Supplementary Information for Counterfactual mobility network embedding reveals prevalent accessibility gaps in U.S. cities

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Supplementary Notes

S1 Implementation Details

S1.1 Counterfactual Random Walk

In each MSA, we construct an urban mobility network comprising nodes that represent POI categories, POIs, neighborhoods, and demographic features. For every category Q, we sample 200,000 random walks, denoted as $Q \to P_i \to C_o \to o$, to observe outcomes represented by C_o. It is important to note that we are interested in five demographic features. Therefore, for each demographic feature, we sample an alternative neighborhood C_a along with its corresponding alternative outcome a. In total, we randomly sample one million pairs of observed-alternative outcomes for each category within each MSA.

S1.2 Network Embedding

In each MSA, we learn an 64-dimensional embedding vector for each of the following entities: four POI categories, all POIs, all neighborhoods, and all treatment levels of five neighborhood demographic features. We use the Adam optimizer to minimize the loss function introduced in the Main Text. The spatial threshold σ_s is set to 2.5 kilometers, and the strength of regularization is 0.0001 and 0.01 for spatial and demographic feature continuity. During the training, we set the batch size as 10,000 and trained for 10,000 epochs. The codes are implemented with PyTorch 1.7 and run on a Linux server with NVIDIA RTX 2080 Ti. Codes for reproducing the CRANE algorithm are available at https://github.com/tsinghua-fib-lab/CRANE.

S1.3 Predictive Analysis

For the within-city prediction task, we conducted a five-fold analysis, repeating the following procedure five times for each POI category in every MSA. We randomly selected 80% of the neighborhoods as the training set, where we performed counterfactual random walks and learned network embeddings to obtain embedding vectors for each category and each level of demographic features. For our Multilayer Perceptron (MLP) regression model, we combined the raw demographic features of each neighborhood (5 dimensions) with the dot products between their corresponding $L(T)$'s embedding vectors and all four category embedding vectors (20 dimensions) as input. The model's output represents the urban facility accessibility of the neighborhood. Specifically, we utilized the "MLPRegressor" function from the Python package "scikit-learn", which includes two hidden layers with dimensions of 32 and 16, respectively, using the sigmoid activation function. To evaluate the model's performance, we tested it on the remaining 20% test set of neighborhoods, calculating the average explained variance as the performance metric.

For the cross-city prediction task, we performed counterfactual random walks and learned network embeddings on the entire urban mobility network consisting of all neighborhoods in an MSA. Then we also combined the raw demographic features of each neighborhood with the dot products between their corresponding $L(T)$'s embedding vectors and all four category embedding vectors to fit an MLP regression model. Subsequently, during the test stage, we applied the learned vectors in conjunction with the demographic characteristics of neighborhoods within the Chicago MSA to predict urban facility accessibility for those specific neighborhoods.

S2 Credibility of Treatment Effects

The goal of PSM is to balance covariates across different treatment levels $L(T)$. Under this scenario, differences in mobility patterns can be attributed to differences in $L(T)$. We report the average relative covariate differences before and after propensity score matching for each treatment variable across six MSAs in Supplementary Figure S2. The covariate differences are lower than 2% after PSM, demonstrating the effectiveness of PSM in balancing covariate and deriving debiased causal effects.

Specifically, we define relative covariate differences after matching as the relative disparities in the average values of covariates between two groups: the high-treatment group, composed of neighborhoods with higher treatment levels in each matched pair, and the low-treatment group, consisting of neighborhoods with lower treatment levels in each pair. Relative covariate differences before matching reflect the correlation between treatment and covariates, indicating the relative disparities in average covariate values between the neighborhoods with the highest and lowest treatment levels.

We observed that propensity score matching (PSM) significantly improved the balance of covariates. Prior to matching, relative covariate differences could exceed 50%, but following matching, they all decreased to less than 2%, as indicated by the red vertical line. This substantial reduction in covariate differences demonstrates the effectiveness of PSM in achieving covariate balance. This, in turn, enhances the reliability of PSM in identifying urban mobility inequalities and analyzing treatment effects.

S3 Complexity analysis

Here we analyze the time complexity of propensity score matching and the counterfactual random walk to demonstrate the efficiency of the CRANE method. In an MSA with N census block groups, the PSM approach first fits an ordinal regression model to estimate propensity scores. The complexity of calculating propensity for all CBG is $\mathcal{O}(N)$. Each CBG is then matched with another CBG with the closest distance defined by propensity score and treatment level, which takes $\mathcal{O}(N)$. The matching procedure takes a complexity of $\mathcal{O}(N^2)$ in total.

In the counterfactual random walk stage, we randomly sample a POI from the starting category, then sample a CBG that has visited the sampled POI, and finally sample an alternative CBG that has covariates identical to the sampled CBG. Each step in the counterfactual random walk is a sampling from a discrete probability distribution. The corresponding time complexity can be reduced to $\mathcal{O}(1)$ if we generate an alias table for each POI category, POI, and covariates combination in advance. In Section 3.2 of the Main Text, we demonstrate that the counterfactual random walk can well approximate PSM with a sample size over 100,000, which is often less than N^2 in big MSAs. Therefore, the counterfactual random walk can efficiently approximate PSM.

Supplementary Figures

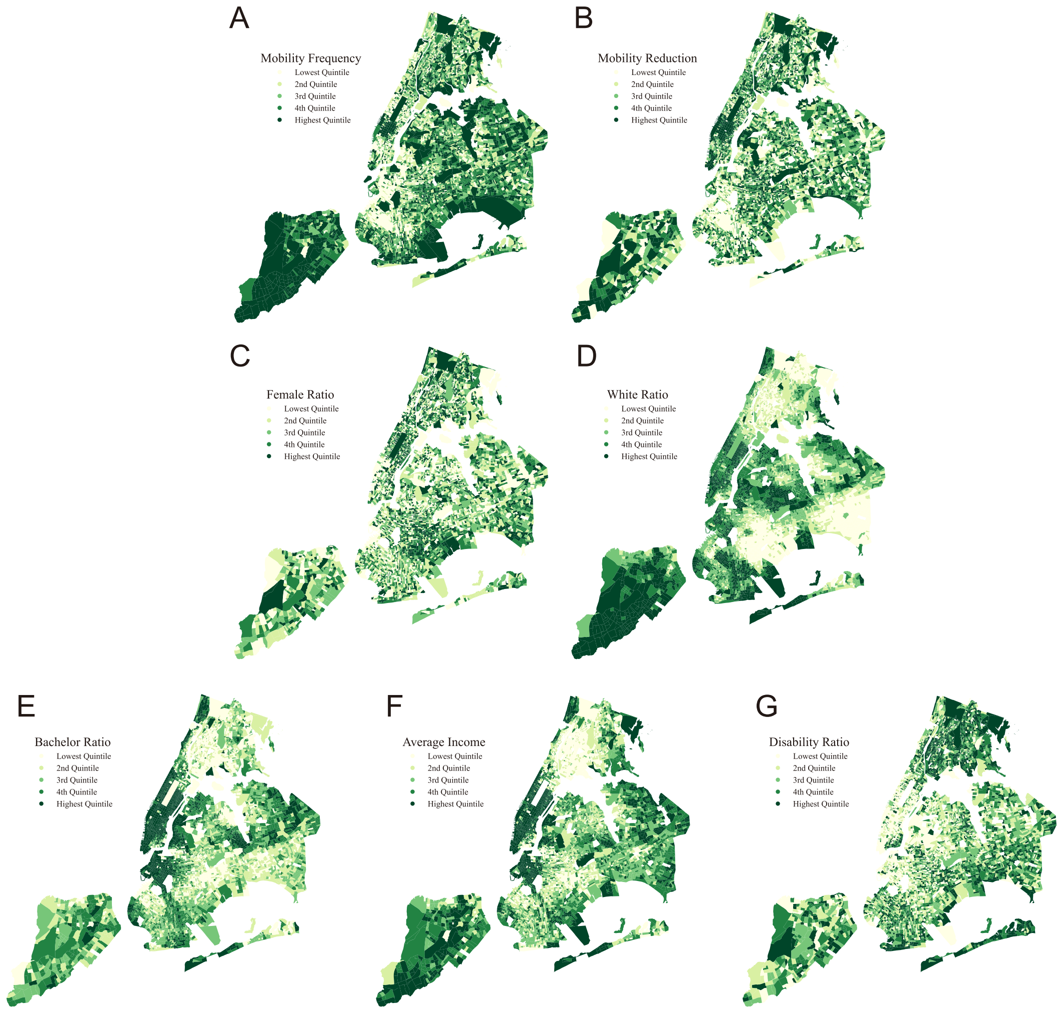

Figure S1. Geographical distribution of neighborhood's features. A-G Mobility frequency and demographic features at the neighborhood level in the New York metropolitan statistical area. Colors indicate the quintile of the neighborhood's features.

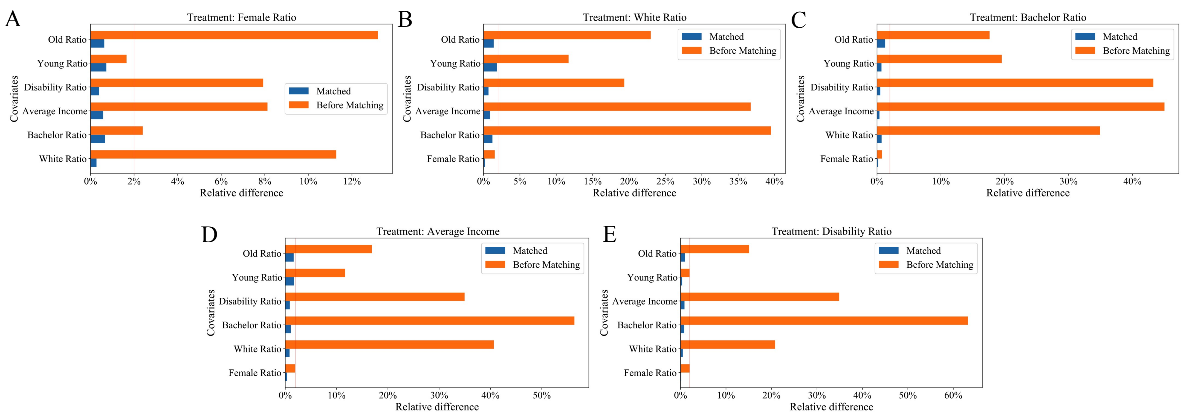

Figure S2. **Effect of propensity score matching on covariate balance. A-E** The relative differences of average confounding variables on high-dose and low-dose groups for each treatment variable. The propensity score matching method can reduce relative differences from over 50% to less than 2%.

Supplementary Tables

Table S1. The correlation between demographic feature and mobility behavior in Los Angeles MSA.

	Female%	White%	Bachelor%	Income	Disability%
N_{M2019}	-0.014	0.160***	0.248***	0.287***	-0.098***
Δ_M	-0.033**	0.071***	0.241***	0.202***	-0.111***
Art%	-0.001	0.199***	0.417***	0.337***	-0.20***
Sports %	0.010	0.410***	0.697***	0.587***	-0.303***
Education %	0.055***	0.084***	0.085***	0.243***	0.007
Health %	0.065***	0.008	-0.019	-0.033**	0.090***

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table S2. The correlation between demographic feature and mobility behavior in Chicago MSA.

	Female%	White%	Bachelor%	Income	Disability%
N_{M2019}	0.009	0.152***	0.192***	0.262***	-0.104***
Δ_M	0.046***	-0.058***	0.304***	0.189***	-0.149***
Art%	-0.017	0.141***	0.399***	0.24***	-0.221***
Sports %	-0.071***	0.492***	0.733***	0.687***	-0.387***
Education %	-0.046***	0.330***	0.083***	0.310***	-0.046***
Health %	0.106***	-0.115***	-0.212***	-0.198***	0.215***

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table S3. The correlation between demographic feature and mobility behavior in Dallas MSA.

	Female%	White%	Bachelor%	Income	Disability%
N_{M2019}	-0.002	0.231***	0.284***	0.376***	-0.136***
Δ_M	-0.014	0.016	0.405***	0.354***	-0.254***
Art%	0.001	0.115***	0.431***	0.351***	-0.19***
Sports %	0.016	0.282***	0.654***	0.635***	-0.325***
Education %	0.042**	0.188***	0.211***	0.366***	-0.012
Health %	0.080***	-0.049**	-0.049**	-0.073***	0.162***

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table S4. The correlation between demographic feature and mobility behavior in Houston MSA.

	Female%	White%	Bachelor%	Income	Disability%
N_{M2019}	0.051**	0.259***	0.260***	0.336***	-0.092***
Δ_M	0.030	-0.006	0.455***	0.35***	-0.281***
Art%	0.004	0.044*	0.314***	0.201***	-0.088***
Sports %	0.036	0.204***	0.753***	0.686***	-0.359***
Education %	0.058**	0.196***	0.208***	0.328***	-0.018
Health %	0.116***	-0.066***	0.072***	-0.002	0.029

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table S5. The correlation between demographic feature and mobility behavior in Washington DC MSA.

	Female%	White%	Bachelor%	Income	Disability%
N_{M2019}	0.019	-0.14***	-0.046**	0.042*	-0.000
Δ_M	0.051**	-0.257***	0.124***	0.044**	-0.069***
Art%	0.015	0.166***	0.397***	0.190***	-0.158***
Sports %	-0.007	0.445***	0.591***	0.579***	-0.258***
Education %	-0.017	0.291***	0.047**	0.298***	0.035*
Health %	0.074***	-0.082***	-0.177***	-0.189***	0.185***

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Table S6. Regression coefficients between each demographic feature and its proximity with POI categories in the embedding space. Significance levels of regression coefficients are listed.

MSAs	New York				Los Angeles				Chicago			
Demographic \ Category	Art%	Sports%	Edu%	Health%	Art%	Sports%	Edu%	Health%	Art%	Sports%	Edu%	Health%
Female%	0.023 (***)	0.018 (***)	0.030	0.003	0.014	0.005	0.018 (**)	-0.001	0.000	-0.002	-0.001	0.014
White%	-0.050 (**)	0.036 (*)	0.021 (*)	0.007	0.015	0.024 (**)	-0.001	-0.006	-0.005	0.041 (*)	0.036 (*)	-0.006
Bachelor%	0.033 (***)	0.031 (*)	-0.027	-0.017	0.044 (**)	0.047 (**)	-0.004	0.000	0.039 (*)	0.054 (***)	-0.010	-0.016
Income	-0.031	0.018	0.032 (*)	0.001	0.025 (*)	0.002	0.025	-0.011	0.011	0.016	0.016	-0.014
Disability%	0.025 (***)	0.002	0.016 (**)	0.016 (***)	-0.024 (***)	-0.016	0.001	-0.000	-0.026 (***)	-0.013	0.002	0.001

MSAs	Dallas				Houston				Washington DC			
Demographic \ Category	Art%	Sports%	Edu%	Health%	Art%	Sports%	Edu%	Health%	Art%	Sports%	Edu%	Health%
Female%	-0.014	0.001	-0.010	0.011	-0.002	0.010	0.005	0.021	0.012	0.019	0.003	0.018
White%	0.018	0.029	-0.003	-0.011	-0.002	0.009	0.021	-0.027	-0.007	0.047 (*)	0.061 (**)	-0.017
Bachelor%	0.051 (**)	0.042 (***)	0.003	0.004	0.059 (**)	0.069 (***)	0.007	0.003	0.062 (**)	0.036 (***)	-0.013	0.008
Income	0.032 (*)	0.022	0.026 (**)	-0.002	0.015	0.041 (*)	0.044	-0.004	0.011	0.034	0.030 (**)	-0.000
Disability%	-0.003	-0.025	0.004	0.011	0.007	-0.009	0.019	0.015	0.006	-0.004	0.006	-0.008

* $p < 0.2$; ** $p < 0.1$; *** $p < 0.05$.