THE UNIVERSITY OF CHICAGO

ESSAYS ON FINANCIAL MARKETS WITH ENDOGENOUS INFORMATION SCREENING CHOICES

A DISSERTATION SUBMITTED TO
THE FACULTY OF THE UNIVERSITY OF CHICAGO
BOOTH SCHOOL OF BUSINESS
AND
THE FACULTY OF THE DIVISION OF THE SOCIAL SCIENCES
KENNETH C. GRIFFIN DEPARTMENT OF ECONOMICS
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

BY
YIRAN FAN

CHICAGO, ILLINOIS
JUNE 2021

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FOREWORDS

It has been my honor to be a co-chair, along with Zhiguo He, of Yiran Fan's PhD dissertation committee.

The loss of our University of Chicago graduate student, Yiran Fan, was indeed tragic. It was the consequence of a senseless act of violence, which has been happening far too frequently. It has been my honor to be a co-chair, along with Zhiguo He, of Yiran's PhD dissertation. We each had the privilege to present a chapter at his posthumous defense/proposal. There was sadness at that event, because it was both a reminder of the loss of someone we knew well on a personal level, and the loss of a truly gifted young scholar who would have wanted the opportunity to make further improvements on an already impressive PhD dissertation.

I have known Yiran since his time as a research professional at the Fama-Miller Center. Even then, his contribution to research projects was special, going well beyond what is expected of research professionals. Moreover, he was an absolute joy for me to know and work with. I was glad to write letters for him to the top PhD programs, and I was pleased that he chose to join the Joint Financial Economics PhD Program.

Right from the start, Yiran established himself as a remarkable PhD student. He won the Lee Prize from the economics department for being a top performing student in the macroeconomics core curriculum. My colleagues and I have been pleased to have him as a teaching assistant. Indeed, he always showed a deep commitment to help his fellow graduate students learn. He has been an invaluable intellectual resource to our economics and finance community. I have had many superb PhD students over the years, but Yiran will stand out as someone truly special.

I am very pleased that the University of Chicago will award a PhD degree to Yiran Fan. This document not only contains some wonderful insights, but it will also serve as a reminder to his family and to his friends of his commitment to intellectual excellence. While his loss is truly sad, the University of Chicago should be very proud to have Yiran as one of its distinguished graduates. No doubt, Yiran would have acknowledged the affection and support of his two very special parents, Chenggang Fan and Chunzhi Xu.

Lars Peter Hansen

ACKNOWLEDGMENTS

For the first chapter, I thank Zhiguo He, Ralph Koijen, Doron Ravid, Zijian Qi, Anthony Zhang, Eric Zwick, participants in the Chicago Booth Third Year Finance Research Seminar, and participants in the University of Chicago Micro Theory Student Group for valuable discussion.

For the second chapter, I am deeply grateful for advice from Veronica Guerrieri and Zhiguo He. I also appreciate very helpful discussions with Douglas Diamond, Lars Hansen, Scott Nelson, Raghuram Rajan, Zijian Qi, Harald Uhlig, and Anthony Zhang.

ABSTRACT

The first chapter studies screening competition under flexible information acquisition and its interaction with price competition. Multiple homogeneous buyers play a game where they simultaneously design independent exams with pre-specified information limit on a binary-type seller. Once observing own exam's outcome, a buyer may choose to bid for the object. This paper shows that under general assumptions on information cost, binary-signal symmetric equilibrium exists and must be supported by mixed strategies. Moreover, equilibrium is inefficient and on average buyers over-reject the seller. Price regulation may help restore efficiency.

The second chapter develops a dynamic general equilibrium model on the interaction of bankers' asset and liability management with liquidity concerns. Bankers screen real production projects and issue deposits. Liquidity concerns stem from endogenized early withdrawals of deposits. To fulfill early withdrawals, bankers sell assets in a secondary market. The paper argues that ex post asymmetric information in the secondary market distorts bankers' incentive in screening ex ante, as bad assets are easier to sell and generate liquidity benefits. Moreover, the general equilibrium feature of the model implies that exogenous aggregate productivity shocks are amplified and booms may lead to busts.

CHAPTER 1

SCREENING COMPETITION UNDER FLEXIBLE INFORMATION ACQUISITION

1.1 Introduction

Many markets have the following structure: a seller wants to sell an indivisible object to multiple potential buyers. Buyers have common valuation on the object, but the value per se is ex ante unknown. In order to distinguish good objects from bad ones, buyers can screen the object beforehand, and then decide whether to bid for the object. Real life examples include, but not limited to, job market candidates who are trying to sell their human capital to employers, inventors who are trying to sell their patent to firms, etc.

Although this kind of market is common in real life, its efficiency is under studied. This paper intends to close the gap by studying a situation where the object's value has binary types. In particular, I formulate a constrained social planner's problem and compare its solution to market equilibrium under quite general assumptions on the screening capacity that each buyer has. Besides, the role of price competition is also taken into consideration. The main conclusion of the paper is that with multiple potential buyers, market equilibrium is inefficient and over-rejects the seller.

The model formulated in the paper is simple. The seller has a reservation price on the object being sold, which is common knowledge. Multiple potential buyers simultaneously screen the object before making purchase decisions, each up to an exogenously specified capacity. The object is either good or bad. Buyers have common priors over the object and ex ante the object's valuation to buyers equals to the seller's reservation price. Screening process is modeled as conducting a test where the two types have different probability distributions over selected signal realizations. Tests across buyers are independent. After seeing own test's outcome, each buyer decides whether to make offer to the seller, and if yes, by how much she would like to pay for the object. The seller then picks up the offer with the highest bid provided that it is no smaller than his reservation price. If multiple buyers bid the highest price, then the seller just randomly chooses one with equal probability. When trade happens, the selected buyer pays her bid.

Armed with the assumption that information cost is posterior-separable, I first show that buyers always have best responses where they use binary-signal exams regardless of what other buyers do. This paper exclusively focus on equilibrium supported by binary-signal tests as this is the most natural equilibrium to start with. Moreover, in any such equilibrium, buyers will always reject the seller following one of the outcomes (henceforth I call the outcome Fail) and bid for the seller following the other outcome (henceforth I can the outcome Pass). The screening choice of each buyer is thus boiled down to a two dimensional vector with each element indicating the probability of passing the exam for each type. With other standard assumptions on information costs, I then characterizes the screening frontier. Ideally, buyers would like to pass good sellers with probability one and bad sellers with probability zero, but due to the capacity constraint this ideal exam is unfeasible. Instead, each buyer has to trade-off on the difficulty of exams. By making exams easier, good seller has higher passing rate, but so does bad seller.

The paper has two major results regarding the equilibrium. First, it characterizes symmetric equilibrium and proves its existence. Buyers' strategy can be thought as choosing an exam and *planning* a bid once Pass realizes from the exam. It turns out that symmetric equilibrium must be supported by mixed strategies. Buyers randomize both their exams and their bidding choices, but given the price planned to bid, a unique exam maximizes buyers' utility. Thus the randomization can be summarized by the distribution of bidding plans. The paper shows that there exists equilibrium where equilibrium screening choice, as a function of bidding plans, can be fully characterized by a first-order ordinary differential equation, while the density function of bidding plans can be solved analytically.[1]

Second, the paper shows that any symmetric equilibrium is ex ante inefficient. Exams conducted in market equilibrium on average are too *hard* relative to the constrained efficiency benchmark. Specifically, conditional on trade does not happen, sellers on average have better quality in market solution than in social planner's solution. The inefficiency is rooted in the competition over potential jointly-offered sellers. A social planner does not care how many offers a seller gets. Rather, she only cares whether the seller has offers since as long as the seller has one offer the trade must happen. Therefore, when dictating exams for buyers, the social planner only considers how a further easing of exams may turn the seller from no offer to having offers. On the contrary, buyers in the market do care how many offers a seller gets especially when they decide to make offers. This is because being jointly offered per se indicates high quality, and hence buyers have strong incentive to win the competition if many other buyers are also making offers. To that end, buyers have intention to increase their bids when they choose to offer. However, since buyers do not know each other's decision when they bid, a buyer who plans to bid high ex post has to harden her exam ex ante to control the loss if it turns out that she is the only, or one of the few, offer-maker(s). In equilibrium, those hardened exams then lead the market to over-reject the seller on average.

In order to further show how the role of competition over potential cross-admissions leads to inefficiency, the paper considers three alternative scenarios. To begin with, if there is only one buyer in the market, then market equilibrium achieves social optimum. As no other buyers exist, cross-admission is void. Second, if all buyers mistakenly believe that they can get the trade only if all other buyers reject the seller, then market equilibrium is efficient. This is because in this hypothetical game, each buyer only thinks how the seller may be turned from having no offers to one when designing the exam, which matches exactly with the social planner's consideration. Lastly, if the seller does not select offers based on prices

1. The paper does not claim uniqueness of symmetric equilibrium, but does provide a set of sufficient and necessary conditions for equilibrium.

but on his own private preference over buyers, then exams in market equilibrium will be too *easy*. Obviously, in this scenario no buyers would ever bid prices larger than seller's reservation price. Since price no longer serves as the tool to compete for potential cross-admissions, the design of exams per se will take that role. Particularly, if all other buyers are following social planner's suggestion, one specific buyer can gain from marginally easing her exam. This deviation increases the probability of offering and hence getting a seller that other buyers are also offering, whose benefit is not considered by the social planner at all.[2] Consequentially, easier exams prevail in equilibrium for this scenario.

Market inefficiency calls for policy intervention. In this model, a price regulation that forces every offer maker to pay a regulated price may help restore efficiency. This regulation on one hand bans price competition; on the other hand it increases buyers' costs on having the object and hence lowers their willingness to pass the seller. By cleverly choosing the regulated price, it can reduce buyers' passing rate by the right amount and overcome the "too easy" problem in the no price competition scenario. In contrast, any price regulation causes inefficiency if there is only one buyer, as market equilibrium per se achieves efficiency for that case.

This paper is closely related to the study of common value auctions with ex ante information acquisition. Similar to this paper, Broecker (1990) considers an environment where multiple buyers screen a binary-type seller. However, unlike the environment considered here, each seller's exam is exogenously specified in Broecker (1990). Hausch and Li (1993), Ruckes (2004), Dell'Ariccia and Marquez (2006) among others allow buyers choose tests ex ante. But they restrict the choice to an *ad hoc* set. In contrast, the choice set of this paper is derived from standard assumptions on information costs. The design of exams is fully *flexible*. Furthermore, unlike those papers, this work holds exam accuracy fixed but emphasizes

2. Clearly, this deviation also increases the probability of offering and getting a seller the no other buyers are offering. But this marginal trade-off has been considered by the social planner and is hence balanced at social planner's solution.

the trade-off of exam difficulty: in order to increase the passing rate of good types, screeners have to tolerate high passing rate of bad types too.

This paper also contributes to the literature on agents' strategic interaction with information costs (e.g. Matějka and McKay (2012), Yang (2015), Matějka (2016), Yang (2020)). Among those works, Yang (2020) considers a similar setup where one seller wants to sell an object to one buyer. There, the object's value is represented by a continuous random variable and the seller and the buyer are allowed to contract ex ante on the realizations of the random variable. Yang (2020) then studies what contract the seller will propose given that the buyer has some screening capacity on the object's value. In contrast, in this paper, possible realizations are binary but there are multiple buyers. The ex post realization of the object's value is not contractable so the seller has to sell the full cash flow right in the trade. Buyers screen the object and propose prices that they would like to pay based on their screening results.

The remaining part of the paper is organized as follow. Section 1.2 lays out the basic environment and characterizes the screening frontier. Section 1.3 solves market equilibrium and compares that to the social planner's benchmark. Section 1.4 discusses the driving force of inefficiency. Section 1.5 studies optimal policy intervention. Section 1.6 concludes. All proofs are given in Appendix A.1.

1.2 The Environment and Preliminary Analysis

A seller is endowed with an indivisible object that values R to the seller. R is common knowledge. $N > 1$ risk neutral potential buyers present in the market and can choose whether to bid for the object. The object has common valuation X to all potential buyers, where X is a random variable that has support $\mathbb{X} \equiv \{x_B, x_G\}$ with $x_B < x_G$ and prior $\mu \equiv \{\mu_B, \mu_G\}$. I assume that

$$\mathbb{E}^{\mu}[X] = R \tag{1.1}$$

Therefore, without screening, buyers have the same ex ante valuation on the object as the seller does. The gain of trade comes from the screening technology that buyers are endowed with. As long as the screening technology can distinguish good type from bad type to some extent, the object may value more to the buyer ex post. Note also that (1.1) also implies $x_B < R < x_G$.

No buyers know X ex ante,[3] but each buyer can choose a test $\pi : \mathbb{X} \mapsto \Delta(\mathbb{S})$ to screen the object. Here, $\mathbb{S} \subseteq \mathbb{N}$ is a set of possible signal realizations. An implicit assumption embedded in the formulation of π is that conditional on $x \in \mathbb{X}$, buyers tests are mutually independent. With abuse of language, sometimes I also call buyers screeners, the seller the candidate, and a test an exam.

The cost of test π is denoted as $C(\pi)$. Each buyer is free to choose any tests from feasible set $\{\pi | C(\pi) \leq A\}$ for some exogenously given $A > 0$. For normalization, no-information tests are normalized to have costs zero. Hence, with $A > 0$ each buyer can distinguish the two types at least to some extent. On other other hand, it is assumed that fully revealing tests are very costly and unfeasible.

Each buyer performs her test simultaneously. After seeing own test's outcome, each buyer makes a decision on whether to bid for the object, and if yes, by how much amount. Without loss, bids are regulated to be at least R since any bids smaller than the seller's reservation price is equivalent to not bidding for the object. If a buyer does not get the object, she gets outside value 0.

On the other side of the market, the seller selects bidding offers. If the seller has only one offer, then the seller will take the offer and sells to the bidder. With multiple offers, the seller always picks up the offer with the highest bid. If more than one offers bid the highest price, the seller will randomly pick up one with equal probability.

Many real life examples fit this framework. For instance, one may read the seller as a

3. Whether the seller knows X is unimportant as it is assumed that the seller has no way to signal the object's quality even if she knows that.

worker who tries to sell his human capital to firms. R is the value that he can get from leisure and unemployment insurance. From firms' perspective, the candidate may either be a productive worker (generate more than R) or an unproductive one (generate less than R). In order to distinguish good from bad, firms simultaneously conduct job interviews on the candidate, and make decisions on whether they would like to hire him. If yes, firms offer wage to the candidate and the candidate would decide which offer to accept. One may also regard the seller as an inventor who is trying to sell his patent to users. The inventor has reservation price R. From users' perspective, implementing the new technology may either be positive NPV or negative NPV. Hence they screen the new technology beforehand. Users then make decisions on whether bid for the patent and if yes by how much amount.

Note that in the model buyers do not need to pay to acquire the screening technology. Instead, they are endowed with it. Also, buyers are not able to pay additional cost to enlarge the accuracy of their tests; rather, their capacity is bounded by a constant A. Thus, the key trade-off studied in this paper is not spending resources to increase the accuracy of the test, but the difficulty level of the test given a fixed accuracy. This trade-off will be made more clear later in Section 1.2.3.

Mathematically, each buyer's strategy space can be represented by

$$\mathcal{S} \equiv \left\{(\pi, \{p(s)\}_{s \in \text{supp}(\pi)}) | C(\pi) \leq A,\ p(s) \in \mathcal{P}\right\}$$

where $\mathcal{P} \equiv [R, x_G] \cup \{-\infty\}$. In particular, $p(s)$ represents the bid that the buyer would choose following signal realization s. $p = -\infty$ indicates that the buyer decides to rejects the seller. Obviously, bidding anything larger than x_G is obviously dominated. Bidding anything below R is equivalent to bidding $-\infty$.

If $s \in \mathbb{S}$ realizes from a buyer's experiment, the buyer then solves

$$V(\bar{\mu}_s) \equiv \max \left\{ 0, \quad \max_{p \in [R, x_G]} \mathbb{E}^{\bar{\mu}_s}[(X - p)\mathbb{I}_p | \sigma^-] \right\} \tag{1.2}$$

where $\bar{\mu}_s$ is the posterior after observing test outcome s, $\sigma^- \in \mathcal{S}^{N-1}$ is the strategy profile chosen by others, $\mathbb{I}_p : \mathcal{P}^{N-1} \times \Omega \mapsto \{0, 1\}$ is a random variable that takes value 1 if a trade happens between the seller and the buyer. Specifically, $\mathbb{I}_p$ maps all other buyers' bidding choice $p^- \in \mathcal{P}^{N-1}$ and $\omega \in \Omega$ to $\{0, 1\}$. If $p > \max\{p^-\}$ then $\mathbb{I}_p$ takes value 1; if $p < \max\{p^-\}$ then $\mathbb{I}_p$ takes value 0; if $p = \max\{p^-\}$ then $\mathbb{I}_p$ takes value 1 if and only if $\omega \in \Omega$ indicates that the seller will pick up the buyer's offer among those highest bids. To fix idea, one may regard ω as a permutation of all buyers. Each permutation has equal probability to realize, and given a permutation ω, the seller picks up the first buyer in the permutation that bid the highest price. Clearly, the mapping relationship from $\mathcal{P}^{N-1} \times \Omega$ to $\{0, 1\}$ is affected by the choice of p. On the other hand, other buyers' strategy profile σ^- will affect the probability distribution of $\mathbb{I}_p$ for any given p, as they affect how p^- may realize.

Given σ^-, it is useful to denote

$$G_\theta(p) \equiv \mathbb{P}\{\mathbb{I}_p = 1 | \sigma^-, X = x_\theta\} \tag{1.3}$$

In word, this is the probability of getting the candidate if a buyer decides to make offer and bid $p \in [R, x_G]$, conditional on the seller's true type is $\theta \in \{B, G\}$. Notice that this value is independent of a buyer's posterior, as it is already conditioned on the true type of the candidate. This definition will be used in later analysis.

Ex ante, each buyer solves

$$\max_{\pi} \mathbb{E}^{\bar{\mu}_s \sim \langle \pi \rangle}[V(\bar{\mu}_s)] \quad \text{s.t.} \quad C(\pi) \leq A \tag{1.4}$$

where $\langle \pi \rangle$ denotes the posterior distribution induced by exam π.

1.2.1 Preliminary Analysis of Buyers' Problem

This section conducts some preliminary analysis on buyers' problem. Results are general and do not depend on assumptions stated in Section 1.2.2.

Lemma 1. *Given any $\sigma^- \in \mathcal{S}^{N-1}$, if $\bar{\mu}_{s'}(x_G) > \bar{\mu}_s(x_G)$, then in problem (1.2)*

1. *it is never optimal to choose $p(s') = -\infty$ and $p(s) > -\infty$ unless*

$$G_B(p(s)) = G_G(p(s)) = 0$$

2. *it is never optimal to choose $-\infty < p(s') < p(s)$ unless*

$$G_B(p(s)) = G_B(p(s')) = 0$$

Lemma 1 describes two strictly dominated strategies. Both are quite intuitive. Item 1 says that if one has decided to bid on the candidate following a worse signal realization, then it is suboptimal to reject the candidate following a better signal realization.[4] Item 2 says that it is suboptimal to bid lower following a better signal realization but bid higher following a worse signal realization, unless by bidding the two prices the buyer can make sure that she can never get the candidate when the candidate is bad. As in equilibrium it is impossible for a buyer to fully screen out bad types by just bidding a price, we thus know,

Corollary 1. *In any equilibrium, it is never optimal for a player to choose $-\infty < p(s') < p(s)$ if $\bar{\mu}_{s'}(x_G) > \bar{\mu}_s(x_G)$ unless $G_B(p(s)) = G_G(p(s)) = G_B(p(s')) = G_G(p(s')) = 0$.*

4. The $G_B(p(s)) = G_G(p(s)) = 0$ clause indicates that the buyer effectively issues a rejection following the worse signal realization s

In a nutshell, in equilibrium, no buyers would bid strictly lower or simply reject the seller after seeing a better outcome unless she is effectively rejecting the seller following both signal realizations.

1.2.2 The Screening Technology

I now state assumptions on the cost function $C(\pi)$ made throughout the rest of the paper.

Assumption 1. $C(\pi)$ *is posterior-separable.*

Lemma 2. *Under Assumption 1, no matter what other buyers do, a specific buyer always has a best response where she conducts a binary test, i.e.,* $\#\text{supp}\{\Delta(\mathbb{S})\} = 2$*.*

The basic idea is that with Assumption 1, the problem can be solved via the concavification approach (Gentzkow and Kamenica (2014)). Since $\#\mathbb{X} = 2$, posterior can be summarized by one scaler. Thus, any point on the concavification can be delivered by a convex combination of at most two posteriors. On the other hand, since by construction (1.2) V must be convex, distinguishing the two types to some extent must be weakly better than acquiring no information as long as they are both feasible. Therefore, one best response must be binary.

Throughout the paper, I focus on symmetric equilibrium where each buyer generates binary signals $\mathbb{S} = \{0, 1\}$ (hereafter I call this type of equilibrium binary-signal equilibrium).[5] Without loss of generality, suppose outcome 1 indicates better quality of the object, i.e., $\bar{\mu}_1(x_G) > \bar{\mu}_0(x_G)$. Armed with that, we can redefine the test as a two-dimensional vector $\pi = (\pi_B, \pi_G) \in [0, 1]^2$ where π_θ represents the probability of type $\theta \in \{B, G\}$ generating outcome 1 in the test. Correspondingly, $1 - \pi_\theta$ is the probability of type $\theta \in \{B, G\}$ generating outcome 0. In this way, the cost function can be read as $C : [0, 1]^2 \mapsto \mathbb{R}_+$. It turns out that

5. As shown below in Section 1.3.2, such equilibrium always exists. Whether there exists other equilibrium is out of the scope of this paper.

Lemma 3. *If $C(\pi_B, \pi_G)$ is continuous in $[0,1]^2$, then under Assumption 1, in any binary-signal equilibrium, $p(0) = -\infty$ and $p(1) > -\infty$.*

The idea behind the lemma is simple. Since the distribution of posteriors must be a mean-preserving spread of μ, after observing outcome 0 the expected quality of X must be worse than $\mathbb{E}^\mu[X] = R$. Naturally, that corresponding signal realization leads to rejection. Similarly, following outcome 1, the posterior will be larger than R. It is then natural to bid for the object. With abuse of language, I also call outcome 0 Fail and outcome 1 Pass.

Next, I assume that

Assumption 2. *$C(\pi)$ respects the Blackwell order.*

With Assumption 2, we have

Lemma 4. *Under Assumption 2,*

1. *For any $\pi_B, \pi_G, \tilde{\pi}, \tilde{\pi}' \in [0,1]$ where $\pi_B \neq \pi_G$ and $\tilde{\pi} \neq \tilde{\pi}'$,*

$$C(\pi_B, \pi_G) > C(\tilde{\pi}, \tilde{\pi}) = C(\tilde{\pi}', \tilde{\pi}')$$

2. *For any $\pi_B, \pi'_B, \pi_G, \pi'_G \in [0,1]$ such that $\pi'_G > \pi_G \geq \pi_B > \pi'_B$,*

$$C(\pi_B, \pi'_G) > C(\pi_B, \pi_G) \quad \text{and} \quad C(\pi'_B, \pi_G) > C(\pi_B, \pi_G)$$

3. *For any $\pi_B, \pi_G \in [0,1]$,*

$$C(\pi_B, \pi_G) = C(1 - \pi_B, 1 - \pi_G)$$

If $\pi_B = \pi_G = \tilde{\pi}$, the screener is not screening at all but just randomly pass the candidate with probability $\tilde{\pi}$ regardless of his type. Item 1 of Lemma 4 says that the cost of the random-passing strategy is the same no matter what probability $\tilde{\pi}$ the screener chooses, and its cost is lower than any strategy that has some distinguish power of the two types. Given the normalization discussed above, all these random-passing strategies have costs zero. The second conclusion in Lemma 4 is also intuitive. It says that in a test where good type has higher passing rate than bad type, further increasing (decreasing) the passing rate of good (bad) type is costly. The third conclusion establishes a symmetry property. It basically says that if one reverses the two labels of possible outcomes, the cost of the test itself will be unchanged.

Lastly, I assume that

Assumption 3. $C(\pi_B, \pi_G)$ *is continuous and strictly convex in* $[0,1]^2$. *Moreover, it is third-order continuously differentiable in* $(0,1)^2$ *with* $\frac{-\partial C/\partial \pi_B}{\partial C/\partial \pi_G}|_{\pi_B \to 0} \to \infty$ *and* $\frac{-\partial C/\partial \pi_B}{\partial C/\partial \pi_G}|_{\pi_G \to 1} \to 0$

The convexity assumption made in Assumption 3 is natural. It implies that the feasible set of tests,

$$\Phi \equiv \left\{ (\pi_B, \pi_G) \in [0,1]^2 \middle| C(\pi_B, \pi_G) \leq A \right\} \tag{1.5}$$

is convex. This assumption ensures that if both tests π^1 and π^2 are feasible, then any convex combination of the two tests are feasible. The assumptions upon third-order continuous differentiability and strictness of convexity are made for technical simplicity. The last part of the assumption helps rule out corner solutions in the following analysis.

One can easily verify that there exists function C that satisfies all Assumptions 1-3, e.g., the widely used entropy reduction cost function.

1.2.3 The Feasible Set of Tests

I now characterize the feasible set Φ as defined in (1.5).

Proposition 1.

1. *For given $\pi_B \in [0,1]$, there exists unique $\overline{\phi}(\pi_B), \underline{\phi}(\pi_B) \in [0,1]$ with $\overline{\phi}(\pi_B) \geq \pi_B \geq \underline{\phi}(\pi_B)$ and the first inequality is strict as long as $\pi_B < 1$ while the second inequality is strict as long as $\pi_B > 0$, such that $(\pi_B, \pi_G) \in \Phi$ if and only if $\overline{\phi}(\pi_B) \geq \pi_G \geq \underline{\phi}(\pi_B)$.*

2. $1 - \overline{\phi}(\pi_B) = \underline{\phi}(1-\pi_B)$ *for all $\pi_B \in [0,1]$*

3. *There exists unique $\varpi \in (0,1)$ such that $C(\varpi, 1) = A$ and $\overline{\phi}(\pi_B) = 1$ for all $\pi_B \geq \varpi$. Moreover, $\overline{\phi}(\pi_B)$ is continuous in $[0,1]$, third-order continuously differentiable at all points in $(0, \varpi)$.*

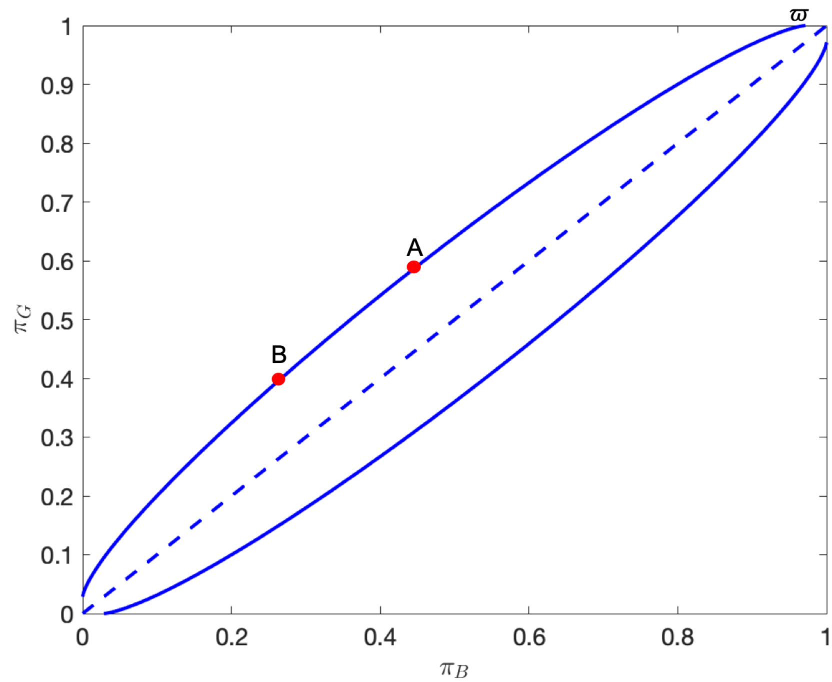
4. *$\overline{\phi}(\pi_B)$ is strictly increasing in $[0, \varpi]$. Moreover, $\lim_{\pi_B \to 0} \overline{\phi}'(\pi_B) = \infty$, $\lim_{\pi_B \to \varpi} \overline{\phi}'(\pi_B) = 0$.*

5. *$\overline{\phi}(\pi_B)$ is strictly concave in $[0, \varpi]$.*

Figure 1.1 draws an illustration of the feasible set. The horizontal axis depicts the passing rate of the bad type π_B; the vertical axis depicts the passing rate of the good type π_G. The dashed line draws the 45-degree line. By normalization, all points on the dashed line have cost zero, and should all be included in the feasible set. Therefore, the upper (lower) bound of the feasible set has to be everywhere above (below) the dashed line (item 1). Due to the continuity assumption of C, the bounds are also continuous (item 3). Due to the symmetry result in Lemma 4, the upper bound and lower bound is symmetric about $(0.5, 0.5)$ (item 2).

The upper bound has to be concave (item 5), otherwise the feasible set cannot be convex (Assumption 3). The upper bound has to be increasing (item 4). If part of the upper bound is decreasing, there must be a following part where the bound is increasing, otherwise it's

Figure 1.1: Feasible Set of Tests



going to cross the dashed line. But the first decreasing then increasing feature contributes to a convexity on the upper bound, which violates Assumption 3. By symmetry, the lower bound must be increasing and convex.

For obvious reason, every screener would like to lower the passing rate of bad type and increase the passing rate of good type. Thus, it is easy to prove that when designing the exam, it is never optimal to choose points off the upper frontier $\overline{\phi}(\pi_B)$. It is also suboptimal to choose any point with $\pi_B > \varpi$ since $(\varpi, 1)$ must be better. Therefore, in what follows, I impose the principle that $\pi_B \in [0, \varpi]$ and $\pi_G = \overline{\phi}(\pi_B)$.

Due to the increasing feature of the upper frontier, the key trade-off faced by each screener is that in order to increase the passing rate of good type, one has to tolerant a higher passing rate of bad type. For two tests on the upper frontier A and B as in Figure 1.1, I say A is an easier test than B since in exam A both good and bad types have higher passing rate. Armed with that terminology, the key trade-off studied by this paper can also be expressed as the trade-off on exam difficulty. Note that along the upper frontier, all exams have the same accuracy since all of them have the same screening cost A.

I now discuss an important implication of the concavity of the frontier:

Lemma 5. *$\frac{\overline{\phi}(\pi_B)}{\pi_B}$ and $\frac{1-\overline{\phi}(\pi_B)}{1-\pi_B}$ are decreasing in $\pi_B \in (0, \varpi)$.*

The decreasing property of $\frac{\overline{\phi}(\pi_B)}{\pi_B}$ can be easily seen from the figure. $\frac{\overline{\phi}(\pi_B)}{\pi_B}$ is just the slope of the segment from origin to the point on the frontier. As one moves up along the frontier, the slope is getting smaller. Clearly, this is driven by the concavity of the frontier. The decreasing property of $\frac{1-\overline{\phi}(\pi_B)}{1-\pi_B}$ can be easily seen due to symmetry.

Note that conditional on passing the exam, Bayes' rule implies that the ex post likelihood ratio of being good over bad is $\frac{\mu_G}{\mu_B}\frac{\overline{\phi}(\pi_B)}{\pi_B}$. According to Lemma 5, if the test becomes easier (i.e., π_B becomes larger), then conditional on passing the exam, the probability of being good will be smaller. This is quite intuitive: passing an easy exam does not mean too much on one's quality, but passing a hard exam is a strong signal of being good. On the other hand, conditional on failing the exam, the likelihood ratio of being good over bad is $\frac{\mu_G}{\mu_B}\frac{1-\overline{\phi}(\pi_B)}{1-\pi_B}$. Lemma 5 then says that failing an easy exam is really a strong indicator of being bad, while failing a hard exam does not mean too much of badness.

1.3 Solution

In this section, I solve the model. In particular, I start by characterizing and solving the problem faced by a constrained social planner, and then compare its solution to market equilibrium.

1.3.1 Social Planner's Problem

A social planner is allowed to dictate exam difficulty for each buyer but has to respect each buyer's information constraint. On top of that, I also assume that the social planner faces a "no discrimination" constraint, in the sense that she has to dictate the same exam to all buyers. This assumption is made so that we can easily compare social planner's solution to *symmetric* market equilibrium. If one lifts this restriction, the social planner might or might not be able to do better depending on the functional form of $\underline{\phi}$ induced by C.[6] Nevertheless, even in the former case, the solution to the restricted problem still serves as a lower bound of efficiency.

The social planner maximizes total welfare of the society. Her problem is

$$\max_{\pi_B \in [0,\varpi], \pi_G = \overline{\phi}(\pi_B)} \mu_B \left(1 - (1-\pi_B)^N\right)(x_B - R) + \mu_G \left(1 - (1-\pi_G)^N\right)(x_G - R) + R \tag{1.6}$$

Note that from the social planner's perspective, as long as the seller gets one offer, the trade will happen. If the seller's object is bad, the probability for the seller to have at least one offer is $1 - (1-\pi_B)^N$. When trade happens, no matter at what price level, the total social welfare increases by $x_B - R$ (a negative number). Similarly, if the seller's object is good, the probability for the seller to have at least one offer is $1-(1-\pi_G)^N$. When trade happens, the total social welfare increases by $x_G - R$. If trade does not happen, the total social welfare just comes from the seller's reservation price R.

The following proposition characterizes the solution to the social planner's problem:

Proposition 2. *The social planner's problem has unique solution, and the solution* (π_B^{**}, π_G^{**}) *is pinned down by*

$$\frac{(R - x_B)\mu_B}{(x_G - R)\mu_G} \frac{(1-\pi_B^{**})^{N-1}}{(1-\pi_G^{**})^{N-1}} = \overline{\phi}'(\pi_B^{**}) \tag{1.7}$$

and $\pi_G^{**} = \overline{\phi}(x_B^{**})$.

6. For instance, either of the following two conditions is sufficient to imply that the social planner would choose same exams for all buyers even if she is allowed not to do so: 1) $\frac{\underline{\phi}'(q)q}{\underline{\phi}(q)}$ is monotone in $q \in (1-\varpi, 1)$; 2) $\underline{\phi}(\sqrt{q_1 q_2}) \leq \sqrt{\underline{\phi}(q_1)\underline{\phi}(q_2)} \quad \forall q_1, q_2 \in (1-\varpi, 1)$.

Equation (1.7) is just the first order condition of the social planner's problem. The proposition verifies its sufficiency. To understand the first order condition, note that on the margin a social planner only cares how an further easing of exams may change the status of the seller from having zero offer to having at least one, since one offer is enough for trade to happen. An ε increase of π_G of one screener affects a good seller's status only if all other buyers have rejected him, which happens with probability $(1-\pi_G^{**})^{N-1}$. As the social planner treats all buyers equally, the total marginal benefit of ε increase of π_G is $(x_G - R)\mu_G N(1-\pi_G^{**})^{N-1}\varepsilon$. On the other hand, since the social planner has to trade-off on the screening frontier, an ε increase of π_G brings an $\varepsilon/\overline{\phi}'(\pi_B^{**})$ increase of π_B, which has total marginal cost $(R - x_B)\mu_B N(1-\pi_B^{**})^{N-1}\varepsilon/\overline{\phi}'(\pi_B^{**})$. At solution, the marginal benefit and marginal cost are balanced.

1.3.2 Market Equilibrium

I now turn to solve the market equilibrium. With the focus on binary-signal symmetric equilibrium, the strategy space of each player can be reinterpreted as choosing a two-dimensional vector $\pi = (\pi_B, \pi_G) \in \Phi$ for screening and *planning* a scalar $p \in [R, x_G]$ for bidding once Pass realizes from the exam. In equilibrium, a specific buyer taking others' strategy profile σ^- as given and solves

$$\max_{\pi, p\in[R,x_G]} \; \mathbb{E}\Big[\mathbb{J}_{\pi,p}(X-p)|\sigma^-\Big] \quad \text{s.t.} \quad C(\pi) \le A \tag{1.8}$$

This problem combines (1.2) and (1.4) together. Here, $\mathbb{J}_{\pi,p}$ is an indicator random variable. It maps $\mathbb{S}\times\mathcal{P}^{N-1}\times\Omega$ to $\{0,1\}$. Specifically, if the buyer sees Fail $\in \mathbb{S}$, then $\mathbb{J}_{\pi,p}$ takes value 0. If the buyer sees Pass $\in \mathbb{S}$, and $p > \max\{p^-\}$ for $p^- \in \mathcal{P}^{N-1}$, then $\mathbb{J}_{\pi,p}$ takes value 1; if $p < \max\{p^-\}$, then $\mathbb{J}_{\pi,p}$ takes value 0. Lastly, if $p = \max\{p^-\}$, then $\mathbb{J}_{\pi,p}$ takes value 1 if and only if $\omega \in \Omega$ indicates that the seller will pick up the buyer's offer among those highest bids. Again, ω can be thought as a permutation of all buyers. Given a realization of ω, the seller chooses the first buyer with highest bids. Clearly, the mapping relation is affected by the choice of p. Given p, the distribution of $\mathbb{J}_{\pi,p}$ is affected by the choice of π as it affects the distribution of $s \in \mathbb{S}$.

Proposition 3.

1. *There is no pure strategy binary-signal symmetric equilibrium.*

2. *In any mixed strategy binary-signal symmetric equilibrium, the distribution of equilibrium bidding plan $p \in [R, x_G]$ following signal Pass has no mass point.*

To get a sense of why the first statement is true, suppose $(\tilde{\pi}_B, \tilde{\pi}_G, \tilde{p})$ is an equilibrium. Now, does each buyer earn positive ex ante profit by playing $(\tilde{\pi}_B, \tilde{\pi}_G, \tilde{p})$? If yes, standard Bertrand competition logic would suggest a deviation $(\tilde{\pi}_B, \tilde{\pi}_G, \tilde{p} + \varepsilon)$. It turns out that this must be a profitable deviation for some $\varepsilon > 0$. If buyers do not earn positive ex ante profit by playing $(\tilde{\pi}_B, \tilde{\pi}_G, \tilde{p})$, then consider deviation $(0, \overline{\phi}(0), \tilde{p})$. With this strategy, the buyer ensures the object is good whenever she observes Pass. As long as $\tilde{p} < x_G$, the deviator must earn positive ex ante profit. Hence, if this is not a profitable deviation, it must be the case that $\tilde{p} = x_G$. But this cannot happen in equilibrium because there is no way for buyers to fully distinguish good from bad. So by bidding $\tilde{p} = x_G$ following outcome Pass, buyers must earn negative profit on average, which is dominated by the strategy that always rejecting the seller.

Since there is no pure strategy symmetric equilibrium, it is natural to consider mixed strategy symmetric equilibrium. Particularly, denote $F(p)$ to be the cumulative density function of the marginal distribution of equilibrium bidding plans. To be clear, before the game starts every buyer chooses an exam and *plans* a price to bid when observing Pass from the exam. F is the distribution of the price they *plan* to bid, not the distribution of actual bidden prices along the equilibrium path. As a buyer who plan to bids p may conduct different exams than a buyer who plans to bid p', the two buyers may differ in the chance of observing Pass. Hence, the ex ante planned distribution may be different from the ex post bidden distribution.

The second claim in Proposition 3 says that F has to be continuous. The intuition for the result is similar to the one that explains why no pure strategy symmetric equilibrium exist. Suppose F does has a mass point $\tilde{p}$. Therefore, whoever bids $\tilde{p}$ has non-zero probability to encounter another offer maker who also bids $\tilde{p}$. If it was the case, why not just bid a price slightly higher than $\tilde{p}$? Standard Bertrand competition logic implies that this deviation must be profitable.

Note also that the support of F must be connected. If not, say the support of F is $[P_0, P_1] \cup [P_2, P_3]$ where $P_2 > P_1$, then why don't the buyer who plans to bid P_2 upon seeing Pass keep her screening choice unchanged but bid $P_1 + \varepsilon$ instead for some $0 < \varepsilon < P_2 - P_1$? We have established that P_2 must not be a mass point. By bidding P_2, the buyer can win the bid if and only if all other buyers reject the candidate or bid less than or equal to P_1; by bidding $P_1 + \varepsilon$, she gets exactly the same probability to win but can pay strictly less when she gets the object.

Therefore, we can denote the support of F to be $[P_{min}, P]$. Note that a screener who plans to bid P_{min} upon seeing Pass can get the candidate only if she is the only offer-maker. Therefore, there is no meaning for her to bid a price larger than R. Hence, we must have $P_{min} = R$. On the other hand, the screener who plans to bid P upon seeing Pass will get the candidate with probability one as long as she makes the offer. Obviously, $P \leq x_G$.

The above analysis characterizes the marginal distribution of bidding plans in equilibrium. Now let's consider the equilibrium distribution of screening strategy (π_B, π_G). It turns out that fixing p, (π_B, π_G) must be a singleton. To see so, recall that in (1.3) we have defined $G_\theta(p)$ to be the probability of getting the candidate if a buyer decides to make offer and bid p, conditional on the candidate's true type is $\theta \in \{B, G\}$, taking other buyers' strategy as given. Thus, the problem to a specific buyer can be represented as

$$\max_{\pi_B \in [0,\varpi], p \geq R} \mu_B \pi_B G_B(p)(x_B - p) + \mu_G \overline{\phi}(\pi_B) G_G(p)(x_G - p) \tag{1.9}$$

To understand the problem, note that with probability μ_θ the screener meets a type $\theta \in \{B, G\}$ seller. With probability π_θ the screener passes the seller. With probability $G_\theta(p)$ the seller chooses the screener's offer. The screener then gets $x_\theta - p$ units of profits.

Due to the strict concavity of $\overline{\phi}$, it is clear that for any given p, problem (1.9) has unique solution π_B and $\pi_G = \overline{\phi}(\pi_B)$. Denote the solution as $\pi_B^*(p)$ and $\pi_G^*(p)$ respectively. Since p is random, the screening strategy is also random. But fixing p, π_B^* and π_G^* are known. A mixed strategy binary-signal symmetric equilibrium can then be fully represented by a tuple $\{\pi_B^*(p), \pi_G^*(p), F(p)\}$.

Let $\bar{\pi}_B^*, \bar{\pi}_G^*$ be the average passing rates of bad type and good type respectively. Namely,

$$\bar{\pi}_B^* \equiv \int_R^P \pi_B^*(p) dF(p)$$
$$\bar{\pi}_G^* \equiv \int_R^P \pi_G^*(p) dF(p)$$

According to the definition of $G_\theta(p)$, it is easy to calculate that

$$G_B(p) = \left(1 - \bar{\pi}_B^* + \int_{\tilde{p}=R}^{p} \pi_B^*(\tilde{p})\, dF(\tilde{p})\right)^{N-1} \tag{1.10}$$

$$G_G(p) = \left(1 - \bar{\pi}_G^* + \int_{\tilde{p}=R}^{p} \pi_G^*(\tilde{p})\, dF(\tilde{p})\right)^{N-1} \tag{1.11}$$

Specifically, to get the candidate, the bidder has to beat all the other $N-1$ potential buyers. The term in the parenthesis calculates the chance of beating one other buyer: with chance $dF(\tilde{p})$, the other buyer plans to bid $\tilde{p}$ upon seeing Pass. If $p > \tilde{p}$, then the bidder must win over this other buyer. If $p < \tilde{p}$, then the bidder can only win over this other buyer if the other buyer gets Fail from her test. In total, the probability of winning over one other buyer is

$$\int_{\tilde{p}=R}^{p} 1 dF(\tilde{p}) + \int_{\tilde{p}=p}^{P} \left(1 - \pi_\theta^*(\tilde{p})\right) dF(\tilde{p}) = 1 - \bar{\pi}_\theta^* + \int_{\tilde{p}=R}^{p} \pi_\theta^*(\tilde{p})\, dF(\tilde{p})$$

The first order condition of problem (1.9) implies that

$$\frac{(p - x_B)\mu_B}{(x_G - p)\mu_G} \frac{G_B(p)}{G_G(p)} = \overline{\phi}'\left(\pi_B^*(p)\right) \tag{1.12}$$

$$\pi_G^*(p) = \overline{\phi}\left(\pi_B^*(p)\right) \tag{1.13}$$

Moreover, in equilibrium, each buyer should be indifferent in randomizing between $p \in [R, P]$. So we know that in equilibrium

$$\mu_B \pi_B^*(p) G_B(p)(x_B - p) + \mu_G \pi_G^*(p) G_G(p)(x_G - p) = Y \tag{1.14}$$

for all $p \in [R, P]$, where $Y \geq 0$ is some constant. Note that here Y must be non-negative since rejecting the candidate for sure is always feasible. In that way, a buyer can get zero.

It is easy to verify that no screener would deviate to bid an even higher price than P.

This is because the only benefit for bidding higher is to increase the probability of getting the candidate when an offer is made. But with P, the screener can get the candidate with probability one as long as she makes offer. So there is no need to bid any price larger than P.

In summary, the equilibrium is characterized by P ,Y, $F(p)$, $G_B(p)$, $G_G(p)$, $\pi_B^*(p)$, $\pi_G^*(p)$ such that $G_B(p)$, $G_G(p)$, $\pi_B^*(p)$, $\pi_G^*(p), F(p)$ satisfy (1.10)-(1.14). Moreover, since F is a cumulative density function, it has to satisfy regulations

$$F(R) = 0; \qquad F(P) = 1; \qquad F(p) \text{ is increasing in } p \tag{1.15}$$

Lastly, P, Y must satisfy

$$x_G \geq P \geq R \tag{1.16}$$

$$Y \geq 0 \tag{1.17}$$

Along the discussion, one can see that conditions (1.10)-(1.17) are both sufficient and necessary for equilibrium.

Theorem 1. *There always exists a mixed strategy binary-signal symmetric equilibrium.*

Along the proof of Theorem 1, one can see that not only there always exists a mixed strategy symmetric equilibrium, but also there always exists one equilibrium such that F has continuous probability density function f. Moreover, in that equilibrium, $\pi_B^*(p)$ satisfies first order ODE:

$$\frac{\pi_B^*(p)}{\overline{\phi}\left(\pi_B^*(p)\right)} = \frac{\frac{d}{dp}\left[\left(\overline{\phi}'\left(\pi_B^*(p)\right)\frac{1}{\varphi\left(\pi_B^*(p)\right)}\frac{1}{(p-x_B)\mu_B}\right)^{\frac{1}{N-1}}\right]}{\frac{d}{dp}\left[\left(\frac{1}{\varphi\left(\pi_B^*(p)\right)}\frac{1}{(x_G-p)\mu_G}\right)^{\frac{1}{N-1}}\right]} \tag{1.18}$$

with boundary condition

$$\frac{P - x_B}{x_G - P}\frac{\mu_B}{\mu_G} = \overline{\phi}'\big(\pi_B(P)\big) \tag{1.19}$$

while the equilibrium $f(p)$ satisfies

$$f(p) = \frac{1}{\overline{\phi}\big(\pi_B^*(p)\big)}\frac{d}{dp}\left[\left(\frac{Y}{\varphi\big(\pi_B^*(p)\big)}\frac{1}{(x_G - p)\mu_G}\right)^{\frac{1}{N-1}}\right] \tag{1.20}$$

where $\varphi(u) \equiv \overline{\phi}(u) - \overline{\phi}'(u)u$. I use these equations to solve the following numerical example.[7]

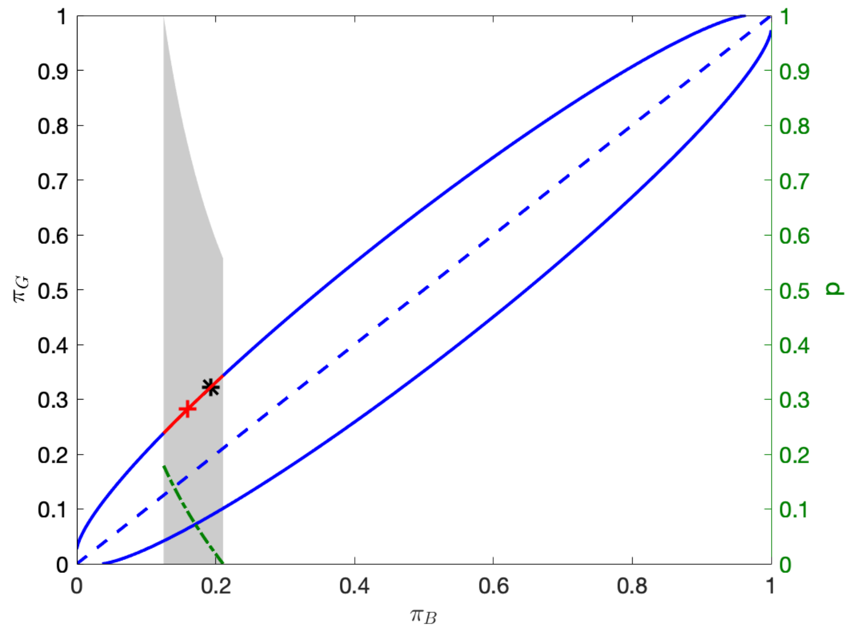
A numerical example

In order to show what the equilibrium looks like, I construct a numerical example. Suppose $\mathbb{X} = \{-1, 2\}$. For application, one may read this as a patent that can generate 3 units profit if it succeeds or nothing otherwise, but the firm needs 1 unit investment to implement that. The inventor of the patent holds reservation price $R = 0$. The prior on $\mathbb{X}$ is $\mu = \{2/3, 1/3\}$. Suppose $N = 2$, so two potential buyers are interested in the patent. The screening cost function is assumed to be the entropy reduction function as in standard literature. Lastly, I let $A = 0.01$. In this way the feasible set is the one plotted in Figure 1.1.

Figure 1.2 illustrates the result. In the equilibrium, screeners are randomizing in screening choices between $(\pi_B = 0.125, \pi_G = 0.238)$ and $(\pi_B = 0.211, \pi_G = 0.343)$ as shown by the red line in the figure. The dash-dot green line plots the bidding plan that pairs with each chosen screening test, ranging from $R = 0$ to $P = 0.179$. The shaded area plots the equilibrium density for each choice of (π_B, π_G). The red cross depicts the market equilibrium average passing rate $\big(\bar{\pi}_B^* = 0.160, \bar{\pi}_G^* = 0.282\big)$. Comparing to the constrained social plan-

7. The numerical procedure goes as follow: 1) Guess P. 2) Use (1.19) to solve $\pi_B(P)$. 3) Use P and $\pi_B(P)$ as boundary condition to solve ODE (1.18). 4) Solve $f(p)$ from (1.20). 5) Verify the guess in step 1) by $\int_R^P f(p)dp = 1$.

Figure 1.2: An Illustration of Mixed Strategy Binary-Signal Symmetric Equilibrium



ner's solution marked by the black star ($\pi_B^* = 0.193, \pi_G^* = 0.322$), the market equilibrium's average passing rate is lower for both types.

Inefficiency

Theorem 2.

1. *Any binary-signal symmetric equilibrium is ex ante inefficient.*

2. *In any binary-signal symmetric equilibrium,* $\frac{1-\bar{\pi}_G^*}{1-\bar{\pi}_B^*} > \frac{1-\pi_G^{**}}{1-\pi_B^{**}}$

To understand the first result, note that the ex ante welfare brought by the equilibrium is

$$\mu_B\left(1-(1-\bar{\pi}_B^*)^N\right)(x_B-R)+\mu_G\left(1-\left(1-\bar{\pi}_G^*\right)^N\right)(x_G-R)+R$$

To be clear, note that with probability $\left(1-(1-\bar{\pi}_\theta^*)^N\right)$, the trade will happen if the seller's type is θ. Whenever it happens, the total social welfare is increased by $x_\theta - R$. If trade does not happen, then the total welfare is just R. Due to Jensen's inequality and $\overline{\phi}$'s strict concavity:

$$\bar{\pi}_G^* < \overline{\phi}(\bar{\pi}_B^*)$$

Hence, the point $(\bar{\pi}_B^*, \bar{\pi}_G^*)$ does not sit on the upper frontier of the feasible set Φ at all. Thus, the ex ante total welfare must be smaller than the one maximized by (π_B^{**}, π_G^{**}).

The second result of the theorem characterizes the sign of the inefficiency. Conditional on the trade does *not* happen, the likelihood ratio of being good over bad induced by the social planner's solution is $\frac{\mu_G}{\mu_B}\frac{(1-\pi_G^{**})^N}{(1-\pi_B^{**})^N}$. On the other hand, the likelihood ratio induced by the market equilibrium $\frac{\mu_G}{\mu_B}\frac{(1-\bar{\pi}_G^*)^N}{(1-\bar{\pi}_B^*)^N}$. According to item 2 of the theorem, the latter is larger than the former. Hence, if one looks at the group of sellers being rejected by the whole society, the leftovers in market equilibrium must have better quality. In that sense, market's exams are too hard compared to social planner's benchmark.

The economic rationale behind the inefficiency is rooted in the competition for jointly offered candidates. Since being joint-offered per se is a strong signal for high quality, buyers would have incentive to increase bidding price to compete for cross-admitted candidate. However, when a buyer makes offer and chooses her bid, she does not know if the candidate truly has many other offers. Thus, a buyer who plans to bid high has to harden her exam ex ante to make sure that she controls the loss once she is the only, or one of the few, offer-maker(s). In equilibrium, this leads the market to over-reject on average.

1.4 Discussion

In order to further show how price competition over potential cross-admitted candidates leads to the inefficiency, in this section I consider three alternative environments: 1) only one screener exists in the market; 2) screeners mistakenly believe that they may only get the candidate when all other screeners reject him; 3) the seller does not select offers according to bids but according to his private preference over offer-makers. As can be seen below, screeners never worry about cross-admission at all in the first two situations, and thus market equilibrium achieves social optimum. In the third scenario, however, buyers still want to compete on potential jointly-offered candidates. But since price cannot be used as the tool for competition, buyers would do so simply by making more offers. Hence, in that case the market over-admits candidates relative to the efficiency benchmark.

1.4.1 One Screener

If there is only one potential buyer in the market, then whenever she makes offer to the seller, she would only bid the seller's reservation price R. The buyer's problem can be summarized as

$$\max_{\pi_B \in [0,\varpi]} \; \mu_B \pi_B (x_B - R) + \mu_G \overline{\phi}(\pi_B)(x_G - R) \tag{1.21}$$

Given the concavity of function $\overline{\phi}$, the first order condition will be sufficient and necessary. The solution π_B^* is pinned down by

$$\frac{\mu_B}{\mu_G} \frac{R - x_B}{x_G - R} = \overline{\phi}'(\pi_B^*) \tag{1.22}$$

On the other hand, vis-a-vis (1.6), in this case the social planner solves

$$\max_{\pi_B \in [0,\varpi]} \; \mu_B \pi_B (x_B - R) + \mu_G \overline{\phi}(\pi_B)(x_G - R) + R \tag{1.23}$$

Obviously, the social planner's maximization problem (1.23) is mathematically equivalent to (1.21). Hence, the market equilibrium reaches constrained efficiency in the one screener case.

Intuitively, as the driving force of inefficiency is the competition over potential jointly-offered candidates, this should not be a problem at all when only one screener exists in the market.

1.4.2 Multiple Screeners but with Mistaken Beliefs

Next consider a hypothetical game where all buyers mistakenly believe that she can get the seller only when she is the only offer-maker. Obviously, with this belief, no buyers would bid higher than R when she makes offer to the seller. Denote $(\hat{\pi}_B, \hat{\pi}_G)$ as the equilibrium of the hypothetical game, then in this equilibrium, each buyer would solve

$$\max_{\pi_B \in [0,\varpi]} \mu_B \pi_B (1-\hat{\pi}_B)^{N-1}(x_B - R) + \mu_G \overline{\phi}(\pi_B)\,(1-\hat{\pi}_G)^{N-1}\,(x_G - R) \tag{1.24}$$

To understand the expression, note that with probability μ_θ the buyer encounters a type $\theta \in \{B, G\}$ seller, with probability π_θ the buyer makes offer, with probability $(1-\hat{\pi}_\theta)^{N-1}$ she is the only offer-maker, and she believes only in this case she can get the trade and earn $x_\theta - R$. Take the first order condition of the this problem, we get

$$\frac{(R-x_B)\mu_B}{(x_G-R)\mu_G}\frac{(1-\hat{\pi}_B)^{N-1}}{(1-\hat{\pi}_G)^{N-1}} = \overline{\phi}'(\hat{\pi}_B)$$

Compared with (1.7), we immediately conclude that $\hat{\pi}_B = \pi_B^{**}$ and $\hat{\pi}_G = \pi_G^{**}$. That is, the market equilibrium of this hypothetical game reaches constrained efficiency.

Since buyers believe that they have no chance to get the seller once other buyers also make offers, they have no incentive to increase price to poach potential jointly-offered candidates at all. In turn, they only consider how a seller may be turned from having zero offer to one offer when choosing their exams' difficulty, which is exactly what the social planner thinks about. Consequentially, buyers' first order condition coincides with the social planner's one, and the market thus achieves optimum.

1.4.3 Multiple Screeners but without Price Competition

Lastly, let's consider a scenario where it is common knowledge that the seller would select offers purely based on his private preference ω over offer-makers as long as they bid at least R. From buyers' perspective, this just means that each offer may be selected with equal probability regardless the bidding amount. Clearly, since price has no role in this case, no buyers will bid prices higher than R. Suppose (π_B^*, π_G^*) is a symmetric market equilibrium for this case. Then in equilibrium, each buyer would solve

$$\begin{aligned}\max_{\pi_B\in[0,\varpi],\pi_G=\overline{\phi}(\pi_B)} &\sum_{\theta\in\{B,G\}} \mu_\theta(x_\theta - R)\pi_\theta \left(\sum_{k=0}^{N-1} \frac{1}{k+1}\binom{N-1}{k}(\pi_\theta^*)^k(1-\pi_\theta^*)^{N-1-k}\right) \\ &= \sum_{\theta\in\{B,G\}} \mu_\theta(x_\theta - R)\pi_\theta \frac{1-(1-\pi_\theta^*)^N}{N\pi_\theta^*}\end{aligned}$$

To understand the expression, note that with probability μ_θ the buyer encounters a type $\theta \in \{B, G\}$ buyer; with probability π_θ the buyer makes offer. In that case, with probability $\binom{N-1}{k}(\pi_\theta^*)^k(1-\pi_\theta^*)^{N-1-k}$ exactly k other buyers are also making offers; and the buyer of interest will have chance $\frac{1}{k+1}$ being selected. If the buyer finally gets the candidate, then she can earn $x_\theta - p$. Some simple calculation leads to the expression given in the second line.

Given the concavity of $\overline{\phi}$, the following first order condition is sufficient and necessary for equilibrium

$$\frac{(R-x_B)\mu_B}{(x_G-R)\mu_G}\frac{\frac{1-(1-\pi_B^*)^N}{N\pi_B^*}}{\frac{1-(1-\pi_G^*)^N}{N\pi_G^*}} = \overline{\phi}'(\pi_B^*) \tag{1.25}$$

It turns out that

Proposition 4. *Market equilibrium (π_B^*, π_G^*) exists. Moreover $\pi_B^* > \pi_B^{**}$ and $\pi_G^* > \pi_G^{**}$.*

Namely, in this case exams in market equilibrium are too *easy* relative to the efficiency benchmark.

To get the intuition, let's consider the simple case where $N = 2$ and call the two screeners A and B. Suppose A and B are both playing the strategy suggested by the social planner. From the social planner's perspective, if she increases A's bad type passing rate by ε, then with probability $(1-\pi_B^{**})\mu_B$, the increase will function on a bad type who does not have offer from B. When this happens, the candidate's status changes from no offer to having one offer. This will contribute a social cost $\varepsilon(1-\pi_B^{**})\mu_B(R-x_B)$. With probability $\pi_B^{**}\mu_B$, the increase will function on a bad type who already gets offer from B. In this case it will not contribute any new social cost to the social planner at all since in the absence of the deviation, trade still happens due to B's offer. Similarly, the marginal benefit brought by the corresponding increase $\varepsilon\overline{\phi}'(x_B^{**})$ in the good type's passing rate would be $\overline{\phi}'(x_B^{**})\varepsilon(1-\pi_G^{**})\mu_G(x_G-R)$. Since the social planner has to treat A and B equally, the total marginal benefits and marginal costs are doubled. At social planner's solution, the marginal benefit equals to the marginal cost.

Now consider market participant A's trade-off: by increasing the passing rate of bad types by ε, with probability $(1-\pi_B^{**})\mu_B$, the increase will change a bad type's status from no offer to having one offer. This will contribute a cost $\varepsilon(1-\pi_B^{**})\mu_B(R-x_B)$ to screener A. With probability $\pi_B^{**}\mu_B$, the increase will change a bad type's status from having one offer to having two offers. From A's perspective this still contributes a cost $\varepsilon\pi_B^{**}\mu_B(R-x_B)\frac{1}{2}$ as A has half chance to trade with the seller in this case. Similarly, the marginal benefit to A is also consisted by two parts: $\varepsilon\overline{\phi}'(x_B^{**})(1-\pi_G^{**})\mu_G(x_G-R)$ brought by the scenario that the seller does not have B's offer, and $\varepsilon\overline{\phi}'(x_B^{**})\pi_G^{**}\mu_G(x_G-R)\frac{1}{2}$ brought by the scenario that the seller does have B's offer. The first components of marginal costs and marginal benefits match exactly with social planner's marginal cost and marginal benefit. They are hence equalized at social planner's solution. The second components of marginal costs and marginal benefits, however, are out of social planner's radar. It turns out that for this part marginal benefit is always larger than marginal cost as

$$\begin{aligned}\frac{\varepsilon\overline{\phi}'(\pi_B^{**})\pi_G^{**}\mu_G(x_G-R)\frac{1}{2}}{\varepsilon\pi_B^{**}\mu_B(R-x_B)\frac{1}{2}} &= \overline{\phi}'(\pi_B^{**})\frac{\pi_G^{**}\mu_G(x_G-R)}{\pi_B^{**}\mu_B(R-x_B)} \\ &= \frac{(1-\pi_B^{**})\mu_B(R-x_B)}{(1-\pi_G^{**})\mu_G(x_G-R)}\frac{\pi_G^{**}\mu_G(x_G-R)}{\pi_B^{**}\mu_B(R-x_B)} \\ &= \frac{(1-\pi_B^{**})}{(1-\pi_G^{**})}\frac{\pi_G^{**}}{\pi_B^{**}} > 1\end{aligned}$$

where the second equality comes from (1.7) and the last inequality follows since $\pi_G^{**} > \pi_B^{**}$. Basically, if the marginal benefit and marginal cost just balance with each other for taking a candidate that is rejected by B, then the marginal benefit must outweigh the marginal cost for getting a candidate that is offered by B. This is just because candidates with B's offer must have better quality in expectation than candidates being rejected by B. Therefore, if B is conducting a test as suggested by the social planner, A would have incentive to deviate and pass more candidates to steal candidates with B's offer. Similarly, B also has incentive to make the exam easier. In result, the market equilibrium features higher admission rate than the social planner's solution.

In a nutshell, as price cannot be used as a tool to compete potential cross-admitted candidates in this case, the design of exams per se will take this role: buyers just make more offers than they should have to steal from each other.

1.5 Optimal Price Regulation

The inefficiency discussed in Section 1.3.2 causes the need for regulation. It turns out that a simple price regulation can restore efficiency:

Proposition 5. *There exists unique* $p^{**} \in (R, x_G)$ *such that if all offer makers are required to bid* p^{**}*, then the market equilibrium would achieve onstrained efficiency.*

Contrast to Section 1.3.2, price competition is banned in this regulation. But different from Section 1.4.3 too, bidders in this regulation are forced to pay a higher bid. The idea is simple. Banning price competition helps the market get rid of inefficiency studied in Section 1.3.2. By forcing buyers to pay more to the seller, the increased cost will incentivize buyers to harden their exams and help the market get out of the "too easy" trap when price competition is absent as studied in Section 1.4.3.

Note that the result is different for the one-screener case. As shown in Section 1.4.1, unregulated market per se already achieves efficiency in that scenario. Hence, any price regulation such that $p > R$ would wrongly disincentivize buyers to pass candidates. To see so, simply note that if one replaces R by p in (1.22), the market equilibrium would feature harder exams than the efficient benchmark.

1.6 Conclusion

This paper studies screening competition under flexible information acquisition and its interaction with price competition. Multiple homogeneous screeners play a game where they simultaneously design independent admission exams on binary-type candidates. After observing own exam outcome, each screener is allowed to bid a price to compete for the candidate if they want. The candidate chooses the offer with highest bid provided that it exceeds a pre-specified reservation price. If multiple offers bid the highest price, the candidate picks one with equal probability.

The information capacity of each screener's exam is exogenously given. Hence, the main focus of the paper is on the trade-off of the difficulty of exams. Specifically, due to the information capacity, a screener who wants to increase the admission rate of good types has also to tolerate a higher admission rate of bad types. The paper first characterizes the frontier of feasible exams under standard assumptions on the screening cost function, and then shows the existence of binary-signal symmetric equilibrium. In particular, equilibrium must be supported by mixed strategies. Due to Jensen's inequality, this means that the equilibrium is ex ante inefficient, as randomizing over exams makes the average screening choice off the feasible frontier.

The paper not only shows the inefficiency of the equilibrium, but also identifies the sign of the inefficiency. Particularly, compared with social planner's solution, candidates that are rejected by the whole market have better quality on average. The fundamental force that drives the inefficiency is the price competition over potential jointly offered candidates. As being jointly offered per se implies high quality, screeners have incentives to bid high prices to increase the chance of getting the candidate when he is cross admitted. However, since screeners do not know if a candidate truly has many other offers, one who plans to bid high upon admission has to harden her exam ex ante so that she can control the loss if she makes the only, or one of the few, offer(s). In equilibrium, those hardened exams lead the market over-reject candidates on average.

Market inefficiency calls for policy intervention. As the inefficiency is rooted in price competition, a natural way is to impose price regulation. However, when price competition is banned, the design of screening test per se may serve as a tool for competing cross-admits.

Screeners have the tendency to make more offers than they should have, hoping to steal candidates that others are offering. Therefore, a cleverly chosen price is needed to restore efficiency. The price has to be high enough so that it reduces screeners' general interests of getting the candidate, and thus they design their exams right to the desired difficulty level.

CHAPTER 2
THE INTERACTION OF BANKERS' ASSET AND LIABILITY MANAGEMENT WITH LIQUIDITY CONCERNS

2.1 Introduction

"When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you have got to get up and dance. We are still dancing."

—Chuck Prince, the then Citibank CEO, July 2007[1]

The 2007-9 Great Recession has attracted many scholars' attention to study the role of financial intermediaries in the macroeconomy (e.g. He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)). Yet it is a question that why bankers did not stop originating subprime loans when early signs of distresses had emerged prior to the crisis. The quotation above from the then Citibank CEO shows that bank managers were aware of potential liquidity pressure right before the crisis. Nevertheless, they chose to continue "dancing" on subprime assets. Eventually, those subprime assets caused large macroeconomic loss.

Behavior literature attributes this to over-confidence or distorted expectations of banking CEOs (e.g. Ma (2015)). This paper, instead, provides a rational framework to explain the phenomenon and further studies its macroeconomic implications qualitatively. The model is built on standard continuous-type signaling literature (e.g. Leland and Pyle (1977), Riley (1979), DeMarzo and Duffie (1999)), but have two major innovations. First, by introducing a macroeconomic environment, the gain of trade, which equals to the liquidity premium that bankers would like to pay for extra cash because of liquidity concerns, is endogenized in general equilibrium. Second, by providing a novel way of modeling assets screening practice with continuous types, the model emphasizes the ex ante effects of the ex post lemon market

1. See https://www.ft.com/content/80e2987a-2e50-11dc-821c-0000779fd2ac

on assets origination. Moreover, it also highlights how the ex ante origination may affect the gain of trade ex post, and hence the signaling game in equilibrium.

The model is consisted by a banking sector and a household sector. Bankers screen real production projects on the asset side and issue deposits on the liability side. The liquidity constraint stems from the time mismatch of assets payout and early withdrawals of deposits (Diamond and Dybvig (1983)). To fill the liquidity gap, bankers sell assets on a secondary market. The key driver of the paper's argument is that with the presence of asymmetric information in the secondary market, bankers have to retain large parts of good assets to signal their quality, whereas bad assets are easier to sell and generate more proceeds for bankers. Therefore, if bankers anticipate worse liquidity situations tomorrow, it is then rational for bankers to screen *less* carefully today. In general equilibrium, assets originated by bankers go into production. The quality of bankers' portfolio thus affects the economy's total productivity. This in turn influences households' labor income and hence their deposit withdrawal behavior, which then endogenizes the size of liquidity that bankers need to raise in the secondary lemon market.

The argument that bad assets are easier to sell is rooted in the literature about adverse selection. The basic idea is that in order to signal the good quality of an asset, informed sellers, who would have sold the whole asset in frictionless world, usually have to abstain from selling the asset in full. The abstention may take various forms: selling only fractions of the asset (Leland and Pyle (1977), DeMarzo and Duffie (1999), and this paper), selling with lower probability (Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014)), delay in the time of selling (Fuchs and Skrzypacz (2015)), etc. But the common observation is that bad assets can be sold without too much abstention. As assets are sold at fair prices in fully separating equilibrium, it turns out that bad assets are able to generate more cash from the secondary market. [2]

2. Several empirical works have shown evidences that the equilibria in various lemon markets are separating and bad assets are easier to sell. See Downing, Jaffee, and Wallace (2009), Begley and Purnanandam

Turning back to the model of this paper. In the model, a mass of long-lived homogeneous competitive bankers, financed by households' deposits, face infinitely many supplied production projects each period. Projects transfer consumption goods from today to tomorrow, but they differ in returns with commonly known ex ante continuous distribution. A banker can exert private screening effort, changing her production portfolio from the ex ante distribution to some other distribution by paying an entropy cost. In next period, before projects pay out proceeds in the afternoon, households may withdraw their deposits in the morning. In order to fulfill that, the banker sells fractions of projects to competitive buyers in the secondary market.

With the presence of asymmetric information in the secondary market,[3] bankers exert less screening effort than the no-friction benchmark. Good assets still carry more weights than bad ones in equilibrium screening outcome, but compared with the no-friction benchmark, more assets with low quality are originated. This is because the extra liquidity benefit brought by bad assets lowers the relative gains of having good ones. A banker who deviates and screens more carefully would end up with a better portfolio. But if she still follows the equilibrium selling strategy to get fair prices, the larger retention on those good assets would disable her from generating enough cash to fulfill her liquidity need. In order to survive, the deviating banker has to sell more fractions of her projects, which will be interpreted as signals for bad quality. The banker is then selling projects at discount and incurring losses. In equilibrium, this loss, together with the higher screening cost, keeps the banker away from deviation.

The projects originated by bankers then go into production. On the other side of the economy, a short-lived household sector inelastically supply labor and earn wage bills propor-

(2017), Adelino, Gerardi, and Hartman-Glaser (2019) for supporting evidence.

3. In this paper, it is assumed that the level of screening does not affect bankers' information advantage over buyers in the secondary market. This assumption is rooted in the monitoring practice conducted by bankers (Diamond (1984)).

tional to the total net output of bankers' production portfolio. Households consume and save for their offspring, but they differ in beliefs about bankers' liquidity situation. Households (whom I call runners) who worry about bankers' capability in fulfilling morning withdrawals run in the morning and call all their parents' bequests back. Households (whom I call non-runners) who are more optimistic just withdraw the amount they want to consume, and automatically rollover the remaining due deposits for their children. Runners in the afternoon may redeposit after they figure out that bankers have survived. An equilibrium deposit rate equalizes the supply of funds with bankers' demand. As deposits cause liquidity burdens, in equilibrium, bankers are compensated by a positive wedge between returns on assets and deposits for providing the deposit service.[4]

When put into a dynamic general equilibrium setting, the model has several implications of the macroeconomy. To begin with, a small aggregate productivity shock may be amplified into a large one. Specifically, suppose in one day, after the liability side has been set but before the origination of production projects, all agents in the economy suddenly learn that a negative aggregate productivity shock hits the economy. The aggregate shock lowers all projects' payoff tomorrow and hence households' labor income. Lower productivity and lower wages then tighten bankers' liquidity constraint via two channels. First, as they earn less wages, households rely more on parents' bequests for consumption. So non-runners would withdraw more. Second, as total productivity is lower, more households become runners as they start to worry about bankers' ability to survive. Knowing that liquidity constraint will be tightened tomorrow, bankers would twist their production portfolio toward bad assets today, as bad assets provide larger liquidity benefits as analyzed above. However, as more assets with low quality are originated, the total productivity tomorrow would be even lower. Households then earn even less labor income. Non-runners withdraw even more, and more households choose to run. This further disincentivizes bankers' screening and exacerbates

4. In the model, proceeds generated by assets are deterministic. Thus, the wedge between asset return and liability return is not due to any risk-return tradeoff.

the reduction in productivity, and thus forms an amplification loop.

The amplification mechanism may have broader applications within and beyond this specific model setup. The key insight is that shocks to bankers' liquidity may be transferred into shocks to real productivity. Within the model, one may consider an liquidity shock that stems from changes in households impatience level. Beyond the model, one may also consider situations where liquidity shocks are caused by alterations in the interbank market. In particular, if liquidity in the interbank market dries up so that bankers have to rely more on the outside secondary market for financing where the asymmetric information is more severe, bankers would then originate more assets with low quality due to their liquidity benefits, which in turn lowers the economy's total productivity.

For the same logic, upon a positive aggregate productivity shock bankers would screen more carefully. However, if the shock is permanent, then in the economy's new steady state bankers' lending standard will be laxer. This is because households accumulate more wealth along the trajectory thanks to the positive shock. As households' wealth sits on bankers' liability, the saving glut eventually burdens bankers' liquidity concern. Bankers respond to that by screening less carefully. If the shock is reversed amid the transition, then the larger-than-origin liability would induce lower-than-origin productivity upon the reversal. The economy thus falls into a bust, even though no negative shocks ever happen at all.

The above rationale is largely related to the growing macro-finance literature on how an economic boom may lead to bust (e.g. Boissay, Collard, and Smets (2016), Gorton and Ordonez (2019)). The key insight of this paper is that as long as banking sector's leverage becomes higher during the boom, then the extra liquidity burden brought by leverage may disincentivize screening and cause a bust. In this model, the higher leverage is rooted in the saving glut during booms. But more broadly, as long as banking sector's leverage is procyclical (Adrian and Shin (2010)), the mechanism applies.

Several other papers in the literature also highlight the ex ante effect of ex post lemon markets (Parlour and Plantin (2008), Chemla and Hennessy (2014), and Vanasco (2017)). This paper differs from the existing literature in two aspects. For one thing, in those papers bankers screen for one asset with binary type while in this paper bankers screen for a portfolio and types are continuous. This paper thus provides a novel framework in modeling the screening practice. More importantly, the gain of trade in the secondary market is exogenous in those papers, whereas in this paper it is endogenized due to the general equilibrium feature. It is the endogeneity that causes the amplification loop and the boom-to-bust dynamics highlighted above.

This paper also relates to the broad macro-finance literature on financial crises and adverse selection (e.g., Eisfeldt (2004), Kurlat (2013), Guerrieri and Shimer (2014), Gorton and Ordonez (2014), Kurlat (2016), Chang (2017), Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019)). This paper contributes to the literature by arguing that adverse selection on the secondary market distorts origination of production projects. The distortion then causes real macroeconomic consequences.

Some other recent works also feature endogenous screening choice with associated macroeconomic dynamics (e.g. Farboodi and Kondor (2020), Fishman, Parker, and Straub (2020)). In those papers, lenders' screening choice today affects borrowers' ex ante quality distribution tomorrow. The variation in the distribution of quality then feeds back into the choice of screening tests. In this work, however, the ex ante distribution of borrowers' quality never changes. It is the time-varying liquidity concerns that drive lenders to make different screening practice.

The remainder of the paper is organized as follow. Section 2.2 describes the model and defines equilibrium. Section 2.3 characterizes the equilibrium. Section 2.4 outlines the balance growth path and provides a numerical example. Section 2.5 discusses the amplification of productivity shocks. Section 2.6 studies the dynamics following productivity shocks. Section 2.7 concludes. All proofs are given in Appendix A.1.

2.2 Model

2.2.1 The Banking Sector

Time is discrete. Each period there are three sessions: morning, afternoon, and evening. A mass of homogeneous competitive bankers exist in the economy. Each banker has log utility with time discount factor $\beta < 1$. Bankers manage both their asset side and their liability side. On the asset side bankers screen production projects; on the liability side bankers issue deposits contracts. The timeline goes as follow: in the evening of each period t, bankers screen production projects and invest. Production projects generate proceeds in the afternoon of period $t+1$ to the banker, but in morning some due deposits are withdrawn. In order to fulfill the withdrawals, bankers sell parts of their assets in a secondary lemon market. In the afternoon of period $t+1$, bankers collect proceeds from retained fractions of projects, adjust liability, and consume. Then time moves to the evening of period $t+1$. I now describe each session in more detail.

The evening session and the screen technology

In the evening of each period t, each banker faces infinitely many supply of production projects. Production projects are the only real technology that transfer consumption goods from t to $t+1$, but they differ in quality. In particular, one unit of project x takes one unit of time t consumption good and generates x units of time $t+1$ consumption goods to the banker in the afternoon of period $t+1$. I assume that $x \in [\underline{x}, \bar{x}] \equiv X$ where $\underline{x} > 0$. The banker does not know the value of x of each project, but she knows that the ex ante distribution of x has probability density function $f(x)$, a common knowledge. I assume that $f(x)$ is strictly positive for all $x \in X$ and

$$\mathbb{E}[x] \equiv \int_{\underline{x}}^{\bar{x}} x f(x) dx \geq 1 \tag{2.1}$$

Hence, ex ante the average *net* output to bankers per unit investment is non-negative.[5]

A banker who wants to invest K units of consumption goods needs K units of projects. Instead of randomly choosing K projects, the banker is able to conduct some costly screening. I model the screening cost in a reduced form way. Specifically, if the banker does not screen at all, then the production portfolio the banker ends up with will be the same as indicated by the ex ante density $f(x)$. If the banker screens, then the banker is twisting her portfolio away from the ex ante density $f(x)$. Suppose the banker ends up with a portfolio where type x project has density $m(x)f(x)$, where m represents a screening outcome. Then, since portfolio weights must add up to one, we have

$$\int_{\underline{x}}^{\bar{x}} m(x)f(x)dx = 1 \tag{2.2}$$

Since the banker cannot short real production projects, we have

$$m(x) \geq 0 \quad \forall x \in X \tag{2.3}$$

Thus, m can be read as a Randon-Nikodym derivative with respect to density f. Hence, a natural way to model the screening cost is to quantify the divergence between the changed density $m(x)f(x)$ against the original density $f(x)$. In general, one can define a divergence measurement $\mathbb{E}\left[g\big(m(x)\big)\right]$ where $g : \mathbb{R}_+ \mapsto \mathbb{R}$ satisfies that

Assumption 4. *1) $g(1) = 0$. 2) g is twice continuously differentiable with $g'' > 0$. 3) $\lim_{m\to 0} g'(m) = -\infty$.*

Condition 1 ensures that if there is no screening at all, i.e. $m(x) = 1 \forall x$, then there is no cost. Condition 2 ensures that as long as there is some screening, the cost is positive due to Jensen's inequality. As shown later in Section 2.3.1, with Condition 3, $m(x) = 0$ will never

5. As detailed below, households' labor income is modeled to be proportional to bankers' net output. This assumption ensures that in equilibrium labor income is always non-negative.

be chosen for any x. That is, it is always too costly for a banker to fully kick out some project types in her production portfolio. Consequentially, Condition 3 implies the support of project returns, X, never changes after screening. This will simplify the analysis quite much for the signaling game in the secondary lemon market.

One can easily check that

$$g(m) = m \log m \tag{2.4}$$

satisfies all the three assumptions, which corresponds to the well-known relative entropy or Kullback-Leibler divergence .[6] I use this specification in the numerical example outlined in Section 2.4. But as shown below, the paper's results do not rely on a specific form of g, as long as Assumption 4 holds.

Two more assumptions are made regarding the screen costs. First, I assume that the screen cost is a real cost that reduces the gross return the banker earns on her asset side. More precisely, if the banker chooses screen outcome m_t in the evening of time t, then the banker's time $t+1$ gross return will be *reduced* by $\xi \mathbb{E}\left[g\big(m_t(x)\big)\right]$. Here, $\xi > 0$ is a parameter of the model that governs the size of screening costs. Second, I assume that individual banker's choice of $m_t(x)$ is unobservable by other agents in the model. This would be one of the key frictions that generate the results.

The morning session and the secondary lemon market

In the morning of period $t+1$, deposits issued at time t incur interests and due. Runners withdraw all their deposits, as they worry about banking failure. In contrast, non-runners are more optimistic and just withdraw the amount they want to consume. Their remaining deposits are automatically rolled over for next period.

Denote R^d_{t+1} as the interest rate of deposits from time t to $t+1$, which is determined at time t. Denote n_t to be the net worth (equity) of a representative banker in period t before

6. For another example of g that satisfies Assumption 4, one may consider $g(m) = -\log m$.

the evening session starts. Denote η_t to be the asset-to-equity ratio chosen by the banker in period t. Denote $\zeta_{t+1} \in [0,1]$ as the fraction of households who choose *not* to run in the morning of period $t+1$. Denote Δ_{t+1} as the withdrawal amount of non-runners per unit of deposits. If $\eta_t > 1$, i.e., if the banker takes leverage, then the total amount of withdrawals in the morning of $t+1$ will be

$$n_t(\eta_t - 1)(1 - \zeta_{t+1})R^d_{t+1} + n_t(\eta_t - 1)\zeta_{t+1}\Delta_{t+1} \tag{2.5}$$

ζ_{t+1} and Δ_{t+1} are decided by households' behavior as detailed in Section 2.2.2. Particularly, I assume that households' savings are well-diversified in the entire banking sector, so that they have no incentive to watch any individual banker's states. Their running and withdrawal behavior only depend on the aggregate states of the economy. That being said, for an individual banker, ζ_{t+1} and Δ_{t+1} are aggregate states that she has no control over. Hence, when making decisions in period t, the banker looks forward and forms rational expectations about ζ_{t+1} and Δ_{t+1} and treats them as given.

As long as the term in (2.5) is positive,[7] the banker has to sell part of her assets in a secondary market to fulfill it.[8] Due to bankers' monitoring practice overnight, it is common knowledge that the banker has full private information on each project's quality when she walks into the secondary market. For simplicity, I assume that the banker cannot commit to pool her assets together and hence has to sell her assets one by one. Furthermore, the banker

7. The term in (2.5) might be negative since that I allow Δ to be negative. Basically, if Δ is negative, it means that instead of withdrawing part of their deposits in the morning, non-runners are adding new deposits to their saving accounts. Just like bankers cannot decline early withdrawals, bankers cannot decline those new savings in the morning by assumption. Indeed, bankers would love to have new savings in the morning as it helps relieving the morning liquidity constraint and bankers do have chances to adjust liability in the afternoon if they really want to get rid of those deposits. However, the implicit assumption is that bankers cannot compete for extra deposits in the morning. This assumption is mainly for the tractability of the model. If bankers can compete for morning deposits, then the morning liquidity premium would also affect the deposit rate promised for next period.

8. Note that in this model, there is no other claims that the banker can hold across periods to hoard liquidity. Bankers are allowed to invest in deposits issued by other bankers. But as explained in Section 2.2.1, this would be modeled as a reduction in liability.

is only allowed to sell a fraction of each asset, and is not allowed to tranche and create new securities. Fractions of selling will be read as signals of quality in equilibrium. Specifically, if fraction α is being sold, buyers will form a belief about the asset's quality according to α, and pay $p(\alpha)$ per unit of the asset.

Denote $\alpha_{t+1}(x)$ to be the fraction that the banker sells for asset type x. The liquidity constraint that the banker faces in the morning of period $t+1$ can be summarized by

$$n_t(\eta_t - 1)(1 - \zeta_{t+1})R^d_{t+1} + n_t(\eta_t - 1)\zeta_{t+1}\Delta_{t+1} \leq \eta_t n_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x) p_{t+1}\left(\alpha_{t+1}(x)\right) m_t(x) f(x) dx \tag{2.6}$$

The left-hand-side (hereafter LHS) is the total withdrawal requests. To understand the right-hand-side (hereafter RHS), recall that $m_t(x)f(x)dx$ is the portfolio weight for asset type x; $n_t\eta_t$ is the total investment made in period t. So the total number of project x is $n_t\eta_t m_t(x)f(x)dx$. $\alpha_{t+1}(x)$ fraction of asset type x is being sold at price $p_{t+1}\big(\alpha_{t+1}(x)\big)$, so the total proceeds generated by sales is given by the RHS of the expression.

Buyers in the secondary market are outside the model. Those buyers only live for one period, born in the morning with deep pocket, and die in the afternoon. Buyers have no time discount between morning and afternoon, so they are willing to pay x in the morning for x units of consumption good due in the afternoon. Competition among buyers makes them break-even in the morning secondary market.

Moreover, like households, it is assumed that buyers' purchases on the secondary market are well-diversified across the entire banking sector. Therefore, they do not watch each banker's individual states. What buyers know is the aggregate states of the economy. Specifically, buyers in the morning of period $t+1$ observe aggregate households behavior $\zeta_{t+1}, \Delta_{t+1}$, aggregate bankers' net worth n^a_t, aggregate banking sector's leverage η^a_t, and aggregate quality of production portfolio $m^a_t(x)$. In equilibrium, $n_t = n^a_t$, $\eta_t = \eta^a_t$, and $m_t(x) = m^a_t(x)$. But if one banker chooses to deviate, buyers on the secondary market would not be able to observe that. In a nutshell, buyers do not post seller-specific pricing functions. Alternatively, one may assume that the secondary market is anonymous.

The afternoon session

In the afternoon of period $t+1$, each banker collects outputs from her retained fractions of projects and adjusts her liability. Specifically, given the market deposit rate R^d_{t+2} from $t+1$ to $t+2$, the banker chooses how much leverage η_{t+1} she would like to take. Recall that the banker inherits some automatically rolled over deposits from the morning session. If the banker wants to reduce her liability, then I assume that she just refers some of her deposits to a banker who is issuing deposits in the afternoon. For now, I impose a restriction $\eta_{t+1} \geq 1$ to govern bankers' leverage choice. That is, bankers are not allowed to hold net saving positions in other banks. In Section 2.3.1 I show that even if bankers are allowed to choose $\eta_{t+1} < 1$, it is never optimal for them to do so. Each banker also makes her consumption decision c_{t+1} in the afternoon. Time then moves to the evening of period $t+1$.

The banker's Bellman equation

The following Bellman equation summarizes the discussion above. Particularly, the value function V is defined right after the afternoon session but prior to the evening session. At that point of time, each banker taking private states n_t and η_t as given, chooses a screening outcome $m_t(x)$ for the evening, a selling scheme $\alpha_{t+1}(x)$ for next morning, and leverage η_{t+1}

and consumption c_{t+1} plans for next afternoon to maximize

$$V_t(n_t, \eta_t) = \max_{m_t(x), \alpha_{t+1}(x), \eta_{t+1}, c_{t+1}} \beta \Big[\log c_{t+1} + V_{t+1}\left(n_{t+1}, \eta_{t+1}\right) \Big] \tag{2.7}$$

$$\begin{aligned} \text{where} \quad n_{t+1} = \; & \eta_t n_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x) p_{t+1}\left(\alpha_{t+1}(x)\right) m_t(x) f(x) dx \\ & + \eta_t n_t \int_{\underline{x}}^{\bar{x}} \left(1 - \alpha_{t+1}(x)\right) x m_t(x) f(x) dx \\ & - \eta_t n_t \xi \int_{\underline{x}}^{\bar{x}} g\big(m_t(x)\big) f(x) dx - n_t(\eta_t - 1) R^d_{t+1} - c_{t+1} \end{aligned} \tag{2.8}$$

s.t. (2.2), (2.3), and (2.6) hold

The banker has to respect four constraints. First is the evolution of the banker's net worth (2.8): next period net worth equals to the proceeds generated in next morning from selling (the first term), plus the remaining proceeds collected in the afternoon (the second term), minus the real costs of screening (the third term), minus the total incurred liability (the fourth term), minus consumption (the fifth term). Second, the banker has to respect the liquidity constraint (2.6). I introduce $\beta V_{1,t+1} \ell_{t+1}$ to be the Lagrangian multiplier of this constraint, where $V_{1,t+1}$ is the partial derivative of V_{t+1} with respect to n_{t+1} evaluated at the solution of the problem. In economic terms, ℓ_{t+1} measures the premium the banker would like to pay for extra liquidity next morning. $\beta V_{1,t+1}$ converts the unit of the liquidity premium from time $t+1$ consumption good to time t utility. Lastly, the banker has to respect the constraint that portfolio weights should add up to one (constraint (2.2)) and that she cannot short a project (constraint (2.3)).

In the following, I may omit time subscripts in notations for simplicity when it causes no confusion.

2.2.2 The Household Sector

I model the household sector in an overlapping generation way. Households born in the morning of period $t+1$ inherit bequests from their parents' deposits. They inelastically provide labor to those projects originated by bankers and earn labor income in the morning. For simplicity, it is assumed that the total labor income earned by households is proportional to the total net output earned by the banking sector. Specifically, define

$$Y_{t+1} \equiv \int_{\underline{x}}^{\bar{x}} x m_t^a(x) f(x) dx \tag{2.9}$$

as the total productivity for period $t+1$, the total labor wage paid to households is assumed to be $\phi n_t^a \eta_t^a (Y_{t+1} - 1)$ where $\phi > 0$ is a constant that governs the proportion. Recall that n_t^a is the aggregate banking sector's wealth and η_t^a is the aggregate banking sector's leverage. So $n_t^a \eta_t^a$ is the total investment made by bankers. $n_t^a \eta_t^a (Y_{t+1} - 1)$ is then the total *net* output to the banking sector.

Households then decide how much to consume and how much to save. Their utility is log in their own consumption and log in the bequests they left for their offspring. Deposits contracts provided by bankers are the only saving technology that households have access to. The consumption-saving problem for generation $t+1$ households is then

$$\max_{\hat{c}_{t+1}} \quad (1-\beta) \log \hat{c}_{t+1} + \beta \log \hat{n}_{t+1} \tag{2.10}$$

$$\text{where} \quad \hat{n}_{t+1} = \phi n_t^a \eta_t^a (Y_{t+1} - 1) + R_{t+1}^d \hat{n}_t - \hat{c}_{t+1} \tag{2.11}$$

Here $\hat{n}$ is the amount of savings chosen by households; $\hat{c}$ is the level of consumption. For simplicity, it is assumed that households have the same time discount factor $\beta < 1$ as bankers.

Households who choose not to run on banks solve problem (2.10)-(2.11) in the morning. Households who choose to run on the banks solve the same problem in the afternoon after they realize that bankers do survive morning runs.[9] Moreover, it is assumed that households' savings are well-diversified in the entire banking sector and hence they treat the banking sector as if it is a conglomerate. That being said, households have no incentive to watch any individual bank. Instead, their running choice is only a function of aggregate states. In the main text, I assume that the fraction of households that choose *not* to run, ζ_{t+1}, is an exogenously given function of total productivity Y_{t+1} and banking sector's interests-included debt-to-asset ratio $\frac{(\eta_t^a-1)R_{t+1}^d}{\eta_t^a}$. The function is denoted as $\zeta\left(Y_{t+1}, \left(1-\frac{1}{\eta_t^a}\right)R_{t+1}^d\right)$, and it is assumed that

Assumption 5. *ζ is monotone increasing in Y, monotone decreasing in $\left(1-\frac{1}{\eta^a}\right)R^d$. Moreover, it is continous in both arguments and the (one-sided) derivatives with respect to Y are bounded.*[10]

Assumption 5 basically says that all else equal, if total productivity is high so that the economy is in good condition, then less people would run (ζ is high); on the other hand, fixing productivity Y, if the banking sector has more liability due relative to assets, then more people may worry and run (ζ is low). The boundedness of $\frac{\partial \zeta}{\partial Y}$ is assumed for technical reasons. In Appendix A.2, I extend the model by introducing a global game to the household sector and provide a functional form of ζ that satisfies Assumption 5. Yet the paper's results do not rely on that specific form. As long as Assumption 5 stands, the analysis will follow.

9. In the model described in the main text, bankers never fail as everything is deterministic. In the extended version of the model described in Appendix A.2, bankers do fail upon some bad fundamental shocks.

10. Throughout the paper, if at a point ζ is nondifferentiable with respect to Y, $\frac{\partial \zeta}{\partial Y}$ is then defined to be the one-sided derivatives with larger absolute value.

2.2.3 Equilibrium

Equilibrium in the Secondary Lemon Market

The secondary lemon market behaves very much like the signaling game modeled in DeMarzo and Duffie (1999). There, an impatient informed seller sells portions of assets to uninformed patient buyers. The seller uses retention fraction to signal quality. The gain of trade is from the relative impatience of the seller. Importantly, the relative impatience is fully understood by both parties. In the model described by this paper, the gain of trade comes from the liquidity premium the banker would like to pay. Basically, one unit of consumption good in the morning values more than one to the banker due to the liquidity constraint. It turns out that the liquidity premium is captured by the Lagrangian multiplier ℓ_{t+1} specified with constraint (2.6).

But unlike the relative impatience parameter in DeMarzo and Duffie (1999) which is exogenously given, the liquidity premium is endogenous in the model and it is not directly known by the buyers. In result, buyers have to form expectations about the liquidity premium. Recall that buyers observe aggregate state $\zeta_{t+1}, \Delta_{t+1}, n_t^a, \eta_t^a, m_t^a(x)$. So buyers' expectation about the liquidity premium, call $\hat{\ell}_{t+1}$, has to be consistent with those observations. Conditional on $\hat{\ell}$ as the gain of trade, I focus on the fully separating equilibrium of the signaling game. Other equilibrium may be ruled out by D1-stable refinement.[11] Particularly, if a banker does have Lagrangian $\hat{\ell}$, program (2.7) implies that when choosing $\alpha(x)$ for given x, the banker will maximize

$$u\big(x, \alpha, p; \hat{\ell}\big) \equiv (1 + \hat{\ell})\alpha p + (1 - \alpha)x$$

One could easily check that this functional form satisfies the key "single-crossing" property

11. For more technical details, see Ramey (1996) and DeMarzo and Duffie (1999).

for signaling games, as

$$-\frac{u_\alpha\big(x,\alpha,p;\hat{\ell}\big)}{u_p\big(x,\alpha,p;\hat{\ell}\big)}$$

is strictly monotone in x.

Definition 1. *Conditional on $\hat{\ell}$, a fully separating Bayes-Nash equilibrium for the signaling game on the secondary market is a pair of functions $(\hat{\alpha},\hat{p})$ such that*

1. $$\hat{\alpha}(x)\in\arg\max_{\alpha} u\big(x,\alpha,\hat{p}(\alpha);\hat{\ell}\big) \tag{2.12}$$

   almost surely.

2. $$\hat{p}\big(\hat{\alpha}(x)\big)=x \tag{2.13}$$

   almost surely.

Condition 2 says that the pricing function $\hat{p}$ chosen by buyers must make them break-even. Note that the pricing function depends on buyers' belief $\hat{\ell}$. Along an off equilibrium path, it is possible that a banker chooses m_t different from m_t^a and hence has different liquidity premium. But along the equilibrium path, they must match. Formally, I make the following definition:

Definition 2. *Given aggregate states $\zeta_{t+1},\Delta_{t+1},R^d_{t+1},n^a_t,\eta^a_t$ and private states n_t,η_t such that $n^a_t=n_t,\eta^a_t=\eta_t$, a partial equilibrium on the asset side for the evening of period t and the morning of period $t+1$ is a collection of banker's screening effort $m(x)$, aggregate production portfolio quality $m^a(x)$, banker's liquidity premium $\tilde{\ell}$, banker's selling function $\tilde{\alpha}(x)$, buyer's expectation about banker's liquidity premium $\hat{\ell}$, buyer's expectation about banker's selling function $\hat{\alpha}(x)$, and pricing function $p(\cdot)$ such that*

1. *Conditional on $\hat{\ell}$, $\big(\hat{\alpha},p(\cdot)\big)$ is a fully separating Bayes-Nash equilibrium for the signaling game on the secondary market as defined in Definition 1.*

2. $\hat{\ell}$ *is consistent with the liquidity constraint*

$$n_t^a(\eta_t^a - 1)(1 - \zeta_{t+1})R_{t+1}^d + n_t^a(\eta_t^a - 1)\zeta_{t+1}\Delta_{t+1} \leq n_t^a \eta_t^a \int_{\underline{x}}^{\bar{x}} \hat{\alpha}(x) p\left(\hat{\alpha}(x)\right) m^a(x) f(x) dx$$

3. *Given pricing function* $p(\alpha)$, $\{m(x), \tilde{\alpha}(x), \tilde{\ell}\}$ *solves the banker's problem (2.7).*

4. *Consistency:*

$$\tilde{\ell} = \hat{\ell} \tag{2.14}$$

$$\tilde{\alpha}(x) = \hat{\alpha}(x) \tag{2.15}$$

$$m(x) = m^a(x) \tag{2.16}$$

For obvious reasons, Conditions 1 and 3 and equation (2.14) imply equation (2.15). Conditions 3 and 4 imply Condition 2. So when solving equilibrium, I only use Conditions 1 and 3 and equations (2.14) and (2.16).

General Equilibrium

Definition 3. *Given initial conditions* $\{n_0, \eta_0\}$, *a general equilibrium is a collection of deposit rates* $\{R_{t+1}^d\}$, *pricing functions* $\{p_{t+1}\}$, *liquidity premia* $\{\ell_{t+1}\}$, *banker's decisions* $\{m_t(x), \alpha_{t+1}(x), c_{t+1}, \eta_{t+1}, n_{t+1}\}$, *household's decisions* $\{\hat{c}_{t+1}, \hat{n}_{t+1}\}$, *and aggregate states* $\{n_t^a, \eta_t^a, m_t^a(x), \zeta_{t+1}, \Delta_{t+1}\}$ *for all* $t \geq 0$ *such that*

1. $\{\hat{c}_{t+1}, \hat{n}_{t+1}\}$ *solve the households problem (2.10).*

2. Δ_{t+1} *is consistent with households' choice.*

3. ζ_{t+1} *is consistent with its functional form.*

4. *Aggregate states consistent with representative banker's choice:*

$$n_t^a = n_t$$

$$\eta_t^a = \eta_t$$

5. *Given* $\zeta_{t+1}, \Delta_{t+1}, R_{t+1}^d$ *and* $n_t^a = n_t, \eta_t^a = \eta_t$*, the collection of* $\{m = m_t, m^a = m_t^a, \tilde{\ell} = \ell_{t+1}, \tilde{\alpha} = \alpha_{t+1}, \hat{\ell} = \ell_{t+1}, \hat{\alpha} = \alpha_{t+1}, p = p_{t+1}\}$ *consists a partial equilibrium on the asset side as defined in Definition 2.*

6. $\{c_{t+1}, \eta_{t+1}, n_{t+1}\}$ *are optimized in the banker's problem.*

7. *The deposit market clears:*

$$(\eta_t^a - 1)n_t^a = \hat{n}_t \tag{2.17}$$

Given that the deposit market clears, Walras' Law implies that the good market also clears.

2.3 Solution

2.3.1 Banking Sector

Lemma 6. *Given any processes* $\{R^d, \zeta, \Delta, p\}$*, bankers' policy function* c *is linear on* n *whereas policy functions* η, m, α *do not depend on* n*. Specifically,* $c = \frac{1-\beta}{\beta} n$*.*

This result is rooted in the log utility feature of the banker.

Solution to the signaling game

The first order condition of problem (2.12) is

$$\left(1 + \hat{\ell}_{t+1}\right) \hat{p}_{t+1}(\hat{\alpha}) + \left(1 + \hat{\ell}_{t+1}\right) \hat{\alpha} \hat{p}'_{t+1}(\hat{\alpha}) = x \tag{2.18}$$

Combine with buyers' break-even condition (2.13), one gets:

$$\left(1+\hat{\ell}_{t+1}\right)\hat{p}_{t+1}(\hat{\alpha})+\left(1+\hat{\ell}_{t+1}\right)\hat{\alpha}\hat{p}'_{t+1}(\hat{\alpha})=\hat{p}_{t+1}(\hat{\alpha})$$

Conditional on the gain of trade is $\hat{\ell}$, this ODE characterizes equilibrium $\hat{p}(\cdot)$ and hence $\hat{\alpha}(x)$ by (2.13). Using the obvious boundary condition

$$\hat{p}_{t+1}(1)=\underline{x} \tag{2.19}$$

i.e., worst projects are always sold in full, we can solve $\hat{p}_{t+1}$ as a function of α, which turns out to be

$$\hat{p}_{t+1}(\alpha)=\underline{x}\alpha^{1/(1+\hat{\ell}_{t+1})-1} \tag{2.20}$$

Boundary condition (2.19) implicitly assumes that the screen technology does not change the support of X, especially the lower bound $\underline{x}$. This will be true if Condition 3 in Assumption 4 holds. In summary, if buyers believe that the banker's liquidity premium is $\hat{\ell}$, they will post the above pricing function (2.20).

On the other hand, suppose $\tilde{\ell}$ is the banker's true liquidity premium, given pricing function $\hat{p}$, the banker's true first order condition with respect to α is

$$\left(1+\tilde{\ell}_{t+1}\right)\hat{p}_{t+1}(\alpha)+\left(1+\tilde{\ell}_{t+1}\right)\alpha\hat{p}'_{t+1}(\alpha)\geq x \qquad \text{with "=" as long as } \alpha<1$$

where the $\geq$ sign captures the fact that α is bounded above at 1.[12] The solution, denoted by $\tilde{\alpha}$, would be

$$\tilde{\alpha}_{t+1}(x)=\min\left\{\left(\frac{x}{\underline{x}}\frac{1+\hat{\ell}_{t+1}}{1+\tilde{\ell}_{t+1}}\right)^{\frac{1}{1/(1+\hat{\ell}_{t+1})-1}},1\right\} \tag{2.21}$$

12. When writing down the first order condition for problem (2.12), I do not write down a $\geq$ sign. This is because there we are solving the fully separating equilibrium conditional on $\hat{\ell}$. The fully separating feature implies that at most one type can choose $\alpha=1$. Here, we are solving banker's optimal choice of α given the pricing function $\hat{p}$.

In equilibrium, $\hat{\ell}_{t+1} = \tilde{\ell}_{t+1} = \ell_{t+1}$, we get

$$\alpha_{t+1}(x) = \left(\frac{x}{\underline{x}}\right)^{\frac{1}{1/(1+\ell_{t+1})-1}} \tag{2.22}$$

Observe that as long as $\ell > 0$, $\alpha(x)$ is decreasing in x. Basically, in equilibrium, in order to signal that an asset's quality is higher, the seller has to retain more fractions of the asset. This observation has been well established in the literature of adverse selection (see Leland and Pyle (1977), DeMarzo and Duffie (1999), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014), Fuchs and Skrzypacz (2015)).

Moreover, note that in equilibrium the total proceeds generated by project x:

$$\alpha(x)p\big(\alpha(x)\big) = \alpha(x)x = \underline{x}\left(\frac{x}{\underline{x}}\right)^{-\frac{1}{\ell}}$$

is also decreasing in x. So indeed in equilibrium, bad assets can generate more proceeds for bankers in the morning. This observation is the main driver of the results of this paper. If bad assets can generate more liquidity, then with the presence of liquidity constraint, the banker will have less incentive to screen. This point will be made clear in Proposition 6.

Note also that in equilibrium the total value that project x generates for the banker is

$$(1+\ell)\alpha(x)x + \big(1-\alpha(x)\big)x = x + \ell\alpha(x)x$$

Specifically, in equilibrium, project x generates proceeds $\alpha(x)x$ in the morning, which the banker values at $(1+\ell)$. The remaining $1-\alpha(x)$ portion held by the banker generates $\big(1-\alpha(x)\big)x$ proceeds in the afternoon, which the banker values at 1. Or writing in a different way, the total value equals to the plain return x, plus a liquidity premium $\ell\alpha(x)x$.

Lemma 7. *$x + \ell\alpha(x)x$ is strictly increasing in x for all $\ell > 0$.*

The x part in the total value is upward sloping in x, whereas the $\alpha(x)x$ part is downward sloping as analyzed above. Lemma 7 implies that in equilibrium the total value is still upward sloping in x. Thus, good projects still provide higher total value to the banker. Hence, we should expect that out of screening, good assets still carry more weights in the production portfolio. This observation will be formalized in Lemma 9. However, as the equilibrium ℓ gets larger, the downward sloping part $\alpha(x)x$ receives more weights in the total payoff $x + \ell\alpha(x)x$. So the relative benefit of having a good project over a bad project, measured by the slope of $x + \ell\alpha(x)x$, is smaller. Formally,

Lemma 8. $\frac{\partial^2}{\partial\ell\partial x}\Big(x + \ell\alpha(x)x\Big) < 0 \quad \forall x > \underline{x}, \ell > 0$

Put differently, the benefit for screening hard is smaller as ℓ gets larger. Consequentially, the banker will have less incentive to screen carefully when the equilibrium ℓ is larger.

Asset management of the banker

Denote $\beta V_{1,t+1}\lambda_t$ as the Lagrangian multiplier for constraint (2.2) and $\beta V_{1,t+1}\mu_t(x)f(x)$ as the Lagrangian multiplier for constraint (2.3). Suppose that the banker has liquidity premium $\tilde{\ell}$ whereas buyers on the secondary market believe that the gain of trade is $\hat{\ell}$ and are posting pricing function $\hat{p}$ as given in (2.20). Then, the first order condition with respect to $m_t(x)$ in problem (2.7) is:

$$(1 + \tilde{\ell}_{t+1})\tilde{\alpha}_{t+1}(x)\hat{p}(\tilde{\alpha}) + \big(1 - \tilde{\alpha}_{t+1}(x)\big)x - \xi g'\big(m_t(x)\big) + \lambda_t + \mu_t(x) = 0$$

Call the solution $\tilde{m}_t(x)$ and rearrange:

$$g'\big(\tilde{m}_t(x)\big) = \frac{1}{\xi}\Big[(1 + \tilde{\ell}_{t+1})\tilde{\alpha}_{t+1}(x)\hat{p}(\tilde{\alpha}) + \big(1 - \tilde{\alpha}_{t+1}(x)\big)x + \lambda_t + \mu_t(x)\Big]$$

As the RHS is finite, by Condition 3 in Assumption 4 we know that $\mu_t(x) = 0 \; \forall x$. Thus, we can rewrite the equation as

$$g'\big(\tilde{m}_t(x)\big) = \frac{1}{\xi}\Big[(1+\tilde{\ell}_{t+1})\tilde{\alpha}_{t+1}(x)\hat{p}(\tilde{\alpha}) + \big(1-\tilde{\alpha}_{t+1}(x)\big)x + \lambda_t\Big] \tag{2.23}$$

Given a functional form of g, this equation pins down the choice of $\tilde{m}$. In particular, $\tilde{\alpha}$ in (2.23) is given by (2.21), $\tilde{\ell}$ in (2.23) can be solved from the liquidity constraint

$$(1-\zeta_{t+1})n_t(\eta_t-1)R^d_{t+1}+\zeta_{t+1}n_t(\eta_t-1)\Delta_{t+1} \leq \eta_t n_t \int_{\underline{x}}^{\bar{x}} \tilde{\alpha}_{t+1}(x)\hat{p}_{t+1}\left(\tilde{\alpha}_{t+1}(x)\right)\tilde{m}_t(x)f(x)dx$$

In equilibrium, $\hat{\ell} = \tilde{\ell} \equiv \ell$, (2.23) then collapses to

$$g'\big(m_t(x)\big) = \frac{1}{\xi}\left[x + \ell_{t+1}\alpha_{t+1}(x)x\right] + \frac{1}{\xi}\lambda_t \tag{2.24}$$

The $\frac{1}{\xi}\lambda_t$ term in (2.24) is some number independent of x that makes sure $m_t(x)$ integrate to one with respect to density f. As $\alpha_{t+1}(x)$ is purely determined by ℓ_{t+1} (see equation (2.22)), so is $m_t(x)$. To that end, sometimes I write $\alpha_{t+1}(x)$ as $\alpha(x;\ell_{t+1})$ and $m_t(x)$ as $m(x;\ell_{t+1})$ to emphasize that they can be read as functions of ℓ.

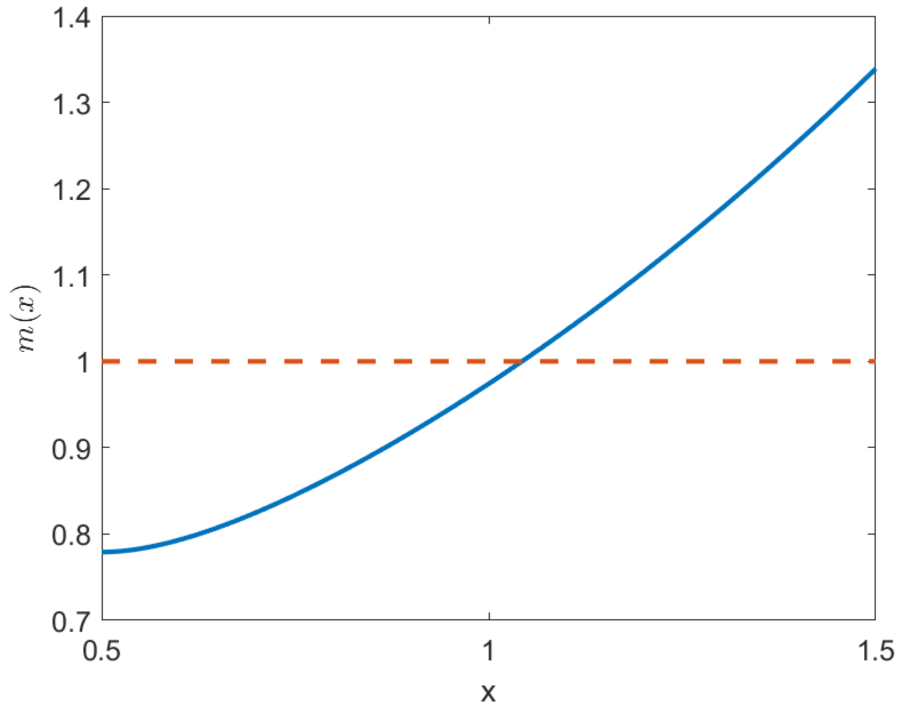
Equation (2.24) characterizes the equilibrium solution of $m_t(x)$. First, observe that

Lemma 9. *$m_t(x)$ is strictly increasing in x.*

For a visualization of the increasing feature of equilibrium $m(x)$, one may refer to Figure 2.1. The red dashed line plots the ex ante density $f(x)$, while the blue solid line plots a numerical example of $m(x)f(x)$, which equals to $m(x)$ in this example as $f(x) = 1\forall x$.

Recall that m can be read as the Randon-Nikodym derivative of the distribution the banker chooses with respect to the ex ante distribution $f(x)$. Lemma 9 implies that the banker must be twisting her portfolio toward more productive assets by screening. This result is indeed driven by the fact that the total value of project x to the banker, $x+\ell\alpha(x)x$,

Figure 2.1: Steady State Screening Choice (blue solid line) versus Ex Ante Density (red dashed line)



is increasing in x, as shown in Lemma 7.

The following proposition conducts a *partial* equilibrium comparative analysis with respect to ℓ:

Proposition 6. *For any* $0 \leq \ell^1 < \ell^2$, *the distribution implied by the change of measure* $m(x; \ell^1)$ *first order stochastically dominates the distribution implied by the change of measure* $m(x; \ell^2)$.

For a visualization of equilibrium m's under different liquidity premium, one may refer to Figures 2.2 (page 59) and 2.3 (page 70). In both figures, the red dashed line corresponds to an m with a smaller ℓ, whereas the blue solid line corresponds to an m with a larger ℓ. As can be seen, when ℓ is larger, m puts more weights on bad assets over good assets. Therefore, m associated with smaller ℓ first order stochastically dominate m associated with larger ℓ.

Corollary 2. *The function*

$$Y(\ell) \equiv \int_{\underline{x}}^{\bar{x}} xm\big(x;\ell\big)f(x)dx$$

is strictly decreasing in ℓ for all $\ell \geq 0$.

Proposition 6 and Corollary 2 are key drivers of this paper's argument. It shows that all else equal, if in period $t+1$ the liquidity constraint is tighter (measured by ℓ_{t+1}), then ex ante in period t, the banker will screen less carefully. Consequentially, the total productivity of the production portfolio will be lower. The main driving force for the result is the fact documented in Lemma 8. Specifically, the relative benefit of a high x project relative to a low x project is smaller when liquidity constraint is tighter, as bad assets are able to generate more cash to fulfill the liquidity need.

As households labor income in period $t+1$ is linear in $Y_{t+1} - 1 = Y(\ell_{t+1}) - 1$, Corollary 2 implies that tighter liquidity constraint hurts the household sector. The following corollary shows that tighter liquidity constraint also hurts the banking sector. Specifically, note that along the equilibrium path projects sold in the morning always get fair price (equation (2.13)). Hence the return to the banker's portfolio is just

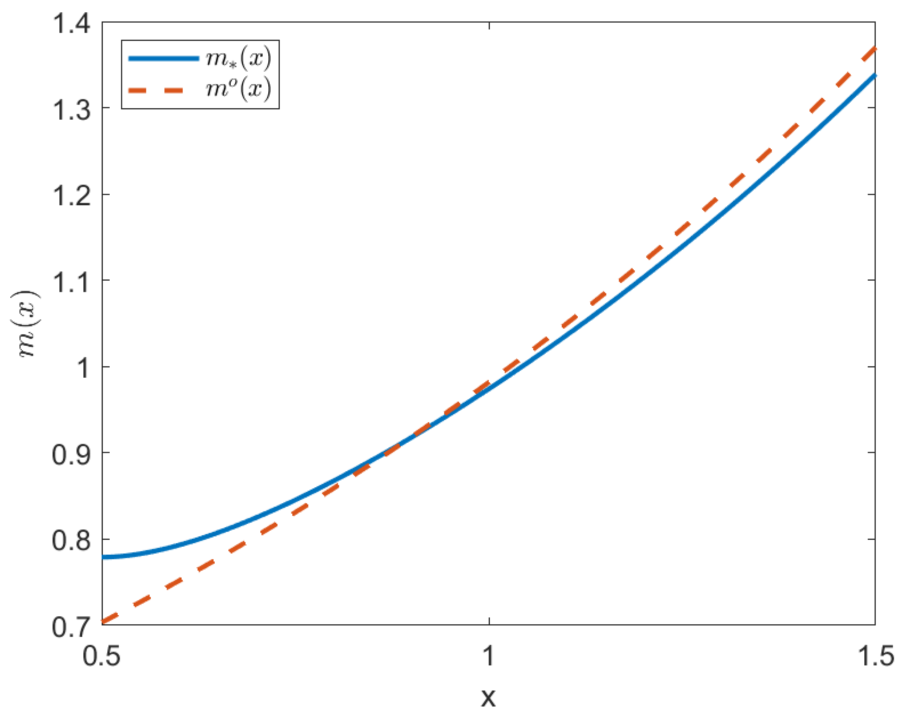
$$R_{t+1} \equiv \int_{\underline{x}}^{\bar{x}} xm(x;\ell_{t+1})f(x)dx - \xi \int_{\underline{x}}^{\bar{x}} g\big(m(x;\ell_{t+1})\big)f(x)dx \equiv R(\ell_{t+1}) \tag{2.25}$$

Corollary 3. *The function $R(\ell)$ is strictly decreasing in ℓ for all $\ell \geq 0$*

One might think that when equilibrium liquidity premium is larger, although the productivity of the production portfolio would be lower, bankers might not be hurt as they also save screening costs. Corollary 3 says that it is not the case: when equilibrium liquidity premium is higher, the return net of screening costs to bankers also decreases. Combine the analysis of Corollary 2 and 3, tighter liquidity constraint hurts both the household sector and the banking sector.

In order to better understand the mechanism, it is useful to consider what the banker would do without any liquidity constraint. In that case, the asymmetric information on the secondary market does not matter at all, as bankers do not need to go there for selling. Hence, bankers will simply choose $m(x)$ that maximizes R_{t+1} as defined in (2.25). Let's call the solution $m^o(x)$. As $m^o(x)$ is the solution associated with $\ell_{t+1} = 0$, by Proposition 6, the equilibrium choice $m_t(x)$ will always contain more bad assets and less good assets compared to the no-friction benchmark $m^o(x)$ as long as the liquidity constraint binds. Note that $m^o(x)$ is also the solution to the banker if liquidity constraint is present but information is symmetric on the secondary market. Figure 2.2 compares the steady state $m(x)$ (blue solid line) with $m^o(x)$ (red dashed line) using the numerical example specified in Section 2.4.

Figure 2.2: Steady State Screening Choice $m_*(x)$ versus No-Friction Screening Choice $m^o(x)$



One might wonder in equilibrium it is what force that stops bankers to screen more carefully when $\ell_{t+1} > 0$. Especially, one might think that as long as the banker can survive, the return to her portfolio is always R_{t+1} (this is indeed wrong), which is maximized at

$m^o(x)$. To get the idea, suppose a banker just deviates to $m^o(x)$. If the banker still sells project x according to the equilibrium selling scheme $\alpha_{t+1}(x) = \left(\frac{x}{\underline{x}}\right)^{\frac{1}{1/(1+\ell_{t+1})-1}}$, then she can generate proceeds

$$n_t\eta_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x)xm^o(x)f(x)dx < n_t\eta_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x)xm_t(x)f(x)dx$$
$$= (1-\zeta_{t+1})n_t(\eta_t-1)R^d_{t+1} + \zeta_{t+1}n_t(\eta_t-1)\Delta_{t+1}$$

The inequality comes from Proposition 6. Since that $m^o(x) = m(x;\ell=0)$, by Proposition 6 the screening outcome associated with $m^o(x)$ first order stochastically dominates the equilibrium $m_t(x)$. As $\alpha_{t+1}(x)x$ is decreasing in x, the inequality follows. The above expression implies that if the banker deviates to $m^o(x)$ and still sells according to the equilibrium scheme α, the banker cannot fulfill her liquidity constraint.

Essentially, as bad assets can generate more liquidity in the morning, the banker cannot fulfill her liquidity constraint just because she does not have enough bad assets. In order to survive, the banker with $m^o(x)$ at hand has to deviate on her selling scheme. Denote $\tilde{\ell}$ again to be the deviating banker's true liquidity premium. From (2.21) we know that the banker will choose the selling scheme

$$\tilde{\alpha}_{t+1}(x) = \min\left\{\left(\frac{x}{\underline{x}}\frac{1+\ell_{t+1}}{1+\tilde{\ell}_{t+1}}\right)^{\frac{1}{1/(1+\ell_{t+1})-1}}, 1\right\}$$

and $\tilde{\ell}$ will be pinned down by

$$(1-\zeta_{t+1})n_t(\eta_t-1)R^d_{t+1}+\zeta_{t+1}n_t(\eta_t-1)\Delta_{t+1} \leq \eta_t n_t \int_{\underline{x}}^{\bar{x}} \tilde{\alpha}_{t+1}(x)\underline{x}\tilde{\alpha}_{t+1}(x)^{\frac{1}{1+\ell_{t+1}}-1} m^o(x)f(x)dx$$

Here, I have substituted in equilibrium pricing equation (2.20). The above expression implies that $\tilde{\ell}_{t+1} > \ell_{t+1}$.[13] Intuitively, as the deviator has difficulty in serving her morning withdrawals, she must feel more liquidity constrained.

Equations (2.20) and (2.21) then imply that the deviating banker indeed sells project x at price

$$\max\left\{x\frac{1+\ell}{1+\tilde{\ell}}\ ,\ \underline{x}\right\}$$

Since $\tilde{\ell} > \ell$, this term is smaller than x for all $x > \underline{x}$. Put different, as the deviator does not have enough bad assets, she has to sell her assets in more portions to generate enough morning proceeds. However, since buyers would interpret more portions of selling as signals for bad quality, the banker's assets are sold at discount, which contributes a loss to the banker. In total, the actual return to the banker can be thought as R_{t+1} minus the selling loss. In equilibrium, although screening more carefully increases R_{t+1}, the selling loss eventually makes such deviation from equilibrium $m_t(x)$ suboptimal.

It is also worth to mention the role of screening cost in preventing deviation. To see so, consider a scenario where one banker is lucky enough to receive a better production portfolio, say $m^o(x)$, by paying the same equilibrium screening costs $\xi\int_{\underline{x}}^{\bar{x}} g\big(m_t(x)\big)f(x)dx$. Will the lucky banker be happy with that? The answer is yes. The lucky banker can just treat those extra good assets as if they were bad and sell them with discount according to the equilibrium selling scheme. This selling practice will generate the same amount of morning proceeds as in equilibrium, but the banker does enjoy extra benefits from the retention parts in the afternoon. Hence in total the lucky banker must be better-off. In summary, the existence of screening costs and potential selling losses jointly prevent deviation. The screening costs lower the gain of screening carefully, such that in equilibrium the gain is not big enough to

13. To see this, first note that as $\ell_{t+1} > 0$, (2.6) must bind. Therefore, the above line implies

$$\int_{\underline{x}}^{\bar{x}} \tilde{\alpha}_{t+1}(x)^{\frac{1}{1+\tilde{\ell}_{t+1}}} m^o(x)f(x)dx \geq \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x)^{\frac{1}{1+\ell_{t+1}}} m_t(x)f(x)dx > \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x)^{\frac{1}{1+\ell_{t+1}}} m^o(x)f(x)dx$$

The last inequality comes from Proposition 6, as $\alpha_{t+1}(x)^{\frac{1}{1+\ell_{t+1}}}$ is strictly decreasing. Compare the first term and last term. It is then easy to conclude $\tilde{\ell}_{t+1} > \ell_{t+1}$.

cover the loss due to selling in discount.

Liability management of the banker

Take the first order condition with respect to η_{t+1} in problem (2.7) and combine with the envelope condition, one gets that in equilibrium

$$R_{t+1} - R^d_{t+1} = \ell_{t+1}\Big((1-\zeta_{t+1})R^d_{t+1} + \zeta_{t+1}\Delta_{t+1} - A_{t+1}\Big) \tag{2.26}$$

where

$$A_{t+1} \equiv \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x) x m_t(x) f(x) dx$$

is the average proceeds that can be generated from the secondary market per unit of assets in equilibrium. As $\alpha_{t+1}(x)$ and $m_t(x)$ are both pure functions of ℓ_{t+1}, sometimes I also write A_{t+1} as $A(\ell_{t+1})$ where $A(\ell) \equiv \int_{\underline{x}}^{\bar{x}} \alpha(x;\ell) x m(x;\ell) f(x) dx$.

To understand equation (2.26), first consider the case if $\ell = 0$. In this setting, (2.26) collapses to $R_{t+1} = R^d_{t+1}$. As everything is deterministic in the model, if there is no liquidity constraint, the two risk-free returns have to match.

If $\ell > 0$, then as shown later in Section 2.3.3, in equilibrium, the RHS term of (2.26) is always positive. This positive term then drives R^d_{t+1} lower than R_{t+1} compared to the no-friction benchmark $R_{t+1} = R^d_{t+1}$. The intuition is straightforward. In this model, deposits contracts issued today create liquidity burdens next morning, so the deposit rate has to be lower than assets' return to compensate bankers. The size of the compensation is related to the liquidity gap created by the deposits. Specifically, one consumption good deposited today, $(1-\zeta_{t+1})R^d_{t+1} + \zeta_{t+1}\Delta_{t+1}$ will be withdrawn tomorrow morning. On the other hand, by investing the one consumption good in production, the banker can generate proceeds A_{t+1} from the secondary market next morning. So the liquidity gap is just $(1-\zeta_{t+1})R^d_{t+1} +$

$\zeta_{t+1}\Delta_{t+1} - A_{t+1}$. In value terms, it is $\ell_{t+1}\Big((1-\zeta_{t+1})R^d_{t+1} + \zeta_{t+1}\Delta_{t+1} - A_{t+1}\Big)$ which shows up in equation (2.26).

As $R^d \leq R$, it is never optimal for a banker to choose $\eta < 1$ even if they are allowed to do so. This justifies the $\eta \geq 1$ restriction introduced in Section 2.2.1.

The liquidity wedge in equation (2.26) echos the discussion of convenience yield provided by banking claims in the literature. The literature has identified that banking claims like deposits typically have lower returns than risk-free rate (e.g. Drechsler, Savov, and Schnabl (2017)). One strand of literature explains this wedge from the demand side (e.g. Greenwood, Hanson, and Stein (2015)), arguing that deposits provide extra transaction convenience or extreme safety to households so that households require a lower return. Another strand emphasizes the monopolistic power that bankers have in the deposits market (e.g. Drechsler, Savov, and Schnabl (2017)). Equation (2.26) provides another angle to explain the liquidity wedge from the supply side. It is deposits contracts that make bankers face liquidity constraints. The banker then has to be compensated for providing the deposit service to households.

2.3.2 Household Sector

Take first order condition with respect to $\hat{c}_{t+1}$ in (2.10), and plug into (2.11):

$$\hat{n}_{t+1} = \beta\phi\eta^a_t n^a_t (Y_{t+1} - 1) + \beta R^d_{t+1}\hat{n}_t \tag{2.27}$$

Basically, the household saves β portion of her total pre-consumption wealth each period. Consumption takes the other $1-\beta$ portion.

Given (2.27), households' total net adjustment in deposits position equals to

$$\hat{n}_t R^d_{t+1} - \beta\phi\eta^a_t n^a_t (Y_{t+1} - 1) - \beta R^d_{t+1}\hat{n}_t$$

Thus, for runners, per unit of last period deposits, they will withdraw R^d_{t+1} in the morning, and then redeposit

$$\frac{\beta\phi\eta^a_t n^a_t(Y_{t+1}-1)+\beta R^d_{t+1}\hat{n}_t}{\hat{n}_t}$$

in the afternoon. For non-runners, per unit of last period deposits, they will directly withdraw

$$\frac{\hat{n}_t R^d_{t+1}-\beta\phi\eta^a_t n^a_t(Y_{t+1}-1)-\beta R^d_{t+1}\hat{n}_t}{\hat{n}_t}$$

in the morning. Using market clearing condition (2.17) and rewrite this term, we then get that in equilibrium

$$\Delta_{t+1}=(1-\beta)R^d_{t+1}-\beta\phi\frac{\eta^a_t}{\eta^a_t-1}(Y_{t+1}-1) \tag{2.28}$$

2.3.3 Summary of Equilibrium

To summarize, the equilibrium defined in Definition 3 is characterized by the following key equations

$$\alpha_{t+1}(x)=\left(\frac{x}{\underline{x}}\right)^{\frac{1}{\frac{1}{1+\ell_{t+1}}-1}} \tag{2.22}$$

$$g'\big(m_t(x)\big)=\frac{1}{\xi}\left[x+\ell_{t+1}\alpha_{t+1}(x)x\right]+\frac{1}{\xi}\lambda_t \tag{2.24}$$

$$R_{t+1}-R^d_{t+1}=\ell_{t+1}\frac{1}{\eta_t-1}A_{t+1} \tag{2.29}$$

$$(1-\zeta_{t+1}\beta)\frac{\eta_t-1}{\eta_t}R^d_{t+1}-\zeta_{t+1}\beta\phi(Y_{t+1}-1)\leq A_{t+1} \tag{2.30}$$

$$\frac{\eta_{t+1}-1}{\eta_t-1}=\frac{R^d_{t+1}+\frac{\eta_t}{\eta_t-1}(Y_{t+1}-1)\phi}{R^d_{t+1}+\eta_t\big(R_{t+1}-R^d_{t+1}\big)} \tag{2.31}$$

(2.22) comes from the first order condition of bankers' signaling choice in the secondary market. (2.24) comes from the first order condition of asset management. (2.29) is a rewritten version of the first order condition of liability management by combining (2.26) with (2.28) and (2.6). As in equilibrium η_t is always larger than 1 by deposit market clearing condition

(2.17), this equation implies that in equilibrium there is always a non-negative wedge between R_{t+1} and R^d_{t+1}. Moreover, the wedge is strictly positive as long as $\ell > 0$. (2.30) is a rewritten version of the key liquidity constraint (2.6) by combining with (2.28). It binds as long as $\ell > 0$. This constraint pins down equilibrium ℓ. (2.31) describes the dynamics of the state variable η. It comes from combining budget constraints of bankers and households.

The above conditions are necessary for equilibrium. The following lemma verifies their sufficiency:

Lemma 10. *The first order conditions on bankers' problem are sufficient.*

The following proposition then characterizes the existence of equilibrium.

Proposition 7. *Given any $\eta_t > 1$, the set of equations (2.22), (2.24), (2.29)-(2.31) always have solutions for α_{t+1}, m_t, R^d_{t+1}, ℓ_{t+1}, η_{t+1}. Moreover, there exists at least one set of solutions where $R^d_{t+1} > 0$ and $\eta_{t+1} > 1$. Therefore, starting at any $\eta_0 > 1$, there exists an equilibrium.*

In principle, solutions to the set of equations (2.22), (2.24), (2.29)-(2.31) may not be unique. Whenever that happens, I focus on the one with the smallest ℓ, i.e, the one where the liquidity concern is least severe. In practice, the solution is actually unique for all economic reasonable parameterization that I have tried, including the one outlined in Section 2.4 below.

2.4 Steady State and a Numerical Example

Proposition 8. *For sufficiently small β, the economy exists a unique balance growth path. Moreover, along the balance growth path, the liquidity constraint binds.*

The presumption that β is sufficiently small is generally satisfied by economic reasonable parameterization. Indeed, for the numerical example outlined below, conclusions in all Propositions 8-12 hold for all $\beta < 1$.

Along the balance growth path, $\ell, \alpha, m, Y, R, R^d, \eta$ will be time invariant. Those objects can be solved from the time invariant version of the set of equations given in Section 2.3.3. In what follows, I denote them with a subscript $*$. Moreover, along the balance growth path, c_t, n_t, $\hat{c}_t$, and $\hat{n}_t$ grow at constant rate G_*. This rate can be backed out from (2.27). With the abuse of language, I also call those solved objects steady states.

I now introduce a numerical example that helps to better illustrate some features of the model. Parameters used for the numerical example are given in Table 2.1. In particular, ϕ is the labor income to capital net output ratio. By setting $\phi = 1$, it is assumed that bankers and labors equally split total net output. β is the time discount factor for both sectors, which I set to 0.95. Regarding that, one may interpret one period as one year. ξ is a measure of the cost of screening. I set $\xi = 1.5$ such that the balance growth rate G_* roughly matches the annual consumption growth rate 2% in the data. Furthermore, I regulate the ex ante distribution of projects quality to be uniform over $X = [0.5, 1.5]$. Thus, without any screening effort, the banker can get ex ante gross return $\mathbb{E}[x] = 1$. $g(m)$ takes functional form (2.4). The non-runner fraction ζ is specified as

$$\zeta\left(Y; \left(1 - \frac{1}{\eta^a}\right) R^d\right) \equiv \min\left\{\frac{\sigma+1}{2\sigma} - \frac{1}{2\sigma}\frac{\left(1-\frac{1}{\eta^a}\right)R^d(2-\beta) - \beta\phi\frac{\sigma}{2}Y + \beta\phi}{2\underline{x} + \beta\phi Y}, 1\right\} \quad (2.32)$$

with $\sigma = 0.04$. This functional form is microfounded in Appendix A.2.

Table 2.1: Parameters for the Numerical Example

Parameter	Value	Economic meaning
ϕ	1	labor income to capital net output ratio
β	0.95	time discount factor of both sectors
ξ	1.5	cost of screening
$\underline{x}$	0.5	lower bound of X
$\bar{x}$	1.5	higher bound of X
f	$f(x) = 1 \forall x \in X$	ex ante distribution of project returns
g	$g(m) = m \log m$	divergence measurement
ζ	see equation (2.32)	non-runner fraction

Table 2.2 summarizes the balance growth path with this set of parameters. The blue solid line in Figure 2.2 (page 59) plots steady state $m_*(x)$ and compares that with first-best benchmark $m^o(x)$ (red dashed line).

Table 2.2: Balance Growth Path

Outcome	Value	Economic meaning
G_*	1.0214	consumption and net worth growth rate
η_*	15.448	bankers' leverage
ℓ_*	0.4016	liquidity premium
Y_*	1.0481	productivity
R_*	1.0271	bankers' assets gross return, net of screening costs
R^d_*	1.0238	deposits gross return
Δ_*	0.0024	non-runner withdrawal per unit deposit
ζ_*	0.8771	non-runner fraction

2.5 Amplification of Productivity Shocks

I now turn to discuss the model's macro implication. To begin with, a productivity shock may be amplified due to the asset management practice of the banker. Suppose the economy has run into the steady state outlined in Section 2.4. In the evening of period $t = 0$, after R^d_1 has been set to R^d_* but before assets origination, all agents in the economy suddenly get known that an aggregate productivity shock hits. A project originally generates x units of time 1 consumption goods would now generate xz for all $x \in X$ for some constant $z > 0$.[14] In this section, I mainly focus on cases where $z < 1$, which can be interpreted as a negative aggregate productivity shock.

It can be shown easily that if the shock is accompanied by a same factor adjustment in the screening cost from ξ to $z\xi$, then Y_1 would go down by the same factor z was the equilibrium ℓ unchanged. This is just the direct effect of the shock on total productivity.

14. Alternatively, the shock changes the support of X from $[\underline{x}, \bar{x}]$ to $[\underline{x}z, \bar{x}z]$ and ex ante density from $f(x)$ to $f(x/z)$.

In what follows, I always accompany a shock with the corresponding adjustment in ξ for mathematical simplicity.

Nonetheless, the total effect goes beyond the direct effect. If (2.30) is binding and Y_1 is lower due to the direct effect of shock z, all else equal (if ζ was unchanged), the LHS of (2.30) would be higher. On the other hand, one can prove that the RHS of (2.30) is increasing in ℓ_1:

Corollary 4. *$A'(\ell) > 0$ for all $\ell \geq 0$.*

Therefore, if LHS of (2.30) is larger because of Y_1 is smaller, ℓ_1 has to increase to rebuild the equality. Indeed, the situation is even worse since the RHS of (2.30) also decreases by a factor z upon the shock, so that ℓ_1 has to increase even more.

But this is not the end of the story. The higher ℓ_1 will feed back into Y_1 and makes Y_1 even lower by Corollary 2. The lower Y_1 further makes the LHS of (2.30) larger, pushing up even more on ℓ_1, and hence forms an amplification loop.

The above amplification mechanism exists even if ζ is constant. But if ζ is endogenized as a function of Y, the amplification will be further reinforced as long as ζ is monotone increasing in Y as assumed in Assumption 5. In this case, ζ is also decreasing as Y is decreasing along the loop. The decreasing ζ further enters the LHS of (2.30), making LHS, and in turn ℓ, even larger. Consequentially, the amplification is exacerbated.[15]

The economic intuition behind the amplification is straightforward. The initial decline in productivity causes tighter liquidity constraint, as non-runners would earn less and save less and hence withdraw more. Knowing that ex ante, bankers originate more projects with low

15. Indeed, with functional form (2.32), z shock also has a direct effect on ζ as it changes $\underline{x}$ to $z\underline{x}$ in (2.32). Briefly, households know that even if bankers sell everything in the secondary market, the maximum amount they can get per unit of project is now $z\underline{x}$ instead of $\underline{x}$. This lowers households' confidence and hence reduces ζ, which makes the situation even worse. In what follows, I assume that in general the direct effect of z on ζ is either zero or positive, i.e.,

Assumption 6. $\frac{\partial \zeta}{\partial z} \geq 0$

This assumption is used in proofs of Propositions 9-10.

quality as argued by Proposition 6. But as more bad projects are originated, the aggregate productivity Y turns to be even lower, which in turn reduces labor income, and increases non-runners' withdrawals even more. The increase in withdrawal once again worsens the banker's liquidity situation, making them originate projects with low quality even more. On top of that, together with the decline in productivity, households become more pessimistic about the banking sector. Consequentially, more households become runners. More runners further burden bankers' liquidity concerns. Bankers respond to that by screening even less carefully. This in turn lowers the total productivity of the economy even further.

The following proposition summarizes the discussion.

Proposition 9. *Suppose the economy runs on its balance growth path up to the afternoon of $t = 0$. In the evening of period 0, a z shock happens. Suppose upon the shock bankers can still satisfy their liquidity constraint. Then for sufficiently small β, $z > 1 \Rightarrow Y_1 > zY_*$ and $z < 1 \Rightarrow Y_1 < zY_*$.*

Table 2.3: Decomposition of the Impact Response of $z = 0.99$ Shock

Stage	ℓ	$Y(\ell)$	Y/Y_*	Y implied Δ	Y implied ζ
Steady state	0.4018	1.0481	1.0000	0.0023	0.8769
Direct effect of z on A and ζ [1]	0.5985	1.0340	0.9865	0.0166	0.7312
1st round amplification [2]	1.0519	1.0287	0.9814	0.0220	0.6993
2nd round amplification [2]	1.3181	1.0268	0.9797	0.0239	0.6878
Further rounds of amplification	...	...	...	...	...
Final outcome	1.5083	1.0258	0.9787	0.0250	0.6814

[1] "Direct effect of z on A and ζ" is calculated in the following way. Put steady state Y and $\frac{\eta-1}{\eta}R^d$ into the LHS of (2.30) and change $\underline{x}$ in the functional form of ζ (2.32) to $z\underline{x}$. Then calculate the new ℓ implied by (2.30) with z's direct effect on A taken into consideration. This solved number goes into the "ℓ" column. Using that ℓ, it is easy to solve $Y(\ell)$ together with z's direct effect. This number goes into the next column. The last three numbers are directly calculated according to their definitions.
[2] "nth round amplification" is calculated in the following way. Put Y and ζ from last row into the LHS of (2.30). Then calculate the new ℓ implied by (2.30) with z's direct effect on A. All the other objects are solved similar as above.

Table 2.3 and Figure 2.3 provide a numerical illustration for $z = 0.99$ under the parameterization given in Table 2.1. As shown in Table 2.3, in this numerical example, a 1%

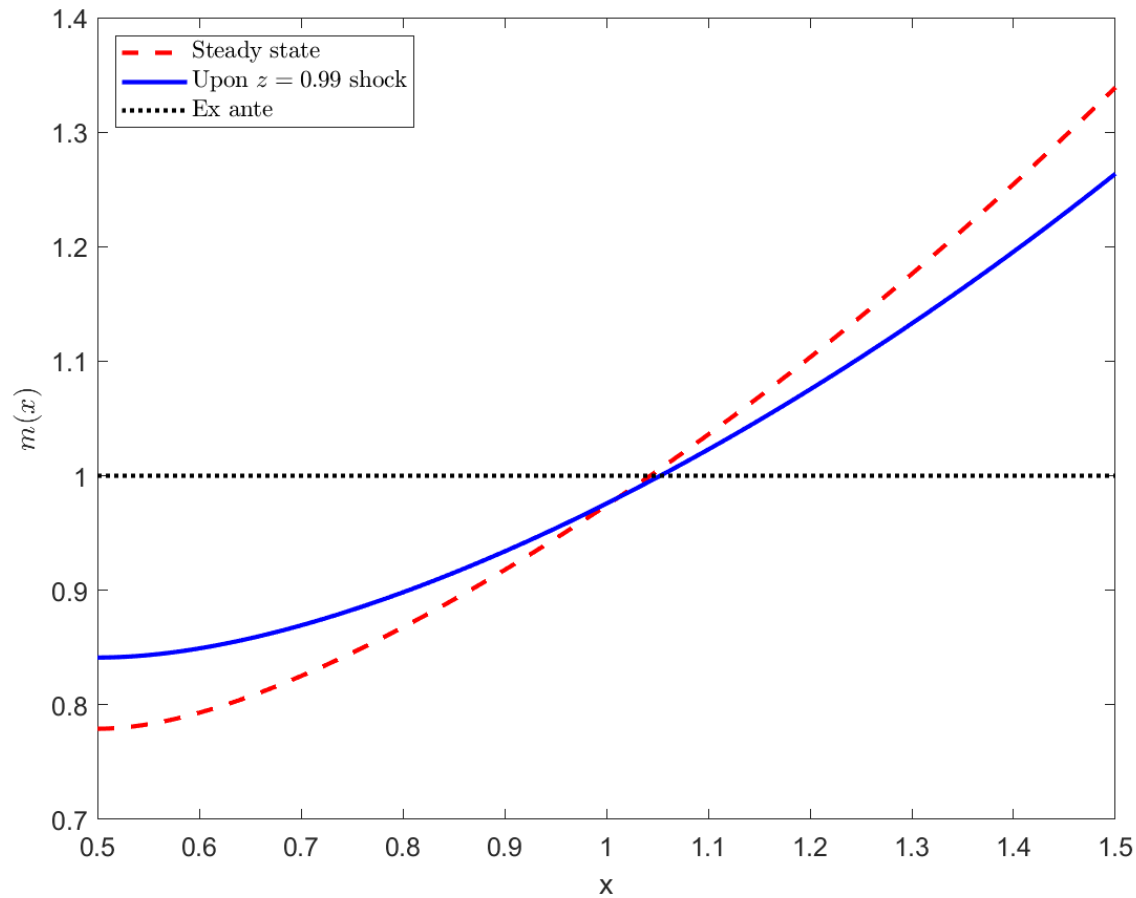


Figure 2.3: Screening Choice $m_t(x)$ upon Negative Shock $z = 0.99$

productivity shock is eventually amplified to more than 2%. Figure 2.3 visualizes the change in screening outcome upon the shock. Was ℓ unchanged, $m(x)$ should stay at its steady state level (red dashed line). However, the increasing liquidity concern distorts bankers' screening effort. As can be seen, compared with steady state, the screening outcome (blue solid line) put more weights toward bad assets and less weights on good assets. In result, total productivity declines more than the size of the shock.

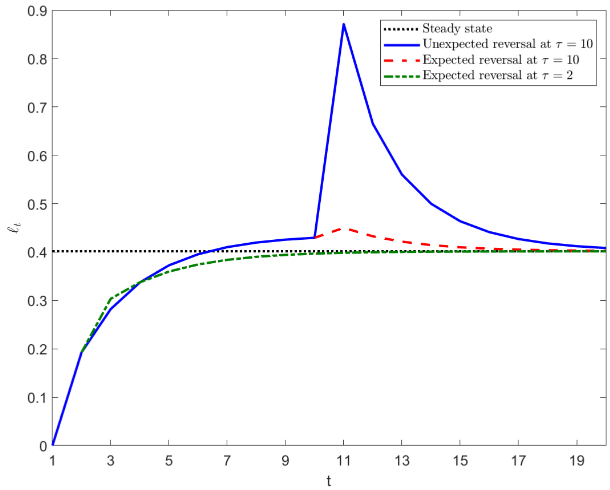
Note that the amplification mechanism described above applies not only to aggregate productivity shocks. The key insight is that any factor that worsens banker's liquidity situation would feed back into a deterioration in productivity. For example, suppose in the evening of period t, instead of the z shock described above, agents suddenly learn that households generation $t+1$ care less about their offspring, i.e, they have a lower β than usual. Consequentially, non-runners would withdraw more next period (an increase in the LHS of (2.30)), which in turn tightens bankers' liquidity constraint next morning. Anticipating that, bankers would reduce screening effort today. In result, the original shock in liquidity would be transferred into a real shock in productivity. Once the productivity is shocked, all the amplification mechanism described above kicks in and will drive productivity even lower.

The insight that liquidity shocks may transfer into productivity shocks may also apply to situations beyond the model. For example, consider situations where liquidity shocks are caused by alterations in the interbank market. In particular, if liquidity in the interbank market dries up so that bankers have to rely more on the outside secondary market for financing where the asymmetric information is more severe, bankers would then originate more assets with low quality due to their liquidity benefits, which in turn lowers the economy's total productivity.

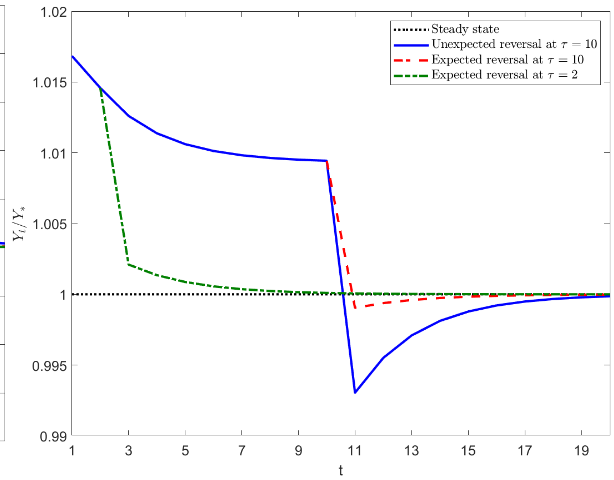
2.6 Dynamics following Productivity Shocks

I now track the dynamics of the economy after it hits by a z shock. Specifically, assume that in the evening of period 0, it is common knowledge that an aggregate productivity shock z hits the economy and it will last for some time. I consider two scenarios: 1) agents believe the shock is permanent, but it is unexpectedly reversed in the evening of period $\tau > 0$; 2) agents know that the shock is temporary and will be reversed in the evening of period τ. For narration simplicity, in this section I mainly focus on a positive shock $z = 1.01$.

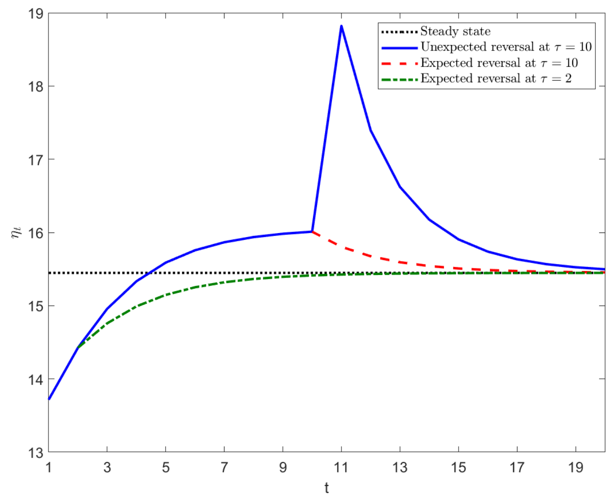
Figure 2.4 shows the dynamics of several key variables of the economy under the parameterization displayed in Table 2.1. Let's start with blue solid lines, which plot the trajectory of unexpected reversal at $\tau = 10$. Due to the amplification mechanism studied in Section 2.5, upon the positive shock bankers face lower liquidity premium (Panel (a)) and thus choose to screen more carefully than in steady state. The total output Y is then boosted more than



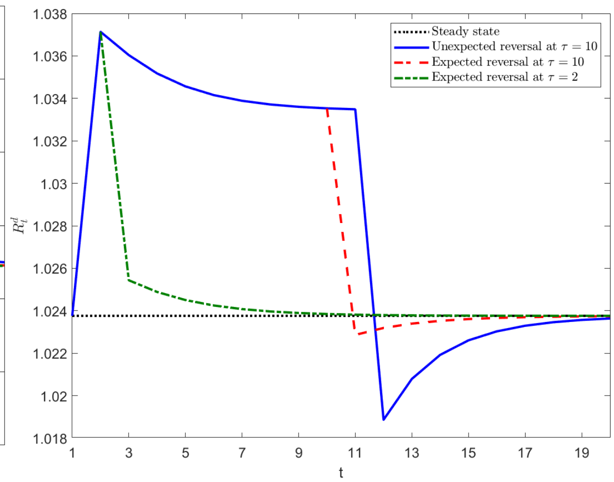
(a) Liquidity premium ℓ_t



(b) Productivity Y_t/Y_*



(c) Leverage η_t



(d) Deposit rate R_t^d

Figure 2.4: Dynamics Along the Trajectory of Selected Variables

1% in period 1 (Panel (b)).

Since the deposit rate from period 0 to 1 is pinned down before agents learn the shock in the evening of period 0, the interests households can earn from deposits in period 1, R_1^d, is unchanged compared to steady state (Panel (d)). As bankers will enjoy higher assets return both because of the positive shock and the lower liquidity premium (Corollary 3), bankers' wealth grows faster than households' wealth. This leads to a lower leverage ratio of the banking sector by the end of period 1 (Panel (c)).

In period 1, the deposit rate for next period, R_2^d, will adjust. As the positive shock drives liquidity premium lower, the net margin $R_2 - R_2^d$ needs not to be as high as before to compensate bankers for taking deposits. Since R_2 must rise thanks to the shock, R_2^d has to increase significantly (Panel (d)). The increase in R_2^d, however, makes bankers face higher liquidity burden in the evening of period 1 than they did in the evening of period 0. This is demonstrated in Panel (a) as $\ell_2 > \ell_1$. In result, $Y_2 < Y_1$ as shown in Panel (b).

The economy then moves toward the new steady state implied by $z = 1.01$. The following proposition characterizes this long-run limit:

Proposition 10. *Consider a sequence of economy indexed by $z > 0$. In economy z, the screening costs parameter is $z\xi$, the return of project x is zx. Suppose for each z the economy has unique steady state (Proposition 8). Let $\ell_*(z), m_*(x; z), Y_*(z), \eta_*(z), R_*^d(z)$ be the steady state liquidity premium, screening choice, productivity, leverage, and deposit rate of economy z respectively. Then,*

1. *$\ell_*(z)$ is increasing in z;*
2. *$z > z' \Rightarrow m_*(x; z')$ first order stochastically dominates $m_*(x; z)$;*
3. *$z > 1 \Rightarrow Y_*(z) < zY_*(1)$; $z < 1 \Rightarrow Y_*(z) > zY_*(1)$;*
4. *$\left(1 - \frac{1}{\eta_*(z)}\right) R_*^d(z)$, the interests-included debt-to-asset ratio, is increasing in z.*

As implied by item 1 of the proposition, along the trajectory ℓ is increasing to a level higher than its initial steady state (Panel (a)). This increasing pattern of liquidity premium is rooted in the growth of bankers' liability. Intuitively, households earn more labor income and accumulates more wealth thanks to the positive shock. This creates a saving glut from the household sector which enlarges bankers' liability. Although bankers' net worth is also growing, item 4 of the proposition demonstrates that after taking deposit interests into consideration, the growth of liability will dominate in the long-run. The larger interests-included leverage then induces higher liquidity burden for the bankers in the new steady state.

The increasing liquidity concern gradually relaxes bankers' lending standard along the trajectory. Eventually, in the new steady state, the lending standard is laxer than its origin (item 2) and hence the productivity grows less than the size of shock (item 3). The prediction that long boom leads to lax lending standard gains various support from empirical research, both on the standard of C&I loans (e.g. Asea and Blomberg (1998), Lisowsky, Minnis, and Sutherland (2017)) and mortgages (e.g. Mian and Sufi (2009), Keys, Mukherjee, Seru, and Vig (2010), Ben-David (2011), Demyanyk and Van Hemert (2011), Loutskina and Strahan (2011), Maddaloni and Peydró (2011), Dell'Ariccia, Igan, and Laeven (2012)). On the other hand, since upon the shock productivity grows more than z (Proposition 9) while in the long-run it grows less than z, the economy also displays an overheating feature along the transition (Panel (b)).

Now let's turn to consider the dynamics upon the reversal of the shock. Recall that in the evening of period $\tau = 10$, z is driven back to 1 unexpectedly. Compared to the initial steady state, interests-included debt-to-asset ratio is at a higher level (item 4 of Proposition 10). Therefore, bankers will have higher-than-initial-steady-state liquidity premium for the morning of period 11 (Panel (a)). In response, they screen less carefully in the evening of $\tau = 10$ and end up with a worse production portfolio. Consequentially, the realized productivity Y overshoots its initial steady state upon the reversal (Panel (b)). As deposit rate is not able to adjust right upon the reversal due to the unexpectedness, bankers suffer a much larger loss than households and their leverage will rise sharply by the end of period 11 (Panel (c)). In period 11, deposit rate for next period will decrease (Panel (d)) so that the net margin is high enough to compensate bankers for the high liquidity burden. The economy then gradually recovers back to its initial steady state.

The dynamics after the reversal contributes to the literature of how an economic boom may eventually lead to a bust (e.g. Boissay, Collard, and Smets (2016), Gorton and Ordonez (2019)). Note that negative shocks ($z < 1$ shock) *never* happen along the path, but the economy's total output is recovered from a level below its steady state. As analyzed above, the key reason behind the bust is that during the boom interests-included leverage is raised to a level larger than its origin. The larger-than-origin interests-included leverage then causes extra liquidity burdens to bankers and disincentivizes their screening practice.

The above discussion can be summarized by the following proposition.

Proposition 11. *Suppose a permanent $z > 1$ ($z < 1$) shock is unexpectedly reversed in the evening of period τ after lasting for long enough time. Suppose also that upon the reversal bankers can still satisfy the liquidity constraint. Then $Y_{\tau+1} < Y_*$ ($Y_{\tau+1} > Y_*$) if β is sufficiently small.*

Indeed, even if the reversal is fully expected, the economy may still overshoot its steady state as long as upon the reversal banking sector's leverage η has passed its initial steady state. Red dashed lines in Figure 2.4 plot the trajectory with expected reversal at $\tau = 10$. Since capital is fully depreciated in the model, knowing ahead that the shock will be reversed in the evening of period $\tau = 10$ does not change the economy's dynamics from time 0 to 10. The key difference, however, is that in period 10 the equilibrium deposit rate for next period, R^d_{11}, will adjust if the reversal is known. Indeed, it has to be much lower to persuade bankers to take the deposit (Panel (d)). Compared to the unexpected case, the lower deposit rate helps partially relieve bankers' liquidity burden. But due to the higher-than-origin leverage (Panel (c)), bankers still face higher-than-origin liquidity concern (Panel (a)). Consequentially, they still choose a worse-than-origin production portfolio upon the reversal and the economy still goes into a bust (Panel (b)). Formally, one can prove that

Proposition 12. *Given any $\eta_t > 1$, the solution to the set of equations (2.22), (2.24), (2.29)-(2.31) has the following feature provided that β is sufficiently small: if $\eta_t > \eta_*$ ($\eta_t < \eta_*$), then $Y_{t+1} < Y_*$ ($Y_{t+1} > Y_*$).*

Nevertheless, thanks to the adjustment in deposit rate, the size of the recession is much smaller.

It is worth to mention that if the positive shock is reversed with full expectation before banking sector's leverage passes its initial steady state, then the economy will not go into a bust upon the reversal of the shock. The green dash-dot lines in Figure 2.4 verify this claim. There, the shock is reversed in the evening of $\tau = 2$. As can be seen from Panel (b), in this case the economy's productivity converges back to its initial steady state from above.

Proposition 12 may have broader implications beyond the specific dynamics described above. The key insight of the proposition is that as long as banking sector's leverage becomes higher during the boom, then the extra liquidity burden brought by leverage may disincentivize screening and cause a bust once the boom period is over. In this model, the higher leverage is rooted in the saving glut during booms. But more broadly, as long as banking sector's leverage is procyclical (see Adrian and Shin (2010) for empirical evidence), the boom-to-bust mechanism applies.

2.7 Conclusion

This paper studies the interaction of bankers' asset management and liability management under the presence of liquidity constraint. On the asset side, bankers costly screen production projects. On the liability side, bankers issue deposits contracts. Deposits can be withdrawn before assets payout. In order to fulfill those early withdrawals, bankers have to sell projects in a secondary market. Because bankers monitor production, whenever they walk into the secondary market, they have superior information than buyers on the quality of each project. Bankers signal each project's quality by retention.

This paper argues that the presence of asymmetric information on the secondary market affects both the banker's asset management and liability management. On the asset side, as bad assets have higher liquidity benefits than good assets, higher ex post liquidity premium disincentivizes banker's screening ex ante. This is because bad assets are easier to sell in the secondary market, and can generate more proceeds for the banker to fulfill the liquidity need. On the liability side, liquidity concerns determine the equilibrium deposit rate. Specifically, in equilibrium bankers earn a compensation for issuing deposits as they cause liquidity burdens.

In general equilibrium, an aggregate productivity shock is amplified. An original negative productivity shock reduces labor income, and hence increases both the fraction of runs and the amount of non-runners' withdrawal. The liquidity constraint is thus tighter, which in turn disincentivizes bankers' screening and further lowers productivity and labor income. Similarly, a positive productivity shock is also amplified. If it is long-lasting, then it also creates a saving glut over time. The saving glut gradually pushes up bankers' leverage and hence liquidity burden which disincentivizes bankers' screening. The economy thus witnesses countercyclical lending standards over the long-run. If the original shock is reversed, the extra liquidity burden leads the economy overshooting its initial steady state. In that sense, the economy would experience a bust after the original boom period is over.

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APPENDIX A

APPENDIX

A.1 Appendix to Chapter 1: Proofs

Proof of Lemma 1

Proof.

1. Suppose there exist such $\bar{\mu}_s$ and $\bar{\mu}_{s'}$ so that it is optimal to take $p(s') = -\infty$ and $p(s) > \infty$. Let $p = p(s)$, then

$$0 \leq \mathbb{E}^{\bar{\mu}_s}[(X-p)\mathbb{I}_p|\sigma^-]$$

and

$$0 \geq \mathbb{E}^{\bar{\mu}_{s'}}[(X-p')\mathbb{I}_{p'}|\sigma^-]$$

for all $p' \in [R, x_G]$. Then obviously,

$$\mathbb{E}^{\bar{\mu}_{s'}}[(X-p)\mathbb{I}_p|\sigma^-] \leq \mathbb{E}^{\bar{\mu}_s}[(X-p)\mathbb{I}_p|\sigma^-]$$

Or

$$\sum_{\theta\in\{B,G\}}(x_\theta - p)G_\theta(p)\bar{\mu}_{s'}(x_\theta) \leq \sum_{\theta\in\{B,G\}}(x_\theta - p)G_\theta(p)\bar{\mu}_s(x_\theta)$$

Therefore, we have

$$\sum_{\theta\in\{B,G\}}(x_\theta - p)G_\theta(p)\Big(\bar{\mu}_{s'}(x_\theta) - \bar{\mu}_s(x_\theta)\Big) \leq 0$$

Note that $(x_B - p)\Big(\bar{\mu}_{s'}(x_B) - \bar{\mu}_s(x_B)\Big) > 0$ and $(x_G - p)\Big(\bar{\mu}_{s'}(x_G) - \bar{\mu}_s(x_G)\Big) \geq 0$. So we know that $G_B(p) = 0$. Moreover, either $p = x_G$ or $G_G(p) = 0$. Obviously, $p = x_G$

contradicts with the conclusion that $G_B(p) = 0$. Therefore, we know $G_G(p) = 0$.

2. Suppose there exist such $\bar{\mu}_s$ and $\bar{\mu}_{s'}$ so that it is optimal to take $-\infty < p(s') < p(s)$. Let $p_1 = p(s')$ and $p_0 = p(s)$. By the definition of G_θ in (1.3), we know $G_\theta(p_0) \geq G_\theta(p_1)$ for both θ.

The optimality of p_0 following signal s implies

$$\begin{aligned}
&\bar{\mu}_s(x_B)(x_B - p_1)G_B(p_1) + \bar{\mu}_s(x_G)(x_G - p_1)G_G(p_1) \\
\leq&\bar{\mu}_s(x_B)(x_B - p_0)G_B(p_0) + \bar{\mu}_s(x_G)(x_G - p_0)G_G(p_0)
\end{aligned}$$

Rewrite the RHS

$$\begin{aligned}
&\bar{\mu}_s(x_B)(x_B - p_0)G_B(p_0) + \bar{\mu}_s(x_G)(x_G - p_0)G_G(p_0) \\
=&\bar{\mu}_s(x_B)(x_B - p_1)G_B(p_1) + \bar{\mu}_s(x_G)(x_G - p_1)G_G(p_1) \\
&+ \Big(\bar{\mu}_s(x_B)(x_B - p_1)G_B(p_0) + \bar{\mu}_s(x_G)(x_G - p_1)G_G(p_0)\Big) \\
&- \Big(\bar{\mu}_s(x_B)(x_B - p_1)G_B(p_1) + \bar{\mu}_s(x_G)(x_G - p_1)G_G(p_1)\Big) \\
&+ \Big(\bar{\mu}_s(x_B)(x_B - p_0)G_B(p_0) + \bar{\mu}_s(x_G)(x_G - p_0)G_G(p_0)\Big) \\
&- \Big(\bar{\mu}_s(x_B)(x_B - p_1)G_B(p_0) + \bar{\mu}_s(x_G)(x_G - p_1)G_G(p_0)\Big)
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
&\bar{\mu}_s(x_B)(x_B - p_1)\Big(G_B(p_0) - G_B(p_1)\Big) + \bar{\mu}_s(x_G)(x_G - p_1)\Big(G_G(p_0) - G_G(p_1)\Big) \\
\geq&\bar{\mu}_s(x_B)G_B(p_0)(p_0 - p_1) + \bar{\mu}_s(x_G)G_G(p_0)(p_0 - p_1) \qquad \text{(A.1)}
\end{aligned}$$

Similarly, the optimality of p_1 following signal s' implies that

$$\begin{aligned}&\bar{\mu}_{s'}(x_B)(x_B - p_1)\Big(G_B(p_0) - G_B(p_1)\Big) + \bar{\mu}_{s'}(x_G)(x_G - p_1)\Big(G_G(p_0) - G_G(p_1)\Big)\\ \leq&\bar{\mu}_{s'}(x_B)G_B(p_0)(p_0 - p_1) + \bar{\mu}_{s'}(x_G)G_G(p_0)(p_0 - p_1) \qquad \text{(A.2)}\end{aligned}$$

Note that $\bar{\mu}_{s'}(x_G) - \bar{\mu}_s(x_G) = \bar{\mu}_s(x_B) - \bar{\mu}_{s'}(x_B) \equiv \Delta\bar{\mu}$. Take the negative of both sides of (A.1) and then plus onto (A.2), we have

$$\begin{aligned}\Big(G_G(p_0) - G_B(p_0)\Big)(p_0 - p_1)\Delta\bar{\mu} \geq (x_G - p_1)&\Big(G_G(p_0) - G_G(p_1)\Big)\Delta\bar{\mu}\\ &- (x_B - p_1)\Big(G_B(p_0) - G_B(p_1)\Big)\Delta\bar{\mu}\end{aligned}$$

Or rather

$$\begin{aligned}\Big(G_G(p_0) - G_B(p_0)\Big)(p_0 - p_1) \geq (x_G - p_1)&\Big(G_G(p_0) - G_G(p_1)\Big)\\ &- (x_B - p_1)\Big(G_B(p_0) - G_B(p_1)\Big) \qquad \text{(A.3)}\end{aligned}$$

Plug (A.3) into (A.1) and use the fact that $\bar{\mu}_s(x_B) = 1 - \bar{\mu}_s(x_G)$, we conclude

$$(x_B - p_1)\Big(G_B(p_0) - G_B(p_1)\Big) \geq G_B(p_0)(p_0 - p_1)$$

Since $x_B < p_1$ but $p_0 > p_1$. The only way to make this true is to have

$$G_B(p_0) = G_B(p_1) = 0$$

□

Proof of Corollay 1

Proof. Suppose $G_G(p(s)) > 0$ and $p(s) < x_G$, in this case, by bidding $p(s)$, a buyer will make strictly positive profits since $G_B(p(s)) = 0$. So in equilibrium nobody should ever reject a candidate. Hence, in equilibrium, the seller sells the object with probability one. So the total surplus for all buyers must be zero. Thus, ex ante, each buyer gets value zero. This directly contradicts with the fact that buyers can earn positively by choosing $p(s)$ even without any screening.

Therefore, we know either $G_G(p(s)) = 0$ or $p(s) = x_G$. If $p(s) = x_G$, then $p(s') < x_G$. By the same logic above, we can conclude $G_G(p(s')) = 0$. If $G_G(p(s)) = 0$, then by the monotonicity of G_G as a function of p, we know $G_G(p(s')) = 0$. Therefore, in any case, we have $G_B(p(s')) = G_G(p(s')) = 0$. Effectively, the buyer rejects the candidate following signal s'. Hence, it is also optimal for the buyer to choose $p(s') = -\infty$ following signal realization s'. By item 1 of Lemma 1, this implies that $G_G(p(s)) = 0$.

□

Proof of Lemma 2

Proof. By Assumption 1, there exists function $K : \Delta(\mathbb{X}) \mapsto \mathbb{R}$ such that $C(\pi) = \mathbb{E}^{\bar{\mu}_s \sim \langle \pi \rangle}[K(\bar{\mu}_s)]$. By Gentzkow and Kamenica (2014), the screener's ex ante problem (1.4) is equivalent to

$$\max_{t \in \Delta\Delta X} \mathbb{E}^{\bar{\mu}_s \sim t}[V(\bar{\mu}_s)] - \xi \mathbb{E}^{\bar{\mu}_s \sim t}[K(\bar{\mu}_s)] \quad \text{s.t.} \quad \mathbb{E}^{\bar{\mu}_s \sim t}[\bar{\mu}_s] = \mu \tag{A.4}$$

where ξ is the Lagrangian multiplier on the constraint that $C(\pi) \leq A$, whose value depends on A. By Kamenica and Gentzkow (2011), the problem can be solved via the concavification approach. Call the solution t^*. Since $\#\mathbb{X} = 2$, so posterior can be summarized by one scaler. Therefore, there exists t^* such that $\#\text{supp}\{t^*\} \leq 2$.

Moreover, any posterior distribution t such that $\#\text{supp}\{t\} = 2$ and $\mathbb{E}^{\bar{\mu}_s \sim t}[\bar{\mu}_s] = \mu$ can be induced by a test $\pi : \mathbb{X} \mapsto \Delta(\mathbb{S})$ with $\#\mathbb{S} = 2$. So the only task remaining is to show that it is never strictly optimal to choose t with $\#\text{supp}\{t\} = 1$. However, $\#\text{supp}\{t\} = 1$ means that $\bar{\mu}_s = \mu$ almost surely, i.e., the screener does not distinguish the two types at all. As V defined in (1.2) must be convex in $\bar{\mu}_s$, by Jensen's inequality we know that any tests that contain two possible signal realizations weakly generate more value to the screener. Hence, a test with $\#\text{supp}\{t\} = 1$ cannot be strictly better than another test with $\#\text{supp}\{t'\} = 2$ as long as the latter is feasible.

□

Proof of Lemma 3

Proof. I first show that in equilibrium, no buyers would choose $p(0) > p(1) > -\infty$.

Suppose not. According to Corollary 1, we must have $G_B(p(0)) = G_G(p(0)) = G_B(p(1)) = G_G(p(1)) = 0$. Therefore, this buyer would get zero ex ante payoff. This then implies that in equilibrium every buyer has ex ante payoff zero.

Since $C(\pi_B, \pi_G)$ is continuous, $C(0,0) = 0 < A$, and $C(0,1) > A$, we know there must exist $\hat{\varpi} \in (0,1)$ such that $C(0,\hat{\varpi}) = A$. Consider strategy $(\pi_B, \pi_G) = (0, \hat{\varpi})$. Under this strategy, whenever the screener gets outcome 1, she would know that the candidate is good with probability one. As we know that by playing this strategy, the screener must get zero or negative ex ante payoff, we then know that $G_G(p) = 0$ for all $p \in [R, x_G)$. This implies that at least one of the other buyers must bid x_G with probability one whenever the object is good. Since no one can fully distinguish good from bad, this then means that the other player must also bid x_G with positive probability when the candidate is bad. Hence, the other buyer's ex ante payoff must be strictly negative. Contradiction.

Hence, we know in equilibrium, every buyer would choose $p(1) \geq p(0) \geq -\infty$. But due to the same logic, it is impossible to for someone to choose $p(1) = p(0) = -\infty$ in equilibrium. Therefore, we conclude $p(1) > p(0) \geq -\infty$.

Using the standard winner's curse argument, $p(1) > p(0)$ then implies that

$$\mathbb{E}^{\bar{\mu}}[X|\mathbb{I}_p = 1, \sigma^-] \leq \mathbb{E}^{\bar{\mu}}[X|\sigma^-] = \mathbb{E}^{\bar{\mu}}[X] \quad \forall p \in [R, x_G]$$

Therefore, for all $p \in [R, x_G]$

$$\begin{aligned}
\mathbb{E}^{\bar{\mu}_0}[(X-p)\mathbb{I}_p|\sigma^-] \leq& \mathbb{E}^{\bar{\mu}_0}[(X-R)\mathbb{I}_p|\sigma^-] \\
=& \mathbb{E}^{\bar{\mu}_0}[(X-R)|\mathbb{I}_p = 1, \sigma^-]\mathbb{P}^{\bar{\mu}_0}\{\mathbb{I}_p = 1|\sigma^-\} \\
\leq& \mathbb{E}^{\bar{\mu}_0}[(X-R)|\sigma^-]\mathbb{P}^{\bar{\mu}_0}\{\mathbb{I}_p = 1|\sigma^-\} \\
<& (\mathbb{E}^{\mu}[X] - R)\mathbb{P}^{\bar{\mu}_0}\{\mathbb{I}_p = 1|\sigma^-\} = 0
\end{aligned}$$

Thus, $p(0) = -\infty$. □

Proof of Lemma 4

Omitted, as the results are very standard.

Proof of Proposition 1

Proof.

1. Fix $\pi_B \in [0,1]$, from Lemma 4 we know that $C(\pi_B, \pi_G)$ is increasing in π_G for all $1 \geq \pi_G \geq \pi_B$. If $C(\pi_B, 1) \leq A$, then we can let $\overline{\phi}(\pi_B) = 1$. Otherwise, by $C(\pi_B, 1) > A > 0 = C(\pi_B, \pi_B)$ and the continuity of C, we know there must exists $1 > \overline{\phi}(\pi_B) > \pi_B$ such that $C\left(\pi_B, \overline{\phi}(\pi_B)\right) = A$. Moreover, by the monotonicity of $C(\pi_B, \pi_G)$ in the

region where $\pi_G \geq \pi_B$, we know that $\overline{\phi}(\pi_B)$ is unique, and $(\pi_B, \pi_G) \in \Phi$ if and only if $\overline{\phi}(\pi_B) \geq \pi_G$. The construction of $\underline{\phi}(\pi_B)$ can be done using exactly the same logic.

2. If $\overline{\phi}(\pi_B) < 1$, we know from item 1 that $C\left(\pi_B, \overline{\phi}(\pi_B)\right) = A$. By the symmetry result in Lemma 4, $C\left(1-\pi_B, 1-\overline{\phi}(\pi_B)\right) = A$. By item 1, we know that $\underline{\phi}(1-\pi_B) = 1-\overline{\phi}(\pi_B)$. If $\overline{\phi}(\pi_B) = 1$, then we know from item 1 that $C\left(\pi_B, \overline{\phi}(\pi_B)\right) \leq A$. By the symmetry result in Lemma 4, $C\left(1-\pi_B, 1-\overline{\phi}(\pi_B)\right) \leq A$. By item 1, we know that $\underline{\phi}(1-\pi_B) = 0 = 1-\overline{\phi}(\pi_B)$.

3. As $C(1,1) = 0$ and $C(0,1) > A$, by the continuity of C we know the existence of ϖ. By the monotonicity implied by Lemma 4, ϖ is unique. By item 1, for all $\pi_B \geq \varpi$, $\overline{\phi}(\pi_B) = 1$. By item 1, for all $\pi_B \leq \varpi$, $C\left(\pi_B, \overline{\phi}(\pi_B)\right) = A$. The continuity of $\overline{\phi}$ directly follows from the continuity of C in $[0,1]$. Since C is third-order continuously differentiable in $(0,\varpi)$, we know that $\overline{\phi}$ is also third-order continuously differentiable in this region.

4. We know that,
$$\overline{\phi}'(\pi_B) = \left.\frac{-\partial C/\partial \pi_B}{\partial C/\partial \pi_G}\right|_{(\pi_B, \overline{\phi}(\pi_B))}$$
Since $\overline{\phi}(\pi_B) > \pi_B$, by Lemma 4, $\partial C/\partial \pi_B < 0$ and $\partial C/\partial \pi_G > 0$. So $\overline{\phi}'(\pi_B) > 0$. The result that $\lim_{\pi_B \to 0} \overline{\phi}(\pi_B) = \infty$, $\lim_{\pi_B \to \varpi} \overline{\phi}(\pi_B) = 0$ directly follows by Assumption 3.

5. For any $\pi_B^1, \pi_B^2 \in [0, \varpi]$ and $\alpha \in (0,1)$, $C\left(\pi_B^1, \overline{\phi}\left(\pi_B^1\right)\right) = C\left(\pi_B^2, \overline{\phi}\left(\pi_B^2\right)\right) = A$. By the strict convexity of C,
$$\begin{aligned} & C\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2, \alpha\overline{\phi}\left(\pi_B^1\right) + (1-\alpha)\overline{\phi}\left(\pi_B^2\right)\right) < A \\ =& C\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2, \overline{\phi}\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2\right)\right) \end{aligned}$$
Obviously, $\alpha\overline{\phi}\left(\pi_B^1\right) + (1-\alpha)\overline{\phi}\left(\pi_B^2\right) \neq \overline{\phi}\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2\right)$. Now, suppose $\alpha\overline{\phi}\left(\pi_B^1\right) +$

$(1-\alpha)\overline{\phi}\left(\pi_B^2\right) > \overline{\phi}\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2\right)$, since $\overline{\phi}\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2\right) > \alpha\pi_B^1 + (1-\alpha)\pi_B^2$ by item 1, from Lemma 4, we get contradiction. Hence

$$\alpha\overline{\phi}\left(\pi_B^1\right) + (1-\alpha)\overline{\phi}\left(\pi_B^2\right) < \overline{\phi}\left(\alpha\pi_B^1 + (1-\alpha)\pi_B^2\right)$$

□

Proof of Lemma 5

Proof. Note that

$$\frac{d}{d\pi}\frac{\overline{\phi}(\pi)}{\pi} = \frac{\overline{\phi}'(\pi)\pi - \overline{\phi}(\pi)}{\pi^2}$$

So it would be sufficient to show that $\overline{\phi}'(\pi)\pi - \overline{\phi}(\pi) < 0$ for all $\pi \in (0, \varpi)$. Note also that

$$\frac{d}{d\pi}\left(\overline{\phi}'(\pi)\pi - \overline{\phi}(\pi)\right) = \overline{\phi}''(\pi) < 0$$

So

$$\overline{\phi}'(\pi)\pi - \overline{\phi}(\pi) \leq \overline{\phi}'(0)0 - \overline{\phi}(0) = -\overline{\phi}(0) < 0$$

The decreasing property of $\frac{1-\overline{\phi}(\pi_B)}{1-\pi_B}$ can be shown by symmetry.

□

Proof of Proposition 2

Proof. To begin with, I first characterize the social planner's iso-welfare curve. Using (1.1), the iso-welfare curve that generates welfare w is

$$\left\{(\pi_B, \pi_G) \in [0,1]^2 | -\mu_B(1-\pi_B)^N(x_B - R) - \mu_G(1-\pi_G)^N(x_G - R) = w\right\} \tag{A.5}$$

As the social planner can always get welfare 0 by choosing any point on the dashed line in Figure 1.1, the solution to the social planner's problem must sit on an iso-welfare curve with $w > 0$. It turns out that For any $w > 0$, the iso-welfare curve is convex.

To see so, rewrite (A.5) as

$$\pi_G = 1 - \left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{1/N}$$

We then have

$$\frac{d\pi_G}{d\pi_B} = -\left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{\frac{1}{N}-1} \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-1} \tag{A.6}$$

$$\begin{aligned}\frac{d^2\pi_G}{d\pi_B^2} &= -\left(\frac{1}{N} - 1\right)\left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{\frac{1}{N}-2} N\left(\frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-1}\right)^2 \\ &\quad - \left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{\frac{1}{N}-1} (N-1)\frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-2}\end{aligned}$$

To show that $\frac{d^2\pi_G}{d\pi_B^2} > 0$ is equivalent to show that

$$\begin{aligned}&(N-1)\left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{\frac{1}{N}-2} \left(\frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-1}\right)^2 \\ &> \left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)^{\frac{1}{N}-1} (N-1)\frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-2}\end{aligned}$$

or rather

$$\begin{aligned}&\left(\frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-1}\right)^2 \\ &> \left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right) \frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^{N-2}\end{aligned}$$

or rather

$$\frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N > \left(-\frac{w}{(x_G - R)\mu_G} - \frac{(x_B - R)\mu_B}{(x_G - R)\mu_G}(1 - \pi_B)^N\right)$$

which is obviously true for all $w > 0$.

Since the social planner's iso-welfare curve is convex and the upper bound of the feasible set is concave, the solution exists and unique. Moreover, the solution is pinned down by the first order condition, which is given by (1.7).

□

Proof of Proposition 3

Proof.

1. Suppose there is a pure strategy symmetric equilibrium where every screener chooses to do $(\tilde{\pi}_B, \tilde{\pi}_G, \tilde{p})$ with $\tilde{\pi}_G > \tilde{\pi}_B \geq 0$ and $\tilde{p} \geq R$. Without loss of generality, consider the problem faced by screener $n = 1$. Rewrite the objective function in (1.8) as

$$\mathbb{E}\left[X - p|\mathbb{J}_{\pi,p} = 1, \sigma^-\right] \mathbb{P}\{\mathbb{J}_{\pi,p} = 1|\sigma^-\}$$

Recall that $\mathbb{J}$ is a random variable, it maps $\mathbb{S} \times \mathcal{P}^{N-1} \times \Omega$ to $\{0, 1\}$. Since this is a pure strategy equilibrium, we know that for $p^- \in \mathcal{P}^{N-1}$, elements in p^- is either $\tilde{p}$ or $-\infty$. In the following, I use notation $\mathbb{J}_{\pi,p}(s, p^-, \omega)$ to denote the mapping result of $(s, p^-, \omega) \in \mathbb{S} \times \mathcal{P}^{N-1} \times \Omega$.

Consider a deviation for screener $n = 1$, $(\tilde{\pi}, \tilde{p} + \varepsilon)$ for some $\varepsilon > 0$. Then,

$$0 = \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(0, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(0, p^-, \omega) \quad \forall p^- \in \{-\infty, \tilde{p}\}^{N-1}, \forall \omega \in \Omega$$
$$1 = \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) \quad \forall p^- \in \{-\infty, \tilde{p}\}^{N-1}, \forall \omega \in \Omega_1(p^-)$$
$$1 = \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) > \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) = 0 \quad \forall p^- \in \{-\infty, \tilde{p}\}^{N-1}, \forall \omega \in \Omega \setminus \Omega_1(p^-)$$

where $\Omega_1(p^-)$ is defined as the subset of Ω such that screener $n = 1$ can win the bid by bidding $\tilde{p}$ given p^-. Obviously, for any p^-, $\Omega_1(p^-) \neq \emptyset$ and has positive measure.

Now, drive $\varepsilon \downarrow 0$, we conclude

$$\lim_{\varepsilon \downarrow 0} \mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon} = 1|\sigma^-\} > \mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}} = 1|\sigma^-\} \tag{A.7}$$

Note also that

$$\mathbb{E}\left[X - \tilde{p} - \varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon} = 1, \sigma^-\right] = \sum_{\omega\in\Omega} \mathbb{E}\left[X - \tilde{p} - \varepsilon|s = 1, \omega, \sigma^-\right] \mathbb{P}\{\omega\}$$

while

$$\mathbb{E}\left[X - \tilde{p} - \varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}} = 1, \sigma^-\right] = \sum_{\omega\in\Omega} \mathbb{E}\left[X - \tilde{p} - \varepsilon|s = 1, p^- \in \Omega_1^{-1}(\omega), \omega, \sigma^-\right] \mathbb{P}\{\omega\}$$

where

$$\Omega_1^{-1}(\omega) \equiv \left\{p^- \in \{-\infty, \tilde{p}\}^{N-1} : \omega \in \Omega_1(p^-)\right\}$$

I will now argue that for any given ω,

$$\mathbb{E}[X|s = 1, \omega, \sigma^-] \geq \mathbb{E}\left[X|s = 1, p^- \in \Omega_1^{-1}(\omega), \omega, \sigma^-\right]$$

If screener $n = 1$ ranks the first in ω, then the two terms equal to each other in the above expression. If screener $n = 1$ is not ranking the first in ω, then this is equivalent to show that

$$\mathbb{E}[X|s = 1, p^- \notin \Omega_1^{-1}(\omega), \omega] \geq \mathbb{E}\left[X|s = 1, p^- \in \Omega_1^{-1}(\omega), \omega\right]$$

But this just equivalent to

$$\begin{aligned}&\mathbb{E}[X|s=1, \text{at least one screener ranked prior to n=1 sees Pass}, \omega, \sigma^-]\\ \geq&\mathbb{E}[X|s=1, \text{all screeners ranked prior to n=1 sees Fail}, \omega, \sigma^-]\end{aligned}$$

This will hold as long as $\tilde{\pi}_G > \tilde{\pi}_B$.

Therefore, we conclude

$$\mathbb{E}\left[X-\tilde{p}-\varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}=1,\sigma^-\right] > \mathbb{E}\left[X-\tilde{p}-\varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right]$$

for all $\varepsilon > 0$. Thus

$$\lim_{\varepsilon\downarrow 0}\mathbb{E}\left[X-\tilde{p}-\varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}=1,\sigma^-\right] \geq \mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right] \tag{A.8}$$

Suppose $\mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right] > 0$ then, by combining (A.7) and (A.8) we know

$$\lim_{\varepsilon\downarrow 0}\mathbb{E}\left[X-\tilde{p}-\varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}=1,\sigma^-\right]\mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}=1|\sigma^-\} > \mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right]\mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}}=1|\sigma^-\}$$

Therefore, there must exist $\varepsilon > 0$ such that $(\tilde{\pi}, \tilde{p}+\varepsilon)$ is a profitable deviation.

Thus, if $(\tilde{\pi}, \tilde{p})$ is an equilibrium it must be the case that $\mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right] \leq 0$. But since rejecting everyone is always feasible, in equilibrium $\mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right]$ cannot be negative. Thus, we conclude

$$\mathbb{E}\left[X-\tilde{p}|\mathbb{J}_{\tilde{\pi},\tilde{p}}=1,\sigma^-\right]=0$$

Now, consider a deviation $\pi_B = 0, \pi_G = \overline{\phi}(0)$ and $p = \tilde{p}$. This deviation will earn the deviator strictly positive profit unless $\tilde{\pi}_B = \varpi, \tilde{\pi}_G = 1$ and $\tilde{p} = x_G$. But this is impossible, as with $\tilde{\pi}_B = \varpi, \tilde{\pi}_G = 1$ and $\tilde{p} = x_G$, $\mathbb{E}\left[X - \tilde{p} | \mathbb{J}_{\tilde{\pi},\tilde{p}} = 1, \sigma^-\right] < 0$.

2. Denote $\mathcal{P}_*$ as the support of the marginal distribution of equilibrium bidding strategy. Suppose the marginal distribution of bidding choice has a mass point $\tilde{p} > -\infty$. Suppose also that $\tilde{\pi}$ is one optimal screening choice align with $\tilde{p}$. Since the cumulative density function on equilibrium price must be right continuous, there must exist $\delta > 0$ such that there is no more mass point in the region $(\tilde{p}, \tilde{p} + \delta)$. Consider $0 < \varepsilon < \delta$ and strategy $(\tilde{\pi}, \tilde{p} + \varepsilon)$. We have

$$
\begin{aligned}
0 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(0, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(0, p^-, \omega) \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega \\
1 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega \quad \text{s.t.} \quad \max\{p^-\} < \tilde{p} \\
1 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega_1(p^-) \quad \text{s.t.} \quad \max\{p^-\} = \tilde{p} \\
1 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) > \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) = 0 \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega \setminus \Omega_1(p^-) \quad \text{s.t.} \quad \max\{p^-\} = \tilde{p} \\
0 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) \quad \forall p^- \in \mathcal{P}_*^{N-1} \quad \text{s.t.} \quad \max\{p^-\} > \tilde{p} + \varepsilon \\
0 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) = \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega \setminus \Omega_2^{\varepsilon}(p^-) \quad \text{s.t.} \quad \max\{p^-\} = \tilde{p} + \varepsilon \\
1 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) > \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) = 0 \quad \forall p^- \in \mathcal{P}_*^{N-1}, \omega \in \Omega_2^{\varepsilon}(p^-) \quad \text{s.t.} \quad \max\{p^-\} = \tilde{p} + \varepsilon \\
1 =& \mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon}(1, p^-, \omega) > \mathbb{J}_{\tilde{\pi},\tilde{p}}(1, p^-, \omega) = 0 \quad \forall p^- \in \mathcal{P}_*^{N-1} \quad \text{s.t.} \quad \tilde{p} < \max\{p^-\} < \tilde{p} + \varepsilon
\end{aligned}
$$

Here, $\Omega_1(p^-)$ is defined as the subset of Ω such that screener 1 can win the bid by bidding $\tilde{p}$ given $\max\{p^-\} = \tilde{p}$; $\Omega_2^{\varepsilon}(p^-)$ is defined as the subset of Ω such that screener 1 can win the bid by bidding $\tilde{p} + \varepsilon$ given $\max\{p^-\} = \tilde{p} + \varepsilon$.

Since we consider $\varepsilon < \delta$, the sixth case and the seventh case have zero measure. When we drive $\varepsilon \downarrow 0$, the measure of the fifth and eighth cases also converge to zero. But as long as the fourth case has strictly positive measure, which is true when $\tilde{p}$ is a mass point, we conclude

$$\lim_{\varepsilon \downarrow 0} \mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon} = 1|\sigma^-\} > \mathbb{P}\{\mathbb{J}_{\tilde{\pi},\tilde{p}} = 1|\sigma^-\}$$

On the other hand, as $\varepsilon < \delta$

$$\begin{aligned}
\mathbb{E}[X|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon} = 1, \sigma^-] &= \mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p} + \varepsilon, \sigma^-] \\
&\xrightarrow{\varepsilon \downarrow 0} \mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p}, \sigma^-] \\
&= \sum_{\omega} \mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p}, \sigma^-, \omega]\mathbb{P}\{\omega\}
\end{aligned}$$

On the contrary,

$$\mathbb{E}[X|\mathbb{J}_{\tilde{\pi},\tilde{p}} = 1, \sigma^-] = \sum_{\omega} \mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p}, p^- \in \Omega_1^{-1}(\omega), \omega, \sigma^-]\mathbb{P}\{\omega\}$$

Using Corollary 1, one can easily show that for any ω,

$$\mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p}, \omega] \geq \mathbb{E}[X|s = 1, \max\{p^-\} \leq \tilde{p}, p^- \in \Omega_1^{-1}(\omega), \omega]$$

as this is just equivalent to

$$\begin{aligned}
&\mathbb{E}[X|s = 1, \text{at least one screener ranked prior to n=1 have better posterior}, \omega, \sigma^-] \\
\geq &\mathbb{E}[X|s = 1, \text{all screeners ranked prior to n=1 have worse posterior}, \omega, \sigma^-]
\end{aligned}$$

Therefore, we conclude

$$\lim_{\varepsilon \downarrow 0} \mathbb{E}\left[X - p - \varepsilon|\mathbb{J}_{\tilde{\pi},\tilde{p}+\varepsilon} = 1, \sigma^-\right] \geq \mathbb{E}\left[X - p|\mathbb{J}_{\tilde{\pi},\tilde{p}} = 1, \sigma^-\right]$$

One can then follow exactly the same argument in part 1 to reach contradiction.

□

Proof of Theorem 1

Proof. In this proof I will show a slightly stronger claim. That is, there always exists a mixed strategy symmetric equilibrium where F has probability density function f. In this case, regulation (1.15) is equivalent to

$$f(p) > 0 \quad \forall p \in [R, P] \qquad \text{and} \qquad \int_R^P f(p)dp = 1 \tag{A.9}$$

In what follows, I omit the $*$ superscript on π_B^* and π_G^* for notation simplicity.

Define function on domain $[0, \varpi]$:

$$\varphi(u) \equiv \overline{\phi}(u) - \overline{\phi}'(u)u$$

Obverse that φ is continuous on $[0, \varpi]$ and is strictly increasing since $\varphi'(u) = -\overline{\phi}''(u)u > 0$ for $u > 0$. Also observe that $\varphi(u) > 0$ for any $u > 0$. This is because $\frac{d}{du}\left(\frac{\overline{\phi}(u)}{\overline{\phi}'(u)} - u\right) > 0$ and $\lim_{u \downarrow 0}\left(\frac{\overline{\phi}(u)}{\overline{\phi}'(u)} - u\right) = 0$ as $\overline{\phi}'(0) = \infty$.

Further define

$$\tilde{G}_B(p) \equiv G_B(p)^{\frac{1}{N-1}}$$
$$\tilde{G}_G(p) \equiv G_G(p)^{\frac{1}{N-1}}$$

I now rewrite equilibrium conditions using the definitions above. Condition (1.10) is equivalent to

$$\tilde{G}'_B(p) = \pi_B(p)f(p) \quad \text{plus boundary condition} \quad \tilde{G}_B(P) = 1 \tag{1.10'}$$

Condition (1.11) is equivalent to

$$\tilde{G}'_G(p) = \overline{\phi}\big(\pi_B(p)\big)f(p) \quad \text{plus boundary condition} \quad \tilde{G}_G(P) = 1 \tag{1.11'}$$

Using (1.12), condition (1.14) is equivalent to

$$\tilde{G}_G(p) = \left(\frac{Y}{(x_G - p)\mu_G\varphi\big(\pi_B(p)\big)}\right)^{\frac{1}{N-1}} \quad \forall p \in [R, P]$$

Since it is true for all p, this is equivalent to

$$\tilde{G}'_G(p) = \frac{d}{dp}\left[\left(\frac{Y}{(x_G - p)\mu_G\varphi\big(\pi_B(p)\big)}\right)^{\frac{1}{N-1}}\right] \quad \text{plus boundary condition} Y = (x_G - P)\mu_G\varphi\big(\pi_B(P)\big) \tag{1.14'}$$

Lastly, Using (1.14), condition (1.12) is equivalent to

$$\tilde{G}_B(p) = \left(\overline{\phi}'\big(\pi_B(p)\big)\frac{Y}{\varphi\big(\pi_B(p)\big)(p - x_B)\mu_B}\right)^{\frac{1}{N-1}} \quad \forall p \in [R, P]$$

or rather

$$\tilde{G}'_B(p) = \frac{d}{dp}\left[\left(\overline{\phi}'\big(\pi_B(p)\big)\frac{Y}{\varphi\big(\pi_B(p)\big)(p - x_B)\mu_B}\right)^{\frac{1}{N-1}}\right] \quad \text{plus boundary condition}$$
$$Y = \frac{\varphi\big(\pi_B(P)\big)(P - x_B)\mu_B}{\overline{\phi}'\big(\pi_B(P)\big)} \tag{1.12'}$$

Plug (1.14') and (1.12') into (1.10') and (1.11'), we get

$$\pi_B(p)f(p) = \frac{d}{dp}\left[\left(\overline{\phi}'\big(\pi_B(p)\big)\frac{Y}{\varphi\big(\pi_B(p)\big)}\frac{1}{(p-x_B)\mu_B}\right)^{\frac{1}{N-1}}\right] \tag{1.12"}$$

$$\overline{\phi}\big(\pi_B(p)\big)f(p) = \frac{d}{dp}\left[\left(\frac{Y}{\varphi\big(\pi_B(p)\big)}\frac{1}{(x_G-p)\mu_G}\right)^{\frac{1}{N-1}}\right] \tag{1.14"}$$

Therefore, if we can find $\pi_B(p), f(p)$ and P, Y such that (1.12") and (1.14") hold and

$$Y = (x_G - P)\mu_G\varphi\big(\pi_B(P)\big) = \frac{\varphi\big(\pi_B(P)\big)(P-x_B)\mu_B}{\overline{\phi}'\big(\pi_B(P)\big)} \tag{A.10}$$

then we must be able to find $\pi_G(p), G_B(p), G_G(p)$ that together with $\pi_B(p), f(p)$ and P satisfy all restrictions for equilibrium, provided that $f(p)$ and P satisfy (1.16) and (A.9). Note that (A.10) implies that (1.17) must hold as we have shown $\varphi \geq 0$ and impose $P \leq x_G$.

Take ratio of (1.12") and (1.14"):

$$\frac{\pi_B(p)}{\overline{\phi}\big(\pi_B(p)\big)} = \frac{\frac{d}{dp}\left[\left(\overline{\phi}'\big(\pi_B(p)\big)\frac{1}{\varphi\big(\pi_B(p)\big)}\frac{1}{(p-x_B)\mu_B}\right)^{\frac{1}{N-1}}\right]}{\frac{d}{dp}\left[\left(\frac{1}{\varphi\big(\pi_B(p)\big)}\frac{1}{(x_G-p)\mu_G}\right)^{\frac{1}{N-1}}\right]} \tag{1.18}$$

This is a first-order ODE on $\pi_B(p)$. The boundary condition is given by (A.10):

$$\frac{P-x_B}{x_G-P}\frac{\mu_B}{\mu_G} = \overline{\phi}'\big(\pi_B(P)\big) \tag{1.19}$$

Rewrite (1.18) as

$$\pi_B'(p) = \frac{(x_G-p)^{-\frac{1}{N-1}-1}[\mu_G\varphi(\pi_B)]^{-\frac{1}{N-1}} + (p-x_B)^{-\frac{1}{N-1}-1}[\mu_B\varphi(\pi_B)]^{-\frac{1}{N-1}}\frac{\overline{\phi}(\pi_B)}{\pi_B}\overline{\phi}'(\pi_B)^{\frac{1}{N-1}}}{\left([(p-x_B)\mu_B\varphi(\pi_B)]^{-\frac{1}{N-1}}\left[\left(\overline{\phi}'(\pi_B)\pi_B\right)^{-1}+\varphi(\pi_B)^{-1}\right]\frac{\overline{\phi}(\pi_B)}{\pi_B}\overline{\phi}'(\pi_B)^{\frac{1}{N-1}} - [(x_G-p)\mu_G\varphi(\pi_B)]^{-\frac{1}{N-1}}\varphi(\pi_B)^{-1}\right)\overline{\phi}''(\pi_B)\pi_B} \tag{1.18'}$$

Observe that take any point $\left(P, \pi_B(P)\right)$ that satisfies (1.19) and $R \leq P < x_G$ and $\pi_B(P) \in (0, \varpi)$, there must exist a closed rectangular neighborhood of the point such that in this neighborhood the RHS of (1.18') is continuous in p and is Lipschitz continuous in π_B. This is because $\overline{\phi}$ is third-order continuous differentiable in $(0, \varpi)$, so the RHS has continuous partial derivative with respect to π_B in the closed rectangular neighborhood. Therefore, by Cauchy-Lipschitz Theorem, we know that the ODE (1.18') locally has unique solution in region $[P - \epsilon_1, P + \epsilon_1]$ for some $\epsilon_1 > 0$.

Use the same argument at point $P - \epsilon_1$, we can extend the unique solution of the ODE back to $P - \epsilon_2$ for some $\epsilon_2 > \epsilon_1$ as long as at $P - \epsilon_1$ we can find a closed rectangular neighborhood so that in the neighborhood $\pi_B \in (0, \varpi)$ and $p > x_B$. Suppose following this procedure we can find the unique solution of the ODE in region $(\hat{P}, P]$ up to point $\hat{P}$. Then it must be the case that at $\hat{P}$, either $\hat{P} = x_B$, or $\lim_{p \downarrow \hat{P}} \pi_B(p) = 0$ or ${}_{p \downarrow \hat{P}} \pi_B(p) = \varpi$. Moreover, following this procedure, we know that in region $(\hat{P}, P]$, π_B never reaches 0 or ϖ. Clearly, $\hat{P}$ is a function of P. Denote this as $\hat{P}(P)$.

Therefore, given any $R \leq P < x_G$, we have found the unique solution of the ODE in region $\left(\hat{P}(P), P\right]$. Call this solution $\pi_B(p; P)$. Armed with $\pi_B(p; P)$, one can also construct $f(p; P)$, $\tilde{G}_B(p; P)$ and $\tilde{G}_G(p; P)$ by (1.14"), (1.10'), and (1.11') respectively. By construction, (1.12) is satisfied.

I now argue that the solved $f(p; P)$ is strictly positive. In order to do so, I first show that $\pi'_B(p; P) < 0$ for all $p \in \left(\hat{P}(P), P\right]$. Note that at P, the term

$$[(P - x_B)\mu_B]^{-\frac{1}{N-1}} \overline{\phi}'(\pi_B)^{\frac{1}{N-1}} \frac{\overline{\phi}(\pi_B)}{\pi_B} - [(x_G - P)\mu_G]^{-\frac{1}{N-1}}$$

which is part of the denominator of (1.18'), is strictly positive according to (1.19). Therefore,

$\pi'_B(P;P) < 0$ since $\overline{\phi}'' < 0$ and all other terms are strictly positive.

Given the continuity of $\pi'_B(p)$ in $\left(\hat{P}(P), P\right]$ (the RHS of (1.18') is continuous), we know that if at some point $\pi'_B(p;P) \geq 0$, then there must be a point where $\pi'_B(p;P) = 0$. This cannot happen at any point p as long as $0 < \pi_B(p) < \varpi$ since the numerator in (1.18') will be strictly positive. Therefore, before reaching $\hat{P}(P)$, we must have $\pi'_B(p) < 0$ for all $\hat{P}(P) < p \leq P$.

By (1.14"), $f(p;P)$ has the same sign as

$$\frac{d}{dp}\left[\left(\frac{1}{\varphi\left(\pi_B(p;P)\right)}\frac{1}{(x_G-p)\mu_G}\right)^{\frac{1}{N-1}}\right]$$
$$=\frac{1}{N-1}[(x_G-p)\mu_G\varphi(\pi_B)]^{-\frac{1}{N-1}-1}\left(\mu_G\varphi(\pi_B)+(x_G-p)\mu_G\overline{\phi}''(\pi_B)\pi'_B(p)\pi_B\right)$$

Thus, $f(p;P) > 0$ in $\left(\hat{P}(P), P\right]$.

For any given P, since $\pi'_B(p;P) < 0$ for all $\hat{P}(P) < p \leq P$, we know that it is impossible to have $\lim_{p\downarrow\hat{P}} \pi_B(p;P) = 0$. Suppose $\lim_{p\downarrow\hat{P}} \pi_B(p;P) = \varpi$. Then by (1.12),

$$\lim_{p\downarrow\hat{P}} \frac{(p-x_B)\mu_B}{(x_G-p)\mu_G}\frac{G_B(p)}{G_G(p)} = \lim_{p\downarrow\hat{P}} \overline{\phi}'\left(\pi_B(p;P)\right) = 0$$

Thus, either $\hat{P}(P) = x_B$, or $\lim_{p\downarrow\hat{P}} \tilde{G}_B\left(p;P\right) = 0$, or $\lim_{p\downarrow\hat{P}} \tilde{G}_G\left(p;P\right) = \infty$. Obviously, it is impossible to have $\lim_{p\downarrow\hat{P}} \tilde{G}_G\left(p;P\right) = \infty$ since along the path $\tilde{G}'_G(p;P) > 0$ by (1.10') and we start from $\tilde{G}_G(P;P) = 1$.

If $\lim_{p\downarrow\hat{P}} G_B(p;P) = 0$, then we know

$$1 = \int_{\hat{P}(P)}^{P} \pi_B(p;P) f(p;P) dp < \int_{\hat{P}(P)}^{P} \overline{\phi}\big(\pi_B(p;P)\big) f(p;P) dp = 1 - \lim_{p\downarrow\hat{P}} \tilde{G}_G(p;P)$$

Thus, $\lim_{p\downarrow\hat{P}} \tilde{G}_G(p;P) < 0$. Thus, there must exist $P > \tilde{p} > \hat{P}(P)$ such that $\tilde{G}_G(\tilde{p};P) = 0$. Thus, at $\tilde{p}$, we should have $\pi_B(\tilde{p};P) = 0$. This contradicts with the conclusion that $\pi_B(p;P)$ is increasing as p decreases from P to $\hat{P}(P)$.

Thus, it must be the case that $\hat{P}(P) = x_B$. That being said, starting from any $P \in [R, x_G)$, the solved path of $\pi_B(p)$ can always go back to x_B. Put differently, for any $P \in [R, x_G)$, there exists a solution $\pi_B(p;P)$ that satisfy (1.18) and (1.19) in domain $[R, P]$.

For any given $\eta > 0$ (a very small number), I now show that $\pi_B(p;P)$ is continuous in $P \in [p, x_G - \eta]$ for any given $p \in [R, x_G - \eta]$. Define the RHS of (1.18') to be $h(p, \pi_B)$. Consider rectangle $[R, x_G - \eta] \times [\pi_B(R;R), \pi_B(x_G - \eta; x_G - \eta)]$. Note that

$$\frac{(P - x_B)\mu_B}{(x_G - P)\mu_G} = \overline{\phi}\big(\pi_B(P;P)\big)$$

so $\pi_B(P;P)$ must be increasing in P. Hence the rectangle covers all $(P, \pi_B(P;P))$ for $P \in [p, x_G - \eta]$.

Since h and $\frac{\partial h}{\partial \pi_B}$ are bounded in the rectangle, h satisfies Lipschitz condition in the rectangle, i.e. $\exists L$ such that $|h(p, \pi_B) - h(p, \pi'_B)| \leq L|\pi_B - \pi'_B|$. Call the bound of $|h|$ value H.

The ODE itself implies

$$\pi_B(p;P) = \pi_B(P;P) + \int_p^P h\big(\tilde{p}, \pi_B(p;P)\big) d\tilde{p}$$
$$\pi_B(p;P') = \pi_B(P';P') + \int_p^{P'} h\big(\tilde{p}, \pi_B(p;P')\big) d\tilde{p}$$

Thus, we have

$$\begin{aligned}
&|\pi_B(p;P) - \pi_B(p;P')| \\
&\leq |\pi_B(P;P) - \pi_B(P';P')| + \int_p^{\min\{P,P'\}} |h\big(\tilde{p}, \pi_B(p;P)\big) - h\big(\tilde{p}, \pi_B(p;P')\big)| d\tilde{p} \\
&\quad + \left| \int_{P'}^{P} |h\big(\tilde{p}, \pi_B(p;P')\big)| d\tilde{p} \right| \\
&\leq |\pi_B(P;P) - \pi_B(P';P')| + \int_p^{\min\{P,P'\}} L|\pi_B(\tilde{p};P) - \pi_B(\tilde{p};P')| d\tilde{p} + H|P - P'| \\
&\leq |\pi_B(P;P) - \pi_B(P';P')| + L\left(\min\{P,P'\} - p\right) + H|P - P'|
\end{aligned}$$

Plug back the last line to the second last line, we get

$$\begin{aligned}
&|\pi_B(p;P) - \pi_B(p;P')| \\
&\leq \Big(|\pi_B(P;P) - \pi_B(P';P')| + H|P - P'|\Big)\Big(1 + L\left(\min\{P,P'\} - p\right)\Big) \\
&\quad + \int_p^{\min\{P,P'\}} L^2 \left(\min\{P,P'\} - \tilde{p}\right) d\tilde{p} \\
&= \Big(|\pi_B(P;P) - \pi_B(P';P')| + H|P - P'|\Big)\Big(1 + L\left(\min\{P,P'\} - p\right)\Big) + \frac{1}{2}L^2 \left(\min\{P,P'\} - p\right)^2
\end{aligned}$$

Plug back for n times, we get

$$\begin{aligned}
&|\pi_B(p;P) - \pi_B(p;P')| \\
&\leq \Big(|\pi_B(P;P) - \pi_B(P';P')| + H|P - P'|\Big) \sum_{m=0}^{n-1} \frac{1}{m!} L^m \left(\min\{P,P'\} - p\right)^m + \frac{1}{n!} L^n \left(\min\{P,P'\} - p\right)^n
\end{aligned}$$

Since $\lim_{n\to\infty} \frac{1}{n!} L^n \left(\min\{P,P'\} - p\right)^n = 0$, we know there must exist $\bar{n}$ such that

$$\frac{1}{\bar{n}!} L^n \left(\min\{P,P'\} - p\right)^{\bar{n}} < \frac{\varepsilon}{2}$$

for a given $\varepsilon > 0$. We can finish the proof by choosing $\delta > 0$ such that $|P - P'| < \delta \Rightarrow \Big(|\pi_B(P;P) - \pi_B(P';P')| + H|P - P'|\Big) \sum_{m=0}^{\bar{n}-1} \frac{1}{m!} L^m \left(\min\{P,P'\} - p\right)^m < \frac{\varepsilon}{2}$ (note that this is feasible since $\pi_B(P;P)$ is continuous in P).

Up to now, I prove that given $\eta > 0$ and $p \in [R, x_G - \eta]$, $\pi_B(p;P)$ is continuous in $P \in [p, x_G - \eta]$. Since this is true for any $\eta > 0$, we conclude that $\pi_B(p;P)$ is continuous in $P \in [p, x_G)$.

The last task is to show that there always exist $P \in (R, x_G)$ such that (A.9) holds. Note that due to the continuity of $\pi_B(p;P)$ in P, $\int_R^P f(p;P)dp$ is continuous in P and $\int_R^R f(p;R)dp = 0$. So if there exists $\tilde{P} \in (R, x_G)$ such that $\int_R^{\tilde{P}} f(p;\tilde{P})dp > 1$, then by continuity there must exist P such that (A.9) holds.

I first argue that $\pi_B(R;R) > \pi_B(R;P)$ for all $P \in (R, x_G)$. Note that

$$\frac{(R - x_B)\mu_B}{(x_G - R)\mu_G} = \overline{\phi}\big(\pi_B(R;R)\big)$$

while

$$\frac{(R - x_B)\mu_B G_B(R;P)}{(x_G - R)\mu_G G_G(R;P)} = \overline{\phi}\big(\pi_B(R;P)\big)$$

where

$$G_B(R;P) = \left(1 - \int_R^P \pi_B(p;P) f(p;P) dp\right)^{N-1}$$
$$G_G(R;P) = \left(1 - \int_R^P \overline{\phi}\big(\pi_B(p;P)\big) f(p;P) dp\right)^{N-1}$$

Therefore, $G_G(R;P) < G_B(R;P)$, which leads to $\overline{\phi}\big(\pi_B(R;P)\big) < \overline{\phi}\big(\pi_B(R;R)\big)$. By monotonicity of $\overline{\phi}$, we conclude $\pi_B(R;P) < \pi_B(R;R)$.

Now, by (1.14"), for any given $P \in (R, x_G)$

$$f(p;P) = \frac{1}{\overline{\phi}\big(\pi_B(p;P)\big)} \frac{d}{dp}\left[\left(\frac{Y}{\varphi\big(\pi_B(p;P)\big)} \frac{1}{(x_G - p)\mu_G}\right)^{\frac{1}{N-1}}\right]$$
$$> \frac{1}{\overline{\phi}\big(\pi_B(R;R)\big)} \frac{d}{dp}\left[\left(\frac{Y}{\varphi\big(\pi_B(p;P)\big)} \frac{1}{(x_G - p)\mu_G}\right)^{\frac{1}{N-1}}\right]$$

Thus,

$$\int_R^P f(p;P) dp > \frac{1}{\overline{\phi}\big(\pi_B(R;R)\big)}\left[1 - \left(\frac{\varphi\big(\pi_B(P;P)\big)}{\varphi\big(\pi_B(R;P)\big)} \frac{x_G - P}{x_G - R}\right)^{\frac{1}{N-1}}\right]$$

Therefore, it is sufficient if we can show there exists $P \in (R, x_G)$ such that

$$\frac{\varphi\big(\pi_B(P;P)\big)}{\varphi\big(\pi_B(R;P)\big)} \frac{x_G - P}{x_G - R} = \big[1 - \overline{\phi}\big(\pi_B(R;R)\big)\big]^{N-1} \tag{A.11}$$

When $P \downarrow R$, the LHS approaches 1. When $P \uparrow x_G$, the term $\frac{x_G - P}{x_G - R}$ approaches 0. On the other hand, since for any $P > R$, $\frac{\varphi\left(\pi_B(P;P)\right)}{\varphi\left(\pi_B(R;P)\right)} < 1$, we know that

$$\lim_{P \uparrow x_G} \frac{\varphi\left(\pi_B(P;P)\right)}{\varphi\left(\pi_B(R;P)\right)} \leq 1$$

Thus, the LHS approaches zero when $P \uparrow x_G$. Since $\left[1 - \overline{\phi}\left(\pi_B(R;R)\right)\right]^{N-1} \in (0,1)$, by continuity we know there must exist $P \in (R, x_G)$ such that (A.11) holds.

□

Proof of Theorem 2

It is useful to start with the following lemma, which is closely related to Corollary 1:

Lemma 11. *In any mixed strategy binary-signal symmetric equilibrium, $\pi_B^*(p)$ and $\pi_G^*(p)$ are decreasing functions of p.*

Proof. Since $F(p)$ is increasing in p, from (1.10) and (1.11), $G_B(p)$ and $G_G(p)$ are increasing in p. So we know that $\mu_B G_B(p)(p - x_B)$ is increasing in p.

By (1.14), we know that

$$\pi_G^*(p) - \frac{\mu_B \pi_B^*(p) G_B(p)(p - x_B)}{\mu_G G_G(p)(x_G - P)} = \frac{Y}{\mu_G G_G(p)(x_G - p)}$$

(1.12) and (1.13) further implies that

$$\overline{\phi}\left(\pi_B^*(p)\right) - \pi_B^*(p)\overline{\phi}'\left(\pi_B^*(p)\right) = \frac{Y}{\mu_B G_B(p)(p - x_B)}\overline{\phi}'\left(\pi_B^*(p)\right)$$

Or rather

$$\frac{\overline{\phi}\left(\pi_B^*(p)\right)}{\overline{\phi}'\left(\pi_B^*(p)\right)} - \pi_B^*(p) = \frac{Y}{\mu_B G_B(p)(p - x_B)} \tag{A.12}$$

As the RHS is decreasing in p, so is the LHS. Note further that

$$\frac{d}{du}\left(\frac{\overline{\phi}(u)}{\overline{\phi}'(u)} - u\right) = \frac{\overline{\phi}'(u)^2 - \overline{\phi}(u)\overline{\phi}''(u)}{\overline{\phi}'(\pi_B^*)^2} - 1 = \frac{-\overline{\phi}(u)\overline{\phi}''(u)}{\overline{\phi}'(u)^2} > 0$$

Thus, the LHS of (A.12) is increasing in π_B^*. Therefore, in order to make sure that LHS of (A.12) is decreasing in p, we must have $\pi_B^*(p)$ is decreasing in p.

□

I now prove Theorem 2.

Proof. The first result holds directly from Jensen's inequality (see main text). So here we focus on the proof of the second result.

By (1.12),

$$\frac{(R - x_B)\mu_B}{(x_G - R)\mu_G}\frac{\left(1 - \bar{\pi}_B^*\right)^{N-1}}{\left(1 - \bar{\pi}_G^*\right)^{N-1}} = \overline{\phi}'\left(\pi_B^*(R)\right)$$

Suppose that

$$\frac{1 - \bar{\pi}_G^*}{1 - \bar{\pi}_B^*} \leq \frac{1 - \pi_G^{**}}{1 - \pi_B^{**}}$$

then

$$\left(\frac{1 - \bar{\pi}_B^*}{1 - \bar{\pi}_G^*}\right)^{N-1} \geq \left(\frac{1 - \pi_B^{**}}{1 - \pi_G^{**}}\right)^{N-1}$$

We then have

$$\overline{\phi}'\left(\pi_B^{**}\right) = \frac{(R - x_B)\mu_B}{(x_G - R)\mu_G}\frac{\left(1 - \pi_B^{**}\right)^{N-1}}{\left(1 - \pi_G^{**}\right)^{N-1}} \leq \frac{(R - x_B)\mu_B}{(x_G - R)\mu_G}\frac{\left(1 - \bar{\pi}_B^*\right)^{N-1}}{\left(1 - \bar{\pi}_G^*\right)^{N-1}} = \overline{\phi}'\left(\pi_B^*(R)\right)$$

where the first equality holds from (1.7). By the strict concavity of $\overline{\phi}$, we then have $\pi_B^{**} \geq \pi_B^*(R)$.

By Lemma 11, as $\pi_B^*(p)$ is decreasing in p, we have $\pi_B^{**} \geq \bar{\pi}_B^*$. Thus,

$$\frac{1-\bar{\pi}_G^*}{1-\bar{\pi}_B^*} > \frac{1-\overline{\phi}\left(\bar{\pi}_B^*\right)}{1-\bar{\pi}_B^*} \geq \frac{1-\overline{\phi}\left(\pi_B^{**}\right)}{1-\pi_B^{**}} = \frac{1-\pi_G^{**}}{1-\pi_B^{**}}$$

where the first inequality follows from the strict concavity of $\overline{\phi}$ in interval $[0, \varpi]$, the second inequality is from Lemma 5. This contradicts with the assumption that $\frac{1-\bar{\pi}_G^*}{1-\bar{\pi}_B^*} \leq \frac{1-\pi_G^*}{1-\pi_B^*}$. So we know that it must be the case

$$\frac{1-\bar{\pi}_G^*}{1-\bar{\pi}_B^*} > \frac{1-\pi_G^{**}}{1-\pi_B^{**}}$$

□

Proof of Proposition 4

Proof. To see the existence, note that from (1.25),

$$\frac{(R-x_B)\mu_B}{(x_G-R)\mu_G} = \overline{\phi}'(\pi_B^*)\frac{1-(1-\pi_G^*)^N}{\pi_G^*}\frac{\pi_B^*}{1-(1-\pi_B^*)^N}$$

When $\pi_B \downarrow 0$, $\overline{\phi}'(\pi_B) \to \infty$, $\frac{1-(1-\pi_G)^N}{\pi_G} \to \frac{1-(1-\overline{\phi}(0))^N}{\overline{\phi}(0)}$, $\frac{\pi_B}{1-(1-\pi_B)^N} \to \frac{1}{N}$, so the RHS is approaching ∞. When $\pi_B \uparrow \varpi$, $\overline{\phi}'(\pi_B) \to 0$, $\frac{1-(1-\pi_G)^N}{\pi_G} \to 1$, $\frac{\pi_B}{1-(1-\pi_B)^N} \to \frac{\varpi}{1-(1-\varpi)^N}$, so the RHS is approaching 0. By continuity, there must exist π_B^* such that the RHS equals to $\frac{(R-x_B)\mu_B}{(x_G-R)\mu_G} > 0$.

I now prove that $\pi_B^* > \pi_B^{**}$. Since the iso-welfare curve of the social planner is convex as shown in the proof of Proposition 2 and the frontier is concave, it is sufficient to show that the iso-welfare curve passes the market equilibrium point (π_B^*, π_G^*) with a larger slope than the tangent line of the frointier at that point.

From (A.6), the slope of the iso-welfare curve at the market equilibrium is

$$\frac{d\pi_G}{d\pi_B} = \frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}\frac{(1-\pi_B^*)^{N-1}}{(1-\pi_G^*)^{N-1}}$$

Now, denote $q_B^* \equiv 1 - \pi_B^*$ and $q_G^* \equiv 1 - \pi_G^*$

$$\frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}\frac{(1-\pi_B^*)^{N-1}}{(1-\pi_G^*)^{N-1}} > \frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}\frac{\frac{1-(1-\pi_B^*)^N}{N\pi_B^*}}{\frac{1-(1-\pi_G^*)^N}{N\pi_G^*}}$$

$$\Leftrightarrow \quad \frac{(q_B^*)^{N-1}}{(q_G^*)^{N-1}} > \frac{\frac{1-(q_B^*)^N}{1-q_B^*}}{\frac{1-(q_G^*)^N}{1-q_G^*}} = \frac{1 + q_B^* + (q_B^*)^2 + ... + (q_B^*)^{N-1}}{1 + q_G^* + (q_G^*)^2 + ... + (q_G^*)^{N-1}}$$

$$\Leftrightarrow \quad \left(\frac{1}{q_G^*}\right)^{N-1} + \left(\frac{1}{q_G^*}\right)^{N-2} + ... + 1 > \left(\frac{1}{q_B^*}\right)^{N-1} + \left(\frac{1}{q_B^*}\right)^{N-2} + ... + 1$$

To show the above inequality, it is sufficient to show that

$$\left(\frac{1}{q_G^*}\right)^n > \left(\frac{1}{q_B^*}\right)^n$$

for all $n \geq 1$. But this is equivalent to show that $\frac{q_B^*}{q_G^*} > 1$, which follows directly from $\pi_G^* > \pi_B^*$.

□

Proof of Proposition 5

Proof. Since we assume that π_B^{**} is an interior, the proposition is equivalent to that there exists $p \in (R, x_G)$ such that

$$\frac{-(x_B - p)\mu_B}{(x_G - p)\mu_G}\frac{\frac{1-(1-\pi_B^{**})^N}{N\pi_B^{**}}}{\frac{1-(1-\pi_G^{**})^N}{N\pi_G^{**}}} = \overline{\phi}'(\pi_B^{**}) = \frac{-(x_B - R)\mu_B}{(x_G - R)\mu_G}\frac{(1-\pi_B^{**})^{N-1}}{(1-\pi_G^{**})^{N-1}}$$

This is equivalent to show that $\exists p \in (R, x_G)$:

$$\frac{(p-x_B)/(x_G-p)}{(R-x_B)/(x_G-R)} = \frac{(1-\pi_B^{**})^{N-1}}{(1-\pi_G^{**})^{N-1}} \frac{\frac{1-(1-\pi_G^{**})^N}{\pi_G^{**}}}{\frac{1-(1-\pi_B^{**})^N}{\pi_B^{**}}}$$

From the proof of Proposition 4, we know that the RHS is some number larger than 1. Observe that the LHS is an increasing function on p and when $p = R$ it has value 1 and when $p = x_G$ is has value ∞. So there must exist $R < p < x_G$ such that it equals to the RHS.

□

A.2 Appendix to Chapter 2: Microfoundation of Non-Runner Fraction ζ

Consider an extension of the model. Specifically, suppose in each morning of period $t+1$, an aggregate productivity shock θ_{t+1} may hit the economy. The aggregate shock changes all projects' gross return from x to $\theta_{t+1}x$, and the screening cost parameter ξ to $\theta_{t+1}\xi$. Bankers and buyers on the secondary market get to know θ immediately once it realizes in the morning, but in period t, agents only know that θ is drawn from a pre-specified distribution. Specifically, I assume that with probability $1-\pi$, θ equals to θ_0. With probability π, θ_{t+1} is drawn uniformly from $\Theta_{t+1} = [\underline{\theta}_{t+1}, \bar{\theta}_{t+1}]$ for some $\underline{\theta}_{t+1} > 0$ and $\bar{\theta}_{t+1} > \theta_0$. $\underline{\theta}_{t+1}$ and $\bar{\theta}_{t+1}$ can be functions of aggregate states $n_t^a, \eta_t^a, R_{t+1}^d, m_t^a$, which are all common knowledge at time t. The model described in the main text corresponds to a situation where $\pi = 0$ and $\theta_0 = 1$. In what follows, I set θ_0 to 1 and mainly consider the $\pi \downarrow 0$ limit.

As buyers on the secondary market also know θ after its realization, in the fully separating equilibrium, they pay θx for project x. Hence, if θ_{t+1} realizes, along the equilibrium path bankers' pre-consumption net worth would be

$$\eta_t^a n_t^a \theta_{t+1} \int_{\underline{x}}^{\bar{x}} x m_t^a(x) f(x) dx - \eta_t^a n_t^a \theta_{t+1} \xi \int_{\underline{x}}^{\bar{x}} g\big(m_t^a(x)\big) f(x) dx - n_t^a(\eta_t^a - 1) R_{t+1}^d \quad \text{(A.13)}$$

Obviously, if θ_{t+1} is too low, then the banker's pre-consumption net worth would go negative. Particularly, I assume that at $\underline{\theta}_{t+1}$ the above term is negative. I further assume that whenever the banker's net worth is negative, she bankrupts and gets a termination utility U_T. One can imagine U_T to be a large negative number, but it is finite. Note that as the θ shock is same to all bankers, bankruptcy is for the entire banking sector.

Besides insolvency, bankers would also go bankrupt if they cannot fulfill the morning withdrawals. Given the structure of the secondary market, the maximum amount of proceeds that bankers can generate from the secondary market is $n_t^a \eta_t^a \theta_{t+1} \underline{x}$, as this is the amount the banker would get if she sells all projects in full. On the other hand, households withdrawal behavior modeled in the main text can be reinterpreted in the following way: all households withdraw in full in the morning, but some of them (non-runners) will immediately deposit back part of their wealth. In this interpretation, the total liability due in the morning is $n_t^a(\eta_t^a - 1)R_{t+1}^d$. Thus, if morning redeposit is smaller than $n_t^a(\eta_t^a - 1)R_{t+1}^d - n_t^a\eta_t^a\theta_{t+1}\underline{x}$ (whenever it is positive), then bankers fail. In the global game shown below, the size of morning redeposits will be a function of θ_{t+1}. Bankers would fail whenever $\theta_{t+1} < \hat{\theta}_{t+1}$, where $\hat{\theta}_{t+1}$ is the threshold for survival endogenized in the model.

Put differently, θ can be read as a fundamental shock to the economy. If the fundamental is too bad, then the entire banking system would fail. It fails either because bankers cannot survive the morning withdrawals, or just because bankers are insolvent. If this happened, then bankers get terminated and the whole economy ends. One can imagine starting next period a new dynasty begins with some newly born bankers.

Bankers' Bellman equation in the extended model can be written as

$$V_t(n_t, \eta_t) = \max_{m_t, \alpha_{t+1}, c_{t+1}, \eta_{t+1}} \beta\mathbb{E}\Big[\log c_{t+1}(\theta_{t+1}) + V_{t+1}\left(n_{t+1}(\theta_{t+1}), \eta_{t+1}(\theta_{t+1})\right) \Big| \theta_{t+1} \geq \hat{\theta}_{t+1}\Big]$$

$$\mathbb{P}\{\theta_{t+1} > \hat{\theta}_{t+1}\} + \beta U_T \mathbb{P}\{\theta_{t+1} \leq \hat{\theta}_{t+1}\}$$

$$\text{where} \quad n_{t+1}(\theta_{t+1}) = \eta_t n_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x;\theta_{t+1}) p_{t+1}\left(\alpha_{t+1}(x;\theta_{t+1});\theta_{t+1}\right) m_t(x) f(x) dx$$

$$+ \eta_t n_t \theta_{t+1} \int_{\underline{x}}^{\bar{x}} \left(1 - \alpha_{t+1}(x;\theta_{t+1})\right) x m_t(x) f(x) dx$$

$$- \eta_t n_t \theta_{t+1} \xi \int_{\underline{x}}^{\bar{x}} g\big(m_t(x)\big) f(x) dx - n_t(\eta_t - 1)R_{t+1}^d - c_{t+1}(\theta_{t+1})$$

$$\text{s.t.} \quad (1 - \zeta_{t+1}(\theta_{t+1}))n_t(\eta_t - 1)R_{t+1}^d + \zeta_{t+1}(\theta_{t+1})n_t(\eta_t - 1)\Delta_{t+1}(\theta_{t+1})$$

$$\leq \eta_t n_t \int_{\underline{x}}^{\bar{x}} \alpha_{t+1}(x;\theta_{t+1}) p_{t+1}\left(\alpha_{t+1}(x;\theta_{t+1});\theta_{t+1}\right) m_t(x) f(x) dx$$

$$\int_{\underline{x}}^{\bar{x}} m_t(x) f(x) dx = 1$$

$$m_t(x) \geq 0 \quad \forall x$$

To be clear, in the evening of period t, the banker taking n_t, η_t, $\hat{\theta}_{t+1}$, $\zeta_{t+1}(\theta_{t+1})$, $\Delta_{t+1}(\theta_{t+1})$, $p_{t+1}(\cdot;\theta_{t+1})$, and R^d_{t+1} as given, chooses $m_t(x)$, $\alpha_{t+1}(x;\theta_{t+1})$, $c_{t+1}(\theta_{t+1})$, and $\eta_{t+1}(\theta_{t+1})$ for all $x \in X$ and $\theta_{t+1} > \hat{\theta}_{t+1}$.

The Household Sector

There is a continuum of households indexed by i. In the morning of period $t+1$, household i earns wage

$$\phi n^a_t \eta^a_t \left(\left(\theta_{t+1} + \varepsilon^i_{t+1} \right) \int_{\underline{x}}^{\bar{x}} x m^a_t(x) f(x) dx - 1 \right)$$

where ε^i_{t+1} are independently drawn from uniform distribution $[-\sigma, \sigma]$. Basically, households have heterogeneity in their labor income, captured by ε^i_{t+1}. But in aggregate, labor income to the entire household sector is still proportional to the total net output of the banking sector. For mathematical simplicity, I allow negative labor income. But the total pre-consumption wealth of household i

$$(\eta^a_t - 1) n^a_t R^d_{t+1} + \phi n^a_t \eta^a_t \left(\left(\theta_{t+1} + \varepsilon^i_{t+1} \right) \int_{\underline{x}}^{\bar{x}} x m^a_t(x) f(x) dx - 1 \right) \tag{A.14}$$

is assumed to be positive even if $\theta_{t+1} = \underline{\theta}_{t+1}$ and $\varepsilon^i_{t+1} = -\sigma$. One can numerically verify that in the numerical examples given in the main text, there exists $\underline{\theta}_{t+1}$ such that the above term (A.14) is positive and the term (A.13) is negative as assumed.

Crucially, households do not know θ_{t+1} in the morning. They only know the aggregate states n^a_t, η^a_t, m^a_t plus their own wage payment. In other words, households get known $\theta_{t+1} + \varepsilon^i_{t+1} \equiv s^i_{t+1}$, but not directly the fundamental θ_{t+1}.

As stated above, households mechanically call back all their parents' bequests in the morning, but they play strategically in choosing when to make their new savings. They can do so either in the morning (AM) or in the afternoon (PM). The difference is that right at noon, households would get known the realization of θ and know whether bankers truly fail.

For simplicity, I assume that no matter the household chooses to save in the morning or afternoon (conditional on the survival of bankers), households always save β fraction of her total wealth. If households save in the morning and bankers survive, then households enjoy an extra utility $\hat{u}_1 > 0$ compared to saving in the afternoon. However, if bankers do fail but households have chosen to save in the morning, then the household would suffer a utility loss $\hat{u}_2(\pi)$. The utility loss is modeled as a function of π, and is assumed to diverge to infinity as $\pi \downarrow 0$. Note that if π goes to zero, the probability of banking failure also goes to zero.[1] The assumption that $\hat{u}_2(\pi) \uparrow \infty$ as $\pi \downarrow 0$ basically says that if the probability of banking failure is very tiny, then once it really happens, the utility loss of participating in the morning would be enormous.

Denote ϖ^i as household i's posterior about banking failure after observing her private signal s^i_{t+1}. Then households would be indifferent in choosing AM or PM if

$$\hat{u}_2(\pi)\varpi^i = \hat{u}_1(1 - \varpi^i)$$

or rather

$$\varpi^i = \frac{\hat{u}_1}{\hat{u}_1 + \hat{u}_2(\pi)} \equiv \hat{\varpi}(\pi)$$

Thus, instead of specifying $\hat{u}_1$ and $\hat{u}_2(\pi)$ respective, I directly specify $\hat{\varpi}(\pi)$. In order to make sure that $\hat{u}_2(\pi) \uparrow \infty$ as $\pi \downarrow 0$. I need $\hat{\varpi}(\pi) \downarrow 0$ as $\pi \downarrow 0$. For simplicity, I just let

$$\hat{\varpi}(\pi) = \kappa\pi$$

for some constant $0 < \kappa < 1$. To summarize, as long as a household believes the probability of banking failure is smaller than $\kappa\pi$, she will participate in the morning.

1. In that case $\theta_{t+1} = \theta_0$ almost surely and I assume that $\theta_0 > \hat{\theta}_{t+1}$ for all t.

Household's global game

In order to make decision about going AM or PM, each household has to infer the realized fundamental θ_{t+1} and other households' behavior. The information households use is the private signal s^i extracted from their wage payment. In that sense, this is indeed a standard global game (e.g. Morris and Shin (2003)). The game has both lower dominance region and upper dominance region. For the former, if a household observes a signal low enough, say $\underline{\theta}_{t+1}$, then even if all other people choose AM, she will choose PM since she knows that the banker will be insolvent anyway. For the latter, if the household observes a signal high enough, then she knows that even if all others choose PM, bankers can still survive morning withdrawals by selling in the secondary market. Hence the particular household would go AM.

Follow standard global game literature (e.g. Morris and Shin (2003)), we then know that the equilibrium is characterized by a threshold k, such that households whoever observes private signal $s^i > k$ chooses AM, otherwise chooses PM. Conditional on a realization of θ, $s^i > k$ is equivalent to $\varepsilon^i > k - \theta$, so the total amount of morning deposits would be

$$\int_{\max\{-\sigma, k-\theta\}}^{\sigma} \left[(\eta^a - 1)n^a R^d \beta + \beta n^a \eta^a \Big((\theta + \varepsilon')Y - 1\Big)\phi\right] \frac{1}{2\sigma} d\varepsilon'$$

Bankers can fulfill morning withdrawal if and only if

$$\int_{\max\{-\sigma, k-\theta\}}^{\sigma} \left[(\eta^a - 1)n^a R^d \beta + \beta n^a \eta^a \Big((\theta + \varepsilon')Y - 1\Big)\phi\right] \frac{1}{2\sigma} d\varepsilon' \geq n^a(\eta^a - 1)R^d - \underline{x} n^a \eta^a \theta$$

Observe that the LHS is increasing in θ while the RHS is decreasing in θ. Therefore, given k, the above expression then implies a threshold $\hat{\theta}$ such that bankers survive if and only if

$\theta_{t+1} > \hat{\theta}_{t+1}$. To be more specific, $\hat{\theta}_{t+1}$ is pinned down by

$$\int_{\max\{-\sigma,k-\hat{\theta}\}}^{\sigma} \left[(\eta^a - 1)n^a R^d \beta + \beta n^a \eta^a \Big((\hat{\theta} + \varepsilon')Y - 1\Big)\phi\right] \frac{1}{2\sigma} d\varepsilon' = n^a(\eta^a - 1)R^d - \underline{x} n^a \eta^a \hat{\theta} \tag{A.15}$$

Note that $\hat{\theta}_{t+1}$ is a function of $\eta_t^a, R_{t+1}^d, Y_{t+1}, k_{t+1}$, and is known to everyone.

In what follows, I focus on the case where π is driven down to zero and θ_{t+1} realizes to be $\theta_0 = 1$. I also assume that $\theta_0 > \hat{\theta}_{t+1}$, so when the shock realizes to be $\theta_0 = 1$, the banking sector does survive. Conditional on the realization is θ_0, households get private signals $s^i \in [\theta_0 - \sigma, \theta_0 + \sigma]$. Household with private signal s^i believes that with probability $1 - \pi$, θ equals to θ_0; with probability π, $\theta \sim U[s^i - \sigma, s^i + \sigma]$. Household i's posterior on banking failure is then $\frac{\hat{\theta} - s^i + \sigma}{2\sigma}\pi$. Given the analysis above, as long as $\frac{\hat{\theta} - s^i + \sigma}{2\sigma}\pi < \kappa\pi$, households would choose AM. At threshold $s^i = k$, people are indifferent. So k must be given by

$$\frac{\hat{\theta}_{t+1} - k_{t+1} + \sigma}{2\sigma} = \kappa \tag{A.16}$$

(A.16) and (A.15) then jointly pin down k and $\hat{\theta}$.

In the numerical example used in the main text, κ is set to $\frac{1}{2}$ for simplicity. In this case, (A.16) implies that $k_{t+1} = \hat{\theta}_{t+1}$. From (A.15) we then have

$$k_{t+1}\eta_t^a \underline{x} + \frac{1}{2}\beta\eta_t^a \phi \left(Y_{t+1}\left(k_{t+1} + \frac{\sigma}{2}\right) - 1\right) = (\eta_t^a - 1)R_{t+1}^d \left(1 - \frac{1}{2}\beta\right) \tag{A.17}$$

We can then solve for k_{t+1}.

Households who receive $s^i > k$ are non-runners. Given the realization of θ is θ_0, the fraction of non-runners is

$$\min\left\{\frac{\sigma + \theta_0 - k_{t+1}}{2\sigma}, 1\right\}$$

Plugging in k_{t+1} solved from (A.17), we have

$$\frac{\sigma+\theta_0-k_{t+1}}{2\sigma}=\frac{\sigma+\theta_0}{2\sigma}-\frac{1}{2\sigma}\frac{\left(1-\frac{1}{\eta^a}\right)R^d(2-\beta)-\beta\phi\frac{\sigma}{2}Y+\beta\phi}{2\underline{x}+\beta\phi Y}$$

ζ is then defined as

$$\zeta\left(Y;\left(1-\frac{1}{\eta^a}\right)R^d\right)\equiv\min\left\{\frac{\sigma+1}{2\sigma}-\frac{1}{2\sigma}\frac{\left(1-\frac{1}{\eta^a}\right)R^d(2-\beta)-\beta\phi\frac{\sigma}{2}Y+\beta\phi}{2\underline{x}+\beta\phi Y},1\right\} \quad (2.32)$$

where I have substituted θ_0 by 1. In the numerical example given in the main text, σ is set to 0.04. This number is chosen such that leverage at steady state is in a reasonable range.

It is obvious from (2.32) that ζ is increasing in Y and decreasing in $\left(1-\frac{1}{\eta^a}\right)R^d$ and is continuous in both arguments. As Y is bounded between $\mathbb{E}[x]$ and Y^o (the productivity under first-best) and $\left(1-\frac{1}{\eta^a}\right)R^d$ is bounded between 0 and R^o (the return under first-best), it is also straightforward to see $\frac{\partial\zeta}{\partial Y}$ is bounded. Hence, the functional form (2.32) satisfies Assumption 5.

It is worth to mention that with this setup, non-runners' early withdrawal amount Δ_{t+1} is slightly smaller than the one derived in (2.28). This is because non-runners are households with better signals hence higher labor income. But in order to be consistent with the main text, this small wedge is ignored. Indeed, if one uses the precise early withdrawal amount of non-runners implied by this extended model, all numerical examples given in the main text are barely affected.

Note also that the θ shock is different from the z shock described in the main text. The θ shock is realized in the morning of period $t+1$, while the z shock is known in the evening of period t. Correspondingly, bankers will take the realized z into consideration when they originate assets in the evening whereas they can only form expectations about θ. Moreover, once z realized in the evening of t, it becomes common knowledge. That being said, households would replace $\underline{x}$ by $z\underline{x}$ in formula (2.32). However, as emphasized above, households do not know θ_{t+1} until noon of period $t+1$.