheng* "notes" is thus discursively contrasted with "triple division with one part subtracted or added" about *lü* "pitch pipes." As it turns out, the discursive distinction between pitch pipes and notes immediately transforms into an epistemic distinction between *shu* "numbers" and *sheng* "sounds," just as in Mao's *Explication.* Indeed, according to the passage, whereas "triple division with one part subtracted or added" is about *zhilü* 制律 "establishing the pitch pipes," specifically their lengths and proportions, "mutual generation at every eighth step" is characterized as being about *shenyin* 審音 "examining the tones," with *yin* 音 here referring to the more "organized" type of sounds. This focus on analyzing the sonic aspects of the tuning pitch pipes means that "having the first and eighth tones [of the diatonic scale] accord with each other so that they become the same note" involves sounding the pitch pipes and listening to the pitches they produce. In other words, although determining the proper proportions and sizes of the twelve tuning pitch pipes and fashioning them out of bamboos might require only mathematical calculations and research in historical metrology on ancient sources, organizing the sounds of these pitch pipes into scales, particularly as successively "generated" cycles of octaves, requires embodied experiences and experiments with sounds.

Did the compilers of *Orthodox Meaning of Pitch Pipes* heed Mao's advice that, in order to produce knowledge about "mutual generation at every eighth step" in relation to the diatonic scale system, one must place *sheng* "sounds" front and center in the knowledge-production process? According to the compilers, they did—even though there is no evidence proving or disproving that they ever read Mao's *Explication* itself. Following the passage above, the

compilers note that all previous writers on *lülü* erred in discussing how the pitches of the twelve tuning pitch pipes might be organized into a system of seven-note diatonic scales:

> […] 蓋各守所傳，固執一理，而未始備制律呂之管以審音也。間嘗截竹為管，詳審其音。黃鐘之半律，不與黃鐘合，而合黃鐘者為太簇之半律。則倍半相應之說在絃音而非管音也，明矣。又黃鐘宮其徵聲不應於林鐘，而應於夷則。則三分損益、宮下生徵之說，在絃度而非管律也，明矣。[…][76]
>
> Indeed, [these previous authors] each stuck to what they had learned [from previous texts] and stubbornly stuck to their own principles, yet they never started to prepare or fashion the actual pipes of the tuning pitch pipes so as to examine the tones [*shenyin*]. Just recently, we once slashed some bamboos into pipes and carefully examined their tones. The [pitch of the] pipe whose length is half that of *huangzhong* does not correspond to [the pitch of] the *huangzhong* pipe, yet what does correspond to [the pitch] of the *huangzhong* pipe turns out to be the pitch of the *taicu* pipe. Thus, it is clear that "the full-length and the half-length correspond to each other [in pitch]" applies only to the tones of strings but not to the tones of pipes. Moreover, if [the pitch of] the *huangzhong* pipe is taken as *gong* [*do*], its respective *zhi* [*sol*] note does not correspond to [the pitch of] the *linzhong* pipe but to [the pitch of the] *yize* pipe. Thus, it is clear that "triple division with one part subtracted or added generates a relation of *gong* and *zhi*" that applies only to the proportions of strings but not to the proportions of pitch pipes. […]

Lest there be any confusion over the content and gravity of what the above passage claims to have observed from "examining the tones" of the twelve tuning pitch pipes, I have not included any translations of the names of pitch pipes to Western note names and will refrain from doing so from this point on: this is necessary precisely because the previous passage effects an epistemic distinction between *lü* and *sheng*, or pitch pipes and the pitches they produce. In all, the last passage above claims to have made four observations.

76 Ibid., vol. 1, Chapter 9, ff. 27r-27v.

First, between the pitch of a *huangzhong* pipe and that of a pipe whose length is half that of *huangzhong*, and thus between two pipes whose lengths are in a 2:1 proportion, there is not an octave correspondence.

Second, between the pitch of a *huangzhong* pipe and that of a pipe whose length is half of *taicu*, and thus between two pipes whose lengths are in a 9:4 proportion (the length of *taicu* is 8/9 that of *hunagzhong*), there *is* an octave correspondence.

Three, between the pitch of a *huangzhong* pipe and that of a *linzhong* pipe, and thus between two pipes whose lengths are in a 3:2 proportion, there is not a perfect fifth, or what would be between a *gong* note and a *zhi* note.

Fourth, between the pitch of a *huanghzong* pipe and that of a *yize* pipe, and thus between two pipes whose length are in a 6561:4096 proportion, there *is* such a perfect fifth.

What do these listening experiments and observations entail? In Table 2-2, I have arranged the twelve tuning pitch pipes—as well as three pipes whose lengths are half that of *huangzhong*, *dalü*, and *taicu*—in decreasing order of their length. In the second column from the left, I have put in the ratio of the length of each pipe to that of *huangzhong*. In the third column from the left, I have put in, following on the four observations above, where the notes of *gong*—with two of them forming an octave correspondence—and where the note of *zhi* that is a fifth above this *gong* fall.

It is the fourth column from the left, however, that presents a critical moment of reckoning. An octave is heard between the two pipes of *huangzhong* and half-length *taicu*, bearing a 9:4 length proportion. If one counts the number of pipes from *huangzhong* to half-length *taicu*, then, one ends up encompassing fifteen pipes constitutive of this octave. In other words, following on the experiments that those compilers of chapter 9, Volume 1 of *Orthodox*

Table 2-2 *A fourteen-fold division of the octave effected in Chapter 9, Volume 1 of* Orthodox Meaning of Pitch Pipes *through listening experiments*

	Length Ratio to *huangzhong*	Note Heard	Pipe Number
黃鐘 *Huangzhong*	1:1	*Gong* [*do*]	1
大呂 *Dalü*	2048:2187		2
太簇 *Taicu*	8:9		3
夾鐘 *Jiazhong*	16384:19683		4
姑洗 *Guxian*	64:81		5
仲呂 *Zhonglü*	131072:177147		6
蕤賓 *Ruibin*	512:729		7
林鐘 *Linzhong*	2:3	[Not *zhi*]	8
夷則 *Yize*	4096:6451	*Zhi* [*sol*]	9
南呂 *Nanlü*	16:27		10
無射 *Wuyi*	32768:59049		11
應鐘 *Yingzhong*	128:243		12
半黃鐘 Half *huangzhong*	1:2	[Not *gong*]	13
半大呂 Half *dalü*	1024:2187		14
半太簇 Half *taicu*	4:9	*Gong* [*do*]	15

Meaning of Pitch Pipes claim to have conducted, these pitch pipes, fashioned according to the Chinese Pythagorean proportions of "triple division with one part subtracted or added," end up somehow dividing the octave into fourteen parts.

Two questions then loom over this experiment. Did they actually ever conduct it such as to produce these extraordinary results? And, seeing that the Kangxi Emperor's technically mistaken definition of "mutual generation at very eighth step" and Mao's epistemological interpretation of it exerted considerable influence in pushing them to conduct these experiments, how did the Emperor get the meaning of that ubiquitous turn of phrase so wrong in the first place, and thus unwittingly herald a fourteen-fold division of the octave within the coming two decades? The answer to both questions lies in a 127-folio manuscript at the National Library of China.

Chapter Three —

Rectifying Pitch Pipes in order to Examine Their Tones:

The Imperial and Empirical Underpinnings of a Tuning Reform

Dated to around 1707, National Library of China (NLC) *Putong Guji* No. 15251 is a manuscript in Chinese of 127 folios, roughly 24 times 12 cm in size, organized in four separate stitched wrapped-back fascicles or *ce* 冊 in one cloth encasement. The cataloguing for the *putong guji* 普通古籍 "ordinary ancient books" collection at the NLC is exceptionally thin on the provenance of its individual holdings, and no preface or postface explains the background for the manuscript's miscellany of various texts, tables, and diagrams. Two pieces of paratextual evidence, however, shed light on how the continuously foliated manuscript came together as a whole. While the manuscript features several different hands in black, a single hand annotated the entire manuscript in vermilion, sometimes in the margins, sometimes between the lines, and other times on pasted pieces of scrap paper. Vermilion or *zhu* 硃, notably, was generally a privileged pigment in the milieu of the Qing court, as the Emperor used it to write his directives in response to *zouzhe* 奏摺 "secret palace memorials" or other communications—such as Zhang Yushu's *Petition*—from government officials. Furthermore, a stamp impression on f. 1r not only confirms the palace and imperial provenance of the manuscript but specifies the identity of the annotator who enjoyed the vermilion privilege. The stamp reads: "Conferred by His Imperial Highness Prince Cheng of the First Rank, Third Son of the Emperor." The title in the stamp, *huang sanzi Cheng qinwang dianxia* 皇三子誠親王殿下, was the official style of Aisin Gioro In-c'i (Yinzhi 胤祉 in Chinese; renamed as Yūn-c'i after 1723, or Yunzhi 允祉 1677-1732), between 1709-1730.

So far, this manuscript has almost completely evaded any scholarly attention. Even its rare mentions in previous literature only consider the first fifty-one folios of the manuscript as a concordance for an otherwise broadly available text, the *Elements of Pitch Pipes* (lülü zuanyao 律呂纂要, or lioi lioi z'uwan yeo bithe in Manchu, c. 1690) I have mentioned in the Introduction.[1] Yet this manuscript contains the answer to the two questions with which I concluded the previous chapter. First, did the compilers of the 1714 treatise *Orthodox Meaning of Pitch Pipes* actually heed the Emperor's lecture-demonstration of *geba xiangsheng* 隔八相生 "mutual generation at every eighth step" as an epistemological lesson in empiricism and conduct experiments in *shenyin* 審音 "examining the tones" of pitch pipes? Second, how could the Emperor have made an error so fundamental as to have mistaken that ubiquitous phrase for octave equivalence back at the court gathering in 1692 in the first place? By using the NLC *Putong Guji* No. 15251 as a critical primary source, I answer these two questions in this chapter and the next, respectively. My analyses emanate from the 127 folios of the manuscript, with other primary sources supplemented as needed, and my goal is also to shed light on some of the broader contexts of the fourteen-tone temperament, ranging from princely rivalry and imperial ethnic politics to singing culture across the Eurasian landmass. While making sense of the tuning reform requires extensive focus on some of the more obscure specificities of Chinese music history and music theory, I also illuminate how much these specificities were deeply embedded

1 To my knowledge, only two previous studies have mentioned *Putong Guji* No. 15251. See Wang Bing 王冰, "徐日昇和西方音樂知識在中國的傳播" ("Tomás Pereira and the dissemination of knowledge of Western music in China"), *Wenhua zazhi* 文化雜誌 47 (2003): 71-91. See also Tao Yabing 陶亞兵, "《律呂纂要》及其與《律呂正義續編》的關係" ("*Elements of Pitch Pipes* and its relation to the last volume of *Orthodox Meaning of Pitch Pipes*"), *Zhongyang yinyue xueyuan xuebao* 中央音樂學院學報 1991, No. 4: 48-53. Both articles only discuss *Putong Guji* No. 15251 to the extent they are related to the copy of *Elements of Pitch Pipes* in ff. 1-51 of the manuscript.

in processes of empire-building and transregional integrations that transcend apparent cultural boundaries. Expanding the scope of my analyses sets the stage for the last chapter of this dissertation, which ask whether the fourteen-tone temperament, particularly its epistemic commitment to *shenyin* "examining the tones" reflected or effected broader global patterns.

Negative Forensics

To recall, Chapter 9, Volume 1 of *Orthodox* claims to have conducted actual listening experiments with tuning pitch pipes and heard the octave not between a *huangzhong* pipe and a half-length *huangzhong* pipe, but between a *huangzhong* pipe and a half-length *taicu* pipe. To describe this observation in numbers, which will become important later in this chapter, it claims to have heard the octave not between a pipe 7.29 *cun* long and one 3.64 *cun* long ,but between that 7.29 *cun*-long pipe and the notably shorter 3.24 *cun* long pipe, with the diameter of all these four pipes being 0.274 *cun*. To translate yet again in terms of proportions, it claims to have heard the octave between two pipes who length ratio is not 2:1 but 9:4, with the diameter of all four pipes bearing a 0.274:7.29 ratio to the length of the first pipe. It is this observation of an octave between the *huangzhong* pipe, the first pipe in a sequence of tuning pitch pipes, and the half-length *taicu* pipe, the fifteenth pipe, that heralds the fourteen-fold division of the octave that would make *Orthodox Meaning of Pitch Pipes* notorious.

Over the past half-century or so, historians of Chinese music have treated this pivotal observation with more than a dose of suspicion. Perhaps drilled with the conception that the ratio 1:2 must correspond to the octave, many have rejected the claimed observation in Chapter 9, Volume 1 of *Orthodox* as being able to have any empirical basis. And it is only a short slip from rejecting the results of an experiment to doubting the very bona fides of the experiments. Yang

Yinliu 楊蔭瀏 (1899-1984), who was arguably a founding figure of modern Chinese musicology, accused the Kangxi Emperor, whom he took as responsible for the fourteen-tone temperament, of outright intellectual dishonesty.[2] The supposed results from *shenyin* "examining the tones" of pitch pipes, he maintained, was fabricated by the Kangxi Emperor simply to distinguish the Qing's official system of musical tuning from that of Zhu Zaiyu, the inventor of twelve-tone equal temperament and prince of the Ming Dynasty, which the Qing overthrew in its conquest of China. Yang even went as far as saying the following:

> […] 明清兩代出了很多樂律著作，其共同之點，是脫離物質，逃避現實，販賣神秘主義，說得玄而又玄，而毫不能解決什麼問題。最不幸的是出現了清康熙皇帝那樣的最高統治者，他插手樂律問題，用復古思想來欺騙人民，鞏固其統治。[…][3]
>
> […] Many works on music and tuning occurred during the Ming and Qing eras, and their common characteristic was that they neglect the material world, evade real practice, peddle mysticism, and explain things in such metaphysical fashion that they could not solve any problem at all. The most unfortunate was that a supreme ruler such as the Kangxi Emperor occurred, who meddled in studies on music and tuning and championed restoring the things ancient in order to deceive the people and fortify his reign. […]

Yang's political interpretation of the Kangxi Emperor's fourteen-tone temperament is too facile. As the conquered Han Chinese continued to use the Ming as a rallying cry for anti-Qing rebellions even into the 19th century, it is admittedly reasonable to ask whether a Qing's attempt to overhaul an area of Chinese scholarship stemmed from a sense of dynastic rivalry and anxiety towards the Ming. Still, I have shown in Chapter One that the Kangxi Emperor openly praised Zhu Zaiyu and his iconoclastic approach to musical scholarship at the court gathering back in

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2 Yang Yinliu 楊蔭瀏, *Zhongguo guodai yinyue shigao* 中國古代音樂史稿 ("Draft History fo Ancient Chinese Music"), 2nd edition (Beijing: Renmin yinyue chubanshe, 1984), Vol. 2, 1012-1014.

3 Ibid., 1012.

1692. In this chapter and the following two, I will show even more evidence that the researchers behind the *Orthodox* project were exceptionally receptive towards Zhu's ideas in light of how much the theory they ended up proposing would differ from Zhu's. Reducing the Qing's tuning reform to an egregious act of deception without any evidence other than some tinted interpretation of the *Orthodox* treatise itself does no justice to either the actual research process of the fourteen-tone temperament or the political and ideological underpinnings of such research.

It should be noted that the above passage came from Yang's general history of Chinese music written in the early 1960s at the dawn of the Cultural Revolution. While Yang's language of materialism and class struggle bear the marks of China at a time when research in the humanities was deemed in and of itself a corrupt "bourgeois" or "feudalist" activity, a contemporary study on the Kangxi Emperor's fourteen-tone temperament in Taiwan—a completely different world, ideologically speaking—reached similar conclusions falsifying the claim of empiricism in Chapter 9, Volume 1 of *Orthodox*. Between 1973 and 1976, Chen Wannai 陳萬鼐 at the National Palace Museum in Taipei actually made tuning pitch pipes whose measurements are exactly as stipulated in *Orthodox*.[4] Using a tuner to gauge the frequency of all these pipes, Chen reported that the pitches produced by them do not support the claim in Chapter 9, Volume 1 of *Orthodox* that an octave is found between the *huangzhong* pipe and the half-length *taicu* pipe, or indeed between any pipe and the fourteenth pipe down the decreasing order of length among the twelve tuning pitch pipes and those that are twice and those who are half as long. While Chen observed that two pipes bearing a 2:1 length ratio with identical diameters do often fall slightly short of producing an octave, the results he recorded did not warrant the drastic

4 Chen Wannnai 陳萬鼐, Qing shigao *yuezhi yanjiu* 《清史稿樂志》研究 ("A Study on the 'Treatise on Music' in *Draft History of the Qing*"), 2nd edition (Beijing: Renmin chubanshe, 2010), 159-217.

measure of locating the octave between the *huangzhong* pipe and the half-length *taicu* pipe, which divide the octave into not twelve but fourteen parts.[5]

Lest one take them at face value, I note that Chen's admittedly fastidious efforts in verifying the empirical claims in *Orthodox* were not adequate replications of the latter's experiments. Whereas *Orthodox* clearly stipulates that the tuning pitch pipes are to be made from bamboos—as in *jiezhu weiguan* 截竹為管 "slashing bamboo stems in order to make pipes"—, Chen built his pipes *not* out of bamboo but out of glass. Even though the absolute and relative sizes of Chen's pipes are identical to those described in *Orthodox*, the difference in materials may affect their porousness and sensitivity to temperature and humidity fluctuations, which would have in turn influenced the pitches they produce.

Furthermore, in addition to making the original twelve tuning pitch pipes, all of which share the same diameter, *Orthodox* also calls for making another 324 pipes. While I explain in more detail later in this chapter, these 324 pipes are effectively constructed in two steps. Step one: a gamut of twenty-seven pipes are created, whose lengths range from double-length *huangzhong* to half-length *huangzhong* and whose diameter also change proportionally from double the diameter of the original *huanghzong* pipe to half thereof. In other word, these are twenty-four *tongxing guan* 同形管 "pipes of the same shape" that bear the same length-to-diameter ratio (7.29:0.274). Step two: each of these twenty-seven pipes are subjected to the "triple division with one part subtracted or added" procedure, thereby generating eleven additional pipes whose lengths are proportional to it in the Chinese Pythagorean manner while their diameters are identical to it.

5 Ibid., 189-211, in which all the "raw data" from Chen's experiments are enumerated.

Chen, however, took what he thought was a shortcut. Rather than troubling to make all those additional *tongjing guan* 同徑管 "pipes of the same diameter" as stipulated in step two, he had twenty-seven cylindrical metal inserts made out of copper alloy, with diameters equal to the (inner) diameter of those twenty-seven pipes of the same shape. When he tested the pitches on one of those twenty-seven pipes, he slid the respective insert into the pipe from its end, so as to create an air column of the desire length.[6] In so doing, he created eleven "virtual" pipes, each featuring an air column whose diameter was identical to that of the physical pipe itself and whose length was, through the sliding insert, adjusted according to the desired "triple division with one part subtracted or added" proportions. Still, even though the air columns for these virtual pipes were mathematically identical to those of the actual pipes, the resonance of the redundant portion of the physical pipe and of the sliding insert and the closing of one open end of the pipe likely affected the pitches these virtual pipes produced and thus Chen's results.

Where Chen tried to disprove the experimental results claimed by Volume 1 of *Orthodox* through his own failure to replicate them, more recently, Weng Panfeng 翁攀峰 approaches the same invalidating impulse not through experiments by through mathematics.[7] In a 2013 article, Weng attempts to calculate the implied pitch frequency relations between the full-length *huanghzong* pipe and the half-length *taicu* pipe by inputting their lengths and diameters into the following equation,

$$f = \frac{c}{2(l + 0.3d + 1.4d)}$$

6 Ibid., 159-164.

7 Weng Panfeng, "關於'康熙十四律'思想來源的初步探討'" ("A Discussion on the Origin of "The Fourteen Temperament [*sic*] of Kangxi") *Wenhua yishu yanjiu* 文化藝術研究 ("Studies in Culture & Art") 6 (2013): 32-41.

where f represents the implied frequency of a pipe, c the speed of sound in air, l the length of the pipe, and d its diameter. This equation, which Weng simply took from a manual of acoustics, notably differs from the standard equation for calculating the implied frequency of an ideal pipe with two open ends:

$$f = \frac{c}{2l}$$

According to Weng, the addition of $0.3d + 1.4d$ to the denominator of this above equation factors in what is referred to as the "end correction" phenomenon, on which I will elaborate in a later part of this dissertation.[8] Put simply, the frequency of the acoustic resonance of a pipe with two open ends is consistently observed as slightly lower than what the second equation above would predict. As a result, in order to make the second equation usable for calculating the actual frequency of a pipe, one must add a "correction" value to the length of the pipe, depending on the number of its open ends. This added value makes the vibrating air column virtually longer than the pipe itself, thus producing an accordingly smaller frequency. By imputing the relevant data of pipe lengths and diameters to the first equation above, Weng claims to show that the implied interval between the *huangzhong* pipe (7.29 *cun* long, 0.274 *cun* wide in diameter) and the half-length *taicu* pipe (3.24 *cun* long, 0.274 *cun* wide in diameter) stipulated in *Orthodox* is notably larger than an octave. Ironically, Weng also claims to show that the implied interval between the same *huangzhong* pipe and the half-length *dalü* pipe (3.413 *cun* long, 0.274 *cun* wide in diameter) stipulated in *Orthodox* do in fact come rather close to producing an octave.[9]

Using nothing but his own calculations, Weng argues that, had the compilers of *Orthodox* actually done the experiments they claim to have in Chapter 9, Volume 1 of *Orthodox*, they

8 Ibid., 40.
9 Ibid., 41-42.

would have observed not a fourteen-fold division of the octave between *huangzhong* and half-length *taicu*. Instead, they would have found a thirteen-fold division between *huangzhong* and half-length *dalü*, the pipe between half-length *huangzhong* and half-length *taicu*. The problem for Weng, however, is that the equation he uses does not appear to have as much scientific validity as he simply assumes.[10] Although "end correction" is a well-recognized phenomenon in modern acoustics, so far, I have not found any broad and empirically based consensus over how to factor it in when calculating the implied frequency of a pipe. As I will show in a later chapter, certain solutions existed even back in the 17th century to nullify this problem when it comes to making pipes in proportion to one another. Yet there simply is not a one-size-fits-all equation for calculating the implied frequency of a pipe with the end-correction phenomenon factored in by simply plugging in its length and diameter while disregarding all other factors.

Thus, neither Yang's political interpretation, nor Chen's experiments, nor Weng's calculations have been convincing in their attempts to dismiss the experimental results in Chapter 9, Volume 1 of *Orthodox* as having no empirical basis. Now, perhaps in order to prove definitely that the researchers behind *Orthodox* did in fact conduct the experiments they described as the basis for their fourteen-fold division of the octave, I ought to harvest some bamboo stems and slash some of them into pipes myself. I argue, however, that this burden of proof is nullified by the rich archive of sources that document how the *Orthodox* project actually came together. Where previous scholars studying the Kangxi Emperor's fourteen-tone temperament typically limit the purview of their sources to the treatise itself, my analysis of how,

10 Weng cites his source as Ma Dayou 馬大猷 et al., *Shengxue shouce* 聲學手冊 ("Manual of Acoustics"), 2nd edition (Beijing: Kexue chubanshe, 2004), 809; the manual specifies that the $0.3d + 1.4d$ added to the denominator of the standard equation for the acoustic resonance of pipes in the ideal situation, $f = \frac{c}{2l}$, is "derived from experience."

by whom, and in what context those experiments in making pitch pipes were carried out shows that the empirical foundation of the claimed observations in Chapter 9, Volume 1 of *Orthodox* is not to be questioned by anyone acting out of epistemological good faith. On the contrary, those experiments in *shenyin* "examining the tones" of pitch pipes were only possible because of the dynastic, scholarly, bureaucratic, manufacturing, and intelligence-gathering network of the Qing Empire.

Mapping the Pipe-lines

The story of the fourteen-tone temperament begins and ends not with the Kangxi Emperor with whom the tuning reform has become associated, but with In-c'i,[11] who is mentioned in the stamp impression on the upper right corner of f. 1r in *Putong Guji* No. 15251. Born in 1677 to a middle-rank concubine, In-c'i was the third-eldest of the Kangxi Emperor's twenty-four sons to live to the age of maturity. Conventionally, In-c'i is portrayed as a prince-scholar more interested in learning than in politics. Indeed, other than being known for fending off attacks against his elder brother In-ceng (Yinreng 胤礽 in Chinese, renamed as Yūn-ceng after 1723, or Yunreng 允礽, 1647-1725)—the primogeniture and Crown Prince from 1675-1708 and 1709-1712—or being imprisoned by his younger brother the Yongzheng Emperor (r. 1722-1735) along with many other sons of the Kangxi Emperor after 1723, In-c'i has received little attention in scholarly or dramatic narratives of the infamous succession strife that mired the last two decades of the Kangxi Emperor. Yet after his two elder brothers were

11 The Manchu naming convention typically does not include the *hala* or clan name in a person's form of address except for population registry purposes. Thus, though In-c'i, as a member of the Qing imperial family, had the clan name Aisin-Gioro, the latter is never included in any mention of him in any record.

stripped of their titles, put under house arrest, and effectively disinherited in 1708 and 1712, respectively, In-c'i became the *de facto* eldest son of the Kangxi Emperor. That made him a factor in the succession fight. In fact, judging by the frequent visits of the Kangxi Emperor to In-c'i's residence during the 1710s and early 1720s and by the number of occasions when the Emperor asked In-c'i to perform ritual duties on his behalf (at least according to those attested to in *Veritable Records of the Kangxi Reign*), In-c'i was arguably one of the frontrunners of the so-called *jiuzi duodi* 九子奪嫡 "nine sons vying for succession" horserace.

Moreover, In-c'i's passion for scholarship was more than likely a strategy for distinguishing himself amid the crowded field of contenders than a distraction from it, or so I argue. Later historians tend to acknowledge the political import of scholarship under the Kangxi reign as a matter of platitude, yet the Yongzheng Emperor clearly considered In-c'i's leadership role in some of the most major publication projects their father sponsored as a grave threat to his ascension to the throne—an ascension that was far from secure, even if only on account of the sheer number of viable and politically active brothers he had. Small wonder that as soon as the Kangxi Emperor passed away towards the end of 1722, the new Yongzheng Emperor immediately robbed In-c'i of his intellectual property consisting of two crowning scholarly achievements of the Kangxi reign: *Imperial Encyclopedia* mentioned in the previous chapter, and *Origins of Cosmological Sciences* including *Orthodox Meaning of Pitch Pipes*. Although *Imperial Encyclopedia* was not compiled by In-c'i himself, its sole author, Chen Menglei 陳夢雷 (1650-1741), was a tutor (*shidu* 侍讀, literally "servant of studies") and close advisor of the prince. In-c'i not only provided Chen with a staff of copyists for the project but also opened up his personal library. Upon the demise of the Kangxi Emperor, the Yongzheng Emepror sent both In-c'i and Chen into exile on account of the former's amicable relationship with In-ceng, the

former Crown Prince. And while *Imperial Encyclopedia* had already received its imperial approbation from the Kangxi Emperor back in 1705, the Yongzheng Emperor reopened the project, erased Chen's name, and transferred it to Jiang Tingxi 蔣廷錫 (1669-1732), the only name that would appear in its version issued by the imperial press in 1726.[12]

Similarly, while In-c'i was the indisputable leader of a team of scholars and literary staff that the Kangxi Emperor put in charge of compiling the 100-volume *Origins of Cosmological Sciences*, he had the grave misfortune of taking just a little too much time to finish this ambitious project. As the Yongzheng Emepror sent In-c'i into exile upon their father's death, he took away the almost-finished *Origins* draft and assigned it to their younger brother In-lu (Yinlu 胤祿 in Chinese, renamed Yūn-lu or Yunlu 允祿 after 1723, 1695-1767). In-lu was an ally of the Yongzheng Emperor who had just made him Prince Zhuang of the First Rank. In 1730, In-lu would even backstab In-c'i by formally impeaching him in front of the Yongzheng Emperor for apparently not appearing grief-stricken enough at the funeral of In-siyang  (Yinxiang 胤祥 in Chinese, 1686-1730), the favorite brother of the Emperor.[13] Thus, when the final draft of *Origins* was imperially sanctioned and published on October 29, 1723, the first name to appear on its list of authors and editors was In-lu, even though no official communication or record of the Qing court or government had ever before recorded In-lu's involvement in the project in any way, shape, or form.[14] What is more, while In-c'i (despite his exile) retained some of his honor

12 See Pei Qin 裴芹, Gujin tushu jicheng *yanjiu* 古今圖書集成研究 ("A Study of *Imperial Encyclopedia of Illustrations and Writings from the Earliest to Current Times*") (Beijing: Beijing tushuguan chubanshe, 2001), 27-42.

13 See Zhao Erxun 趙爾巽 et al., *Qing shigao* 清史稿 ("Draft History of the Qing," 1928), Vol. 220.

14 In-c'i et al., *Lüli yuanyuan* 律曆淵源 ("Origins of Cosmological Sciences," 1723), "List of Personnel" (*zhiming* 職名), f. 1r.

by still being ranked as second in the list of editors, in the preface he penned from *Origins*, the Yongzheng Emperor blatantly lies in stating that the Kangxi Emperor put "Prince Zhuang [In-lu] et al." (*zhuang qinwang deng* 莊親王等) in charge of the treatise—never mind that In-lu was less than eighteen years old when the Emperor commissioned the project in 1712 and was never granted any title, not even a *doro-i beile* ᡩᡠᠣᡵᠣ ᠪᡝᡳᠯᡝ (duoluo beile 多羅貝勒 in Chinese) or Prince of the Third Rank, the entry-level title for imperial sons, until his father's death.[15]

The extremes to which the Yongzheng Emperor was willing to go in humiliating his elder brother as a rival in the succession strife reflected the immense amount of political capital In-c'i had accumulated from spearheading imperially sponsored scholarly projects such as *Origins of Cosmological Sciences*. In fact, as the first part of *Origins* to be completed, *Orthodox Meaning of Pitch Pipes*, which would become volumes 43 to 47 of *Origins*, was commissioned in 1713 before the entire *Origins* project was formally conceived as a three-part treatise on astronomy, music theory, and mathematics. This timing was significant. Just the year before, in 1712, the Kangxi Emperor had deposed In-ceng as Crown Prince for the second and the last time, and In-c'i had just begun to communicate with his father on behalf of all is brothers as the *de facto* eldest imperial prince. A string of imperial edicts and palace memorials from 1713 to 1714 between the Kangxi Emperor and In-c'i shed light on the infrastructure and context for the *Orthodox* project. According to *Veritable Records of the Kangxi Reign*, the Kangxi Emperor formally commissioned *Origins* on July 22, 1713—that is, less than nine months after he deposed In-ceng:

> 諭和碩誠親王允祉：
> 律呂、演算法諸書，應行修輯。今將朕所制律呂演算法之書發下。爾率領庶起士何國宗等，即於行宮內立館修輯。[16]

15 Ibid., 2r.
16 *Veritable Records of the Kangxi Reign*, Vol. 254. KX 52.6.2.

> Edict to Prince Cheng [In-c'i] of the First Rank:
> The various books on pitch pipes and mathematics ought to be compiled. Now, We issue [to you] the books We have made on pitch pipes and mathematics. May you lead associate scholar [*shujishi*] He Guozong and others, establish an office in the temporary imperial residence, and compile them.

While this edict specifically asked In-c'i to "establish an office" (*liguan* 立館) inside a *xinggong* 行宮, which may refer to any temporary or seasonal imperial residence, it would seemingly take another three months before there was any concrete movement on the project. It was only on November 7, 1713 that the Kangxi Emperor issued another edict to In-c'I, one that finally appropriated the necessary resources fitting for the project:

> 諭和碩誠親王允祉等：
> 修輯律呂、演算法諸書，著于蒙養齋立館，並考定壇廟宮殿樂器。舉人照海等四十五人，系學習演算法之人。爾等再加考試。其學習優者，令其于修書處行走。[17]

> Decreed to Prince Cheng [In-c'i] of the First Rank and others:
> Regarding the compilation of various books on pitch pipes and mathematics, may [you] establish an office at the Study of Receiving Nourishment and also investigate and determine the musical instruments of sacrificial mounts, temples, palaces, and halls. Provincial Examination Graduate [*juren*] Giohai and forty-four others are students of mathematics. May you all give them additional examinations. Have those who are well-studied be adjuncts at the office of book compilation.

The Study of Receiving Nourishment (*mengyang zhai* 蒙養齋) was located in the Garden of Refreshing Spring (*Changchun yuan* 暢春園), a now ruined imperial garden to the northwest of Beijing where the Kangxi Emperor typically resided for several months a year and would eventually pass away in 1722. Besides granting In-c'i and He Guozong access to this workspace, the Emperor also assigned to them a cohort of well-educated *juren* 舉人 "Provincial

17 Ibid., Vol. 255. KX 52.9.20.

Examination Graduates," holding the third-highest degree in the civil examinations.[18] Granted, *yansuan fa* 演算法 "methods of calculation" or mathematics was almost never a test subject for the civil examinations, whose curriculum focused on Neo-Confucian interpretations of a selection of Confucian classics. Since its conquest of China, however, the Qing had established inside or near the Forbidden City various schools—typically known as *guanxue* 官學 "official schools"—for boys of Manchu and of bannermen-Chinese and bannermen-Mongol households. Besides providing both literary and martial education for the male children of the Qing's conquest elites, these schools prepared them for various positions in the Qing court and government that were reserved for Manchus and bannermen by quota, at the exclusion of the Han Chinese. Some of these positions were in the Bureau of Astronomy, or *qintian jian* 欽天監 (literally "Imperial Monitor of the Heaven"), which, besides finding the most auspicious time and location for a variety of ritual events, maintained the Chinese lunisolar calendar and predicted eclipses through celestial navigation and computations. Thus, since the 1660s, the Bureau of Astronomy regularly trained young Manchu and bannermen-Chinese and -Mongol students to perform the Bureau's technical duties.[19] This stream of adequately skilled mathematicians constituted a significant part of In-c'i's staff, which featured a number of Manchu and Mongol students besides Han Chinese examination graduates and candidates.

While In-c'i's support staff drew on technical talents from outside the Han Chinese literati as the body politic of Confucianism and civil examinations, his high-level assistants—

18 For more information on these students whom the Kangxi Emperor assigned to In-c'i as his support staff, see Jami, *The Emperor's New Mathematics*, 267-268.

19 See Huang Yinong 黃一農, "清初欽天監中各民族天文家的權力起伏" ("The ebb and flow of the power of astronomers of various ethnic groups at the Bureau of Astronomy in the early Qing era"), *Xin shixue* 新史學 2, No. 2 (1991): 75-108, particularly 99-106.

those whose names ended up being mentioned at the front of the list of authors and editors in the final published version of *Origins*, except for the undeserving In-lu—were fed from yet another pipeline: the tutelage of Li Guangdi. Besides workforce and workspace, according to *Draft History of Qing* (qingshi gao 清史稿, 1928), it was also in November 1713 that the Kangxi Emperor ordered Mei Juecheng 梅穀成 (1681-1764) and Wang Lansheng 王蘭生 (fl. 18th century) to join He Guozong in helping In-c'i.[20] Notably, He Guozong ascended to the Qing court through the "ordinary" channel of civil examinations. Having earned a *jinshi* degree just a year before in 1712, he was then assigned by the Kangxi Emperor to study *suanfa* "methods of calculation" or mathematics before joining In-c'i at the highest editorial level for the *Origins* project in 1713.[21] In stark contrast, neither Wang Lansheng nor Mei Juecheng had ever even sat in the examinations before joining In-c'i's editorial team. Known particularly for his knowledge of music theory and phonetics—subjects that were irrelevant to one's success in the examinations—Wang was a student of Li Guangdi's, and the latter specifically recommended Wang to the Kangxi Emperor for the *Origins* project. It was only after his assignment to help In-c'i that the Kangxi Emperor granted Wang the rather basic *juren* "Provincial Examination Graduate" degree , waiving the normal course of the examinations.[22]

Meanwhile, Mei Juecheng's path towards joining the *Origins* project reflected a broadly representative trajectory that brought numerous Han Chinese scholars into serving their Manchu conquerors. Mei Juecheng's grandfather, Mei Wending 梅文鼎 (1633-1721), was an exceptionally prolific Chinese scholar on mathematics and the calendar. Like many scholars and literati established in the Yangtze River Delta area, Mei Wending chose not to participate in the

20 Zhao Erxun et al., *Draft History of the Qing*, Vol. 94.
21 Ibid., Vol. 283.
22 Ibid., Vol. 290.

Qing's civil examinations after it conquered China and overthrew the Ming, albeit for apparently non-ideological reasons. It appeared that Mei Wending would have spent the rest of his life in southern China, disseminating his scores of treatises through the network of printing houses, libraries, and private academies burgeoning in the area. In 1689, however, the directors of the Qing's *History of the Ming* project invited Mei Wending to come to Beijing and take charge of the compilation of its "Treatise on Celestial Patterns" (*tianwen zhi* 天文志; *tianwen* 天文 "celestial patterns" had since become the default translation for "astronomy" in Chinese) and "Treatise on Calendar" (*lizhi* 曆志).[23] As I mentioned in the first chapter of this dissertation, because the paradigm of official dynastic histories stipulated a series of *zhi* "treatises" on practical matters of statecraft, the compilation of *History of the Ming*—its first phase lasting from 1678 to 1703—recruited many Han Chinese scholars vested in *shixue* "concrete learning," including the calendar, mathematics, and astronomy. Because *History of the Ming* commemorated the Chinese regime the Qing helped overthrow during its conquest of China, furthermore, it also served the political purpose of drawing into the Qing government and institutions those who had previously boycotted service to the conquest regime out of loyalty to the Ming.

It was during his sojourn in Beijing for the *History of the Ming* project between 1689 and 1693 that Mei Wending ingratiated himself with Li Guangdi, arguably the most influential Han Chinese scholar in the Kangxi Emperor's retinue and thus the most important patron for any Han Chinese scholar who wished to get imperial attention. As it happened, Li Guangdi did relay some of Mei Wending's treatises on the calendar to the Kangxi Emperor.[24] In fact, according to Li in

23 See Jami, *The Emperor's New Mathematics*, 83-86.
24 Ibid., 214-218.

his posthumous *Quotations*, during the 1692 court gathering, the Kangxi Emperor first berated the Han Chinese for "knowing nothing about mathematics" but then added: "There is this Mei guy from the Yangtze River Delta region who knows some [mathematics], but even he is quite bewildered."[25] Given that the Kangxi Emperor deliberately wanted to dress down his Han Chinese courtiers at the court gathering, his remark on Mei Wending's work was as good a compliment as he was willing to give anyone. And while the Kangxi Emperor rejected Zhang Yushu's *Petition* that he commission treatises on mathematics, the calendar, and music theory in 1692, over the next decade or so, the Emperor became more open to the suggestion. Just as he granted an audience to Mao Qiling and received a copy of the latter's *Explication of the Sagely Instruction on the Essence of Music* during a tour to the Yangtze River Delta region in 1696, the Kangxi Emperor finally met Mei Wending in person during another southern tour in 1703. In addition to rewarding Mei Wending with gifts and imperial calligraphy, the Kangxi Emperor decided to take into his inner court Mei Wending's grandson, Mei Juecheng, who had been studying mathematics and related topics with his grandfather.[26] Thus, just as Wang Lansheng's ascent to the retinue of the Kangxi Emperor bypassed the normal course of advancement for Han Chinese literati, namely the civil examinations, by relying on the patronage network of Li Guangdi, so did Mei Juecheng's.

In addition to Manchu, bannermen-Chinese, and bannermen-Mongol studies forming a support staff and Li Guangdi's protégés taking high-level editorial positions, the Kangxi Emperor brought yet another network of personnel and expertise on board with the *Origins* project: namely, artisans employed or contracted by the Qing court and its southern outposts.

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25 Li Guangdi, *Quotations of Sir Rongcun: A Sequel*, Volume 17, f. 25v.
26 Zhao Erxun et al., *Draft History of the Qing*, Vol. 506.

Compared to the edict issued on July 22, 1713, the November 7, 1713 edict expanded the scope of In-c'i's commission: in addition to compiling a treatise on *lülü* "[the twelve] tuning pitch pipes" and *yansuanfa* 演算法 "methods of computation" or mathematics, the Kangxi Emperor also tasked In-c'i with applying his studies on musical tuning to organology. A few months afterwards, in yet another edict to his son dated on March 2, 1714, the Emperor also revised his request from *kaoding* 考定 "investigating and determining" various instruments as stipulated in the November 7, 1713 edict to *chongxiu* 重修 "mending anew."[27] In other words, besides descriptions and designs of different classes of instruments that would be duly included in *Orthodox Meaning of Pitch Pipes*, the Kangxi Emperor asked In-c'i to oversee the actual making of new musical instruments, now that a new system of musical tuning was expected.

I have not discovered the specific trove of court records documenting how In-c'i managed to replace all the instruments used for rites and ceremonies at the many *tan* 壇 "sacrificial mounts," *miao* 廟 "temples," *gong* 宮 "residential palaces," and *dian* 殿 "ceremonial halls" located throughout the Forbidden City, the city of Beijing, and the various imperial gardens and retreats. Typically, such a project would have been handled by the Imperial Household Department (*dorgi baita be uheri kadalara yamun* ᡩᠣᡵᡤᡳ ᠪᠠᡳᡨᠠ ᠪᡝ ᡠᡥᡝᡵᡳ ᠺᠠᡩᠠᠯᠠᡵᠠ ᠶᠠᠮᡠᠨ, *neiwu fu* 內務府 in Chinese), which managed the internal affairs of the Qing imperial family and its estates. Specifically, the Department ran two agencies inside the Forbidden City that would have provided vital services to In-c'i's project: the *zaobanchu* 造辦處 or Imperial Workshops, located near the Hall of Mental Cultivation (*yangxin dian* 養心殿), manufactured myriad artefacts, from lamps, clocks, and containers to maps, pistols, and equine equipment; and the *xiushuchu* 修書處

27 *Veritable Records of the Kangxi Reign*, Vol. 257, KX 53.1.17.

or Imperial Publishing House, located at the Hall of Martial Valor (*wuying dian* 武英殿), copyedited, printed, and promulgated almost all imperially commissioned book projects.[28]

Outside the Forbidden City, the Imperial Household Department also supervised the three *zhizao* 織造 or Imperial Silk Factories located in Nanjing (also known as Jiangning), Suzhou, and Hangzhou in the Yangtze River Delta area.[29] Besides regular orders of sumptuous textiles for the Qing imperial family, the Department frequently outsourced highly specialized handicraft projects to the Silk Factories. Their location at the cultural and consumerist center of China proper allowed both easy access to the highly skilled artisans and laborers of the area and convenient shipment to Beijing via the Grand Canal. For example, when the Qianlong Emperor (r. 1736-1796), grandson of the Kangxi Emperor, ordered new sets of jade chimes to furnish the various ceremonial venues of the court, special grade Hotan nephrites were excavated by military outposts in Turkestan with the help from local *begs*, transferred to Beijing via the Grand Council (*junji chu* 軍機處, literally the "office of military secrets"), marked up with designs at the Imperial Workshops in the Forbidden City, and ultimately assigned to the Silk Factory in Suzhou for carving, polishing, and finishing.[30]

In addition to fulfilling the consumerist needs of the Qing court, the Silk Factories also functioned as what may be best described as personal liaison offices for the Qing Emperor in the

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28 See Zhang Naiwei et al., *Accounts and Knowledge of the Qing Palace*, vol. 1, 261-262 and 295-298.

29 See Qi Meiqin 祁美琴, *Qingdai neiwufu* 清代內務府 ("Imperial Household Department of the Qing Era") (Shenyang: Liaoning minzu chubanshe, 2008), 189-195.

30 See, for example, First Historical Archives of China 中國第一歷史檔案館 *Junjichu manwen lufu zouzhe* 軍機處滿文录副奏摺 ("Copies of Secret Palace Memorials to the Grand Council"), 03-0180-2024-013 (QL 27.10.25), 03-0180-2024-004 (QL 28.3.12), and 03-0180-2041-001 (QL 28.6.25). See also Zhou Xiaojing 周曉晶, "清代和田玉的開發與使用" ("Excavation and Use of Hotan Jade in the Qing Era"), *Liaoningsheng bowuguan guankan* 辽宁省博物馆馆刊 No. 1 (2010): 340-352.

Yangtze River Delta area. Besides more benign functions, such as coordinating the many imperial southern tours or sometimes accommodating and entertaining the imperial household during such tours, the managers of the Silk Factories were charged with gathering information on behalf of the Emperor himself. Like the most high-ranking civil and military officials during and after the mid-Kangxi era, the managers of the Silk Factories enjoyed the privilege of secret palace memorials, which allowed them to communicate directly with the Emperor without any intermediary or reader. While many of their memorials to the Forbidden City relayed weather conditions, harvests, food prices, and public opinions, others reported back on investigations conducted upon specific prior instructions by the Emperor to no one else's knowledge: background checks on particular officials, inquiries into certain rumors, or other intelligence-gathering critical to the security of the Qing's grip on the Han Chinese heartland against any possible sedition.[31]

Thus, just as the Qing's unique institution of Manchu and bannermen conquest elites curtailed and circumvented the entrenched power of the Han Chinese bureaucracy in its rule of China, the Silk Factories provided yet a further check on the formal government institutions of the Qing Empire. This check was waged exclusively and personally by the Emperor himself through the Imperial Household Department. Indeed, during the 17th and 18th centuries, some of the most prominent managers of the Silk Factories were neither Han Chinese scholar-officials who joined government ranks through civil examinations, nor scholarly services to the Qing government, nor ordinary Manchus or bannermen who rose through military services (including

31 See Qi Meiqin 祁美琴, *Qingdai neiwufu* 清代內務府 ("Imperial Household Department of the Qing Era"), 212-216.

the imperial palace guards) or other pipelines reserved for them as the Empire's conquest elites.[32] Instead, they came from the Emperor's household, though not as his sons, brothers, or cousins but as his *booi aha* ᠪᠣᠣᡳ ᠠᡥᠠ (*baoyi* 包衣 in Chinese) or bondservants.[33] Literally meaning "household slaves," the *booi aha* were lifetime hereditary servile people whose entire families were attached to the household of the commander of one of the Eight Banners. Whereas ordinary Manchus—all of whom were by default registered in one of the Eight Banners—and the Chinese and Mongol bannermen were prepared and destined for serving the Qing military, the *booi aha* primarily provided household services for their masters. And even though they faced no legal restrictions in taking government positions and even advancing to bureaucratic or military high ranks, their status as *booi aha* to their original household never dissolved. Because the Qing Emperor was the commander of three of the Eight Banners, furthermore, the Imperial Household Department had attached to it thousands of *booi aha* households whose members served the Qing imperial family and estates for generation after generation. By naming *booi aha* as managers of the Silk Factories, then, the Emperor put someone whose entire family and lineage were legally his personal bondservants in charge of some of the most lucrative posts in the Empire and also of a shadowy yet influential network of espionage surveilling the Han Chinese cultural and economic heartland on behalf of the Qing conquest regime. He thus used the personal and politically informal master-servant relationship to rein in and spy on the more institutionalized infrastructure of Qing rule, be it the Han Chinese dominated bureaucracy or the Manchu and bannermen network of viceroyalties and garrisons.

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32 See Chang, *A Court on Horseback*, 18-27. See also Elliott, *The Manchu Way*, 200-207.

33 See Qi Meiqin 祁美琴, *Qingdai neiwufu* 清代內務府 ("Imperial Household Department of the Qing Era"), 14-28, 217-235.

As it happens, the two managers of Silk Factories who facilitated In-c'i's project of compiling *Orthodox Meaning of Pitch Pipes* and refurbishing all the musical instruments at the Qing court came from *booi aha* families attached to the Imperial Household Department. Though Cao Yong 曹顒 (1689-1715) served as manager of the Silk Factory of Jiangning (Nanjing) for a mere three years between 1712 and 1715, he came from a *booi aha* family that virtually monopolized the Silk Factory there from the mid 17th to the early 18th century. Both his grandfather Cao Xi 曹璽 (1629-1684) and his father Cao Yin 曹寅 (1658-1712) managed the Silk Factory of Jiangning from 1663 to 1684 and from 1692 to 1712 respectively. Cao Xi's wife (Cao Yong's mother) was also the Kangxi Emperor's wet nurse, bringing the Cao family even closer to the Emperor on a personal level.[34] Meanwhile, Cao Yin's brother-in-law, Li Xu 李煦, who also came from a *booi aha* family attached to the Imperial Household Department, managed the Silk Factory at Suzhou between 1693 and 1724.[35] Cao Yin and Li Xu's tenures witnessed the most glorious decades of the Silk Factories. Together, they coordinated four out of the Kangxi Emperor's six imperial tours to the region, which became a critical institution throughout the late 17th and 18th centuries for the Qing Emperors to perform their at times contradictory role both as the conqueror of the Han Chinese and as subjects of their Confucian tradition of rulership dating back to the ancient sage kings. Besides performing their official duties of managing the Silk Factories and gathering whatever information the Emperor desired, they themselves also became influential patrons and trend-setters at the geographic center of Han Chinese scholarly and cultural production. Not only did they support scholars who established themselves in southern

34 Ibid., 237-238.

35 See anon. ed., *Suzhou zhizao Li Xu zouzhe* 蘇州織造李煦奏摺 ("Secret Palace Memorials of Li Xu, Manager of the Suzhou Silk Factory") (Taipei: Wenhai Chubanshe, 1975), i-iii.

China in lieu of joining the Qing government in Beijing, but their lavish expenditures for the imperial tours and for their own social and familial occasions also contributed to the flourishing of opera, which would in turn influence the taste and soundscape at the Qing court precisely through the Silk Factories as a cultural as well as political and economic interface between the Forbidden City and the Yangtze River Delta area.[36]

Thus, it was not surprising that, as the *Orthodox* project unrolled in 1713, the Kangxi Emperor demanded the assistance of Li Xu as the manager of the Silk Factory of Suzhou and Cao Yong as that of Jiangning. On September 27, 1713, two months after the Kangxi Emperor first commissioned In-c'i to compile a treatise on music theory, Li and Cao received the following decree from the Emperor:

> 諭李煦、曹顒：
> 朕集數十年功，將《律曆淵源》書將近告成，但乏做器好竹。爾等傳於蘇州清客周姓的老人，他家會做樂器的人，並各樣好竹子多選些進來，還問他可以知律呂有人一同送來。但他年老了走不得，必打發要緊人來攙好。
> 特諭。[37]

> Decreed to Li Xu, Cao Yong:
> Having accumulated several decades of effort, We are close to completing *Origins of Cosmological Sciences*, except that good bamboos for the purpose of making instruments are lacking. Summon the old freelance guest Mr. Zhou in Suzhou and those from his family who know how to make musical instruments. Also select many good bamboos of varying kinds and send them in, and ask [Mr. Zhou] whether he knows anyone who knows about the tuning pitch pipes [*lülü*] and send them in all together. However, he is old and cannot walk, make sure you send for reliable people to help him by the arm.
> It has been specially decreed.

36 See Chang, *A Court on Horseback*, 297-304.

37 Anon. ed., *Suzhou zhizao Li Xu zouzhe* 蘇州織造李煦奏摺 ("Secret Palace Memorials of Li Xu, Manager of the Suzhou Silk Factory"), 146. KX 52.9.18.

According to this decree, more than a month before he actually equipped In-c'i with the necessary personnel and workplace in November 1713, the Kangxi Emperor already began coordinating with the Silk Factories in southern China to harvest raw materials and recruit artisanal experts for *Orthodox* and the affiliated instrument-upgrade program. Notably, the Emperor appears personally familiar with the urban world of the Yangtze River Delta region in which Li Xu and Cao Yong lived and operated the Silk Factories. Aside from the unceremonious tone of the decree, which was typical of his communications to these two bondservants of his household, the Emperor was able to recall the surname of a *qingke* 清客 "freelance guest" in Suzhou whose family apparently specialized in making musical instruments. Typically, a *qingke* was patronized by the most affluent families to provide company and entertainment at their parties. Many *qingke* would have had a literary education, yet their life paths deviated from the established course of Han Chinese advancement, that of the civil examinations. Their education in the classics and poetry served them well as conversationalists, but a *qingke* also needed expertise in matters that, though irrelevant to the study of Confucianism, would prove critical for their success in the socio-cultural world of accomplished scholar-officials, landed gentry, and merchants: drinking, board games, table tricks, flattery, humor, and, indeed, singing opera arias.[38]

38 The best description of the kinds of activities a *qingke* would associate themselves with is found in the (in)famous homoerotic novel by Chen Sen 陳森 (19th century)'s *Treasured Mirror of Connoisseurs of Flosers* (c. 1849), which touches on various aspects of urban culture. See Chen Sen 陳森, *Pinhua baojian* 品花寶鑑 ("Treasured Mirror of Connoisseurs of Flowers," c. 1849), Han Ping ed. (Beijing: Huaxia chubanshe, 2016), 175-177. For scholarly studies on *qingke* "freelance guests" and freelance providers of musical services and lessons in general, see Xu Peng, "The Music Teacher: The Professionalization of Singing and the Development of Erotic Vocal Style During Late Ming China," *Harvard Journal of Asiatic Studies* 75:2 (2015): 259-297, and Lu Eting 陸萼庭, *Kunju yanchu shigao* 崑劇演出史稿 ("A Draft History of the Performance of *Kunqu* Opera") (Shanghai: Shanghai wenyi chubanshe, 1980), 76-88.

It might not be entirely surprising that the Kangxi Emperor took an interest in a professional socialite whom he might have even met in person and remembered from his latest tour to Suzhou in 1705, which Li Xu as manager of the Silk Factory there helped coordinate. Remarkably, however, the Emperor asked about this Mr. Zhou not only regarding music-making or even instrument-making but also regarding *lülü* "tuning pitch pipes." Specifically, the Emperor sought a recommendation from Mr. Zhou for people "who know about *lülü*" and directed Li Xu and Cao Yong to send them in along with the bamboo stems he demanded. The fact that the Emperor asked Li Xu and Cao Yong to send both bamboos and experts on *lülü* from southern China to Beijing to support the *Origins* project indicates that the purpose of his decree on September 27, 1713, issued before the compilation of *Origins* even started full steam, was neither musical instruments in general nor musical tuning in general but rather *lülü* in the literal and specific sense of the pitch pipes. This specificity became clearer in the two *booi aha*'s subsequent responses to the Emperor's decree. On November 5, 1713, a month after they received the Emperor's order for bamboos and *lülü* experts, Li Xu and Cao Yong wrote to the palace:

> […] 臣等遵即傳蘇州清客周啟蘭，著他選擇做樂器的人。周啟蘭年老不能行走，謹舉薦錢君達、張玉成二人知道律呂，會做樂器。臣等差家人護送上京，伏候諭旨，並將各樣竹子進呈。第此等竹子，俱產浙江，必於冬間取起方好。今年來蘇州的俱已買完，一時未有佳者。目下正值冬天，臣已差人往產竹地方前去尋覓，俟一得，隨即星齎進上。[…][39]
>
> […] In compliance, we your subjects immediately summoned the freelance guest in Suzhou, Zhou Qilan, and asked him to select makers of musical instruments. Zhou Qilan is old and cannot walk, and he respectfully recommended Qian Junda and Zhang Yucheng, two people who know the tuning pitch pipes [*lülü*] and how to make musical instruments. We your subjects will send our own

39 Anon. ed., *Suzhou zhizao Li Xu zouzhe* 蘇州織造李煦奏摺 ("Secret Palace Memorials of Li Xu, Manager of the Suzhou Silk Factory"), 146. KX 52.9.18.

> domestic helpers to escort them to Beijing, as we know, prostrating, await Your Decree, and will send in all kinds of bamboos. However, this grade of bamboos are all produced in the Zhejiang Province and must be harvested in the middle of the winter for them to be good. Those that have come to Suzhou this year have already sold out, and at the moment there aren't any good ones. Now it is winter time, and we your subjects have dispatched people to search in places that produce bamboos. As soon as we get them, we will immediately send them all in with lightening speed. […]

Alas, even though the final version of *Origins* credits as many as forty-seven people ranging from the two imperial princes (In-lu and In-c'i) to low-degree members of the support staff, no further mentions of the names Qian Junda or Zhang Yucheng is to be found in any official Qing record or publication. Still, according to Li Xu and Cao Yong's secret palace memorial above, the freelance guest from Suzhou, Zhou Qilan, recommended Qian and Zhang specifically for their knowledge of *lülü* "tuning pitch pipes" as well as their expertise in making musical instruments. The new instruments In-c'i would make required a variety of other raw materials such as metals for bells, jade for chimes, and fine wood and silk strings for zithers, yet the trouble the Kangxi Emperor caused Li Xu and Cao Yong in pursuing bamboos and nothing else shows his urgent need for pipes, which are the only thing for which bamboos are useful in instrument-making. From Li Xu and Cao Yong's memorial above, it seems that the Emperor rushed to order bamboos for In-c'i's work on *Origins* and *Orthodox* before it even began, and for a pretty good reason. As the two *booi aha* reported, winter, being the harvest season for bamboos, was the best time to get hold of the best quality bamboos. And just as Mao Qiling used bamboos from Mount Kuaiji in the province of Zhejiang to conduct his listening experiments with tuning pitch pipes, as he would describe in *Explication*, Li Xu and Cao Yong similarly insisted on procuring bamboos from Zhejiang, apparently the only grade of bamboos that could suffice for the making of pipes. While they promised swift action to the Emperor, it would take

the two managers of Silk Factories three months to send dispatchers to Zhejiang in search of bamboos, send them back to Suzhou for a quality check, then ship them along the Grand Canal to Beijing. At last, they delivered: on February 8, 1714, Li Xu and Cao Yong sent a secret memorial to the Emperor, alerting him that a shipment of 7,100 bamboos, all inspected by the freelance guest Zhou Qilan at Suzhou, was on its way. The Emperor replied that, not having seen the bamboos, he could not gauge their quality—the subtext being that he could not yet shower his two *booi aha* with any praise.[40] As I will show in the next section of this chapter, these 7,100 bamboos turned out to be critical for In-c'i's research on the twelve tuning pitch pipes, for which they designed and made many more pitch pipes than twelve. Thus, through the Silk Factories, which functioned as manufacturing, liaison, and intelligence-gathering outposts of the Qing's inner court in the Yangtze River Delta area, the research and compilation of *Orthodox* as the Qing's official treatise on musical tuning became a transregional project astride two different power centers of the Empire.

By the end of 1714, or just four hundred ten days after he assembled his editorial team and support staff, In-c'i delivered on both the music theory treatise and the new musical instruments for various ritual and ceremonial venues of the court. On December 22, 1714, the Kangxi Emperor personally performed the sacrificial rites at the Altar of Heaven (*tiantan* 天壇), the same place from which, he had searched for the *di* flute and the *se* zither for his lecture-demonstration at the court gathering back in 1692. Being the Winter Solstice of the northern hemisphere, the day marked the beginning of a new tropical year or a new cycle of *qi* 氣 "solar periods" in the lunisolar Chinese calendar. Additionally by virtue of featuring the longest night

40 Anon. ed., *Suzhou zhizao Li Xu zouzhe* 蘇州織造李煦奏摺 ("Secret Palace Memorials of Li Xu, Manager of the Suzhou Silk Factory"), 151-152. KX 52.12.24.

and shortest day in the northern hemisphere and thereby the highest *yin* 陰 "feminine" cosmic principle and the lowest *yang* 陽 "masculine" principle, Winter Solstice also heralds the decline of the *yin* and the ascent of the *yang*, with the latter being embodied not in the least by the Emperor himself as the epitome of the masculine principle. According to *Veritable Records*, accompanying this annual sacrifice to the Heaven was *xinding yuelü* 新定樂律 "the newly determined musical tuning."[41] Here, *yuelü* 樂律 combines the modifier *yue* "music" with the main noun *lü* "pitch pipes" to refer to musical tuning in general, and it further confirms that In-c'i and his team not only devised a new system of tuning but also applied it specifically to the ritual and ceremonial performances of music. It was only on the day after the Qing court and officials heard the fruits of their musical research at the Altar of Heaven that In-c'i and his team formally submitted to his father the treatise describing the "newly determined" musical tuning and instruments:

> 和碩誠親王允祉等以御製《律呂正義》進呈。得旨:
> 律呂、曆法、演算法三書，著共為一部。名曰《律曆淵源》。[42]

> [December 23, 1714] Prince Cheng [In-c'i] and others submitted the imperially compiled *Orthodox Meaning of Pitch Pipes*. To them was decreed:
> The three books on pitch pipes, calendar, and mathematics shall be integrated into one. It shall be named *Origins of Cosmological Sciences*.

This was how both *Origins* and its music theory portion, *Orthodox Meaning of Pitch Pipes*, entered the formal records of the Qing Empire with their imperially endowed names.

41 *Veritable Records of the Kangxi Reign*, Vol. 260. KX 53.11.5.
42 Ibid. KX 53.11.6.

This series of historical records shows that the compilation of *Orthodox* and the related refurbishment of musical instruments drew on many different pipelines of personnel and material resources from what may be referred to as the imperial infrastructure of the Qing Empire. The political capital created by ambitious research projects such as *Origins* not only served the Manchu conquest regime as a way to appropriate the Chinese tradition of Confucian scholarship, but also played a part in the succession strife between the Kangxi Emperor's many sons in the 1710s and 1720s. As the Chinese imperial succession norm of primogeniture came into conflict with the Manchu's Inner Asian political tradition that was much more egalitarian towards the imperial sons regardless of their age or the status of their mothers, Chinese scholarship ironically became a crucial tool for any ambitious Manchu prince to raise their profile above their fraternal competitors. Large-scale book compilation projects also allowed the Qing to recruit into its political system the many Han Chinese scholars who did not sit in the civil examinations, either for ideological reasons or for the competing scene of privately own printed houses and academies in the Yangtze River Delta region. These recruitment efforts created a fast-track for many scholars such as Mei Juecheng and Wang Lansheng to bypass the normal course of advancement. They also allowed a few established Chinese scholar-officials such as Li Guangdi to wage significant influence at the court where much power was still vested in traditional Manchu institutions and bannermen elites.

Yet the compilation of *Orthodox* as the Qing's official treatise on music tuning and as the music theory portion of *Origins* did not only reflect a rising influence of Chinese scholarship or scholars in the Manchu conquest regime. As I have also shown, two networks of sources unique to the Qing Empire played an indispensable role in the project. Bannermen schools trained Manchu, bannerman-Chinese, and bannerman-Mongol students for technical positions reserved

for them by quota at various parts of the Qing bureaucracy. Students of *suanfa* or mathematics trained for the Bureau of Astronomy thus became a convenient reserve of support staff for *Origins* whose contents heavily rely on mathematical skills. Meanwhile, the Qing imperial family's consuming and manufacturing needs and the Empire's concern for engaging with and surveilling the Yangtze River Delta region made the Silk Factories in Suzhou and Jiangning pivotal links in the Qing's rule over the Han Chinese cultural and commercial heartland. It was these informal yet multivalent outposts of the Imperial Household Department headed by the Kangxi Emperor's own bondservants that allowed the research team of *Orthodox* to, as they put it in Chapter 9, Volume 1 of the treatise, *jeizhu weiguan* 截竹為管 "slash bamboo stems into pipes" and *shenqi shengyin* 審其聲音 "[in order to] examine their sounds and tones," by providing them with harvested bamboo stems with two artisans skilled in making pitch pipes.[43]

While my above analysis situates the *Orthodox* project within the expansive political structures of the Qing Empire and reveals actual efforts of making numerous pitch pipes on the part of its researchers, they beg the question: what was In-c'i's role in all this? Although I have begun my analysis with the prince as the commissioned leader of the *Origins* and *Orthodox* projects, from the specific documentations I have examined, it would appear that In-c'i's involvement was miniscule at best. It was Li Guangdi who ended up packing the editorial team full of his own protégés. It was his father, the Kangxi Emperor, who let Li exert considerable influence on the project, assigned an entire staff of Manchu, bannermen-Chinese, and bannermen-Mongol students to him, and personally coordinated the Imperial Household Department and its Silk Factories in southern China to recruit experts in making pitch pipes and instruments and raw materials—namely a whopping 7,100 bamboos. So, for the purpose of this

43 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Volume 1, Chapter 9, f. 27r-27v.

dissertation, I am compelled to ask: did In-c'i's nominal leadership, recorded in Qing official archives and even in the final published version of the treatise, actually mean anything for the treatise of *Orthodox* and particularly its proposal for a fourteen-tone temperament? What did In-c'i bring to the table, after his father assigned him the task of compiling the Qing Empire's official treatise on musical tuning and music theory, both in terms of the content of the treatise and in terms of the institutional setup of the working process?

The answer, I have discovered, begins with f. 119 of the manuscript *Putong Guji* No. 15251.

"Rectifying the Pitch Pipes in order to Examine Their Tones," A Walkthrough

Before diving into the content of this single folio in the NLC manuscript, I should reiterate my earlier statement that no part of the manuscript explains the rationale for its miscellany of various apparently unrelated materials or describes the background history of this assemblage. Although the text in f. 119 of *Putong Guji* No. 15251 bears the title "A Postface on Pitch Pipes" (*lülü houxu* 律呂後序) and appears towards the end of the 127-folio manuscript, it does not at all address all, or indeed much, of the content of the entire manuscript. Rather, according to the text itself, which is dated at its end to the summer of 1707, it was intended as a postface to a treatise entitled *A Glimpse through the Pitch Pipes* (lülü guankui 律呂管窺).[44] This title appears to be a pun on the word *guankui* 管窺, which literally means "to look [at the sky] through a pipe" but figuratively means "to have a restricted overview of a subject," here used as a gesture of humility. The title does not turn up in the "Summary Catalogue and Digest" of the 18th-century book collection *Emperor's Complete Library in Four Sections*, a catalogue that

44 National Library of China, *Putong Guji* No. 15251, f. 119v.

includes not only titles that were eventually incorporated into the collection but also those that were rejected. Nor have I found it in the National Library of China or the Palace Museum in Beijing or the National Palace Museum Library in Taipei, three institutions whose holdings collectively cover almost all the books ever owned by the Qing court.

Fortunately, the text in f. 119 of the manuscript *Putong Guji* No. 15251 provides in its second half a detailed description of what the content of *A Glimpse through Pitch Pipes* would have looked like. This description turns out to be a perfect summary of the first volume of *Orthodox Meaning of Pitch Pipes*. As I will show through a comparison between Volume 1 of *Orthodox* analyzed in this section and f. 119 of *Putong Guji* No. 15251 examined in the next, the glaring similarity between this two-volume treatise and the twelve-chapter Volume 1 of *Orthodox* appears on two levels. First, the argumentative structure of both texts embodies the epistemic distinction between *shu* "numbers" and *sheng* "sounds" that Mao Qiling had teased out in his *Explication* from analyzing the Kangxi Emperor's lecture-demonstration of "mutual generation at every eighth step" back in 1692. Second, although the postface in f. 119 of *Putong Guji* No. 15251 is dated to the summer of 1707 and thus more than six years before the Kangxi Emperor even commissioned In-c'i to work on musical tuning or *Orthodox*, it already contains all the key findings that would define the Qing's official music theory treatise, including its proposal for a fourteen-tone temperament. In order to illustrate the crossovers of these key findings between Volume 1 of *Orthodox* and the postface to *A Glimpse through the Pitch Pipes*, then, I will first analyze the technical content of the former, emphasizing both its specific arguments and its overall structure.

To recall: as I have shown towards the end of the previous chapter of this dissertation, Chapter 9, Volume 1 of *Orthodox* adopts the Kangxi Emperor's technically mistaken definition

of "mutual generation at every eighth step" to stake an epistemological claim regarding the study of *lülü* "tuning pitch pipes" or music theory. Specifically, established discourse took both *sanfen sunyi* 三分損益 "triple division with one part subtracted or added" and *geba xiangsehng* 隔八相生 "mutual generation at every eighth step" as monikers for the Chinese Pythagorean tuning method and thus typically used them in close proximity.[45] Yet Chapter 9 identifies "triple division […]" as a method of *zhilü* 制律 "establishing the pitch pipes" yet "mutual generation […]"—meaning octave equivalence—as a method of *shenyin* 審音 "examining the tones [of said pitch pipes]." To wit, "triple division with one part subtracted or added" spells out the mathematical proportions whereby the length of the *huangzhong* pipe can be used to generate the lengths of all the other eleven tuning pitch pipes through compounding ratios of 2:3 and 4:3—in a process that involves only computations and no sound. Meanwhile, according to Chapter 9, Volume 1 of *Orthodox*, "mutual generation at every eighth step" spells out the sonic patterns whereby the pitches of the twelve tuning pitch pipes, having been generated and proportioned according to the Chinese Pythagorean method, can be organized into various iterations of the seven-tone diatonic scale, that is, "the five proper notes and the two altered notes"— in a process that involves actually listening to the octave equivalences between the sounds produced by the pitch pipes.

This epistemological contrast between the silent and mathematical process of "making pitch pipes" and the sonorous and auditory process of "examining the tones [of said pitch pipes]" is not only spelled out in Chapter 9, Volume 1 of *Orthodox* in distinguishing the two ubiquitous phrases in musical scholarship, "triple division […]" and "mutual generation […]," but also

45 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Volume 1, Chapter 9, f. 26r.

reflected in the subtitles of the entire treatise. The five-volume *Orthodox* is divided into three *bian* 編 or parts. The *shangbian* 上編 "former part" comprising Volumes 1 and 2 is entitled *zhenglü shenyin* 正律審音 "Rectifying the Pitch Pipes in order to Examine Their Tones," and presents the fourteen-tone temperament as a theory of tuning and a system of pitch organization. The *xiabian* 下編 "latter part" comprising Volumes 3 and 4 is entitled *hesheng dingyue* 和聲定樂 "Harmonizing the Notes in order to Establish the Institution of Music," and applies the aforementioned tuning and pitch organization to redesigning thirteen different types of musical instruments used in the court music ensembles. The final *xubian* 續編 "appended part," comprising only Volume 5, is entitled *xieyun duoqu* 協均度曲 "Coordinating the Note-Series in order to Compose Opera Tunes," and presents a rather novel system of musical pedagogy and notation aimed at reforming the compositions and performances of Chinese opera (more on this in the next chapter).

Thus the summary title of Volumes 1 and 2 constituting the "former part" of *Orthodox* directly combines the two knowledge-producing processes mentioned in Chapter 9, Volume 1: *zhilü* 制律 "establishing the pitch pipes"—which is slightly altered into *zhenglü* 正律 "rectifying the pitch pipes," since the word *zheng* 正 "orthodox" echoes the title of the treatise *Orthodox Meaning of Pitch Pipes*—and *shenyin* "examining the tones." In fact, the entire Volume 1 of *Orthodox* is organized around this distinction, with none other than Chapter 9 functioning as the pivot between the *shu* or mathematical processes of making pitch pipes and the *sheng* or sonorous and auditory processes of examining their pitches and organizing them into scales. Below are listed and translated the titles of all the twelve chapters of Volume 1 of *Orthodox*:

[1] 黃鐘為萬事根本
[2] 黃鐘理數

[3] 黃鐘轉生律呂
[4] 黃鐘律分
[5] 定黃鐘縱長體積面冪周徑
[6] 定律呂之長損益相生
[7] 定律呂之積損益相生
[8] 度量權衡
[9] 審定十二律呂五聲二變
[10] 審定十二律呂高低字譜
[11] 十二律呂同徑倍半生聲應五聲二變
[12] 黃鐘加分減分比例同形得聲應五聲二變[46]

[1] The *huangzhong* pipe is the root and essence of all things
[2] The principle and number [*shu*] of the *huanghzong* pipe
[3] The *huangzhong* pipe generating the other pitch pipes
[4] The *huangzhong*-pipe metrology
[5] Determining the length, volume, base surface area, and base circumference of the *huangzhong* pipe
[6] Determining the "one part subtracted and added" and "mutual generation" of the lengths of pitch pipes
[7] Determining the "one part subtracted and added" and "mutual generation" of the volumes of pitch pipes
[8] Length, volume, weight, and balance
[9] Examining and determining the "five proper notes and two altered notes" of the twelve tuning pitch pipes
[10] Examining the high and low notation syllables of the twelve tuning pitch pipes
[11] How the notes generated by the twelve tuning pitch pipes whose circumferences are the same and whose lengths are twice and half as long [as the original tuning pitch pipes] correspond to the "five proper notes and two altered notes"
[12] How the notes obtained by proportionally enlarging or shrinking the *huangzhong* pipe while maintaining the same shape correspond to the "five proper notes and two altered notes"

In this progression of topics and arguments, Chapters 1 to 8 focus on *zhenglü* "rectifying the pitch pipes" whereas Chapter 9 to 12 focus on *shenyin* "examining the tones [of said pitch pipes]." Chapter 1 also opens the volume and thus the entire treatise of *Orthodox Meaning of Pitch Pipes* with the words: "Oh! How grand is the *huangzhong* pipe, the essence of all things!"

46 Ibid., ff. 1r-1v.

(大哉黃鐘萬事之本也).[47] And just as this maxim of *huangzhong* being the "root and essence" of all things refers to the function of the twelve tuning pitch pipes as the foundation of metrology, the first eight chapters of Volume 1 precisely present a full narrative arch starting with the *huangzhong* pipe itself and concluding with *du liang quan heng* 度量權衡 "length, volume, weight, and balance." Chapter 2 lays out the various *shu* "numbers" associated with the *huangzhong* pipe in the oldest extant sources, including *Chronicles of Master Lü*, *Writings of Prince Huainan*, *Records of the Grand Historian*, and *Book of Han*: the length of the *huangzhong* pipe as "9 *cun*," its base as "9 *fen*," and its volume as "180 *fen*."[48] Chapter 3 presents the "triple division with one part subtracted or added" method, introducing the other eleven tuning pitch pipes and explaining how their lengths are proportionally related to that of *huangzhong*.[49] And Chapter 4 surveys different systems of metrology from the past and establishes the relationship between *guchi* 古尺 "ancient measurement standards" and *jinchi* 今尺 "modern [current] measurement standards." Specifically, the chapter claims to have determined the conversion rate between ancient and modern units of length by projecting each onto a string of *shu* 黍 "grains of millet" arranged in a straight line, since several of the early extant sources describe different units of measurements in relation to grains of millet (see Illustration 3-1).[50] Regardless of whether the text's researchers had actually used grains of millet

47 Ibid., f. 3r.

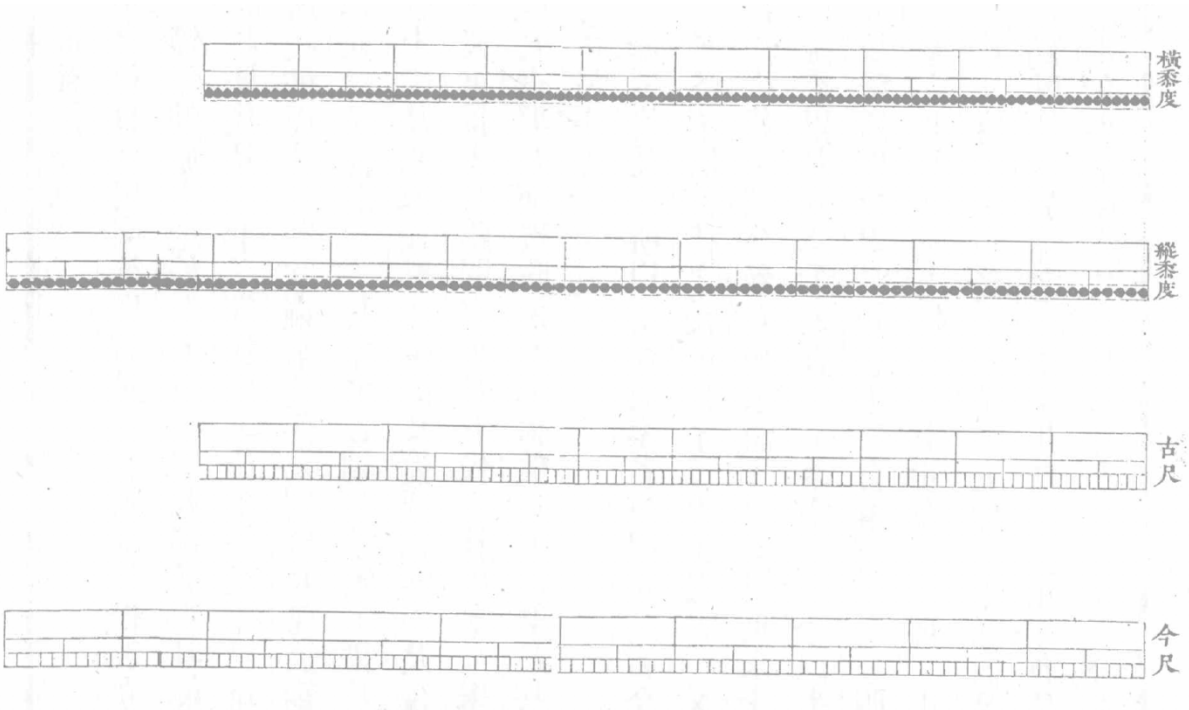
48 Ibid., ff. 5r-6r.

49 Ibid., ff. 7r-9v.

50 Ibid., ff. 10r-12v. In Illustration 3-1, taken from f. 11*bis*, the compilers of *Orthodox* compares the ancient *cun* with the Qing's modern *cun*. Combining analyses of textual sources with supposed observations with grains of millet, they argue that 10 ancient *cun* equals 100 grains of millet arranged in a row from belly to belly, whereas 10 modern *cun* equals 100 grains of millet arranged in a row from tip to tip. As a result, they observe and establish the conversion rate between ancient *cun* and modern *cun* as 100 ancient *cun* = 81 modern *cun*. Thus 9 ancient *cun*, the length of *huangzhong*, becomes 7.29 modern *cun*.

in this process or whether their analyses of ancient metrological records were correct, the chapter concludes with the conversion that 10 ancient *cun* equals 8.1 modern *cun*. Thus the length of the *huangzhong* pipe, which all of the oldest extant sources recorded as 9 ancient *cun* in length, is accordingly 7.29 modern *cun*, or 7 *cun* 2 *fen* 9 *li* 釐 (a tenth of a *fen*, which is a tenth of a *cun*) in the Qing's *jin* 今 "modern" metrology system.

***Illustration 3-1 Ancient and Qing's modern standards of length compared and measured with grains of millet, Chapter 4, Volume 1 of* Orthodox, *f. 11*bis**



Whereas Chapters 2 to 4 focus primarily on deriving *shu* "numbers" as textually transmitted historical data on the absolute and relative sizes of the twelve tuning pitch pipes, Chapters 5 to 8 process these data through *shu* "numbers" as mathematical computations. Using the aforementioned data on the length, base, and volume of the *huangzhong* pipe and the established conversion rate between ancient and modern units, Chapter 5 computes the length,

volume, base surface area, base circumference, and base diameter all in modern units.[51] Chapter 6 inputs the length of *huangzhong* into the compounding series of 2:3 and 4:3 proportions to generate the lengths of the other eleven pitch pipes,[52] concluding with a full illustration of the twelve tuning pitch pipes (see Illustration 3-2), and Chapter 7 accordingly computes the volumes of all twelve tuning pitch pipes—an easy feat, since they all bear the identical circular base.[53] Finally, Chapter 8 takes these newly calculated magnitudes of the twelve tuning pitch pipes as the basis for various units of measurements.[54]

The last third of Volume 1 transitions from *zhenglü* "rectifying the pitch pipes" to *shenyin* "examining the tones." Taking those "orthodox" twelve tuning pitch pipes whose absolute and relative sizes have supposedly been "rectified" so as to conform to ancient sources, Chapters 9-12 play them to produce sounds and organize the pitches of those sounds according to the seven-note diatonic scale system, that is, the "five proper notes and the two altered notes." As I showed towards the end of the previous chapter of this dissertation, Chapter 9 claims to have observed that when the *huangzhong* pipe as reconstructed in Chapters 1-8 is cut in half, the pitch of the resulting half-length *huangzhong* pipe—1/2 the length of *huangzhong*, with the same diameter—does not correspond to that of the original full-length *huangzhong* pipe in an octave identity. Meanwhile, it also claims to have observed that when the *taicu* pipe as reconstructed in Chapters 1-8 is cut in half, the pitch of the resulting half-length *taicu* pipe—4/9 the length of *huangzhong* with the same diameter—does in fact correspond to that of the original full-length *huanghzong* pipe. Because, as I have shown in Table 2-2 of Chapter 2 above, the octave heard

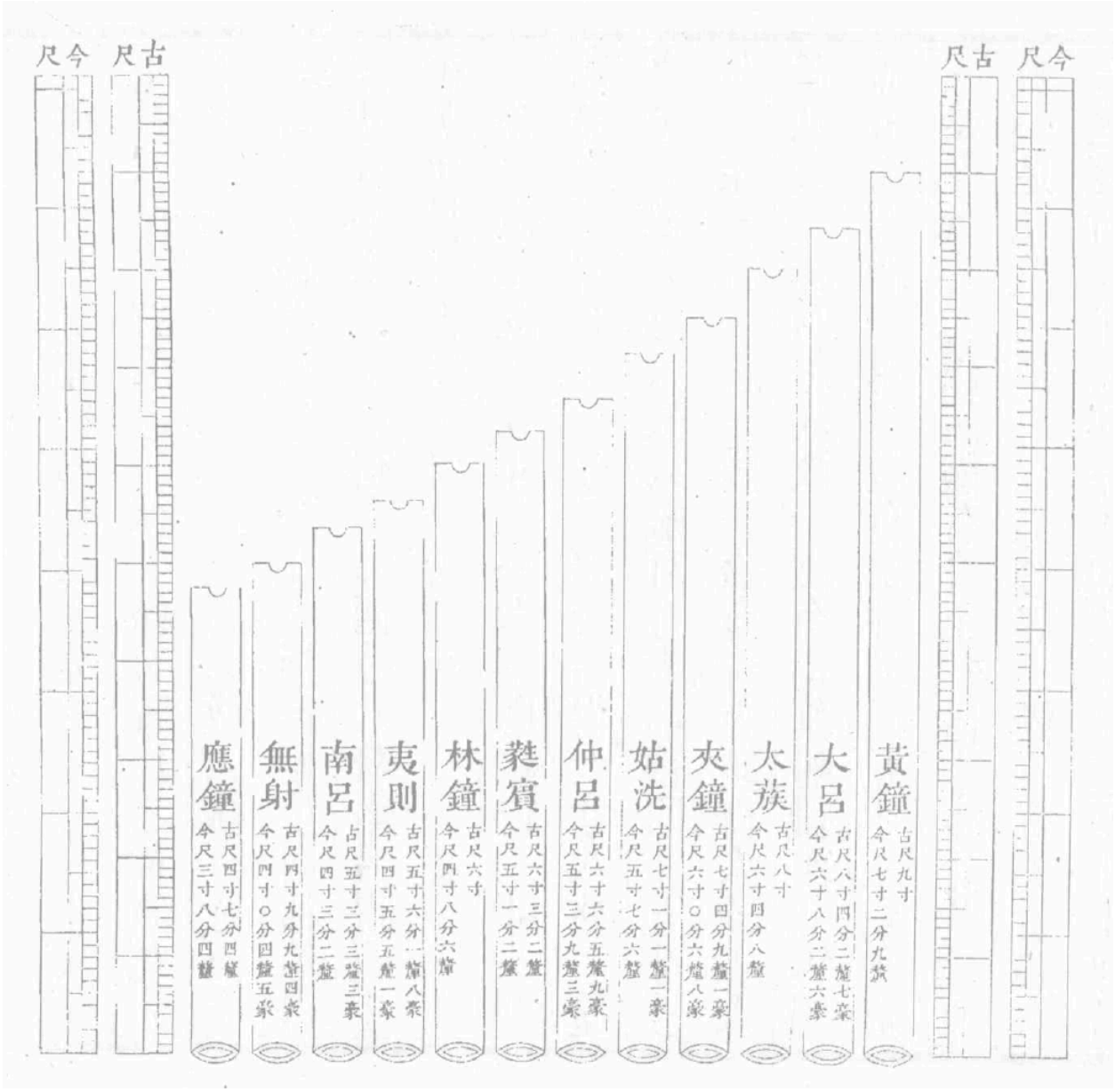
51 Ibid., ff. 12r-14r. Note that all the *jing* 徑 "diameters" in *Orthodox* refer to the inner diameter of pipes, which are effectively the air columns whose vibrations produce their pitches.

52 Ibid., ff. 15r-17v.

53 Ibid., ff. 18r-22r.

54 Ibid., ff. 23r-25v.

***Illustration 3-2 The twelve tuning pitch pipes proportioned according to "triple division with one part subtracted or added" in Chapter 6, Volume 1 of* Orthodox, *f. 17*bis**



between the full-length *huangzhong* pipe and the half-length *taicu* pipe encompasses fifteen tuning pitch pipes, what results from the experiments with *shenyin* "examining the tones" of the twelve tuning pitch pipes made according to the historical data and computations in Chapters 1-8 is a fourteen-fold division of the octave.

At this stage, I should acknowledge that the term "fourteen-tone temperament," though a convenient moniker for the tuning reform, can be rather misleading. For one, some may wonder, is the tuning reform proposed in *Orthodox* actually a "temperament"? Indeed, in Western European music theory, temperament refers to altering the actual tuning proportions away from the historically established and supposedly more natural Pythagorean tuning system based on the perfect-fifth ratio of 2:3. Following this definition, for example, since twelve-tone equal temperament alters the ratio corresponding to a perfect fifth from 2:3 in Pythagorean tuning to 1: $\sqrt[12]{2^7}$, it is said that the perfect fifth is thereby "tempered." The so-called "fourteen-tone temperament," by contrast, does not change at all the actual proportions whereby the *huangzhong* pipe can generate the remaining eleven tuning pitch pipes; instead, the same Chinese Pythagorean tuning method of "triple division with one part subtracted or added" is used to compute the lengths of these other pitch pipes from that of *huangzhong*, while the base diameter of *huangzhong* (ergo also the base circumference) is also used for these other pipes.

What is confusing about the tuning reform in *Orthodox* with respect to the question of temperament, however, is precisely our old friend, the tension between *shu* "numbers" and *sheng* "sounds." Although, as examined in the above paragraph, the *shu* "numbers: in the forms of length ratios used to proportion the twelve tuning pitch pipes are in no way "tempered," the *sheng* "sounds" produced by the resultant tuning pitch pipe in the forms of a series of notes spanning an octave do effect a *de facto* temperament, which is the reconfiguration of the twelve-tone octave into a fourteen-tone one. For anyone familiar with the rudiments of tuning and acoustics in the Western European and modern-scientific traditions, it could seem utterly oxymoronic that a non-tempered set of Pythagorean proportions could produce a tempered set of intervals and sonic patterns, let alone something as unusual as a fourteen-tone division of the

octave. I note, however, that the *shu* "numbers" bespoken in *Orthodox* and in the Chinese tradition of *lülü* "tuning pitch pipes" writ-large concern neither the varying frequencies of pitches as they typically do in modern acoustics nor the varying lengths of otherwise identical strings as they typically do in Western music theory up to the 17th century, specifically the monochord. Instead, they concern the length of pitch pipes, specifically pitch pipes whose base diameters (and thus circumferences) are all identical. As I have briefly mentioned in an earlier section of this chapter and will reprise with more detail in Chapter Five, tuning these types of pipes of identical diameters poses a tricky problem due to the so-called "end correction" phenomenon. Specifically, whereas the proportion of between the different lengths of two otherwise identical strings perfectly translates into the inversion of the proportion between their different fundamental frequencies, such is not the case for the length proportion between two pipes. For example, a 1:2 length proportion between two strings of the same tension, material, and thickness produces a 2:1 proportion between the frequencies of their fundamental pitches, which corresponds to an octave, and a 2:3 length proportion produces a 3:2 proportion between their frequencies, which corresponds to a perfect fifth; both sets of correspondences between length ratios of strings and intervals were commonly attributed to Pythagoras in the Western European tradition of music theory, whereas the concept of vibration frequency was formulated in the 17th century. Yet if we were to translate the claimed observation in Chapter 9, Volume 1 of *Orthodox* into the terms of frequencies, the researchers of the Qing treatise argue that a 1:2 length proportion between two bamboo pitch pipes of the same diameter does not produce a 2:1 proportion between the frequencies of their pitches, which is an octave between two *gong* or *do* notes, though a 4:9 length proportion apparently does, and that a 2:3 length proportion between two pipes of the same diameter does not produce a 3:2 proportion between the frequencies of

their pitches, which is a perfect fifth between the *gong* or *do* and the *zhi* or *sol* notes, though a 4096:6451 proportion apparently does. Thus, while I will continue to use the term "fourteen-tone temperament" both for the sake of convenience and for the lack of any better alternative, it is worth reiterating that the very essence of this so-called temperament is not the tempering of any ratios or even any deliberate re-division of the octave. Rather, as I will show in the rest of this dissertation, the Qing's tuning reform resulted from attempts to make sense of complex interactions between *shu* "numbers," *sheng* "sounds," and *lü* "pitch pipes" qua a type of *qi* "instruments," whereby mathematically non-tempered pitch pipes proportioned in strict compliance with the Chinese Pythagorean tuning method were nonetheless observed to produce the effectively tempered sonic patterns in the form of a fourteen-tone octave.

In fact, even the "fourteen-tone" part of the term is misleading. Granted, the observation in Chapter 9, Volume 1 of hearing an octave between the full-length *huanghzong* pipe and the half-length *taicu* pipe that is 4/9 the former's length logically and necessarily results in a fourteen-tone division of the octave, again, as shown in Table 2-2 of Chapter 2 above. But the problem is that *Orthodox* does not actually describe any fourteen-tone octave *per se*. Instead, the goal of *shenyin* "examining the tones" of pitch pipes is to sort out how the sounds they produce can be mapped onto various patterns of the seven-note diatonic scale, that is, the "five proper notes and two altered notes" of *gong* (roughly *do*), *shang* (*re*), *jue* (*mi*), *bianzhi* (*fa*# or *fi*), *zhi* (*sol*), *yu* (*la*), and *biangong* (*ti*). For what concerned the researchers of *Orthodox* was not into how many parts the octave is divided, given those pitch pipes constructed in Chapters 1-8 of Volume 1; instead, it was how the pitches of those pipes should be matched with various notes of the diatonic scale, so that they could be used as tuning standards for other musical instruments and actual musical practice.

To understand the treatise's intervention in pitch organization, it is necessary to explain a concept that is as difficult to translate as it is important: *yun* 均, which I have somewhat awkwardly rendered as "note-series."[55] In some ways, the notion of *yun* refers to "keys" in Western musical nomenclature, yet in a strictly non-modal sense. For example, the "key" of C major refers to the series of notes C-*do*, D-*re*, E-*mi*, F-*fa*, G-*sol*, A-*la*, and B-*ti*, specifically with C, *do*, as the tonic or modal final. In other words, C major means C-Ionian, as a key refers to a series of diatonically-related notes and to a structural hierarchy between the notes that is expected to manifest melodically or harmonically. If one were allowed to indulge in a cross-cultural music-theoretical thought experiment, a *yun* of C would also refer to a series of seven notes, C-*gong*, D-*shang*, E-*jue*, F#-*bianzhi*, G-*zhi*, A-*yu*, and B-*biangong*, yet there would no longer be any hierarchy between these notes. While C being the *gong* note determines onto which specific pitches the other six notes will fall, it only functions as a signifier that imposes the entire series of seven diatonically-related notes onto specific pitches without being the most important note, so to speak, within the series. Indeed, the concept of *yun* in Chinese music theory actually encompasses the concept of *diao* 調 "modes": under the *yun* of C, for example, each of the seven notes would take turn being the tonic or final of a mode, so that a *yun* would comprise all the seven modes that are constituted by the seven diatonically-related notes it entails. Similarly, a *yun* of G would refer to the series of seven notes G-*gong*, A-*shang*, B-*jue*, C#-*bianzhi*, D-*zhi*, E-*yu*, and F#-*biangong*, encompassing the seven modes that arise from each of these seven notes taking its turn as the final. Thus, to recalibrate in purely Chinese musical terms, a *yun* refers to a series of seven diatonically-related notes that result from placing the *gong*

55 See also Tong Zhongliang 童忠良 et al., *Zhongguo chuantong yuexue* 中國傳統樂學 ("Traditional Chinese Musicology") (Fuzhou: Fujian jiaoyu chubanshe, 2004), 129-135.

note at a certain pitch (pipe). Because, conventionally, an octave is divided into twelve parts, there are accordingly twelve possible *yun*'s, with each of the twelve pitches taking turn being the *gong* of the *yun*. Each each of these twelve *yun*'s, furthermore, the seven constituent notes further take turn being the tonic or final of a mode. Since there are twelve *yun*'s in which there are seven modes, the total number of modes in the conventional pitch organization system is eighty-four.

However, this system of twelve pitches, ergo twelve *yun*'s, seven notes per *yun*, and consequently eighty-four modes no longer holds when Chapter 9, Volume 1 of *Orthodox* observes what is effectively a fourteen-fold division of the octave. Indeed, the chapter describes the pitch-organization consequence of its observation of an octave heard between the full-length *huangzhong* and half-length *taicu* pipes not as a fourteen-tone octave *per se* but as two complementary seven-note *yun*'s. As Table 3-1 below shows, from the full-length *huangzhong* to the half-length *jiazhong* pipes, all the odd-numbered pipes form one *yun*, whereas all the even-numbered pipes form a complementary *yun*. To recall, in addition to hearing an octave between full-length *huangzhong* and half-length *taicu* pipes, which are the 1st and the 15th pipes respectively, Chapter 9 also claims to have heard a *gong-zhi* or perfect fifth between the *huangzhong* pipe and the *yize* pipe, the 9th pipe, whose length is 4096/6451 that of *huangzhong* (rather than the *linzhong* pipe, 2/3 of *huangzhong*'s length). From this, Chapter 9 claims that the *huangzhong* (the 1st pipe), *taicu* (3rd), *guxian* (5th), *ruibin* (7th), *yize* (9th), *wuyi* (11th), half-length *huangzhong* (13th), and half-length *taicu* (15th) form one *yun* in which the eight pitch pipes correspond to *gong*, *shang*, *jue*, *bianzhi*, *zhi*, *yu*, *biangong*, and clear *gong* in order. In contrast, in the conventional pitch organization system, the *yun*, in which *gong* falls on *huangzhong* comprises *huangzhong* (1st) as *gong*, *taicu* (3rd) as *shang*, *guxian* (5th) as *jue*, *ruibin* (7th) as

bianzhi, *linzhong* (8th) as *zhi*, *nanlü* (10th) as *yu*, *yingzhong* (12th) as *biangong*, and half-length *huangzhong* (13th) as clear *gong*.

Table 3-1 A fourteen-tone octave comprising two complementary diatonic* yun*'s separating odd-numbered from even-numbered pipes

No.	Tuning Pitch Pipes	Length to *huangzhong*	Odd-No. *yun*	Even-No. *yun*
1	黃鐘 *Huangzhong*	1:1	*Gong* [*do*]	
2	大呂 *Dalü*	2048:2187		*Gong* [*do*]
3	太簇 *Taicu*	8:9	*Shang* [*re*]	
4	夾鐘 *Jiazhong*	16384:19683		*Shang* [*re*]
5	姑洗 *Guxian*	64:81	*Jue* [*mi*]	
6	仲呂 *Zhonglü*	131072:177147		*Jue* [*mi*]
7	蕤賓 *Ruibin*	512:729	*Bianzhi* [*fa#*]	
8	林鐘 *Linzhong*	2:3		*Bianzhi* [*fa#*]
9	夷則 *Yize*	4096:6451	*Zhi* [*sol*]	
10	南呂 *Nanlü*	16:27		*Zhi* [*sol*]
11	無射 *Wuyi*	32768:59049	*Yu* [*la*]	
12	應鐘 *Yingzhong*	128:243		*Yu* [*la*]
13	半黃鐘 Half *huangzhong*	1:2	*Biangong* [*ti*]	
14	半大呂 Half *dalü*	1024:2187		*Biangong* [*ti*]
15	半太簇 Half *taicu*	4:9	*Gong* [*do*]	
16	半夾鐘 Half *jiazhong*	8192:19683		*Gong* [*do*]

Meanwhile, all the remaining even-numbered pipes form their own *yun* in which *gong* falls onto *dalü*: *dalü* (2nd) as *gong*, *jiazhong* (4th) as *shang*, *zhonglü* (6th) as *jue*, *linzhong* (8th) as *bianzhi*, *nanlü* (10th) as *zhi*, *yingzhong* (12th) as *yu*, half-length *dalü* (14th) as *biangong*, and half-length *jiazhong* (16th) as clear *gong*. In contrast, in the conventional organization system, the *yun* in which *gong* falls onto *dalü* comprises *dalü* (2nd) as *gong*, *jiazhong* (4th) as *shang*, *zhonglü* (6th) as *jue*, *linzhong* (8th) as *bianzhi*, *yize* (9th) as *zhi*, *wuyi* (11th) as *yu*, half-length *huangzhong* (13th) as *biangong*, and half-length *dalü* (14th) as clear *gong*. What results, then, can best be described as two complementary seven-note diatonic scales, one encompassing the odd-

numbered pipes and the other encompassing the even-numbered pipes. This oxymoronic system is embodied in the sixteen-pipe *paixiao* or pan-flute I presented in the introduction to this dissertation through the imagined perspective of the French Jesuit Jean-Joseph Marie Amiot in composing his *De la musique moderne des chinois* (1754).

Granted, this rather unique system of two complementary diatonic *yun*'s was a logically necessary outcome of fitting a seven-note scale onto a fourteen-tone octave: after all, fourteen divided by seven equals two, so that the two separate diatonic scales will never overlap, as if they were whole-tone scales or some other series of limited transpositions. What makes Chapter 9 of *Orthodox* even more idiosyncratic if not outright ludicrous from the perspective of the established discourse of *lülü* "tuning pitch pipes" qua musical tuning is that it argues that this oxymoron of two complementary diatonic scales was the very system used by the ancient sages. On face value, such an argument could be easily disproved by a flood of historical sources. While the earliest documented articulation of the system of twelve tones or *yun*'s per octave, seven notes per *yun*, and eighty-four modes took place no earlier than the 6th century CE, earlier sources such as *Records of the Grand Historian* (2nd century BCE) and *Writings of Prince Huainan* (3rd century BCE) all stipulate the *yun* of *huangzhong* as *huangzhong* (1st) as *gong*, *taicu* (3rd) as *shang*, *guxian* (5th) as *jue*, *ruibin* (7th) as *bianzhi*, *linzhong* (8th) as *zhi*, *nanlü* (10th) as *yu*, *yingzhong* (12th) as *biangong*, and half-length *huangzhong* (13th) as clear *gong*. That is, not only are there only twelve tones in an octave, but the *gong* and the *zhi* notes also fall onto two pipes whose lengths specifically bear the 2:3 ratio; this latter ratio is the cornerstone of the Chinese Pythagorean "triple division with one part subtracted or added" method and the phenomenon of "mutual generation at every eighth step" in the received meaning of the phrase as circle of fifths. This correspondence between the *gong-zhi* interval and the 2:3 ratio was even

confirmed in *Writings of Master Guan* (guanzi 管子, c. ? 5th century BCE), which was commonly attributed to Guan Zhong 管仲 (725-645 BCE) and thus revered as the earliest text to discuss music after the Confucian canons attributed to the ancient sages themselves—even though the text does not discuss the twelve tuning pitch pipes at all but only "the five proper notes and the two altered notes."[56]

So how does Chapter 9, Volume 1 of *Orthodox* fend off these broadly respected historical sources in championing its new proposal of two complementary diatonic *yun* constituting a fourteen-tone octave as the pitch organization system of the ancients? One explanation *Orthodox* puts forward, which I will not be able to elaborate within the span of this dissertation, is that the conventional twelve-tone octave and location of the *gong-zhi* interval between the *huangzhong* and *linzhong* pipes bearing a 2:3 length ratio apply only to Pythagorean-proportioned strings, not to pitch pipes. While this explanation implicates the subject of organology and the mechanisms of sound production in different types of instruments, it also allows the compilers of *Orthodox* to frame pitch organization as a philological problem. Like any other series study on musical tuning after the 12th century, Chapter 9 openly cites *Master Guan*, *Chronicles of Master Lü*, *Records of the Grand Historian*, and *Book of Han*. Yet it argues that their information on pitch organization, though not incorrect in itself, fails to either understand or specify that it applies solely to strings, not pitch pipes.[57]

In addition to upholding those earliest extant historical sources on *lülü* "tuning pitch pipes" while simultaneously bracketing them off as incomplete or misinformed, Chapter 9, Volume 1 of *Orthodox* also invokes two positive pieces of evidence to prove its newly proposed

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56 See Guanzi 管仲, *Guanzi* 管子 ("Writings of Master Guan," c. ? 5th century BCE), Chapter 58 "Measuring the Earth" (*diyuan* 地員).

57 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, ff. 26r-26v.

system of two complementary diatonic *yun*'s as the actual practice of the ancient sages. Each of these two proofs revolves around a single phrase. The first one concerns the very term *lülü* 律呂 "tuning pitch pipes" itself, which actually consists of two characters: *lü* 律 (pronounced with the falling tone in Standard Mandarin) specifically refers to the odd-numbered ones of the twelve tuning pitch pipes, and *lü* 呂 (pronounced with the low dipping tone) the even-numbered ones. Because odd numbers are associated with the masculine cosmic principle or *yang* and even numbers with the feminine cosmic principle or *yin*, it follows that the *lü* 律 pipes are associated with the *yang* principle and *lü* 呂 pipes with the *yin* principle. This distinction might seem trivial until one surveys the Confucian canons and some of their earliest extant annotations—which is precisely what Chapter 9 does. As it turns out, immediately after describing the results of their supposed experiments, in which they heard an octave between the full-length *huangzhong* pipe and the half-length *taicu* pipe and thus begot an effectively fourteen-fold division of the octave, the researchers of *Orthodox* write:

> […] 古聖人審定律呂，陰陽各六。陽則為律，陰則為呂，意固有在也。《孟子》曰：“不以六律不能正五音。”鄭康成《大司樂》註：“六律合陽聲，六呂合陰聲。”《國語》以六呂為六間。非陰陽分用之證耶？[…][58]
>
> […] When the ancient sages examined and determined the twelve tuning pitch pipes, there were six masculine [*yang*] ones and six feminine [*yin*] ones. The *yang* ones are the odd-numbered *lü* [律], the *yin* ones are the even-numbered *lü* [呂], and these have their proper meanings. *Master Mencius* [372-289 BCE] says: "without using the six odd-numbered *lü*, the five notes cannot be rectified." Zheng Kangcheng [127-200 CE] annotated the chapter "The Grand Steward of Music" from *Rites of Zhou*, saying: "The six odd-numbered *lü* correspond to the *yang* notes, the six even-numbered *lü* correspond to the *yin* notes." *Discourses of the States* takes the six even-numbered *lü* as the six intermediary ones. Are these not

58 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, ff. 27r-27v.

> the proofs that the *yin* and the *yang* pipes ought to be used separately? […]

All the sources invoked in this passage predate (and were understood as such in the Qing era) *Chronicles of Master Lü* and the other 3rd-century BCE or later sources commonly featured in studies on *lülü* "tuning pitch pipes." To be sure, during the 16th to 18th centuries, *Chronicles*, *Records*, and the likes were the earliest extant sources that discussed music theory in significant detail, whereas *Rites of Zhou* (zhouli 周禮, c. ? 8th-5th century BCE), *Discourses of the States* (guoyu 國語, c. 5th century BCE), and *Master Mencius* (mengzi 孟子, c. 4-3rd century BCE) only mention the technical matters of tuning, scales, and pitch organization in passing. Moreover, though the notion of *liulü* 六律 "the six odd-numbered tuning pitch pipes" in these sources was not at all unfamiliar to subsequent discourse, it was always used either metonymically or metaphorically but never literally. Even when writers used *liulü* "the six odd-numbered tuning pitch pipes" as a metonym for all the twelve tuning pitch pipes, or when treatises from Cai Yuanding's *New Treatise on Pitch Pipes* to Zhu Zaiyu's *Essential Meaning of Pitch Pipes* described the neat symmetry between the six odd-numbered *yang* pipes and the six even-numbered *yin* pipes, they did not at all take this symmetry to have any actual bearing on musical tuning, pitch organization, or musical practice. In fact, the only discourse that considered the *yang-yin* symmetry between the odd-numbered and even-numbered tuning pitch pipes meaningful was not that of *lülü* or musical tuning but geomancy.[59]

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59 In fact, a student of Zhu Xi and thus fascinated by the Neo-Confucian musings on cosmological correspondences, Cai Yuanding fills his *New Treatise on Pitch Pipes* with references to *yin* and *yang* with respect to the twelve tuning pitch pipes — even then, of course, he does not take the rhetorical distinction between the odd-numbered and the even-numbered pipes to have any musical applications. See Cai, *New Treatise on Pitch Pipes*, Volume 1, Chapter 3 "*Huangzhong* generating eleven pitch pipes" (黃鐘生十一律第三).

Furthermore, following on "mutual generation at every eighth step" in the received definition, whereby the 1st pipe and the 8th pipe are always in a 2:3 length proportion, the conventional system that associates the *gong*-*zhi* interval with the 2:3 length proportion necessarily mixes and matches the *yang*/odd-numbered with the *yin*/even-numbered pipes in any seven-note diatonic scale: for example, *huangzhong* (1st) as *gong*, *taicu* (3rd) as *shang*, *guxian* (5th) as *jue*, *ruibin* (7th) as *bianzhi*, *linzhong* (8th) as *zhi*, *nanlü* (10th) as *yu*, *yingzhong* (12th) as *biangong*, and half-length *huangzhong* (13th) as clear *gong*. Thus, from the perspective of Chapter 9, Volume 1 of *Orthodox*, the conventional pitch organization of twelve *yun*'s and seven notes per *yun* has been violating the symmetry of the *yang* and *yin* pipes mentioned in *Rites of Zhou*, *Discourses of the States*, and *Master Mencius*. Not only did these texts predate *Chronicles of Master Lü* and the like by centuries, but *Rites of Zhou* and *Master Mencius* also counted among the Thirteen Confucian Canons (*shisan jing* 十三經), recognized as the very corpus of Confucianism from the time of the 12th century. In this way, by proposing a system of two complementary diatonic *yun*'s following on a fourteen-tone octave, Chapter 9 claims to have finally found a way for the system of pitch organization in musical practice to comply with the *yin*-*yang* symmetry of the tuning pitch pipes, so that "the *yin* and the *yang* each follow their own categories without intermingling" (陰陽以類相從而不雜).[60]

If the pursuit of a symmetrical segregation between odd-numbered and even-numbered pitch pipes in organizing their pitches into diatonic scales might strike many readers as frivolous, if not ludicrous, the second phrase that Chapter 9, Volume 1 of *Orthodox* invokes in order to support its oxymoronic system of two complementary diatonic *yun*'s might appear even more far-off. As I briefly mentioned in the previous chapter, *xuangong* 旋宮 "rotating the *gong* note"

60 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, f. 28r.

is more or less comparable to the notion of transposition in Western music theory. Adding it to the concept of *yun* that I just introduced, "rotating the *gong* note" means moving the *gong* from one pitch pipe to another so that the entire *yun*, whose other six notes are diatonically related to *gong*, would move to other pitch pipes accordingly. For example—and again, to use Western pitch names so as to avoid confusion over twelve-tone vs. fourteen-tone octaves—the *yun* of C comprises the notes of C-*gong*, D-*shang*, E-*jue*, F#-*bianzhi*, G-*zhi*, A-*yu*, and B-*biangong*, and also encompasses the seven modes that are constituted by these seven notes, with each note taking its turn being the tonic or modal final. This "taking its turn being the tonic or modal final" is referred to as *zhuandiao* 轉調 "shifting the mode," and is comparable to the concept of modal mutation in Western music theory—for example, switching from C-Ionian to D-Dorian simply shifts or mutates the final of the mode without altering their constitutive pitches, which remain C, D, E, F, G, A, and B. When the *gong* note is *xuan* 旋 "rotated" from C to G, however, the *yun* changes accordingly into C#-*bianzhi*, D-*zhi*, E-*yu*, F#-*biangong*, G-*gong*, A-*shang*, and B-*jue*; here I have kept the order of the Western letter names so as to deemphasize the importance of their ordering, since within a *yun* or seven-note pitch collection, the notes do not bear any hierarchical relationship to one another. And the seven modes encompassed by this new *yun* are also constituted with these new seven notes—that is, with C#-*bianzhi* as opposed to C-*gong*—with each note taking its turn being the tonic or modal final. In Western parlance, then, to "rotate the *gong*" means to transpose all seven diatonic modes from one key to another.

And yet Chapter 9, Volume 1 of *Orthodox* espouses a completely heterodox definition of *xuangong* "rotating the *gong* note." When commenting on the conventional system of pitch organization, it states:

> […] 黃鐘為宮、太簇為商、姑洗為角、蕤賓為變徵、林鐘為徵、南呂為羽、應鐘為變宮，至半黃鐘復為清宮。[…] 夫正律

> 為宮，至半律而仍為宮；正律為商，至半律而仍為商。則宮商一定，而旋宮之義已失。[…][61]
>
> […] The *huangzhong* pipe is *gong* [*do*], *taicu* is *shang* [*re*], *guxian* is *jue* [*mi*], *ruibin* is *bianzhi* [*fa#*], *linzhong* is *zhi* [*sol*], *nanlü* is *yu* [*la*], *yingzhong* is *biangong* [*ti*], and the half-length *huangzhong* pipe is, again, the clear *gong*. […] To wit, when the original pipe is *gong*, the half-length pipe is still *gong*; when the original pipe is *shang*, the half-length pipe is still *shang*. And thus *gong* and *shang* are fixed, so that the meaning of "rotating the *gong*" has been lost. […]

This passage observes that, in the conventional system, the 1:2 length ratio always corresponds to the recurrence of the same note in the seven-note cycle: that is, both *huangzhong* and half-*huangzhong* pipes are *gong*'s, both *taicu* and half-*taicu* pipes are *shang*'s, etc. In other words, the 1:2 length ratio corresponds to an octave equivalence—or indeed to what both the Kangxi Emperor and Chapter 9, Volume 1 of *Orthodox* (mis)understood "mutual generation at every eighth step" to mean. Yet curiously, the passage above blames this correspondence between 1:2 pipe length ratio and octave equivalence on the loss of a true understanding of *xuangong* "rotating the *gong*," a claim that begs of the question of what the writers of *Orthodox* understand the term to mean here. As it turns out, a later passage in the chapter explains:

> […] 故今所定黃鐘為首音宮聲，次太簇為二音以商聲應、姑洗為三聲以角聲應、蕤賓為四音以變徵聲應、夷則為五音以徵聲應、無射為六音以羽聲應、半黃鐘為七音以變宮聲應。此陽律之五聲二變也。至半太簇為清宮，而與黃鐘應，則陽律之旋宮之義見焉。[…][62]
>
> […] Therefore, today, we determine that the *huangzhong* pipe is the first tone corresponding to the *gong* note, *taicu* pipe the second tone corresponding to the *shang* note, *guxian* pipe the third tone corresponding to the *jue* note, *ruibin* pipe the fourth tone corresponding to the *bianzhi* note, *yize* pipe the fifth tone corresponding to the *zhi* note, *wuyi* pipe the sixth tone corresponding to the *yu* note, and half-*huangzhong* pipe the

61 Ibid., f. 26v.
62 Ibid., f. 27v.

> seventh note corresponding to the *biangong* note. These are the "five proper notes and two altered notes" of the *yang* [odd-numbered] pitch pipes. The half-*taicu* pipe is the clear *gong* note, and it corresponds to the *huangzhong* pipe, and thus the meaning of "rotating the *gong*" among the *yang* pitch pipes is manifest. […]

Because the chapter claims to have heard an octave equivalence between the full-length *huangzhong* pipe and the half-length *taicu* pipe, it follows that whenever the note of *gong* falls onto the pitch of the *huangzhong* pipe, the clear *gong* sounding an octave above will fall onto the pitch of the half-*taicu* pipe. This also means that whenever one moves from one *gong* to the *gong* an octave above, one will necessarily move from one pitch pipe to another pitch pipe that bears a completely different name: e.g. from (full-length) *huangzhong* to (half-length) *taicu*. According to the passage above, this octave shift of the *gong* note from one class of pitch pipes bearing the same name to another class of pitch pipes bearing a different name is the true meaning of *xuangong* "rotating the *gong*." In the conventional pitch organization following the twelve-tone octave, a shift in the octave of the *gong* note does not land on another class of pitch pipes. In contrast, the new system of two complementary diatonic *yun*'s following the fourteen-tone octave necessarily effects such a "rotation" from one class of pitch pipes to another at every new cycle of "the five proper notes and the two altered notes" at the octaves.

To summarize, Chapter 9, Volume 1 of *Orthodox*, arguably the crux of the entire treatise, not only observes a fourteen-tone octave through experiments with "making pitch pipes in order to examine their tones," but also turns this observation into a novel system of pitch organization. Since an octave is heard between the full-length *huangzhong* pipe and the half-length *taicu* pipe, and since a *gong-zhi* perfect fifth is heard between the *huangzhong* and the *yize* pipes, therefore the twelve tuning pitch pipes and the half-length ones implicated in these listening experiments are separated accordingly into odd-numbered pipes and even-numbered pipes, with each forming

its own diatonically-related series of notes, or *yun*. What result, then, are two non-overlapping *yun*'s or two complementary seven-note diatonic scales, one formed only by odd-numbered pipes and another only by even-numbered pipes, so that the *yang* pipes and the *yin* pipes actually end up segregated from each other. Furthermore, because an octave is heard between the first and the fifteenth pipe rather than between the first and the thirteenth pipe among the twelve tuning pitch pipes and their respective half-length pipes, any octave displacement from *gong* to clear *gong* will trigger a displacement from one class of pitch pipes bearing the same name and bearing various degrees of 1:2 length ratios to another class of pitch pipes: that is, from full-length *huangzhong* as *gong* to half-length *taicu* as *gong*.

As Chapter 9 has not only transitioned from "rectifying the pitch pipes" to "examining the tones [of said pitch pipes]" but had also turned experiments in sounding and listening into a new system of pitch organization, Chapters 10 and 11 merely build on it. Chapter 10 asks how the twelve tuning pitch pipes that had been fashioned in Chapters 1-8 and organized into notes, scales, and *yun*'s in Chapter 9 relate sonically to the *xiao* 簫 or end-blown pipe and the *di* 笛 or traverse flute.[63] Finding where the different notes on these two wind instruments fall in the newly developed scale system of the fourteen-tone temperament is crucial. For one, the *xiao* end-blown pipe and the *di* flute collectively lay the foundation for the *gongche* 工尺 syllables—*shang* 上, *che* 尺, *gong* 工, *fan* 凡, *he* 合 (or *liu* 六), *si* 四 (or *wu* 五), and *yi* 乙 (or *yi* 一), which were defined through the fingering holes of these two instruments.[64] Known sometimes and deployed in the title of Chapter 10 as *zipu* 字譜 "notation using characters," *gongche* syllables were

63 Ibid., f. 26v.

64 See Wu Xiaoping 吳曉萍, *Zhongguo gongchepu yanjiu* 中國工尺譜研究 ("A Study of the Chinese *Gongche* Notation"), 20-24.

broadly used by singers and instrumentalists, particularly in performing and notating opera. In this connection it is noteworthy that from a practical standpoint that the *di* flute was the instrument *de rigueur* of Chinese opera before the 18th century, particularly in *kunqu* 崑曲 opera, which had been popular among the Han Chinese cultural elites in the Yangtze River Delta area and was increasingly so at the Qing court and in Beijing since at least the 16th century. In some ways comparable to the *basso continuo* in Baroque music, the *di* flutist typically plays throughout an entire opera or a selection of scenes or arias, usually doubling the singer, except for purely percussive passages that punctuate entrances, exits, and other stage actions or spoken parts. Establishing correspondences between the pitches produced through various fingerings on the *xiao* end-blown pipe and the *di* flute on the one hand and the twelve tuning pitch pipes as organized in the two complementary diatonic scales and *yun*'s is thus critical in applying the tuning pitch pipes and their system of pitch organization to actual musical practice.[65]

Thus, Chapter 10 claims to have conducted additional listening experiments, which match the pitch of the *huangzhong* pipe fashioned according to the data in Chapters 1-8 with the *gong* 工 fingering on the *xiao* end-blown pipe, or the *si* 四 fingering on the *di* flute. These correspondences in turn allow the seven *gongche* syllables and their respective fingerings on the two wind instruments to be matched one-by-one to "the five proper notes and the two altered notes" as they have been matched with the pitches of the twelve tuning pitch pipes in Chapter 9. What results, then, is a system of pitch organization that organizes the tuning pitch pipes into two complementary diatonic scales, so that they can be matched with such diatonic scales used to design instruments such as the end-blown pipe and the flute and to guide musical practice such as opera. Chapter 11 completes this pitch organization system by expanding its gamut of pitch

65 See Liang Mingyue, *Music of the Billion*, 241.

Illustration 3-3 Two* yun*'s of one octave plus a perfect fifth (lower* zhi-*high* shang*), one each for the odd-numbered and even-numbered pipes in Chapter 11, Volume 1 of* Orthodox*, f. 36

pipes in both directions. Whereas Chapters 9 and 10 so far only discussed the sixteen pipes from full-length *huangzhong* to half-length *jiazhong* pipes—that is, the two *yun*'s each encompassing an octave, one from full-length *huangzhong* to half-length *taicu*, the other from full-length *dalü* to half-length *jiazhong*—Chapter 11 incorporates the six pipes longer than *huangzhong* from double-length *ruibin* to double-length *yingzhong* and the two more pipes shorter than half-length *jiazhong*, that is, half-length *guxian* and half-length *zhonglü*, thus producing an entire gamut of twenty-four pipes. By applying the same pattern of pitch organization whereby the odd-numbered pipes are grouped into one diatonic *yun* and the even-numbered pipes are grouped into the complementary diatonic *yun*, Chapter 11 extends both *yun* up by one note beyond the high

gong and down by three notes beyond the low *gong*. The table concluding Chapter 11, shown in Illustration 3-3 and translated in Table 3-2, presents this final gamut.

Table 3-2 Illustration 3-3 translated[66]

Odd-Numbered Pipes		**Even-Numbered Pipes**	
倍蕤賓 Double-length *ruibin*	下徵乙字 Low *zhi* [*sol*]	倍林鐘 Double *linzhong*	清下徵高乙字 Clear low *zhi* [*sol*+]
倍夷則 Double-length *yize*	下羽上字 Low *yu* [*la*]	倍南呂 Double *nanlü*	清下羽高上字 Clear low *yu* [*la*+]
倍無射 Double-length *wuyi*	變宮尺字 *Biangong* [*ti*]	倍應鐘 Double *yingzhong*	清變宮高尺字 Clear *biangong* [*ti*+]
黃鐘 *Huanghzong*	宮聲工字 *Gong* [*do*]	大呂 *Dalü*	清宮高工字 Clear *gong* [*do*+]
太簇 *Taicu*	商聲凡字 *Shang* [*re*]	夾鐘 *Jiazhong*	清商高凡字 Clear *shang* [*re*+]
姑洗 *Guxian*	角聲六字 *Jue* [*mi*]	仲呂 *Zhonglü*	清角高六字 Clear *jue* [*mi*+]
蕤賓 *Ruibin*	變徵五字 *Bianzhi* [*fa#*]	林鐘 *Linzhong*	清變徵高五字 Clear *bianzhi* [*fa#*+]
夷則 *Yize*	徵聲乙字 *Zhi* [*sol*]	南呂 *Nanlü*	清徵高乙字 Clear *zhi* [*sol*+]
無射 *Wuyi*	羽聲上字 *Yu* [*la*]	應鐘 *Yingzhong*	清羽高上字 Clear *yu* [*la*+]
半黃鐘 Half-length *huangzhong*	變宮尺字 *Biangong* [*ti*]	半大呂 Half-length *dalü*	清變宮高尺字 Clear *Biangong* [*ti*+]
半太簇 Half-length *taicu*	少宮工字 High *gong* [*do*]	半太簇 Half-length *taicu*	清少宮高工字 Clear high *gong* [*do*+]
半姑洗 Half-length *guxian*	少商凡字 High *shang* [*re*]	半仲呂 Half-length *zhonglü*	清少宮高凡字 Clear high *shang* [*re*+]

Here, the twelve odd-numbered pipes from double-length *ruibin* to half-length *guxian* are shown in the first row, and the twelve even-numbered ones in the second row. By nature of the complementarity of the two diatonic *yun*'s, their respective note names ("the five proper notes and the two altered notes") and *gongche* syllables are completely identical—except that the note

66 Besides translating Illustration 3-3, this table rotates it counterclockwise by 90 degrees, so as to better fit the typesetting standards of English prose.

names of the even-numbered pipes are all attached with a prefix *qing* 清 "clear" and their *gongche* syllables attached with a prefix *gao* 高 "high." Notably, here, *qing* "clear" no longer means an octave above, which is what the character typically means in the context of scales and notes, but is used in the sense of an upward finetuning of a pitch in contrast to *zhuo* 濁 "muddy." Meanwhile, whereas *xia* 下 "inferior" remains the prefix to note names that indicate an octave lower, *shao* 少 "youthful" is used instead of *qing* "clear" to indicate an octave above.

While Chapters 1-8 in Volume 1 of *Orthodox* make the twelve tuning pitch pipes through textually transmitted historical data and mathematical computations, and Chapters 9-11 listen to the pitches of these pipes and organize them into two complementary *yun*'s of "five proper notes and two altered notes," Chapter 12 concludes the volume by integrating these two distinct knowledge-producing processes that the volume's title succinctly summarizes as "Rectifying the Pitch Pipes" in order to "Examine their Tones."[67] Specifically, Chapters 1-11 first determine the proper sizes of the tuning pitch pipes and only then listen to their pitches and pattern them according to the seven-note diatonic scale, while Chapter 12 conversely takes the newly proposed pitch organization system and makes a new set of pitch pipes to accord with it. The difference is that the pipes involved throughout Chapters 1-11 are what the treatise refers to as *tongjing guan* 同徑管 "pipes of the same diameter." Indeed, all pipes mentioned in Chapters 1-11, regardless of their lengths, bear the same base diameter as the *huangzhong* pipe: 0.274 modern *cun*, or 2 *fen* 7 *li* and 4 *hao* 毫 (which is 1/1000 of a *cun* in the Qing's measuring system). This diameter is in turn determined in Chapter 5 through a combination of historical records and calculations. Since at least the time of Ban Gu's *Book of Han* from the 1st century

67 Ibid., ff. 37r-72v.

CE, it has almost become a mantra that "the *huangzhong* pipe is 9 *cun* and its hollow enclosure is 9 *fen*." As mentioned in the previous chapter of this dissertation, not only did scholars debate the exact sizes of the ancient *cun* or ancient *fen* as well as the conversion between these two ancient units of length (whether 1 *cun* = 9 *fen* or 1 *cun* = 10 *fen*), they also disagreed over whether this *kongwei* 空圍 "hollow enclosure" that is 9 ancient *fen* in magnitude refers to the circumference of the base of the *huangzhong* pipe or its surface area. Zhu Zaiyu, for example, subscribes to the former interpretation in his *Essential Meaning of Pitch Pipes*, so that for him the diameter *d* of the *huangzhong* pipe can effectively be expressed in ancient *fen* as[68]:

$$d = \frac{9}{\pi}$$

In contrast, the compilers of *Orthodox* concur with Cai Yuanding's *New Treatise on Pitch Pipes*, which in turn seizes on a rather vague line in the "Treatise on Pitch Pipes and Calendar" in Ban's *Book of Han* that identifies 810 *fen* as the *shi* 實 "concrete" of *huangzhong*. According to Cai, this *shi* refers to the volume of the *huangzhong* pipe, that is, the product of multiplying its 9 *cun* length with its 9 *fen* base surface area, with 9 *cun* equal to 90 *fen* in the supposed ancient system of metrology. And thus, in concurring with Cai, Chapter 5, Volume 1 of *Orthodox* calculates the diameter *d* of the *huangzhong* pipe in ancient *fen* as[69]:

$$d = 2 \times \sqrt{\frac{9}{\pi}}$$

68 Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, Volume 1, f. 33r.

69 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Volume 1, f. 12r. It is worth repeating here that all the discussions on diameters in *Orthodox* refer to the inner diameter of pipes.

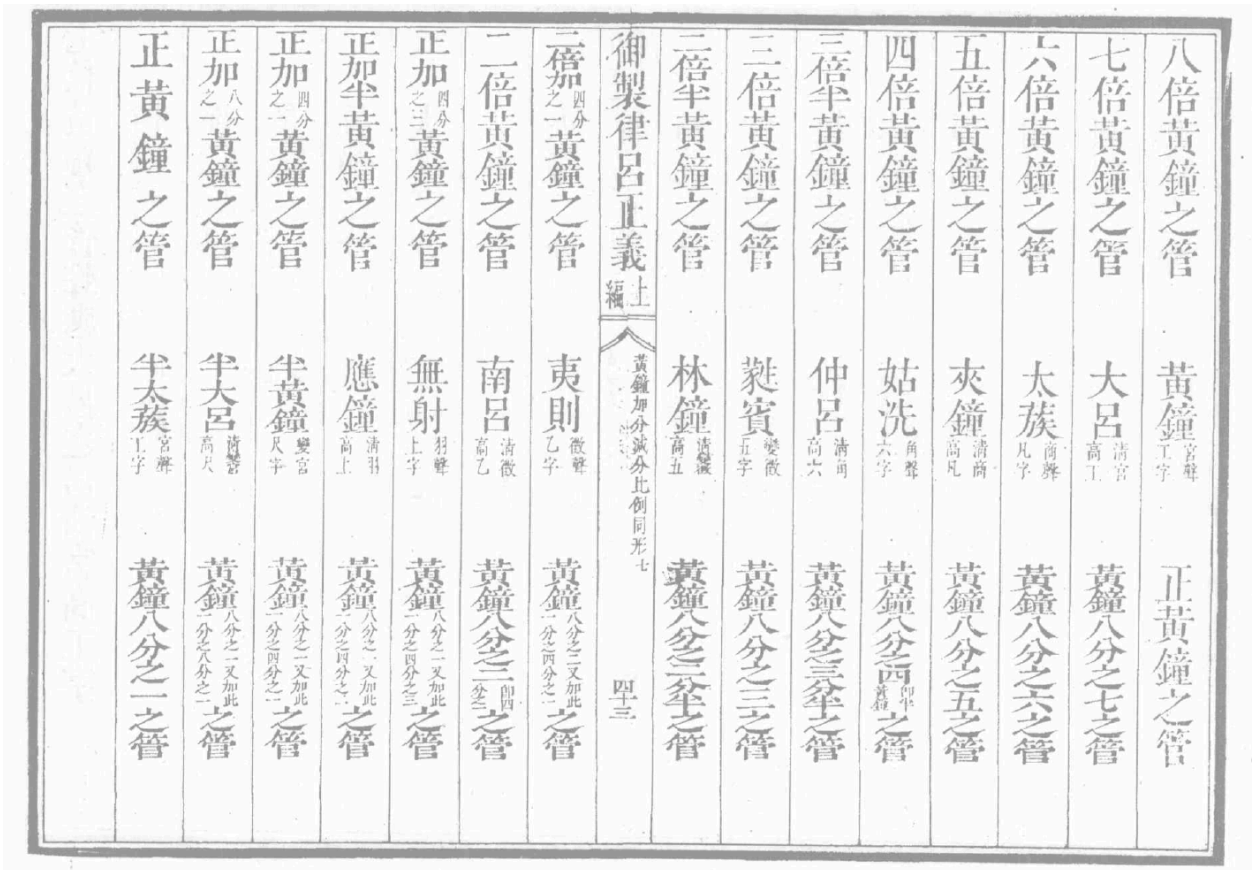
Chapter 5 computes this value rather accurately as 3.38 ancient *fen*. And as the 9 ancient *cun* length of *huangzhong* translates to 7.29 modern *cun* in Qing's metrology standards, 3.38 ancient *fen* in turn translates to 2.74 modern *fen*, or 0.274 modern *cun*.

Notably, while Chapter 6 applies the ratios of 2:3 and 4:3 following "triple division with one part subtracted or added" to the length of *huangzhong* in order to generate the lengths of the other eleven tuning pitch pipes, it maintains the same diameter—that is, 2.74 modern *fen*—for all of them; see, for example, the illustration of all twelve tuning pitch pipes at the end of Chapter 6 (see Illustration 3-2 above). Even when it comes to the *bei* 倍 "double" pipes, which are twice as long as the *zheng* 正 "original" pipes bearing the same name, and the *ban* 半 "half" pipes, which are half as long as the original pipes bearing the same name, the diameter and thus circumference of their bases remain constant, or equal to that of the original *huangzhong* pipe. Therefore, by using only *tongjing guan* "pipes of the same diameter," Chapters 1-11 of Volume 1 of *Orthodox* stay true to the notion of *jiezhu weiguan* 截竹為管 "slashing bamboo stems in order to make pitch pipes"; after all, slashing a bamboo stem into however many pipes would only produce pipes whose circumferences and diameters are identical.

And yet Chapter 12 of Volume 1 examines and makes a completely different type of pipes: *tongxing guan* 同形管, which literally means "pipes of the same shape." This *xing* 形 "shape" refers not to, say, straight pipes vs. bent or circular pipes; instead, it refers to the ratio between a pipe's length and its diameter (and thus its circumference). For example, if a double-length *huangzhong* pipe, full-length *huangzhong* pipe, and half-length *huangzhong* pipe are "pipes of the same diameter," their lengths will be 14.58 modern *cun*, 7.29 modern *cun*, and 3.645 modern *cun*, respectively, and their diameters will all be 0.274 modern *cun*. On the other hand, if these were to be "pipes of the same shape," their lengths would still be 14.58 modern

cun, 7.29 modern *cun*, and 3.645 modern *cun*, respectively, yet their diameters would no longer be identical, yet 0.548 modern *cun*, 0.274 modern *cun*, and 0.137 *cun*, respectively. In other words, whereas the constant for "pipes of the same diameter" regardless of their changing lengths is the diameter of 0.274 modern *cun*, the constant for "pipes of the same shape" regardless of their changing lengths *and* diameters (and ergo circumference) is the proportion of 7.29:0.274 between any pipe's length and diameter.

Illustration 3-4 Twenty-seven* "huangzhong" *pipes of the same shape forming two fourteen-tone octaves, collated with the fourteen original tuning pitch pipes also forming a fourteen-tone octave, in Chapter 12, Volume 1 of* Orthodox, *f. 43



Curiously, however, Chapter 12 does not create a set of twelve tuning pitch pipes "of the same shape" to contrast or complement the twelve tuning pitch pipes "of the same diameter" created and analyzed in Chapters 1-11. In fact, its decision not to do so is quite remarkable, as I will reflect in a richer context in a later chapter of this dissertation. Instead, Chapter 12 concludes

Table 3-3 Illustration 3-4 translated[70]

Pipes of the Same Shape	**Original Tuning Pitch Pipes of the Same Diameter**	**Pipes of the Same Shape**
八倍黃鐘之管 8-fold *huangzhong*	黃鐘 宮聲工字 *huangzhong-gong* [*do*]	正黃鐘之管 original *huangzhong*
七倍黃鐘之管 7-fold *huangzhong*	大呂 清宮高工 *dalü*-clear *gong* [*do*+]	黃鐘八分之七之管 7/8-times *huangzhong*
六倍黃鐘之管 6-fold *huangzhong*	太簇 商聲凡字 *taicu-shang* [*re*]	黃鐘八分之六之管 6/8-times *huangzhong*
五倍黃鐘之管 5-fold *huangzhong*	夾鐘 清商高凡 *jiazhong*-clear *shang* [*re*+]	黃鐘八分之五之管 5/8-times *huangzhong*
四倍黃鐘之管 4-fold *huangzhong*	姑洗 角聲六字 *guxian*-clear *jue* [*mi*]	黃鐘八分之四之管 4/8-times *huangzhong*
三倍半黃鐘之管 3.5-fold *huangzhong*	仲呂 清角高六 *zhonglü*-clear *jue* [*mi*+]	黃鐘八分之三分半之管 3.5/8-times *huangzhong*
三倍黃鐘之管 3-fold *huangzhong*	蕤賓 變徵五字 *ruibin-bianzhi* [*fa#*]	黃鐘八分之三之管 3/8-times *huangzhong*
二倍半黃鐘之管 2.5-fold *huangzhong*	林鐘 清變徵高五 *linzhong*-clear *bianzhi* [*fa#*+]	黃鐘八分之二分半之管 2/8-times *huangzhong*
二倍加四分之一黃鐘之管 2.25-fold *huangzhong*	夷則 徵聲乙字 *yize-zhi* [*sol*]	黃鐘八分之二又加此一分之四分之一之管 2.25/8-times *huangzhong*
二倍黃鐘之管 2-fold *huangzhong*	南呂 清徵高乙 *nanlü*-clear *zhi* [*sol*+]	黃鐘八分之二之管 2/8-times *huangzhong*
正加四分之三黃鐘之管 1.75-fold *huangzhong*	無射 羽聲上字 *wuyi-yu* [*la*]	黃鐘八分之一又加此一分之四分之三之管 1.75/8-times *huangzhong*
正加半黃鐘之管 1.5-fold *huangzhong*	應鐘 清羽高上 *yingzhong*-clear *yu* [*la*+]	黃鐘八分之一又加此一分之四分之二之管 1.5/8-times *huangzhong*
正加四分之一黃鐘之管 1.25-fold *huangzhong*	半黃鐘 變宮尺字 half-*huangzhong-biangong* [*ti*]	黃鐘八分之一又加此一分之四分之一之管 1.25/8-times *huangzhong*
正加八分之一黃鐘之管 1.125-fold *huangzhong*	半大呂 清變宮高尺 half-*dalü*-clear *biangong* [*ti*+]	黃鐘八分之一又加此一分之八分之一之管 1.125/8-times *huangzhong*
正黃鐘之管 original *huangzhong*	半太簇 宮聲工字 half-*taicu-gong* [*gong*]	黃鐘八分之一之管 1/8-times *huangzhong*

70 Besides translating Illustration 3-4, this table rotates it counterclockwise by 90 degrees, so as to better fit the typesetting standards of English prose.

Volume 1 of *Orthodox* by taking the pitch organization system proposed in Chapters 9-11 and making two gamuts of fifteen pipes of the same shape, each sonically matching a fourteen-tone octave. As shown in Table 3-3, which is rendered from Illustration 3-4 drawn from Chapter 12, the middle column featuring the original tuning pitch pipes of the same diameter runs from the full-length *huangzhong* pipe to the half-length *taicu* pipe, thus encompassing a fourteen-tone octave. Meanwhile, the column on the right begins with the same full-length *huangzhong* pipe, 7.29 modern *cun* in length and 0.274 modern *cun* in diameter. Instead of generating the shorter pipes by applying the "triple division with one part subtracted or added" method to the 7.29 *cun* length while maintaining the identical 0.338 *cun* diameter, however, the column on the right features only pipes of the same shape, so that both the length and the diameter are being slightly reduced in proportion with one another from the full-length *huangzhong* pipe as the first pipe in this column to a so-called "1/8 *huangzhong* pipe" as the fifteenth and last pipe of the column. Supposedly, according to the internal logic of the table, this 1/8 *huangzhong* pipe produces exactly an octave—a fourteen-tone octave—above the full-length *huangzhong* pipe, and thus it should sound the same as the half-length *taicu* pipe.

Notably, the name of this pipe, "1/8 *huangzhong* pipe," does not refer to its length in relation to the full-length *huangzhong* pipe. In fact, whereas the pipes of the same diameter in the middle column bear the original names of the twelve tuning pitch pipes—*huangzhong*, *dalü*, half-length *huanghzong*, half-length *dalü*, etc.—the pipes of the same shape in the column on the right, despite matching those in the middle column in pitch, are labeled as the fraction of their volume to that of the full-length *huangzhong* pipe. For example, whereas the last pipe in the middle column, half-length *taicu*, is 4/9 as long as the full-length *huangzhong* pipe and is as wide in diameter as is the latter (0.274 modern *cun*), the last pipe in the column on the right is half as

long and as wide as the full-length *huangzhong* pipe. Because a 1:2 ratio in length and in diameter (thus also in circumference) between two cylinders of the same shape necessarily translates into a 1:8 ratio in volume, the last pipe in the column on the right can only be properly called a 1/8 *huangzhong* pipe in terms of volume. Such is the case for all the columns in Table 3-3 except, of course, that the full-length *huangzhong* pipe is the first pipe both in the middle column and in the column on the right. Even though the two pipes in the middle column and the column on the right in any column sound the same, they are different pipes both in themselves and in relation to the other pipes in their respective columns. In themselves, the two pipes bear different lengths, different diameters, and different length-to-diameter ratios, albeit sounding the same pitch. In relation to other pipes in their columns, the original twelve tuning pitch pipes in the middle column keep the same 0.274 *cun* diameter throughout the changing lengths, whereas the pipes of the same shape in the column on the right keep the same 7.29:0.274 length-to-diameter ratio of the full-length *huangzhong* pipe throughout their changing lengths and diameters. Also, the column on the left of Table 3-3 features yet another gamut of fifteen pipes of the same shape. Each of these pipes bears the very same 7.29:0.274 length-to-diameter ratio, except that they are all twice as long, twice as wide in diameter, and thus accordingly eight times as large in volume as their respective pipes in the column on the right. This latter fact is reflected in the labels for these pipes: column on the right begins with the original, full-length *huangzhong* pipe and ends with the 1/8 *huangzhong* pipe as its fifteenth pipe, the column on the left begins with the "8-fold *huangzhong* pipe" and ends with the full-length *huanghzong* pipe as its fifteenth pipe. As a result, the twenty-seven pipes in the columns on the left and on the right of Table 3-3 duplicate the length-diameter ratio of the original *huangzhong* pipe in their physical dimensions

and match the pitch patterns of the original twelve tuning pitch pipes of the same diameter in the middle column, encompassing two fourteen-tone octaves.

Now, in some sense, all these twenty-seven pipes of the same shape can be referred to as "*huangzhong* pipes," and they are labeled as *huangzhong* pipes, albeit each prefixed with the fraction of their volume to that of the original, full-length *huangzhong* pipe, from "8-fold" to "1/8." They are all "*huanghzong* pipes" in that they all bear the same length-to-diameter ratio as the original *huangzhong* pipe, though in twenty-seven different orders of magnitude. And in fact, Chapter 12 concludes by taking each and every one of these twenty-seven "*huangzhong*" pipes and doing to each of them what Chapters 1-11 have done to the original full-length *huangzhong* pipe: it applies the "triple division with one part subtracted or added" procedure to each of their varying lengths while keeping constant each of their varying diameters. It thus generates for each of these twenty-seven "*huangzhong* pipes" eleven other pipes of the same diameter, whose length proportions to that *huangzhong* pipe is the same compounding ratios of 2:3 and 4:3 as are those of the eleven original tuning pitch pipes to the original *huangzhong* pipe. What results, then, is a total of 324 pipes (12 times 27 equals 324), which Chapter 12 presents through twenty-seven tables, each table featuring twelve pipes of the same diameter.[71] Thus concludes Volume 1 of *Orthodox*, with this interlocking system of 324 pipes that would in turn be used to tune various musical instruments in later parts of the treatise.

The Phantom Treatise of 1707

As part of the much larger treatise *Origins of Cosmological Sciences*, *Orthodox Meaning of Pitch Pipes* does not have its own preface or postface. Instead, of its three constitutive parts—

71 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Volume 1, ff. 44-72.

Volumes 1 and 2 “Rectifying the Pitch Pipes in order to Examine their Tones,” Volumes 3 and 4 “Harmonizing the Notes in order to Establish the Institution of Music,” and Volume 5 “Coordinating the Note-Series in order to Compose Arias”—only the last part, Volume 5, has any paratext, here specifically in the form of a preface, which I will examine in the next chapter. The only summary of the content of *Orthodox* available in *Origins*, particularly the content of the four volumes, is the aforementioned preface to the entire *Origins* treatise penned by the Yongzheng Emperor upon its formal publication in 1723. And even though the preface summarizes each of the three parts or *bian* “series” of *Orthodox*, the summaries hardly add any new information beyond the titles themselves.

As it turns out, however, there is available an exceptionally detailed summary of Volume 1 of *Orthodox*, which, as I showed above, lays out the entire fourteen-tone temperament reform from the absolute and relative sizes of the twelve tuning pitch pipes to a new pitch organization system of two complementary diatonic *yun*’s forming a fourteen-tone octave. This summary found nowhere in *Origins* but instead in f. 119 of the NLC manuscript *Putong Guji* No. 15251, in the second half of the supposed postface to the treatise *A Glimpse through the Pitch Pipes*.

> 伏睹：上卷《本源考正》，則定黍尺、辨管長、研損益、考算積。不恥下問，俾參末議。至若黃鍾九寸，在今尺止七寸二分九釐。又，仲呂轉生，積至四十八律，則黃鍾全成大呂，而知京房六十律之故；皆特出睿見推定，非末學荒陋所能贊一辭也。
>
> 下卷《制器審音》，則截竹為管、絜短長、驗圍徑、聽聲音。或同徑而異形，或同形而異徑，由倍至半，由數倍以至幾何之幾。定律呂五聲二變，知工字即宮聲，凡字即商聲，六字即角聲之類，而證俗譜、古譜之皆非。用律呂實積、密律、比例，製為簫、笛、笙、簧、琴、瑟，應乎皆合，而知昔人用角八倍縱長之非。又，律倡呂和，共叶一聲，而知昔人每聲獨用一管之非。又，一均五聲，陽律隨陽，陰呂隨陰，而知黃鍾均以呂隨律，用林鍾、南呂為徵、羽，大呂均以律隨呂用，夷則、無

射為徵、羽之非。又，半律之聲，不應全律，而為變聲，以成旋宮，而知昔人半律與全律相應，不能成旋宮之非。[…][72]

Prostrating, I have read: the first volume entitled "Investigating and Rectifying Essence and Origins" determines the length measurements of grains of millet, distinguishes the lengths of pipes, studies the method of "with one part subtracted or added," and investigates the calculations of volumes. [His Imperial Highness] is not ashamed to seek answers from those inferior to him, and he takes advice from the humble and debates with the lowly. As for the length of the *huangzhong* pipe being 9-*cun*-long [in ancient measurements of length], this translates to merely 7 *cun* 2 *fen* and 9 *li* in modern measurements of length. What is more, if one continues to generate [new pipes through 2/3 and 4/3 proportions] from *zhonglü* [roughly equivalent to E#], after forty-eight more pipes, one will have started from the *huangzhong* pipe [C] and ended with the *dalü* pipe [C#], and thus the reason for Jing Fang [77-37 BCE]'s system of sixty pitch pipes can be known; this was all deduced by the Wisdom [of His Imperial Highness], and it is not a subject to which an unlearned or ignorant person could add anything.

The second volume entitled "Establishing Instruments in order to Examine their Tones" [*shenyin*] slashes bamboos into pipes, measure their lengths, verifies their circumferences and diameters, and listens to their sounds [*sheng*] and tones [*yin*]. Some of these pipes have the same diameter yet different shapes, while some have the same shape [i.e. length-to-diameter ratio] yet different diameters, from pipes whose lengths are twice as long [as the original twelve tuning pitch pipes] to those that are half as long, and from those that are several times as large to those that are a fraction of a fraction as large. In determining the "five proper notes and two altered notes" of the tuning pitch pipes, one finds that the *gong* syllable is really just the *gong* note [*do*], the *fan* syllable is really just the *shang* note [*re*], the *liu* syllable is just the *jue* note [*mi*], and so on, revealing that both the popular and the [supposedly] ancient notation systems are wrong. When the volumes, circumference-to-diameter ratios, and proportions of the tuning pitch pipes are used to make end-blown pipes [*xiao*], flutes [*di*], mouth organs [*sheng*], pipe reeds [*huang*], seven-string zithers [*qin*], and twenty-five-string zithers [*se*], [the resulting instruments] all match and correspond to [instruments in actual use], revealing that past scholars who took the length of the eight-times-long *jue* [*mi*] pipe [as a standard] were wrong. What is more,

72 National Library of China, *Putong Guji* No. 15251, f. 119v.

> each odd-numbered pipe is paired with an even-numbered pipe and they collectively match one note [*sheng*], revealing that past scholars were wrong in using only one pipe for each note. Moreover, in each note-series [*yun*] comprising five proper notes, the odd-numbered pipes are grouped together and the even-numbered pipes are grouped together, revealing that it is wrong to use even-numbered pipes for the note-series based on *huangzhong* [roughly C], such as *linzong* [G] as *zhi* [*sol*] and *nanlü* [A] as *yu* [*la*], and wrong to use odd-numbered pipes for the note-series based on *dalü* [C#], such as *yize* [G#] as *zhi* and *wuyi* [A#] as *yu*. Furthermore, the note of the half-length pipe does not correspond to that of the full-length pipe, yet it becomes the matching *biangong* note [of the full-length pipe] and thus rotates the *gong*, revealing that past scholars were wrong to posit that the full-length and the half-length pipes correspond to each other so that they cannot rotate the *gong* note anymore. […]

Even the two-volume structure of *A Glimpse through the Pitch Pipes* described in this postface resembles that of Volume 1 of *Orthodox* inasmuch as Chapters 1-8 focus on *shu* "numbers" by "rectifying the pitch pipes" through historical data and mathematical computations, whereas Chapters 9-12 focus on *sheng* "sounds" by "examining the tones" of pitch pipes through listening experiments that in turn churn out a system of pitch organization. Indeed, not only does the first volume of *A Glimpse through the Pitch Pipes* focus on reconstructing the absolute and relative sizes of the twelve tuning pitch pipes, it even mentions what is arguably the most important finding in Volume 1 of *Orthodox* regarding "rectifying the pitch pipes": identifying the conversion rate between ancient *cun* and modern *cun* as 9 ancient *cun* equals 7.29 modern *cun*. Indeed, this conversion rate proposed in Chapter 4, Volume 1 of *Orthodox* is essentially not found in any other studies on *lülü* "tuning pitch pipes" or musical tuning, from Cai Yuanding's 12th-century *New Treatise on Pitch Pipes* to Zhu Zaiyu's 16th century *Essential Meaning of Pitch Pipes*.

The second volume of *A Glimpse through the Pitch Pipes* parallels Chapters 9-12 of Volume 1 of *Orthodox* even more closely. Not only is the title of this volume, *zhiqi shenyin* 制器

審音 "Establishing Instruments in order to Examine their Tones," effectively identical to the title of Volumes 1-2 of *Orthodox*, *zhenglü shenyin* 正律審音 "Rectifying the Pitch Pipes in order to Examine Their Tones," with the emphasis on *shenyin* "examining the tones," but it also focuses on the questions of pitch organization—the *sheng* "notes," *yun* "note-series," *zi* "characters" or *gongche* notation syllables, and *xiangying* 相應 "mutual correspondence" or octave equivalence. Virtually every major or idiosyncratic finding of Chapters 9-12, Volume 1 of *Orthodox* is found in the summary of the second volume of *A Glimpse of Pitch Pipes* in the above postface. Proceeding in the order of the postface, just as the concluding Chapter 12 of Volume 1 of *Orthodox* presents an entire set of three-hundred and twenty-four tuning pitch pipes by multiplying twenty-seven sets of twelve pipes of the same diameter with twelve sets of twenty-seven pipes of the same shape, the postface mentions that some of the pipes studied in *A Glimpse through the Pitch Pipes* are pipes of the same diameter while others are pipes of the same shape. Just as the largest pipe computed in Chapter 12 is the 8-fold *huangzhong* pipe (twice as long and twice as wide in diameter as the full-length *huangzhong* pipe) and the smallest pipe the 1/8 *yingzhong* pipe (one-eighth as long as the original full-length *yingzhong* pipe and 1/8 as wide in diameter as the original full-length *huangzhong* as well as *yinzhong* pipes), the postface mentions that the pipes studied in *A Glimpse through the Pitch Pipes* range from the double-length to the half-length and from the several times as large in volume to several times as small.

Next, just as Chapter 10 through experiments matches the pitch of the full-length *huangzhong* pipe being the *gong* 宮 note in the *yun* of odd-numbered pitch pipes with the pitch of the fingering of the *gong* 工 syllable on the *xiao* end-blown pipe, the postface argues that "the *gong* syllable is really just the *gong* note" when matching "the five proper notes and two altered notes" with the *gongche* syllables. Chapter 9 proposes a pitch organization system in which the

odd-numbered pipes form one diatonic *yun* and the even-numbered ones form a complementary, non-overlapping *yun*. As a result, every odd-numbered pipe and its adjacent even-numbered pipe down in the direction of decreasing length necessarily bear the same note in their respective *yun*'s: both *huangzhong* (1st) and *dalü* (2nd) are *gong* in their own *yun*, both *taicu* (3rd) and *jiazhong* (4th) are *shang*, and so on. Similarly, the postface argues that "in each note-series [*yun*] comprising five proper notes, the odd-number pipes are grouped together and the even-number pipes are grouped together," thus segregating the odd-numbered *yang* pipes and even-numbered *yin* pipes into two distinct *yun*'s. In contrast, the conventional pitch organization system that mixes the odd-numbered with the even numbered—such as *huanghzong* (1st) as *gong* yet *linzhong* (8th) as *zhi* in the same *yun*—violates this *yang*-*yin* symmetry. Furthermore, because, every two pipes are assigned the same note in their respective *yun*'s, beginning with *huangzhong*, conversely, each of the seven notes is imposed onto two pipes, no more and no less. Thus, as the postface writes, "each odd-numbered pipe is paired with an even-numbered pipe and they collectively match one note."

And, finally, in what is perhaps the "crown jewel" of Chapters 9-12 of Volume 1, *Orthodox*, if not even of the entirety of *Orthodox*, Chapter 9, claims to have heard an octave between full-length *huangzhong* and half-length *taicu*, or indeed between any one pipe and the fourteenth pipe down the direction of the decreasing order of length. As a result, the octave displacement of any note would necessarily move it to a pipe bearing a different name, e.g. *gong* from full-length *huangzhong* to half-length *taicu*, which Chapter 9 further takes as the true meaning of *xuangong* "rotating the *gong* note." In addition, whereas both full-length *huangzhong* and half-length *taicu* pipes, given their octave correspondence, bear the *gong* note, the half-length *huangzhong*, being the thirteenth of all the tuning pitch pipes or the seventh of the odd-

numbered pipes, necessarily bears the *biangong* note, which is the note one step lower than *gong* in "the five proper notes and the two altered notes." Thus, halving the length of any pitch pipe would not produce a pitch that sounds an octave above, but would produce one that is the *biangong* to the *gong* of the pitch of the original pipe. Accordingly, the postface writes: "the note of the half-length pipe does not correspond to that of the full-length pipe, yet it becomes the matching *biangong* note [of the full-length pipe] and thus rotates the *gong*."

Before concluding this cross-comparison of the postface to *A Glimpse through the Pitch Pipes* with *Orthodox*, I note that the postface speaks of using the twelve tuning pitch pipes as the standards for making instruments. Here, notably, the pitch pipes function not only as tuning standards to whose pitches the sounds of the instruments must be adjusted in order to make them accord, but also as standards of length, as if they were themselves rulers and yardsticks. Thus, the postface speaks of making *xiao* end-blown pipes, *di* flutes, *sheng* 笙 mouth organs, *huang* 簧 reeds, the seven-string *qin* zither, and the twenty-five-string *se* zither specifically with the *shiji* 實積 "concrete volumes," *milü* 密律 "precise algorithms," and *bili* 比例 "proportions" of the twelve tuning pitch pipes. And indeed, Volumes 3 and 4 of *Orthodox* design those very same instruments whose various notes are tuned to certain ones of the three-hundred and twenty-four pitch pipes established in Chapter 12, and whose tangible physical sizes are also measured in reference to the lengths of certain ones of those pitch pipes.

Therefore, although the postface of *A Glimpse through the Pitch Pipes* in f. 119 of *Putong Guji* No. 15251 is explicitly dated to the summer of 1707, it already summarizes virtually all the key materials and findings that would constitute Volume 1 of *Orthodox*, the foundation of the treatise's proposal for a fourteen-tone temperament, whose compilation would not start until more than six years later. While the treatise *A Glimpse through the Pitch Pipes* itself has not

survived, or may not have ever been written at all, I have discovered that the thirty-five folios immediately preceding the postface in f. 119 of *Putong Guji* No. 15251 contain what appear to be notes, tables, and diagrams whose content perfectly matches that of the postface and thus also that of *Orthodox*. Folios 84-118 do not bear any title—neither *A Glimpse through the Pitch Pipes* nor *Orthodox Meaning of Pitch Pipes*—and in fact do not even feature any significant amount of prose at all, but I argue that these were precisely the draft materials intended for compiling *A Glimpse through the Pitch Pipes* which, whether ever compiled or completed, ended up becoming the very foundation for *Orthodox Meaning of Pitch Pipes*.

Below is a summary of content of ff. 84-119 of *Putong Guji* No. 15251. Although these folios do not feature any table of contents, here, I use headings that are found in the actual manuscript. Most of these headings are written either on the top-right corner of their respective folios or on the right side of the folding edge of a wrapped-back page. Many are also written in vermilion pigment, which I have indicate below in red:

f. 84r	表
f. 84v	[Outline of findings]
ff. 85-90	同徑異形
ff. 91-95	同形
ff. 96-99	笙
ff. 100-101	管
ff. 102-105	琴
ff. 106-107	瑟
ff. 108	排簫
ff. 109-116 *bis*	簫笛
f. 117	[left blank]
f. 118	奉先殿鐘磬
f. 119	律呂後序

f. 84r	*biao* “Supplication”
f. 84v	[Outline of findings]
ff. 85-90	[Pipes of] the same diameter and varying shapes
ff. 91-95	[Pipes of] the same Shape

ff. 96-99	*Sheng* mouth organs
ff. 100-101	*Guan* end-blown pipes
ff. 102-105	Seven-string *qin* zithers
ff. 106-107	Twenty-five-string *se* zithers
ff. 108	Pan-flutes
ff. 109-116	*Xiao* end-blown pipes and *di* flutes
f. 117	[left blank]
f. 118	Bells and chimes at Hall to the Imperial Ancestors
f. 119	Postface to *[A Glimpse through the] Pitch Pipes*

Even from this patchy summary, it is clear that ff. 84-118 of *Putong Guji* No. 15251 contain the very preparatory materials for the treatise as described in the postface in f. 119 and thus indeed for *Orthodox Meaning of Pitch Pipes*. At the outset, these folios can be divided into two parts: the first part, ff. 84-95, concern the tuning pitch pipes, whereas ff. 96-118 concern a variety of musical instruments. Of course, a bipartite structure that first constructs a standard of tuning through pitch pipes and then applies it to making all classes of instruments is not at all limited to the *Orthodox*. Nonetheless, other than *xun* 壎, a type of oval-bodied vessel flute made from earth, and *chi* 篪, a type of traverse flute that differs from *di* in having a closed end, all the different types of pitched instruments (that is, excluding *gu* 鼓 "drums" and the likes) found in Volumes 3 and 4 of *Orthodox* are also studied in ff. 96-118 of the manuscript. Notably, most of the individual instruments did not end up in *Orthodox*, as their measurement specifications differ notably from their closest corresponding counterparts in the latter treatise. And since ff. 98-99, 101, 102-103, 106, and 110-115 are all dedicated to mathematical reckonings for the lengths of strings, dimensions of sound boards, lengths and diameters of various wind instruments, and distances between their fingerholes, the tables and diagrams of musical instruments in ff. 96-118 are more likely new designs of instruments whose measurements need to be computed, rather than descriptions of ones that were in use at the time. The only exception appears to be the *bianzhong* "arranged bells set" and *bianqing* "arranged chimes set" featured in f. 118, which, as

indicated in its heading, describes instruments currently in use at the Hall of Imperial Ancestors (*fengxian diao* 奉先殿), the family ancestral shrine of the Qing's ruling Aisin-Gioro clan located in the inner quarters of the Forbidden City.

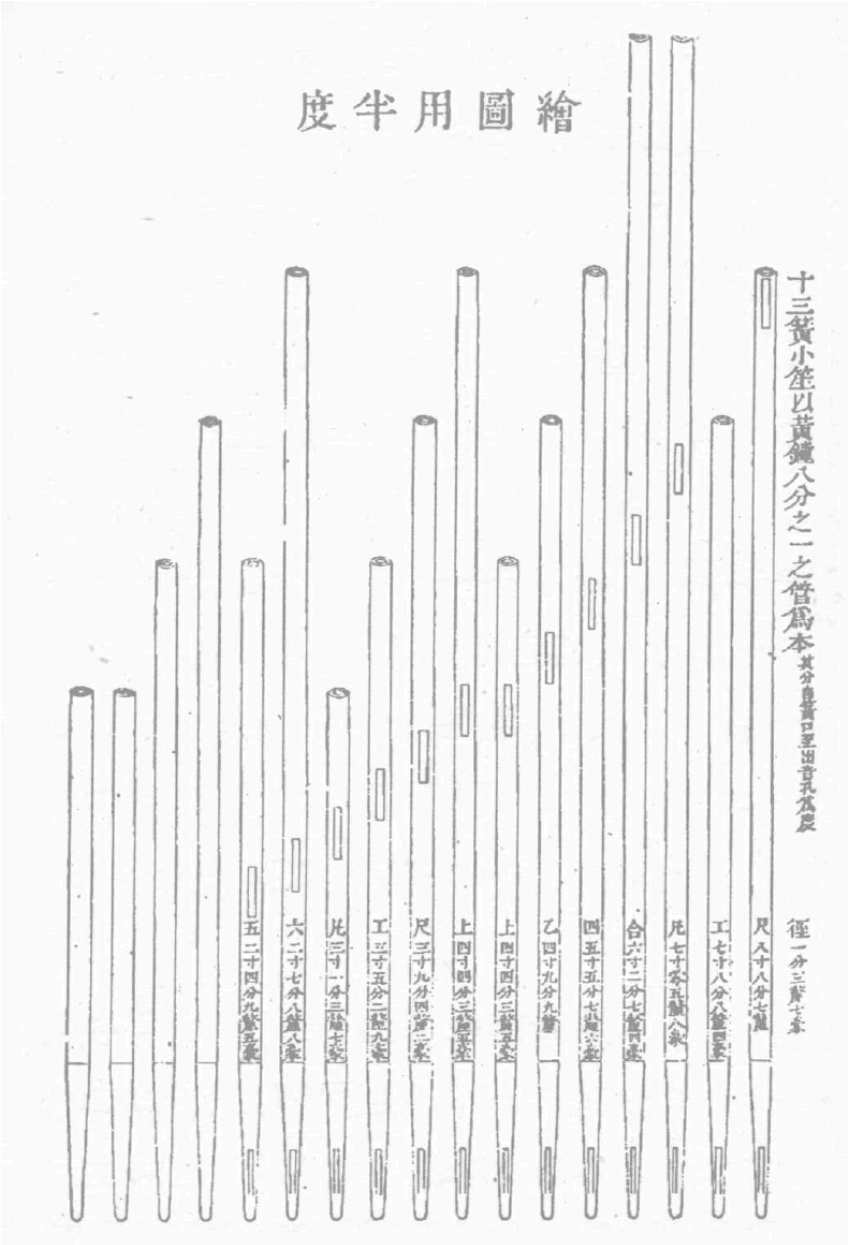
Table 3-4 Two small mouth organs of thirteen reed-pipes from* Putong Guji *No. 15251 and* Orthodox, *the length of their pipes compared

Pipe Length in *Putong Guji* No. 15251, f. 97*bis*-v	Pipe Length in Chapter 5, Volume 3 of *Orthodox*, f. 51*bis*-2
	8.87 *cun*
7.88 *cun*	7.884 *cun*
7.04 *cun*	7.058 *cun*
6.26 *cun*	6.274 *cun*
5.56 *cun*	5.576 *cun*
4.98 *cun*	4.99 *cun*
4.42 *cun*	4.435 *cun*
4.18 *cun*	4.435 *cun* [*sic*]
3.98 *cun*	3.942 *cun*
3.73 *cun*	[N/A]
3.52 *cun*	3.529 *cun*
3.13 *cun*	3.137 *cun*
2.76 *cun*	2.788 *cun*
2.49 *cun*	2.49 *cun*

Still, the discrepancies between *Putong Guji* No. 15251 and *Orthodox* in terms of instrument designs might only mean that the former served as a basis for In-c'i's later research in compiling *Orthodox* and refurbishing the Qing court's instruments without becoming the actual research in and of itself. A few instruments newly designed in ff. 96-118 of *Putong Guji* No. 15251 did anticipate their corresponding ones in *Orthodox*. Folio 97*bis*-v, for example, features the diagram of a small mouth organ of thirteen reed-pipes (*shisanhuang xiaosheng* 十三簧小笙). The lengths and *gongche* syllables of each pipe are identical—except for two pipes and the last digit of some pipe lengths—to those of the same instrument featured in Volume 3 of *Orthodox* (see Table 3-4 comparing the small mouth organs of thirteen reed-pipes in the two texts, as well

as Illustration 3-5 of the mouth organ from *Orthodox*), which features three additional mouth organs of different numbers and lengths of pipes.[73]

Illustration 3-5 The pipes of the small mouth organ of thirteen reed-pipes in Chapter 5, Volume 3 of* Orthodox, *f. 51*bis-*2



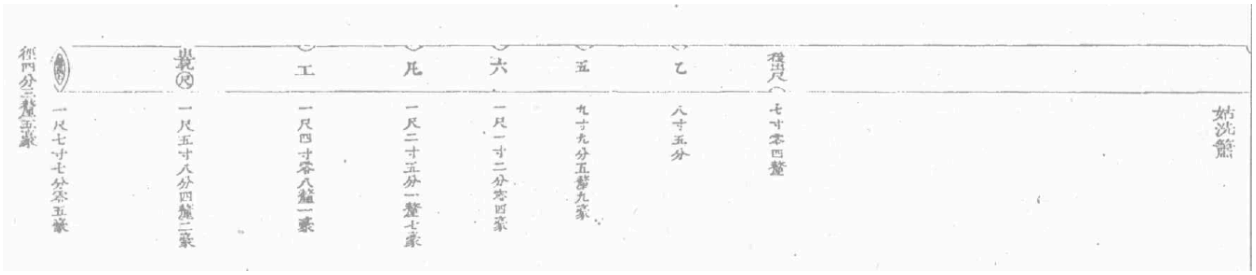
73 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, Volume 3, f. 51*bis*.

Similarly, f. 109 features the diagrams of an "end-blown pipe of the 4-fold *huangzhong* pipe" (*sibei huanghzong xiao* 四倍黃鐘簫) and "a traverse flute of the 4-fold *huangzhong* pipe" (*sibei huangzhong di* 四倍黃鐘笛). The lengths, diameters, and fingerhole configurations of both these two wind instruments are identical to their counterparts in Volume 3 of *Orthodox* (see Tables 3-5 and 3-6 comparing each instrument in the two respective texts, as well as Illustrations 3-6 and 3-7 of the two instruments in *Orthodox*), which also features three other designs of end-blown pipes and one other design of a flute, all of different lengths and diameters.

Table 3-5 Two* xiao *end-blown pipes from* Putong Guji *No. 15251 and* Orthodox, *the configurations of their fingerholes compared

Distance of fingerholes in *Putong Guji* No. 15251, f. 109 Length: 17.7 *cun* Diameter: 0.4355 *cun*	Distance of fingerholes in Chapter 3, Volume 3 of *Orthodox*, f. 27*bis*-2 Length: 17.705 *cun* Diameter: 0.435
15.84 *cun*	15.842 *cun*
14.08 *cun*	14.081 *cun*
12.51 *cun*	12.517 *cun*
11.21 *cun*	11.204 *cun*
9.96 *cun*	9.959 *cun*
8.39 *cun*	8.5 *cun*
7.04 *cun*	7.04 *cun*

Illustration 3-6 The* xiao *end-blown pipe in Chapter 3, Volume 3 of* Orthodox, *f. 27*bis-*2



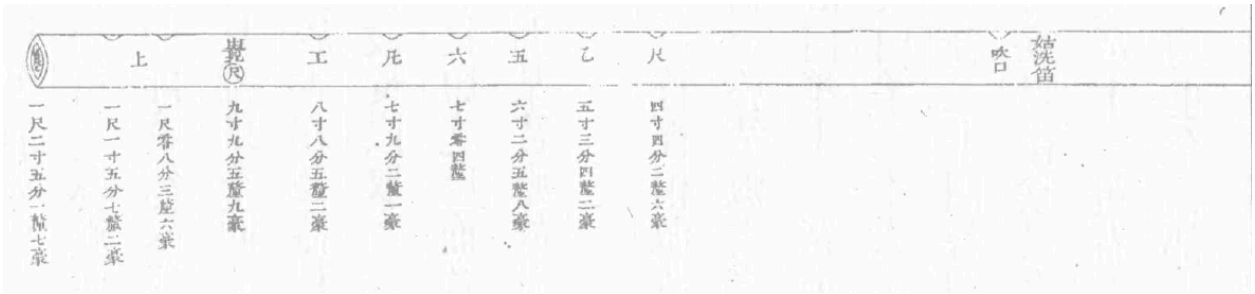
Notably, f. 109 already labels its designs for the end-blown pipe and the traverse flute mentioned above in reference to a certain "4-fold *huangzhong*" pipe. The name of this latter pipe complies with the rather unique naming patterns of those twenty-seven pipes of the same shape

computed and presented in Chapter 12, Volume 1 of *Orthodox*, pipes whose lengths and diameters all differ yet whose length-to-diameter ratio is all identical and equal to that of the original, full-length *huangzhong* pipe. But upon closer examination, although the "end-blown pipe of the 4-fold *huanghzong* pipe" and the "traverse flute of the 4-fold *huangzhong* pipe" have different lengths, they share the same diameter, which is specified in f. 109 as 4.355 *fen* and thus exactly identical (except for the last decimal) to the "4-fold *huangzhong* pipe" in Chapter 12, Volume 1 of *Orthodox*: that is, a pipe whose length-to-diameter ratio is the same as that of the original full-length *huangzhong* pipe and whose volume (*not* length or diameter) is four times as large as that of the said *huangzhong* pipe.

Table 3-6 Two* di *traverse flutes from* Putong Guji *No. 15251 and* Orthodox, *the configurations of their fingerholes compared

Distance of fingerholes in *Putong Guji* No. 15251, f. 109 Length: 12.5 *cun* Diameter: 0.4355 *cun*	Distance of fingerholes in Chapter 4, Volume 3 of *Orthodox*, f. 36*bis*-1 Length: 12.517 *cun* Diameter: 0.435
11.57 *cun*	11.572 *cun*
10.83 *cun*	10.836 *cun*
9.96 *cun*	9.959 *cun*
8.85 *cun*	8.852 *cun*
7.92 *cun*	7.921 *cun*
7.04 *cun*	7.04 *cun*
6.25 *cun*	6.258 *cun*
5.29 *cun*	5.342 *cun*
4.42 *cun*	4.426 *cun*

Illustration 3-7 The* di *traverse flute in Chapter 4, Volume 3 of* Orthodox, *f. 36*bis-*1



This identity between the diameter of the two "4-fold *huangzhong*" wind instruments and that of the 4-fold *huangzhong* pitch pipe from *Orthodox* offers two critical clues. First, *Putong Guji* No. 15251 adopts exactly the same conversion rate between the supposedly ancient units of length and the Qing's modern units. Since the critical piece of historical data —"the *huangzhong* pipe is 9 *cun*, and its hollow enclosure is 9 *fen*"—is recorded under a defunct if not truly "ancient" system of metrology, it is only by adopting the same 7.29 modern *cun* = 9 ancient *cun* conversion rate stipulated in Chapter 4, Volume 1 of *Orthodox* that *Putong Guji* No. 15251 could have possibly arrived at the same value as would *Orthodox* for what is effectively $\sqrt[3]{4}$ times the diameter of the original, full-length *huangzhong* pipe. Second, when the compilers of the *Putong Guji* No. 15251 manuscript began working on their designs for the instruments, they must have already established the very same system of twelve tuning pitch pipes, the very same pitch organization system of the fourteen-tone octave, and the very same system of 324 pipes interlocking twenty-seven series of twelve pipes of the same diameter with twelve series of twenty-seven pipes of the same shape or length-to-diameter ratio, all presented in Volume 1 of *Orthodox*. Without any one of these systems, labeling a pipe as "4-fold *huangzhong*" in reference to the volume as opposed to any one-dimensional size of the pipe would not have even made any sense, let alone led to arriving at the same results in calculating the diameter of a pipe that (1) has the same length-to-diameter ratio as does one whose length is 9 *cun* long and the surface area of whose base is 9 *fen*, and (2) is four times as large in volume as is such a pipe. To reiterate my point from the earlier comparison of the postface in f. 119 to the content of Volume 1 of *Orthodox*, the conversion rate of 9:7.29 between ancient and modern units of length, the pitch organization system based on a fourteen-tone octave divided into two complementary diatonic *yun*'s, the gamut of twenty-seven "*huangzhong*" pipes of the same shape whose pitches

encompass two fourteen-tone octaves, and the entire system of 324 pipes interlocking twelve pipes of the same diameter with twenty-seven pipes of the same shape are all absolutely unique to *Orthodox*. In other words, it appeared as though, as many as seven years before In-c'i and his team submitted the completed *Orthodox* for imperial approbation, those who worked on the instrument designs in ff. 96-118 of the NLC manuscript *Putong Guji* No. 15251 already knew of the treatise's key findings.

As it turns out, this was exactly the case, as proven by ff. 84-95 of the same manuscript, immediately preceding the organological designs. As shown in my above outline of the headings of ff. 84-119 of *Putong Guji* No. 15251, ff. 85-95 concerning the tuning pitch pipes consist of two parts. The first part, ff. 85-90, has the heading *tongjing Yixing* 同徑異形 "[pipes of] the same diameter and varying shapes" written in vermilion on every single one of these folios, whereas the second part, ff. 90-95, has the word *tongxing* 同形 "[pipes of] the same shape" in the headings of every single folio, also written in vermilion pigment; for the second part. This bipartite structure mirrors that of Volume 1 of *Orthodox*, of which Chapters 1-8 focus on computing the original twelve tuning pitch pipes of the same diameter, whereas Chapters 9-12 first construct the pitch organization system of the fourteen-tone octave and then compute a novel gamut of twenty-seven "*huangzhong*" pipes of the same shape to encompass two such octaves, before combining these two different types of pipes into a whole system of 324 pipes. While there is no prose text whatsoever in ff. 85-95, f. 84v features what may best be described as a summary of key findings for the entirety of ff. 85-118 of the *Putong Guji* No. 15251 manuscript. For the sake of legibility, I have rendered the passage as a numbered list:

1. 今尺累縱黍一百，古尺當今尺八寸一分。
2. 黃鍾之長，古尺九寸，在今尺為七寸二分九釐。
3. 黃鍾實積，古尺八百一十分，今尺為四百三十分有奇。

4. 十二律圍徑皆同，亦依今尺之數。不用班氏、諸家之說。
5. 十二律呂相生，皆用實數密律損益，不拘舊數。
6. 七聲字譜不依十二律應配，以便轉旋。如黃鍾為宮，時譜合字，古譜上字，今宜為工字。不可極定。
7. 制樂器，皆用各律實積比例推算。如小笙用黃鍾四之一；排簫用十二律全數；笛用黃大四倍；簫用姑仲八倍之類。
8. 古人旋宮，各律只用一管起宮，其餘七聲皆然。今以一律一呂兩管相兼而用，使一均之中，眾律咸備。
9. 變宮、變徵，古律逼近宮、徵正聲，致簫管布孔，長從不勻。今既兩管相兼，亦取正、變勻列，以免凌混。
10. 舊定八十四聲，往而不返。今別依字譜，使八十四聲運于十二管七周而復，且倍半迭用，升降轉旋自然，變化相和。[74]

1. One hundred grains of millet arranged in one straight line equal 1 modern *chi* [10 modern *cun*], and the ancient *chi* [10 ancient *cun*] equals 8 *cun* 1 *fen*.
2. The length of the *huanghzong* pipe is 9 ancient *cun* and 7 modern *cun* 2 *fen* 9 *li*.
3. The volume of *huangzhong* is 810 ancient *fen* and 430 and more modern *fen*.
4. The twelve tuning pitch pipes all have the same base enclosure and diameter, and they are also measured according to modern standards. The theories of Ban Gu and others are not to be followed.
5. The mutual generations of the twelve tuning pitch pipes all use concrete numbers and precise proportions for “one part subtracted or added” and are not fixated on old calculations.
6. The *gongche* notation of the seven notes is not assigned in synchronization with the twelve tuning pitch pipes, in order to facilitate [modal] mutations and transpositions. For example, when *huangzhong* is *gong* [*do*], the current *gongche* notation would solmize it as *he* [*liu*], the ancient *gongche* notation would solmize it as *gong*. This cannot be inflexible.
7. When making musical instruments, they are all computed from the volumes and proportions of the various pitch pipes. For example, the small mouth organ uses the 1/4 *huangzhong* pipe as its basis; the pan-flute uses all of the twelve tuning pitch pipes; traverse flutes use the 4-fold *huangzhong* and 4-fold *dalü* pipes; end-blown pipes use 8-fold *guxian* and 8-fold *zhonglü* pipes, etc.

74 National Library of China, *Putong Guji* No. 15251, f. 84v.

8. When the ancients rotate the *gong* note, among all the pitch pipes, only one pipe is used as *gong*, and the same is true for all the seven notes. Now, one odd-numbered pipe and one even-numbered pipe are used simultaneously, so that all the pipes are used within one *yun*.
9. Within the ancient [use of] pitch pipes, the *biangong* [*ti*] and *bianzhi* [*fa#*] notes are exceedingly close to the proper *gong* [*do*] and *zhi* [*sol*] notes, so that the fingerholes of end-blown pipes and flutes are not even. Now, given that two pitch pipes share one note, these proper and altered notes are evenly distributed in order to avoid chaos and confusion.
10. In the old system, the eighty-four transposed notes cannot return to where it has begun. Now, using the *gongche* syllables, the eighty-four transposed notes are embedded in the seven cycles of the twelve tuning pitch pipes, and the double-length pipe and the half-length pipe are used alternately, the rising and lowering and the mutations and transpositions naturally take place, and transformations will correspond to each other.

Compared even to the summary of the supposed content of *A Glimpse through Pitch Pipes* in the postface in f. 119, this enumeration of findings does not feature a laser-sharp focus on the most pivotal or idiosyncratic interventions of the fourteen-tone temperament tuning reform. Still, and yet again, virtually all matters that would be discussed in Volume 1 of *Orthodox Meaning of Pitch Pipes* are featured in this list. To begin with, items 1-3 of the list bespeak of the reconstruction of the supposedly ancient measuring standards and the various physical dimensions of the *huangzhong* pipe. Here, in complete parallel to Chapters 1-4 of Volume of *Orthodox*, the conversion rate of 10 ancient *cun* to 8.1 modern *cun* is established (item 1), so that the length of *huangzhong* pipe, being 9 ancient *cun* is established as 7.29 modern *cun* (item 2). Next, items 4-5 concern the lengths of the other eleven tuning pitch pipes computed in proportion to the length of the *huangzhong* pipe. And while the embrace of the Chinese Pythagorean "triple division with one part subtracted or added" could be more conventional, the clarification in item 4 that all these twelve pipes have the same enclosure and

diameter hints that the distinction between pipes of the same diameter and pipes of the same shape would become important.

Granted, the list of the findings does not explicitly feature the twenty-seven "*huangzhong*" pipes whose length-to-diameter ratio is the same as the original full-length *huangzhong* pipe and whose pitches encompass two fourteen-tone octaves. Nonetheless, item 7 speaking of making various wind instruments from *sheng* or mouth organs to *di* or traverse flutes mentions several ones of the 324 pipes that multiply the original twelve tuning pitch pipes of the same diameter with the twenty-seven pipes of the same shape: the "1/4 *huangzhong*" (*huangzhong sizhiyi* 黃鐘四之一) pipes, whose length-to-diameter ratio is the same as the original *huangzhong* pipe and whose volume is 1/4 of the latter pipe; the "4-fold *huangzhong* and *dalü*" (*huangda sibei* 黃大四倍), whose length-to-diameter ratios are the same as the original *huangzhong* and *dalü* pipes and whose volumes are 4 times those latter two pipes respectively; and the "8-fold *guxian* and *zhonglü*" (*zhonggu babei* 仲姑八倍) pipes whose length-to-diameter ratios are the same as the original *guxian* and *zhonglü* pipes and whose volumes are 8 times those latter two pipes, respectively. Clearly, by saying that "all [musical instruments] are computed from the volumes and proportions of the various pitch pipes," item 7 refers to using the total system of 324 pipes mirroring that of Chapter 12, Volume 1 of *Orthodox* as a measurement as well as a tuning standard for making instruments, indeed anticipating the organological ff. 96-118 of the *Putong Guji* No. 15251 that I have analyzed above.

The remaining items 6 and 8-10 all concern the pitch organization system. Here, again, the list may seem to be demurring on the most egregious aspect of the fourteen-tone temperament. Yet item 8 hints at a fourteen-tone octave organized into two diatonic *yun*'s, one comprising only odd-numbered pipes and the other even-numbered pipes, albeit from a different

angle. As I showed in the previous section, because the two *yun*'s each comprising the seven notes of the diatonic scale are completely complementary, conversely, each note of the scale is always assigned to two adjacent pipes, specifically an odd-numbered pipe and the shorter even-numbered pipe adjacent to it. Both *huangzhong* (1st pipe) and *dalü* (2nd) are *gong*'s in their respective *yun*'s and both *taicu* (3rd) and *jiazhong* (4th) are *shang*'s, for example—or, as item 8 puts it, "one odd-numbered pipe and one even-numbered pipe are used simultaneously" for each note. Similarly, item 9 hints at the overhaul of pitch organization whereby odd-numbered and even-numbered pipes are segregated. In the conventional pitch organization system, no matter in what *yun*, the *biangong* (roughly *ti*) or "altered" *gong* note and the proper *gong* (*do*) note are always immediately adjacent among the twelve tuning pitch pipes, and so are the *bianzhi* (*fa#*) or "altered" *zhi* note and the proper *zhi* (*sol*) note: for example, when *huangzhong* (1st/12th pipe) is *gong* in the conventional pitch organization system, the corresponding *biangong* note is found at *yingzhong* (11th pipe), and the corresponding *bianzhi* and *zhi* notes are found at *ruibin* (8th) and *linzhong* (9th) pipes, respectively. Following the complete segregation between the odd-numbered or *yang* pipes and the even-numbered or *yin* pipes under the new pitch organization system, however, neither *biangong* and *gong* nor *bianzhi* and *zhi* are ever to be found adjacent among the twelve tuning pitch pipes. Instead, they will always be one pipe apart, so that the *biangong*-*gong* and *bianzhi*-*zhi* intervals in terms of the number of pitch pipes they encompass are indistinguishable from all the other intervals between adjacent notes in the diatonic scale. Thus, as item 9 says, *biangong* and *bianzhi* are no longer "exceedingly close" to *gong* and *zhi*, so that "[the] proper and altered notes are evenly distributed" among the twelve tuning pitch pipes constituting a fourteen-tone octave divided into two complementary diatonic *yun*'s.

Lastly, items 6 and 10 concern the (re)interpretation of the concept of *xuangong* "rotating the *gong*." Whereas item 6 simply points to the location of the *gong* 工 syllable of the *gongche* notation at the *gong* 宮 note, thus in parallel to Chapter 10, Volume 1 of *Orthodox*, item 10 actually provides more information than do the relevant sections in the postface in f. 119. To recall, because the new pitch organization system groups the odd-numbered ones of the twelve tuning pitch pipes into one seven-note *yun* and the even-numbered ones into the complementary seven-note *yun*, every octave displacement will "rotate" the *gong* note and indeed the entire *yun* by two pipes in the direction of such displacement. For example, whereas *gong*, *shang*, *jue* are found at *huangzhong* (1st pipe), *taicu* (3rd pipe), and *guxian* (5th pipe) in one octave, at the next octave higher, because the octave is heard between full-length *huangzhong* and half-length *taicu*, the same notes *gong*, *shang*, and *jue* will be found at half-length *taicu* (15th pipe), half-length *guxian* (17th pipe), and half-length *ruibin* (19th pipe), respectively. As a result, the *gong* note will be "rotated" from a *huangzhong* pipe to a *taicu* pipe. But because there are only six odd-numbered pipes among the original twelve tuning pitch pipes, this means that the seventh of such rotations of the *gong* note will necessarily bring it back to a *huangzhong* pipe. Thus, through octave displacements towards the half-length pipes, the *gong* note can be rotated only seven times in the *yun* of the odd-numbered pipes before it will "return to where it has begun," namely to the *huangzhong* pipe, and the same can be inferred for the *yun* of the even-numbered pipes. As a result, there are in total two times seven times six or eighty-four such *gong* rotations possible, and it can be further inferred that, when the octave displacements take place towards the double-length pipes, the same *gong* rotations will take place except in the reverse direction. Or as item 10 puts it, "the eighty-four transposed notes are embedded in the seven cycles of the twelve tuning pitch pipes" by the very nature of octave displacement and combinatorics.

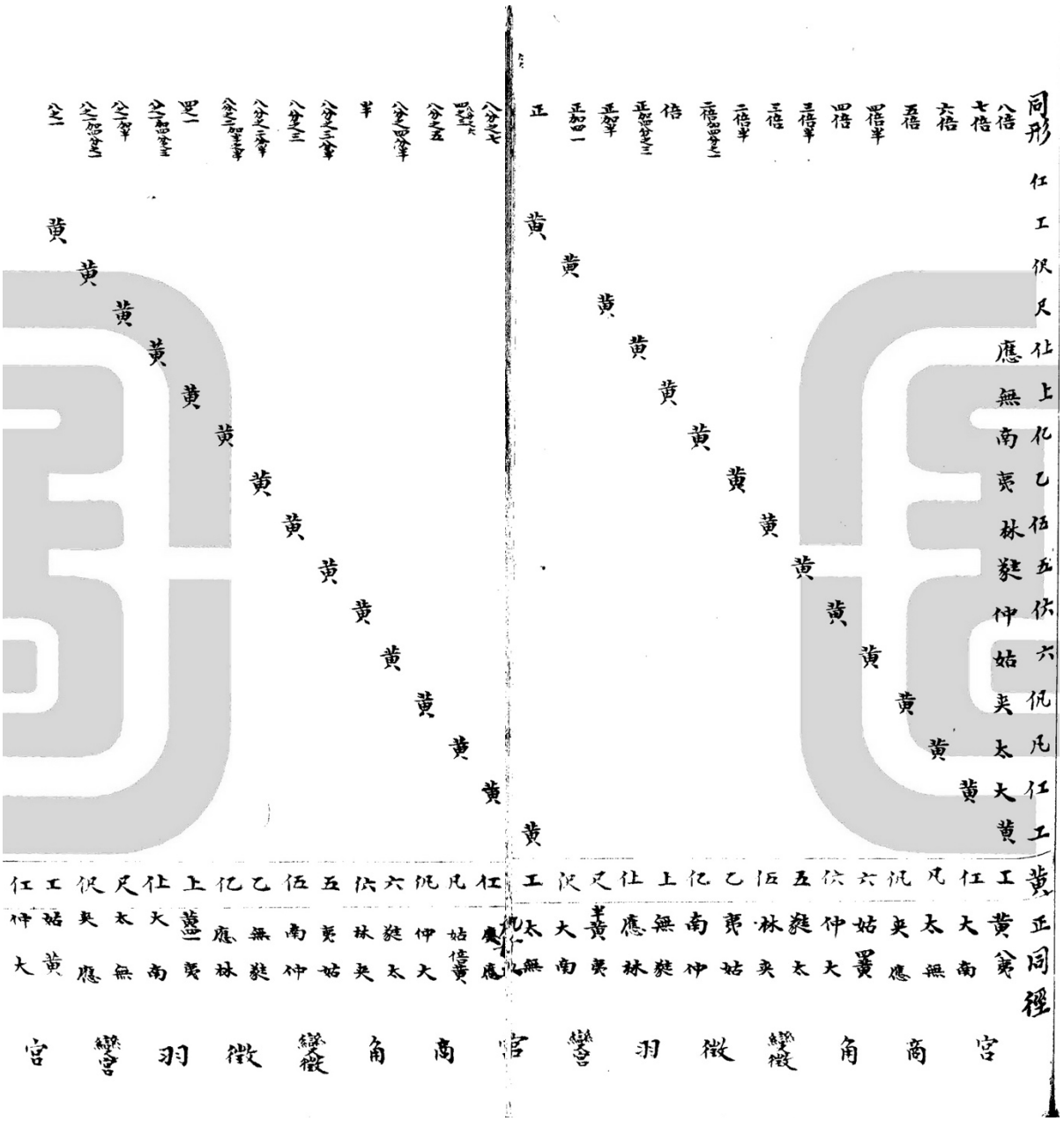
Therefore, not only does the list of findings in 84v of *Putong Guji* No. 15251 parallel the summary of *A Glimpse through the Pitch Pipes* in f. 119 or give a good outline of Volume 1 of *Orthodox*, but several of its items, such as item 7 on making instruments and items 8-10 on the pitch organization system and "rotating the *gong*," appear not to be mere conjectures or proposals but thought-out analyses of the implications and applications of the fourteen-tone temperament. This means that, as this list of findings is placed at the very start of ff. 84-119, by the time this latter portion of the continuously foliated *Putong Guji* No. 15251 manuscript was compiled, the research for *A Glimpse through the Pitch Pipes*, whose postface is dated to 1707, had already taken shape. Still, as evinced in my earlier analysis of the organological portion in ff. 96-118, the entirety of ff. 85-118 consists not of any prose text at all, but only tables, diagrams, graphs, and mathematical reckonings. On the other hand, the handwriting is uniformly neat and never hurried throughout these folios, except for the ample amount of annotations in both black and in vermilion pigments and the few pieces of scrap paper pasted on. This suggests that ff. 85-118 were compiled not as the draft of any treatise but as what may best be described as a series of finished "lab notes." Instead of description, reasoning, historical sources, or even analyses of varying types of evidence, all of which would require blocks of prose texts, the materials in these folios focus on presenting the "raw" results of research, be they mathematical computations, empirical experiments, or new organological designs and specs. In all likelihood, they were intended as progress reports on a research project to its patron who, though not personally involved in conducting those calculations or experiments, kept a close eye on the process—close enough, at least to want to see the actual calculations and observations and to occasionally weigh in on the new designs for instruments with a vermilion writing brush.

Indeed, the lab notes contain the results of the most critical experiment for not only the entire manuscript, but also the supposed *A Glimpse through the Pitch Pipes* treatise, and *Orthodox Meaning of Pitch Pipes*: the experiment of listening to the twelve tuning pitch pipes that led to a fourteen-tone octave. To recall, Chapter 9, Volume 1 of *Orthodox*, claims to have actually made the twelve tuning pitch pipes according to the measurements computed in Chapters 1-8 in order to *shenyin* "examine [their] tones." It claims to have heard an octave between the full-length *huangzhong* pipe and a half-length *taicu* pipe, and it is from this observation that, among the twelve tuning pitch pipes and those that are half and double their lengths, the new pitch organization system outlined in Chapters 9-11 locates the octave between any one pipe and the fourteenth pipe down the decreasing order of pipe length, effectively incurring a fourteen-fold division of the octave.

Furthemore, the long-overlooked *Putong Guji* No. 15251 actually contains iron-clad evidence that there were experiments conducted in listening to the twelve tuning pitch pipes, as well as twelve additional pipes that are half as long and twelve twice as long. To recall, whereas ff. 96-118 feature new designs of instruments following the fourteen-tone temperament, ff. 85-95 focus on the tuning pitch pipes themselves. These folios further divide into two sections, ff. 85-90 under the rubric of pipes of the same diameter, and ff. 91-95 under that of pipes of the same shape. The latter, ff. 91-95, a detailed analysis of which I will have to save for another time , focus on the rather complicated mathematics of computing the twenty-seven "*huangzhong*" pipes of the same shape encompassing two fourteen-tone octaves and generating from each of them eleven additional pipes of the same diameter through "triple division with one part subtracted or added." As shown in Illustration 3-8, a summary table in ff. 95r-95v concludes these folios with a summary table, showing the 324 pipes through the twenty-seven pipes of the same shape in the

horizontal axis and the twelve pipes of the same diameter in the vertical axis, and this table clearly anticipates the series of tables that present the system of 324 pipes at the end of Chapter 12, Volume 1 of *Orthodox*.

Illustration 3-8 The table of 324 pipes, multiplying twenty-seven pipes of the same shape (horizontal axis) by twelve pipes of the same diameter (vertical axis) in* Putong Guji *No. 15251, ff. 95r-95v



It is in the former section, ff. 85-90, however, that, in perfect parallel to Chapters 1-11, the manuscript reconstructs the twelve tuning pitch pipes, observes the octave relations between their pitches, and comes up with the new pitch organization system based on a fourteen-tone octave divided into two complementary *yun*. First, ff. 85-87 compute the length, base surface area, diameter, and volume of the *huangzhong* pipe and use "triple division with one part subtracted or added" to derive the lengths and volumes of the other eleven tuning pitch pipes proportional to those of the *huangzhong* pipe, using both the supposedly ancient and the Qing's modern units of measurements. Notably, these folios do not stop at simply computing the relevant data of the original, full-length twelve tuning pitch pipes alone. Instead, f. 85v calculates the lengths of not only the full-length twelve tuning pitch pipes—that is, from full-length *huangzhong* (7.29 modern *cun*) to full-length *yingzhong* (3.84 modern *cun*)—but also the double-length twelve tuning pitch pipes and the half-length twelve tuning pitch pipes, always keeping the very same diameter of 0.274 modern *cun*. It thus produces the lengths of thirty-six tuning pitch pipes of the same diameter, from the longest double-length *huangzhong* (14.58 modern *cun*) to the shortest half-length *yingzhong* (1.92 *cun*).

Why did compilers of these folios take pains to enumerate the *bei* 倍 "double-length" pipes and the *ban* 半 "half-length" pipes in addition to the *zheng* or original, full-length ones, even though the double-length and the half-length can easily be derived from a simple multiplication or division by two? The table in ff. 88r-88v, shown in Illustration 3-9 (in two parts), offers a clue. Granted, without any legend or text explaining its setup or rationale, the table can strike us as very confusing, and there are various aspects of the table that I still do not quite understand. Yet the core information is clear. In this table of ten columns by thirty-six rows—only thirty-six rows contain any information at all—each row represents one of the thirty-

Illustration 3-9 (Part I) – Finished "lab notes" of listening to the twelve tuning pitch pipes and those twice and half as long in* Putong Guji *No. 15251, f. 88r (the binding is on the righthand side of this image)

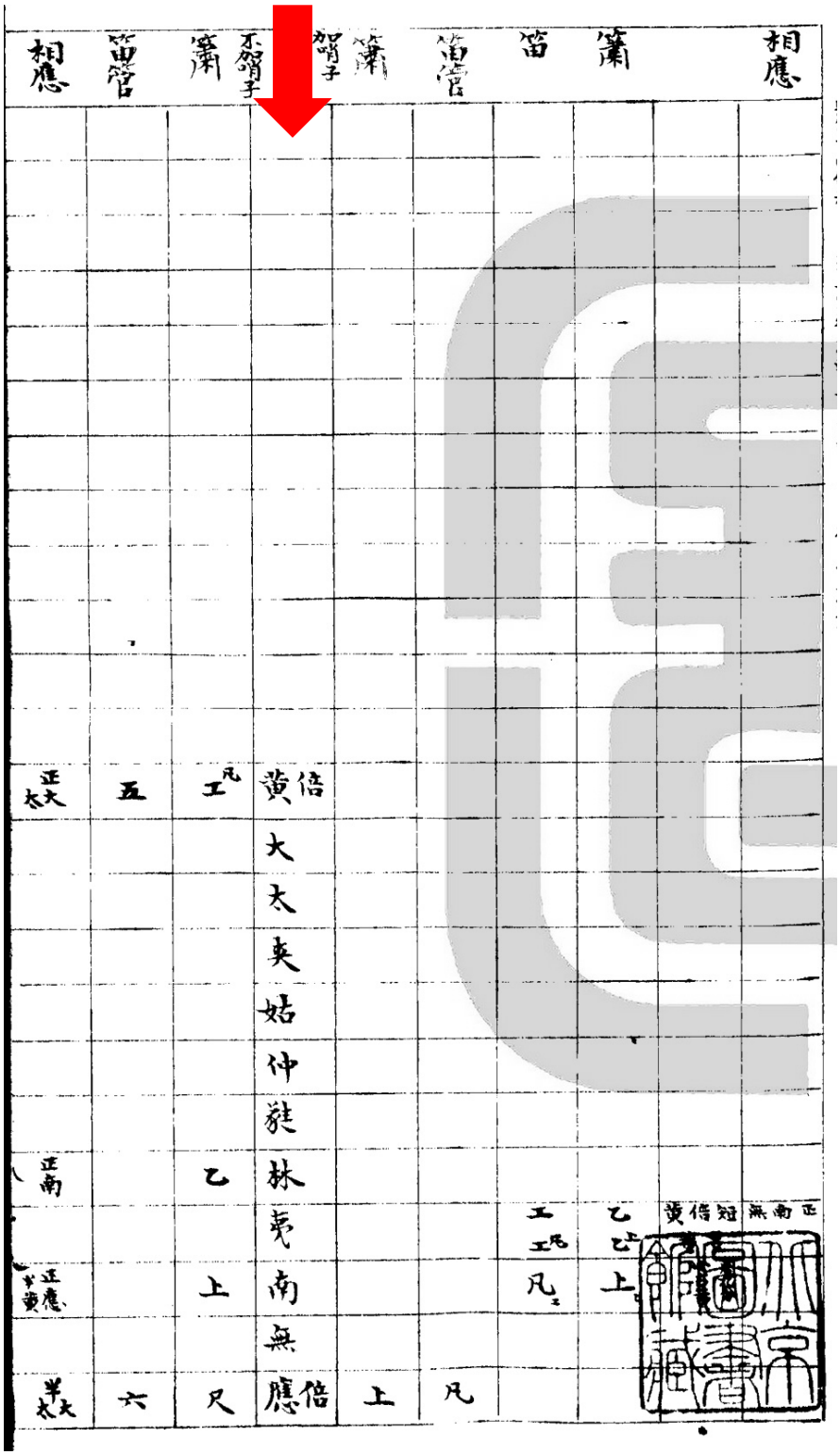
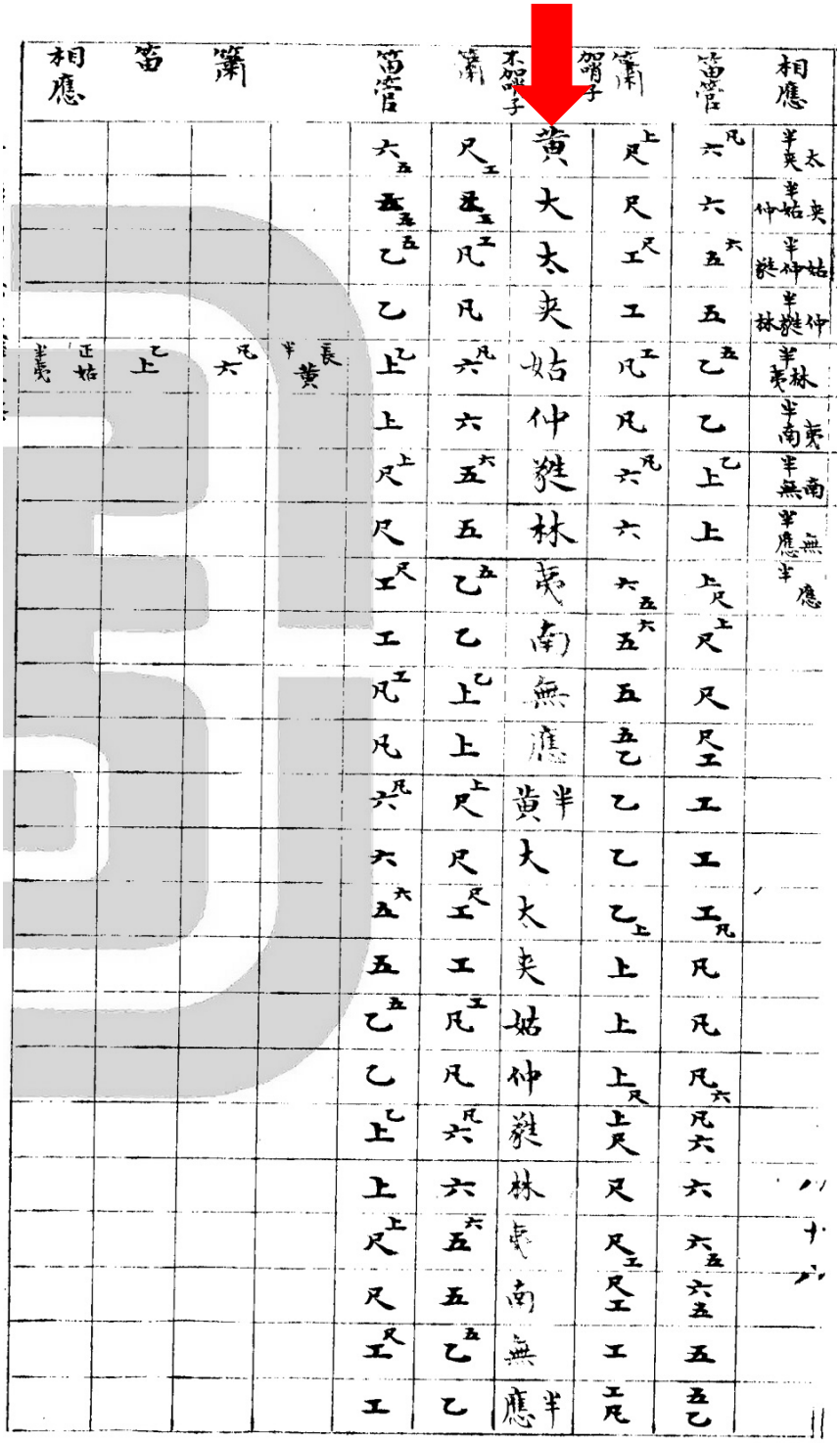


Illustration 3-9 (Part II) – Finished "lab notes" of listening to the twelve tuning pitch pipes and those twice and half as long in* Putong Guji *No. 15251, f. 88v (the binding is on the lefthand side of this image)



six tuning pitch pipes computed in f. 85v. Whereas the other columns of the table are all written in black, the fourth column (counting from the left in f. 88r or Part I of Illustration 3-9, and from the right in f. 88v or Part II of Illustration 3-9) featuring the names of the thirty-six pitch pipes—distinguished by the twelve names and by the prefix of *bei* "double-length" and *ban* "half-length" for the first and the last twelve pipes—is written completely in vermilion; I have therefore indicated this column in both parts of the table. Proceeding horizontally towards the edges of the table in both directions, this more-or-less middle column featuring the names of the thirty-six tuning pitch pipes is flanked on both sides by two columns featuring various *gongche* notation syllables. The headings of these four flanking columns read, from the edge of the folio towards the binding, *diguan* 笛管 "traverse flute and short end-blown pipe," *bujia shaozi xiao* 不加哨子簫 "end-blown pipe without reed-whistle," *jia shaozi xiao* 加哨子簫 "end-blown pipe with reed-whistle," and, again, "traverse flute and short end-blown pipe." In every row, these columns all feature different *gongche* notation syllables. Notably, many cells feature more than one *gongche* syllable, some with a main syllable and another one as a superscript or subscript, and others with two syllables juxtaposed vertically within the same cell. While it remains unclear what the difference is between the two columns that feature different notes in every row yet still bear the same heading, "traverse flute and short end-blown pipe"—it can be inferred that they record the results of listening experiments: one person would play a tuning pitch pipe to produce a pitch, and another person would try to find the note on a traverse flute, end-blown pipe, or short end-blown pipe (*guan* 管) that matches that pitch. In other words, the columns reflect attempts to match the pitches of those newly reconstructed twelve tuning pitch pipes and their double-length and half-length duplicates with instruments that were already in use. The coexistence of different *gongche* syllables in one cell, whether in the form of superscripts,

subscripts, or vertical juxtaposition, likely indicates that the pitch produced by the pitch pipe falls somewhere in between these notes on the wind instrument in question.

The fascinating information, however, comes from the two columns on the outside of the table. Both columns bear the heading *xiangying* 相應 "mutual correspondence." I argue that just as the four columns flanking the one featuring the names of the thirty-six tuning pitch pipes record unison correspondences between the latter and notes on the various wind instruments, the two outer columns record octave correspondences among the thirty-six tuning pitch pipes themselves, which is to say *mutual* correspondences. What is more, while there are two columns recording such mutual correspondences, not only does the column on the binding side of the folio contain significantly less information than that on the outer edge (only three out of thirty-six cells of the former have any information at all, compared to twelve out of thirty-six cells of the latter), but, for the two rows that do feature information in both columns, they also do not contradict each other—that is, unlike the two "traverse flute and short end-blown pipe" columns, as explained above. Thus, while I cannot quite explain the difference between the two columns that ostensibly bear the very same heading "mutual correspondence," it can also be safely inferred that these two columns record the results of listening experiments that attempt to locate octave relations among the tuning pitch pipes themselves.

The results completely vindicate the pivotal claim from Chapter 9, Volume 1 of *Orthodox* against any accusation of failure: an octave is never found between the pitch of any one pipe and that of the twelfth pipe either up or down the order of length, that is, between any two of the pipes whose length is in a 1:2 (or 2:1) ratio. Most often, furthermore, an octave is found instead between a pipe and fourteenth pipe up or down the order of length, if not even further. Starting from the longest pipes, for the double-length *huangzhong* pipe, the table puts both the full-length

dalü pipe (13th pipe) and the full-length *taicu* pipe (14th pipe) as sounding an octave above. If it is difficult to determine the exact significance of this record featuring two pipes, it seems fair to presume that it likely implies either that the pitch is found somewhere between those of the two pipes, or that the pitch wavers within this range in multiple experiments, given that end-blown pipes can produce highly unstable pitches depending on the breath and embouchure, and that bamboos are highly sensitive to humidity and temperature conditions. Continuing towards the shorter pipes, for the double-length *yize* pipe, the table puts the full-length *nanlü* pipe (13th pipe) and the full-length *wuyi* pipe (14th pipe) as sounding an octave above. For the double-length *nanlü* pipe, the table puts both the full-length *yingzhong* pipe (14th pipe) and the half-length *huangzhong* pipe (15th pipe) as sounding an octave above. For the double-length *yingzhong* pipe, the table puts both the half-length *dalü* pipe (14th pipe) and the half-length *taicu* pipe (15th pipe) as sounding an octave above.

Here, even just among the double-length pipes, a pattern has already surfaced. It appears that the longer the pipe, the closer to it lies the pipe that sounds an octave above. For example, whereas the octave correspondence of the double-length *huangzhong* pipe is found at the 13th and 14th pipes in the decreasing order of length, that of the double-length *yingzhong* pipe, which is almost half the length of the double-length *huangzhong* pipe, is found at the 14th and 15th pipes in the decreasing order of length. When it comes to the full-length pipes, or the original twelve tuning pitch pipes, their octave-correspondence pipes tend to move even farther down the decreasing order of length. Whereas for the full-length *huangzhong* pipe, the table puts the half-length *taicu* (14th pipe) and half-length *jiazhong* (15th) pipes as sounding an octave above, for the next three pipes *dalü*, *taicu*, and *jiazhong*, the table all puts their respective 14th, 15th, and 16th pipes as sounding an octave above. Furthermore, for the remaining pipes *guxian*, *zhonglü*,

linzhong, and *yize*, the table all puts their respective 15th and 16th pipes as sounding an octave above (only the 15th pipe for *yize*).

Of course, these data do not produce a uniform answer for the main question: where is the octave found among the thirty-six tuning pitch pipes that comprise the original twelve tuning pitch pipes and those that are half and twice as long? At some point, an "executive decision" must have been made to fix the octave between one pipe and the fourteenth pipe up and/or down the order of length. One can criticize this decision for not being precise enough, or even for not coming up with a more complex mathematical equation to address the fact that the octave-correspondence pipe becomes farther and farther away as the pitch pipe under question becomes shorter and shorter—an equation that would have required a certain amount of calculus, except, alas, neither Newton nor Leibniz lived within commuting distance to Beijing at the time. Still, the convoluted and at times tentative table in ff. 88r-88v should preclude any doubt as to whether the pivotal claim in Chapter 9, Volume 1 of *Orthodox* regarding the fourteen-tone octave had any empirical basis. The answer is a simple yes. At some point, certainly before the postface to the phantom treatise *A Glimpse through the Pitch Pipes* was penned in the summer of 1707, research was carried out that involved listening for octave relations among the twelve tuning pitch pipes and their full-length and half-length duplicates, with all these pipes having been computed and made following the same research in historical metrology that would end up becoming Chapters 1-4 of Volume 1 of *Orthodox*.

And it was based on this research that a new system of pitch organization of two complementary seven-note diatonic *yun*'s constituting a fourteen-tone octave was developed. In fact, immediately after the table in *Putong Guji* No. 15251, ff. 89-90 lay out, presumably for the first time ever on paper, just this pitch organization system. In particular, ff. 89r-89v feature a

Illustration 3-10 Table of 181 tuning pitch pipes of the same diameter ranging from 16-times-length* huangzhong *to 1/16-length* nanlü *pipes in* Putong Guji *No. 15251, ff. 89r-89v

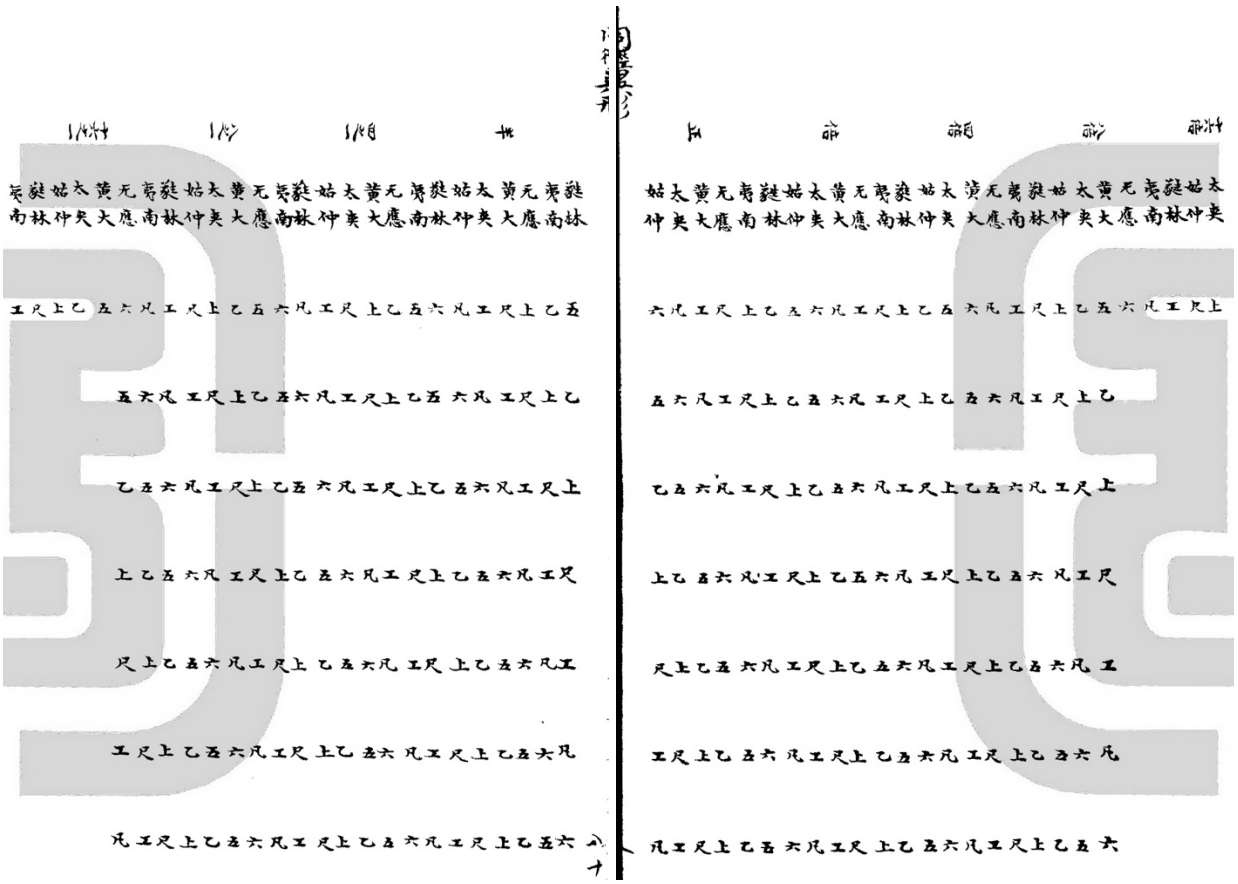
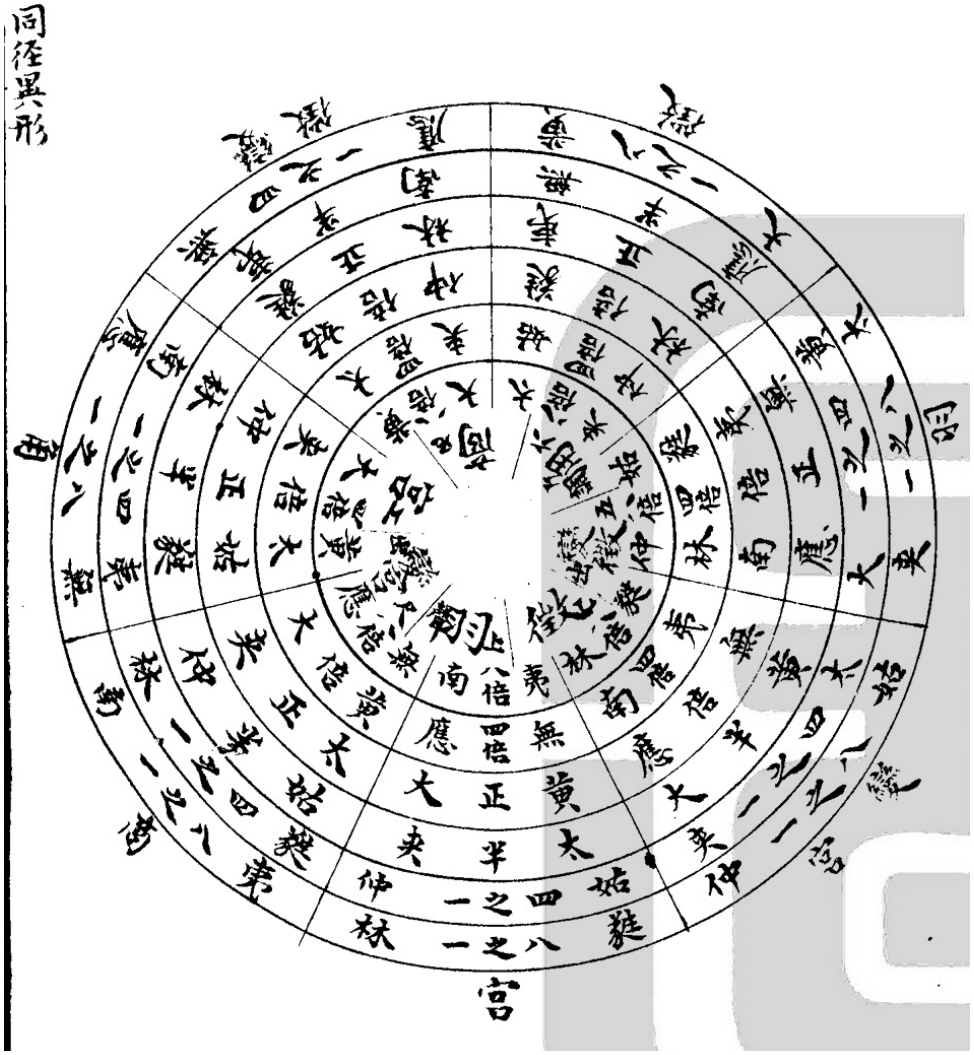


table of 181 tuning pitch pipes, all of the same diameter (see Illustration 3-10). In addition to the original full-length, the half-length, and the double-length pitch pipes, the table also includes the four-times-long, eight-times-long, and sixteen-times-long pipes, as well as the 1/4-length, 1/8-length, and 1/16-length in the reverse direction. Already, however, the table separates the one-hundred-and-eight tuning pitch pipes into two rows, the top one including only all the odd-numbered pipes and the bottom one including only the even-numbered pipes. Below the two rows, the table features the seven *gongche* syllables repeating in cycles, showing how an octave displacement will necessarily "rotate" the syllables down to a different position. And f. 90r ever renders this linear-form table into a circular slide chart, or volvelle (see Illustration 3-11). Whereas the movable circle in the middle of the chart has written on it "the five proper notes and

Illustration 3-11 Circular slide-chart or volvelle, matching a gamut of pipes from 8-times-long* huangzhong *to 1/8-length* yingzhong *pipes with the seven-note diatonic scale in* Putong Guji *No. 15251, f. 89r



two altered notes," all evenly distributed along the circle, the six outer and unmovable layers are also divided into seven equal segments, with each segment being occupied by two pipes, one odd-numbered and one even-numbered. The pipes featured in this chart run from the longest eight-times-long *huangzhong* and *dalü* pipes to the shortest 1/8-length *wuyi* and *yingzhong* pipes, thus comprising a total of seven series of twelve tuning pitch pipes, with the original set in the middle, or a total of eighty-four pipes. To recall, the list of findings at the beginning of this

section of *Putong Guji* No. 15251 in f. 84v touts that, following the new system of pitch organization and the new interpretation of the meaning of *xuangong* "rotating the *gong*," "the eighty-four transposed notes are embedded in the seven cycles of the twelve tuning pitch pipes." In other words, as shown in the linear table in ff. 89r-89v and the sliding chart in f. 90r, "rotating the *gong*" becomes an inherent part of the new pitch organization system, in which, following the mathematics of twelve tuning pitch pipes divided into the odd-numbered and the even-numbered, the seven-note diatonic scale will naturally transpose itself seven times along the two complementary sets of six tuning pitch pipes before it "returns to where it has begun."

❦

In sum, from the metrology and mathematics in ff. 85v-87 to the table of experiments listening for octave relations among the tuning pitch pipes in ff. 88r-88v, from the two tables laying out the two complementary diatonic *yun*'s forming a fourteen-tone octave in ff. 89-90r to the intricate computations of the twenty-seven "*huangzhong*" pipes of the same shape in ff. 91-94, from the total system of three-hundred and twenty-four pipes presented in the summary table in ff. 95r-95v to the application of these pipes to designing various instruments in ff. 96-118, the entire historical, mathematical, empirical, and organological research that would form the basis of *A Glimpse through the Pitch Pipes* and *Orthodox Meaning of Pitch Pipes* is found in the NLC manuscript *Putong Guji* No. 15251. By tracing Volume 1 of *Orthodox* back to the summary of *A Glimpse through the Pitch Pipes* in f. 119 and by tracing the latter further back to the "finished" lab notes, reckonings, and instrument designs in ff. 84v-118, I have established a "paper trail," so to speak, of the Kangxi Emperor's fourteen-tone temperament. Particularly, I have shown that the tuning reform's seemingly preposterous claim of hearing an fourteen-tone octave out of the

Pythagorean-proportioned twelve tuning pitch pipes did in fact come from actual experiments of *zhilü* “establishing the pitch pipes” in order to *shenyin* “examine their tones.”

But how does any of this have to do with In-c’i?

Chapter Four

Singing *Ut*, *Re*, *Mi*, Singing the Empire:

A Princely Vision for Qing Imperial Music Theory

As I have said near the beginning of the previous chapter, figuring out In-c'i's role in the fourteen-tone temperament begins with f. 119, a key link in the chain of textual sources that connects the finished lab notes in *Putong Guji* No. 15251 to the final *Orthodox* treatise. Yet to use the summary of the phantom treatise *A Glimpse through the Pitch Pipes* in f. 119 to reflect on the prince's contribution to the Qing's tuning reform requires looking beyond the specific content of that treatise or the fourteen-tone temperament research itself. Thus having belabored the point that the experiments behind the fourteen-tone octave described in Chapter 9, Volume 1 of *Orthodox* did in fact take place, I turn in this chapter to the two questions of how: that is, in what context and using what infrastructure did the experiments and the broader research for *A Glimpse through the Pitch Pipes* and *Orthodox* take place, at least six years before the Kangxi Emperor even commissioned In-c'i to head the *Origins* and *Orthodox* projects? And what inspired these specific experiments with listening specifically for octave equivalences using pitch pipes—which both the Kangxi Emperor in 1692 and Chapter 9, Volume 1 in *Orthodox* in 1714 refer to as *geba xiangsheng* "mutual generation at every eighth step?"

Answering of these two questions requires that I reconstruct how the entirety of *Putong Guji* No. 15251 came together as the most extensive and critical collection of preparatory materials that would feed into the fourteen-tone temperament tuning reform. Because there is no paratextual information for the manuscript as a whole other than the stamp impression mentioning In-c'i's name on the upper-right corner of f. 1r, the best option for approaching the

question is first to gauge how the miscellaneous components of the manuscript each came about and then to infer how their different contents and raisons d'être fit in with one another. In this regard, the provenance of the "lab notes" for the pitch pipes experiments and the instrument designs in ff. 84v-118 is the least ambiguous element. Granted, *Orthodox* and the related refurbishment of all the ritual musical instruments at the Qing court are well-documented as an imperial project commissioned and closely supervised and coordinated by the Kangxi Emperor. The various records I have examined from between 1713-1714 also show that the Kangxi Emperor not only commissioned but personally supervised and coordinated the *Orthodox* compilation. Nonetheless, my analysis of *Putong Guji* No. 15251 in the previous chapter has shown clear parallels between ff. 84v-118, *A Glimpse through the Pitch Pipes* as summarized in the postface in f. 119 dated to 1707, and the treatise of *Orthodox* completed in 1714. Additionally, two blocks of extensive prose texts that bracket ff. 84v.-118 in *Putong Guji* No. 15251 make it unequivocally clear that the research that would eventually become *Orthodox* began squarely under the aegis and patronage of In-c'i himself. In other words, the so-called Kangxi Emperor fourteen-tone temperament had actually taken full form as a princely reform to music theory and musical tuning by the emperor's son years before it would be subsumed by his father's *yuzhi* 御製 "imperially made" project.

By using these two blocks of text in ff. 84r-84v and 119 as springboards, I will expand my analysis to include the entire *Putong Guji* No. 15251 as a manuscript that was once owned, read, and annotated by In-c'i. This allows me to reconstruct a grand vision In-c'i had for the music and music theory of his father's Empire, a vision that long survived the lifespan of both his father and himself yet had long been overlooked by modern historians. Beginning with the seemingly unimaginative problem of making and listening to the twelve tuning pitch pipes, this

princely vision set its sight on harnessing for Qing imperial advantage what was arguably the most influential genre in the 17th and 18th centuries: Chinese opera. As it turns out, it was precisely this operatic vision that could also explain how the Kangxi Emperor could have possibly misunderstood *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence, thereby triggering the epistemological reinvention of music theory that would one day be reflected in the pivotal Chapter 9, Volume 1 of *Orthodox*.

Progenitors Ancient and Modern

In-c'i's "ownership" of the research for the fourteen-tone temperament that went from ff. 84-118 of *Putong Guji* No. 15251 to *A Glimpse through the Pitch Pipes* as summarized in f. 119 and finally to *Orthodox Meaning of Pitch Pipes* as Volumes 43 to 47 in *Origins of Cosmological Sciences* is nowhere more manifest than in the extraordinary text in ff. 84r-84v of *Putong Guji* No. 15251. As I will show, the text turns out to be no more than a preface to the phantom treatise *A Glimpse through the Pitch Pipes*, yet its rhapsodic grandiloquence and floridity rival the most solemn instruments issued by the Empire. It is not difficult to understand why. The text bears no title but only an indicator of its literary genre and register: *biao* 表 "supplication." In the world of bureaucracy, this genre was similar to the aforementioned *shu* "petition" in its political function in that officials used both to formally bring a matter to the ruler's attention. During the Qing era, this formality contrasted with the secret palace memorial, a relatively new institution that guaranteed direct and personal, that is, "informal," communications between the Emperor himself and the privileged officials bypassing much of the institutional bureaucracy. Yet there is a critical difference between *shu* "petitions" and *biao* "supplications" in their conception as literary genres. According to Liu Xie 劉勰 (465-522 CE), whose *The Literary Mind and the*

Carving of Dragons (wenxin diaolong 文心雕龍, 5th century CE) set the towering paradigm for Chinese literary criticism, the purpose of a *biao* is "to express one's own feelings" (*biao yi chenqing* 表以陳情).[1] In other words, if *shu*, which literally means "to dredge" or "to clear away," emphasizes the conveyance of information and opinions, *biao*, which literally means "to express" or "to profess," serves to envoice affect.

As it turned out, the *biao* in ff. 84r-84v in *Putong Guji* No. 15251 did not express itself to either the Kangxi Emperor or in fact anyone living. In addition to being political instruments for communicating oneself to the sovereign, the *biao* also served as a ritual instrument whose significance was less about its contents and more about its act of proclaiming and articulating the rapport of the document's originator with his superior addressee. Just as officials, vassal polities, and newly conquered territories submitted *biao*'s extolling the Emperor in order to reaffirm and perform their allegiance to him on important ritual occasions, the Emperor as the embodiment of the Heavenly mandate and his subjects also had to regularly "express" and enact his submission to Heaven, Earth, the Ancestors, and the Soil and Grain (*sheji* 社稷, a metaphor for the entire realm), as well as a variety of other deified or canonized entities or figures, such as Confucius. Thus, a *biao* written especially for an occasion almost always counted among the sacrifices and other goods the Emperor would solemnly present at the altar at a sacrificial rite or other ritual occasion for offering, after an elaborate preparation that was meticulously codified in imperially commissioned ritual manuals.[2]

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1 See Liu Hsieh, *The Literary Mind & the Carving of Dragons*, Vincent Yu-Chung Shih trans., 3rd edition (Hong Kong: The Chinese University of Hong Kong Press, 2015), 182.

2 On the performative and embodied nature of ritual texts during the Qing era, see Zito, *Of Body & Brush*, 153-184.

What makes the particular *biao* in ff. 84r-84v even more intriguing is not only its deified addressees but also its addresser, who turned out not to be the Emperor as someone who occupied a privileged position mediating between the deified and the mundane:

> 表
> 伏惟，　皇天眷德萬年，永臻清晏之休；　至聖傳心千古，重仰和平之聽。既功成而治定，宜樂備而禮明。敢竭愚忱，敬求神告。臣　皇三子多羅貝勒，　齋沐虔誠，稽首頓首，昭告黃帝有熊氏、　先師伶倫、　先師后夔之神：[…][3]
>
> Supplication
> Prostrating, I contemplate: the SOVEREIGN HEAVEN keeps the virtuous for tens of thousands of years, and it perpetually attains the happiness of refreshing tranquility; the UTMOST SAGE [likely referring to Confucius] passes on the mind for millennia since ancient times, and he especially venerates the audition of the harmonizing calmness. Now that the deed has been achieved and the rule has been established, it is appropriate to perfect the music [*yue*] and clarify the rites. Dare I exhaust my foolhardy sincerity and respectfully entreat the Deities to declare themselves. Your subject, THIRD SON OF THE EMPEROR, PRINCE OF THE THIRD RANK, having FASTED AND BEEN CLEANSED, piously bowing and knocking his head on the ground, perspicuously announce to the Spirits of the YELLOW EMPEROR of the Youxiong Clan, the PIONEERING MASTER Ling Lun, and the PIONEERING MASTER Houkui: […]

It is impossible to know the literalness of "having fasted and been cleansed" (*zhaimu* 齋沐), a ritual to which an emperor would have been subjected days beforehand in preparation for a sacrificial rite, since the turn of phrase—like so many others in Classical Chinese—had long been adopted as a metaphorical usage in heightened literary registers, to express sincerity and piety.[4] Nor is it clear whether the "knocking [one's] head on the ground" had literally ever taken place in relation to this *biao*. Though the Qing Emperors regularly dispatched officials to make

3 National Library of China, *Putong Guji* No. 15251, f. 84r.
4 Zito, *Of Body & Brush*, 155-165.

offerings on their behalf at the Mausoleum of the Yellow Emperor in the province of Shaanxi during the 17th and 18th centuries, I have not found any textual component or remnant of these offerings that bespeaks anything as specific as an endeavor to "perfect the music and clarify the rites." On the other hand, there might have just been a place in Beijing where such a *biao* could be solemnly presented not only to the Yellow Emperor but also to Ling Lun and Houkui who, despite their well-documented deeds in assisting the Yellow Emperor and Emperor Shun in *Chornicles of Master Lü* and *Canon of History*, respectively, were quite obscure as far as imperially venerated figures went: the Temple of Emperors of the Past Dynasties (*lidai diwangmiao* 歷代帝王廟).[5] Originally built in the 16th century and dedicated to the past rulers of China and noteworthy officials, the Temple does include the spirit tablets of both the Yellow Emperor and Houkui. Perhaps Ling Lun's subsequent association with the socially marginalized court entertainers, comedians, and minstrels, who were literally referred to as *ling* 伶 after his name, precluded his chances of being counted among the elevated rank reserved for mandarins and generals deemed most worthy of moral emulation. Yet the *Comprehensive Rites of the Great Qing* (daqing tongli 大清通禮, 1736), the primary codification of rituals of the Qing Empire, keeps only a list of the emperors and not the officials commemorated in that Temple; and other historical sources recording the various expansions of its register in the 17th and 18th centuries were uniformly silent on names.[6] Thus, even though it is inconceivable for the Yellow Emperor not to be included in this register in any point of the Temple's history, it remains impossible to

5 This information is according to the current registry of spirit tablets maintained by the Temple as a historical site. See http://www.lddwm.com/info/monarchs/index.shtml, accessed April 19, 2019.

6 Laibao et al., *Daqing tongli* 大清通禮 ("Comprehensive Rites of the Great Qing," 1736), vol. 9.

gauge whether Houkui's name had already been included by the turn of the 17th into the 18th century.

Regardless of whether this *biao* ever functioned as a ritual instrument, one thing is as remarkable as it is unambiguous: it was In-c'i, the third of the Kangxi Emperor, a Prince of the Third Rank or *doro-i beile* between 1699 and 1709, who is presenting himself to the three spirits, be it literarily or literally. It was In-c'i who submitted to them on behalf of the Qing Empire that it had "achieved the deeds [*gong* 功]" and "established the rule [*zhi* 治]," which clearly refers to the Qing's conquest of China proper in the mid-17th century and its subsequent suppression of renegade regimes and major rebellions, such as the Southern Ming (eradicated in 1666), the Three Feudatories (subdued in 1681), and the Koxinga regime in Taiwan (conquered in 1683). It was In-c'i who announced to the three ancient progenitors of *yue* "music" and *lülü* "tuning pitch pipes" his intention to "perfect the music and clarify the rites." Indeed, had In-c'i's mode of self-address been replaced with the Kangxi Emperor's, the entire *biao* would still have made perfect sense, if not much more than that. As I argued in the introduction to this dissertation, *yue* "music" was a critical institution of the state. Not only was it broadly construed to reflect and also affect the moral well-being of a ruler, his regime, and his realm, but the very endeavor of reconstructing ancient music and musical tuning was critical for the ruler in performing his role in the lineage of righteous rulers in the Confucian historical imagination that traced back to the ancient sages. Nonetheless, whereas the Kangxi Emperor often sent his sons to make offerings to the Qing's imperial mausoleums on his behalf, nowhere in this *biao* does In-c'i mention any order from his father to pay tribute to the Yellow Emperor, Ling Lun, or to Houhui for their roles in establishing the ancient institution of the twelve tuning pitch pipes and music in general. In fact, even though In-c'i does evoke his father at one point, he limits his mention to a generic

praise of his father's sage governance, without ever speaking of any commission or mandate from him to conduct research into music or music theory. Instead, coming right after a formulaic lament of how the perfect institution of music under the ancient sage kings had been lost for subsequent millennia, the passage in question paints In-c'i, not the Emperor, as the hero who would specifically make music great again. Indeed, if there is anyone to whom In-c'i makes himself accountable, it is not his father but the three ancient spirits:

> […] 我國家宅中無外，德洽神人；我　皇上出治開天，功龍參贊。是誠宇宙太和之會，君師道統之全矣。審樂，知政典莫重于格　天；審聲，知音孝孰先于祫　祖。圜丘方澤，將考正其宮懸；　列聖同堂，必鏗鎬乎雅奏。思以管窺之見，上報高旻，蠡測之微，添流瀛海。豈謂聰明是炫，必使功自己成？譬諸耳目之官，庶幾各效仿其職。但生徒聚訟，迄少折衷；理數乖睽，類多虛語。自度量權衡之遞變，致參銖毫，忽微異宜。娶妻生子，秖屬卮言，而面冪體分，終非密率。調分五旦，誰為雅頌之音？韻攝四聲，半入琵琶之譜。若將據陳弋獲，慮儒者泥于典籍之未符；更歟抉摘諸訛，又工師苦于傳習之未慣。吹灰則葭管之飛難據，累黍或上黨之粒可憑。是用按史、準經、即數、揆理，猶恐昔與今之不相及，理與器之未相孚。爰列諸條，敬祈　神聽。[…][7]

> Our Empire stands at the Center without bounds, and its virtue accords with the divine and the human; Our EMPEROR excels in his reign comparable to the Cosmic Creation, and the meritorious and the valiant are all serving in his counsel and assistance. Truly, this is the moment of the Supreme Harmony of the cosmos and the perfection of the Righteous Lineage. From examining [*shen*] music [*yue*], I know that there is nothing more important than studying the HEAVEN in governance and laws; from examining [*shen*] sounds [*sheng*], I know that there is nothing more primary in tones [*yin*] or filial piety than making sacrifices to the ANCESTORS. At the circular Mount of Heaven and at the quadrilateral Mount of Earth, I will examine and rectify the instruments. At the halls dedicated to the VARIOUS SAGES, I will have the Elegant [*ya*] Music played with the clanging of bells and beating of drums. I realize that I am honoring the HEAVEN ON HIGH with my narrow view through the pipes and feeding into the immense ocean with my measuring through a ladle. How can I say that my eyes

7 National Library of China, *Putong Guji* No. 15251, ff. 84r-84v.

> and ears are so bright, that the exploit will necessarily be accomplished by me alone? For example, when it comes to the eyes and the ears, perhaps they can each perform their own duties. Yet young scholars clamor with different views and, so far, very rarely reach any compromise. Principles and numbers are perverted and deviant, and they mostly resemble empty words. The successive alteration of measurements of length, volume, weight, and balance, governance and laws, as since caused the confusion of various measuring units, and these units all have their different proper uses. "A gentleman takes a wife to generate an heir" has become nothing but malarkey, and the surface areas and volumes [of pitch pipes] are still not calculated with the precise algorithms. Modes are distinguished through the five note-series, yet which ones are the tones [*yin*] for Elegant Music? Rhymes are arranged according to the four tonal patterns [*sheng*], yet not all of them are reflected in the notation of *pipa* music. If I just put forth what I have discovered, I fear scholars will be obsessed with its deviations from the classical books. I also like to expose others' mistakes, yet practitioners will suffer from not being used to what they are learning [anew]. The flying out reed ashes from pitch pipes [while "awaiting the *qi*" breath] is baseless, yet arranging grains of millet from Shangdang County into a straight line [as a standard of length] is warranted. Therefore, I follow historical records and comply with the canons, I stay close to the numbers and probe the principles, yet I still fear that the past and the present do not concur, and the principles and the instruments do not yet match each other. Therefore, I am listing several items, and I respectfully entreat that the SPIRITS listen. […]

By "listing several items," In-c'i is referring precisely to the list of ten findings in ff. 84v. After the main text of the *biao* draws to an end—notably, the text does not feature a single paragraph break—a new section immediately begins with the following rubric:

> 論律呂以蔡氏《新書》為主，考正微有異同，開列于後。[8]
>
> The discussion of the tuning pitch pipes will primarily follow Cai Yuanding's *New Treatise on Pitch Pipes*, [yet] the investigations and rectifications have both similarities and slight differences. They are listed one after another hereafter.

8 Ibid., f. 84v.

In further proof that the *biao* running from f. 84r to 84v and the list of ten findings on f. 84v actually belong to one document, with the latter as an attachment to the former, the last thirty-three characters of the list of findings in f. 84v (making up exactly three quarters of item 10) are written in a font that is significantly smaller than the rest of the texts in ff. 84r-84v. I infer from this that all the content in f. 84 was copied from one integral text, yet as the copyist approached its end he realized he would run out of space and therefore switched to a smaller font to keep the text within the span of a single folio.

The fact that In-c'i's *biao* dedicated to the three ancient progenitors of music concludes with a list of ten specific findings that perfectly parallel the content of ff. 85-118 and the summary of *A Glimpse through the Pitch Pipes* in its supposed postface in f. 119 sheds major light on this past part of *Putong Guji* No. 15251 as the basis for the 1714 *Orthodox Meaning of Pitch Pipes*. To begin with, since the *biao* was already submitting much of the fourteen-tone temperament tuning reform for Heavenly review, so to speak, via the list of ten findings, clearly the research, included as lab notes, mathematical reckonings, and new instrument designs, laid out in ff. 85-118 of the manuscript, must have been concluded by the time In-c'i penned this supplication to the ancient spirits. As In-c'i was demoted from a Prince of the Second Rank to that of the Third Rank in 1699 and would only be made a Prince of the First Rank in 1709, the range of possible dates for this *biao*, 1699-1709, aligns perfectly with the date, summer 1707, in the postface on f. 119. Consequently, I argue, just as f. 119 summarizing the content of ff. 85-118 claims to be a postface to *A Glimpse through the Pitch Pipes*, the *biao* including both the main text in ff. 84r-84v and the attached list of ten findings in 84v was very likely conceived as a preface to the same treatise.

Flanking the research materials that would eventually become *Orthodox Meaning of Pitch Pipes*, both the *biao* as preface in f. 84 and the postface in f. 119 prove that the fourteen-tone temperament tuning reform developed solely under In-c'i's initiative and command, albeit from different angles. Instead of dedicating the as yet unwritten, or perhaps never written, *A Glimpse through the Pitch Pipes* to his father the Emperor In-c'i's *biao* describes the project as mediating between him and the ancient sages alone. While there was nothing unusual about addressing a *biao* to a supernatural being or ancient spirit, using such a ritual instrument—even if it was never literally used as such—to open a treatise of scholarship had very few other models, to my knowledge. Almost all *yuzhi* "imperially made" or *qinding* "imperially authorized" treatises from the Qing era begin with a *yuzhixu* 御製序 "imperially made preface," yet in those the texts adopt the typical imperial voice that speaks to all precisely by speaking to no entity or individual in particular. In contrast, not only does In-c'i's *biao* per the characteristic of its genre pointedly address the three ancient progenitors of music and tuning pitch pipes, it also lays bare in front of them the course of the research project as if it were his personal commitment and struggle. In many ways, In-c'i styles the last passage above as a call for help. In particular, he portrays his effort to "examine and rectify the instruments" and "have the Elegant [*ya*] Music played with the clanging of bells and beating of drums" at the various solemn ritual venues such as the Mount of Heaven and the Mount of Earth as *guankui* 管窺 "seeing through a pipe" and *lice* 蠡測 "measuring with a ladle." Granted, these two phrases, whose full forms are *yiguan kuitian* 以管窺天 "to see the sky through a pipe" and *yili cehai* 以蠡測海 "to measure the ocean with a ladle," are commonplace metaphors of self-deprecation. Although Classical Chinese can be frustratingly ambiguous about the subject or topic of a sentence, these two phrases make it unambiguous that In-c'i is not speaking of—and certainly not deprecating—his father here. And

yet not only does the former phrase echo the title of the treatise *A Glimpse through the Pitch Pipes* (lülü guankui 律呂管窺), with *lülü* "pitch pipes" resonating and thus amplifying the literal meaning of *guan* 管 "pipes," but the references to *ce* 測 "measuring" and *li* 蠡 "[handle-less] ladle [usually made of gourd]" in the latter phrase echo the topic of metrology, which is critical for the broader endeavor of restoring the proper musical institution and tuning of the ancients.

As the passage progresses, In-c'i turns these two formulations of self-deprecation into a performance of vulnerability, enumerating all the struggles and anxieties he faces in pursuing the various aspects of *guyue* "ancient music" before its three progenitors. Admittedly, all the challenges In-c'i moans about are stock formulations both in monographic treatises on music and musical tuning and in *zhi* "treatises" in official dynastic histories. By surveying a few of these texts, one can see that scholars of music theory were always quarrelsome students of Confucian canons, always stubborn with the texts, and music practitioners always oblivious or resistant to change, whereas standards of metrology were never properly restored *à l'ancienne*, phrases recorded in historical sources were never correctly understood, and calculations were never sufficiently precise. Now in a standard treatise or essay on music theory, the next move after such a litany of complaints would be to praise the current text, its author, or the regime or era they represent for having finally gotten all the right solutions to the millennia-old deterioration of and confusion over music. In-c'i, by contrast, opts for the complete opposite. While "I follow historical records and comply with the canons, [and] I stay close to the numbers and probe the principles" might sound like a humble brag, he concludes these strings of scholarly bona fides by expressing yet again his fear that the results are not good enough—thus heralding the list of the ten finding in f. 84v attached to the end of the *biao* and beseeching the three ancients to enlighten him in their regard.

Here, I argue, that being overly concerned with the extent to which the affect of vulnerability and worry that the *biao* effuses was "real" or "performed" would miss the mark. Instead, by the *biao* "expressing" these emotions along with the core findings of the fourteen-tone temperament research, In-c'i submits himself to the judgment of the three progenitors of *guyue* "ancient music" and thus incorporates himself into the Confucian imagination of music history and of civilization writ large, which posited them as the ancient gold age. He incorporates himself into an age-old tradition. In so doing, In-c'i presents himself not only as being accountable only to the ancients but also as being the *only* one accountable to the ancients within the scope of *A Glimpse through Pitch Pipes*, which is thereby cast as an attempt to rectify the pitch pipes, examine the tones, and thus restore ancient music. Seen in this light, the entreaty to the ancient spirits makes clear both his attachment to the fourteen-tone temperament tuning reform and his understanding of its political potential, for it allows him to occupy a position in the lineage of sage guardians of the ancient institution of music that traces itself back all the way to the ancient kings, as if he himself were a Confucian ruler.

As it turns out, In-c'i was quite ambitious. Whereas the *biao* qua preface to *A Glimpse through the Pitch Pipes* in ff. 84r-84v in *Putong Guji* No. 15251 conveys to the ancient spirits In-c'i's aspiration to restore their practices of musical tuning, the postface to the same treatise on f. 119 and the rest of the manuscript reflect an even grander ambition of this Prince of the Third Rank: namely, to become the progenitor of an all-inclusive treatise on music and music theory on behalf of the entire Qing Empire. In fact, as I will show in the next section, it appears that In-c'i had already articulated that grand vision for a Qing imperial music theory long before the key findings of the fourteen-tone temperament would coalesce.

Servants and Guests

While In-c'i calls all the attention to himself in the *biao* preface to *A Glimpse through the Pitch Pipes*, the postface, penned by an anonymous scholar who almost certainly worked on the project, describes the setting in which In-c'i's research leading to the fourteen-tone temperament took place. Here is the beginning part of the postface:

> 皇三子殿下秉天挺之睿思，承　皇宮之　聖訓。翱翔德藝，綜貫天人，而又虛懷好學，下士求賢。公士大夫，一善必錄，凡被　容接，皆為不世之遇。風聲所被，靡間遐邇。某南國編氓，朽樗善病，不能與懷鉛夾槧之子競短長于一日，潦倒無成，風塵白首。　京華旅食，獲覲　龍光，辱以學問之道，商榷觀摩，因得仰闚高深，領會萬一。德業經綸之餘，詩文翰墨而外，妙算逾乎祖衝，測量侔于姬旦。一切鐘漏儀器，發　內府之秘藏，極人世之偉觀矣。榮喜逾分，屬疾還山，過蒙　垂慈，使命不絕。　賜示《律數論》十餘篇，莊誦環周，霍然病已。又以為可教，趣召赴　都，廩之舍館。政事閒暇，輒訪求今古，辨析異同。久而彌勤，日新月異，積成卷軸，統名《律呂管窺》，謙詞也。將勒為一編，　命序次其顛末。[…][9]
>
> His Imperial Highness the Third Son of the Emperor [In-c'i] is endowed with Wise Introspection from Heaven and inherits the SAGELY INSTRUCTION from the IMPERIAL PALACE. His virtue and his art hover on high and encompass the heavenly and the mundane, yet he is also humble and diligent, and he reaches below his rank to look for the able and virtuous ones. He would take into his service any scholar or official who turns out to be good, and once they are TAKEN IN BY HIM, they would make the encounter of their lifetime. The reputation he enjoys is carried far afield. I was just a commoner from the south, useless and prone to illness, and I could not compete even for a day with those students who are so prepared and hardworking that they carry writing utensils and tablets with them all the time. Thus, dejected and with no achievements, I was wandering here and there and my hair grew grey. When I travelled to the CAPITAL, I was granted a glimpse of the DRAGONLY luster [of the Prince]. I was indebted [to the Prince] for the way of scholarship, and by debating and watching, I was able to behold the height and depth [of knowledge] and understand one of every ten thousand things. In addition to moral self-cultivation and governance, in addition to composing

9 Ibid., 119r.

> poetry and writing essays, he is better than Zu Chongzhi [429-500 CE] in his mathematical ingenuity and rivals the Duke of Zhou [d. ? 1032 BCE] in testing and measuring. All the clocks and instruments [he uses] come from the secret treasures of the INNER COURT, and they are the epitome of the great things this mundane world has seen. I was so beyond honored and ecstatic that I became ill and returned to my hometown, yet I was so greatly endowed with BENEVOLENCE that my mission did not end there. The Prince GRANTED me some ten chapters of his *On the Numbers of the Huangzhong Pipe*, and as I was solemnly reading it aloud again and again, my illness suddenly subsided. What is more, deeming me worthy to be taught, the Prince hastily summoned me to the CAPITAL and put me up in the guest apartment of his residence with a pension. During leisure time off from his governance duties, he always investigates the ancient and the modern in order to distill and analyze their similarities and differences. He became more and more diligent as time went by, and his studies advanced every day and every month. These accumulated into volumes and scrolls, and they were summarily given the name *A Glimpse through the Pitch Pipes*, a self-deprecating title. He took them to compile into one work and ORDERED that I write a postface to be attached to its end. […]

Coming from a Chinese literati culture in which self-deprecation was the ultimate form of virtue-signaling, the postface cannot be taken at face value in reconstructing the profile of its anonymous author. For example, it is hard to know to what extent they were really "dejected and with no achievements" by the time they aged to the point of *baishou* 白首 or all their hair turning white, a euphemism for old age. Indeed, how could a mere *bianmeng* 編氓 "commoner" (literally "one enrolled in the household register"), who had also achieved nothing in his aging life, have gotten a chance not only to travel to Beijing from his hometown in southern China, but also to meet the Third Son of the Emperor and Prince of the Third Rank? Still, two aspects of this brief autobiography, no matter how far from the truth, potentially sheds light on the personnel and infrastructure for the actual research that led to the fourteen-tone temperament under In-c'i—that is, as opposed to the compilation of *Orthodox*, of which almost all the major findings were already written down in ff. 85-118 of *Putong Guji* No. 15251.

First, by saying that they could not *jing* 競 "compete" with those who *huaiqian xieqian* 懷鉛夾槧 "carry lead [pencils] in their bosom and tablets [for writing] under their arms," the author implies that he did not sit or advance far enough in the civil examinations. In this regard, he resembles the many scholars the Qing employed for its major scholarly publications. While He Guozong was the only one of the high-level editorial team as well as the support staff for *Origins of Cosmological Sciences* who had actually achieved the terminal *jinshi* degree before joining the project, many scholars recruited for *History of the Ming* did not sit in any Qing-sponsored civil examinations, let alone advancing to the *jinshi* level.

Second, while participating in imperially sponsored scholarly projects offered many Han Chinese literati such as Mei Juecheng a "fast track" to bypass the established pipeline of civil examinations, that appears not to have been the case for the anonymous author of the postface. Whereas He Guozong, Mei Juecheng, Wang Lansheng, and all the support staff for *Origins* occupied institutionalized positions in various sections of the Qing government and were assigned to work under In-c'i by an explicit decree from the Kangxi Emperor, the author was employed, salaried, and even accommodated in Beijing by the prince, instead of either the Qing court or government. In this regard, the author of the preface appears to have been in a different kind of position than the one previously mentioned Chen Menglei, the original author of *Imperial Encyclopedia of Illustrations and Writings from the Ancient to Modern Times.* A *jinshi* laureated in 1670, Chen had served as a *shidu* "tutor" for In-c'i since 1698, a position that made him the closest advisor to the prince to the point that the latter's fall from grace in 1722 after his brother the Yongzheng Emperor assumed the throne also brought Chen down, sending him into exile in Manchuria at the age of seventy-four. Still, even though Chen was assigned to the retinue of a prince, as a *shidu* he was a permanent official of the associate fourth rank (*cong sipin* 從四

品) under the Imperial Academy, where scholars who had typically just received a *jinshi* degree in the civil examinations provided scholarly, clerical, and literary services for the Qing court before many of them would be assigned to administrative posts in the Qing government.[10] In contrast, according to his own account, not only was the author of the postface summoned to Beijing by the prince himself, after somehow getting an opportunity to meet him during his first trip to the imperial capital, he was also *lin* 廩 "given a pension" (the word literally means "granary" or "stockpile of grain") and even resided in the *sheguan* 舍館 "guest apartment" of the prince's residence. From this, it might be inferred that the author did not take any formal post in the Qing government but was hired by In-c'i as a member of his personal and domestic staff. Whereas Chen as a ranked official would have been at least nominally accountable to the Qing government, the author of the postface in all likelihood only needed to respond to the prince himself.

Regardless of their official status (or lack thereof) or the trajectory that brought them to In-c'i's service, it would appear that In-c'i had surrounding him a potentially sizeable retinue of scholars and literary staff who could assist him in his studies or perhaps even be commissioned to conduct research on his behalf. As mentioned in an early section of this dissertation, although In-c'i never claimed any intellectual ownership over Chen's *Imperial Encyclopedia*, the latter work clearly grew under the aegis of In-c'i, who provided Chen with copyists and his own library, two kinds of resources that would have been the most serviceable for compiling a *leishu* "encyclopedia by category," which amasses numerous citations of varying lengths from preexisting books. Similarly, the author of the postface describes *A Glimpse through the Pitch*

10 See Pei Qin 裴芹, Gujin tushu jicheng *yanjiu* 古今圖書集成研究 ("A Study of *Imperial Encyclopedia of Illustrations and Writings from the Earliest to Current Times*"), 31-42.

Pipes as the result of the prince's studies and research with help from literary staff members like themselves. As he took a sick leave from Beijing back to his hometown, the author writes, In-c'i sent him a tract entitled *On the Numbers of the Huangzhong Pipe* (huangzhong lüshu lun 黃鐘律數論) amounting to some ten chapters.

By all indications, this text has not survived. Likely due to the circumstances of the last few years of his life under the reign of his brother the Yongzheng Emperor, In-c'i never issued any anthology of his writings. As it turns out, though, ff. 120-125 in *Putong Guji* No. 15251 immediately after the postface on f. 119, features an eleven-part text with an introduction and ten numbered sections, which bears the title on f. 120r "On the Numbers of the *Huangzhong* Pipe,"—identical to what is mentioned in the postface. Upon closer examination, this text turns out not to be In-c'i's original tract, but rather a series of summaries and commentaries on each of its constituent parts without quoting the respective part in full. On three occasions in the text, it directly addresses "Your Imperial Highness" with praises of the latter's enlightening teachings.[11] On two occasions it alludes to its current section as being "in this chapter" (*cipian* 此篇), clearly referring to the original tract on which it is commenting.[12] In commenting on the second chapter of the original tract, the text even raises a point of criticism to the prince, writing in a smaller font.[13]

As the author of the postface on f. 119 mentions reading *On the Numbers of the Huangzhong Pipe* and mentions that a commentary of the latter text immediately follows the postface in ff. 120-125 of *Putong Guji* No. 15251, it is reasonable to infer that this latter commentary was penned by the same author who wrote the postface. Though In-c'i's short tract

11 National Library of China, *Putong Guji* No. 15251, ff. 122r, 122v, 123v.
12 Ibid., f. 121r (two occasions on this page).
13 Ibid..

is limited in scope to reconstructing the exact length, diameter, and volume of the ancient *huangzhong* pipe, sorting out the various historical sources on the topic, and critiquing prior studies on the tuning pitch pipes from Cai's *New Treatise on Pitch Pipes* to Zhu Zaiyu's *Essential Meaning of Pitch Pipes*, it turns out to have laid a significant foundation for *A Glimpse through the Pitch Pipes* and the entire fourteen-tone temperament project.

Among the many concurring arguments between *On the Numbers of the Huangzhong Pipe* and the various manifestations of In-c'i's research, from ff. 85-118 of *Putong Guji* No. 15251 to the final *Orthodox* treatise, two particular ones stand out. First, on a more specific matter, according to its commentary on ff. 120-125 of *Putong Guji* No. 15251, in particular f. 124v, In-c'i's *On the Numbers of the Huangzhong Pipe* posits a conversion rate of 9 ancient *cun* = 7.29 modern *cun* in its ninth numbered section. Given that the tract is mentioned in a postface to *A Glimpse through the Pitch Pipes* dated to 1707, it can be safely presumed to be the very first source for the eventual adoption of this conversion rate in *Orthodox*.

Second, on a more general note, the eighth numbered section of In-c'i's original tract appears to have anticipated the very structure of the pivotal Volume 1 of *Orthodox*. Unlike all the other sections of *On the Numbers of the Huanghzong Pipe*, at least according to the summary and commentary of it in ff. 123v-124r, this section does not concern anything specific regarding the *huangzhong* or the other eleven tuning pitch pipes or historical metrology. Instead, it opines on how to achieve the broader endeavor of reconstructing the ancient tuning pitch pipes and thereby restoring ancient music. After lamenting, formulaically, the loss of ancient music and the failures of previous scholars in its resurrection, the commentary directly quotes In-c'i:

> […] 今　殿下毅然斷之曰：“姑守累黍之法為律尺大分之資，則實學、實用一言而綱維大定矣。古者，先有律而後有尺，固也。今，律不可考，而尺在，溯流尋源一間耳。古者，既有度，而以黍校之，固也。今，度屢變而黍又是。考異黍，同一

間耳。明乎此而可以無畏難之慮。此制樂下手第一義也。”又斷之曰：“參古人耳聽心會之術，為考音命尺之本。則精義入神，一言而蹟隱無遺矣。古人患律之難定，欲多截管以候氣。而候氣之說，後儒闢其荒唐 […]。故不若多截管以審音。審音似乎難矣，而無難也。今世絲竹登歌，一聲出律，即戾音調，伶人俗工悉能知之，不必延、涓、摯、曠也。然後節其高下，以考中和。審其叶應，以調子母。其理則謀之學士大夫，其器則謀之伶官樂師。其是非則公之天下，萬世明乎此，又何患乎眾論之搖奪？此制樂下手第二義也”。[…][14]

[…] Now, HIS IMPERIAL HIGHNESS resolutely arbitrates [on the matter], saying: “For the time being, if the method of arranging grains of millet in a row is kept as the main principle for pitch pipes-based metrology, then the general frameworks of concrete learning [*shixue*] and concrete applications can be accomplished through just one utterance. During ancient times, there were first the pitch pipes and then metrology as a matter of course. Today, however, pitch pipes themselves can no longer be investigated, yet metrology still exists, and tracing the stream in order to locate its source is not a distant excursion. During ancient times, there were first measurements and then graints of millet were used to check them as a matter of course. Today, however, measurements constantly change, yet grains of millet are still here. Investigating the different [numbers of] grains of millet is similarly not a distant excursion. Upon understanding this, one can use it to dispel any worry of difficulty. This is the first meaning of taking initiative in establishing the institution of music [*yue*].” Then, [the prince] also judges [on the matter], saying: “Contemplating the art of the ancients in listening with one’s ears and understanding with one’s mind is the essence of investigating the tones [*yin*] and determining the system of metrology. Thus, whenever the essential meaning reaches the heart of the mind, the obscure traces of things can be dispeled by just one utterance. Past scholars worried that pitch pipes are hard to determine and wanted to slash bamboos into many pipes in order to “await the *qi*.” Yet the theory of “awaiting the *qi*” has been refuted by later scholars as nonsense […]. Therefore, it is better instead to slash bamboos into many pipes in order to examine their tones [*shenyin*]. Examining the tones [of pitch pipes] may seem difficult, yet it is not. In this day and age, whenever string and wind instruments are played to accompany singing at ritual events, whenever one note [*sheng*] deviates from the pitch pipes [*lü*] and violates the tones and the modes, even musical practitioners and unlearned players can know the

14 Ibid, ff. 123v-124r.

> deviation, not to mention the Yellow Emperor's court musician Master Yan, Duke Ling of Wei's court musician Master Juan, the Kingdom of Lu's court musician Master Zhi, or the Kingdom of Jin's court musician Master Kuang. Afterwards, one can pattern the high and low [notes] of the tuning pitch pipes in order to investigate their harmoniousness and examine the correspondences between them, in order to sort out the various modes. One can discuss their principles with scholars and officials, their instruments with court musicians and practitioners. One can proclaim the correct and the mistaken to all that is under Heaven, so that thousands upon thousands of generations in posterity will understand them, and so what is to fear regarding the swaying and wavering of public opinion? This is the second meaning of taking initiative in establishing the institution of music." […]

To recall, Volumes 1 and 2 of *Orthodox*, which presents the so-called Kangxi Emperor's fourteen-tone temperament, are collectively labelled with the heading *zhenglü shenyin* "Rectifying the Pitch Pipes in order to Examine Their Tones." Chapter 9, Volume 1 of *Orthodox*, the pivotal chapter that first reports the experiments of listening to the twelve tuning pitch pipes whose initial results were recorded in the table in ff. 88r-88v in *Putong Guji* No. 15251, lays out a bipartite epistemic structure of the tuning reform, with each part identified with a commonly used turn of phrase in the study of *lülü* "tuning pitch pipes" or musical tuning. The first part, *zhenglü* "rectifying the pitch pipes," is explained as "triple division with one part subtracted or added": the exact measurement of the ancient *huangzhong* pipe is established through research into historical metrology and the Chinese Pythagorean tuning ratios of 2:3 and 4:3 are applied in successive alternation to the length of this *huangzhong* pipe in generating those of the other eleven tuning pitch pipes. The second part, *shenyin* "examining the tones" of said pitch pipes, is explained as "mutual generation at every eighth step" following on the Kangxi Emperor's mistaken gloss of it as octave equivalence: the twelve tuning pitch pipes as well as those that are double and half their lengths are played to produce pitches, so that these pitches can be patterned

in cycles of octaves each constituted with "the five proper notes and two altered notes" of the seven-note diatonic scale.

Here, already in this commentary on *On the Numbers of the Huangzhong Pipe* that was most likely written years before the 1707 date of the postface in f. 119, In-c'i is recorded as having articulated this bipartite structure, a structure, furthermore, that not only mirrors the setup of Volume 1 of *Orthodox* but also that of *A Glimpse through the Pitch Pipes,* whose two volumes are summarized in its postface on f. 119. Whereas the "first meaning" of establishing the institution of music pertains to the epistemically silent procedure of reconstructing the exact sizes of ancient measuring units—particularly through the arrangement of millet grains, a method that Chapter 4, Volume 1 of *Orthodox* pursues—the "second meaning" involves the epistemically sonorous procedure of *shenyin* "examining the tones," which In-c'i even names as such. By equating the sounds produced by the twelve tuning pitch pipes in a scholarly setting to those produced by singers and instrumentalists during actual performances of music, In-c'i demystifies listening as a critical yet matter-of-course procedure in studying tuning pitch pipes as the foundation for a pitch organization system. Even at the nascent stage of *On the Numbers of the Huangzhong Pipe*, a short tract that focuses only on the *huangzhong* pipe itself and does not address issues of scales, *yun*'s, or modes, In-c'i already elevates *shenyin* "examining the tones" as being on par with the historicist and mathematical studies on the absolute and relative sizes of the twelve tuning pitch pipes.

In sum, as reflected in these series of exchanges between In-c'i and his anonymous household staffer over *On the Numbers of the Huangzhong Pipe*, the seed for listening experiments that would lead to a fourteen-tone octave comprising two complementary seven-

note diatonic *yun*'s was already sown even before the author of the postface formally joined the prince's staff.

From Metrology to Musical Theater, and Beyond

Indeed, according to the author of the supposed postface to *A Glimpse through the Pitch Pipes*, *On the Numbers of the Huangzhong Pipe* was only the beginning of In-c'i's interests in musical tuning. After the author of the postface read this short tract—which apparently cured his illness—the prince summoned him to Beijing and took him into his retinue. From there, the author claims to have witnessed the prince's research "advance every day and every month" to the point of amounting to "volumes and scrolls," as he dedicated all his leisure time to "investigating the ancient and the modern in order to distill and analyze their similarities and differences." According to the author's postface, the "volumes and scrolls" that In-c'i accumulated, presumably over a span of quite some years, would become *A Glimpse through the Pitch Pipes*, which their summary later in the postface would reveal as a forerunner to the fourteen-tone temperament tuning reform in *Orthodox*. Nonetheless, I would argue that both In-c''s accumulated studies and his research vision for musical tuning went far beyond ff. 85-118 as a series of preparatory materials for *A Glimpse through Pitch Pipes* and indeed the final version of *Orthodox Meaning of Pitch Pipes*. Granted, a series of successive overhauls of the *huangzhong*-based metrology, the pitch organization system and division of the octave, and the designs for various musical instruments used for courtly rites and ceremonies might already seem to have been quite thorough and impressive in delineating collectively what a treatise on tuning reform could achieve in four volumes. But as the rest of the *Putong Guji* No. 15251 manuscript would show, even before the core research leading to the fourteen-tone temperament

had begun, In-c'i had set his eyes on a grander ambition: to reform and harness Chinese opera in the service of the Qing Empire.

This vision, which sought to move from rather parochial studies of historical sources on metrology all the way to the genre of musical theater that defined the soundscape of the Qing court as well as urban centers and rural market and temple fairs across China during the late 17th and 18th centuries, bore its earliest testament in ff. 52v-83 of *Putong Guji* No. 15251. Those folios immediately precede the *biao* in ff. 84r-84v, followed by the lab notes, reckonings, and instrument designs in ff. 85-118, the postface in f. 119, and the commentaries and summaries of *On the Numbers of the Huangzhong Pipe* in ff. 120-125. A brief foreword on f. 52v explains the background and raison d'être of its materials, here transcribed and translated in full:

> 樂律之說，古今沿革，見于史志及專家之學。考求訂正，難于書一。夫是非辨晳，固宜各舉其全；而實用發凡，又在先提其要。茲奉　令，纂輯入用之法，務取易簡，以便施行，不敢以虛文雕繪，塗飾耳目。凡古人議論，是者存之；有未備或舛譌者，間以蘁蕘，次第闡明。謹擬末議并圖，先進上若干首，以備採擇。至其詳細，另具全書，不敢多贅。[15]

> When it comes to theories of music [*yue*] and pitch pipes [*lü*], its various transmissions and developments from the ancient to the modern eras are seen both in historical records and in specialist studies. Investigating, verifying, and correcting them are very difficult to do in an integrated manner. The truth is, in distinguishing the correct from the mistaken, it is of course better to fully enumerate [the arguments on] both sides. Nonetheless, in applying them to use and in positing their main points, it is conversely about summarizing the gist of things above all. Now, in compliance with [the Prince's] ORDER, we compile methods that are applicable to use, striving for simplicity and straightforwardness so that they can be easily implemented, and daring not to embellish them with empty words or to dress up what one sees or hears. Of whatever past scholars have debated on, we include the correct ones; as for those that are imperfect or erroneous, we sparingly insert our unenlightened opinions and explain them one after another. We respectfully pull together our

15 Ibid., f. 52v.

> unworthy discussions in drafts with illustration and submit several pieces in advance in so that they might be ready for selection and adoption. As for their details, the full books are available separately, so we dare not be redundant.

Because the word *ling* 令 "ORDER" is preceded in this text with a typographical blank space known as *taitou* 抬頭 "raising the head," I have rendered it using all capital letters for the words that have triggered it (as in my previous translations of parts of *Putong Guji* No. 15251). It represents an order that necessarily came down from a person of imperial status. Given the broader context of the manuscript, that person was almost certainly In-c'i. The foreword conveys three pieces of information regarding the *zuanji* 纂輯 "gathering and compiling" previous studies on music and pitch pipes it introduces. First, the compilation focused more on quantity than on quality. As the foreword reports, even when a source is deemed problematic or outright wrong, the compilers do not exclude it but simply add their own opinions as needed. Second, the compilation was conceived much more as a digest than an anthology or a *leishu* "encyclopedia by category," which comprises direct citations from existing works that can often be quite extensive. In addition to incorporating their own commentaries from time to time, the compilers considered it more productive to keep the work simple, straightforward, and focused on the *yao* 要 "main points" or "gist" of the cited texts, rather than indulging in details, which, as they point out, would otherwise be available in the original titles anyway. Third, and most importantly, the compilation had a clear, forward-looking function, indeed a goal to which it could be *shiyong* 實用 or *ruyong* 入用, both of which mean "applicable for use." As the conclusion of the foreword suggests, the compilers submitted to In-c'i this series of digests, summaries, and commentaries on a broad variety of prior writings on music and tuning pitch pipes so that they "might be ready for selection and adoption." In other words, the compilation, with all its focus on

comprehensiveness, concision, and applicability, was in all likelihood conceived as a reference book that served a large research and publication project on musical tuning and music writ large.

Surveying the content of ff. 53-83 following immediately after the foreword in *Putong Guji* No. 15251 gives a sense of just how large the scale of this project was envisioned to be. The reference book comprises six numbered sections, each with a succinct title:

ff. 53-54	一議黃鐘之管
ff. 55-58	二議生十二律
ff. 59-60	三議審音
ff. 61-64	四議起聲調
ff. 65-70	五敷古今宮調
ff. 71-83	六議定樂譜

ff. 53-54	One: Discussion of the pipe of *huangzhong*
ff. 55-58	Two: Discussion of generating the twelve tuning pitch pipes
ff. 59-60	Three: Discussion of examining the tones [*shenyin*]
ff. 61-64	Four: Discussion of the initial note of each mode
ff. 65-70	Five: Covering all the modal systems from ancient to modern
ff. 71-83	Six: Discussion of determining the system of notation

Even with this scanty list of the headings of each of its six constitutive sections, it is more than clear that the first half mirrors the subsequent research for *A Glimpse through the Pitch Pipes* and the eventual treatise of *Orthodox Meaning of Pitch Pipes*, at least in structure. As would be the case with the latter two works, the first half of the reference book proceeds by reconstructing the exact sizes of the *huangzhong* pipe through historical metrology in the first section (c.f. Chapters 1-4, Volume 1 of *Orthodox*), to generating the twelve tuning pitch pipes proportionally to such a *huangzhong* pipe in the second section (c.f. Chapters 5-8, Volume 1 of *Orthodox*), and finally to "examining the tones" of these pitch pipes in order to develop a system of pitch organization (c.f. Chapters 9-12, Volume 1 of *Orthodox*).

Notably, unlike either the final treatise of *Orthodox*, the summary of *A Glimpse through the Pitch Pipes* in f. 119 of *Putong Guji* No. 15251, or the lab notes, reckonings, and designs in ff. 85-118 of the same manuscript, the first three sections of the reference book do not put forward any specific argument, or indeed any of the key findings that would set In-c'i's tuning reform apart from many previous studies. For example, while I have shown that the conversion rate of 9 ancient *cun* = 7.29 Qing modern *cun* is a characteristic finding of In-c'i's research from *On the Numbers of the Huangzhong Pipe* to *Orthodox*, nowhere is such an argument to be found in the first section of the reference book. Instead, the latter strings together a variety of historical sources and prior studies on the subject of the size of the *huangzhong* pipe and the *huangzhong*-based metrology, ranging from *Writings of Prince Huainan* on the earliest end to Zhu Zaiyu's much more recent *Essential Meaning of Pitch Pipes.* The second and third sections of the reference book appear to be similarly open-minded as they too focus much more on amassing various sources and theories on the tuning pitch pipes and music theory in general than on advancing any single solution in particular. Among all the sources it surveys, section two on using *huangzhong* to generate the length of the other eleven tuning pitch pipes focuses especially on three: Sima Qian's 2nd-century BCE *Records of the Grand Historian*, Cai Yuanding's *New Treatise on Pitch Pipes*, and Zhu Zaiyu's *Essential Meaning of Pitch Pipes*. Remarkably, the section devotes most of its attention to this latter work and appears almost convinced by Zhu Zaiyu's ostensible rejection of "triple division with one part subtracted or added" method and proposal for a twelve-tone equal temperament, in stark contrast to the full embrace of the conventional Chinese Pythagorean tuning ratios in the eventual *A Glimpse through Pitch Pipes* and *Orthodox*. Section three on "examining the tones" similarly concludes with an apparent "deviation" from what is arguably the most critical component of the fourteen-tone temperament.

Rather than segregating the odd- and even-numbered pipes in two complementary diatonic *yun*'s and thus effecting a fourteen-fold division of the octave, the section ends with a diagram of the conventional pitch organization system, one in which the *gong* (*do*) and the *zhi* (*sol*) notes fall respectively on the *huangzhong* (1st) and *linzhong* (8th) pipes.

The absence of all the arguments that would define the eventual fourteen-tone temperament and *Orthodox* treatise might date the reference book to even before In-c'i's *On the Numbers of the Huangzhong Pipe*. But that does not at all mean that it was irrelevant to the tuning reform that the prince spearheaded on his own initiative before bringing it into his father's *Origins* project. Despite its general emphasis on comprehensiveness over argumentation, the first three sections of the reference book are exceptionally clear on one matter: the bipartite epistemological structure summed up in the title of Volumes 1 and 2 of *Orthodox*: *zhenglü shenyin* "Rectifying the Pitch Pipes in order to Examine Their Tones." Not only does the progression of content in this first half of the reference book mirror that of Volume 1 of *Orthodox*, as I have shown earlier, it also explicitly articulates the importance of *shenyin* "examining the tones [of pitch pipes]," which, again, is associated in the pivotal Chapter 9, Volume 1 of *Orthodox* with the Kangxi Emperor's mistaken gloss of "mutual generation at every eighth step" as octave equivalence. After the foreword quoted in full above speaks only of the rationale and background of the reference book, the beginning of the first section on *huangzhong* doubles as an introduction to the entire reference work:

> 樂律始于伶倫截竹，為黃鍾之宮，轉生各律。其後，夔典樂，至三代而大備。秦滅舊章，樂經殘缺，漢儒綴輯遺文。歷代互有因革，數千年迄無定論。皆由工師僅習鏗鏘而不明其理，儒者止爭文義而不適于用。是以議論滋多，成績日鮮。古聖人制器尚象，本非僅託空言。 故正樂貴乎審音，先宜制律。[…][16]

16 Ibid., f. 53r.

> Music [*yue*] and pitch pipes [*lü*] began when Ling Lun harvested bamboos, made the *huangzhong* pipe as the *gong* [*do*] note, and in turn generated all the [other] tuning pitch pipes. Subsequently, Kui established the institution of music, which became grand and perfect during the Three Dynasties. After the Qin exterminated the old writings, the Confucian *Canon of Music* only survived in fragments, leaving Confucian scholars of the Han era to cobble together what was left. As each era continued and changed what it had gotten from the previous one, there was no consensus for millennia until now. This was all because musical practitioners only study the ringing and clanging of their instruments without understanding their principles, and because Confucian scholars only fight over the meaning of texts without making them applicable to use. Therefore, as quarrels increasingly proliferate, achievements become increasingly rare. Originally, however, when the ancient sages made instruments to heed the patterns [of the cosmos], they did not only rely on empty words. Therefore, the importance of rectifying music resides in examining the tones, and it is appropriate to first make the pitch pipes. […]

Granted, blaming the ignorant musical practitioner and the pedantic Confucian scholar collectively for ruining any chance of restoring the perfect music of the ancients was clichéd in standard narratives about the state of musical scholarship. By contrasting the onomatopoeic *kengqiang* 鏗鏘 "[the bright metallic] ringing and clanging [of musical instruments" of the musical practitioners with the *wenyi* 文義 "meaning of texts" of the scholars, however, the compilers of the reference book articulate a position whereby these two inherently parochial sources of knowledge can and must be combined. Indeed, even though it was the emphasis on *shenyin* "examining the tones" that would lead to the listening experiments and in turn the fourteen-fold division of the octave in all the subsequent stages of musical research under In-c'i, still the study of *shu* "numbers" through historical sources as well as mathematical computations would serve as the proper knowledge-producing process insofar as it concerns *zhenglü* "rectifying the pitch pipes."

As I will elaborate more in the next chapter, what distinguishes the epistemic logic of the fourteen-tone temperament tuning reform is not simply its commitment to sounding and listening; instead, it is its commitment to both historicism and empiricism that simultaneously conjoins and segregates these two different modalities of knowledge production. Indeed, though literally entitled "Three: Discussion on Examining the Tones," the third section of the reference book reiterates that making tuning pitch pipes through sources and mathematics is just as important for musical tuning as is listening to the sounds of these pipes in order to establish a pitch organization system. The introduction to this section concludes: "granted, without establishing the pitch pipes, there is of course no basis from which to examine the tones [*shenyin*], yet without examining the tones, there is also no basis from which to determine the pitch pipes" (夫不制管固無由以審音，不審音亦無由以定律). Thus, already in this early stage, the musical research In-c'i sponsored had begun to negotiate the epistemic relations between the textual and mathematical reconstructions of the twelve tuning pitch pipes as historicist objects and the empirical experiences of their sounds as audible phenomena.

In addition to addressing the broader epistemological questions related to *shenyin* "examining the tones," the third section also highlights its specific importance in resolving two problems.[17] Almost certainly unknown to compilers of the reference book at the time, these two problems would have major implications in the fourteen-tone temperament reform. First, the section notices contradictory resources in previous studies over whether the diameters of the twelve tuning pitch pipes should be the same or should be proportional to their lengths. Put differently, should the original, full-length twelve tuning pitch pipes and those that are half or twice as long be *tongjing guan* "pipes of the same diameter" or *tongxingguan* "pipes of the same

17 Ibid., ff. 59r-60r.

shape," which is to say identical in length-diameter ratio? Second, as a corollary to the first question, the section devotes yet again much room to Zhu Zaiyu, who claims that when two pipes of the same diameter bear a 1:2 length proportion, their pitches do not produce an octave, whereas when two pipes of the same shape bear a 1:2 length *and* diameter proportion, their pitches do end up producing an octave. As a result, Zhu argues, all the tuning pitch pipes should be fashioned not as pipes of the same diameter, but as pipes of the same shape. Again, consistent with its overall value of inclusiveness over judgement, the section refrains from deciding which sides of the two questions are correct, that is, whether the twelve tuning pitch pipes should be pipes of the same diameter or pipes of the same shape, and whether two pipes of the same diameter in a 1:2 length proportion do in fact create an octave. Instead, it opines: "none of these questions can be fought over with empty words alone; one must make pitch pipes and listen to them, and only then will the right and the wrong be decided" (凡此皆不可以空言爭，必制管聽之而是非始定).[18] Thus, already in this part of the reference book, which might have dated to even before In-c'i's *On the Numbers of the Huangzhong Pipe*, a concrete plan to conduct listening experiments looking for octave relations between pitch pipes (cf. Chapter 9, Volume 1 of *Orthodox*) and to determine whether pipes of the same shape should have any role in the system of pitch pipes in addition to those of the same diameter (cf. Chapters 12, Volume 1) had already taken place.

Whereas its first half clearly parallels in structure and content In-c'i's subsequent reform of musical tuning in *A Glimpse through the Pitch Pipes* (as well as the lab notes and reckonings in ff. 85-118 of *Putong Guji* No. 15251) and *Orthodox*, the latter three sections of the reference book contain materials that would not end up in the Qing's official music theory treatise as

18 Ibid., f. 59v.

Volumes 43-47 of *Orthodox*. Nonetheless, because these three sections immediately follow those preparatory materials for In-c'i's research on musical tuning, I argue, they actually reveal In-c'i ultimate agenda: a reform to Chinese opera.

Though the Qing's Manchu rulers governed China as a minority conquest regime, they arguably surpassed all preceding Chinese regimes in opera patronage. Not only did opera's penchant for historical themes help the Manchus appropriate the Chinese historical imagination, but sponsoring these immensely popular theatrics also brought the court closer to the Chinese landed gentry and merchant guilds, the Empire's bread basket and tax base, many of whom were opera aficionados themselves. Then too, though the Qing remained on the *su* 俗 "vulgar" end of the echelon of genres in contrast to the *yayue* "elegant music" that Confucianism stipulated for a virtuous ruler, opera dominated all court rituals of the Qing, from monthly feasts to diplomatic receptions and military triumphs. And while theater troops from across China performed for special occasions and imperial tours, the court also established an academy of music, *nanfu* 南府, to oversee regular opera performances and train eunuch performers, who had been responsible for all types of music at the court since the 17th century.[19]

The reform to opera envisioned in the second half of the reference book, furthermore, concerns the *diao* or modes most particularly. Ever since the genre matured in the 14th century, most Chinese operatic traditions had been deriving their melodic materials almost exclusively from preexisting *qupai* 曲牌 "fixed tunes." Comparable to tunes used for contrafacta in the west,

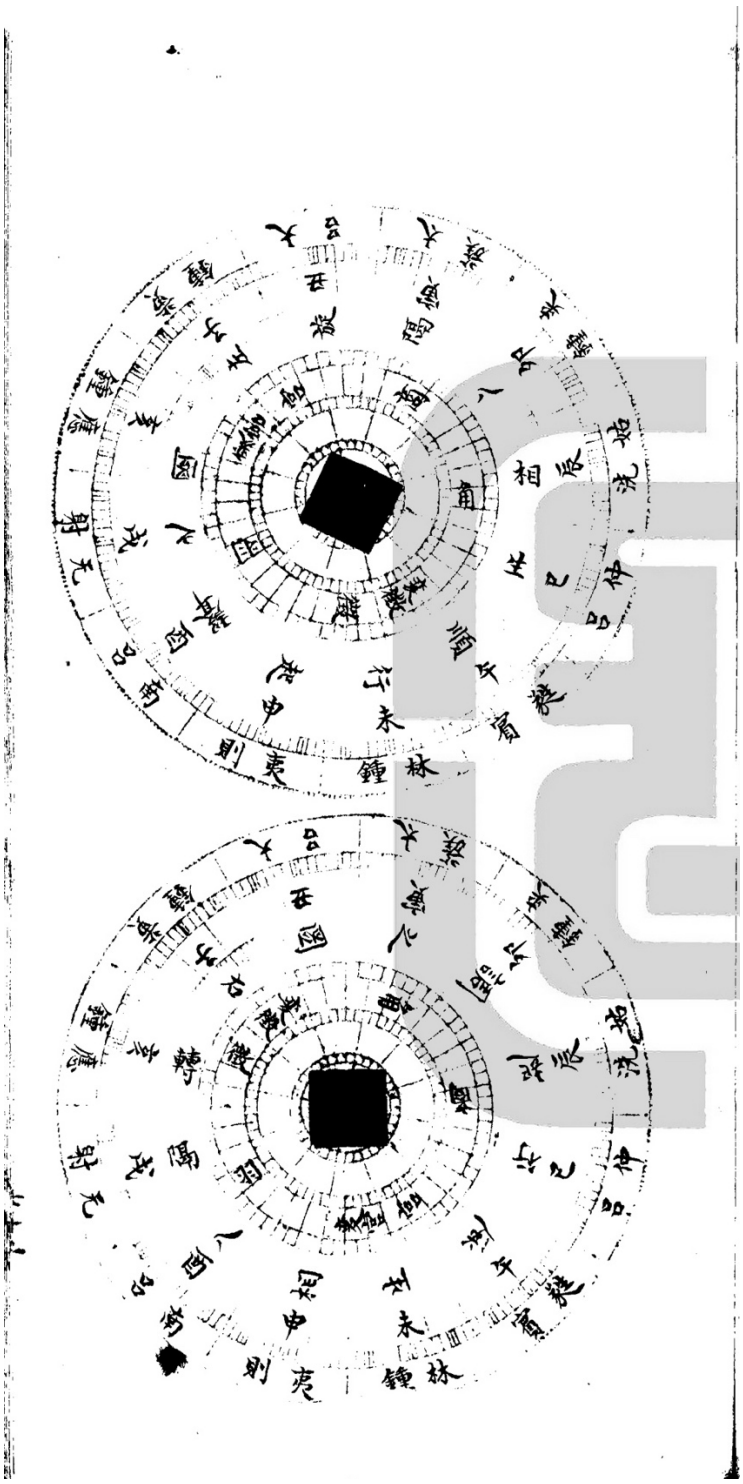
19 See Ye Xiaoqing, *Ascendant Peace in the Four Seas: Drama and the Qing Imperial Court* (Hong Kong: The Chinese University Press, 2012), 15-29; Zhu Jiajin 朱家溍 and Ding Ruqin 丁汝芹, *Qingdai neiting yanju shimo kao* 清代內廷演劇始末考 ("Investigation on the history of drama in the Qing inner court") (Beijing: The Forbidden City Publishing House, 2014), 21-36.

each fixed tune features a characteristic metric and rhyme scheme for fitting new lyrics and a melody that is malleable to the tonal patterns of different words. Even the composition of new operas did not involve creating any significant new melodic materials. Instead, the creative process typically focused on selecting and fitting those preexisting and relatively fixed tunes to the newly written plays and lyrics, which not only posed aesthetic challenges but also technical ones, including outfitting them with phonetics, versification, and, indeed, musical modes. In fact, the theoretical backbone for these thousands of fixed tunes used in opera as well as in a variety of singing genres was precisely the system of eighty-four modes.[20] To recall, this system of eighty-four modes combine the (conventional) twelve-tone octave with the seven-note diatonic scale through *xuanggong* "rotating the *gong*" or transposition and *zhuandiao* "shifting the mode" or modal mutation. Specifically, in each seven-note *yun*, the seven notes *gong* (*do*), *shang* (*re*), *jue* (*mi*), *bianzhi* (*fa#*), *zhi* (*sol*), *yu* (*la*), and *biangong* (*ti*) take turn being the modal final, and this entire *yun* is also transposed so that the *gong* would be matched with each of the twelve tuning pitch pipes.

Thus, since the third section of the reference book concludes with the conventional pitch organization system, in particular the *yun* of *huangzhong* (1st pipe)-*gong*, *taicu* (3rd)-*shang*, *guxian* (5th)-*yue*, *ruibin* (7th)-*bianzhi*, *linzhong* (8th)-*zhi*, *nanlü* (10th)-*yu*, and *yingzhong* (12th)-*biangong*, the fourth section puts "rotating the *gong*" and "shifting the mode" in action through two slide charts or volvelles on f. 62r (see Illustration 4-1). Each of these two charts comprises one fixed wheel labeled with the twelve tuning pitch pipes on the outside and one movable wheel labeled with the seven notes of the diatonic scale on the inside. Of course, since this was before

20 See for reference Liang Mingyue, *Music of the Billion: An Introduction to Chinese Musical Culture* (New York: Heinrichshofen, 1985), 234-243.

Illustration 4-1 Two rotating slide charts of volvelle showing how to "rotate the* gong*" and "shift the mode"*, Putong Guji *No. 15251, f. 62r[21]



21 The difference between these two charts is that the moving circle in the upper chart rotates counterclockwise, whereas the moving circle in the lower chart rotates clockwise.

ff. 85-118, where the fourteen-fold division of the octave and the segregation of odd- and even-numbered pipes into two complementary diatonic *yun*'s was first recorded, the outer wheel is divided not into fourteen equal radians but twelve, and the seven notes on the inner wheel are not evenly distributed but distributed with the distance between *biangong* and *gong* and *bianzhi* and *zhi* half as large as are those between all the other notes. The rest of the fourth section enumerates each of the twelve *yun*'s, whereby the *gong* of the *yun* is successively "rotated" from *huangzhong* (1st pipe) to *dalü* (2nd pipe) and eventually all the way to *yingzhong* (12th pipe), and each of the seven modal mutations under each *yun*, whereby each of the seven notes, assigned to a particular pitch pipe under the *yun*, takes turn being the tonic or final of a mode.[22]

By virtue of combinatorics (twelve "rotations" of *yun* times seven "shifts" of the modal final), the system of eighty-four modes presented in the fourth section of the reference book is the necessary outcome of a pitch organization system that specifically combines a twelve-fold division of the octave with the seven-note diatonic scale by locating the *gong* and *zhi* note—the perfect fifth—onto every first and eighth of the twelve tuning pitch pipes. Whereas the fourth section exhausts all the modal transpositions and mutations, however, the fifth section studies the modes that were actually documented as having been used.[23] Unsurprisingly, of all the eighty-four modes, most of them were never more than theoretical possibilities. First, we read that right before the Tang era (618-907), only sixteen modes were in use (ff. 68r-68v), as the *yun* is rotated only to four of the twelve tuning pitch pipes, and only four of the seven notes of the diatonic

22 National Library of China, *Putong Guji* No. 15251, ff. 63r-64v.

23 For the exceptionally convoluted history in which the system of modes (both theoretical and practical) developed at the Tang court would later develop and evolve, see Qiu Qiongsun 丘瓊蓀, *Yanyue tanwei* 燕樂探微 ("Investigating the details of music of the Tang court") (Shanghai: Shanghai guji chubanshe, 1985).

scale were used as modal finals. Furthermore, although court musicians of the Sui (581-618) and early Tang eras first articulated the eighty-four-mode system, only twenty-eight of them were in use (f. 68v), wherefore the *yun* is now rotated to seven out of the twelve tuning pitch pipes. While Song-era (960-1279) sources still recorded the twenty-eight-mode system (ff. 69r-69v), ever since the Mongol rule of China, or the Yuan era (1259-1368)—arguably the first golden age of Chinese opera that set the stage for the genre up to the 18th century—only seventeen modes had been in use for *beiqu* 北曲 "northern opera" (f. 70r) and only thirteen in *nanqu* 南曲 "southern opera" (ff. 70r-70v). This discussion of the use of modes in the context of operas continues into the sixth and last section of the reference book. After first discussing the *gongche* notation syllables,[24] which were primarily used by opera performers, the sixth section catalogues nearly a thousand fixed tunes used in operas[25]—all ordered according to their modes—and concludes by discussing the contours, cadences, permissible mutations, and affective characteristics of each mode.[26]

The studies on modes and fixed tunes in operas in the reference book serve two purposes. First, as the reference book claims, many of the modes still used in northern and southern operas at the time had remained in name only. Though these proper names could be traced back to those mentioned in historical sources from as early as the Tang era, their musical features as used in operatic practices had long since deviated from their original forms. What is more, the names of those modes often did not contain any clear information on where the mode belongs in the eighty-four-mode system combining the twevel pitches with the seven-note diatonic scale—just as the Greek names "Dorian" or "Lydian" are in and of themselves poor indices of which notes

24 National Library of China, *Putong Guji* No. 15251, ff. 71r-73v.
25 Ibid., ff. 76r-81v.
26 Ibid., ff. 82r-8rv.

of the diatonic scale are their respective modal finals. Additionally, changes in the ordering of modes across different periods also caused confusion. Thus, by fleshing out how each of the seventeen modes used in northern opera and thirteen modes in southern opera had evolved from the original system of eighty-four modes first articulated at the court of the Sui and Tang eras, the reference book sought to bring the use of modes in opera to conform to their proper melodic and intervallic features as stipulated by the eighty-four-mode system as the theoretical backbone.

Second, the work on modes and fixed tunes in operas in the reference book is imbued with a reconstructionism that goes beyond the modes themselves, indeed paralleling the yearning for a broader reconstruction of *guyue* "ancient music" in the scholarly paradigm of *lülü* "tuning pitch pipes" or musical tuning. As the opening prose text of the fifth section of the reference book explains, opera, despite their association with the *su* "vulgar," actually descended from a genealogy of sung genres that dated back all the way to—you guessed it—the ancients:

> […] 樂有情，情成文，所謂詩言志歌永言也。樂有聲，聲為譜，所謂聲依詠，律和聲也。古者未有譜，而先有其情。發為歌謠，一唱三歎。聖人體天地之和，因制聲調，自然中節。長言不足，至於手舞足蹈，而樂成焉。後人不能述作，僅就工師所傳，鏗鏘鼓舞之節，按譜填詞。不數傳而宮調殘闕，愈變愈下。故自三百篇，而為樂府，而詩，而詩餘，而北調，而南詞。今南北曲，亦僅存續餘，自詩餘以上，皆不能入律。救弊之道，宜先求之聲譜，從今之詞曲，逆推而上，考其同異。如南詞九宮十三調，較之北曲十七宮調，同者十一調。[…][27]
>
> […] Music [*yue*] has feelings, feelings become words, and thus [Emperor Shun said]: "may poetry express the will and songs chant such expression." Music [*yue*] has notes [*sheng*], notes become notated, and thus [Emperor Shun said]: "may notes follow such chanting and pitch pipes accord with such notes." During the ancient times, there was no notation, but there were first feelings. When feelings were expressed into songs and ditties, one person sang and three others cheered. The Sages recognized the harmoniousness between Heaven and Earth [in such singing], and

27 Ibid., f. 65r.

> used it to establish systems of notes and modes, and [the singing] was naturally compliant with the rhtythm. [As the preface to the Confucian *Canon of Songs* says,] when long tones became inadequate, to the point that [the singer] would wave their hands and swing their feed, music [*yue*] thus became complete. Later people could neither articulate nor create [this music]; they could only take what the musical practitioners had transmitted in the form of instrumental melodies and rhythms, and could only take the notated [tunes] and fill in new lyrics. Within just a few generations of transmission, the system of modes fell into disrepair, and it has become worse and worse. Therefore, songs recorded into the *Canon of Songs* deteriorated into those folk tunes collected by the Bureau of Music [of the Qin and Han eras, c. 3rd century BCE to 3rd century CE], which deteriorated into metered poetry [of the Six Dynasties, Sui and Tang eras, c. 3rd century to 10th century], which deteriorated into lyrics [of the Song era, c. 10th century to 13th century], which eventually deteriorated the northern and southern opera tunes. Even the northern and southern operas today are only what have lingered on from their original forms, and anything that came from before the lyrics [of the Song era] could no longer be sung to music. In order to reverse these defects, it is appropriate to first figure out the notation of notes and then trace the opera tunes of the present day back to the previous eras and investigate their similarities and differences. For example, southern operas today have thirteen total modes, nine of which are *gong* modes; compared to the seventeen modes of northern operas, eleven are identical. […]

It is only after narrating this millennia-long history of how *qu* "operas" "tunes" of the 17th and 18th centuries originated from the singing and dancing of the ancients that the reference book begins to discuss modes in the context of opera tunes and the importance of reconstructing their proper musical features. The passage above reveals that reforming the use of modes in opera was but part of a much more ambitious agenda: to use opera to resurrect the prefect music of the ancients specifically as a form of immediate, emotive singing. The beginning of the passage combines both Emperor Shun's words recorded in the Confucian *Canon of History* and the "Records on Music" (*yueji* 樂記) from *Records of Rites* (liji 禮記, c. 1st century BCE), which is one of the three constituent texts of the Confucian *Canon of Rites*, Narrating the origins of *yue*

"music" as when *qing* "feelings" became so overwhelming that not only did they outpour words (*wen* 文, literally "written glyphs") and morphed into songs, but the singing also aroused the arms to wave, feet to swing, and the entire body to dance. This literally embodied act of sounding among the ancient progenitors of music is in turn contrasted with those on the other side of the ancient-modern wedge. Whereas the embodied music-making of the ancients relied only on outward expressivity and nothing of the sorts of written notation, modern musicians could only take the preexisting fixed tunes and fill them in with new lyrics. In other words, whereas the ancients were able to sing, dance, and make *yue* "music" as a thoroughly integrated and embodied form of direct expression of the mind, the moderns, no longer able to create anything anew, could only rely on the traces of singing, which are notations of preexisting fixed tunes, and add new lyrics to them in order to sing anything at all.

Adding further to the cataclysmic "fall" from the ancients' perfect singing, the moderns, as it turned out, could not even get the notation or fixed tunes rights. As a result, the passage narrates a series of deteriorations from the folk tunes of the Western Zhou era (c. 1046-771) all the way to the northern and southern opera tunes of the present day. This chain of successive "falls" spanning two millennia links all the "signature" lyrical genres of each era: the *yuefu* 樂府 songs of the Qin and Han eras that were either collected from the fields or composed in imitation of their styles; the *lüshi* 律詩 or metered poetry that flourished among literati circles beginning with the Six Dynasties period (c. 220-589) and reached its peak during the Tang era; the *ci* 詞 or lyrical tunes, sometimes known as *shiyu* 詩餘 "what is left from metered poetry," that developed during the Tang and climaxed during the Song era; and finally the *qu* 曲 or opera tunes, sometimes known as *ciyu* 詞餘 "what is left from lyrical tunes," which began to flourish since the 13th and 14th centuries. In fact, the passage argues, even the opera tunes themselves have

deteriorated somewhat, thus extending the chain of degeneration into the contemporary moment. What is more, though *Canon of Songs*, *yuefu* songs from the Qin and Han era and metered poetry from the Six Dynasties, the Sui and the Tang all used to be sung, as the passage suggests, the knowledge of how to sing them had been completely lost, leaving only their lyrics as literary texts.

As I will show in the last chapter of this dissertation, this linear history of successive deterioration of songs from the ancients' spontaneous singing and dancing all the way to early modern operas was a widespread narrative in writings on songs, operas, and folk tunes as well as Confucian scholarship on *Canon of Songs* during the 15th to 18th centuries. Here, by positing contemporary opera tunes as the however imperfect remainder, the trace, or all that has been left from the ancient songs, the reference book identifies opera tunes as one end of a thread of singing and songs by which one can trace back to the other end, which was the perfect music and songs of the ancients. Insomuch as it could bring contemporary opera tunes and song cultures to conform to earlier practices if not to the earliest ancient songs, a reform to the use of modes and fixed tunes in opera as a remnant of ancient singing may constitute the first step in reverting the *longue durée* loss and degeneration of ancient music.

Given that the structure of the reference book proceeds from making the twelve tuning pitch pipes (first and second sections) to deriving a system of pitch organization from listening to the sounds produced by these twelve tuning pitch pipes (third section), to deriving the system of modes from this pitch organization system (fourth section), to finally examining the use of some of these modes in the context of fixed tunes and opera (fifth and sixth sections), it is clear that the ultimate research agenda laid out in the reference book is not musical tuning *per se* but how tuning could be used to reform the theory and practice of modes in opera and to use opera as the

lingering thread to trace back to the embodied songs of the ancients. Granted, whereas the *biao* in ff. 84r-84v formally declared—to the spirits of three ancient progenitors of music, not in the least—his intention to reform musical tuning, no such declaration regarding opera reform could be found either within or outside the reference book in *Putong Guji* No. 15251. Yet the traces left by In-c'i's readership of the reference book in *Putong Guji* No. 15251 suggests the importance of opera reform as that final objective of the research into music and music theory conducted under his aegis. To recall, In-c'i left annotations in vermilion throughout the entire manuscript. While he pasted additional papers into the fifth and sixth sections of the reference book just to accommodate his profuse comments on the fixed tunes and modes, he appeared to have read through the first three sections on tuning pitch pipes too cursorily even to punctuate them. This discrepancy of "wear and tear" between these two parts of the reference book suggests that even before his research on musical tuning and eventually the fourteen-tone temperament began to take shape, In-c'i had already set his eyes on the ambitious objectives of opera reform and even going back further in the history of songs.

As it happened, just as the materials on musical tuning in the first half of the reference book laid a foundation for later research that would in turn lead to the fourteen-tone temperament tuning reform and to the 1714 treatise *Orthodox Meaning of Pitch Pipes*, the second half of the reference book, aimed at reforming fixed tunes and modes in opera, would also bear fruit in imperially sponsored scholarship. At the same time In-c'i led his team of scholars working on *Orthodox*, he oversaw two modally ordered compendia of fixed tunes: *The Emperor's Library of Lyrical Tunes* (yuding cipu 御定詞譜, 1715) featuring tunes from before the 13th century, and *The Emperor's Library of Opera Tunes* (yuding qupu 御定曲譜, 1715) featuring tunes from after

the 13th century.[28] These two anthologies of tunes clearly built on the latter three sections of the reference book, particularly the sixth section, which features a modally ordered catalogue of fixed tunes. Indeed, even after In-c'i's political demise in 1722 after his brother the Yongzheng Emperor rose to the throne, his brother In-lu, to whom the *Origins* project was granted, also assumed In-c'i's library and continued his research agenda. This culminated in *Grand Compendium of Northern and Southern Opera Tunes in All Modes* (jiugong dacheng nanbeici gongpu 九宮大成南北詞宮譜, 1746), which features more than two thousand tunes, four thousand musical scores, and extensive discussions of the history, usage, and characters of the modes.[29] It was also around the same time that the Qing court would codify the ritual use of opera, assigning different ceremonial functions to some two hundred newly composed *chengying xi* 承應戲 "on-demand intermezzi."[30]

From *Ut*, *Re*, *Mi* to Fourteen-Tone Temperament

As it turns out, opera was not only the ultimate objective of research by In-c'i into music and musical tuning that led to the fourteen-tone temperament, but also a significant (if not decisive) factor that brought about the Kangxi Emperor's misunderstanding of the term *geba xiangsheng* 隔八相生 "mutual generation at every eighth step." To figure out where the Kangxi Emperor could have possible learned the definition of this term as octave equivalence in

28 In-c'i et al. *Yuding qupu* 御定詞譜 ("*The Emperor's Library of Lyrical Tunes*," 1715), and In-c'i et al. *Yuding qupu* 御定曲譜 ("*The Emperor's Library of Opera Tunes*," 1715). See *Kangxichao hanwen zhupi zouzhe* 康熙朝漢文硃批奏摺 ("Complete Translation of Vermilion-Annotated Memorials in Manchu from the Kangxi Era"), Vol. 8 (1985), 1178-1180; No. 3115.

29 In-lu et al. *Jiugong dacheng nanbeici gongpu* 九宮大成南北詞宮譜 ("Grand Compendium of Northern and Southern Opera Tunes in All Modes," 1746).

30 See Ye Xiaoqing, *Ascendant Peace in the Four Seas*, 57-128.

egregious contradiction to every Chinese-language source on music theory, one only needs to look at the earlier ff. 1-51 of *Putong Guji* No. 15251, specifically f. 5v:

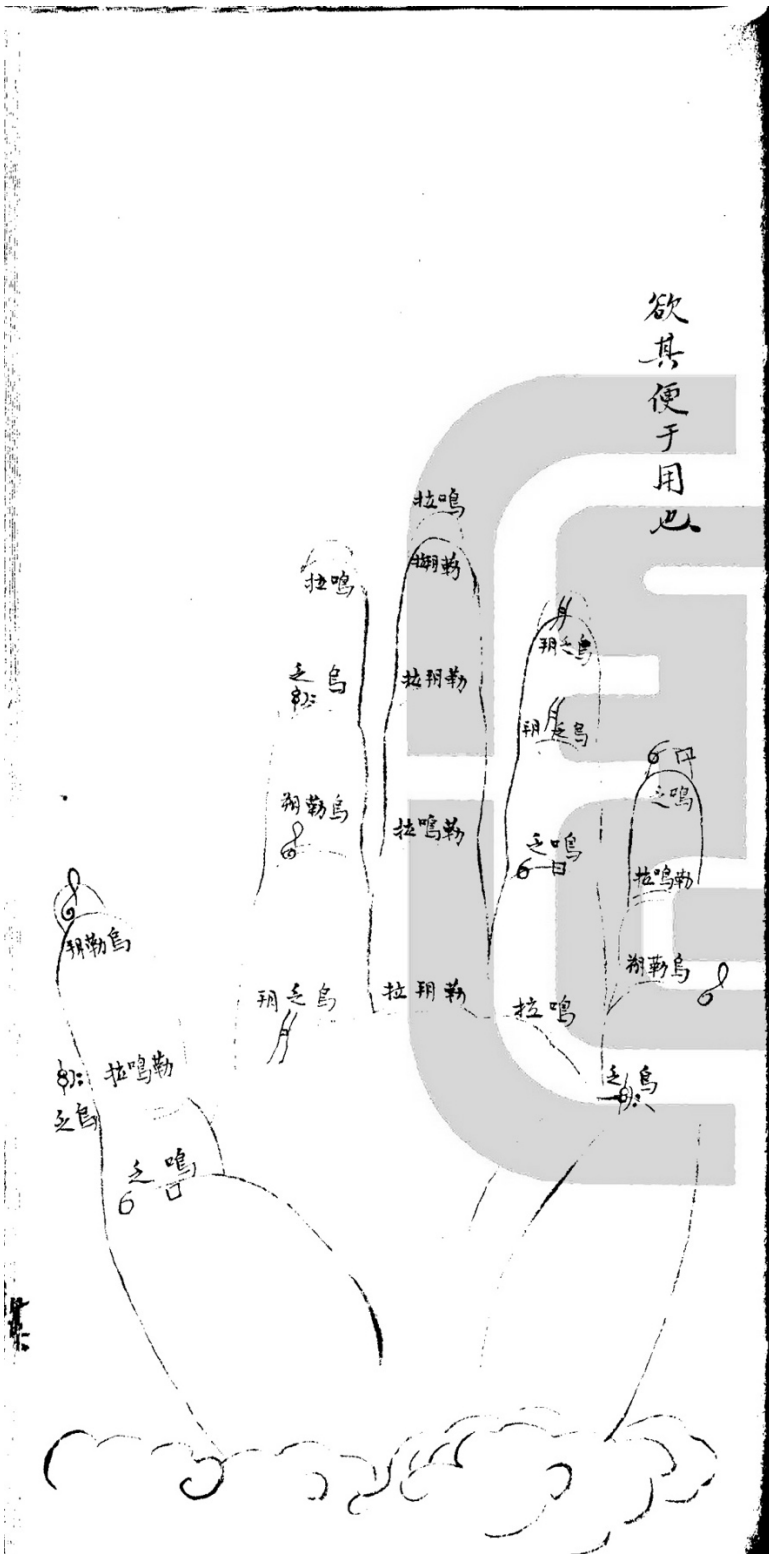
> 樂音說
> 樂音者，樂之種種高下粗細之音聲也。奏樂時，無論起於何音，或自下上讀，或自上下讀，其自然連貫者，止有八音。第八音者，“隔八相生”決定與首音合。又九音之合於第二音，十音之合於第三音，亦如之。[…][31]
>
> On Musical Tones [*yin*]
> Musical tones are the various high and low, thick and thin tones and sounds of music. When music is played, from no matter what tone or whether it is read from low to high or from high to low, the naturally continuous series [of notes] only features eight tones. The eighth tone, as determined by “mutual generation at every eighth step,” corresponds with the first tone. And it is in the same way that the ninth tone corresponds to the second tone and the tenth corresponds to the third. […]

To my knowledge, this is the only Chinese-language text before 1692 that uses the term “mutual generation at every eighth step” to mean octave equivalence—the caveat being, however, that the content of the text itself might not be fully Chinese. For ff. 1-51 contain a copy of *Elements of Pitch Pipes*, the earliest Chinese-language treatise on Western music. It particularly focuses on the *musica practica* topics of the Guidonian hexachords and staff notation; see Illustration 4-2 for an image in *Putong Guji* No. 15251 of the iconic Guidonian hand.

There is no definite record on by whom or when this treatise was completed and submitted to the inner court circle of the Qing Empire, nor any preface or postface to the treatise itself that explains its background or provenance. Yet one piece of evidence was provided by the two French Jesuits Jean-François Gerbillon (1654-1707) and Joachim Bouvet (1656-1730). Since the late 1680s, the two Jesuits had been giving regular lessons to the Kangxi Emperor on mathematics. And they kept a detailed diary, now Bibliothèque nationale de France, MS

31 National Library of China, *Putong Guji* No. 15251, f. 5v.

Illustraiton 4-2 The image of a Guidonian hand with the solfège syllables arranged onto the various joints and tips of the five fingers, **Putong Guji** ***No. 15251, f. 16r***



Français 17240, in which they recorded the Emperor's progress and some of their other engagements at the Qing court, they wrote for May 1, 1690 (see also Illustration 4-3):

> May. Le 1er. […] Ensuite, Sa majesté a feuilleté un livre des principes de musique que le P. Pereyra luy a fait autre fois tourner en chinois, et a ordonné a ce Pere de les examiner et de voir s'il y manquait quelque chose. L'Empereur nous fit demander ce jour precedent si nous pouvions les mettre en Tartare, nous nous en sommes excuses, disant que nous n'avions jamais pas appris la musique pratique. Il s'est fait apporter apres cela un petit orgue que le P. Pereyra luy a fait autre fois, Sa majesté l'a touché devant nous pendant quelque temps.[32]

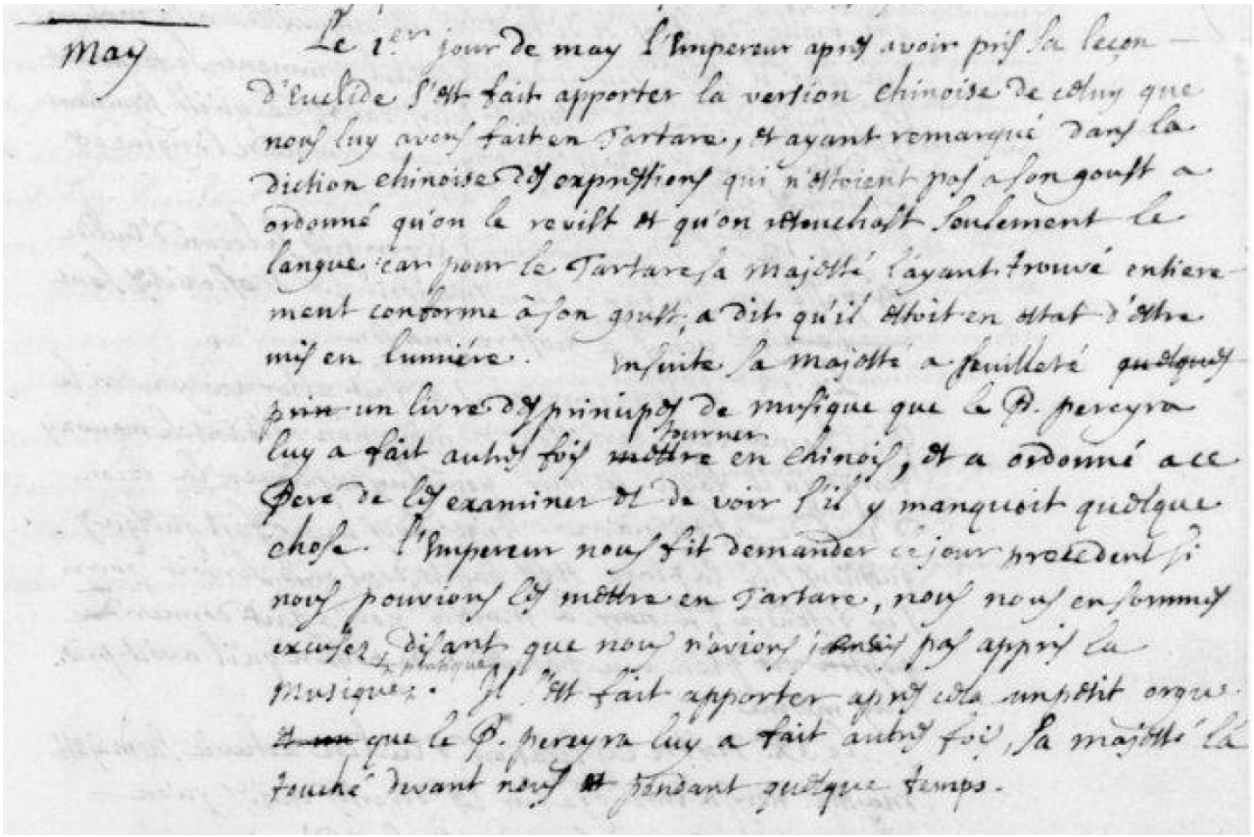
> May. 1st. […] Afterwards, His Majesty leafed through a book on the principles of music that Father Pereira had at one point written for him, and he ordered him to check and see if anything was missing in it. The Emperor had someone asked us on the previous day whether we could translate these principles [of music] into Manchu, and we offered our excuses [for declining to do so], saying that we had never learned practical music. Afterwards, [the Emperor] had someone bring to him a small organ that Father Pereira built for him at one point, [and] His Majesty played it in front for us for some time.

The following pieces of information can be derived from the entry above. First, matching the content of *Elements of Pitch Pipes*, the "book on the principles of music" written for the Kangxi Emperor by this Portuguese Jesuit Tomás Pereira (1645-1708) concerns *musique pratique*, as the two French Jesuits suggested. Second, though it remains unclear how early the text was written, but Pereira's treatise was certainly done by May 1, 1690. Third, the Kangxi Emperor had the intention of translating this treatise, written originally in Chinese, into Manchu; there are currently two manuscripts of *Elements of Pitch Pipes* in Manchu.[33]

32 Bibliothèque nationale de France, MS *Français* 17240, f. 277r.

33 Beijing, Palace Museum Library, *shu* 21781, and National Library of China, *Shanben* I22 J632 0512.

Illustration 4-3 Bibliothèque nationale de France, MS* Français *17420, f. 277r, Gerbillon and Bouvet's entry on May 1, 1690



May

Le 1.er jour de may l'Empereur aprés avoir pris la leçon d'euclide s'est fait apporter la version chinoise de celuy que nous luy avons fait en Tartare, et ayant remarqué dans la diction chinoise des expressions qui n'etoient pas à son gousts a ordonné qu'on le revist et qu'on retouchast seulement le langue car pour le Tartare, sa majestté l'ayant trouvé entierement conforme à son gusts, a dit qu'il etoit en etat d'etre mis en lumiére. ensuite, la majestté a feuilleté quelques prin un livre des principes de musique que le P. pereyra burnet luy a fait autrefois mettre en chinois, et a ordonné aux Pere de les examiner et de voir s'il y manquoit quelque chose. l'Empereur nous fit demander ce jour precedent si nous pouvions les mettre en Tartare, nous nous en sommes excusez disant que nous n'avions jamais pas appris la Musiques. Il s'est fait apporter après cela un petit orgue que le P. pereyra luy a fait autrefois, sa majestté la touché devant nous pendant quelque temps.

What is more, Gerbillon and Bouvet's description of that day also gives a rather representative snapshot of how the Kangxi Emperor studied Western music. Since the 1660s, the Emperor had taken great interests in Western learning, particularly subjects related to mathematics, astronomy, and engineering; in turn, Jesuit missionaries were happy to provide the Emperor with lessons and translated texts, identifying his interests in European formulations of the natural sciences as a convenient gateway for converting not just himself but also his entire realm.[34] Yet Gerbillon and Bouvet's description concurs with various other archival records of

34 Numerous monographs and articles have discussed this subject. For historical details of the Kangxi Emperor's studies themselves, see Jami, *The Emperor's New Mathematics*, 139-159. For the Jesuit's strategy of using Western learning to advance their proselytizing cause, a strategy that panned out over more than a century, as well as the mutual ideological "accommodations" between the Qing and the Jesuits, see Elman, *On Their Own Terms*, 107-189.

Pereira's musical lessons to the Kangxi Emperor in showing that the mode of pedagogy for these lessons was significantly different from that of the Emperor's lessons on Euclidean mathematics, for example, which consisted primarily of the Jesuit-tutor and the Emperor-student working their way through a translated text. Instead, all of Pereira's musical interactions with the Kangxi Emperor on record centered on singing, listening, and playing instruments,[35] and both the treatise and the various instruments Pereira made were serviceable to this practically oriented musical pedagogy: indeed, the treatise *Elements of Pitch Pipes* addresses nothing with respect to tuning but focuses solely on how to identify different notes of the scale and how to read Western musical notation. The practical nature of Pereira's musical activities at the Qing court was further attested to and immortalized by a joint eulogy written by his fellow Jesuits in China to Rome after both Pereira and Antoine Thomas (1644-1709) passed away within a year:

> […] atque insuper Talentis in Musicā, in Mathematicā, aliisque scientiis curiosis non tamtum speculatione, sed, quod hic plausibilius est, praxi inventivā et executivā (in quibus verō excellens erat Pr. Thomas Pereyra) planē insignibus […][36]
>
> […] and furthermore, the talents [of Pereira and Thomas were] thoroughly distinguished in Music, in Mathematics, and in other curious learnings not as much in philosophizing [*speculation*] but, a fact that is more praiseworthy, in creative and executive practice (matters in which Father Thomas Pereira was truly excellent) […]

Therefore, even though the very term *geba xiangsheng* "mutual generation at every eighth step" is used as octave equivalence in Pereira's treatise *Elements of Pitch Pipes*, more likely than not, the Kangxi Emperor did not develop his misunderstanding of the Chinese term from reading this particular text. After all, his eventual lecture in 1692 to his Han Chinese

35 See, for example, several records of Pereira's musical activities in Rome, Archivum Romanum Societatis Jesu, *Jap. Sin.* 117, *Sinarum Litterae annuae 1610-1770*, ff. 161-182.

36 Rome, Archivum Romanum Societatis Jesu, *Jap. Sin.* 173, f. 152r.

courtiers on "mutual generation at every eighth step" as octave equivalence took the specific form of a lecture-demonstration on a zither and a flute, and both Pereira's musical interactions with the Emperor and the treatise he wrote emphasized musical practice. As a result, the Emperor probably gained an understanding of "mutual generation at every eighth step" through embodied processes of music-making. His interactions with instruments and learning the rudiments of Western music through singing solfège syllables and scales likely predisposed him to understand "mutual generation at every eighth step" intuitively as octave equivalence. In contrast, in order to understand the phrase "correctly" as cycle of fifth, he would have needed a thorough understanding of the Chinese Pythagorean tuning method "triple division with one part subtracted or added" and its implications for patterns of proportions among the twelve tuning pitch pipes thus generated. This would not have been possible, since the Emperor's lessons in music (as well as astronomy and mathematics) were virtually monopolized by the Jesuits. And part of his political objective of holding the court gathering on February 20, 1692 is to signal to his Han Chinese scholar-officials that, despite their guardianship of the Confucian tradition, they were not at all the Emperor's sole source of knowledge.[37]

Furthermore, although the Kangxi Emperor took his practice-oriented music lessons with Pereira around the same time he was studying mathematics and other topics through translated treatise with Gerbillon, Bouvet, and Thomas, the political utility and ideological underpinnings of Western *musica practica* differed from those other realms of Jesuit-articulated European learning. Just like the rest of *Putong Guji* No. 15251, the copy of Pereira's *Elements of Pitch Pipes* in this manuscript would also feed into *Orthodox Meaning of Pitch Pipes* as the Qing's official treatise on music theory. It was adapted into the fifth and last volume of *Orthodox*.

37 See Jami, *The Emperor's New Mathematics*, 229-235.

Notably, the title of this volume is *xieyun duqu* "Coordinating the Note-Series [*yun*] in order to Compose Arias." Not only does *xieyun* 協均 "coordinating the *yun*" implicate this rather unique concept of the Chinese modal system, but *duqu* 度曲 literally refers to composing opera arias using fixed tunes. It was precisely for its potential usefulness for Chinese opera that *musica practica* morphed from a personal proclivity of the Kangxi Emperor starting in the 1670s into a set of concerns significant enough to be markedly incorporated in the Qing's official music theory treatise in the early 18th century. Although European missionaries bragged only about their musical tutelage of the Kangxi Emperor and his sons, an edict on August 2, 1714 indicates that most of their pupils were actually eunuchs. Addressing the chief eunuch, the Emperor asked that eunuch musicians learn their "*u le ming fa shuo la* (i.e. *ut re mi fa sol la*)" from the Italian Lazarist Teodorico Pedrini (1671-1746) in order to master not only their voice and instruments but also *lülü* "music theory."[38]

And the type of music theory they should master by studying *ut re mi fa sol la* was exactly the use of modes in opera. Published less than five months after the above edict, *Orthodox* does not feature any preface or postface for the entire five-volume treatise, as mentioned before. However, the last volume on Western music, designated as the *xubian* 續編 "appended tract," does open with a preface that takes its reader to a time and place quite distant from the Qing court or even its Jesuit missionaries. Below is the preface translated in full:

38 I have not seen this source myself, only as cited in various credible secondary sources. Purportedly it is located at the Propaganda Fide archives in Rome. See François Picard and Fañch Thoraval, "*Musica mecanica, practica & speculative*: de Pereira à Pedrini, la musique européenne à la cour de Kangxi," in Luciane Beduschi et al ed., *Musica, sive liber amicorum Nicolas Meeùs, Mélanges offerts au professeur Nicolas Meeùs* (Paris: Presses Universitaires de Paris-Sorbonne, 2014), 453-492; see also aforementioned Fang Hao, *Zhongxi jiaotong shi* 中西交通史 ("History of the Exchanges between China and the West"), 901-902

嘗觀《隋書音樂志》，柱國沛公鄭譯云：“考尋樂府鐘石，律呂皆有宮、商、角、徵、羽、變宮、變徵之名。七聲之內，三聲乖應。每恆求訪，終莫能通。先是周武帝時，有龜茲人曰蘇祇婆，善胡琵琶。聽其所奏，一均之中間有七聲。因而問之，答云’調有七種’。以其七調勘校七聲，冥若合符。就此七調，又有五旦之名，旦作七調。以華言譯之，旦者，則謂均也。其聲亦應黃鐘、太簇、姑洗、林種、南呂五均。以外七律，更無調聲。譯遂因其所捻琵琶絃柱，相引為均，推演其聲，更立七均，合成十二，以應十二律。律有七音，音立一調，故成七調十二律，合八十四調。旋轉相交，蓋皆和合。”此唐宋而後，雅俗樂部旋宮轉調之大綱也。惜乎！後此之從事者未嘗發明其旨。遂成史志虛文。我朝定鼎以來，四海盡入版圖。遠人慕化而來者漸多。有西洋波爾都哈兒國人徐日昇者，精於音樂。其法專以絃音清濁二均遞轉和聲為本。其書之大要有二：一則論管律、絃度生聲之由，聲字相合不相合之故；一則定審音合度之規，用剛、柔二記以辨陰、陽二調之異，用長、短、遲、速等號以節聲字之分。從此法入門，實為簡徑。後相繼又有壹大里呀國人德理格者，亦精律學，與徐日昇所傳，遠流無二。以其所講聲律、節奏，覈之經史所載律呂宮調，實相表裏。故取其條例、形、號，分配於陰陽二均高低字譜，編集成圖，使談理者有實據，而入用者亦有所持循云。[39]

Once we read the “Treatise of Music” from The *Book of Sui* where Zheng Yi [540-501], Duke Pei of the Country of Zhu, said: “When I examine the bells and stones of the imperial Bureau of Music, their tunings all have the names of *gong* [*do*], *shang* [re], *jue* [*mi*], *zhi* [*sol*], *yu* [*la*], *biangong* [*fa#*], and *bianzhi* [*ti*]. Among these seven scale-degrees, three of them are out of tune. I have always been trying to solve this, but still to no avail. During the reign of Emperor Wu of the previous Zhou dynasty, there was a man from Kucha named Suzhipo who was good at playing the hu pipa. Listening to what he plays, I discovered that the seven sounds are found in each *yun*. I thus asked him, and he replied: ‘there are seven kinds of modes’ [*diao*]. Collating his seven modes to the seven scale-degrees, I found as though they match each other. In addition to these modes, there are also what are called the five *dan*, and each *dan* yields seven modes. Translated into the Chinese language, *dan* means *yun*. Their sounds also correspond to those of the *yun*’s based on *huangzhong* [C], *taicu* [D], *guxi* [E], *linzhong* [G], and *nanlü* [A]. The remaining seven pitch pipes [lü] do not have their corresponding modes or *yun*’s. Thus, I followed the strings of the pipa that Suzhipo played, retuned each of them

39 In-c’i et al., *Orthodox Meaning of Pitch Pipes*, vol. 5, part 1, ff. 2r-3r.

> alternately as characteristic pitches of a *yun*, deduced their sounds, further established seven *yun*'s, and combined them with the previous five to match the twelve pitches. Since each pitch produces a *yun* consisting of seven pitches, and each pitch within this *yun* generates a mode, and therefore there are, with seven modes in each of the twelve pitches, eighty-four modes. The seven modes and the twelve pitches rotate and combine with each other, and everything matches." This was the general framework for mutations and transpositions in both elegant [*ya*] and vulgar [*su*] musics since the Tang and Song dynasties. What a pity! Practitioners afterwards did not discover or understand its essence. Thus it became empty words only logged in historical account. Since the establishment of our dynasty in Inner China, the territories of all four directions have been incorporated into the realm, and people from far away who come to us in desire of being civilized are increasing in number. There was a westerner Xu Risheng [Tomás Pereira] of Portugal who was learned in music. His teachings are specialized in the mutations between clear [raised; with b-natural] and muddy [lowered; with b-flat] *yun*'s and harmony in stringed music. There are two main points in his books: one is the causes of the sounds of pipes and strings, and the reasons for their pitches' being in tune or being out of tune; the other is establishing the rules of examining sounds and measuring proportions, by using the hard and soft signs to differentiate the two modes of yin and yang, and by using long, short, slow, fast, and other signs to differentiate between durations of notated pitches. Beginning the study of music through these methods is really effective. Later, after him there was also De Lige [Teodorico Pedrini] of Italy who was also specialized in the science of tuning, and what he taught compared to what Xu Risheng taught does not differ at all with respect to their origin and genealogy. Comparing what he taught to what the classics and histories say about tunings and modes shows that they are really just form and essence to one another. Therefore, we took his rules, shapes and signs and distributed them across exemplary scores in the *yin* and *yang yun*'s and high and low frequencies and compiled them into illustrative examples, so that the theoreticians have actual evidence to draw on and practitioners have something to use and follow.

This rich and exceptionally imaginative passage deserves a thoroughly analysis for its ideological rhetorics and historiographic narrations that would go far beyond the scope of this chapter, let alone this current section. It suffices to note that the Qing Empire's official treatise on musical tuning prefaces its volume on Western *musica practica* in reference to a famous

passage from *Book of the Sui* the bespeaks the very origins of the Chinese modal system. It prefaces describes how Suzup (f. 568), a *pipa* player from Kucha in the Tarim Basin, helped Chinese musicians of the Sui Dynasty deveop the eighty-four-mode system, combining the seven notes of the diatonic *yun* with the twelve tuning pitch pipes. It even praises this system for setting the "general framework for [modal] mutations and transpositions in both elegant and vulgar musics since the Tang and Song dynasties," with the "vulgar" referring clearly to vernacular song traditions such as opera tunes. Thus, drawing a parallel between the 6th-century Suzup from *xiyu* 西域 "Western Region" or Central Asia and the 18th-century European missionaries from *xiyang* 西洋 "Western Ocean" or Europe, the preface portrays the latter's Guidonian hexachords as an effective pedagogy for the former's modal system. And just as Suzup's Central Asian music helped rescue Chinese music from centuries of disrepair at his time, the preface argues, Guido's hexachords promised to rescue Suzup's system of eighty-four modes from a millennium of misuse in Chinese opera after the Sui and Tang eras, to the point that it had become "empty words in history books."

The proposed use of Guido's system of solfège sylalbles, hexachords, and scales to aid in relation to Chinese opera brings us back in full circle. This proposal was likely fully conjectural, except for the Kangxi Emperor's 1714 order that his eunuch musicians learn Western *musica practica* mentioned above, and I am yet to chance upon any 18th-century Chinese score of opera tunes notated with Guido's syllables. Still, even though no cross reference exists between the copy of the "Western" music treatise *Elements of Pitch Pipes* in ff. 1-51 and the "Chinese" content in the rest of *Putong Guji* No. 15251, both components of the manuscript appear to have been compiled and consumed with a common ultimate objective: to perfect the use of modes in opera, so as to better harness their political power in the service of the Qing Empire. The

entanglement both inside and outside the manuscript between the Qing court's operatic ambitions and its interest in Western *musica practica* show that the latter played a far more significant role in the vision to reform Chinese opera than merely that of foreign exotica.[40]

Further, a significant by-product of this avid engagement with Western music theory as a practical, embodied system of musical pedagogy was the Kangxi Emperor's misunderstanding of "mutual generation at every eighth step" as octave equivalence. It was this misunderstanding, I argue, one to which the Emperor was predisposed through his lessons in singing and music-making with Pereira, that would not only make listening for octave relations among the twelve tuning pitch pipes as a critical part of In-c'i's subsequent research into musical tuning, but would also elevate sounding and listening as a crucial knowledge-producing process. It would not be far-fetched to say that, in some way, the introduction of *ut re mi* to the Qing court triggered a series of events that led to the Kangxi Emperor's fourteen-tone temperament.

Yet the soflège syllables and the intuitive sense of recurrent cycles of octaves embedded in Western *musica practica* alone could not have caused an overhaul to Chinese musical tuning as major and unconventional as was the Kangxi Emperor's fourteen-tone temperament. Like any trigger, it exerted its effect within and in tandem with broader transformations at work. And it is in attempt to shed light on these transformations that I shall turn to in the last two chapters of this dissertation.

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40 It may seem ironic that the Kangxi Emperor's misunderstanding of *geba xiangsheng* "mutual generation at every eighth step" as octave equivalence arose partly from a system of solfège that only spelled six syllables instead of eight. This irony, however, would be misguided. First, as a system of mnemonics and pedagogy, Guido's six syllables never posited "sixth-ness" over octave equivalence as the primary building block of the scale system, and it was only 15th- and 16th-century writers such as Franchino Gafurio (1451-1522) and Gioseffo Zarlino (1517-1590) who read theoretical and tonal significance out of the "sixth-ness" of Guido's six syllables and hexachords; see Mengozzi, *The Renaissance Reform of Medieval Music Theory*, 110-114, 181-252.

Chapter Five

An Easier Path Not Taken:

Historicism vs. Empiricism, Records vs. Sounds

Let us return to Chapter 9, Volume 1 of *Orthodox Meaning of Pitch Pipes* and indulge in the following thought experiment reconstructed from the content of the chapter. By that point in the treatise, the researchers had already figured out (at least according to them) the exact length and diameter of the ancient's original *huangzhong* pipe. They had further computed the lengths of the eleven remaining tuning pitch pipes in proportion to that of *huangzhong* through the Chinese Pythagorean "triple division with one part subtracted or added." With the help of artisans working in the Imperial Household Department, they had even constructed a set of thirty-six bamboo pipes following exactly the measurements they had reconstructed, a set comprising the original twelve tuning pitch pipes, twelve pipes that are twice as long as the original twelve and twelve that are half as long.

Now, inspired by the Kangxi Emperor's mistaken gloss of "mutual generation at every eighth step" as octave equivalence back at the court gathering in 1692, the researchers set out to *shenyin* "examine the tones" of these pitch pipes, trying to locate exactly where those patterns of octave equivalence exist among these thirty-six pipes of the same diameter that they have just made. First they pick up the *huangzhong* pipe and the half-length *huangzhong* pipe. They assume these two pipes should produce an octave. In the conventional system of pitch organization, full-length and half-length pipes bearing the same name are assigned the same note in the diatonic scale. When the *gong* (*do*) note falls on the pitch of the *huangzhong* pipe, for example, the matching *shang* (*re*) note will fall onto that of the *taicu* (3rd) pipe, *jue* (*mi*) the *guxian* (5th) pipe,

bianzhi (*fa#*) the *ruibin* (7th) pipe, *zhi* (*sol*) the *linzhong* (8th) pipe, *yu* (*la*) the *nanlü* (10th) pipe, *biangong* (*ti*) the *yinghzong* (12th) pipe, and finally the *gong* back onto the half-length *huangzhong* (13th) pipe, according to the conventional system. However, when one of them plays the *huangzhong* pipe and the other the half-length *huanghzong* pipe, the interval the two pitches form sounds notably smaller than an octave. Befuddled by this phenomenon, they scramble to pick out a pipe that does sound an octave above the *huangzhong* pipe, the "root and essence of all things." Somehow, they chance upon playing the *huangzhong* pipe simultaneously with the half-length *taicu* pipe. In the conventional system of pitch organization, if the pitch of the *huangzhong* pipe is *gong*, that of the half-length *taicu* pipe would be the matching "clear *shang*," or the *shang* an octave above the one right after *gong*. Nonetheless, what the researchers actually hear is an octave between these two pipes, even though their lengths bear a 9:4 ratio.

At this point, the researchers put the two pipes, *huangzhong* and half-length *taicu*, back in the row of thirty-six tuning pitch pipes and begin to realize the gravity of what they have just observed. Starting from the *huangzhong* pipe and moving towards the shorter pipes, they count atotal of fifteen pipes up to the half-length *taicu* pipe, which they have just found to be sounding exactly an octave above *huangzhong*. By playing each of the fifteen pipes from *huangzhong* to half-length *taicu*, they realize what their careful efforts in reconstructing the twelve tuning pitch pipes (and their half-length and double-length duplications) exactly as stipulated in the oldest extant records on the ancients' pipes have led to: a fourteen-fold division of the octave.

Bracketing the dramatic restaging of this process that my prose attempts here, the series of observations involved in *shenyin* "examining the tones" in my thought experiment certainly did take place in one form or another in the early 18th century. My discovery of *Putong Guji* No. 15251 at the National Library of China reveals the actual "lab notes" recording these

experiments in listening to the twelve tuning pitch pipes as they happened under In-c'i's patronage at some point before 1707. My pursuit of the phrase *geba xiangsheng* 隔八相生 "mutual generation at every eighth step," from the Kangxi Emperor's lessons in Western music with the Jesuits to his lecture-demonstration in front of his Han Chinese courtiers and finally to the pivotal Chapter 9, Volume 1 of *Orthodox* shows the provenance of this commitment to listening—listening for octave relations, in particular—as it emerges from the broader ideological directive of the Qing Empire in harnessing the power of opera scholarship and opera reform. My analysis of the personnel and material resources provided to In-c'i and his team at various stages of the fourteen-tone temperament project also illustrates the embeddedness of their research in *zhilü* "making the pitch pipes" and *shenyin* "examining their tones" deep within the unique political structures of the Qing Empire as a conquest regime ruling China.

But why, we can now ask, did the observation of a fourteen-tone octave as described in ff. 88r-88v of *Putong Guji* No. 15251 and Chapter 9, Volume 1 of *Orthodox* lead to a complete overhaul of the system of pitch organization, based on a fourteen-tone octave comprising two complementary diatonic scales? It might seem natural that an observation of a fourteen-tone octave ought to result in a system of pitch organization and scales based on such a fourteen-tone octave. Yet there was nothing natural about it, because the researchers of the fourteen-tone temperament always knew of an exceptionally easy fix that would allow them to adjust the tuning pitch pipes in such a way that the pitches they produce would comply with the conventional pitch organization system in which both *huangzhong* and half-length *huanghzong* pipes would produce the same note, in which all pipes whose lengths are in a 1:2 proportion would produce the same note, and in which the tuning pitch pipes would not be not segregated into complementary diatonic scales, as if they were some type of heptatonic whole-tone scales.

Nullifying the "End Correction" Problem

What is this easy fix? I will approach to it below through two paths that will soon converge. First, recall what I have briefly touched on in Chapter 3 regarding the "end correction" phenomenon. First articulated in its current form by Isaac Newton, the implied frequency of the fundamental pitch of a pipe open on both ends is calculated as[1]:

$$f = \frac{c}{2l}$$

in which f stands for the frequency, c the speed of sound in the medium (air), and l the length of the pipe. Nonetheless, the actual frequency of the fundamental pitch of such a pipe is consistently observed to be slightly lower that what is calculated through the above formula. One explanation has been that the vibrating air column that produces the pitch actually extends beyond the two open ends of the physical pipe, so that the actual length of the vibrating body is slightly longer than the length of the pipe itself. As a result, in order to use the above equation to calculate the frequency of the fundamental pitch of a pipe more accurately, an "end correction" is applied that extends the length used for such calculation, as in the equation below[2]:

$$f = \frac{c}{2l + \Delta l}$$

in which Δl stands for the added end correction. So far, I have not been able to locate any scientific consensus regarding exactly how this end correction value of Δl should be calculated. However, a typical solution estimates this end correction value as a linear function of the

1 See William M. Hartmann, *Principles of Musical Acoustics* (New York: Springer, 2013), 77-78.

2 Ibid., 80-82.

diameter of the pipe, which, notably, is not taken into account in the first equation used above to calculate the ideal situation. Put into a mathematical equation, $\Delta l = nd$, in which n is a constant.

There seems, again, to be no scientific consensus over what this constant n should be. I have seen values from 0.61[3] to 1.7, the latter being Weng Panfeng's case examined in Chapter 3.[4] Yet there is a curious consequence of estimating the end correction value Δl as nd, regardless of what exactly the constant n is. Consider in case in which we have two *tongjing guan* 同徑管 "pipes of the same diameter": pipe No. 1 is twice as long as pipe No. 2, yet their diameter is identical. To express this in mathematical terms:

$$l_1 = 2l_2$$

$$d_1 = d_2$$

Then, if we factor in the end correction of both pipes as $\Delta l = nd$, we can calculate the implied and "end-corrected" frequency of both pipes, f_1 and f_2 as:

$$f_1 = \frac{c}{2l_1 + \Delta l_1} = \frac{c}{2l_1 + nd_1}$$

$$f_2 = \frac{c}{2l_2 + \Delta l_2} = \frac{c}{2l_2 + nd_2}$$

The frequency ratio between the two pipes, or f_1:f_2, becomes:

$$\frac{f_1}{f_2} = \frac{\frac{c}{2l_1 + nd_1}}{\frac{c}{2l_2 + nd_2}} = \frac{2l_2 + nd_2}{2l_1 + nd_1}$$

This last equation is very curious, for it shows multiple consequences regarding the end correction phenomenon. First, if we were to ignore the end correction altogether, the end

3 Ibid., 81.

4 See footnote No. 10 in Chapter 3 above. Note that we are uniformly concerned only with pipes with two open ends here, since the *lülü* "tuning pitch pipes" were almost always treated as having two open ends in Chinese musical scholarship.

correction value for both pipes $\Delta l_1 = nd_1$ and $\Delta l_2 = nd_2$ would be zero, since there would be no need to extend the length of the vibrating air column beyond the physical ends of the pipe. In that case, the implied frequency ratio between the two pipes would be:

$$\frac{f_1}{f_2} = \frac{2l_2 + 0}{2l_1 + 0} = \frac{2l_2}{2l_1} = \frac{l_2}{l_1} = \frac{1}{2}$$

Or, put differently, the ratio of frequency between two pipes will be the inverse of the ratio of length between them: as in, if one pipe produces the pitch A=440 Hz, the pipe that is half as long—ignoring end correction—will produce the pitch A1=880 Hz, or two times as great the frequency.

Second, for the same equation with end-correction factored in,

$$\frac{f_1}{f_2} = \frac{2l_2 + nd_2}{2l_1 + nd_1}$$

if we apply the fact that $l_1 = 2l_2$ (following the length ratio of the two pipes) and that $d_1 = d_2$ (following their identical diameter), their frequency ratio can be calculated as:

$$\frac{f_1}{f_2} = \frac{2l_2 + nd_2}{2l_1 + nd_1} = \frac{2l_2 + nd_2}{4l_2 + nd_2}$$

Now, adding the same positive value to both the numerator and the denominator of a fraction that is larger than zero yet smaller than one increases the value of the fraction. Therefore, whenever $n > 0$, it must necessarily be true that:

$$\frac{2l_2 + nd_2}{4l_2 + nd_2} > \frac{2l_2}{4l_2} = \frac{1}{2}$$

In other words, when the end correction is factored in, the frequency ratio between one pipe and a shorter pipe, with their diameter being the same, is necessarily larger than the inverse of the ratio between their lengths. To put this in specific terms, the frequency ratio between one pipe and the pipe half as long, with their diameter being the same, is necessarily larger than the

inverse of the ratio between their lengths, which is 1:2. As a result, the intervallic distance between them is necessarily smaller than what is implied by 1:2, or the octave—or indeed what the whole fuss about "end correction" and "octave not being found between *huangzhong* and half-length *huangzhong* pipe" are all about.

Third, suppose that the ratio, *k*, between the diameter and the length of the first pipe is calculated as:

$$k_1 = \frac{d_1}{l_1}, \therefore d_1 = k_1 l_1$$

Suppose also that the ratio between the diameter and the length of the second pipe is calculated as:

$$k_2 = \frac{d_2}{l_2}, \therefore d_2 = k_2 l_2$$

The implied frequency ratio between these two pipes, with end correction factored in, becomes:

$$\frac{f_1}{f_2} = \frac{2l_2 + nd_2}{2l_1 + nd_1} = \frac{2l_2 + nk_2 l_2}{2l_1 + nk_1 l_1} = \frac{l_2}{l_1} \times \frac{2 + nk_2}{2 + nk_1}$$

Now we have a neat formula. On the left side, we have the frequency ratio between the two pipes, $f_1 : f_2$. On the right side, we have the inverse length ratio between the two pipes, $l_2 : l_1$. In the "ideal" situation in which end corrections are not taken into account, the two should equal each other. In other words, in the last equation above, the "end correction" factor is the multiplier $\frac{2+nk_2}{2+nk_1}$, which is a function of the diameter-to-length ratio of both of the two pipes, k_2 and k_1. As a result, if we can make $\frac{2+nk_2}{2+nk_1} = 1$, we will have:

$$\frac{f_1}{f_2} = \frac{l_2}{l_1} \times \frac{2 + nk_2}{2 + nk_1} = \frac{l_2}{l_1} \times 1 = \frac{l_2}{l_1}$$

Voilà: the ideal situation, in which the frequency ratio and length ratio are but the mathematical inverse of each other, without having to trouble over diameters or end corrections.

So how do we make $\frac{2+nk_2}{2+nk_1}$ equal to 1? The answer is by having *tongxing guan* 同形管 "pipes of the same shape," or pipes of the same length-to-diameter ratio. Suppose that both the diameter-to-length ratios of the two pipes are actually equal:

$$k_1 = \frac{d_1}{l_1} = k_2 = \frac{d_2}{l_2} = q$$

As a result,

$$\frac{f_1}{f_2} = \frac{l_2}{l_1} \times \frac{2+nk_2}{2+nk_1} = \frac{f_1}{f_2} = \frac{l_2}{l_1} \times \frac{2+nq}{2+nq} = \frac{f_1}{f_2} = \frac{l_2}{l_1}$$

Thus, using "pipes of the same shape" where the lengths and the diameters are proportional nullifies the problem of "end correction."

What this series of mathematical deductions shows is this. If one is given a specific frequency ratio (such as 1:2) and desires specifically that the interval corresponding to this ratio (the octave) be produced by two pipes whose lengths bear the inverse of this ratio (2:1) one should not use "pipes of the same diameter" but rather "pipes of the same shape." This is because having two pipes bear the same length-to-diameter ratio would nullify the need to factor in any end correction, presuming that such a correction is a linear function of a pipe's diameter. To put this in the context of Chapter 9, Volume 1 of *Orthodox*, let us try again to imagine the researchers behind the treatise, when they first discovered the octave to be not between the pitches of the *huangzhong* and half-*huangzhong* pipes bearing a 1:2 ratio, but rather between the pitches of the *huangzhong* and half-length *taicu* pipes bearing a 4:9 ratio. If they were very keen on keeping 1:2 as the ratio of the octave, keeping the twelve-fold division of the octave, and indeed keeping the conventional pitch organization system in which two pipes bearing the same

name and a 1:2 length proportion necessarily takes the same note of the seven-note diatonic scale, they had a simple option they could pursue: namely, to remake the twelve tuning pitch pipes (as well as those twice and half as long) into pipes not of the same diameter but of the same shape. Had they done so, they would have had the chance to keep exactly the same conventional system of pitch organization, exactly the same twelve-tone octave, and would have saved themselves a lot of trouble devising a new system of two complementary seven-note diatonic scales, reinventing the system of modes, and refashioning and retrofitting various instruments to these new systems. But they decided otherwise.

Strings vs. Pipes

Admittedly, it would be anachronistic to posit that the goal of those researchers under In-c'i was or could ever have been to match the proportions of their pitch pipes to some preconceived or idealized system of frequency ratios. Though they did turn out to know quite a lot about the vibration theory of sound-production, largely through their Jesuit interlocutors, the notions of frequencies or frequency ratios, from the lab notes in *Putong Guji* No. 15251 to Volume 1 of *Orthodox*, never played any role at any stage of the fourteen-tone temperament research process. But there existed a more materially concrete body of ratios that occupy a homological position, a body of ratios that would certainly have prompted the researchers of the fourteen-tone temperament to think twice before they decided to head down the path of devising an entirely new system of pitch organization, and that was the Chinese Pythagorean tuning method of "triple division with one part subtracted or added," specifically as applied to the lengths of strings. Recall the pivotal passage in Chapter 9, Volume 1 of *Orthodox* describing the experiments in *shenyin* "examining the tones" of pitch pipes.

> […] 蓋各守所傳，固執一理，而未始備制律呂之管以審音也。間嘗截竹為管，詳審其音。黃鐘之半律，不與黃鐘合，而合黃鐘者為太簇之半律。則倍半相應之說在絃音而非管音也，明矣。又黃鐘宮其徵聲不應於林鐘，而應於夷則。則三分損益、宮下生徵之說，在絃度而非管律也，明矣。[…][5]
>
> Indeed, [these previous authors] each stuck to what they had learned [from previous texts] and stubbornly stuck to their own principles, yet they never started to prepare or fashion the actual pipes of the tuning pitch pipes so as to examine the tones [*shenyin*]. Just recently, we once slashed some bamboos into pipes and carefully examined their tones. The [pitch of the] pipe whose length is half that of *huangzhong* does not correspond to [the pitch of] the *huangzhong* pipe, yet what does correspond to [the pitch] of the *huangzhong* pipe turns out to be the pitch of the *taicu* pipe. *Thus, it is clear that "the full-length and the half-length correspond to each other [in pitch]" applies only to the tones of strings but not to the tones of pipes*. Moreover, if [the pitch of] the *huangzhong* pipe is taken as *gong* [*do*], its respective *zhi* [*sol*] note does not correspond to [the pitch of] the *linzhong* pipe but to [the pitch of the] *yize* pipe. Thus, it is clear that "triple division with one part subtracted or added generates a relation of *gong* and *zhi*" that applies only to the proportions of strings but not to the proportions of pitch pipes. […] [emphasis added]

Here, the passage recognizes a critical observation that I have only mentioned in passing in previous chapters: unlike the length ratio between two pipes of the same diameter, the length ratio of 1:2 does in fact produce an octave between two strings, and the length ratio of 2:3 does in fact produce a *gong-zhi* interval, or a perfect fifth. In addition, as mentioned in Chapter 3 above, Chapter 9, Volume 1 of *Orthodox* justifies its complete overhaul to the conventional system of pitch organization by arguing specifically that those well-respected old sources positing an octave as corresponding to a 1:2 length ratio and the *gong-zhi* perfect fifth as corresponding to a 2:3 or "triple division with one part subtracted" ratio were actually speaking

5 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, vol. 1, Chapter 9, f. 27r-27v.

not about pipes but about strings. The passage below immediately follows the discussion in Chapter 9, Volume 1 on "mutual generation at every eighth step" as octave equivalence:

> […]但五聲二變，有施於管律者，有施於絃度者。其生聲取分各有不同。自漢唐以後，皆宗司馬氏、淮南子之說，以三分損益之術，誤為管音五聲二變之次。復執《管子》絃音五聲度分而牽合十二律呂之重。故管律、絃度，俱不可得而明。[…][6]
>
> […] Yet "the five proper notes and the two altered notes" are sometimes applied to the proportions of pitch pipes, sometimes applies to those of strings. Their production of musical notes [*sheng*] and the assignment [of such notes] to certain proportions differ from one another. Since the Han [206 BCE-220 CE] and Tang eras, scholars all followed the theories of Sima Qian [in his *Records of the Grand Historian*] and Prince Huainan [in his *Writings of Prince Huainan*]. They took the method of "triple division with one part subtracted or added" and mistook it for the order whereby the "five proper notes and the two altered notes" are also generated. They further confused the proportions between the five proper notes on strings in *Master Guan* [c. ? 5th century BCE] with the proportions of the twelve tuning pitch pipes. As a result, they could neither understand the proportions of pipes nor those of strings. […]

Some explanation is due here. The chapter "Measuring the Earth" (*diyuan* 地員) in the c. 5th-century *Writings of Master Guan* features the following passage:

> […] 凡將起五音，凡首，先主一而三之。四開以合九九，以是生黃鐘小素之首以成宮。三分而益之以一，為百有八，為徵。不無有三分而去其乘，適足，以是生商。有三分而復於其所，以是成羽。有三分去其乘，適足，以是成角。[7]
>
> […] Whenever one wants to find the five [proper] notes, for the first note, take the number 1 and multiply it by 3. Do it four times in order to compute 9 times 9 [equals 81], in order to generate *huangzhong* and use it as *gong* [*do*]. After "triple division with one part added," [81] becomes 108, which is *zhi* [*sol*; here the *sol* is lower than the *do*]. After "triple division with one part subtracted," it becomes exactly [72, which is] *shang* [*re*]. After "triple division with one part added," it becomes [96, which is] *yu* [*la*]. After

6 Ibid., vol. 1, Chapter 9, f. 26r-27r.

7 Guan Zhong, *Writings of Master Guan*, Chapter 58.

> "triple division with one part subtracted," it becomes [64, which is] *jue* [*mi*].

This passage therefore spells out a "harmonic number" for each of the five proper notes of the diatonic scale: lower *zhi* (*sol*) as 108, lower *yu* (*la*) as 96, *gong* (*do*) as 81, *shang* (*re*) as 72, and *jue* (*mi*) as 64. What is more, it describes the process of "triple division with one part subtracted or added" not through the twelve tuning pitch pipes but through the proper notes of the diatonic scales: *gong* generates lower *zhi* through 3:4, lower *zhi* generates *shang* through 3:2, *shang* generates lower *yu* through 3:4, and lower *yu* generates *jue* through 3:2. As a result, Chapter 9, Volume 1 of *Orthodox* argues that subsequent scholars conveniently matched these proportions between the notes of the scale recorded in *Master Guan* with those proportions between the pitch pipes recorded in *Chronicles of Master Lü*, *Records of the Grand Historian*, and *Writings of Master Huainan*. For example, because *gong* (96) and *zhi* (54; 108 divided by 2) forms a 3:2 proportion in *Master Guan*, it was accordingly assumed by later scholars to corrrespond to the 3:2 proportion between the *huangzhong* pipe and the *linzhong* pipe.

What Chapter 9, Volume 1 of *Orthodox* observes through listening to the pitch pipes, of course, is that those correspondences simply do not exist: the ratio 1:2 simply does not produce an octave among the twelve tuning pitch pipes, whereas 4:9 does. Similarly, the ratio 2:3 simply does not produce a perfect fifth or *gong-zhi* interval among the twelve tuning pitch pipes, whereas 4096:6451 (the length ratio between the *yize* and *huanghzong* pipes) does. To reconcile its empirical observations with the textual records of *Master Guan*, which dated back even before *Chronicles of Master Lü*, the earliest source to speak of "triple division with one part subtracted or added" in the specific context of pipes, Chapter 9, Volume 1 argues that *Master Guan* was speaking about those proportions in terms of strings, if not explicitly. In fact, the entire *Orthodox Meaning of Pitch Pipes* is always careful to distinguish between strings and

pipes regarding what length proportions generate what kind of intervals expressed through two notes of the diatonic scale. When describing the listening experiments with tuning pitch pipes, for example, Chapter 9, Volume 1 phrases its observations specifically with reference to the difference between strings and pipes:

(1) "It is clear that [the rule] 'the full-length and the half-length correspond to each other [in pitch]' applies only to the tones of strings, not to the tones of pipes."

(2) "It is clear that [the rule] 'triple division with one part subtracted or added generates a relation of *gong* and *zhi*' applies only to the proportions of strings, not to the proportions of pitch pipes."

In other words, the conventional system of pitch organization based on the twelve-tone octave, the identification of 1:2 as the octave, and the identification of 2:3 as perfect fifth or *gong-zhi* works completely fine with strings whose lengths are proportioned according to the Chinese Pythagorean "triple division with one part subtracted or added." It just does not work for pipes.

So now the question becomes: why didn't researchers for the fourteen-tone temperament change their twelve tuning pitch pipes from "pipes of the same diameter" to "pipes of the same shape"? Had they done so, they could have applied the same Chinese Pythagorean tuning ratios to both strings and pipes and then applied the same conventional system of pitch organization to both those Pythagorean-proportioned strings and to pipes. They could have ended up with a world in which the same length ratio among strings and pipes corresponds to the same intervals in the diatonic scale: for example, the *gong-zhi* perfect fifth would have corresponded to both a 1:2 length ratio between two strings and a 1:2 length and diameter ratio between two pipes of the same shape.

Why didn't they change their pipes?

Two Sources

As it turns out, not only did they know that pipes of the same shape was a solution to the problem of "1:2 ratio not corresponding to an octave between two pipes," but they even knew about this solution through two independent sources. Their first source was *Essential Meaning of Pitch Pipes*, in which Zhu Zaiyu made very similar observations. In the fifth chapter of the treatise, Zhu describes his experiments with making and listening to different pitch pipes of the same length in his search for the octave:

> […] 以竹子或筆管，製黃鐘一律。一樣二枚。截其一枚。分作兩段。全律半律各令一人吹之。聲必不合矣。此昭然可驗也。又製大呂之律。一樣二枚。周徑與黃鐘同。截其一枚。分作兩段。全律半律各令一人吹之。則亦不相合。而大呂半律乃與黃鐘全律相和。略差不遠。是知所謂半律者。皆下全律一律矣。[…][8]

> […] Take a bamboo stem, or the stem of a writing brush. Let it be the pipe of *huangzhong*. Let there be two identical ones of such a pipe. Cut one of them into two halves. Have one person blow the *huangzhong* pipe and another person blow the pipe that is thus half the length of *huangzhong*. It will necessarily turn out that the notes they produce do not match. This is self-evident and one can verify it. Next, make the pipe of *dalü* [proportional to the aforementioned pipe of *huangzhong*]. Let there be two identical ones of such a pipe. Cut one of them into two halves. Have one person blow the *dalü* pipe and another person blow the pipe that is thus half the length of *dalü*. Again, the notes they produce do not match. Yet, the pipe whose length is half that of *dalü* does indeed match the *huangzhong* pipe—not exactly, but well enough. And thus, it can be deduced that the half-length pipe always sounds one step lower than the full-length pipe. […]

A few caveats need acknowledging. First, given Zhu's twelve-tone equal temperament, the two pipes between which he heard the octave, *huangzhong* and half-length *dalü*, bear the length ratio of $\frac{\sqrt[12]{2^{11}}}{2} : 1$, which he calculates as 943,874,312:2,000,000,000, or about 0.4719. This is slightly

8 Zhu Zaiyu, *Essential Meaning of Pitch Pipes*, vol. 1, ff. 31r-31v.

larger than the length ratio between the Pythagorean-proportioned *huangzhong* and half *taicu* pipes heard as an octave in Chapter 9, Volume 1 of *Orthodox*, which is 4:9 or about 0.4444. Second, because Zhu Zaiyu interprets the historical record alleging that "the *huangzhong* pipe is 9 *cun*, and its hollow enclosure is 9 *fen*" differently from *Orthodox*, the length-to-diameter ratio of the *huangzhong* pipe he made is much larger than the *huangzhong* pipe in *Orthodox*, with the result that his pipes are proportionally narrower. Still, Zhu's observations are basically identical to those in Chapter 9, Volume 1 of *Orthodox*: namely, that the 1:2 length proportion does not produce an octave between two pipes of the same diameter, but rather an interval notably smaller than an octave. As a result, in order to produce that desired octave between two pipes of the same diameter, the length of the shorter pipe must be significantly shorter than half that of the longer pipe.

Yet what did Zhu Zaiyu do? If we were to repeat the same thought experiment with which I opened this chapter, Zhu could have taken this observation of the octave between *huangzhong* the 1st pipe and half-length *dalü* the 14th pipe and construct based thereon a new system of pitch organization that divides the octave into thirteen, not twelve portions. In this way, Zhu's modern reputation as the ingenious inventor of twelve-tone equal temperament would have been overshadowed by some quirky "thirteen-tone temperament." Fortunately for him, Zhu did not go down the same path as the researchers for the Qing's tuning reform were to do. Fittingly, as I mentioned in Chapter 1 above, the very chapter of *Essential Meaning of Pitch Pipes* in which he describes the experiments in question is entitled: "The Same Circumference or Diameter Is Not to Be Applied to All Tuning Pitch Pipes" (不取圍徑皆同). In his iconoclastic fashion, Zhu argues that previous scholars have erred in assuming that the diameters of all twelve tuning pitches pipes as well as those that are twice and half as long are identical. Instead, he

applies the ratios of twelve-tone equal temperament not only to the length but also to the diameter of the *huangzhong* pipe, thereby generating all the tuning pitch pipes in *Essential Meaning* as "pipes of the same shape" or the same length-to-diameter ratio without exception. In so doing, he effectively nullifies the "end correction" problem, thereby maintaining the same conventional system of pitch organization and octave division for both strings and pipes, as well as applying the same length proportions to each of them.

The second source for researchers working under In-c'i and the eventual compilers of *Orthodox* was, perhaps surprisingly for many readers, the two-part treatise *Harmonie universelle* (1636) by the Marin Mersenne (1588-1648). Reflective of the polymathic scope of Mersenne's scholarship, *Harmonie universelle* is an immense treatise on mechanics, acoustics, tuning, practical music theory, organology, and cosmology. Building on the vibration theory of sound production of Galileo Galilei (1564-1642), Mersenne articulates a series of laws regarding the frequency of oscillation of a string, which will later be known as Mersenne's Laws. According to Mersenne's Laws, the fundamental frequency of a string is inversely proportional to its length, proportional to the square root of its tension or stretching force, and inversely proportional to the square root of the mass per unit length of its material. In addition to using mechanical reasoning and mathematical calculations, Mersenne claims to have derived these laws from extensive experiments, and he shows his empirical bona fides by frequently displaying the raw data.

Compared to his treatment of strings, to which he dedicates an entire *livre* or constituent book of his *Harmonie*, Mersenne's discussion of pipes is quite scanty. Whereas he examines the property of vibrating strings in an abstract context in addition to using specific instruments, most of his analyses of aerophone acoustics appears in his book on organs in Part II of *Harmonie*. In particular, Proposition No. 13 is entitled "To determine in what ratio the lengths of pipes of the

same width [i.e. diameter] should be in order to produce the required sounds and intervals, and whether one can make an organ of which all the pipes have the same width."

> Puis que la seule raison de l'homme ne peut trouver les proportions des corps & des sons, comme j'ai montré dans le second livre des mouvements, où l'on void que les cylindres de même grosseur, & des différentes longueurs ne font pas des sons qui gardent la même raison entr'eux que leurs longueurs: par exemple, que celui qui est double en longeur ne fait pas l'Octave, comme plusieurs croyent, il faut encore ici consulter experience, afin de remarquer que les sons ne suivent pas aussi la raison de la longueur des tuyaux de même grosseur […]. Mais ayant fait l'expérience en de plus gros tuyaux qui parlent mieux, j'ai remarqué qu'il s'en faut ordinairement un demi-ton, ou près d'un ton, que le tuyau double en longeur ne face l'Octave. […][9]

> Since the reason of humans cannot alone reveal the proportions of [vibrating] bodies and sounds, as I showed in the second book on movements [i.e. "Book on the Movements and Sounds of Strings"], where one sees that cylinders of the same width and different lengths do not produce the sounds that keep the same ratio among them as do their lengths—for example, those that are twice in length do not make an octave, as many believe—here, it is also necessary to consult experience, in order to notice that the sounds do not follow the ratio of the length of pipes of the same width either […]. But, having conducted the experiment with the largest pipes that sound better [than the small ones], I have noticed that the pipe double in length misses the octave by a semitone, or almost a whole-tone. […]

The beginning of this passage refers to Proposition No. 10 "To find out if, given two or more sounds, one can find the sonorous cylinders that produce the said sounds, or if, given the cylinders, one can know their sounds, where one will see many marvelous experiments," and to Proposition No. 11 "To explain what lengths and widths the cylinders should have in order to produce such sounds of which one can discern the high and the low, and why they do not follow

9 Marin Mersenne, *Harmonie universelle contenant la théorie et la pratique de la musique* (Paris, 1636), Part II, "Libre VI Des Orgues," 333.

the ratios of strings" in Book III, Part I of *Harmonie universelle*.[10] While Book III is dedicated to the property of vibrating strings, Mersenne dedicates Propositions No. 10 and No. 11 to an excursion on metallic cylinders. Here, he observes "through marvelous experiments" that, even if one applies the same ratios to the lengths of strings and to the lengths of cylinders of the same width or diameter, one will end up getting different intervals. In particular, Mersenne notices that the 1:2 length ratio does not produce an octave between two cylinders of the same diameter as it does with two strings. In contrast to his systematic studies of vibrating strings, however, Mersenne does not produce any definite laws or equations regarding how the length and width of a metallic cylinder relate to the frequency of its vibration. In fact, Mersenne explicitly laments in Proposition No. 11 that he could not establish any mathematical relation between the physical dimensions of a cylinder and its frequency, despite trying both linear and non-linear functions. Instead, he simply enumerates the various observations he made through the experiments, concluding "it suffices to have given these real appearances [*apparences*] of these cylinders, in order to inspire excellent minds to find their causes."[11]

By drawing attention to his failure to establish any definite mathematical relations regarding metallic cylinders, Mersenne frames his Proposition on organ pipes of the same width in the same manner. First, he observes that the same ratios that produce certain intervals when applied to the lengths of strings do not end up producing the same types of intervals when applied to the lengths of these pipes of the same width or diameter. Again, he singles out how 1:2 does not produce an octave among such organ pipes. Curiously, he notices that, if one doubles the length of a pipe, the resulting pitch descends only a major seventh or sometimes only a minor

10 Ibid., Part I, "Libre III Des mouvements de toutes sortes de corps," 175-180.
11 Ibid., 178-179.

seventh, or, as he puts it, a semitone or a whole-tone short of an octave; this observation is rather similar to that in Chapter 9, Volume 1 of *Orthodox*, which posits that the pitches of the *huangzhong* and half-length *huangzhong* pipes do not correspond to *gong* (*do*) and clear *gong* (*do* the octave above), but rather to *gong* (*do*) and *biangong* (*ti*). Second, just as with metallic cylinders, Mersenne does not spell out any mathematical equations that relate the intervals between two pipes of the same diameter to their lengths or width. Instead, he displays a series of data from his observations with pipes of varying sizes without trying to establish even a semblance of a pattern as to what ratios consistently produce what types of intervals.

Yet Mersenne's musings on pipes of the same diameter is just as much a digression from his discussion on organ tuning as has been his excursion into metallic cylinders amid his treatment of vibrating strings. In response to the question of "whether one can make an organ of which all the pipes have the same width," his answer is a resounding no. Instead, all the organ pipes for which Mersenne actually spells out lengths and widths are pipes whose lengths and circumferences bear a constant proportion: as early as in Proposition No. 4 in Mersenne's book on organs, he reports that the "Facteurs" or instrument makers typically proportion the circumference of an organ pipe as anywhere from a quarter, a third, or even two-fifths of its length.[12] In Proposition No. 14, which comes immediately after Proposition No. 13 on pipes of the same diameter, Mersenne confirms that the diameters or circumferences of organ pipes must change accordingly as their lengths vary, because "the pipes make just the interval one wants if

12 Ibid., Part II, "Libre VI Des Orgues," 318-321.

their heights and their widths have the same ratio as the said interval."[13] Notably, for Mersenne, the "ratio of an interval" refers not just abstractly to the ratio of vibration frequencies but also to the ratio of the lengths of strings, whose vibrations produce such an interval. Following the tradition of Western music theory, Mersenne's primary "instrument for music theory" was the single-string monochord (ergo its name). Even though Book III of Part 1 "On the Movements and Sounds of Strings" only discusses strings in the abstract and does not even include an illustration of the monochord, he develops his entire theory of sound-producing vibrations, including how certain ratios of vibration frequencies correspond to certain intervals, based solely on strings. Thus, just like Zhu Zaiyu, Mersenne makes organ pipes not as pipes of the same diameter but as pipes of the same shape (or constant length-to-circumference ratio), so that he can apply the same ratios to both strings and pipes and produce the same intervallic relations among these two types of vibrating bodies.

Scientia juris non excusat

By the time In-c'i likely began envisioning a reform to musical tuning as well as to Chinese opera and music theory as a whole, both Zhu Zaiyu and Marin Mersenne had been deceased for several decades. However, there is concrete archival and bibliographic evidence that researchers of the fourteen-tone temperament had complete and solid knowledge of both Zhu's and Mersenne's solution of using pipes of the same shape to nullify the "end correction"

13 Ibid., Part II, "Libre VI Des Orgues," 334-337. It should be noted, however, that in his ultimate design of organ pipes, the length-to-circumference ratio is *not* a fixed one. Rather, this ratio becomes larger as the pipe becomes shorter—put differently, the pipes become proportionally even thinner as it becomes shorter. Still, the important point here is that Mersenne, having observed the problems with pipes of the same diameter (thus the same circumference), is completely fine with addressing these problems by altering the diameters (thus circumferences) of organ pipes as their lengths change.

problem they observed. As I showed in Chapter 1, the Kangxi Emperor spoke positively about Zhu Zaiyu's tuning theory during the court gathering on February 20, 1692. Though no one even mentioned the twelve tuning pitch pipes at the court gathering, let alone the issue of pipes of the same diameter vs. pipes of the same length, the latter topic, both in general and as specifically theorized by Zhu Zaiyu, appears in several parts of the reference book in ff. 52v-83 of the *Putong Guji* No. 15251 manuscript.

Also, as mentioned in the previous chapter above, the third section of the reference book entitled "Discussion of Examining the Tones" or *shenyin* 審音 explicitly mentions Zhu Zaiyu's *Essential Meaning of Pitch Pipes*. In fact, the section directly quotes the passage cited above from Chapter 5, "The Same Circumference or Diameter Is Not to Be Applied to All Tuning Pitch Pipes," Volume 1 of *Essential Meaning*, in which Zhu describes in his experiments with pipes of the same diameter an encounter with what is essentially the "end correction" problem. The section in the reference book continues to cite in block quotes Zhu's argument that, in order to apply the same ratios to strings and to pipes to produce the same intervallic relations, one must opt for pipes of the same shape in lieu of pipes of the same diameter. While taking Zhu Zaiyu's case for pipes of the same diameter seriously, the reference book goes one step further in the direction of *shenyin* "examine the tones," following the very title of this section with this:

> […]凡此，皆不可以空言爭，必制管聽之，而是非始定。[…]隨用一尺，以量其縱長得，即用以量其空徑。先依圍徑皆同之說，制正、倍、半三等律管。聽其半律與正律合否，正律與倍律合否，倍律與半律合否。又聽其一律之均，五聲二變相叶否。依次圍徑增損之說，亦制正、倍、半三等律管，一一聽之。則長短之清濁，圍徑之異同與上下相生之法皆定。[…][14]
>
> […] All these [disagreements over pipes of the same diameter vs. pipes of the same shape] cannot be argued about with empty

14 National Library of China, *Putong Guji* No. 15251, f. 59v.

> words; instead, one must make pipes and listen to them and one will [thereby] start to determine which is right and which is wrong. […] Take whatever standard of length and use it to measure both the lengths and the inside diameter of pipes. First, follow the theory of "[pipes] of the same diameter," make a set of [twelve] full-length pipes, a set of [twelve] double-length pipes, and a set of [twelve] half-length pipes. Listen to whether a half-length pipe corresponds to the full-length pipe [bearing the same name], whether a full-length pipe corresponds to the double-length pipe [bearing the same name], and whether a double-length pipe corresponds to the half-length pipe [bearing the same name]. Also listen to whether among one set of pitch pipes, the "five proper notes and two altered notes" of the *yun* are in tune. Next, following the theory of the "circumference and diameter increase and decrease [proportionally with the length]" [i.e. pipes of the same shape]," similarly, make full-length, double-length, and half-length, in sum three sets of pipes, and listen to them one after the other. In this way, the correspondences between lengths and pitch heights, the equality or inequality of the pitch pipes' diameters, and the methods of upward and downward mutual generations [as cycles of octaves] can all be determined. […]

Remarkably, the third section of the reference book here in *Putong Guji* No. 15251 uses Zhu Zaiyu's experiments regarding pipes of the same diameter vs. pipes of the same shape to devise its own plan of *shenyin* "examining the tones" of pitch pipes. The experiments proposed for both types of pipes in the passage above match exactly those whose results were recorded in the finished lab notes in ff. 88-95 later in the manuscript and those described in Chapter 9, Volume 1 of the eventual *Orthodox* treatise. In addition to investigating whether an octave is found between two pipes whose lengths are in a 1:2 (or 1:4) ratio in either type of pipes, the proposal also suggests listening for whether the pitches of these different sets of twelve tuning pitch pipes are in tune when compared to the "five proper notes and two altered notes" of the diatonic scale. In other words, the proposal intends to discover whether pipes of the same diameter or pipes of the same shape, when both proportioned in length according to "triple division with one part subtracted or added," would produce pitches whose intervallic patterns can

be perfectly matched with the conventional system of pitch organization, whereby an octave is divided into twelve parts and the "five proper notes and two altered notes" are bound within one set of twelve tuning pitch pipes (as opposed to, for example, the *gong* being displaced from *huangzhong* to half-length *taicu* in the fourteen-tone temperament system).

One thing proves beyond reasonable doubt that the researchers of the Qing's tuning reform were fully aware of pipes of the same shape as a potential solution to the "octave not found between two pipes bearing a 1:2 ratio" problem: namely, that *shenyin* "examining the tones"—arguably the central tenet of the fourteen-tone temperament project—was framed in the project's preparatory reference book specifically as verifying the "end correction" problem that Zhu Zaiyu raised in *Essential Meaning* and evaluating his advocacy for pipes of the same shape over pipes of the same diameter. In fact, the prospects of using pipes of the same shape instead of pipes of the same diameter are raised as early as in the first section of the reference book. Entitled "Discussion of the pipe of *huangzhong*," this section opens what can be considered an introduction to the entire reference book. As I observed in Chapter 4 above, the opening lines lay out the bipartite research program of *zhilü* 制律 "making the pitch pipes" and *shenyin* "examining the tones" of said pitch pipes. Immediately afterwards, the section dives into the specifics of the precise measurements of the tuning pitch pipes:

> […] 夫一黃鍾也，司馬遷以為百分，淮南子以為八十一分，劉歆以為九十分。通長之數，多寡不齊。若此不制管而審之，何由辨其孰是乎？班固、司馬彪、鄭康成謂：凡律空圍九分。孟康謂：林鍾圍六分，太簇圍八分。一則律管雖有長短而圍徑皆同，一則圍徑隨長短而亦異。不制管而審之，何由辨其孰是乎？[…][15]
>
> The same *huangzhong* pipe, Sima Qian said it to be 100 *fen* [in length in *Records of the Grand Historian*], while Prince Huainan

15 Ibid., f. 53r.

> said it to be 81 *fen* [in *Writings of Prince Huainan*], and Liu Xin [46 BCE-23 CE] [is recorded in Ban Gu's *Book of the Han* to have] said it to be 90 *fen*. The recorded numbers of its length vary from long to short. How can one ever determine what is correct if not by making pitch pipes and examining them? Ban Gu [in his *Book of the Han*], Sima Biao [c. 240s-306 CE] [in his *Continued Book of the Han*], and Zheng Kangcheng [127-200] [in his annotations of the Confucian canons] said: "every pitch pipe has a 9 *fen* hollow enclosure." Yet Meng Kang [3rd century] [in his annotations to Ban Gu's *Book of the Han*] said: "the enclosure of *linzhong* is 6 *fen*, of *taicu* 8 *fen*." One theory posits that even though the pitch pipes vary in length, their enclosures are the same. The other theory posits that the enclosures also change according to the variance of lengths. How can one ever determine what is correct if not by making pitch pipes and examining them?

In further confirmation of the researchers' awareness of the problem of pipes of the same diameter vs. pipes of the same shape, this passage describes the procedure of *zhilü* "making pitch pipes" as not one but two procedures. In addition to figuring out the exact length and diameter of the single *huangzhong* pipe through research into historical metrology, the passage stipulates a second problem: whether the diameters of the other eleven tuning pitch pipes should remain the same or should change proportionally to their respective lengths.[16] Here, the passage cites early sources—early here typically meaning sources from the Han era (206 BCE-220 CE) and earlier—in support of each option. Sources stipulating pipes of the same diameter include Ban Gu, Sima Biao 司馬彪, author of *Continued Book of the Han* (xu hanshu 續漢書, c. 3rd century), and Zheng Kangcheng 鄭康成, whose annotations of the Confucian canons would become important during the Qing era, which advocated for interpretations of Confucianism by Han-era scholars over those by Song-era scholars such as Zhu Xi. The one source the passage cites in support of pipes of the same shape is Meng Kang 孟康's *Pronunciations and Glosses of Ban Gu's Book of*

16 Again, the diameters concerned here are typically always the inner diameters of pitch pipes.

the Han (hanshu yinyi 漢書音義, 3rd century), which had long been lost by the Qing era and was available only through subsequent quotations and commentaries. Meng (or sources reporting on his work) calculates the enclosure of *huangzhong* of 9 *fen*, *linzhong* as 6 *fen*, and *taicu* as 8 *fen*, the three numbers bearing the same ratios as do the lengths of the three pipes following the Chinese Pythagorean method: 6:9 equals 2:3 or "triple division with one part subtracted", 8:6 equals 4:3 or "triple division with one part added." Thus, in the very likely case that Meng interprets the *kongwei* 空圍 "hollow enclosure" as the circumference and not the surface area, he effectively describes the twelve tuning pitch pipes as pipes of the same shape, whose length-to-diameter ratios are identical. The passage concludes by anticipating the third section of the reference book, arguing that the debate over pipes of the same diameter vs. pipes of the same shape can only be settled by "making pitch pipes and examining them."

A significant fact I have withheld until now, however, is that the passage above, after it was copied out with the rest of the reference book, would later be tampered with. According to the preface to the reference book, f. 52v, the reference book was *jinshang* 進上 "submitted [to someone superior" to In-c'i, and thus was most likely written out as a fair if not presentational copy. While In-c'i annotated various parts of the reference book with his vermillion paintbrush and while a few traces of copyediting in dark ink here and there, the passage cited above from the beginning of the first section is the only one that was at any point heavily redacted and revised, to the point of altering its core meaning. Below, I repeat the same passage, marking all the parts that were eventually crossed out with a double strikethrough and with the added parts shown in bold fonts and with underlining:

> […] 夫一黃鍾也，司馬遷以為百分，淮南子以為八十一分，劉歆以為九十分。通長之數，多寡不齊。若此不制管而審之，何由辨其孰是乎？班固、司馬彪、鄭康成~~謂~~**<u>凡</u>**律**<u>管</u>**空圍九

分之數。~~孟康謂：林鍾圍六分，太簇圍八分。一則律管雖有長短而圍徑皆~~各家不同~~，一則圍徑隨長短而亦異~~。不制管而審之，何由辨其孰是乎？[…][17]

The same *huangzhong* pipe, Sima Qian said to be 100 *fen* [in length in *Records of the Grand Historian*], while Prince Huainan said it to be 81 *fen* [in *Writings of Prince Huainan*], and Liu Xin [46 BCE-23 CE] [is recorded in Ban Gu's *Book of the Han* to have] said it to be 90 *fen*. The recorded numbers of its length vary from long to short. If not by making pitch pipes and examining them, how can one ever determine what is correct? Ban Gu [in his *Book of the Han*], Sima Biao [c. 240s-306 CE] [in his *Continued Book of the Han*], and Zheng Kangcheng [127-200] [in his annotations of the Confucian canons]**'s numbers for** ~~said:~~ "every pitch pipe has a 9 *fen* hollow enclosure~~." Yet Meng Kang [3rd century] [in his annotations to Ban Gu's *Book of the Han*] said: "the enclosure of *linzhong* is 6 *fen*, of *taicu* 8 *fen*." One theory posits that even though the pitch pipes~~ vary in length **from one author to another.*** ~~, their enclosures are the same. The other theory posits that the enclosures also change according to the variance of lengths.~~ If not by making pitch pipes and examining them, how can one ever determine what is correct?

[*The revised sentence reads: Ban Gu, Sima Biao, and Zheng Kangcheng's numbers for "every pitch pipe has a 9 *fen* hollow enclosure" vary from one author to another.]

Besides the remarkable amount of cross-outs, at one point, whoever revised the passage inserted the three characters *gejia bu* 各家不 in the mix of an extensive line that got crossed out. In and of themselves, these three characters are nonsensical and grammatically incomplete, as they literally mean "each author not." It is only by preserving a single character *tong* 同 "same" from that crossed-out part and grafting the three nonsensical characters onto this character that the revision makes any sense at all: *gejia butong* 各家不同 "… vary from one author to another." On a broader level, the second half of the passage that originally discusses pipes of the same diameter vs. pipes of the same shape has essentially been written anew. The reference to

17 Ibid.

Meng Kang's pipes of changing diameters is effaced. The contrast between "one theory" favoring pipes of the same diameter and "the other theory" favoring pipes of the same shape is likewise effaced. The scholarly debate over *weijing* 圍徑 "enclosure and diameter" is no longer about the base diameters of the other eleven tuning pitch pipes to that of *huangzhong*'s—that is, whether they are all identical or change according to their Pythagorean-proportioned lengths. Instead, it is about the different "numbers" various authors came up with in their interpretation of the dictum "every pitch pipe as a 9 *fen* hollow enclosure."

Pipes of the same diameter are being recast as a scholarly consensus, rather than a matter of dispute.

It is difficult to date the heavy-handed revision of this passage, since the reference book itself does not bear a date either. Yet the erasure of pipes of the same shape from what is effectively the introduction to this preparatory compilation of sources for In-c'i's research into musical tuning anticipated what would await those pipes of the same shape in the eventual *Orthodox* treatise. Here, I turn to Mersenne, the second source from which researchers for the fourteen-tone temperament knew about pipes of the same shape as a solution to their observation of "1:2 length ratio not corresponding to an octave between two pipes of the same diameter." So far, I have not found any record of Mersenne's *Harmonie universelle* appearing in the circles of the Kangxi Emperor or his sons. There is proof, however, that another Jesuit-authored monument in music did reach the desk of the Emperor at some point even before the 1692 court gathering: *Musurgia universalis* (1650) by Athansius Kircher (1602-1680). Heavily based on Mersenne's *Harmonie universelle*, especially in acoustics and organology, *Musurgia* played a major role in shaping various parts of the *Orthodox* project, albeit not in the fourteen-tone temperament *per se*.

As I showed in the previous chapter, according to the diary of the two French Jesuits Jean-François Gerbillon and Joachim Bouvet, on May 1, 1690, the Portuguese Jesuit Tomás Pereira (1645-1708), who had been giving the Kangxi Emperor musical lessons, presented to him "a book on the principles of music." The two French Jesuits specify in their diary that it was a treatise on "la musique pratique"—the characterization that allowed them to excuse themselves from translating it into Manchu—which I interpret to refer to what would eventually become *Elements of Pitch Pipes*, the prototype of the fifth and last volume of *Orthodox* on Western *musica practica*. Pereira's treatise appears to have lit a spark in the Emperor's head. Just a few days later, the French Jesuits recorded in their diary (see also Illustration 5-1):

> Le 8^{e}. 9^{e}. 10^{e}. 11^{e}. l'Empereur s'est fait expliquer par nous l'ordinaire sa leçon d'Euclide et il s'y est raisonablement appliqué. Il s'est amusé une fois fort long temps a feuilleter un gros volume du pere Kirker qui traite des sons et de tout ce qui appartient a là musique. un jour il nous demanda en detail comment se formoient les songes, et quelles estoient leurs causes ce qu'ils signifioient et divers autres points de physique comme l'origine des fontaines &c.[18]

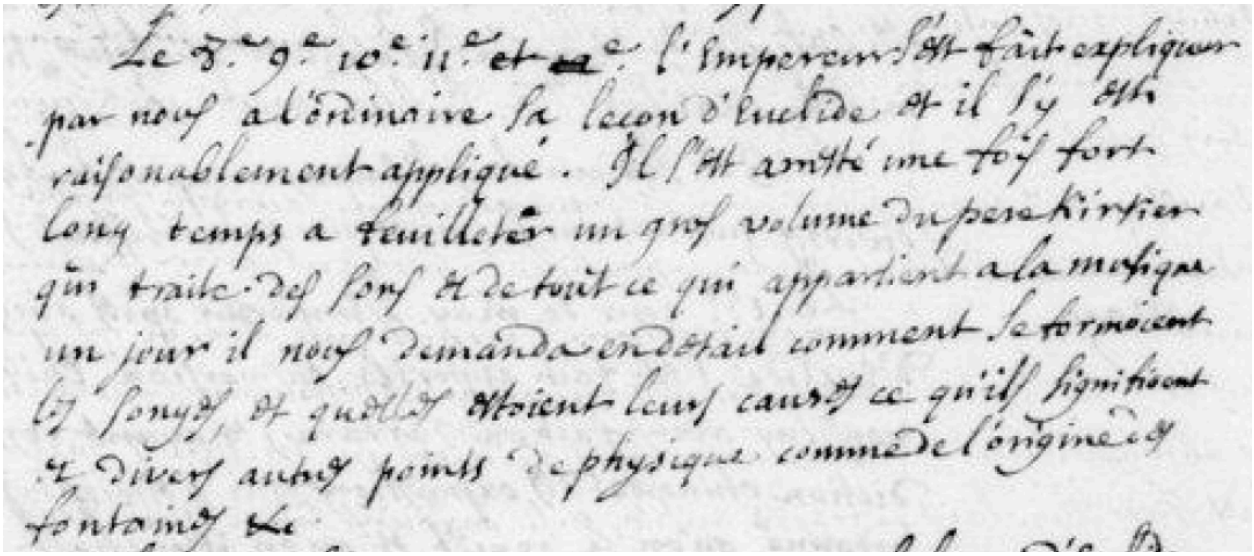
> The 8th, 9th, 10th, and 11th [of May]. The Emperor had us explain, per the usual, his lesson on Euclid, and he devoted himself fairly hard to the matter. Once, he spent a very long time leafing through a thick volume by Father Kircher that discusses sounds and all that pertains to music. One day he asked us in detail how dreams are formed, what their causes are, and what they represent, and various other points of physics such as the origin of fountains.

There is no other candidate for "a thick volume by Father Kircher that discusses sounds and all that pertains to music" other than *Musurgia universalis*. From their failure to mention the title of this famous treatise, it was likely not the two French Jesuits who brought this work to the Kangxi Emperor's attention. As early as the 1660s, as many as twenty-four copies of *Musurgia*

18 Bibliothèque nationale de France, MS Français 17240, f. 277v.

were brought to China, as Kircher used the network of Jesuits to disseminate his *magnum opus* on music on a global scale.[19] Pereira's newly submitted treatise on Western music might have reminded the Emperor of the existence in his library of a copy of *Musurgia*, a treatise whose sumptuous visuality would have struck even those who are illiterate in Latin.

Illustration 5-1 Bibliothèque nationale de France, MS* Français *17240, f. 277v



Because the polyglottism of the Kangxi Emperor did not include Latin, and because he did not ask the two French Jesuits to explain to him the content of *Musurgia* as they had been doing with Euclidean mathematics for some time, it was hardly possible that the Emperor while "leafing through" Kircher's monument understood any of its contents. Still, this turned out not to be the first time that the Kangxi Emperor encountered *Musurgia*. In fact, Pereira's new treatise may have reminded the Emperor of that previous occasion in which he was presented another treatise on Western music as well, one that turned out to be a version of *Musurgia*. That occasion took place at some point after the summer of 1685. That summer, on August 1, 1685, the

19 See Noël Govlers, "Two Overlooked Letters of Ferdinand Verbiest to Athanasius Kircher. How a Missionary Project was Shaped, and Kircher's Books were Received in Mid-17th-Century Spain and Portugal," *Humanistica Lovaniensia* 54 (2005): 267-284.

Flemish Jesuit Ferdinand Verbiest wrote to the Superior General of the Society of Jesus in Rome, Charles de Noyelle:

> […] Nunc etiam artem musicae nostrae Europaeae, quam plurimum laudat, ipsemet addiscit eamque nomine suo in lucem edere desiderat, atque hanc ob causam quando hoc anno, die 2 Julii calores aestivos declinandi gratiâ in Tartariam Occidentalem profectus est, P. Thomam Pereira Lusitanum in musica practica egregie peritum secum duxit. Hanc ipsam ob causam ego modo ex **Musurgia** [emphasis original] P. Kircheri rariora aliqua tam theoretica quam practica in idioma Sinicum verto et in exigui tractatûs libellum compono, Imperatori cum reversus fuerit offerendum, ut scilicet illam quam de scientiis Europaeis concepit opinionem vel hoc modo utcumque confirmem.[20]
>
> […] Now, indeed [the Emperor] himself is learning the art of our European music, which he praises very much, and he wants to bring it forth into the public view by his own name. And for that reason, when he set out for Eastern Tartary on July 2 of this year in order to avoid the summer heat, he brought with him Father Thomas Pereira the Portuguese, [who is] excellently skilled in practical music. For this very reason, I am currently translating in the Chinese idiom some scattered passages from Father Kircher's ***Musurgia***—[passages that are] theoretical as well as practical—and composing them into a small book of a humble treatise. It will be offered to the Emperor when he returns, namely so that I can fortify the opinion that he has formed about European learnings, in this or whatever way.

This letter from Verbiest further confirms that Tomás Pereira's role as the Kangxi Emperor's music teacher was limited to *musica practica*. As I argued in the previous chapter, except for *Elements of Pitch Pipes*, which is a primer on the rudiments of Western music theory as applied to musical practice, most of Pereira's musical engagements with the Kangxi Emperor took place in the form of music-making and face-to-face interactions. Here, Verbiest further describes Pereira's expertise in "musica practica" as "peritus," which connotes knowledge

20 Archivum Romanum Societatis Iesu, *Jap. Sin.* 145, f. 82v, in Verbiest's letter to Charles de Noyelle in Rome on August 1, 1685.

specifically in the sense of having experience and practical skill (compare *peritus* to *experior* "to try," "to experience," "to test"). By emphasizing the practical focus of Pereira's musical activities and output, the Flemish Jesuit and Chief Astronomer (*qintian jian jianzheng* 欽天監監正) of the Qing Empire got to set himself apart. Indeed, it does appear Verbiest felt his status as the Kangxi Emperor's favorite European threatened by the quick rise of Pereira at the court. Since 1677, the Kangxi Emperor and his court would go on a summer retreat for several months every year to "Eastern Tartary" or the area intersecting Manchuria and Mongolia to the northeast of China beyond the Great Wall. Whereas the Kangxi Emperor had brought Verbiest to these retreats for several years in a row, in 1685 he ordered Verbiest to stay in Beijing and brought Pereira instead, precisely in order to continue their musical lessons.

No archival or bibliographic record mentions whether Verbiest did submit his short selective translations of Kircher's *Musurgia* to the Kangxi Emperor when the court returned at the end of the summer of 1685, or where the treatise ended up. Yet I would argue that a treatise in five volumes entitled *Proportions of Pitch Pipes* (lülü jieyao 律呂節要, or lioi lioi jiyei yoo bithe ᠯᡳᠣᡳ ᠯᡳᠣᡳ ᠵᡳᠶᡝᡳ ᠶᠣᠣ ᠪᡳᡨᡥᡝ in Manchu), which currently exists in several copies in Chinese and Manchu, was at least based on Verbiest's rendition of *Musurgia*. Its fairest extant copy in Chinese being at a length of 81 folios, *Proportions* does fit Verbiest's description of a *libellum* "short tract": in comparison, a copy of *Elements of Pitch Pipes* produced with the same *mise-en-page* as that 81-folio copy of *Proportions* spans 80 folios, and its content would become one of the five volumes of *Orthodox Meaning of Pitch Pipes*. Further, though I cannot elaborate on this here, a significant part of *Proportions* can only be traced back to Kircher's *Musurgia* as its source. A treatise focused on tuning and acoustics, *Proportions* discusses the general properties of sounds in Volume 1, the mechanism of sound production in strings and the ratios and causes of

consonance vs. dissonance between different strings in Volumes 2 and 3, and the mechanisms of sound production in pipes and the ratios and causes of consonance and dissonance between different pipes.[21] Besides its mention of *tiesi qin* 鐵絲琴 "metallic-string zither" or violin and *feng qin* 風琴 or organ,[22] the treatise's discussion of the vibration theory of sound production, the vibratory-coincidence theory of consonance and dissonance, and a variety of finer points such as the speed of sound suggest that it incorporated some of the latest developments on acoustics in Western Europe. While anyone ranging from Galileo Galilei to Marin Mersenne could have been the treatise's source for this information, Volumes 2 and 3 of *Proportions* uses a rather peculiar model to explain the causes of consonance and dissonance in strings that can only be traced back to Book IX of Kircher's *Musurgia*. Put simply, both Kircher at one point in *Musurgia* and

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21 Conventionally, the treatise is assumed to be authored by Tomás Pereira; see, for example, Weng Panfeng 翁攀峰, "關於'康熙十四律'思想來源的初步探討" ("A Discussion on the Origin of "The Fourteen Temperament [*sic*] of Kangxi"), *Wenhua yishu yanjiu* 文化藝術研究 6 (2013), 33. This assumption, however, is based solely on the preface to the fifth and last volume or *xubian* "appended tract" of *Orthodox Meaning of Pitch Pipes*, which I have translated in full in Chapter 4, pages 344-345 of this dissertation. The preface mentions "two main points" of Pereira's "book" on music: (1) "the causes of the sounds of pipes and strings, and the reasons for their pitches' being in tune or being out of tune," and (2) "establishing the rules of examining sounds and measuring proportions, by using the hard and soft signs to differentiate the two modes of *yin* and *yang*, and by using long, short, slow, fast, and other signs to differentiate between durations of notated pitches." In terms of content, the first main point matches very well the text of *Proportions of Pitch Pipes* that has survived, whereas the second main point matches that of *Elements of Pitch Pipes*, the basis of this fifth and last volume of *Orthodox* on music theory. Still, as I have shown in Chapter 4, the earliest recorded submission by Tomás Pereira of any book on music to the Kangxi Emperor was May 1, 1690. Meanwhile, Ferdinand Verbiest worked on his translation of Kircher's *Musurgia* in the summer of 1685 and explicitly wrote about his plan to submit it to the Kangxi Emperor upon his return from Jehol to Beijing. It is probable that the compilers of *Orthodox* could have misattributed *Proportions of Pitch Pipes* to Tomás Pereira a few decades later, especially since the other text, *Elements of Pitch Pipes*, does appear to have been authored and submitted to the Kangxi Emperor by Pereira, according to Gerbillon and Bouvet's entry in their diary on May 1, 1690; see Chapter 4, pages 339-441 of this dissertation.

22 Ferdinand Verbiest (?), *Lülü jieyao* 律呂節要 ("Proportions of Pitch Pipes"), vol. 1, Chapter 1.

Proportions, in its Volumes 2 and 3, analyze the sound-producing vibrations of strings not as the oscillation of the body of the string itself as a transverse wave but rather the back-and-forth journey of the vibration itself between its two ends along its linear body as though a longitudinal wave.[23]

What tied *Proportions* into the *Orthodox* project, however, was mainly its Volumes 4 and 5 on pipes, specifically the pipes of organs. Since Pereira was assigned by the Society to Beijing on account of his mechanical skills in clockwork and instrument-making, he was responsible for building several organs of varying sizes, some for the churches in the city and others for the Kangxi Emperor. In addition to their impact on Beijing's soundscape and the curiosity they aroused in some Han Chinese literati, organs also played a role in studies of musical tuning at the Qing court.[24] According to the epigraph engraved on the stele by the entrance to his graveyard, Wang Lansheng received "the imperially made [i.e. commissioned] pipe organs and several other instruments" (御製管風琴諸器) when the Kangxi Emperor asked him to join the team of scholars assisting In-c'i in compiling *Orthodox* and the entire *Origins* treatise.[25]

Further in a secret palace memorial to his father dated on January 15, 1714, or about two months after his father assigned to him a support staff and workspace for the compilation project, In-c'i mentions they he had specifically been adapting *Proportions of Pitch Pipes* for writing

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23 See Athanasius Kircher, *Musurgia sive Ars Magna consoni et dissoni* (Rome, 1650), Book IX "Magia Consoni & Dissoni," 207-208.

24 See David Francis Urrows, "The Pipe Organ of the Baroque Era in China," in Hon-Lun Yang and Michael Saffle ed., *China and the West: Music, Representation, and Reception* (Ann Arbor: University of Michigan Press, 2017), 21-48.

25 Wang Lansheng 王蘭生, *Jiaohe ji* 交河集 ("Anthology of Sir Wang from Jiaohe," 18th century), vol. 1, f. 3r.

Chapter 1, Volume 2 of *Orthodox*.[26] An exceptionally rich and equally bizarre chapter that would take more space than is available here to unpack, the central tenet of this chapter is evident in its title: "Explaining the Differences between Matching Proportions with the 'Five Proper Notes and Two Altered Notes' on Pitch Pipes vs. Strings" (明管絃律度五聲二變取分之不同).[27] This chapter borrows from *Proportions* on several levels. First, it takes the latter treatise's explanation of sound production on strings and pipes to argue that the latter two are fundamentally different sounding mechanisms. Strings, it argues, produce sound through the active vibrating of its own concrete bodies, yet pipes produce sound by being passively bombarded on their inner wall by the flow of air.

Illustration 5-2 Air producing sound in a pipe by colliding on and reflecting of its inner wall, showing why two pipes whose lengths and diameters bear a 1:2 ratio are in accord, Chapter 5, Volume 4 of* Proportions of Pitch Pipes *(c. 1690)

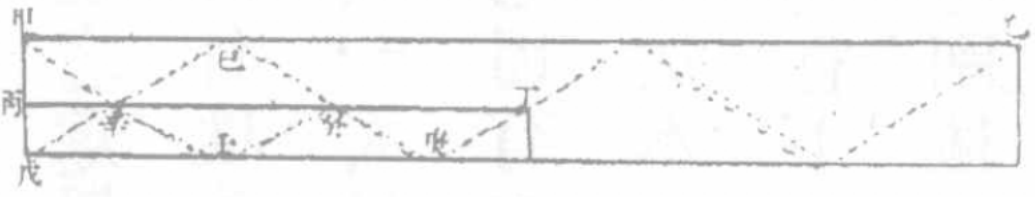
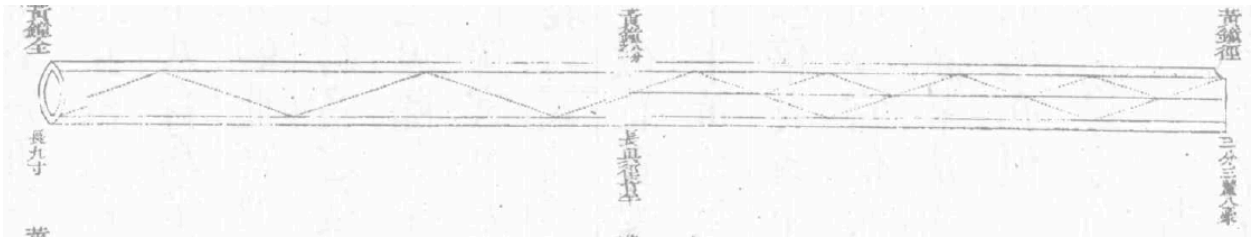


Illustration 5-3 The same illustration above adapted in Chapter 1, Volume 2 of* Orthodox, *f. 3bis

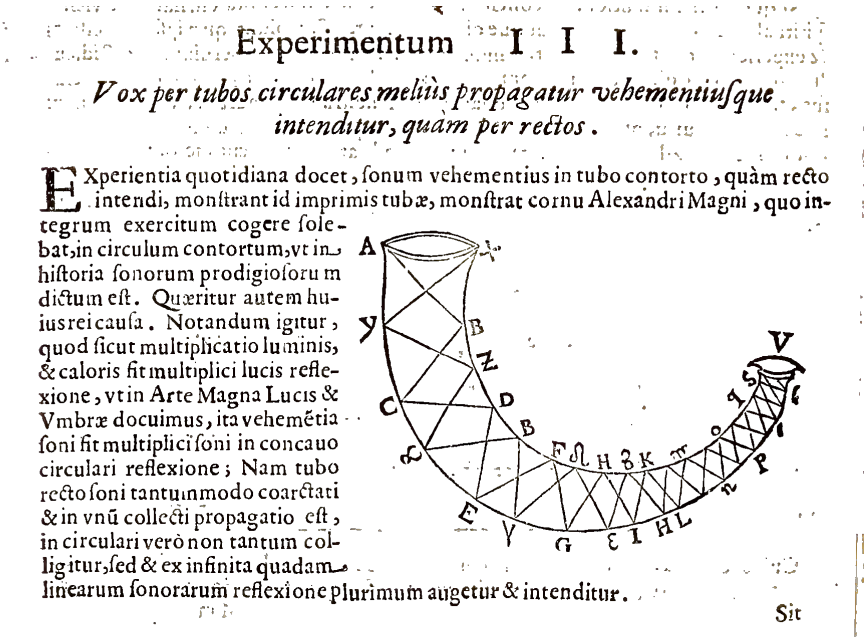


26 See *Kangxichao Manwen Zhupi Zouzhe Quanyi* 康熙朝滿文硃批奏摺全譯 ("Complete translations of imperially annotated palace memorials in the Manchu language from the reign of Kangxi") (Beijing: Zhongguo shehui kexue chubanshe, 1996), Vol. 3, item No. 2307, 914. See also Weng Panfeng, "關於'康熙十四律'思想來源的初步探討" ("A Discussion on the Origin of "The Fourteen Temperament [*sic*] of Kangxi"), 34-41; Weng, to my knowledge, was the first to realize this connection between *Proportions* and *Orthodox*.

27 In-c'i et al., *Orthodox Meaning of Pitch Pipes*, vol. 2, ff. 1r-3v.

Second, Chapter 1, Volume 2 of *Orthodox* takes the latter treatise's explanation of the causes of consonances vs. dissonances between the pitches of different pipes. It is difficult to make sense of much of this explanation, yet it essentially tries figure out how the incident angle by which the air enters a pipe and the length of the pipe may affect the number of times the air will collide with the inner wall of the pipe and produce sound (See Illustrations 5-2 and 5-3). While I am yet to locate any corroborative evidence, this unique model using the collision of air and breath to explain the production of sound in a pipe and of consonances vs. dissonances between different pipes might have taken some inspiration from Kircher's fascination with echoes in Book IX of *Musurgia*,[28] which is the same part of his treatise from which *Proportions of Pitch Pipes* has borrowed a similarly peculiar explanation for consonances in strings, as mentioned above. Illustration 5-4, for example, shows a circular pipe Kircher whereby the

Illustration 5-4 A semicircular pipe designed to amplify the human voice, in Kircher,* Musurgia *(1650), Book IX, 247



Experimentum III.

Vox per tubos circulares meliùs propagatur vehementiusque intenditur, quàm per rectos.

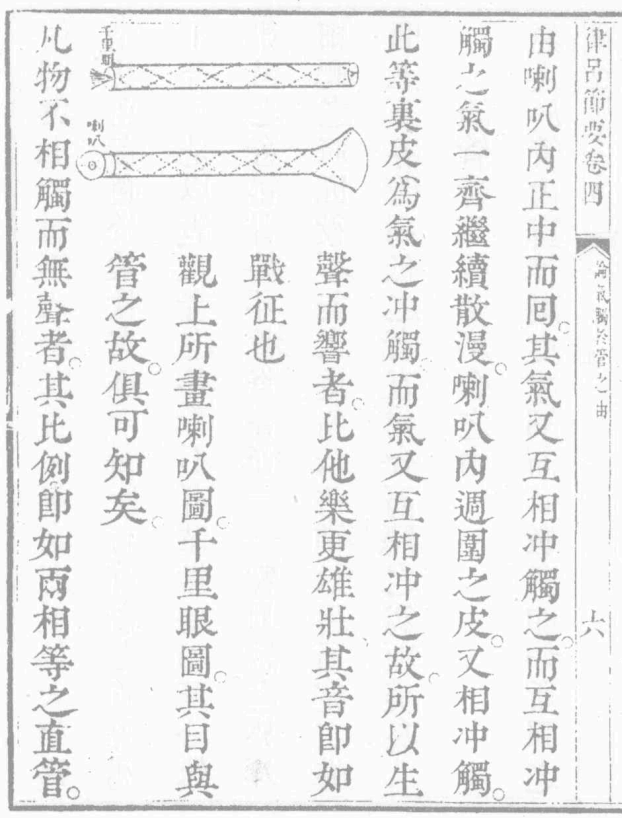
EXperientia quotidiana docet, ſonum vehementius in tubo contorto, quàm recto intendi, monſtrant id imprimis tubæ, monſtrat cornu Alexandri Magni, quo integrum exercitum cogere ſolebat, in circulum contortum, vt in hiſtoria ſonorum prodigioſorum dictum eſt. Quæritur autem huius rei cauſa. Notandum igitur, quod ſicut multiplicatio luminis, & caloris fit multiplici lucis reflexione, vt in Arte Magna Lucis & Vmbræ docuimus, ita vehemẽtia ſoni fit multiplici ſoni in concauo circulari reflexione; Nam tubo recto ſoni tantummodo coarctati & in vnũ collecti propagatio eſt, in circulari verò non tantum colligitur, ſed & ex infinita quadam linearum ſonorarum reflexione plurimum augetur & intenditur.

Sit

28 See Kircher, *Musurgia*, Book IX, 247-308.

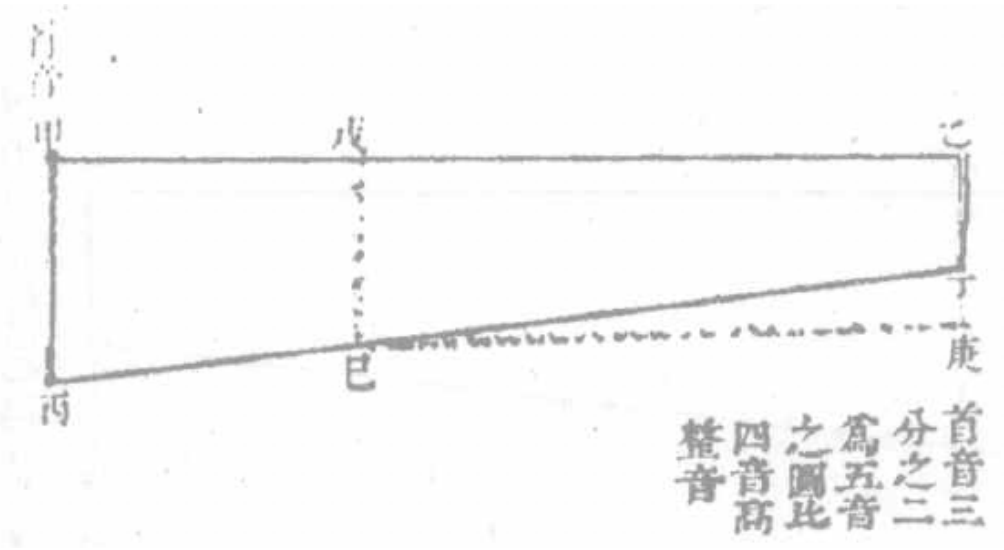
human voice is amplified through back-and-forth movements colliding and reflecting on the pipe's inner wall. Of course, there is an obvious difference between the sound-producing pipes in *Proportions* and the voice-amplification pipes designed by Kircher: the active agent in the former is the human breath or air (*qi* 氣, or *sukdun* ᡧᡠᡴᡩᡠᠨ in the Manchu version of the text), whereas that in the latter is the human voice. Still, not only is the mechanic process of collisions and reflections formally identical, but *Proportions of Pitch Pipes* also borrowed Kircher's light-sound analogy that precisely drove the latter's fascination with echoes in *Musurgia*. As shown in Illustration 5-5, when explaining how sound is produced in a *laba* 喇叭 (also called a *suona* 嗩吶, a type of key-less conical reed pipe), which, unlike the pipe presented earlier in Illustration 502, does not feature its embouchure on one side of its body but right in the middle, *Proportions*

***Illustration 5-5 An explanation of how sound is produced in a conical reed pipe in comparison to the mechanism of the telescope, Chapter 3, Volume 4 of* Proportions**



analogizes it to a *qianli yan* 千里眼 "an eye that can see across a thousand miles"—or a telescope, one of the recurrent instruments related to calendar-keeping and celestial navigation European missionaries used to gain favor from the Kangxi Emperor.[29] While the text leaves its readers to fill in the detail of this analogy, it explains that the reed pipe produces sound through the collision and reflection of the breath on its inner wall in all directions so that, rather like the telescope amplifying an image also through analogous movements of light within its walls, the sound of the reed pipe is "grander and more powerful than that of the others, and its tones are warlike."[30]

***Illustration 5-6 A trapezoid reference graph for calculating the circumference of a* sol *pipe in proportion to its length qua 2/3 of that of the* do *pipe, Chapter 3, Volume 5 of* Proportions**



Three, and most importantly, Chapter 1, Volume 2 of *Orthodox* takes both the explanation and the conclusion from Volume 4 of *Proportions of Pitch Pipes* that, in order to produce an octave between two pipes whose lengths are in a 1:2 proportion, one must also

29 See Jami, *The Emperor's New Mathematics*, 33-34 and 111-116.
30 Verbiest (?), *Proportions of Pitch Pipes*, vol. 4, Chapter 3..

reduce the diameter of the shorter pipe to half of that of the longer pipe—or, one must make organ pipes of the same shape, not the same diameter. The question of how to make organ pipes of the same shape takes up much of Volumes 4 and 5 of *Proportions*, which discuss aerophone acoustics and organ pipes. In a direct quote from Kircher's *Musurgia*, which in turn borrows heavily from Mersenne's *Harmonie*, Volume 4 of *Proportions* suggests that the circumference of the organ pipes must not remain constant but should be 1/4 or 2/5 of their lengths, or anywhere between these two ratios.[31] Notice that, in contrast to the *tongjing guan* "pipes of the same diameter" vs. *tongxing guan* "pipes of the same shape" in which the role of *jing* 徑 "diameter" is at play, here, all three texts bespeak of circumferences: after all, unlike *lülü* "tuning pitch pipes" made of bamboo stems, organ pipes were made of rolled-up metal sheets that can be measured unrolled and flattened as a rectangle. And Volume 5 of *Proportions* devises a series of reference graphs, all in the shape of a right trapezoid that would help an organ maker obtain both the circumference of an organ pipe and its length by applying to them the same proportions (see Illustration 5-6). These individual trapezoid graphs were clearly derived from the large trapezoid graph found in Problemas No. 1 and 2, Chapter III, Part III of Book Six of Kircher's *Musurgia*, which was in turn based on the many organ-proportioning trapezoid graphs in Mersenne's *Harmonie* (see Illustrations 5-7 and 5-8).

31 Ibid., Chapter 6. See Kircher, *Musurgia*, Book VI "De Musica instrumentali," 507. Notice that, like Mersenne, Kircher does not keep the same length-to-diameter ratio for all his organ pipes; instead, this ratio becomes larger as the pipes become shorter—so that the pipes become even proportionally thinner as they become shorter. The same is true for the pipes in *Proportions*.

Illustration 5-7 Trapezoid reference graph for calculating the circumferences of organ pipes in relation to their lengths, Book VI of* Musurgia, *508

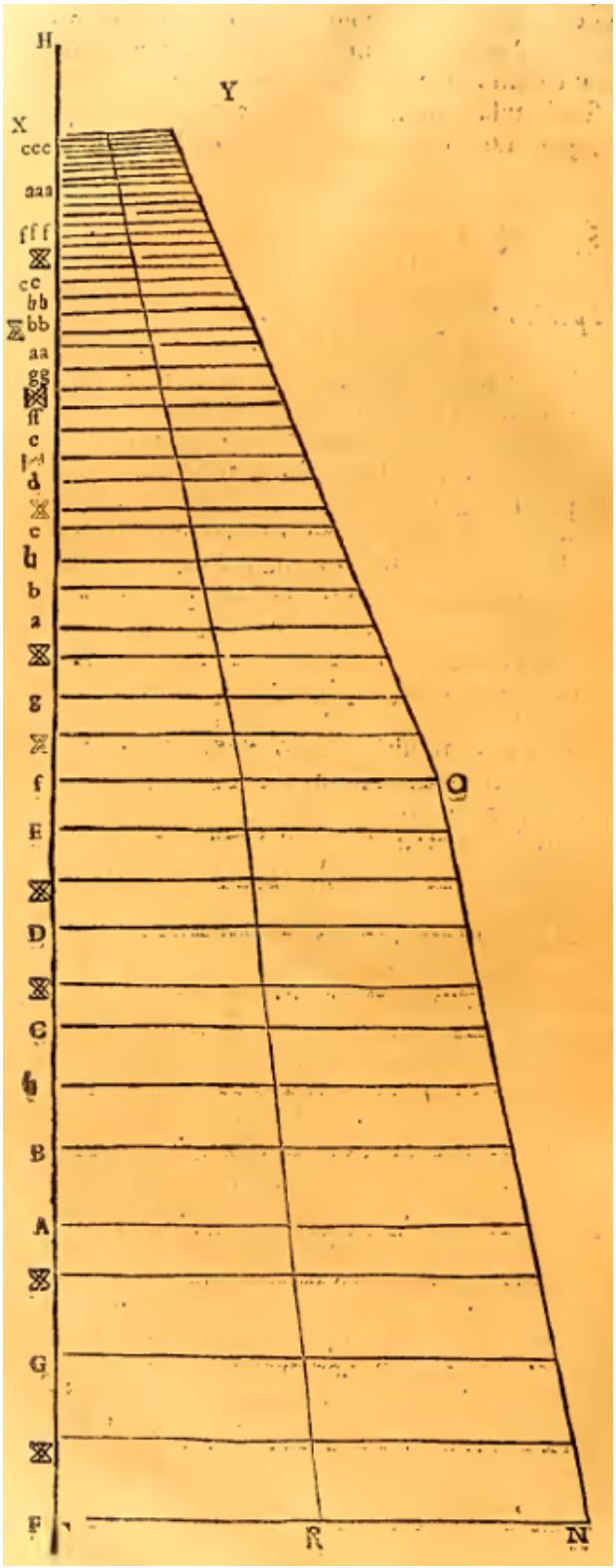
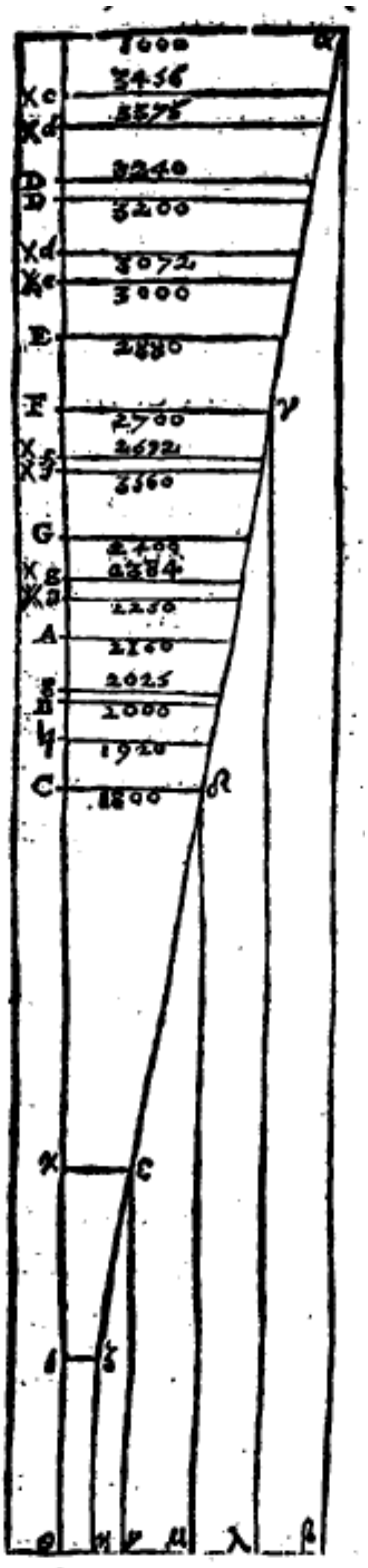


Illustration 5-8 Trapezoid reference graph for calculating the circumferences of organ pipes in relation to their lengths, Livre IV "Des Orgues," Part II of* Harmonie, *336



Doubling Down

The trajectories whereby both Zhu Zaiyu's and Mersenne's studies of the tuning irregularities of pipes of the same diameter reached the researchers behind the Kangxi Emperor's fourteen-tone temperament make very clear the following: they knew very well that using pipes of the same shape, or tampering in whatever manner with the diameter of the tuning pitch pipes, as their lengths vary, could solve the "octave not corresponding to 1:2 length proportion" problem they observed in the experiments described in Chapter 9, Volume 1 of *Orthodox*. They knew very well that, if they simply relented on their adherence to pipes of the same diameter, they would not have to take their observation of the fourteen-fold division of the octave so seriously as to devise an entirely new system of pitch organization based on two complementary diatonic *yun*'s. Instead, they would simply have treated that observation as a glitch, which they could easily have fixed by making pipes of proportional diameters, so that the same Chinese Pythagorean "triple division with one part subtracted or added" method would create for both strings and pipes the same patterns of octave equivalences and diatonic intervals.

Yet, not only did researchers choose not to adopt this solution presented to them from two independent sources, they even doubled down on their commitment to pipes of the same diameter and to the fourteen-tone octave they have heard from these pipes. To recall, Volume 1 of *Orthodox* does not actually hide the fact that its compilers know full well that once one reduces both the length and the diameter of a pipe by half, one will produce another pipe that sounds exactly an octave above the original one. To revisit Illustration 3-3 and Table 3-2 from Chapter 3, it is clear that the twenty-seven "*huangzhong*" pipes of the same shape created in the concluding Chapter 12, Volume 1 of *Orthodox* are based on this very notion that, put simply, pipes of the same shape nullify the "end correction" problem. The longest pipe, 8-fold

huangzhong, is "8-fold" because it is both twice as long and twice as wide in diameter as is the original, full-length *huangzhong* pipe, thus having a volume that is eight-times as large. The shortest pipe, 1/8-times *huangzhong*, is "1/8-times" because it is both half as long and half as wide in diameter as is the original, full-length *huangzhong* pipe, thus having a volume that is 1/8 as large. As the twenty-seven pipes of the same shape encompass two octaves, the pitches of these three pipes are taken as octave equivalences. Indeed, the left column of Table 3-2 first matches the 8-fold *huangzhong* and original full-length *huangzhong* pipes to the pitch of the original full-length *huangzzong* and the original half-length *taicu* pipes (of the same diameter), respectively, with the two pipes sounding an octave apart from each other. In turn, the right column of the Illustration also matches the full-length *huangzhong* and 1/8 *huangzhong* pipes with the same octave between the original full-length *huangzhong* and half-length *taicu* pipes.

Now, by this stage, the researchers have three pipes that not only sound an octave apart from one another but also bear the length ratio of 4:2:1 in correspondence to such octaves. In other words, as stipulated in the third section of the reference book in *Putong Guji* No. 15251, they have obtained the *zheng* "original," *bei* "double-length," and *ban* "half-length" pipes of *huangzhong*, all of the same shape or length-to-diameter ratio, rather than the same diameter. They have established that Zhu Zaiyu's *Essential Meaning of Pitch Pipes* was indeed correct. All they needed to do was to find the other eleven tuning pitch pipes of the same shape between the double-length and double-diameter *huangzhong* pipe and the original full-length *huangzhong* pipe, as well as the eleven tuning pitch pipes of the same shape between the full-length *huangzhong* pipe and the half-length and half-diameter *huangzhong* pipe, both using "triple division with one part subtracted or added" to proportion the lengths and the diameters. In so

doing, they would have created twenty-three pipes spanning two twelve-tone octaves, compliant with the conventional system of pitch organization.

Instead of creating eleven pitch pipes of the same shape between each of the two *huangzhong* pipes of the same shape, the researchers created thirteen. Instead of using "triple division with one part subtracted or added," which would have generated the proportional lengths and diameters of those eleven pipes, they concocted a novel and bizarre quasi-exponential series of integers from 8 to 1 and from 1 to 1/8 in order to proportion not the lengths or diameters but the volumes of thirteen pipes of the same shape in between each pair of *huangzhong* pipes of the same shape. Instead of making pipes of the same shape for two twelve-tone octaves that match the conventional system of pitch organization, they made pipes of the same shape for two fourteen-tone octaves that match the new system they had devised from none other than those original pipes of the same diameter.

The contrast, then, is clear. Zhu Zaiyu, Mersenne, and researchers for the Qing's fourteen-tone temperament noticed the tuning irregularities of pipes of the same diameter. Namely, if one applies to them the same ratio as one does normally to the lengths of strings, be it twelve-tone equal temperament (Zhu Zaiyu), twelve-tone mean-tone temperament (Mersenne), or twelve-tone Pythagorean tuning (Qing researchers), one ends up begetting some unconventional octave relations that would divide the octave into more than twelve parts among those parts. Yet their reactions to these irregularities could not be more different. Zhu and Mersenne took these observations with pipes of the same diameter as a problem needing a solution. And they solved it by changing the way in which pipes are made, that is, by changing their diameters proportionally to the variance of their lengths. The Qing researchers did the exact opposite. Though cognizant that pipes of the same shape could solve the problem in pitch

organization created by pipes of the same diameter, they not only stayed loyal to pipes of the same diameter but even made a series of pipes of the same shape to conform to that new fourteen-tone octave. They took what could have been the solution and made it part of the problem.

The deliberate refusal on the part of the Qing's researchers to alter the manner in which they *zhilü* "made tuning pitch pipes" is confounding—and it is worth repeating my argument back from Chapter Three that there is no easily political or ideological explanation for this. Not only was the Kangxi Emperor not involved in the research that years before his commission of *Orthodox Meaning of Pitch Pipes* had already produced the key findings for the fourteen-tone temperament under the direction of In-c'i, but there is also no obvious advantage for the Qing Empire's rule over China for it to contemplate a deliberate re-division of the octave from twelve-fold into fourteen-fold, or to be so stubborn with pipes of the same diameter despite a much easier alternative path. To be sure, following the rich historical records on the topic, making pitch pipes in and of itself is largely an epistemically silent endeavor, inasmuch as determining the exact lengths and diameter(s) of the tuning pitch pipes depends significantly on how one interprets the at-times contradictory sources and how one conducts one's calculations. But what is the point of the empiricism embedded in *shenyin* "examining the tones" of pitch pipes if one is absolutely unwilling to change the pitch pipes themselves based on one's observation of the patterns of their pitches? For Zhu Zaiyu, the empirical experience of making and listening to pipes of the same diameter compelled him to decide against that particular design of tuning pitch pipes and opt for pipes of the same shape, despite his own acknowledgement that historical sources are not on his side in this regard. This willingness to use empirical experiments to decide or at least guide his historicist analyses of sources is evident also in Zhu's signature development

of twelve-tone equal temperament.[32] As a version of this temperament has long been practiced by real-life musicians, Zhu argues, this twelve-tone equal temperament is more likely the actual tuning system the ancients used and has come down through generations and generations of oral transmission. Meanwhile, the profuse records of "triple division with one part subtracted or added" found in historical texts, which most musical practitioners cannot or do not care to read, is more likely just an imprecise approximation of the fluid, intuitive, and experience-based tuning that musicians had been practicing for ages.

A philological decision—that the 2:3 and 4:3 ratios in sources dating from two millennia ago are not meant to be taken as literal computations but only as estimates of real-life practices—has thus been effected by empirical observations.

This feedback loop between the historicist and epistemically silent processes of "making the pitch pipes" and the empirical and sonorous processes of "examining the tones" of said pitch pipes appears also in the third section of the Qing tuning reform's preparatory reference book in the *Putong Guji* No. 15251 manuscript. Near the start of this section entitled "Discussion of Examining the Tones," it writes: "in truth, without establishing the pitch pipes, there is of course no basis from which to examine the tones [*shenyin*], yet without examining the tones, there is also no basis from which to determine the pitch pipes" (夫不制管固無由以審音，不審音亦無由以定律)—a passage I briefly touched on in the previous chapter. The dialectical and reciprocal logic of this statement is hard to miss.[33] Without pipes being made through properly determined numbers and units of measurement, there is no way to examine any tones of theirs. By virtue of the single adverb *gu* 固 "admittedly," however, the second part of the phrase takes is

32 See Zhu Zaiyu, *A New Theory of the Study of Pitch Pipes*, vol. 1, ff. 5r-7r.
33 National Library of China, *Putong Guji* No. 15251, f. 59r.

emphasized: without examining their tones, there is no way to *dinglü* 定律 "determine the pitch pipes." This latter formulation can be rather ambiguous, yet the strong concessive character of the phrase suggests that the empirical process of *shenyin* "examining the tones" should be able to exert the same *sine qua non* influence on the historicist and epistemically silent process of *zhiguan* 制管 "making the pipes."

By the time the finished lab notes had more or less formulated the entire fourteen-tone temperament as first and foremost a reform to the system of pitch organization based on a fourteen-tone octave heard among pipes of the same diameter, this reciprocal feedback loop was broken up into a chain. As indicated by the title of Volumes 1 and 2 of *Orthodox*, *zhenglü shenyin* 正律審音 "Rectifying the Pitch Pipes in order to Examine Their Tones," once the absolute and relative sizes of the twelve tuning pitch pipes had been determined in Chapters 1-8 of Volume 1, including the fact that they will have the same diameter, no changes whatsoever can be made to alter them. No matter what egregious things one might hear in the experiments described in Chapter 9 with those pipes fashioned in Chapters 1-8, one must not dare to change a thing about them. Instead, one must take whatever pattern of octave division or pitch organization these pipes produce as givens. A steadfast fundamentalist conservatism with textual sources of *shu* "numbers" is thus conjoined with an equally radical iconoclasm with the organization of *sheng* "sounds" or "musical notes."

So, to what extent was the fourteen-tone temperament a historicist exercise, and to what extent was it an empiricist one?

Chapter Six

Shenyin 審音 "Examine the Tones": A Phonological Revolution?

In 1758, Paris found itself in the middle of two literary quarrels. One, sparked by a staging of Giovanni Battista Pergolesi's operatic intermezzo *La serva padrona* at the Académie royale de Musique at Palais-Royal on August 1, 1752, fought over the merits of Italian *opera buffa* vs. the prized operatic tradition of France, *tragédie en musique*. Though this "Querelle des Bouffons" had reached its peak in 1754, it reverberated throughout the decade thanks particularly to a fierce partisan for the Italians, Jean-Jacques Rousseau (1712-1778). In the aftermath of his public crossfires against Jean-Philippe Rameau (1683-1764), the chief apologist and himself a prolific composer for French opera, Rousseau penned his *Essay on the Origin of Languages*,[1] which, especially thanks to Claude Lévi-Strauss and Jacques Derrida, would become one of the most widely interpreted texts in Western philosophy.[2] Around the time Rousseau was writing the *Essay*, another quarrel broke out at the Académie des Inscriptions et Belles-Lettres at the Louvre on November 14, 1758, when curator of Eastern manuscripts of the Bibliothèque du Roi, Joseph de Guignes (1721-1800), delivered a lecture arguing that China originated as a colony of the ancient Egyptians.[3] De Guignes' preposterous hypothesis was as popular as it was controversial,

1 The translation I am using is Jean-Jacques Rousseau, *Essay on the Origin of Languages and Writings Related to Music*, translated by John T. Scott (Hanover, NH: University Press of New England, 1998), 289-332.

2 Derrida openly accords to Rousseau a "privileged place … in the history of logocentrism" and dedicates the entire Part II of his *Of Grammatology* to interpreting Rousseau's *Essay*. See Jacques Derrida, *Of Grammatology*, translated by Gayatri Chakravorty Spivak (Baltimore and London: The John Hopkins University Press, 1976), 97-100.

3 The paper was later published in several editions. See Joseph de Guignes, *Mémoire dans lequel on prouve que les chinois sont une colonie Egyptienne* (Paris: Desaint & Saillant, 1759).

and his lecture would eventually go through several editions at the press. In fact, the young Orientalist only amplified a century-long debate over the relationship between what Enlightenment Europe then considered two of the world's oldest civilizations. Largely instigated by the Jesuit polymath Athanasius Kircher (1602-1680) in his *China Illustrata* (1667), which followed his monumental Egyptological treatises, the debate never subsided among later Enlightenment *philosophes*, colonial officers, and Jesuits in the Qing Empire (1636-1911), until the discovery of the Rosetta Stone in 1799 and its subsequent decipherment altered the paradigms of Egyptology.[4]

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For a dissertation dedicated to a reform to Chinese musical tuning under the late 17th and early 18th-century Qing Empire, these two literary quarrels of 1750s Paris might seem irrelevant. Even though the proposed connections between Ancient Egypt and Ancient China inspired European *philosophes* to look towards the other end of the Eurasian continent, nowhere did the fourteen-tone temperament or even the twelve *lülü* "tuning pitch pipes" in general play any role in these two quarrels. To recall from the introduction to this dissertation, not even Amiot himself understood that the sixteen-pipe pan-flute he copied from Volume 3 of *Orthodox Meaning of Pitch Pipes* into his own *De la musique des chinois* (1754) actually embodies a fourteen-fold division of the octave into two complementary diatonic scales. For what is worth, because of the sheer size and diversity of the 18th-century Parisian intelligentsia and the network of scholars and

4 For the "pre-Champollion" Egyptology and its interweaving with contemporary Sinology, see Don Cameron Allen, "The Predecessor of Champollion," *Proceedings of the American Philosophical Society* 104 (1960): 527-547. Alexander Rehding explores the musical and particularly music-theoretical resonances of early modern European craze for things Chinese and Egyptians in "Music-Historical Egyptomania, 1650-1950," *Journal of the History of Ideas* 75 (2014): 545-580, particularly 550-566.

texts it connected beyond the city, the two quarrels that broke out at two *académies royales* just a street across from each other nonetheless never interacted with each other in any substantial way.

However, the lack of explicit or apparent connections between the Kangxi Emperor's fourteen-tone temperament, the War of the Buffoons, and the Egypt-China debate does not preclude their coeval embedment into certain structural transformations that encompassed each of their seemingly well-contained spheres of discourse. And as this dissertation is drawing to its close, it is precisely a structural transformation that I hope to shed some light on, —not necessarily to provide any complete picture but to point out a direction to look further and deeper into. The transformation concerns that keyword I have been examining ever since the second chapter of this dissertation: *shenyin* 審音 "examining the tones." In Chapters Two, Three, and Four above, I have shown this term to be a critical methodological rubric in the various stages of the fourteen-tone tuning reform, from the reference book that In-c'i had his staff compile (ff. 52-83 of *Putong Guji No. 15251*) to the final treatise of *Orthodox*. Inspired by the Kangxi Emperor's misunderstanding of *geba xiangsheng* 隔八相生 "mutual generation at every eighth step" as octave equivalence, which the Emperor himself picked up from his musical lessons with the Jesuits, the tenet of "examining the tones" in In-c'i's musical research posits that sounding and listening are the only appropriate method of knowledge-production in organizing the pitches of the twelve tuning pitch pipes into a system of scales and modes. As a result, "examining the tones" becomes a linchpin connecting the tuning pitch pipes as silent objects of historicist reconstructions to the embodied activities of music-making through singing or instruments.

The chapter preceding this one further pins down the nuanced significance of *shenyin* "examining the tones." For one, the empirical underpinnings of listening to the sounds of pitch pipes do not at all replace or invalidate the historicist and textual studies that constitute *zhenglü*

正律 “rectifying the pitch pipes.” In his *Explication* commenting on a report of the Kangxi Emperor’s demonstration of “mutual generation at every eighth step,” Mao Qiling pushes for a reinvention of musical scholarship that discards the *lü* “pitch pipes” and their epistemically silent *shu* “numbers” in the form of computations and historically transmitted data, and instead places *sheng* “sounds” and the empirical experiences of listening to and producing *sheng* “musical notes” at the center of producing musical knowledge. Similarly, Chapter 9, Volume 1 of *Orthodox Meaning of Pitch Pipes* uses the Kangxi Emperor’s definition of the phrase to emphasize the importance of *shenyin* “examining the tones” specifically as listening for recurrent patterns of notes such as cycles of octaves and the seven-note diatonic scale. Yet clearly the Qing’s official treatise on musical tuning did not at all forsake the textual records through which it reconstructs the exact absolute and relative sizes of the twelve tuning pitch pipes, or at least so it claims. From the preparatory reference book (ff. 52v-83) and finished lab notes (ff. 84-119) in *Putong Guji No. 15251* to the final *Orthodox* treatise, the juxtaposition of *zhenglü* “rectifying the pitch pipes” or *zhilü* “making pitch pipes” with *shenyin* “examining the tones” conveys a consistent attempt on the part of the Qing researchers to balance the two knowledge-producing processes, one historicist and silent, the other empirical and sonorous.

Yet the balancing act between the historicism of textual studies and the empiricism of sounding and listening was simultaneously a segregation. As I have shown in Chapter Five, even when provided with the option of using pipes of the same shape in lieu of pipes of the same diameter and thus annulling the need to revise the conventional system of pitch organization, the researchers for the Qing’s tuning reform refused to change a thing about the tuning pitch pipes once their physical parameters had been established through the historicist process of *zhenglü* “rectifying the pitch pipes.” This religious adherence to the exact teachings of those oldest extant

and best-respected sources on the length and diameter of the ancients' twelve tuning pitch pipes was met with an equally fanatical determination to ignore those very same sources with respect to the organization of the *sheng* "musical notes" as well as "sounds" of the twelve tuning pitch pipes into systems of *yun*'s, scales, and modes. To be sure, always avid readers of historical texts, the researchers found convenient ways to dismiss the fact that sources upholding the conventional pitch organization system were implicitly speaking only about strings yet not pipes. As I have shown in Chapter 3, they also exaggerated the literalness of the rhetorical notion of *liulü* 六律 "the six odd-numbered pipes" and invented a new interpretation of *xuangong* 旋宮 "rotating the *gong* note" to justify the peculiar system of pitch organization they devised in adherence to the fourteen-tone octave they heard through experiments. Still, a rather clear epistemic line was drawn between making the pitch pipes and listening to them. Making pitch pipes can only draw upon textual studies, whereas organizing the sounds of pitch pipes into patterns of octaves and scales can only draw upon empirical experiments, and no crossover shall ever be allowed. Trying to solve the octave equivalence and pitch organization problems by using pipes of the same shape violates this boundary in both directions. Forcing the empirically observed fourteen-tone octave to conform to written records on the conventional system of pitch organization based on the twelve-tone octave violates the epistemic sanctity of sounding and listening in the realm of pitch organization with historicist textual studies. Conversely, modifying the twelve tuning pitch pipes from pipes of the same diameter to pipes of the same shape so as to maintain the conventional system of pitch organization also subjugates the historicist studies on the sizes of the tuning pitch pipes to the empirical observations of sounds, *yun*'s, diatonic scales, and octave equivalences.

Contradictorily, an absolute segregation between the text-based historicist studies on reconstructing the exact sizes of the tuning pitch pipes and the sound-based empiricist experiments on patterning the pitches of said pitch pipes simultaneously effects a radically transparent interface between the two segregated parts. The research program stipulated in the title of Volumes 1 and 2 of *Orthodox*, *zhenglü shenyin* 正律審音 "Rectifying the Pitch Pipes in order to Examine Their Tones" is only possible if there is a pathway whereby the first part of the program can turn into the second part. Because the two parts of the research program are each shoehorned into a distinct and mutually exclusive knowledge-producing process—textual studies of numbers and metrology vs. empirical observations of sounds— it follows that the epistemic segregation between these two parts must presuppose a fundamental self-evidence of knowing on the threshold between the segregated epistemes. Put differently, there is a profound optimism at play, which presumes that textual studies would by themselves reveal exactly how long and how wide in diameter the twelve tuning pitch pipes ought to be without help from any other source of knowledge, and that empirical observations of sounds produced by those pipes would by themselves reveal exactly how their pitches ought to be organized into cycles of octaves and notes without any external reference. Further, in order that a transference of knowledge can take place from the former's historicism and the latter's empiricism, in order that the epistemically silent texts can lead to a resonant experience of sounds, it must be that texts and sounds, despite their diametric opposition, can immediately translate into one another. As a result, *zhenglü* "rectifying pitch pipes" and *shenyin* "examining the tones" of said pitch pipes are simultaneously two segregated processes of knowledge production and one single chain of self-revelation: once the texts are properly interpreted, there shall be properly patterned sounds.

Triangulation

To shed light on this critical episteme of *shenyin* "examining the tones" in the Kangxi Emperor's fourteen-tone temperament, I will attempt in this concluding chapter to situate it in a much broader transregional context. The analyses here are as audacious as they are sketchy. The goal, however, is to allow my previous examination of the Qing's tuning reform to shed light on what were potentially some global epistemological transformations in the early modern era. To do so, this chapter does not actually situate "examining the tones" in any transregional or global context *per se* but does the exact reverse: it uses the episteme of "examining the tones"—in its various manifestations, as I will later show—to contextualize and in fact triangulate some other events that took place more or less contemporaneously on the other end of Eurasia: namely the two Parisian quarrels I mentioned at the start of this chapter. I argue that both quarrels ultimately sought to rewrite the genealogy of the world's peoples and to recuperate lost knowledge of and/or about the ancestors of all humans against what they perceived as a *longue durée* degeneration. Specifically, both sought to achieve this restorationist agenda through a praxis of linguistics whereby the two quarrels actually embodied an increasingly powerful dichotomy in the conceptualization of language in the early modern era. In time, this dichotomy would serve as a pillar for how "modernity" is being understood in the humanities in the 20th and 21st centuries, particularly after the advent of the (perhaps by now clichéd?) post-structuralist linguistic turn: the dichotomy between speech and writing.

Let me unpack this latter point before moving on to my objectives with respect thereto. Rising out of his musical polemics during the "Querelles des Bouffons" and his *Discourse on the Origin and Basis of Inequality among Men* (a.k.a. *Second Discourse*, 1755), Rousseau's *Essay* constructs a universal history of humankind by narrating a degeneration of speech. At first, the

mild and carefree clime of the south produced a melodious and passionate speech-song in the tongues of the primitive savages.[5] As humans moved to the harsh and depriving climes of the north that demanded precise communications of needs and laws, however, the primitive speech-song deteriorated into the drawn-out and monotonous languages of the moderns. For Rousseau, salvaging residues of the primitive speech-song in certain styles of modern music, specifically of Italian opera, does more than simply allow for better expressions of human emotions compared to the *ramiste* kind of French music.[6] Rather, restoring a prosodic and harmonious speech conducive to the immediate conveyance of passions in civil society also provides the key to returning humankind to their freedom in nature, away from the shackles of arbitrary laws and institutions. For Rousseau, these latter formulations of human societies are akin to the arbitrary conventions of writing, that is, of signs, spelling, and grammar: the latter serve to supplement the lack of inflections and passions in modern speech but, due to their very supplementarity, fall short of fully re-presenting speech and continue to inflict that very lack upon speech.[7]

Though sharing neither the revolutionary aspirations nor the universal scope of Rousseau's *Essay*, the Egypt-China quarrel and the field of Egyptology at large in early modern Europe rested on a comparable restorationist premise and promise: namely, to uncover the secret

5 Rousseau, *Essay on the Origin of Languages and Writings Related to Music*, "Chapter IX: Formation of the Southern Languages," 305-315.

6 Ibid, "Chapter XIV: On Harmony," 321-323.

7 My admittedly brisk summary of Rousseau's *Essay* owes much to a succinct yet thorough unpacking of its key ideas, particularly the "supplement," in Gary Tomlinson, *The Singing of the New World: Indigenous Voice in the Era of European Contact* (Cambridge, UK: Cambridge University Press, 2007), 11-18.

wisdom of the ancient Egyptians.[8] Besides the rising circulations of ancient artefacts and woodcut printings of hieroglyphs, ancient Egypt captured the European imagination because of the position it occupies on the broadly recognized family tree of human knowledge. Falling between Noah as the (re)progenitor of all humankind after the Deluge and the ancient Greeks as progenitors of Western civilization and their disciples, the ancient Egyptians were widely held in 17th- and early 18th- century Europe for divine revelations even closer to the original Adamic religion than to the Israeli one. In contrast to Rousseau's quest for the primitive speech-song, however, the Egyptological quest for this lost ancient enlightenment took the path not of speech but of writing. Hardly interested in the spoken languages of ancient Egyptians, scholars like Kircher and de Guignes devoted themselves fully to deciphering the hieroglyphs on obelisks and temple façades. And hence the traction of the hypothesis that the Chinese were actually descendants, at least civilizationally, of the ancient Egyptians. Indeed, Chinese writing not only struck many Europeans as pictographic and thus akin to the Egyptian hieroglyphs, but also possible to learn. Themselves competent in the language and mingling daily with Chinese scholar-officials, Catholic missionaries in the Qing Empire regularly shipped cultural and

8 These "secrets" could either be esoteric or antiquarian. That is, they were pursued both by those who believed that the Egyptians possessed the otherwise lost knowledge of the perennial religion the Abrahamic God bestowed onto Adam, and by those who simply wanted to know more about the ancient Egyptians as the earliest human civilization. In particular, many scholars of ancient Egyptian used Hermetic and other esoteric writings *not* as religious but as historical sources. For this subtle distinction, see Daniel Stolzenberg, *Egyptian Oedipus: Athanasius Kircher and the Secrets of Antiquity* (Chicago and London: The University of Chicago Press, 2013), Chapter One "Esoteric Antiquarianism," 36-70.

pedagogical materials to Europe and provided expert opinions on academic debates.[9] Were Chinese writing really to turn out to be a degenerate form of Egyptian hieroglyphs, it would mean that one could appropriate the still legible Chinese writing along with the enormous documentation of its evolutions through at least two millennia. And thus, the century-long dream of deciphering the no longer legible Egyptian hieroglyphs and in turn uncovering the secret teachings they had been transmitting from the pre-Pythagorean high priests might finally come true.

In this chapter, then, I seek to contextualize globally this dichotomy between speech and writing as embodied in the two 18th-century Parisian quarrels. To do so, I study the two quarrels as restorationist praxes of writing universal history through linguistics and unpack the epistemic functions of speech, sounds, writing, and traces in their respective paradigms. I then compare them to a series of loosely defined phenomena that took place some thousands of miles away across the Eurasian continent during what I call a "Phonological Revolution" that unfolded in the 16th to 18th centuries, between the dawn of the Qing conquest of China and the height of its empire-building across eastern and inner Eurasia. Specifically, I examine two well-studied cultural events: the "folksong mania" in 16th- and early 17th-century China, and the rise of historical philology under the 17th- and 18th-century Qing Empire. Focusing on the oft-overlooked interstices between the singing-and-speaking voice and its textual traces in each of

9 On the deepening integrations between Qing and Western European scholarly communities in the early modern era, see Nicolas Standaert, *The Intercultural Weaving of Historical Texts: Chinese and European Stories about Emperor Ku and His Concubines* (Leiden and Boston: Brill, 2016), particularly Chapter 2 "Jesuit Accounts of Chinese History and Chronology and Their Chinese Sources," 94-163. For an example of Jesuits in China participating in European Egyptological and Sinology debates, see Jean-Jacques d'Ortous de Mairan, *Lettres de. M. de Mairan au R. P. Parrenin, Missionaire de la Compagnie de Jesus, à Pékin* (Paris: Desaint & Saillant, 1759), based on de Mairan's letters with the Jesuit Dominique Parrenin during the 1730s on questions about the Egypt-China hypothesis.

these two hitherto discrete events in Chinese intellectual history, I explore how the two events reflected a broader shift in the roles of sounds in the nexus of historical imaginations and imperial expansions. And it is in this Phonological Revolution that our old friend *shenyin* "examining the tones" appears at yet another critical juncture of paradigm shift.

As a final note before I dive into the two Parisian quarrels, my choice for triangulating the two Parisian quarrels with the rather *longue durée* development of Chinese scholarship on songs and historical linguistics serves two purposes. One, inasmuch as the two quarrels directly or indirectly implicated the "Tartar-Manchu Empire" as it was referred to in 18th-century France,[10] such a triangulation allows me to put the evolving discourses of speech, writing, song, voice, and sound on both ends of Eurasia into some kind of transregional perspective. This perspective can be as simple as the recognition of their coevalness, a coincidence that may raise the possibility of making further comparisons in the future. It can also be concrete mappings of their co-constitutive integrations into the material and conceptual ecologies of the early modern world, of which I will make some initial attempts. Two, the Qing side of the triangulation allows me to explore a nexus of *logos* (meaning, presence), *gramma* (shape, writing), and *phonē* (voice, speech) that can be productively compared and contrasted with the purported logocentrism or phonocentrism in Western metaphysics that, at least according to Derrida, reached a critical juncture through one of the two Parisian quarrels.[11] In so doing, I hope to raise a broader question the answers to which will go far beyond the frame of this dissertation: how can understanding the varying relations between speech and writing on a global scale—particularly

10 This is evident in the title of the most influential treatise on China in 18th-century Europe, Jean-Baptiste Du Halde, *Description geographique, historique, chronologique, politique, et physique de l'Empire de la Chine et de la Tartarie Chinoise*, in four volumes (Paris: P. G. Le Mercier, 1735), which highlights "Tartarie" as part of the regime that ruled China.

11 Derrida, *Of Grammatology*, 97.

in the context of early modern imperial expansions and colonialisms—deepen our conceptualizations of modernity or early modernity, insofar as we continue to deem such concepts critically valuable, beyond being mere markers of periodization?

Grammatological vs. Phonological Restorationism

Let me begin, then, with the two Parisian quarrels, through which I will tease out what I respectively call grammatological and phonological modes of restorationism. As I have argued, the traction of Egypt-China debates in 17th- and 18th-century Europe rested upon the possibility of unraveling the secrets transmitted in the no longer legible Egyptian hieroglyphs. Yet the hope that Chinese writing could in any way inform the decipherment of the hieroglyphs also relies on a specific gramma-centric (or writing-centric) mapping of the linguistic relationship between speech, writing, and meaning for the two writing systems. Indeed, European scholars singled out Chinese writing for embedding Egyptian influence and Egyptological potentials because they considered Chinese and Egyptian writing systems to be exceptionally logographic, where written signs directly represent meanings [*logos*]. In contrast, they considered most of the world's writing systems to be phonographic, where written signs represent sounds [*phonē*], which in turn represent meanings. Now, of course, we know that neither Chinese nor Egyptian writing is completely logographic, as each employs an enormous number of phonographs as well as a much more limited number of logographs (more on this later).[12] What mattered, however, is that this perceived logographic similarity became both the premise and the promise of the Egypt-

12 The myth of a purely ideographic or logographic script completely bypassing sound, spoken words, speech, or phonography is a very persistent one, particularly when it comes to Chinese writing. See John DeFrancis, *Visible Speech: The Diverse Oneness of Writing Systems* (Honolulu: The University of Hawaii Press, 1989), in particular "Writing as Visible Speech," 248-252. Put simply, there is no writing without phonography.

China hypothesis. It was the premise inasmuch as this similarity in writing was often cited as the most convincing evidence that the Chinese have must have derived their core institutions and mores, including such exceptional logographic writing, from the ancient Egyptians; in other words, the study of writing or grammatology became a concrete way to organize the world's peoples and civilizations into a universal genealogy, an endeavor that engrossed a variety of 17th- and 18th-century European scholars from geologists to biblical scholars. It was the promise inasmuch as the hypothesis of China being a descendent of ancient Egypt provided a concrete research paradigm for 17th- and 18th-century Egyptology. By tracing back specific Chinese characters to specific Egyptian hieroglyphs, the former's individual pairings of *gramma* with *logos* could be used to decipher Egyptian texts and inscriptions as a string of such logograms, thus uncovering the secret teachings that were themselves recognized as a mediating stage between the biblical revelations to Noah and the earliest Greek philosophers, particularly Pythagoras.

That both this premise of producing a universal genealogy of human civilizations and promise of uncovering lost ancient Egyptian wisdoms rested specifically upon the study of writing or grammatology was most telling from the works of Athanasius Kircher. Curiously, Kircher originally proposed two methods of reconstructing the meanings of the Egyptian hieroglyphs. First of all, being one of the first Europeans outside the Ottoman Empire to know Coptic, Kircher argued in one of his earliest treatises on Egypt, *Prodromus coptus sive Aegiptiacus* (1636), that one could reconstruct spoken ancient Egyptian based on this modern descendent thereof and in turn reanimate the written hieroglyphs through this reconstructed

spoken language.[13] Later, however, after his *magnum opus* on ancient Egypt, *Oedipus Aegiptiacus* (1652), Kircher planted the seed for the second method in *China Illustrata* (1667), raising the prospect of using modern Chinese writing as a degenerate residue of ancient Egyptian writing to reconstruct the latter.[14] Of the two methods Kircher proposed, only the second one based on grammatology captured the scholarly imagination. In 1761, for example, J. Tuberville Needham (1713-1781) claimed to have found inscriptions on a bust at the ducal museum at Turin that, he argued, were the direct ancestors of both Egyptian hieroglyphs and Chinese characters.[15] The supposed discovery renewed the international debate over similarities and connections between the two writing systems; see Illustration 6-1, for an example of a Jesuit contribution to this debate.[16] In fact, not even the detractors of the Egypt-China hypothesis in the 17th and 18th centuries immediately recognized the spoken language of Coptic as a potential alternative thread—even though Coptic would end up being part of the toolkit that allowed Champollion to recognize the systematic phonographic use of hieroglyphs and to decipher the Rosetta Stone in

13 See Stolzenberg, *Egyptian Oedipus*, Chapter 2 "How to Get Ahead in the Republic of Letters," 71-103, and Alastair Hamilton, *The Copts and the West, 1439-1822: The European Discovery of the Egyptian Church* (Oxford: Oxford University Press, 2006), Chapter 12 "Athanasius Kircher and His Shadow," 195-228. Indeed, compared to this interpretation of Egyptian hieroglyphs as "nondiscursive" and non-phonographic signs that directly refer to things in nature themselves, Kircher's work on the Coptic language has enjoyed much more favorable approval among current scholars.

14 See Athanasius Kircher, *China monumentis qua sacris qua profanis nec non variis naturae & artis spectaculis aliarumque rerum memorabilium Illustrata* (Amsterdam: Joannes Jansson, 1667), Part VI "De Sinensium Literatura," 225-237. Kircher himself never used Chinese characters to decipher Egyptian hieroglyphs, believing there was no need after he had already deciphered the latter through his Egyptological *magnum opus* in the 1650s. Yet the comparisons and connections he drew between the two writing systems inspired 18th-century scholars, especially since most were not at all convinced by Kircher's interpretations of hieroglyphs.

15 Allen, "The Predecessors of Champollion," 540-542.

16 See Anonymous [Pierre-Martial Cibot], *Lettre de Pekin sur le génie de la langue chinoise et de la nature de leur écriture symbolique comparée avec celle des anciens égyptiens, en réponse à celle de la Société Royale des Sciences de Londres, sur le même sujet* (Paris: J. L. de Boubers, 1773).

Illustration 6-1 Bibliothèque-l'Institut de France, MS 1524, f. 11[17]

17 The illustration is taken from the manuscript Cibot sent to Paris from China, which features the various diagrams comparing Egyptian hieroglyphs and Chinese characters, now in the Bibliothèque de l'Institut de France as MS 1284 B. From the left, the first and fourth columns feature "modern" Chinese characters; the second and the fifth "ancient" Chinese characters; and the third and sixth Egyptian hieroglyphs. The logic of visual juxtaposition implying a hypothetical common origin leaps from the page here.

the 1810s and 1820s. In other words, while both Chinese writing and the spoken Coptic language were recognized as descendants of the nexus of meaning, sound, and shape in the ancient Egyptian language, Chinese writing clearly enjoyed more traction than the Egyptian spoken language in the research paradigm of 17th- and 18th-century Egyptology.

Moving to Rousseau's *Essay on the Origin of Languages*, Chapter Five "On Writing," we see that it not only encapsulates neatly his linguistic history of humankind but also directly counters what I have just summarized about the grammatological mode of restorationism in 17th- and 18th-century European Egyptology. The chapter opens with a narrative of the degeneration of speech:

> "[…T]he more voices become monotone, the more consonants multiply, and that as accents are eliminated and quantities are equalized, they are replaced by grammatical combinations and new articulations […]. In proportion as needs increase, as affairs become entangled, as enlightenment extends, language changes character; it becomes more precise and less passionate; it substitutes ideas for feelings, it no longer speaks to the heart but to reason. As a result, […] language becomes more exact and clearer, but more drawn out, more muted, and colder."[18]

Following this narrative of the gradual degeneration of speech from the primitive humans, however, is a narrative of the gradual perfection of writing.[19] It begins with the logographic writing of the Egyptians (and the indigenous Central Americans):

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18 Rousseau, 296.

19 Among the many other sources that could have informed Rousseau on this narrative (for example, Giambattista Vico's 1725 treatise on historiography and human knowledge, *Scienza nuova*), I highlight William Warburton's *Divine Legation of Moses*. Not only does Warburton map out and identical progression of writing from pictograms to hieroglyphs to characters and finally to alphabets, but, like Rousseau, he also juxtaposes this history of writing to the history of language, emphasizing the coeval and mutually transformative evolution of writing and speech (though his emphasis is not on the voice but on figures of speech and rhetoric). He even concurs with Rousseau in implying that "letter" is a form of "art" imposing itself on spoken sounds. See Warburton, *Divine Legation of Moses Demonstrated* (1738), 10th ed. (London: Thomas Tegg, 1846), Vol II, 201-218.

> […] The cruder the writing, the more ancient the language is. […] The first manner of writing is not to depict sounds but the objects themselves, whether directly as the Mexicans did, or by allegorical figures as the Egyptians did of old. This state corresponds to passionate language, and already presupposes some degree of society and some needs to which the passions have given rise.
>
> The second manner is to represent words and propositions by conventional characters, which can be done only when the language is completely formed and when an entire people is united by common Laws; for there is already here a double convention. Such is the writing of the Chinese: this is truly to depict sounds and to speak to the eyes.
>
> The third is to break down the speaking voice into a certain number of elementary parts, whether vowels or articulations, with which one could form all imaginable words and syllables. This manner of writing, which is our own, must have been devised by commercial peoples who, traveling in several countries and having to speak several languages, were forced to invent characters that could be common to all of them. This is not precisely to depict speech, it is to analyze it.[20]

Notably, as did most Egyptologists at the time, Rousseau considered both Egyptian writing logographic, or representing meaning without mediation of sound. What set Rousseau apart, however, was the relationship between this narrative of the history of writing vis-à-vis that of speech. Whereas the Egypt-China hypothesis posited a genealogy of writing (from Egyptian hieroglyphs to Chinese characters) separate from the genealogy of speech (from ancient spoken Egyptian to modern Coptic), here, Rousseau defines the development of writing in relation to speech: "the cruder the writing, the more ancient the language is." Purely logographic writing, Egyptian, is deemed crude by Rousseau precisely because it does not represent the sound of speech but only "the objects themselves," whether "directly" or "allegorically." Chinese, about which Rousseau concurs with Egyptologists in considering it the next stage of grammatological development after Egyptian hieroglyphs, occupies such a middling position precisely because it

20 Rousseau, 297.

has begun to represent the sound of speech via a "double convention": although the writing is by no means alphabetical, there is nonetheless a conventional connection between each character and its agree-upon sound parallel to the conventional connection between each character and its agree-upon meaning. And when writing achieves perfection with alphabetical writing of the (European) moderns, it is precisely because the devices of letters and syllables have begun to "analyze" the sound and only the sound of speech into increasingly precise components, so that writing has become completely phonographic, completely occupied with sounds.

For Rousseau, this gradual incorporation of the visual writing into the aural and sonic speech spells an ultimate degeneration: the gradual perfection of writing was for him the deepening of its representational violence upon speech, and one can perhaps interpret the "breaking down" of the speaking voice and the "analyzing" (literally "cutting loose") of speech by writing as such a form of violence.[21] As the perfect primitive speech-song lost its power of immediately conveying the passions, writing rose as a "supplement" that attempts to recuperate the loss passionate effects of the primitive speech. Such an attempt can only be futile, however, because the logic of supplementarity and representation central to Rousseau's understanding of writing automatically voids any possibility of the immediate communication of passions once afforded by perfect speech. Indeed, writing for Rousseau does not simply mean visual representation. Instead, the rise of writing as a way to visually represent sounds was linked directly to the rise of "common Laws," so that for Rousseau, the conventions of writing (spelling, articulation, etc.) whereby visual marks could begin to arbitrarily break down speech into sonic components and represent them are akin to social, political, and legal conventions that

21 Tomlinson, *The Singing of the New World*, 17-18.

increasingly mediated and rationalized human interactions at the expense of their immediate passionate communications once afforded by the primitive speech.

It is worth remembering that Rousseau's musings on the origin of languages, though not published until posthumously in 1781, arose specifically from his public polemics against Rameau in the 1750s. And indeed, in the later chapters of his *Essay*, Rousseau turns his critique of alphabetical writing to attack the *ramiste* kind of harmony precisely as such a form of writing. Just as alphabetic writing subjects speech to its muted conventions of orthography based on but a paltry number of letters,

> by […] shackling the melody, [harmony] deprives it of energy and expression, […] eliminates passionate accent in order to substitute the harmonic interval for it, […] effaces and destroys multitudes of sounds and intervals that do not enter into its system; in a word, it separates song from speech so much that these two languages combat one another, contradict one another, deprive each other of every characteristic of truth and cannot be united in a pathetic subject without being absurd.[22]

Whereas the primitive speech-song could immediately convey human passions through "multitudes of sounds and intervals," a clear reference to microtonal variations afforded by monophony, the artificial laws or "system" of harmony deprives melody of such "passionate accent." In other words, harmony drains out melody in the same way that alphabetic writing emaciates speech, so that the two conventions join forces in corrupting and dividing up the primitive speech-song. And while Rousseau does not use any music-theoretical terms, there is little doubt that the "system" of harmony here refers to the theory of "triple progression" of Rameau, whom he does mention by name. In contrast to Rousseau's penchant for the natural beauty of melody, Rameau dedicated his lifetime to formulating a natural law of harmony. The linchpin for Rameau was the *corps sonore*, a vibrating string that produces a series of harmonic

22 Rousseau, 321-322.

overtones above its fundamental pitch.[23] Observing that the two lowest overtones sound an octave and a perfect-fifth-plus-an-octave above the fundamental pitch and that their vibrating frequencies bear a 2:1 and 3:1 proportion to that of the string, Rameau sanctifies the duple and triple ratios as the basis of harmony. In particular, by repeating the 3:1 proportion several times, Rameau shows how the resultant chains of perfect fifths—that is, …-F-C-G-D-A-E-B…—can in varied ways generate the Greek tetrachords, Chinese pentatonic scales, European diatonic scales, the major and minor triads, and even the dominant-seventh chord. Thus, for Rameau, these *progressions triples* "triple progressions" became a universal foundation of music theory. Besides using them to justify the use of harmony as simply the foregrounding of those naturally-embedded overtones, Rameau argues that "triple progressions" provide the key to uniting all the musical systems of the world and thereby tracing them back to their common origin. Thus, though they disagreed whether Egypt or China was the closest to this origin, both Rameau and Pierre-Joseph Roussier, a major follower of Rameau's ideas and author of *Mémoire sur la musique des anciens* (1770), used "triple progressions" to sketch a universal genealogy of music. They hoped that such a genealogy could compel modern musicians to return their art to its original perfection at the time of Adam and Noah.[24]

In a clear rejection of Rameau's ambition of restoring ancient music through his "triple progressions" theory of harmony, Rousseau argues in his *Essay* that neither the Ancient Greeks nor the "American savages" had anything to do with Rameau's modern conventional harmony in their music. Here, Rousseau is referring not only to the pervasive monophony in ancient Greek music as recorded in Classical sources or in Native American songs as told in colonial

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23 See Thomas Christensen, "Eighteenth-Century Science and the 'Corps Sonore': The Scientific Background to Rameau's 'Principle of Harmony," *Journal of Music Theory* 31 (No. 1): 23-50; 23 and 41-42.

24 See Rehding, "Music-Historical Egyptomania, 1650-1950," 563-566.

travelogues, but also to their use of "inflections"—again meaning microtonal variations—"which we call false because they do not enter into our system and because we cannot *notate* them [emphasis mine]."[25] By reducing harmony to *notation* and thus to a form of writing, Rousseau compares the study of harmony as a form of grammatology: an ultimately futile attempt to recuperate the lost primitive speech-song of early humans, Rousseau argues, precisely because of the representational violence writing/harmony continues to inflict on speech/melody by the logic of their supplementarity. What Rousseau did share with Rameau and his partisans, however, was the restorationist ambition. Because Rousseau consistently relates the rules of harmony and writing as laws, conventions, and even "shackles," his linguistic-musical restorationism also bore a clear revolutionary undertone, culminating in the last chapter of his *Essay*, "Relationship of Languages to Governments."[26] Here, Rousseau argues that in the process of the degeneration of speech-song and its usurpation by writing and harmony, public oration as well as democratic organizations of society based on immediate communication of passions between humans also degenerated into tyranny, which, like the drawn-out and muted modern speech, is governed by arbitrary conventions. "There are languages favorable to liberty," Rousseau writes, "and these are sonorous, prosodic, harmonious languages, in which discourse can be made out from a distance. Ours [modern speeches] are made for the murmuring in sultans' Council-chambers."[27] Thus, unlike the grammatological restorationism sketched by the 18th-century Egyptologists and by Rameau, Rousseau's politicization of the degeneration of speech implies a phonological mode of restorationist praxis, one that does not try to restore a system of writing but to restore the original unified state of speech and song. Whereas the Egypt-China hypothesis posits that the

25 Rousseau, 321-322.
26 Rousseau, 328.
27 Ibid., 332.

genealogy of writing-at-large—from Egyptian hieroglyphs to Chinese characters, from Greek tetrachords to Chinese pentatonic scales—offers a direct thread for tracing back to the lost knowledge of/about the ancient Egyptians or ancient humans in general, by making writing a supplement to speech, Rousseau locates the key to recapturing the liberal organization of human society by resurrecting a passionate and immediate form of speech that is precisely free from the conventions and representational violence of writing. It is phonological, because the hopes of ever returning to a primitive state or reacquiring some lost knowledge rests not with writing or its trace [*gramma*], but with speech and the speaking voice [*phonē*].

The *Jouissance* of Chinese Scripts?

In sum, the two Parisian quarrels embodied a speech-writing dichotomy in their shared restorationist ambition after a perceived linguistic degeneration and loss. The Egypt-China hypothesis strove to recuperate the hieroglyphic writing of ancient Egyptians so as to regain the latter's secret knowledge, whereas Rousseau's *Essay* aspired to recuperate the primitive speech-song so as to reestablish a state of natural freedom. As I hope to have illuminated through the above sketches, however, both the grammatological restorationism in the Egypt-China hypothesis and the phonological restorationism in Rousseau's *Essay* derived from a linear narrative of the history of writing. It was only by recognizing Chinese characters as a descendent of Egyptian hieroglyphs that scholars like Kircher and de Guignes could raise the prospect of deciphering the latter through the former. Similarly, it was only in opposition to the gradual perfection of writing whose signs increasingly encroach on speech that Rousseau could posit the degeneration of the passionate speech-song of the primitive savages by means of the supplementarity of written signs.

Still, what was it that convinced 17th- and 18th-century European scholars of this linear grammatological historiography? I argue that it was Chinese sources on the "internal" history of Chinese writing that substantiated the evolution from picto- and logography to phonography as the broadly agreed upon universal historiography of writing in early modern Europe. Circulating between Europe and China through the various East India companies, these sources gave European scholars the impression that Chinese writing itself had at one point been more "naturalistic" and pictographic, indeed comparable to the Egyptian hieroglyphs that, according to them, directly imitated things in nature. They also convinced them that Chinese writing gradually evolved from its pictographic beginning towards their current schematic shapes whereby the signifying connections between characters and their meanings (as well as sounds) no longer depend on natural resemblances but on social conventions. It was only a small step to extend this evolution from pictographs to characters further towards phonographic alphabets whose sound-representing signs are even more conventionalized and streamlined. It was also only a small step to expand this linear progression from the internal history of Chinese writing to the universal history of human grammatology, with Egyptian hieroglyphs preceding and alphabetic letters following.[28]

Two types of Chinese sources on grammatological history furnished early modern Europeans with this evolutionary history of Chinese writing. First, as the Chinese tradition attributed the invention of writing to the earliest ancient kings, chronicles detailing their legendary deeds often contain traces of the history of writing. These chronicles were important for European scholars, since the supposedly uninterrupted Chinese record-keeping allowed them

28 While the narrative of the evolution of writing from pictures to alphabets is rather sketchy in Rousseau's *Essay*, Warburton thoroughly unpacks the idea in *Divine Legation of Moses Demonstrated*, 172-218.

to reconstruct a universal history in conjunction with biblical historiography.[29] Most of these chronicles, however, derived their fleeting records on writing from *Explicating Glyphs and Analyzing Characters* (shuowen jiezi 說文解字) by Xu Shen 許慎 (c. 58-c. 148)'s, a 2nd-century dictionary that has remained the authoritative source for studying early Chinese writing even today. Xu writes in the postface to his dictionary:

> 古者庖犧氏之王天下也。仰則觀象於天，俯則觀法於地。視鳥獸之文，與地之宜。近取諸身，遠取諸物。於是始作《易》八卦，以垂憲象。及神農氏，結繩為治而統其事，庶業其繁，飾偽萌生。黃帝之史倉頡，見鳥獸蹄迒之跡，知分理之可相別異也。初造書契，百工以乂，萬品以察。[…]以迄五帝三王之世，改易殊體。封於泰山者，七十有二代，靡有同焉。
>
> In ancient times, Emperor Fuxi governed all that was under Heaven. He looked up and observed phenomena in the Heaven, faced down to observe principles of the Earth. He saw the patterns of birds and beasts, the structures of the landmass. By modeling what was near after his own body and what was far after various things, he created *Canon of Change* and the Eight Trigrams to manifest predictions of the Heavenly course. Later, Emperor Shennong kept records by tying knots on ropes in governing and presiding over all affairs. Worldly matters proliferated, and ornateness and falsehoods also burgeoned. During the Yellow Emperor's time, his scribe Cang Jie saw the footprints of birds' feet and beasts' hooves. He understood that he could distinguish between the various types of birds and beasts by differentiating between the patterns of their footprints. In so doing, he invented writings and inscriptions. Hundreds of professions were carried out and thousands of kinds of things were studied with the help of this new invention. […] Through the later Five Sovereigns and then the Three Dynasties, the strokes of some characters were changed, and the shapes of some characters were altered. As a result, at Mount Tai, none of the inscriptions left by the seventy-two successive rulers who had performed rites there resembles another.[30]

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29 Standaert, *The Intercultural Weaving of Historical Texts*, 303-314.

30 Xu Shen 許慎, *Shuowen jiezi* 說文解字 ("Explicating Glyphs and Analyzing Characters," 121 CE), Xu Xuan et al ed. (986 CE), facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=en&res=77351, accessed October 22, 2018), Part 4, 108-109.

Granted, at the very beginning of the invention of writing, Xu describes an evolution from Fuxi's abstract trigrams of *I-Ching* (yi jing 易經, also translated as the *Canon of Change*) and Shennong's tying knots on ropes—which 18th-century European scholars compared to Peruvian *quipu*[31]—to Cang Jie's naturalistic pictographs. The *longue durée* narrative, however, still describes a process of simplification and schematization, as pictures of "footprints of birds' feet and beasts' hooves" were gradually replaced with strokes and dots through "seventy-two successive rulers" of the Three Dynasties (c. 2000s BCE-771 BCE).

Second, ever since a 3rd-century BCE grammatological reform that completed the schematization of Chinese characters, Chinese literati had taken great delight in searching for and imitating ancient character forms or scripts (known as the *zhuan* 篆 scripts).[32] Collecting supposedly ancient scripts became, I argue, a form of grammatological historiography, and the few collections that circulated to Europe were indeed taken as such by European scholars impressed at their visuality. In his *China Illustrata*, for example, Kircher incorporates sixteen woodcuts, each featuring a group of five characters first written in a supposedly ancient Chinese script and then written in the modern script (see Illustration 2 for the 2nd, 6th, and 7th of these woodcuts).[33] What is more, each group of five characters describes the supposed history and origin of the very script in which they are written. For example, the five characters in the 2nd

31 See Warbuton, *Divine Legation of Moses Demonstrated*, Vol. 2, 180.

32 This was the shift from *zhuan* 篆 "seal" script to *li* 隸 "clerical" script. The former derived mostly from inscriptions on bronzes and stones and were variegated, due to the lack of a powerful central authority. The latter, in contrast, served to facilitate handwriting on bamboo strips (and later paper) for administrative and clerical duties. Thanks to the centralization efforts by the Qin (221-207 BCE) and Han (206 BCE-220 CE) empires, the *li* or clerical script not only became the official script but would also serve as the origin from which all subsequent scripts developed. It is thus possible establish a divide between the "ancient" and the "modern" right at the transition from the *zhuan* "seal" script to the *li* "clerical" script.

33 Kircher, *China … Illustrata*, Part VI, Chapter 3 "Characterum antiquissimorum Chinensium explication," 228-232.

woodcut reads "the script in the form of wheat ears was created by [Emperor] Shennong," who legendarily invented agriculture and herbal medicine; accordingly, the characters written in this script feature strokes that resemble plants of millet or wheat. The five characters in the 6th woodcut reads "Cang Jie created [this script] based on the footprints of birds," a clear reference to the narrative form Xu Shen quoted above; accordingly, the characters written in this script consist of strokes that resemble the footprints of birds. For a last example of these fascinating illustrations, the five characters in the 7th woodcut reads "[King] Yao made [this script] because a tortoise emerged [from River Luo]," which Chinese traditions esteemed as an exceptionally auspicious sign; accordingly, the characters written in this script consist of strokes that resemble tortoises.[34]

Kircher's sixteen woodcuts proved very influential, perhaps more so than his *China Illustrata* itself. Even though his translations of the explanations of these scripts are thoroughly imprecise,[35] seeing the supposedly ancient scripts juxtaposed with the modern ones sufficed for his readers to conclude that Chinese writing evolved from naturalistic pictographs to conventional-schematic characters. Indeed, though the English theologian William Warburton (1698-1779) systematically refuted Kircher's interpretation of Egyptian hieroglyphs and his Egypt-China hypothesis, he nonetheless employed these woodcuts precisely to extend the

34 Not specified in Kircher's source, legend has it that the tortoises that miraculously emerged in River Luo during King Yao's time had inscribed on their shell mysterious writings. And thus, in the same way that the script in the 6th woodcut not only harks back to Cang Jie's creation of writing using bird's footprints but also claims to be that very script he created, the script in the 7th woodcut styles itself as the very mysterious and auspicious writing inscribed on the back of those miraculously appearing tortoises in River Luo.

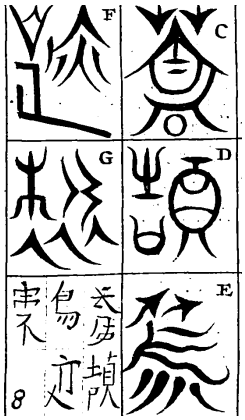
35 Kircher mistranslated the five characters in the 2nd woodcut as "The *Book of Agriculture* that King Shennong wrote" (there is no reference to any *Book of Agriculture*), the 6th woodcut as "Cang Jie wrote books with the small wings of birds" (as opposed to their footprints), and the 7th woodcut as "King Yao wrote this script with turtle shells" (as opposed to in the form of tortoises).

***Illustration 6-2 Kircher*, China … Illustrata *(1667), 229*[36]**

2nd woodcut

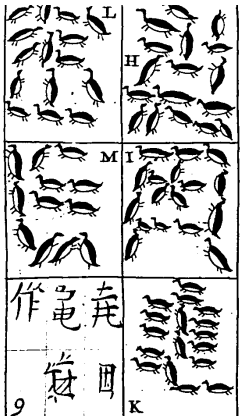
穗農
書作
神

6th woodcut



跡倉
製頡
鳥

7th woodcut



出堯
作因
龜

internal evolution of Chinese writing into a linear history of human writing evolving from Mexican pictograms to Egyptian hieroglyphs, Chinese characters, and finally to alphabetic letters.[37] In turn, Rousseau would build upon Warburton's narrative of grammatological evolution in his *Essay*.

As it turned out, Kircher copied these sixteen woodcuts from a 16th-century *riyong leishu* 日用類書 "encyclopedia for everyday use" marketed to scholar-officials and literati whose

36 The illustration shows the 2nd, 6th, and 7th of the sixteen woodcuts of supposedly ancient Chinese scripts Kircher incorporates in *China ... Illustrata* (1667). As the modern characters in Kircher's original (written in the bottom right square of the 2nd woodcut and the bottom left squares of the 6th and the 7th) are not very legible, I write them out in printed types in at the bottom of each woodcut, in the same alignment as are the ancient-script characters.

37 Warburton, *Divine Legation of Moses Demonstrated*, Vol. 2, Plate 6 between 180 and 181, which contains a direct replica of a woodcut from Kircher's *China ... Illustrata*, 227.

administrative and social duties demanded extensive literary and historical knowledge.[38] It was sent to him by Michał Boym (1612-1659), a Polish Jesuit in southern China.[39] What neither Kircher nor Boym might have known, however, was that an even more visually enthralling genre called *za zhuan* "[text set to] a miscellany of ancient scripts" had been popular among Chinese literati since at least the 10th century, thanks particularly to the invention of woodblock printing around that time. Typically, such a compilation features only a single text—a favorite was the *Diamond Sūtra* (jingang jing 金剛經) in the Chinese Chan Buddhist tradition—yet adapts it to eighteen, thirty-two, or even fifty-six different scripts, with each script accompanied with a gloss on its supposed origin.[40] These compilations of texts set in various different ancient scripts belonged to the broader genre of *tie* 帖 "copybooks" in Chinese calligraphy, which contain examples by celebrated masters for scholar-officials to imitate in practicing their handwriting (with brushes).[41]

Of course, while a small number of a typical "miscellany of ancient scripts" compilations did come from inscriptions on ancient bronzes, stone drums, and steles, anyone even slightly

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38 Kircher's source was discovered by Haun Saussy to be *Tianxia bianyong Wenlin miaojin wanbao quanshu* 天下便用文林妙錦萬寶全書 ("A Universally Convenient and Applicable Compendium of Ingenious Ideas and Ten Thousand Treasures for Literati"), dated to about the late Ming (1368-1644) era. The particular exemplar is now Vatican Library, Barb. orient. 139. See Saussy, "The Prestige of Writing, 文 [wen], Letter, Picture, Image, Ideography," in *Great Walls of Discourse and Other Adventures in Cultural China* (Cambridge, UK: Harvard University Asia Center, 2001), 35-74; 50-55.

39 Kircher, *China ... Illustrata*, 225.

40 See Yu Kuo-ching 游國慶, "三十二體篆書金剛經之研究" ("The Study of Thirty-Two Chuan-Shu Style Script Writing of the Diamond Sutra"), *Tushu yishu xuekan* 圖書藝術學刊 4 (2008), 77-142.

41 For example, the Jesuit Jean-Joseph Marie Amiot sent back to Paris an exemplar of Zhang Yingzhao, *Lidai diwang mingchen fa tie* 歷代帝王名臣法帖 ("Fac-similé d'autographes des Empereurs, Rois et fonctionnaires célèbres"), 10 volumes (1615), Paris, Bibliothèque nationale de France, MS Chinois 1213-1222.

familiar with Chinese writing would know that most of them were but later concoctions (oracle bones script, which is currently the earliest known form of Chinese writing, would not be discovered until the turn of the 19th into the 20th century). In fact, to be precise, they cannot even be properly referred to as "scripts" but should only be called "fonts": they simply took an actual ancient script and replaced its strokes and dots with idiosyncratic motifs: tortoises, wheat ears, tadpoles, footprints of birds, knots of strings, fish, and so forth. These motifs, however, are not at all random. Rather, they all make specific references to well-recorded deeds of the ancient kings—it being a different question whether those records were accurate—and thus claim to be the very scripts invented by these kings. As shown in my above analysis, for example, the wheat-ear script in Kircher's 2nd woodblock claims to have been created by Shennong, the inventor of agriculture. And the bird-footprint script in the 6th woodblock even claims to be the very first glyphs Cang Jie invented by imitating bird footprints, precisely as narrated in Xu Shen's postface quoted above. Thus, whether participants in this "miscellany of ancient script" genre knew or cared about the inauthenticity of most of these supposedly ancient scripts, writing, compiling, engraving, publishing, and imitating them still constituted a material and embodied way of narrating and imagining the history of writing.

Indeed, one such compilation adapting a single text to multiple ancient scripts did make its way from 18th-century China to Europe. In the winter of 1743, the Qianlong Emperor (1711-1799) visited Mukden, the capital of the Qing Empire in Manchuria before it invaded and conquered China proper in the mid 17th century. In commemorating this first trip he took as Emperor to the native land of the Manchus, the Qianlong Emperor composed a *Rhapsody of Mukden* (mukden i fujurun bithe , shengjing fu 盛京賦). Perhaps to resonate with the *fu* "rhapsody" genre, which was the dominant genre in the long-ago Han era (202 BCE-220

Illustration 6-3 Manchu word* mukden ᠮᡠᡴᡩᡝᠨ *("Mukden") and Chinese character* yi 翼 *("wings") in thirty-two ancient scripts taken from* Rhapsody of Mukden *(1748)

(*Mukden* "the rising capital [name of the Qing's first capital]")

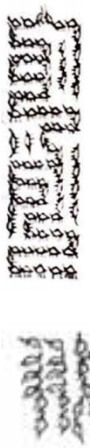
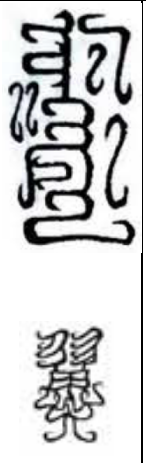
翼

(*yi* "wing")

Jade Chopstick	Fantastic Glyph	Grand Seal	Small Seal	Square Seal	Tomb	Wheat Ear	Hanging Chives
							
Willow Leaves	Astrology	Sesame Buds	Green Jade	Dripping Dew	Dragon Claw	Low Clouds	Tadpole
							

(continued in the next page)

(Illustration 6-3 continuing from the previous page)

Bird Footprint	Corrosive Worms	Unicorn	Goose Head	Birds	Phoenix	Tortoise	Dragon
							
Scissors	Knots of Ropes	Needle Puncture	Lance	Metal Wetstone	Inscribed Icon	Brushing Broom	Bronze and Bell
							

CE) and was renowned for its grand scale, exhaustive details, and profuse rhetorical devices,[42] the Qianlong Emperor ordered that his *Rhapsody* be printed in thirty-two different ancient scripts. What resulted, then, was *Rhapsody of Mukden Written in Thirty-Two Ancient Scripts*

42 For the *fu* "rhapsody" genre and its aesthetics, see David Knechtges, "'Have You Not Seen the Beauty of the Large?'": An Inquiry into Early Imperial Chinese Aesthetics," in *Wenxue, wenua yu shibian: Disanjie guoji Hanxue huiyi lunwenji* 文學、文化與世變：第三屆漢學會議論文集 ("Literature, Culture, and the Changing World: Proceedings of the Third Sinology Convention") (Taipei: Institute of Literature and Philosophy, Academia Sinica, 2002), 41-66.

(1748).[43] In this thirty-two-volume publication, each volume adapts the Emperor's five-thousand-character rhapsody to one of the thirty-two scripts (see Illustrations 6-3).[44] Each also explains the history of the script it uses (still written in that very script), explanations that resemble the five-character glosses in Kircher's woodcut, though far more extensively.

What makes the Qianlong Emperor's *Rhapsody of Mukden* exceptional in the long tradition of multi-script compilations, however, is that the Emperor had also published a Manchu version of the same text adapted to the very same thirty-two supposedly ancient scripts. In fact, through the French Jesuit Jean-Joseph Marie Amiot (1718-1793), both the Chinese and the Manchu versions of *Rhapsody of Mukden Written in Thirty-Two Ancient Scripts* reached Paris and are now found at the Bibliothèque nationale.[45] That the Qing would publish the same text in two different languages was by no means exceptional. Both Manchu and Chinese were administrative languages of the Empire: the former was the native tongue of its ruling military class, and the latter was the language of the Han Chinese scholar-officials, bureaucrats, and

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43 See Yu Kuo-Ching, "古漢字與雜體篆—以三十二體篆書盛京賦為例" ("A Study on Ancient Chinese Characters and Mixed-seal Characters"), *Tushu yishu xuekan* 圖書藝術學刊 2 (2006), 71-94.

44 In Illustration 6-3, Images of the Manchu word *mukden* written in the thirty-two scripts are taken from the first folio of each of the thirty-two volumes of Paris, Biliothèque Nationale de France, MS Madnchou 110. Images of the Chinese word *yi* 翼 ("wing") written in the thirty-two scripts are taken from Yu (2006), 81-86, which are in turn taken from an exemplar of the Chinese version of *Rhapsody of Mukden in Thirty-Two Ancient Scripts* (1748) at the National Palace Museum in Taipei.

45 Amiot sent to Paris both an exemplar of the Chinese version of *Rhapsody of Mukden written in Thirty-Two Ancient Scripts* and one of the Manchu versions. The former is now Paris, Bibliothèque nationale de France, MS Chinois 1578-1581. The latter is now Paris, Bibliothèque nationale de France, MS Mandchou 110. The latter is the source I am using to analyze this work, though many more exemplars exist in libraries around the world. In addition, Amiot also composed a translation of the Emperor's *Rhapsody*, together with its preface, postface, and the explanations for the thirty-two supposedly ancient scripts. See Amiot, *Éloge de la Ville de Moukden et de ses Environs; poème compose par Kien-Long, Empereur de la Chine & de la Tartarie, actuellement regnant* (Paris: N. M. Tilliard, 1770).

landed gentry.[46] Indeed, the Empire constantly erected literary and material monuments in as many as five languages (Manchu, Chinese, Mongolian, Tibetan, and Chagatay) to emblematize the plurality of peoples and cultures under its rule.

The problem, however, is that the Manchu script, unlike Chinese characters, is phonographic through and through. Each letter in its alphabet represents a unit of sound. The alphabet was developed in 1599 out of the Mongolian alphabet, which derived in the 13th century from the Old Uyghur alphabet, and the latter can be traced further back to Phoenician, perhaps the mother of all alphabets, through Syriac and Aramaic. Indeed, the thousands of imperially or privately printed Manchu primers and dictionaries from the 17th and 18th centuries reflect the widespread understanding that the Manchu letters serve to represent the sounds of spoken words. A few studies on Chinese phonetics and phonology even adopted Manchu letters to transcribe the sounds of Chinese characters and to manifest the minute differences between certain phonemes.[47] Many Christian missionaries also learned Manchu, deeming its alphabet far easier than Chinese characters.

But how could it make sense to write the Manchu version of *Rhapsody* in exactly the same thirty-two ancient Chinese scripts or fonts? To reiterate, the discursive and cultural

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46 See aforementioned Crossley and Rawski, "A Profile of the Manchu Language in Ch'ing History," 63-102.

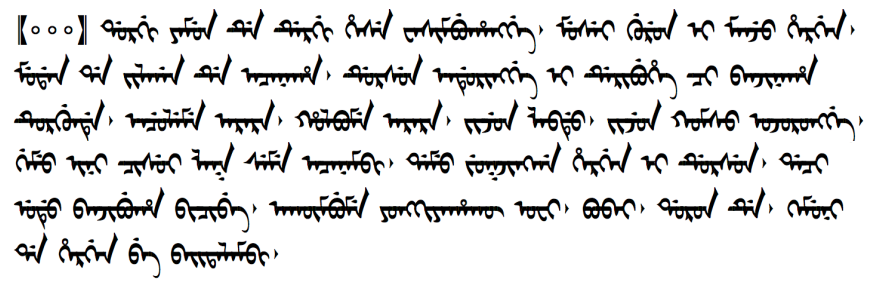
47 One example is In-lu, Li Guangdi, Wang Lansheng et al., *Yinyun chanwei* 音韻闡微 ("Manifesting the Nuances of Pronunciations and Rhymes") (1726), which uses Manchu phonetics and pedagogy to revise the traditional *fanqie* 反切 method of representing the pronunciation of a monosyllabic character by taking the initial consonant of one character and the post-initial-consonant sounds (the glide, the vowel, and the final) of another character. Another example is Anon., *Yuanyin zhengkao* 圓音正考 ("The Orthodox Investigation on Affricates," 1743), which aims at rectifying the Manchu transliteration of Chinese proper nouns and in so doing uses the Manchu alphabet and phonetics to discern subtle differences between various affricates, or what are now known as the *jiantuan yin* 尖團音 "alveolar and alveolo-palatal series consonants" problem.

function of *za zhuan* "miscellany of ancient scripts" compilations was not to propose any empirically substantiated theory on how writing evolved or how writing works. Rather, by simply adding naturalistic motifs to modern Chinese characters, those supposedly ancient scripts visually juxtaposed through woodblock printing incorporated Chinese literati in the imagined genealogy of Chinese writing, harking back to its supposed naturalistic and pictographic origin. For the Manchu script, however, there was no such origin to hark back to. While the "invention myth" of Chinese characters such as Xu Shen's *Explicating Glyphs* described the ancients imitating the visual shapes of things in nature, the imperially canonized history of the Manchu script recounted Nurhaci (1559-1626), founder of the Qing's ruling clan, using Mongolian letters to spell spoken words in Manchu. *Veritable Records of Nurhaci* (taizu wu huangdi shilu 太祖武皇帝實錄, 1636) even quotes him saying; "Put the letter *a* (ᠠ) and add a *ma* (ᠮ) to it, isn't this *ama* (ᠠᠮᠠ) 'father'? Put the letter *e* (ᡝ) and add a *me* (ᠮ) to it, isn't this *eme* (ᡝᠮᡝ) 'mother'?"[48]

So why did the Qianlong Emperor adapt an alphabetical writing system that had originated in and continued to represent the sounds of spoken words to thirty-two supposedly ancient scripts that hark back to the pictographic origins of Chinese characters? Why did he ask compilers of the Manchu version of *Rhapsody of Mukden Written in Thirty-Two Ancient Scripts* to add the same motifs of wheat ears, tadpoles, willow branches, knotted ropes, bird footprints, tortoises, dragon claws, and so on to Manchu letters, even though these motifs narrate a naturalistic origin of pictograms that violated the accepted history of Manchu writing? Clues, I argue, can be found in the paratexts to the multi-script *Rhapsody of Mukden*. The preface, penned by the Emperor in name, is an imperial edict formally sponsoring the project, dated to November

48 Gang-lin et al., *Qing Taizu Wuhuangdi Shilu* 清太祖武皇帝實錄 ("Veritable Records of the Taizu Emperor Wu [Nurhaci] of the Qing," 1636), Vol. 4.

2, 1748 in the Gregorian Calendar. It begins by explaining why Manchu writing should be adapted to the supposedly ancient Chinese scripts:



[…] Imperial edict decreed to the Grand Secretariat: "The sounds of the Manchu writing of our country accord to the *ur*-sound of the cosmos, and its shapes were sagely created and established. Because of this, [the letters] can be separated or conjoined, and [a written word] can be complex or simple in terms of its strokes. [The writing] perfectly matches the self-so-ness of all things. But then, when it comes to the shapes of ancient scripts, although there had been a few established at the beginning, because they have never been thoroughly completed, imperial and official seals still use the original script. […][49]

Lest anyone be confused about the alphabetical and phonographic nature of Manchu writing, the edict begins by praising the Manchu script for "according to the *ur*-sound of the cosmos." In the Chinese version of this edict, "cosmic *ur*-sound" (*da jilgan* ᡩᠠ ᠵᡳᠯᡤᠠᠨ, literally "original sound") is translated as *yuansheng* 元聲 ("original sound"), an expression most often used to describe the perfect musical tuning whose foundational pitch pipe *huangzhong* 黃鐘 ("yellow bell") was thought to produce a sound that corresponds with cosmic forces. Indeed, as the opening chapter of *Orthodox Meaning of Pitch Pipes* says, "when the pipe of *huangzhong* is rightly established, the cosmic *ur*-sound will be harmonious."[50] And speaking of the cosmological connections between musical tuning and calendar-keeping, Zhu Xi 朱熹 (1130-

49 Paris, Bibliothèque nationale de France, MS *Mandchou* 110-1, ff. 1r-2r.
50 In-c'i et al. *Orthodox Meaning of Pitch Pipes*, vol. 1, f. 2r.

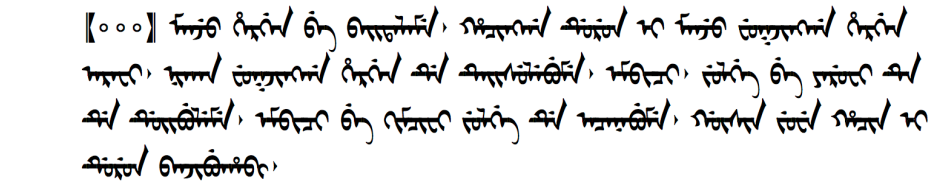
1200), arguably the most influential interpreter of Confucianism in the 2nd millennium CE, concurs: "For scholars of tuning and calendar, the cosmic *ur*-sound is the most important, for when it is ascertained everything else will fall into place, and when it is misjudged everything else will err."[51] Curiously, however, according to the edict, what accords to the cosmic *ur*-sound is not the sounds of the spoken Manchu language (*manju gisun*) but rather the sounds of the Manchu writing system (*manju hergen*). In other words, the superiority of Manchu (implicitly to Chinese) celebrated here is not about its sounds *per se* but rather the sound-transmitting nature of its script. Thus, before even addressing the project of adapting Manchu writing to supposedly ancient Chinese scripts, the edict underlines phonography as the most essential characteristic of the "writing of our country."

Moreover, regarding the reason for adapting the phonographic Manchu writing to Chinese scripts whose naturalistic motifs hark back to a pictographic origin, the edict suggests that it was only for making Manchu writing stylistically comparable to Chinese characters in seal inscriptions. As emblems of imperial and state power, official seals that stamped edicts, laws, orders, permits, and passports were consistently inscribed in ancient scripts under Chinese regimes, demonstrating a ruler's reverence for the ancients and their supposedly perfect statecraft. Since Manchu and Chinese were the two main administrative languages of the Qing Empire, official seals of the latter were inscribed in two languages. As the edict suggests, however, up to the mid 18th century, a stark visual disjuncture marred the Qing's seals: whereas the Chinese portions were inscribed in all kinds of ancient and supposedly ancient scripts, the Manchu portions could only be written in that same ordinary script, which I have used in the

51 Zhu Xi, *Zhuzi yulei* 朱子語類 ("Thematically Arranged Quotations of Master Zhu," 1270) Chapter 92, "Yue gujin" 樂古今 ("Music ancient and modern").

Manchu quote above. Thus, it was only for the visual unity and prestige of the Empire's seal inscriptions that the Qianlong Emperor had invented "ancient Manchu scripts" whose naturalistic motifs were completely irrelevant to the understood phonographic origins and functions of the Manchu alphabet.

Written by Fuheng  (Fu Heng 傅恆, 1720-1770) and others in charge of the project, the more detailed postface to *Rhapsody of Mukden* concurred that the sole purpose for these new Manchu scripts was making seals. After articulating the same problem of juxtaposing ancient Chinese scripts with the not-so-ancient-looking Manchu script in seal inscriptions, the postface describes how the Emperor (no doubt embodying the team of scholars who did the actual work) invented those new Manchu scripts:

> 〖○○○〗 
>
> […] Using Manchu writing, [Our Emperor] created Manchu scripts of various shapes. In order to match the ancient Chinese scripts, he either cited ancient [scripts] to compare them to the modern [scripts] or studied modern [scripts] to accord them to the ancient [scripts]. He thus created thirty-two types of scripts.[52]

Thus, different from those ancient Chinese scripts, which, regardless of their authenticity, embodied the pictographic and naturalistic origins of Chinese writing, the Qianlong Emperor's addition of Manchu to the *za zhuan* "miscellany of ancient scripts" genre did not at all strive to hark back to any such origins. In fact, the Emperor's edict calls the "non-ancient" standard Manchu script in use the *da hergen*  "original script"—*da* means "head" or "origin," as in the *da jilgan* "cosmic *ur*-sound" to which the Manchu script accord—implying that making those

52 Paris, Bibliothèque nationale de France, MS *Mandchou* 110-32, appendix ff. 4r-4v.

thirty-two "ancient" Manchu scripts was in fact not an act of historically narrating resurrection but rather a marked act of innovation. And the only reason for such a creation is "[to] match the ancient Chinese scripts" and its pictographic invention myths.

Personally, I cannot help but read a sense of irony in these paratexts of *Rhapsody of Mukden*. The Emperor's edict-qua-preface opens by praising the Manchu alphabet for signifying cosmos-corresponding sounds, only to end up subjecting this phonographic alphabet to the pictographic grammatological imaginations of those supposedly ancient Chinese scripts. While it would have been a massive undertaking to invent thirty-two kinds of Manchu scripts, translate the Emperor's lengthy and deliberately obscure *Rhapsody* into Manchu, and print the text in thirty-two volumes each in a different script, the final product ends up citing the stylistic integrity of seal inscriptions as its only *raison d'être*. What is more, the postface written by scholars who worked on the multi-script project even criticizes the fact that "people who study ancient [writings]" only cared about hoarding those preciously few ancient bronzes and stones that bear supposed ancient inscriptions but did not care at all about actually verifying or investigating them.[53]

This lack of verification (*kimcimbi* ᡴᡳᠮᠴᡳᠮᠪᡳ) and investigation (*baicambi* ᠪᠠᡳᠴᠠᠮᠪᡳ), however, was no small criticism in the 18th century. Since the Qing conquered China proper in the mid 17th century, the *kaozheng* 考證 or evidential learning movement had come to dominate the scholarly pursuits of Han Chinese literati.[54] This new strand of scholarship emphasized "investigating" (*kao*) and "verifying" (*zheng*) concrete demonstrable evidence in reconstructing the exact institutions and mores of the ancient sages through their textual legacies. To say that scholars of

53 Ibid., 1v-2r.

54 See Elman, "Early Modern or Late Imperial Philology? The Crisis of Classical Learning in Eighteenth Century China," 3-25.

certain research do not actually verify or investigate was to completely invalidate the paradigm of their research—and in fact, I argue, it had become a broad consensus among 18th-century scholars to dismiss the compilation of those supposedly ancient Chinese scripts in exactly such negative light. Indeed, in 1815, a scholar named Duan Yucai (1735-1815) published *Annotations of Explicating Glyphs and Analyzing Characters* (shuowen jiezi zhu 說文解字註), in which he furnished every entry in Xu Shen's 2nd-century dictionary with extensive explanations and references. In his preface to Duan's *Annotations*, Wang Niansun (1744-1832) praised the work precisely by disparaging not simply the visual and material *jouissance* of concocting, compiling, and copying or carving scores of supposedly ancient scripts but actually the entire pursuit of grammatology *in toto*:

> 蓋千七百年來無此作矣。若夫辨點畫之正俗，察篆隸之繁省，沾沾自謂得之，而於轉注假借之通例，茫乎未之有聞。是知有文字而不知有聲音、訓詁也。其視若膺之學，淺深相去爲何如耶?
>
> For the one thousand and seven hundred years since Xu Shen's *Explicating Glyphs*, there has never been a work like this! As for those who think highly of their achievements in differentiating between standard and vulgar characters through their strokes and points and in observing the simplification process from the ancient script to the modern script, these people have never heard anything about those common instances of cognate derivatives or rebus usages of characters. They only know about writing but nothing about sounds or glosses. What a great difference between the shallowness of their learning and the depth of the learnedness of Ruoying [the courtesy name of Duan]![55]

Though it may be understandable that Wang used the most glowing terms to describe the work of his friend, it is puzzling how Wang could praise an annotation to Xu Shen's *Explicating*

55 Wang Niansun, "Preface" to Duan Yucai, *Shuowen jiezi zhu* 說文解字註 ("Annotations of *Explicating Glyphs and Analyzing Characters*," 1815), facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=81427, accessed October 22, 2018), Part 1., 2-3.

Glyphs and Analyzing Characters by dismissing as insufficient and frivolous the grammatological study of character shapes and their evolutions. Not only was Xu Shen's postface to his dictionary the canonic text on Chinese grammatology, but subsequent scholars also revered Xu's dictionary precisely because it "explicated" and "analyzed" the origins and signifying mechanisms of each character. Wang, however, suggests that studying Chinese characters and their evolution from naturalistic pictograms to schematic signs reveals little if anything about Chinese writing. Thus, whereas 17th- and 18th-century European scholars took Chinese sources on the "internal" history of Chinese characters as the very basis for a narrative on the overall evolution of human writing from pictographic to phonographic, contemporary scholars under the Qing Empire like Wang and Duan had only come to dismiss the usefulness of such narratives. Still, just like their European counterparts who embraced the imagined natural progression of writing from pictures to alphabets only to develop grammatological vs. phonological modes of linguistic restorationism, 18th-century Qing scholars rejected the gramma-centric historiography of human language only to further negotiate the complex interstices between phonology and grammatology. And they did so by widely opening their eyes—in order to listen to sounds coming from the page.

Folksong Mania

To recall, both the Egypt-China hypothesis and Rousseau's speculation on the origin of languages strove to mitigate what they perceived as a loss of knowledge of and/or about early humans. Proponents of the Egypt-China hypothesis hoped to borrow Chinese writing to decipher the no-longer legible Egyptian hieroglyphs and reveal the esoteric or antiquarian information they transmitted, and Rousseau aspired to counter the social enslavement of humans from their

natural freedom as well as the degeneration of their speech from the passionate primitive speech-song by parsing the corrupting supplementarity of writing. Likewise, as I have shown in previous chapters, degeneration and loss had in fact been a fundamental narrative in the political as well as philosophical traditions of Confucianism. Confucian scholar-officials romanticized the time of the legendary ancient kings—including Fuxi, Shennong, and Yellow Emperor mentioned above—and the so-called Three Dynasties, i.e. Xia (c. 2070-c. 1600 BCE0, Shang (c. 1600-c. 1046 BCE), and Former Zhou (c. 1046-771 BCE). During these eras, they believed, sage rulers governed their realms and edified their peoples by employing a variety of institutions, including rites and music, yet they were later lost in civil wars and foreign invasions.[56]

This Confucian restorationism was only heightened in the 17th century when China fell to the supposedly "barbarian" Manchus, the semi-nomadic people who founded the Qing Empire in northeast Asia in 1636. Following this political shock, many Chinese literati not only made it an urgent task to preserve the textual sources of Confucianism under the foreign regime, but also called for stricter adherence to the recorded ways of the ancients. Accordingly, they blamed previous generations for deviating from the ancients' mores and thus bringing about the humiliation of Manchu conquest. Many particularly rallied against the so-called *xinxue* 心學 "learning of the mind," a strand of Confucianism that began to flourish in the 15th century.[57] Often characterized by modern scholars as a form of radical subjectivism, learning of the mind emphasized self-truthfulness: since the self is an integral part of the cosmos, it contends, one only needs to look inward in order to acquire the most fundamental moral and ethical principles of the universe.[58] In other words, the best way of emulating the ancient sages who conformed

56 See, for example, *Records of Rites*, Chapter 19 "Record of Music" (*yueji* 樂記).
57 See Peterson, "Confucian Learning in Late Ming Thought," 716-728.
58 Ibid., 719-722.

with these cosmic principles lies not with any rational study or analysis of external canons, classics, or texts, but rather with the attainment of authenticity of the self.

It was in the context of this discourse of subjective authenticity that 16th-century Chinese literati saw a craze for collecting and emulating what may be best characterized as "folksongs." Granted, the latter term carries specific connotations of 19th-century European nationalism and romanticism, and I am only using it for the sake of convenience. Still, though there was not a singular term or category from the time that could be deemed a non-problematic equivalence of "folksong" or even "folk" or "song," a discursive unity of literati collections and emulations of "folksongs" was furnished by the aforementioned *Canon of Songs*. Traditionally, it was posited that Confucian himself compiled and edited this volume of some 305 songs (only the lyrics were written down), which comprised three genres or categories: *feng* 風 "regional songs," *ya* 雅 "elegant songs," and *song* 頌 "hymns." *Feng* literally means "wind" and metonymically means "customs" and "regional songs," a connotation alluding to the perceived connections between the geographic features of a region, the comportment of its inhabitants, and the characteristics of their songs. It was believed that the ancient sage kings right up to the Former Zhou Dynasty sent officials to collect songs from the commoners of their realms in order to inquire about the temperaments and livelihoods of their subjects and the state of their governance. In contrast, *ya* "elegant songs" and *song* "hymns" were refined tunes for sacrificial rites and courtly ceremonies. The word *ya* "elegant" or "edified" had come to be juxtaposed against the word *su* 俗 "vulgar" or "unrefined," and the latter was subsequently combined with *feng* as *fengsu* to mean "customs" of the common folk.

Indeed, the 16th- and 17th-century discourse of folksongs and collections and emulations of them typically made direct reference to the "regional songs" in the *Canon of Songs* as an

ancient precedent of gathering songs from the commoners. Such is exemplified in the preface by Feng Menglong 馮夢龍 (1547-1646) to his *Mountain Songs* (shan'ge 山歌, c. 1630s), a collection of folk songs (though almost certainly incorporating his own emulations):

> 書契以來，代有歌謠。太史所陳，並稱"風""雅"。自楚騷唐律，爭妍競暢；而民間性情之響，遂不得列於詩壇。於是別之曰"山歌"。 […] 今雖季世，而但有假詩文，無假山歌。則以山歌不與詩文爭名，故不屑假。苟其不屑假，而吾藉以存真，不亦可乎？抑今人想見上古之陳於太史者為彼，而近代之留於民間者如此，倘亦論世之林云爾。
>
> Since the invention of writing and inscription, each of the ancient dynasties had their own songs and ditties. They were collected by the grand historians [*taishi*] and were called "local songs" [*feng*] and "elegant songs" [*ya*]. Over the following millennia, emotive songs of the Chu style and highly regulated lyrics of the Tang era competed against each other in showing off their beauty; meanwhile, the sounds of the temperaments and affections from among the people were no longer admitted to the world of poetry but were instead differentiated and called "mountain songs." […] Although the present day finds itself at the declining end of an era, there is only inauthentic poetry and prose, but no such thing as an inauthentic mountain song. This is because mountain songs do not compete for renown against poetry or prose and thus disdain to feign. So, shouldn't I be permitted to seize upon them in order to preserve the authentic? Today, people want to behold those songs from ancient times that were collected by the grand historians, yet following are the more recent songs that have remained among the people, and perhaps the latter should also count among sources through which we gauge the ethos of an age.[59]

Granted, prefacing a publication project aimed at commercial success, Feng understandably indulges in hyperboles to make the songs he collected as ideologically and aesthetically significant as possible. It is still notable, however, that he recognizes songs from *minjian* 民間 "among the common folk"—understood as outside the literate world of the

59 Feng Menglong, *Shan'ge* 山歌 ("Mountain Songs," c. 1630s), a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=4670, accessed October 22, 2018), Part 1, 2-4.

Confucian scholar-officials—in the modern era to be analogous to the ancient *feng* "local songs" and *ya* "elegant songs" whose textual remnants have been transmitted through written records in the *Canon of Songs*. This argument is crucial, for it puts the oral practices of singing from among the illiterate masses on an equal footing with the prized textual tradition of the literati elites. Since both the "authentic" singing voice of the people and the written traces of Confucian classics such as *Canon of Songs* are recognized residues of ancient times—the former embodied, the latter textual—both could potentially be utilized for restoring the proper institutions and mores of the ancient sage kings.

The preface by Li Mengyang 李夢陽 (1472-1529) to a collection of his own poems went even farther in this regard. Li was an interesting figure. An adamant advocate for a *fugu* 復古 "restoring the ancient" movement in poetry in the 15th and early 16th century, Li argued with his comrades that literary poets should emulate historical poetic styles so as to avoid what they perceived as a decadent ornate style of their time. Importantly, living a century before Feng's *Mountain Songs* qua the epitome of the 16th- and 17th-century "folksong mania," Li never collected or emulated folk tunes, that is, contemporary songs drawn from illiterate commoners. Rather, all of the poems in his collection were written in emulation of poetry found in written sources passed down from literati poets from different time periods. The preface to the anthology of his own works, however, betrays a bizarre sense of insecurity, describing a conversation between Li and a friend, Wang Shuwu, on whom information is lacking. The preface portrays Wang schooling Li not only on the value of folk tunes, which Li completely ignored in his poetic compositions, but also on the fundamental inadequacy and uselessness of emulating historical poetry from written sources. The beginning of the preface quotes Wang saying:

> 夫詩者，天地自然之音也。今途咢而巷謳，勞呻而康吟，一唱而群和者，其真也，斯之謂風也。孔子曰：“禮失而求之野”。今真詩乃在民間，而文人學子顧，往往為韻言。
>
> [Wang said:] Poetry is the natural sound of Heaven and Earth. Today, when someone roars on the side of a road or sings in an alley, when the belabored one groans or the happy one chants, when one sings and a crowd responds, it is an authentic song, and it is called a “local song” [f*eng*]. Confucius once said: “when the proper rites are lost, go find them among the wild countryside!” Today, the authentic poetry exists among the people, whereas the literati and the learned often versify only for the sake of rhyming.[60]

“When the proper rites are lost, go find them among the wild countryside!” is a quote that 16th century scholars commonly attributed to Confucius and used to argue that the illiterate common folks outside the materially and culturally privileged scholar-official and landed-gentry classes nonetheless possessed valuable wisdom that could help restore the lost knowledge of and/or about the ancient sages. As mentioned in the previous chapter, for example, Zhu Zaiyu used precisely this quote to justify the sources of inspiration for his twelve-tone equal temperament, which is commonly credited as the world’s first: according to Zhu, he got the idea when he observed some professional musicians slightly adjusting the time-honored Chinese Pythagorean tuning method when tuning their instruments.[61] By using this quote, Wang argued that folksongs are not just one kind of poetry but actually the only “authentic poetry” that still existed. In fact, so ashamed did Li become of his own inauthentic poems that Li actually held off publishing his anthology for more than two decades, as he describes at the end of the preface.

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60 Li Mengyang wrote the preface for a collection of his own poems entitled *Hongde ji* 弘德集 (“Anthology of poems written during the Hongde era”), of which I have not been able to find an exemplar. I have based this translation on Li’s preface as anthologized in Liu Shilin ed., *Ming wen yu* 明文霱 (“Auspicious Clouds of Ming Prose,” 1634), a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=en&res=92044&remap=gb, accessed October 22, 2018), Part 2, 29-33.

61 Zhu Zaiyu, *A New Theory of Pitch Pipes*, vol. 1, Chapter 3 “密律律度向求” (“Calculating the Precise Proportions of Pitch Pipes”), specifically ff. 5r-5v.

Thus, whereas Feng Menglong's preface to his *Mountain Songs* argues that the oral practice of contemporary folk tunes, thanks to their authentic expressions of the self, are as good a thread for restoring the ancient poetic ethos as textual, literary sources, Li's preface goes much further in arguing that the orality of folk tunes, being the only authentic form of literary expression that is left, is the *only* possible route towards such a restoration.

From *Kaogu* "Investigating the Ancient" to *Shenyin* "Examining the Tones"

With the core tenet of radical subjectivism and "authenticity" under attack in the 17th century after the Manchus subjugated China and incorporated it into their Qing Empire (1636-1912), the craze for folksong collections suddenly subsided.[62] Apparently, literati no longer romanticized the illiterate and unrefined masses as being any closer to the innocent, original, or authentic condition of the ancients. Instead, Han Chinese scholars turned to the careful guardianship and study of ancient texts, and many embraced the aforementioned "evidential learning" movement in this process. The latter differed not only from Ming-era radical subjectivism in its prizing of studying the textual sources of Confucian canons and classics over inward examining of the authentic self, but it also differed from earlier traditions of textual studies from the Song era (960-1279). Whereas Song-era literati emphasized understanding a text through close reading and meditation, Qing-era literati emphasized verifiable proofs and evidence of exegeses and glosses.[63] As a result, the experience of reading and explicating a

62 Though the "folksong mania" subsided in the Qing era, collections of regional opera arias blossomed during the 18th century, following the general popularity of music theaters across China. See Deng Xiaodong 邓晓东, "清代民歌小曲的文獻考察" ("A Textual Investigation of Qing Dynasty Folk Ballads"), *Nanjing shifan daxue wenxueyuan xuebao* 南京師範大學文學院學報 (2015): 59-65.

63 Elman, "Early Modern or Late Imperial Philology?," 16-18.

Confucian classic largely shifted from philosophical speculations of abstract moral principles of the cosmos to philological analyses of the concrete meanings and references of each word of a text. Thus, philology or *xiaoxue* 小學 "little learning" (little, because it served the "greater learning" of interpreting classics, rather than being a discipline on its own) became the most important discipline in an ethos of textual optimism: so long as one has the right understanding of language and writing, one can correctly interpret a historical Confucian text. And so long as one can arrive at such a correct interpretation, one can ultimately restore the institutions of the ancient sages and their proper order of governance.[64]

Different from previous philologists, however, 17th- and 18th-century philologists were not simply grammatologists but were distinct for being predominantly phonologists. Up until the 17th century, historical philology in China largely followed a grammatological paradigm—indeed quite like 17th- and 18th-century Egyptology and the Egypt-China debate—in trying to restore the meaning of ancient texts by establishing secure pairings of characters [*gramma*] and meanings [*logos*]. Most reflexive of this paradigm, of course, was Xu Shen's aforementioned *Explaining Graphs* and its globally influential postface. As I have analyzed, Xu's narrative of the origins of writing and its later development mentions nothing whatsoever about speech, sound, or the speaking voice. Rather, it begins by tracing a purely visual history, in which writing arose from the Eight Trigrams of *I-Ching* that reflect the observable patterns of the universe to the earliest characters that depict the traces of footprints left by birds and beasts. Continuing from this invention of writing, furthermore, the narrative morphs into one of degeneration and loss, so that, by the era of his time, Xu argues, knowledge of the most original writing of the ancients tied to

64 On the embodiment of philological research in the Qing-era and the "textual optimism" of the ultimate ability to restore the exact meaning of ancient texts, see Zito, *Of Body and Brush*, Chapter 4 "Writing the Ritualist Metaphysics: Self and the World," 96-117.

the propagation of their sagely wisdom and edification was already lost. And it is through this loss of grammatological knowledge that Xu defines his intervention: by creating a dictionary in which each entry not only lists the various meanings of a character but also "explicates" and "analyzes" the logic and rationale whereby its character [*gramma*] has come to signify its meanings [*logos*].

A dramatic shift away from this grammatologically oriented historical philology took place in the 17th century in what I term a "Phonological Revolution." Three important changes occurred: first, the realization that the pronunciations of characters changed over time; second, the consequent establishment of historical phonology as a discipline and the complete reorientation of historical philology (and therefore the exegeses of Confucian classics) from historical grammatology to this new phonological discipline; and third, the rewriting of the myth of how language and writing were invented, so that writing became a consequence of and supplement to spoken language or speech.

For languages whose writing system has always remained alphabetical or phonographic, the notion that spoken languages change over time is perhaps a non-starter. For a language whose writing system relies heavily on many signs that do not bear a direct relationship with phonemes, however, the lack of "orthographical" changes can easily obscure the fact that the same character could have been pronounced in drastically different ways across time (and space, as in dialects). And here, the *Canon of Songs* as a collected body of once-sung lyrics played a crucial rule in revealing historical changes in the pronunciation of written characters. Since at least the 10th century, scholars had noticed irreconcilable anomalies in the rhymes of these songs. For example, Illustration 6-4 shows the lyrics for the second and third stanzas of a song supposedly collected from the Zhaonan region and included in the *feng* "regional songs" section

of *Canon of Songs*.[65] Each stanza bears an *ababb* rhyme scheme, which produces a paradox: in the second stanza, the character 家 ought to rhyme with 角 *jiao*; in the third stanza, the same character 家 ought to with 牙 *ya*. If one were to uphold the modern pronunciations of these characters with which 家 ought to rhyme (that is, *jiao* and *ya*), one would have to pronounce the same character bearing the same meaning differently in the same poem. And this is but one of the thousands of apparent rhyming anomalies if one were to recite *Canon of Songs* according to more recent pronunciations of the characters.

***Illustration 6-4 the 2nd and 3rd Stanza of "Dew drops on the road" from* Canon of Songs**[66]

誰謂雀無角	jiao	*Who says birds don't have beaks?*
何以穿我屋	wu	*How did they poke through into my room?*
誰謂女無家	jia	*Who says you don't have a household?*
何以速我獄	yu	*Why do you drag me to the authorities?*
雖速我獄	yu	*Even if you drag me to the authorities,*
室家步足	zu	*I will not marry you.*

誰謂鼠無牙	ya	*Who says mice don't have teeth?*
何以穿我墉	yong	*How did they get through my walls?*
誰謂女無家	jia	*Who says you don't have a household?*
何以速我訟	song	*Why do you sue me in court?*
雖速我訟	song	*Even if you sue me in court,*
亦女不從	cong	*I will not follow your wishes.*

65 The poem is the 6th poem "Xing lu" 行露 ("Dew drops on the road") in Chapter 2 "Zhaonan" 召南 ("Songs from the southern outskirts of Zhao") of the *Feng* "local songs" proportion of the *Canon of Song* as edited by Mao Heng (c. 3rd century BCE), the received edition of the *Canon*.

66 This illustration shows the apparent rhyming irregularities if one reads the poem in the current received Mandarin pronunciations of the characters — though these irregularities would have also occurred when scholars after the 10th century read this poem in the received pronunciations of their time.

For centuries, scholars have proposed different solutions to this problem, the most popular being that of *xieyun* 叶韻 "vowel harmonization," which posits that the same character was simply pronounced differently *ad hoc* to fulfill specific rhyming needs.[67] Following this solution, one would pronounce the first *jia* in the poem above as *jiao* in order to rhyme with the last word *jiao* of the first line of the stanza, while still pronouncing the second *jia* as *jia* in order to rhyme with the last word *ya* of the first line of the stanza. Another popular explanation was *tongzhuan* 通轉 "conversion and transformation," which posits that, compared to modern poetic rules, the ancients were simply much more permissive when it came to rhyming, so that sounds like *ya*, *jia*, and *jiao* were actually considered similar enough to rhyme.[68]

Throughout the 17th century, however, a new explanation became increasingly popular: many characters occupy the same rhyme position in the songs in *Canon of Songs* but do not rhyme when pronounced in the received manner of the 17th century because their pronunciations have changed since ancient times.[69] Following this new explanation, the apparent rhyming defects of lyrics from *Canon of Songs* when recited according to modern pronunciations became the strongest proof of not only the difference between ancient and modern pronunciations of the same characters but also a system of ancient phonology completely different from that of the

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67 An epitome of this theory is *Shiji zhuan* 詩集傳 ("A collective interpretation of the *Canon of Song*") by Zhu Xi (1130-1200), who was also the most highly regarded scholar of the speculative and philosophical type of Confucianism that Qing-era scholars would later turn against.

68 First systematically proposed by Wu Yu (c. 1100-1154) in his *Yunbu* 韻補 ("Supplementing the Rhymes"), the theory of ancients using loose rhymes that could easily transform or convert into one another remained popular until the mid 18th-century. See aforementioned Zhang Minquan, *Qingdai guyinxue yanjiu* 清代古音學研究 ("The Studies of the Archaic Chinese Phonology in Early Qing Dynasty"), Vol. 1, 42-88; Vol. 2, 135-153.

69 The first recorded formulation of this hypothesis was seen in Chen Di, *Maoshi guyin kao* 毛詩古音攷 ("An Investigation on the Ancient Pronunciations of *Canon of Songs* as edited by Mao Heng," 1606).

modern. And the thousands of rhyming lines from *Canon of Songs* became the most important primary source for studying this ancient phonology. Thus, thanks to their purportedly historical origins as sung lyrics that rhyme, the written texts of *Canon of Songs* [*gramma*] became the database upon which historical pronunciations [*phonē*] of characters were reconstructed.[70]

This "discovery" of an ancient phonology distinct from the modern one and of the sonic rather than visual-grammatological aspect of explicating ancient Chinese texts should not be understood as part of a teleological progress, as it tends to be narrated in scholarship on 17th- and 18th-century Chinese linguistics. Indeed, almost a millennium passed from Wu Yu 吳棫 (c. 1100-1154)[71] who first attempted to systematically address the apparent rhyming irregularities in *Canon of Songs* to Duan Yucai 段玉裁 (1735-1815) whose categories of ancient vowel types in and application thereof to philology are broadly considered the epitome of Qing-era research on ancient linguistics.[72] And though Chen Di 陳第 (1541-1617) conjectured in the late 16th century that ancient pronunciations of the same words and characters were systematically different from those of the moderns,[73] it was only by the mid-18th century that this "ancient phonology" hypothesis became the generally accepted explanation for the apparent rhyming irregularities in *Canon of Songs* and was gradually adopted for studying ancient philology and explicating ancient texts. The reason is that up to the early to mid 18th century, debates over whether ancient

70 The first extant study that used the rhyming data in *Canon of Songs* to reconstruct ancient pronunciations of characters—specifically by grouping together characters that were pronounced with the same vowel—was Gu Yanwu 顧炎武, *Yinxue wushu* 音學五書 ("Five treatises on the study of phonology," 1667)

71 Wu Yu (c. 1100-1154) in his *Yunbu* 韻補 ("Supplementing the Rhymes," c. 12th century).

72 Duan Yucai, *Shuowen jiezi zhu* 說文解字註 ("Annotations of *Explicating Glyphs and Analyzing Characters*," 1815)

73 Chen Di, *Maoshi guyin kao* 毛詩古音攷 ("An Investigation on the Ancient Pronunciations of *Canon of Songs* as edited by Mao Heng," 1606).

phonology was systematically different from its modern counterpart *in toto* only drew upon textually transmitted rhyming data, namely the various rhyming relations between thousands of characters extracted from the lyrics of *Canon of Songs*. As a result, the hypothesis that the ancients pronounced the same written characters differently from the moderns was but one method for making sense of the rhyming data. A competing hypothesis, for example, was that the ancients did not at all pronounce the characters themselves differently from the moderns, but were simply much looser in their pronunciations and their use of rhymes than later rules of poetry demanded.[74]

So what eventually made the "ancient phonology" hypothesis win out at the end of the day? The answer lies in exactly the same term with which *Orthodox Meaning of Pitch Pipes* frames its experiments of listening to pitch pipes as the sole empirical basis for reforming the systems of pitch organization around the fourteen-tone octave: *shenyin* 審音 "examining the tones." A major figure in the evidential learning movement with a special proclivity for phonology and musical tuning, Jiang Yong 江永 (1681-1762) lamented in his paradigm-shifting *Standards of Ancient Rhymes* (guyun biaozhun 古韻標準, 1771) that previous scholars who had embraced the "ancient phonology" explanation of the apparent rhyming irregularities in *Canon of Songs* accomplished much in *gaogu* 考古 "investigating ancient things" but ignored *shenyin* "scrutinizing the sounds."[75] What he proposed, then, was to introduce the modern study of *dengyun* 等韻 ("classified rhymes") or ahistorical phonetic principles (tone shifts, rhyme

74 See Zhang Minquan, *Qingdai guyinxue yanjiu* 清代古音學研究 ("The Studies of the Archaic Chinese Phonology in Early Qing Dynasty"), Vol. 1, 42-88; Vol. 2, 135-153.

75 Jiang Yong, *Guyun biaozhun* 古韻標準 ("Standards of Ancient Rhymes," 1771), a facsimile version available on available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=5452, accessed October 22, 2018), Part 1, 14.

transformations, prenuclear glides, places of articulation, vowel finals, etc.) to organize the rhyming data from *Canon of Songs*.

Granted, the study of *dengyun* or phonetics originated only in the 9th century and used as their primary materials what is now known as Middle Chinese, which was considered part of the "modern" in the Qing era. Jiang, however, argued that the rhyming data extracted from the texts of *Canon of Poetry* are ultimately meaningless in reconstructing the ancient language unless scholars borrow the methods of modeling tone shifts, rhyme transformations, prenuclear glides, places of articulation, and so on from the study of modern phonetics—or, in his words, unless they start "scrutinizing the sounds." Jiang even suggested that proving the differences between ancient and modern pronunciations should ultimately rely not on extracting rhyming patterns from characters featured in *Canon of Poetry* but on examining these patterns through phonetic principles derived from studying modern pronunciations.

It was by using modern phonetics to guide the analyses of ancient rhyming data and the reconstructions of ancient pronunciations that Jiang and his student Dai Zhen 戴震 (1727-1777) put a decisive end to the centuries-long debate over whether the ancients pronounced the same written characters differently (the answer is yes).[76] Granted, previous proponents of the "ancient phonology hypothesis" managed to distill various vowel groups that comprise characters pronounced with different vowels in modern speech. Nonetheless, every attempt before Jiang to rationalize the apparently irregular rhyming patterns in *Canon of Songs* by organizing ancient pronunciations of characters into different vowel groups only ended up creating more irregularities, as characters from different ones of these newly constructed ancient vowel groups

76 See Wang Li 王力, *Qingdai guyin xue* 清代古音學 ("Ancient Phonology in the Qing Era") (Beijing: Zhonghua shuju, 1990/2012), 140-141.

could still rhyme on occasion. These newly created irregularities actually became a major weakness of the "ancient phonology hypothesis," exploited by those who believed that the ancients did not pronounce any characters differently but were simply much freer in rhymes and pronunciations in poetry. Rather than inventing even more patterns to give order to ancient rhyming data, however, Jiang and Dai focused on examining the principles whereby different vowels—ancient or modern—could transform into one another on the ahistorical phonetic level. In so doing, they constructed an intricate system detailing under what conditions characters from different reconstructed ancient vowel groups can still be taken to rhyme through shifts in tones, glides, and allophones. Thus, their endeavor in "scrutinizing the sounds" cleared a major hurdle that had hitherto prevented the "ancient phonology hypothesis" from becoming the dominant narrative of Qing-era phonology.

What exactly, however, did "scrutinizing the sounds" mean for Jiang Yong as a way of knowing-in-sound and knowing-about-sound? Indeed, Jiang was certainly not the first person to heed the importance of sounds in studying *Canon of Songs* with respect to ancient pronunciations. For the centuries before and after him, entering students of Chinese literature have been reciting those lyrics and casually changing the pronunciation of individual words when they apparently do not rhyme as they should in whatever dialect they speak. And it could only have been from reciting those lyrics—that is, treating the texts of *Canon of Songs* as teleologically or potentially sonorous traces of poetry-as-enunciated-sounds—that scholars at the turn of the 17th century first hypothesized that the ancient pronunciations of the same characters were decidedly different from the modern ones. What changed in Jiang's advocacy of "scrutinizing the sounds" *id est* using modern models and principles of phonetics to give structure and sense to ancient rhyming data, I argue, was how sounding and listening functioned

as a way of knowing—indeed as Feld's acoustemology. Before, when patterns of vowel groups were established solely from making sense of the textual rhyming data of *Canon of Songs* themselves and ultimately fed back to the sonorous recitation of those texts, sounds were integrated and embedded in written texts, characters, and graphemes. The 18th century, by contrast, saw sounds disconnected from texts and becoming a prior epistemological entity.

For evidential scholars trying to recuperate lost ancient knowledge through philology, the importance of "examining the sounds" and the newly established epistemological priority of pronunciation and sounds meant that understanding an ancient text requires getting at the sounds of the characters: it is these sounds, not the shapes of the characters themselves, that actually hold the key to revealing the meaning of the ancients. In other words, the relationship between the meaning, pronunciation, and shape of a character was no longer a grammatological one, in which the shape could independently represent meaning regardless of pronunciation. Instead, it became a phonological one, whereby the shape can represent any meaning only in so far as it represents certain units of sound. This phonological shift is evident in Wang Niansun's preface to Duan Yucai's *Annotations of Explicating Glyphs and Analyzing Characters* (1815), partly quoted above. Indeed, Wang's preface challenges the grammatological mode of historical philology that was first established by the very dictionary whose annotations by Duan the preface introduces. As Wang explains, interjecting the new scholarship on historical phonology into the exegeses of historical texts reveals a completely new sonic dimension: rebuses. In brief, many Chinese characters function not as logographs or pictographs—that is, directly representing any meaning—but instead as phonographs, or rebuses. For example, the character 止 *zhi* originated as the shape of the foot ( ) and means "foot" or "toe." Around the 4th century BCE , it was increasingly used as a rebus for a *zhi* sound in speech, meaning "to halt," which at that time

lacked its own corresponding logographic character. During that period, then, 止 *zhi* could be said to properly mean "foot" or "toe" and to be used as a rebus to mean "to halt." Over time the rebus usage of this character unrelated to its shape took over its original, pictographic meaning and became the character's only received meaning, to the point that writers began using another homophonous character 趾 "toe" to fill in the original proper signification of the logograph 止 that had been dropped by the rebus or phonograph 止.

It is therefore no wonder that, in my earlier quotation from his preface, Wang praises Duan's work by disparaging the traditional grammatologists, "those who think highly of their achievements in differentiating between standard and vulgar characters through their strokes and points and in observing the simplification process from the ancient script to the modern script." By focusing only on shapes, these grammatologists were chained to the proper logographic meaning or pictographic origins of a characters. In contrast, equipped with knowledge in historical phonology and pronunciations, scholars like Duan Yucai showed that many characters in ancient texts actually function as homophonous rebuses for another word. By locating the level of signification not on the logographic or pictographic link between shape and meaning but on the phonographic link between shape and sound, they revealed meanings of characters not directly suggested by their shapes but evident in their reconstructed ancient pronunciations.

The result, then, was not only the discovery of many rebus characters in ancient texts resolving many once puzzling passages, but also a reinvention of the entire field of historical philology in ways that still inform its research today. The phonological revolution was made plain in Duan Yucai's 1795 preface to Wang Niansun's *Commentaries and Proofs for Towards Elegance* Extended (guangya shuzheng 廣雅疏証). The original next, Towards Elegance *Extended* (guangya 廣雅) was a 3rd-century expansion of *Towards Elegance* (er ya 爾雅), which

was a glossary typically dated to the 1st millennium BCE. Taking a *longue durée* perspective, Duan writes in his preface to Wang's *Commentaries and Proofs*:

> 聖人之製字有義而後有音，有音而後有形。學者之考字，因形以得其音，因音以得其義。治經莫重於得義，得義莫切於得音。
>
> When the sages created the characters, first there was meaning and then there was pronunciation, and first there was pronunciation and then there was shape. The historical investigation of characters by scholars employs the shape of characters to acquire the pronunciation of characters, and employs the pronunciation of characters to acquire the meaning of characters. In studying the classics, there is nothing more important than seeking the meaning of characters. In seeking the meaning of characters, there is nothing more important than seeking the pronunciations of characters.[77]

Not only did Duan argue that philology should treat shapes or written characters as stand-ins—or, perhaps borrowing Rousseau's term, supplements—for spoken words, but he also reversed the order of the invention of language as originally narrated in Xu Shen's postface to *Explicating Graphs*: that is, shapes or written characters were no longer invented on their own, but only invented *after* spoken words as, again, supplements to them. Thus, from the origin myth of language to the academic paradigm of interpreting historical texts in order to restore the ancient wisdom they transmit, the Phonological Revolution elevating spoken words over written characters was complete.

77 Duan Yucai, "Preface" to Wang Niansun, *Guangya shusheng* 廣雅疏証 ("Commentaries and Proofs for '"Towards Elegance" Extended,'" 1795), a facsimile version available on the Chinese Text Project website (https://ctext.org/library.pl?if=gb&res=1490, accessed October 22, 2018), 3.

Some Closing Remarks

In 1822, just seven years after Duan Yucai's *Annotations of Explicating Glyphs and Analyzing Characters* (1815) with Wang Niansun's preface, Jean-François Champollion (1790-1832) wrote a letter to Bon-Joseph Dacier (1742-1833), secretary of the French Académie des inscriptions et belles-lettres. In the letter, Champollion not only reconstructs the phonetic values of the Egyptian hieroglyphs on the Rosetta Stone, but also posits the phonographic functions of hieroglyphs, challenging the ingrained European perception of them as pure pictographs. In the following decades, Egyptologists transcribed more and more hieroglyphic inscriptions and discovered more and more recognizable words thanks to their knowledge of Coptic, a language that had been quietly preserved in European academies during the 18th century.[78]

Of course, knowledge of how to read the hieroglyphs had been lost since the 4th century,[79] whereas most 18th-century Chinese scholars could read even the earliest scripts known at the time. Still, I argue that Champollion's decipherment of the Egyptian hieroglyphs was in many ways comparable to the Phonological Revolution in 18th-century Chinese historical phonology. Both discovered that written signs once thought to be purely logographic and/or phonographic actually functioned predominantly as phonographs, specifically homophonic rebuses. The hieroglyph  , for example, most frequently functions not as a pictograph for "goose," but actually as a phonograph for the sound *sa*, which could mean either "goose" or "son or daughter." Similarly, the Chinese character 又, for example, most frequently functions not as a

78 See John Ray, *The Rosetta Stone and the Rebirth of Ancient Egypt* (London: Profile Books, 2007), 38-79. See also Andrew Robinson, "Styles of Decipherment: Thomas Young, Jean-François Champollion, and the Decipherment of the Egyptian Hieroglyphs," *SCRIPTA* 3 (2011): 123-132.

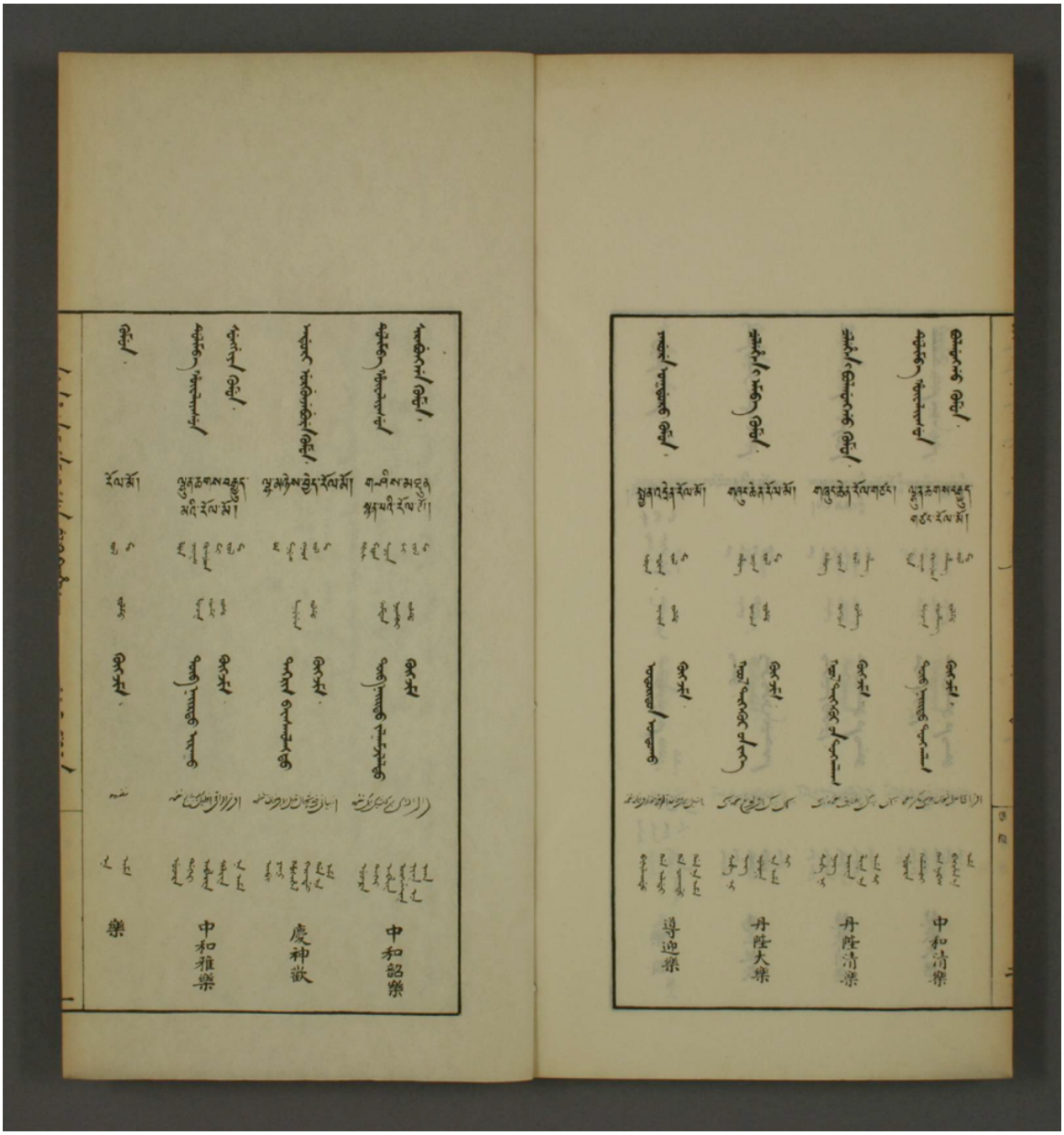
79 Robert Solé and Dominique Valbelle, *The Rosetta Stone: The Story of the Decoding of Hieroglyphs*, trans. Steven Rendall (London: Profile Books, 1999), 16.

pictograph for "to grab something by the right hand," whose shape the character was first created to imitate, but actually as a rebus for the sound *you*, which could mean either "right hand" or "again" in the spoken language.

The sixty-four-thousand-dollar question, of course, is whether the two "breakthroughs" in historical phonology—breakthroughs that took place by emphasizing the sonic dimensions of historical writing—were purely coincidental or pointed to some deeper, structural similarities or integrations. Through this chapter that concludes my examination of the Qing's fourteen-tone temperament, I hope to have sketched out some parallels between 18th-century France and Qing regarding the relationships between phonology and grammatology, particularly regarding the notion of "examining the tones" as an empirical knowledge-producing exercising having to do with sounds. Still, my comparison of historical philology or linguistic restorationism at the two ends of Eurasia in the early modern era should only be an initial step towards a much more nuanced and grounded narrative. For one, within the scope of this chapter, I have not managed to trace the consequences of the two 18th-century Parisian quarrels all the way to Champollion's new Egyptological paradigm as much as I have detailed the origins and backgrounds of the Phonological Revolution in 18th-century China. Besides, there is little reason why the comparison ought to be limited to Eurasia. Indeed, European discourses on language and writing were materially situated upon its colonial projects around the world, particularly the Americas and South Asia.[80] Chinese historical philology also developed in tandem with the colonial

80 Indeed, there have been several pioneering studies on writing and the voice in colonial Americas: Tomlinson, *The Singing of the New World* (2006), Olivia Bloechl, *Native American Song at the Frontiers of Early Modern Music* (Cambridge: Cambridge University Press, 2008), and Ana María Ochoa Gautier, *Aurality: Listening and Knowledge in Nineteenth-Century Colombia* (Durham and London: Duke University Press, 2014).

***Illustration 6-5 The Emperor's Pentaglot Dictionary* (1794), *Vol. 7, f. 1v-2r, the entry on "music"*[81]**



81 Illustration 5 is a page from *The Emperor's Manchu Pentaglot Dictionary* (han i araha sunja hacin i hergen kamciha manju gisun i buleku bithe ᡥᠠᠨ ᡳ ᠠᡵᠠᡥᠠ ᠰᡠᠩᡤᠠ ᡥᠠᠴᡳᠨ ᡳ ᡥᡝᡵᡤᡝᠨ ᡴᠠᠮᠴᡳᡥᠠ ᠮᠠᠨᠵᡠ ᡤᡳᠰᡠᠨ ᡳ ᠪᡠᠯᡝᡴᡠ ᠪᡳᡨᡥᡝ; yuzhi wuti qingwenjian 御製五體清文鑒, 1794). The first column from the left displays, from top to bottom, the Manchu word *kumun* ᡴᡠᠮᡠᠨ, the Tibetan word *rol mo* རོལ་མོ།, the Manchu transliteration of the Tibetan word, the Manchu transcription of the Tibetan word, the Mongolian word *högžim* ᠬᠥᠭᠵᠢᠮ, the Chagatai word *naghma* نغمه, the Manchu transcription of the Chagatai word, and finally the Chinese word *yue* 樂. All these five words can be translated, in this or that way, as "music" in English.

expansion of the Qing Empire, which spurred the studies of many more languages besides Chinese and Manchu—see Illustration 6-5, for example, for a page from the *Emperor's Pentaglot Dictionary* (1794), a thematically organized vocabulary in Manchu, Tibetan, Mongolian, Chagatai, and Chinese that was but one of the Qing's many multilingual linguistic projects.

Even more intriguing, of course, is the question of why. Why did these two and potentially more early modern frontiers of imperialism and colonialism simultaneously turn to phonology in trying to recuperate lost knowledge of and/or about the ancients, their writings, and their languages? Are some of the comparable features of historical philology or perhaps a simultaneous turn towards spoken words and voice (Rousseau's *Essay* may be particularly comparable to the "phonological revolution") the result of some shared experience at the two ends of Eurasia, some material processes that co-constituted the intelligentsia in 18th-century Paris and Beijing? If so, how can we trace these processes, so they may in turn shed new light on the significance (or lack thereof?) of the "early modern" or the "modern" on a global scale? Perhaps, more than half a century after the post-structuralist "linguistic term," the time is finally ripe to examine yet again the formulations of logocentrism and phonocentrism in Jacques Derrida's reading of Rousseau's *Essay* through Lévi-Strauss—except this time on a truly transregional and global scale.[82] And that, I argue, would be the global acoustemologies reflected and embodied in the Qing's reform to musical tuning, far more than is the intriguing

82 See Adriana Cavarero, *For More than One Voice: Toward a Philosophy of Vocal Expression*, trans. Paul A. Kottman (Stanford, CA: Stanford University Press, 2005), "Appendix: Dedicated to Derrida," 213-241, particularly 227-234 where Cavarero directly takes down Derrida's phonocentric characterization of Ancient Greek philosophy.

and coincidental passage from the Kangxi Emperor's studying of *ut re mi* to In-c'i's fourteen-fold division of the octave.

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