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RETAIL TRADING AND ASSET PRICES:
THE ROLE OF CHANGING SOCIAL DYNAMICS

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FULIN LI

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This dissertation is dedicated to my parents, Yeqing Li and Lixiang Zhang, for their unwavering love, support, and encouragement.

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ABSTRACT

Social-media-fueled retail trading poses new risk to institutional investors. This paper examines the origin and pricing of this new risk. I first present stylized facts on prices, quantities, and retail investors' beliefs for a set of meme stocks. I establish that aggregate fluctuations in retail sentiment originated from a growing and concentrated social network. The retail sentiment fluctuations induced changes in investor composition. As sentiment increased throughout 2020 and 2021, retail investors built up long positions, while price-sensitive long-only institutions have gradually exited the market since early 2020. Short interest stayed high in 2020, but dropped sharply following the price surge in January 2021 and remained low for the rest of the year. Motivated by these facts, I develop a model of the interaction between three groups of investors – retail investors, long-only institutions, and short sellers. I calibrate the model to match the price, quantity, and retail sentiment dynamics during this period. Then I use the calibrated model to demonstrate that social network dynamics shape the distribution of retail sentiment and have an economically large impact on asset prices. In the model, retail investors participate in a social network with concentrated linkages. This implies that their idiosyncratic sentiment shocks can lead to aggregate fluctuations in retail sentiment. Aggregate retail sentiment shocks shift investor composition, which in turn determines the price of retail sentiment risk. Following an increase in the aggregate retail sentiment, price-sensitive long-only institutions first hit their short-sale constraints, leading to a decrease in the aggregate demand elasticity in the market for an individual stock. Then a "small" positive retail sentiment shock can have a "large" price impact and even squeeze the short sellers.

CHAPTER 1

RETAIL TRADING AND ASSET PRICES: THE ROLE OF CHANGING SOCIAL DYNAMICS

1.1 Introduction

Retail trading accounts for an increasing share of U.S. equity trading activity. In the first quarter of 2021, new brokerage accounts opened by retail investors hit a record high.[1] This flood of new investors, many of whom are young first-time traders, have transformed social media platforms (e.g., Reddit, Twitter, and TikTok) into virtual trading clubs, where they share investment ideas and coordinate actions against institutional short sellers. In the short squeeze episode of January 2021, their sudden coordinated actions resulted in significant volatility in individual stocks, with the price of GameStop surging by 1500% within a few weeks. Additionally, short sellers have stayed out of the market for GameStop since the January 2021 short squeeze, as illustrated in Figure 1.1.

How does social-media-fueled retail trading have such a large and persistent impact on asset prices? In this paper, I demonstrate that the topology of social connections among retail investors and their interaction with institutional investors are two critical factors in answering this question.

First, the social network topology of retail investors shapes their aggregate sentiment. Specifically, retail investors cluster around a handful of "influencers" on the network. This concentration of network linkages implies that idiosyncratic shocks to retail investors' beliefs can lead to aggregate fluctuations in retail sentiment, which is effectively an aggregate demand shock to retail investors. Moreover, the growth of the network further amplifies the aggregate retail sentiment shock.

1. Fitzgerald, M. (2021, April 15). Schwab adds 3.2 million new brokerage accounts in first quarter – more than all of 2020. *CNBC*. `https://www.cnbc.com/2021/04/15/retail-trading-boom-schwab-first-quarter-2021-earnings.html`.

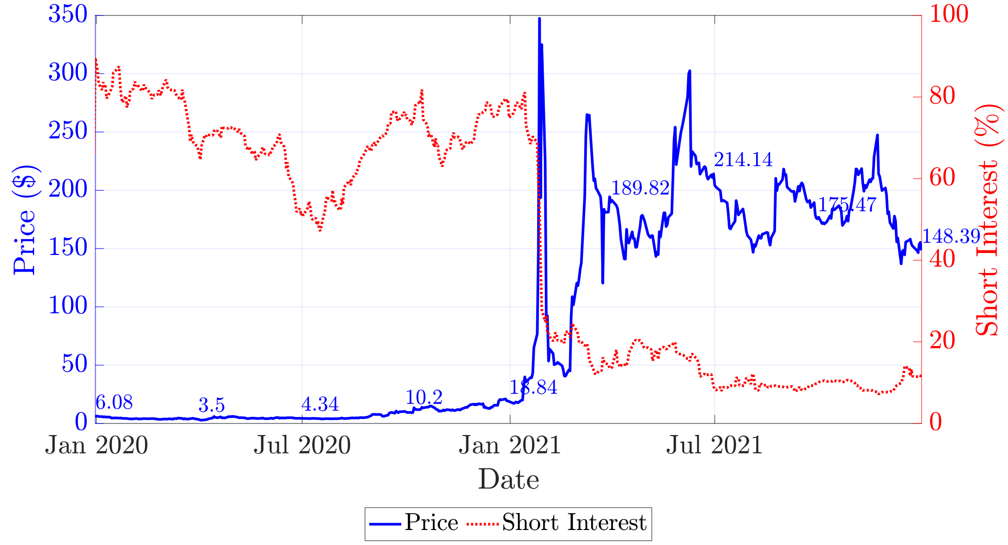


Figure 1.1. Price and short interest of GameStop. This figure shows the daily close price (left y-axis) and the daily short interest (right y-axis) of GameStop, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price. The dotted red line plots the short interest, which is defined as the ratio of the number of shares sold short to the number of shares outstanding (equation (1.6)). Data on the number of shares sold short is from IHS Markit.

Second, the price impact of the aggregate retail sentiment shock depends on the interaction between retail investors and other market participants – the long-only institutions and the short sellers. As retail sentiment increases, price-sensitive long-only institutions gradually exit the market, leading to a decrease in the aggregate demand elasticity in the market for an individual stock. Consequently, even a moderate retail sentiment shock can drive up the stock price and put institutional short sellers at risk.

I present empirical evidence on the above mechanisms using the January 2021 short squeeze and develop a model to quantify the role of the contributing factors. To analyze the dynamics of retail sentiment, I obtain data from Reddit's WallStreetBets forum (hereafter WSB). This dataset allows me to recover the communication network of a representative group of retail investors and quantify the sentiment (or belief) of each individual investor.

I then combine this data with stock prices, short interest, and portfolio holdings of various classes of long-only investors. Using this comprehensive dataset, I present four facts on prices, quantities and retail investors' beliefs in the context of GameStop short squeeze.

Fact 1 establishes the relationship between GameStop's stock price and retail sentiment from Reddit's WSB forum. I document that the average retail sentiment of GameStop had been steadily increasing since the beginning of 2020, while the WSB discussion volume on GameStop spiked in January 2021. The spike in the discussion volume resulted from a sudden expansion of the WSB social network and coincided with the price surge of GameStop.

The increase in average sentiment and the expansion of the network collectively lead to an increase in the aggregate retail sentiment, effectively shifting the aggregate demand curve of the retail investors. The price impact of this demand shift crucially depends on those investors who take the other side of the trade, which I explore in the next two facts.

Fact 2 establishes that retail investors gradually built up their positions in GameStop throughout 2020 and early 2021 compared to long-only institutions, and retail investors' relative positions remained constant for the rest of 2021. This suggests that retail investors were more optimistic than long-only institutions. Furthermore, long hedge funds also increased their positions in 2020 but then liquidated almost all their long positions in 2021 Q1. This suggests that long hedge funds were initially riding the price increase. But their long strategies may not have been profitable after the price surge in January 2021 as they may have expected the price to quickly fall back to the pre-January-2021 level.

Fact 3 establishes that the short interest of GameStop increased from 60% to 80% from mid- to late 2020. But then it dropped sharply in January 2021 and stayed at below 20% throughout the rest of the year. This is consistent with the narrative that short sellers were squeezed and forced to cover their short positions.

Long-only institutions and short sellers are the two groups of investors who take the other side of the trade against retail investors. However, they are both constrained in terms of taking (large) short positions. Long-only institutions do not short for institutional reasons,[2] and short sellers face margin constraints. If retail sentiment keeps rising and drives up the stock price, both groups of investors will hit their portfolio constraints. In particular, when short sellers hit their margin constraints, they will be forced to cover their short positions, and the stock price will rise even further.

Next, I examine the role of the WSB social network in driving retail sentiment fluctuations, in particular, how individual retail investors' opinions factor into the aggregate retail sentiment. To do so, I construct daily WSB communication networks from users' conversations and measure each user's influence based on their network connections.

Fact 4 establishes that the WSB communication network is highly concentrated, with a few influencers dominating the discussions. The influence distribution across users is highly right-skewed, indicating that the sentiment of a small number of highly influential users can significantly impact the sentiment of the broader community. This suggests that the influencers' idiosyncratic sentiment shocks would propagate strongly through the social network, generating sizable aggregate fluctuations in retail sentiment.

Motivated by these observations, I develop a model to demonstrate how social network topology of retail investors and their interaction with other market participants can lead to significant fluctuations in asset prices. Importantly, the model explains why social-media-fueled retail trading can permanently change the behavior of other market participants and thus have long-lasting effects on asset prices. Through a simple calibration of the model, I can explain the joint dynamics of prices, quantities, and retail sentiment in the GameStop short squeeze. I use the calibrated model to conduct counterfactuals and quantify the role of social network topology in driving asset price swings.

The model features three groups of investors: a large number of unconstrained retail

2. For example, Almazan et al. (2004) show that mutual fund managers may be restricted by their investors from shorting. An et al. (2021) argue that long-short mutual funds may not be attractive to investors because they hoard cash to absorb fluctuations in capital flows and thus underperform long-only indices.

investors, one long institution facing short-sale constraint, and one short institution facing margin constraint. The three groups of investors trade one risky asset and hold heterogeneous beliefs (i.e., sentiment) about the asset's payoff.

The aggregate fluctuations in retail sentiment originate from a social network with highly concentrated linkages. A subset of retail investors participate in the social network. They draw idiosyncratic sentiment shocks and communicate according to their network connections. The concentration of the network implies a right-skewed influence distribution among these retail investors, with influencers' views carrying a disproportionately high weight in the aggregate view. This leads to aggregate fluctuations in retail sentiment, as idiosyncratic sentiment shocks do not "average out."

The price of retail sentiment shock depends on the composition of investors in the market, who face heterogeneous financial constraints. As retail sentiment fluctuates over time, these constraints may bind for a sub-group of investors and make them price-inelastic. This drives the time variation in the aggregate price elasticity in the market for the risky asset and thus determines the price impact of an aggregate retail sentiment shock.

To fix ideas, suppose that retail investors (in aggregate) are relatively more optimistic than institutional investors. The two institutions have the same beliefs and only differ in their financial constraints – the long institution cannot short and thus faces a "tighter" constraint than the short institution. As retail sentiment increases and drives up the price, the long institution gradually reduces the long positions in the risky asset until hitting the short-sale constraint. Once the constraint binds, his demand does not respond to price changes, which translates into a decrease in the aggregate demand elasticity in the market for the risky asset. Consequently, even a "small" positive shock to retail sentiment can have a "large" price impact and even squeeze the short seller.

Retail sentiment fluctuations also redistribute wealth across investors with heterogeneous beliefs, as those who happen to make the "right" bets gain wealth at the expense of others.[3] The aggregate demand elasticity is a wealth-weighted average of individual investors' demand elasticities. Hence, wealth redistribution generates time variation in aggregate demand elasticity, thereby changing the price impact of retail sentiment shocks. For example, as price increases, the short institution loses wealth and carries a smaller weight in the aggregate demand elasticity. In the extreme scenario where the short institution loses all the wealth, he exits the market, and the remaining investors determine the aggregate demand elasticity. If these investors are sufficiently price-inelastic, this leads to a decrease in the aggregate demand elasticity in the market for the risky asset.

Thus, the model offers a unified explanation for the retail sentiment fluctuations originated from the social network, the price impact of the retail sentiment shocks, and the quantity dynamics resulting from the retail sentiment fluctuations. Through a simple calibration, I demonstrate that the model can generate the price and quantity movements observed in the data.

Using the model, I conduct counterfactuals to answer some key questions surrounding the GameStop short squeeze episode. First, I consider a scenario where the WSB discussion volume did not spike in January 2021. In the model, a smaller discussion volume corresponds to a smaller subset of retail investors participating in the social network, i.e., a smaller network size. I show that the realized aggregate retail sentiment would be lower, and the short seller would not hit the margin constraint or get squeezed. Network concentration plays a fundamental role in driving the wedge between sentiment realizations under different network sizes. If network linkages are not concentrated, idiosyncratic sentiment shocks always average out, regardless of the network size.

Second, I consider the case where short sellers updated their perceptions of retail

3. Martin and Papadimitriou (2022) study the implications of the same mechanism on volatility and speculation, in a dynamic setting where agents do not face financial constraints. However, in my context, it is necessary to introduce financial constraints to rationalize the quantity dynamics observed in the data.

sentiment risk after observing the influx of retail investors to WSB in January 2021. I demonstrate that the change in their risk perceptions can help explain why they exited the market after the short squeeze.

The findings in this paper have broader implications on the evolving market dynamics, beyond a specific short squeeze episode or a specific set of meme stocks. Social media has fundamentally changed the nature of retail trading as a source of risk. In a world with financial constraints, even moderate fluctuations in retail sentiment can have significant consequences for institutional players in the market. Retail sentiment risk from social media and investor composition change are two new risks that institutional short sellers must adapt to.

My paper contributes to the empirical literature that examines the effects of retail trading on asset prices. Recent studies[4] have focused on the the trading patterns of retail investors identified from the TAQ data (Barber et al., 2021a; Barber et al., 2021b; Boehmer et al., 2021; Eaton et al., 2022) or the Robinhood data (Welch, 2022), how social media affects their behavior (Cookson and Niessner, 2020; Hu et al., 2021; Allen et al., 2022; Cookson et al., 2022a), as well as how institutional investors respond to retail investor activities on social media (Cookson et al., 2022b). I present new facts on the interaction between retail investors and institutional investors. In particular, I show that retail trading can drive price-sensitive long-only institutions out of the market, resulting in a decrease in aggregate demand elasticity in the market for an individual stock. This mechanism explains the price impact of retail trading observed from the data.

My paper highlights how social media has fundamentally changed the nature of retail trading as a source of risk. I show how the dynamics of social connections shape the

4. The literature on retail trading and its impact on asset prices is extensive, with studies using proprietary data from various markets including the U.S. (Barber and Odean, 2000; Barber et al., 2008; Kaniel et al., 2008; Barber et al., 2009; Kaniel et al., 2012; Kelley and Tetlock, 2013; Kelley and Tetlock, 2017). For a comprehensive review of this literature, see Barber and Odean (2013). Recent work has employed the algorithm developed by Boehmer et al. (2021) to identify retail trades from the TAQ data.

distribution of retail sentiment risk. This connects to the growing literature on social finance (Hirshleifer, 2020; Kuchler and Stroebel, 2021), which emphasizes the role of social interactions in shaping financial outcomes. For example, Bailey et al. (2018a) and Bailey et al. (2018b) examine the role of Facebook friendship network in driving economic decisions in the housing market and various other contexts. Compared with the Facebook friendship network, the Reddit discussion network evolves much faster, since it is visible to all market participants including those who are not yet on the network. This fast-evolving nature of Reddit makes it harder to predict retail sentiment movement and is crucial for understanding the nature of retail sentiment shocks.

Recent papers have explored various features of the Reddit community and its asset pricing implications. Bradley et al. (2021) focus on the due diligence reports on Reddit's WSB forum. Hu et al. (2021) combine the information from Reddit's WSB with data on stock prices, shorting flows, and retail order flows. They study the impact of social media activity on retail trading and asset prices using a cross-sectional approach. Allen et al. (2022) conduct a comprehensive analysis of the short squeeze episode in January 2021, using social media data from Reddit and data on stock prices, shorting activities, and retail trading of equities and options. Bryzgalova et al. (2022) document the relation between the number of mentions on Reddit's WSB and options trading activities by retail investors. In contrast, my paper brings in additional institutional holdings data to study the interaction between retail investors and institutional investors. This is a unique setting where I observe prices, quantities, and retail investors' beliefs. I demonstrate that the information embedded in quantities (i.e., holdings by the long-only institutions and the short sellers) is important for understanding the asset pricing implications of social-media-fueled retail trading. Moreover, I establish a direct mapping from network geometry to the asset price movements, both theoretically and empirically.

Retail investors are commonly viewed as noise traders (De Long et al., 1989; De Long et al., 1990), and their sentiment or beliefs are often considered a “black box.” My paper opens up the “black box” by empirically measuring the sentiment of individual investors and examining the changing social network structure that drives the day-to-day sentiment fluctuations. Through the lens of the model, I demonstrate that market participants can better predict retail sentiment movement by opening up this “black box.”

My model combines the insights from the literature of learning on networks and the asset pricing literature on limits to arbitrage.

First, I microfound the retail sentiment dynamics using a model of naive learning on social networks, which builds on the DeGroot-type models of social learning (DeGroot, 1974; DeMarzo et al., 2003; Golub and Jackson, 2010; Pedersen, 2022). My model extends the DeGroot-type framework to allow for interim sentiment shocks and time-varying network size.[5] These extensions are essential for establishing retail sentiment as a source of risk.

I argue that the interim idiosyncratic sentiment shocks can lead to aggregate fluctuations in retail sentiment. I borrow this idea from the literature that studies the effect of granularity (Gabaix, 2011) and network geometry (Acemoglu et al., 2012) on idiosyncratic shock aggregation. I apply this idea to the aggregation of idiosyncratic shocks to investor sentiment (or beliefs) and provide a statistic that captures the coordination among investors.

Second, my model for pricing sentiment risk ties to the literature on disagreement and limits to arbitrage (Miller, 1977; Scheinkman and Xiong, 2003). In my model, there are two types of institutions: a long institution facing short-sale constraint and a short institution facing margin constraint (Brunnermeier and Pedersen, 2009; Gârleanu and Pedersen, 2011). The heterogeneity in financial constraints, together with the heterogeneity in beliefs, can

5. Network size in the model corresponds to the discussion volume on Reddit’s WSB forum in the data. The discussion volume on a particular stock can also be interpreted as retail investors’ attention on that stock. Hence, my model speaks to the relation between investor attention and equilibrium asset prices (Barber and Odean, 2008; Da et al., 2011; Barber et al., 2021a). My model also separates the effect of average sentiment from the effect of network size (or investor attention), and thus can be a useful framework to analyze the comovement of attention and disconnect of sentiment across various social media platforms, as documented in Cookson et al. (2022c).

reconcile the price, quantity and retail sentiment dynamics observed from the data.[6]

Finally, my model also fits into a broader theme of how heterogeneity matters for pricing (Panageas, 2020; Caballero and Simsek, 2021; Gabaix and Koijen, 2022). The retail sentiment shocks in my context is a particular type of "flow" in Gabaix and Koijen (2022)'s definition. In my model, aggregate demand elasticity is one statistic that is closely related to the pricing of this "flow," which is consistent with Gabaix and Koijen (2022)'s argument. Moreover, my model microfounds the time variation in aggregate demand elasticity by introducing heterogeneous financial constraints and wealth effects (Xiong, 2001). I illustrate that the time-varying aggregate demand elasticity is important for understanding the price of retail sentiment risk.

1.2 Data and methodology

1.2.1 Reddit data

Sample construction

I retrieve historical data on Reddit submissions and comments from the Pushshift API, using the Python Pushshift Multithread API Wrapper (PMAW). I restrict the data download to the subreddit r/wallstreetbets (WSB) and to the period from January 2020 to December 2021.

Occasionally, the Pushshift API does not return any submissions or comments for a given day, due to API outages. The missing data can be retrieved from the Pushshift dump files.[7] For any date that the Pushshift API returns zero submissions or comments, I pull data from these dump files.

6. The literature has studied the interaction between retail investors and institutional investors under other types of financial constraints. For example, Basak and Pavlova (2013) present a model where the institutional investors face benchmarking constraints.

7. See `https://files.pushshift.io/reddit/`.

In the raw data from Pushshift, submissions and comments are labeled with a UTC (Coordinated Universal Time) timestamp, which I convert to the New York time zone – a difference of five hours during Eastern Standard Time and four hours during Daylight Saving Time.

Next, I construct a sample that includes submissions and comments about CRSP common stocks. To do so, I first obtain the list of tickers for CRSP common stocks, and then tag each submission with stock tickers through an iterative process of searching for tickers in the title and body text. If a submission is tagged with a ticker, then the associated comments are also tagged with the same ticker. Note that a submission or a comment can be associated with multiple stock tickers. Appendix C.2.2 provides further details on the sample construction and the tagging algorithm.

Network construction

The WSB user network on day t can be represented by a directed graph $\mathcal{G}_t = (\mathcal{V}_t, \mathcal{E}_t)$, where $\mathcal{V}_t = \{1, 2, \cdots, N_t\}$ is the set of users (or nodes, vertices) on the network, and $\mathcal{E}_t \subseteq \mathcal{V}_t \times \mathcal{V}_t \backslash \mathcal{D}_t$ is the set of directed edges between users, with $\mathcal{D}_t = \{(i, i) : i \in \mathcal{V}_t\}$ denoting self-loops.

To construct the node set $\mathcal{V}_t$ for day t, I select submissions and comments about CRSP common stocks,[8] made within the time window $[t - 30, t - 1]$. I define the node set $\mathcal{V}_t$ as the set of unique users who are authors of these selected submissions and comments. Hence, the nodes of the network are the users that have ever participated in the discussion of CRSP common stocks, during the 30-day window $[t - 30, t - 1]$.

To construct the edge set $\mathcal{E}_t$, I start by representing conversation threads as comment trees. A conversation thread consists of a particular submission and the associated comments. Figure A.1 shows an example of a conversation thread. This thread consists of a submission

8. The network constructed in this section is common to all stocks. Alternatively, one could also construct stock-specific networks, by selecting submissions and comments about a specific stock ticker and performing the rest of the construction in a similar way.

made by the user Deep*******Value and the comments (on this submission) made by five other users. In particular, two of the users, YoloFDs4Tendies and FroazZ directly commented on the submission made by Deep*******Value. The other three users, smols1, GrowerNotAShower11, and DingLeiGorFei commented on FroazZ's comment. This thread is represented as a comment tree on the left side of Figure A.2 panel (a). The comments made by YoloFDs4Tendies and FroazZ are called level-1 comments, since they were directly replying to the submission. The comments made by smols1, GrowerNotAShower11, and DingLeiGorFei are called level-2 comments, since they replied to a level-1 comment. The right side of Figure A.2 panel (a) shows another tree, with quantkim being the author of the submission. FroazZ is the common user across the two trees.

I simplify each comment tree following Gianstefani et al. (2022). Specifically, I assume that any level-k comment is a direct reply to the submission, even if the comment was originally replying to some other comments. Figure A.2 panel (b) shows the simplified trees that correspond to the original ones in Figure A.2 panel (a).

I construct one simplified tree for each selected submission within the $[t-30, t-1]$ time window. The nodes in each tree are the users who authored the submission or the associated comments. The set of directed edges are from users who commented on the submission to the user who authored the initial submission.

Finally, I define the edge set $\mathcal{E}_t$ as the union of the directed edges of all conversation trees. For example, the two trees of Figure A.2 panel (b) have a common user FroazZ. After I take the union of the two trees, two edges come out of FroazZ – one points to Deep*******Value (who is the author of the submission in the first conversation), and the other points to quantkim (who is the author of the submission in the second conversation). Figure A.2 panel (c) shows the resulting network. Note that there are also cases where two distinct users i and j belong to multiple trees, and there is a directed edge from user i to user j in each tree. Then I only keep one edge from i to j in the edge set $\mathcal{E}_t$.[9] Furthermore, I drop self-loops, i.e., any edge from a user to himself.

To summarize, the user network on day t consists of node set $\mathcal{V}_t$ and edge set $\mathcal{E}_t$. The node set $\mathcal{V}_t$ is the set of unique users who are authors of the selected submissions and comments. The edge set $\mathcal{E}_t$ captures the connections between users. For any two distinct users $i, j \in \mathcal{V}_t$, if user j made a submission within the $[t-30, t-1]$ time window and user i commented on that submission, then there is a directed edge from i to j, i.e., $(i, j) \in \mathcal{E}_t$.

Influence measure

Based on the day-t network, I can measure the "influence" of each user on the network. In Section 1.3.4, I explore the properties of the influence distribution in the cross section of users.

First define the adjacency matrix $\boldsymbol{A}_t = \left(a_{ij,t}\right)$, which is an $N_t \times N_t$ square matrix with

$$a_{ij,t} \equiv \begin{cases} 1, & (i,j) \in \mathcal{E}_t \\ 0, & \text{otherwise} \end{cases}. \tag{1.1}$$

In other words, in the day-t network, there is a directed edge from user i to user j if and only if $a_{ij,t} = 1$. Hence, the adjacency matrix encodes the same information about user connections as the edge set $\mathcal{E}_t$. $a_{ij,t} = 1$ indicates that user i "listens to" or "attends to" user j, in the sense that i has commented on j's submission during the past 30 days.

Then I normalize the rows of the adjacency matrix to be 1 to get the weighting matrix

9. Alternatively, one could assign a positive weight to the edge from i to j, where the weight corresponds to the number of trees that have an edge from i to j, which is also the number of times user i commented on user j's submissions within the specific time window.

$\boldsymbol{W}_t = \left(\omega_{ij,t}\right)$, where

$$\omega_{ij,t} \equiv \frac{a_{ij,t}}{\sum_{j=1}^{N_t} a_{ij,t}}. \tag{1.2}$$

I define the in-degree of user j on day t as

$$d_{j,t}^{in} \equiv \sum_{i=1}^{N_t} \omega_{ij,t}. \tag{1.3}$$

I call $d_{j,t}^{in}$ the "influence" of user j on day t. Intuitively, $\omega_{ij,t}$ captures the weight that user i assigns to user j, among all users that i listens to. Then $d_{j,t}^{in}$ sums up the weights that user j gets from all other users. A higher value of $d_{j,t}^{in}$ indicates more users listen to or attend to j, and thus j is more influential.

Retail sentiment measures

For each submission (or comment), I conduct textual analysis on its augmented body text,[10] using the Python sentiment analysis tool Valence Aware Dictionary and sEntiment Reasoner (VADER). VADER is a sentiment analysis tool attuned to social media text (Hutto and Gilbert, 2014). Its lexicon includes emojis and emoticons. Following Mancini et al. (2022), I further augment the VADER dictionary with the WSB-specific jargons listed in Table B.1.

For a submission (or comment) l about stock n made by user j on day t, VADER returns a weighted composite sentiment score $Sent_l$ normalized to the range $[-1, 1]$.[11] A score in $[-1, -0.05]$ indicates the submission has a negative tone, a score in $[0.05, 1]$ indicates a positive tone, and a score in $(-0.05, 0.05)$ indicates a neutral tone.

I aggregate the sentiment scores to stock-day level. I first compute an equal-weighted

10. A submission has its title and body text. I obtain the augmented body text by appending the body text to the title, separated by a white space. A comment only has body text (without title).

11. I use the compound score returned from VADER.

sentiment measure for stock n on day t, defined as

$$Sent_t^{EW}(n) \equiv \frac{1}{|\mathcal{L}_t(n)|} \sum_{l \in \mathcal{L}_t(n)} Sent_l, \tag{1.4}$$

where $\mathcal{L}_t(n)$ is the set of submissions and comments about stock n that came out within the window (4pm on day $t-1$, 4pm on day t], and $|\mathcal{L}_t(n)|$ is the number of submissions and comments in this set. For Monday sentiment, in addition to including the 4pm-midnight articles from Sunday, I also include articles from 4pm to midnight on the prior Friday.

Then I construct an influence-weighted sentiment measure, $Sent_t^{IW}(n)$, for stock n on day t. It is the average sentiment across users weighted by their influence,

$$Sent_t^{IW}(n) \equiv \frac{1}{|\mathcal{J}_t(n)|} \sum_{j \in \mathcal{J}_t(n)} d_{j,t}^{in} \cdot Sent_{j,t}(n), \tag{1.5}$$

where $Sent_{j,t}(n) \equiv \frac{1}{|\mathcal{K}_{j,t}(n)|} \sum_{l \in \mathcal{K}_{j,t}(n)} Sent_l$ is the average sentiment of all submissions and comments about stock n made by user j on day t, $\mathcal{K}_{j,t}(n)$ is the set of submissions and comments about stock n made by user j on day t, $\mathcal{J}_t(n)$ is the set of users who made submissions or comments about stock n on day t, and $d_{j,t}^{in}$ is the influence of user j on day t defined in equation (1.3).

I use $Sent_t^{EW}(n)$ and $Sent_t^{IW}(n)$ as measures of retail investors' sentiment about stock n on day t. By construction, both measures are within the range $[-1, 1]$.

1.2.2 Financial data

I obtain data on stock prices and shares outstanding from CRSP, short interest data from IHS Markit and Compustat, holdings data of 13F institutions from FactSet, and retail order flow data from TAQ.

Short interest

I obtain the daily number of shares sold short from IHS Markit. I also obtain mid-month and end-month number of shares sold short from Compustat.

Short interest of stock n on day t, $SI_t(n)$, is defined as the ratio of the number of shares sold short to the number of shares outstanding

$$SI_t(n) = \frac{S_t^{short}(n)}{S_t^{out}(n)}, \tag{1.6}$$

where $S_t^{short}(n)$ is the number of shares sold short, and $S_t^{out}(n)$ is the number of shares outstanding.[12]

Institutional and household holdings

I retrieve quarterly portfolio holdings of 13F institutions from FactSet. Following Gabaix and Koijen (2022) and Koijen et al. (2022), I classify 13F institutions into five groups: Hedge Funds, Brokers, Private Banking, Investment Advisors, and Long-Term Investors. I then compute the total number of shares held by the institutions in each group. Appendix C.3 includes further details on the data construction.

I back out household holdings from the market clearing condition, as in Mainardi (2022). I assume households do not short, and short sellers is a separate group of investors that are distinct from households and the long-only institutions in the 13F data. Then for stock n at the end of quarter t, the market clearing condition can be written as

$$Q_t^{\text{Households}}(n) + \sum_{g \in G} Q_t^g(n) = S_t^{out}(n) + S_t^{short}(n). \tag{1.7}$$

$Q_t^{\text{Households}}(n)$ is the number of shares held by households. $Q_t^g(n)$ is the total number of

12. Figures C.1 and C.2 compare the short interest measure constructed from IHS Markit data versus that from Compustat data.

shares held by the 13F institutions in group $g \in G$, where $G = \{$Hedge Funds, Brokers, Private Banking, Investment Advisors, Long-Term Investors$\}$. $S_t^{out}(n)$ is the total number of shares outstanding, and $S_t^{short}(n)$ is the number of shares sold short from Compustat.

Equation (1.7) is an accounting identity. It says the total number of shares held by long-only investors is equal to the number of shares outstanding plus the additional supply of shares from short selling. In the data, I observe the holdings of long-only institutions $\{Q_t^g(n)\}_{g\in G}$, the number of shares outstanding $S_t^{out}(n)$, and the number of shares sold short $S_t^{short}(n)$. Hence, I can back out the number of shares held by households from equation (1.7)

$$Q_t^{\text{Households}}(n) \quad = \quad S_t^{out}(n) + S_t^{short}(n) - \sum_{g\in G} Q_t^g(n)\,. \tag{1.8}$$

For each investor group $k \in G \cup \{\text{Households}\}$, I construct two measures of their percentage holdings.

- Shares held by investor group k as a percentage of the number of shares outstanding:

$$q_t^k(n) \quad \equiv \quad \frac{Q_t^k(n)}{S_t^{out}(n)}. \tag{1.9}$$

- Shares held by investor group k as a percentage of the sum of the number of shares outstanding and the number of shares sold short:

$$\hat{q}_t^k(n) \quad \equiv \quad \frac{Q_t^k(n)}{S_t^{out}(n) + S_t^{short}(n)}. \tag{1.10}$$

  Note $\sum_k \hat{q}_t^k(n) = 1$.

For the rest of the paper, I treat households and retail investors as the same group of investors, and I use household holdings as a measure of retail investors' positions.

Figures C.3 and C.4 plot the total institutional holdings versus the sum of the number of shares outstanding and the number of shares shorted for GameStop and AMC. After correcting for the additional supply from short selling (equation (1.7)), the total institutional holdings do not exceed the total supply.

Retail order flow

The household holdings above is an indirect measure of retail investors' positions. In this section, I present a direct yet noisy measure based on retail order flow. It serves as a cross-check of the indirect measure.

Boehmer et al. (2021), referred to as BJZZ hereafter, propose an algorithm to identify off-exchange trades made by retail investors, based on sub-penny price improvement. Importantly, they assume the bid-ask spread is equal to one cent, and thus the price improvement has to be a fraction of one cent. If a trade was executed at less (more) than 0.4 (0.6) of a cent, they label it as a retail sell (buy) trade. Barber et al. (2022) modify the BJZZ algorithm to take into account the cases where the bid-ask spread is much larger than one cent. Bernhardt et al. (2022) further examine wholesalers' decisions to internalize retail orders and the effect on the retail order imbalance measure from BJZZ.

I use the modified BJZZ algorithm in Barber et al. (2022) to identify retail buy trades and sell trades. Appendix C.4 includes details of the algorithm. For stock n on day t, I first compute the total volume of retail buy orders $Mrbvol_t\,(n)$ and the total volume of retail sell orders $Mrsvol_t\,(n)$. Then I define cumulative net retail buy volume of stock n on day t as the cumulative difference between the two

$$\textit{Cum Net Retail Buy Vol}_t\,(n) \equiv \sum_{s=0}^{t} Mrbvol_s\,(n) - Mrsvol_s\,(n)\,. \tag{1.11}$$

Finally, I define cumulative net retail flow of stock n on day t as the ratio of the cumulative net retail buy volume to the sum of the number of shares outstanding and the number of shares shorted

$$Cum\ Net\ Retail\ Flow_t\,(n) \;\equiv\; \frac{Cum\ Net\ Retail\ Buy\ Vol_t\,(n)}{S_t^{out}\,(n) + S_t^{short}\,(n)}. \tag{1.12}$$

1.3 Stylized facts: prices, quantities, and retail investors' beliefs

1.3.1 Price and aggregate retail sentiment

On January 28, 2021, GameStop hit an intra-day high price of $483, compared to a price of less than $20 throughout 2020. This price surge was believed to be driven by retail investors who communicated on WSB. So I begin by analyzing the relationship between GameStop's stock price and the aggregate retail sentiment from WSB.

Figure A.3 plots the daily close price of GameStop (solid blue line), together with the equal-weighted retail sentiment from WSB (dotted red line).[13] The equal-weighted sentiment started at close to 0 in 2020 Q2, steadily increased to 0.2 till 2021 Q1, and remained stable for the rest of 2021. Recall from Section 1.2.1 that a sentiment score in $[0.05, 1]$ indicates an optimistic tone. Then the average sentiment level of 0.2 in 2021 suggests that retail investors were indeed optimistic, but far from extremely optimistic.

More importantly, at different points in time, the same change in average retail sentiment had dramatically different price impact. For example, the equal-weighted sentiment increased by 15% from mid- to late December 2020, and also from early to late January 2021. Yet the price of GameStop increased by 1700% in the latter period, compared to 36% in the former. Moreover, the average retail sentiment of GameStop was stable in the latter half of 2021, but despite that, the price of GameStop still exhibited substantial volatility.

The price impact of average retail sentiment shocks not only had significant time

13. In Figure A.3, I plot 30-day moving averages of the daily sentiment series.

variation, but also differed across stocks. Figure A.4 panel (a) compares the equal-weighted sentiment of GameStop with that of two technology stocks – Amazon and Microsoft.[14] From late 2020 to early 2021, retail sentiment of Amazon and Microsoft had a similar increase as that of GameStop. However, Figures C.5 and C.6 show that the prices of the two stocks did not soar as GameStop did in January 2021.

Aggregate retail sentiment is a combination of the average sentiment across users and the number of users who participate in the discussions on the social network. Figure A.5 shows that, despite the moderate increase in average retail sentiment, the discussion volume about GameStop spiked in January 2021. Hence, the aggregate retail sentiment increased more than the average retail sentiment.

The change in aggregate retail sentiment effectively shifted the aggregate demand curve of retail investors, and its price impact crucially depends on the demand of investors who took the other side of the trade. In the extreme case where other investors (who traded GameStop) are perfectly price-elastic, they would willingly take the other side and prices would be unaffected, then retail sentiment change would have zero price impact. Conversely, a lack of price-elastic investors in this market could help explain the price surge of GameStop in late January of 2021. In Sections 1.3.2 and 1.3.3, I present facts on who took the other side of the trade and how their positions changed over time.

As a robustness check, I plot the price and sentiment of AMC in Figure C.7. The price of AMC had a similar spike in late January of 2021, and its equal-weighted sentiment had a similar steadily increasing trend.

I summarize the findings of this section into the following fact.

Fact 1: In the time series, the average retail sentiment of GameStop has been steadily increasing since the beginning of 2020, while the discussion volume on WSB about GameStop spiked in January 2021. The spike in discussion volume coincided with the price surge of

14. In Figure A.4, I plot 30-day moving averages of the daily sentiment series.

GameStop. In the cross section, there are technology stocks that had similar trends in the average sentiment but did not have a price surge as GameStop did.

With a large number of retail investors participating in the social network, the conventional wisdom is that idiosyncratic shocks to their beliefs should "average out" and should not lead to fluctuations in aggregate retail sentiment. The average retail sentiment should remain neutral, and so should the aggregate sentiment. However, this conventional wisdom does not hold in the data – the average sentiment has been positive and steadily increasing throughout 2020 and 2021.

In Section 1.3.4 below, I demonstrate that the concentration of Reddit's WSB social network can resolve this puzzle. If investors update their beliefs according to their network connections and the network linkages are highly concentrated around a few "influencers," idiosyncratic belief shocks do not average out. In particular, when the influencers happen to be optimistic, retail investors on average will also be optimistic. As more investors participate in the discussion and adopt the influencers' views, the aggregate optimism will be further amplified. A concentrated network allows the average retail sentiment to build up in the first place.

1.3.2 Positions of long-only investors

Figure A.6 plots the quarterly holdings of households and long-only institutions of GameStop, as a fraction of the number of shares outstanding plus the number of shares sold short (equation (1.10)). From 2020 Q1 to 2021 Q1, households (blue shaded area) gradually built up their positions in GameStop, relative to long-only institutions. Households' relative positions remained constant for the rest of 2021. This suggests that households (or retail investors) were relatively more optimistic than long-only institutions, and the dynamics of household holdings is consistent with the dynamics of retail sentiment documented in Section 1.3.1.

Interestingly, long hedge funds (red shaded area) also built up their positions in 2020, but then liquidated almost all their long positions in 2021 Q1. One story is that long hedge funds were initially riding the price increase in 2020.[15] But after the price surge in January 2021, their initial long strategies may not be profitable, as they may have expected the price to quickly fall back to the pre-January-2021 level.

Figure A.7 panels (d), (e), and (f) plot the holdings of households, investment advisors, and hedge funds using the number of shares outstanding as the denominator (equation (1.9)). These figures show the "absolute" holdings of each group of investors, which had similar patterns as the relative holdings in Figure A.6 and Figure A.7 panels (a)-(c). For AMC, Figures C.8 and C.9 show similar patterns in the holdings of households versus long-only institutions.

In Figure A.8 (and Figure C.10 for AMC), I compare the quarterly household holdings measure in equation (1.10) with the daily cumulative net retail flow measure in equation (1.12). Both measures exhibit an increasing trend, though the latter has a temporary drop in late January of 2021, and the change in the latter from early 2020 to late 2021 is only half of the change in the former.

I summarize the key results in the following fact.

Fact 2: Households built up their positions in GameStop from 2020 to 2021, while long-only institutions reduced their positions. As a notable exception, long hedge funds initially built up their positions throughout 2020 then liquidated almost all their positions after 2021 Q1.

1.3.3 Positions of short sellers

Section 1.3.2 documents that the long-only institutions reduced their positions in GameStop, possibly because they thought the price was "too high" in January 2021, and it would quickly

15. Brunnermeier and Nagel (2004) document similar behaviors of hedge funds during the technology bubble.

drop to the pre-January-2021 level. If short sellers (e.g., short hedge funds) held the same belief, they would short more of GameStop in January 2021, hoping to profit from the subsequent price drop.

However, the data suggest the opposite. Figure 1.1 plots the daily short interest of GameStop (dotted red line) together with the price of GameStop (solid blue line). Short interest started out high at 80% of the outstanding shares till the end of 2020. But surprisingly, it dropped sharply in January 2021 and stayed at below 20% throughout 2021.[16] Given the high price of GameStop in 2021, it would be profitable for short sellers to take even larger short positions. But instead, they seem to have dropped out the market since January 2021.

Anecdotally, some short sellers were squeezed and lost capital. For example, Melvin Capital was forced to cover its short positions in GameStop and lost 53% on its investments in January 2021.[17] If these short sellers account for a large fraction of the short positions opened before January 2021, then the sharp drop in short interest is consistent with the fact that they lost capital and exited the market.

The short squeeze might have been triggered by the 15% retail sentiment increase from early to late January 2021 (see Section 1.3.1). Consider a short seller who already had a large short position in GameStop prior to January 2021 and who faced a margin constraint. A 15% increase in the average retail sentiment could make the margin constraint bind and force the short seller to close part of the short position.

However, the remaining question is why "sophisticated" short sellers failed to anticipate

16. A short interest of 20% of outstanding shares is still considered high relative to an average stock. So the puzzle here is not the absolute level of the short interest in January 2021, but the time series patterns of the short interest.

17. Adinarayan, T. (2021, January 28). Explainer: How retail traders squeezed Wall Street for bets against GameStop. *Reuters.* `https://www.reuters.com/business/retail-consumer/how-retail-traders-squeezed-wall-street-bets-against-gamestop-2021-01-27/`; Chung, J. (2021, February 1). Melvin Capital Lost 53% in January, Hurt by GameStop and Other Bets. *WSJ.* `https://www.wsj.com/articles/melvin-capital-lost-53-in-january-hurt-by-gamestop-and-other-bets-11612103117`.

the increase in retail sentiment and still maintained a large short position till January 2021. In Section 1.3.4, I explore the changing social dynamics on Reddit's WSB, which likely led to an "unexpected" retail sentiment increase from the short sellers' perspective.

I sum up the findings of this section into the following fact.

Fact 3: The short interest of GameStop started out high at 80% of the outstanding shares until the end of 2020. But then it dropped sharply in January 2021 and stayed below 20% throughout the rest of the year.

Long-only institutions and short sellers are the two groups of investors who can take the other side of the trade against retail investors. However, they are both constrained in terms of taking (large) short positions. Long-only institutions like Fidelity do not short for institutional reasons, while short sellers like Melvin Capital face margin constraints. If retail sentiment keeps rising and drives up the price, both groups of investors will hit their constraints at some point. Once short sellers hit their margin constraints, they will be forced to cover their short positions, and price could rise even further. In Section 1.4, I present a model to formalize this idea.

1.3.4 Changing social dynamics on Reddit's WallStreetBets forum

In this section, I document the changing dynamics of Reddit's WSB community leading up to the short squeeze in January 2021. If short sellers failed to anticipate these changes, they would likely make "mistakes" in opening or covering their short positions, or even get squeezed.

I first examine the aggregate dynamics of Reddit's WSB community. Figure A.10 presents some descriptive statistics of daily submissions, comments, and user activity on WSB. Panel (a) shows that the number of subscribers to WSB (solid blue line) grew exponentially in late January of 2021, and then the growth rate reverted back to its pre-January-2021 level. Consistent with the growth of subscribers, there was a concurrent surge in the daily number of new submissions (panel (b) solid blue line), the daily number of new comments (panel (b) dotted red line), and the daily number of users who participated[18] in the discussions of CRSP stocks (panel (c)) in late January of 2021. Moving to the subjects of the discussions, panel (d) shows that the number of stock tickers mentioned (on a given day) also spiked in late January of 2021 – over 700 tickers were mentioned on a given day, compared to less than 200 tickers before January 2021.

These facts suggest that WSB users became more engaged in the discussions in January 2021, and the engagement coincided with the price surge of GameStop. But how exactly did individual users' engagement translate into "collective actions" that could squeeze short sellers? How is it related to the 15% sentiment increase from early to late January of 2021?

To answer these questions, I inspect the day-to-day activities of WSB users, and in particular, how influential users manage to spur others. Figure A.11 shows the user communications on January 14, 2021.[19] Panel (a) plots the user activities from 6-9am right before the market opened. Each node represents a unique user who made a new submission or comment within this three-hour window. For any two users i and j in this figure, if i commented on j's submission (within the three-hour window), I draw a directed edge from i to j. For example, the largest red dot represents the AutoModerator, and the dots clustered around it represent the users who commented on AutoModerator's submission.

The AutoModerator created "Daily Discussion Thread for January 14, 2021" at 06:00:18 on January 14, 2021. This thread quickly became the center of WSB discussions, as it received 46,228 comments, which is 94.26% of the comments received by new threads that came out between 6am and 9am. A similar discussion "hub" emerged right after market closed: at 16:00:16 on the same day, the AutoModerator started another thread titled "What

18. I define "participation" as follows: A user participated in the discussions about CRSP stocks on a given day, if and only if he made at least one new submission or one new comment about CRSP stock(s) on that day.

19. This figure is inspired by Mancini et al. (2022).

Are Your Moves Tomorrow, January 15, 2021." Like the morning discussion thread, this afternoon thread was the dominant thread on WSB between 4pm and 7pm (Figure A.11 panel (b)), which received 80.28% of the comments.

These two types of threads are routine discussions on WSB. On each weekday, the AutoModerator will publish a new "Daily Discussion Thread" before market opens and a new "What Are Your Moves Tomorrow" thread after market closes. Users typically discuss the market conditions and their trading strategies under these threads (Boylston et al., 2021; Mancini et al., 2022).

"Daily Discussion Thread" and "What Are Your Moves Tomorrow" are two prominent examples of "megathreads" on WSB, which are user-initiated discussions designated for a specific topic or issue. There are other megathreads for discussing individual stocks, e.g., GME megathreads. Figure A.12 plots the WSB discussions between 6am and 8am on January 21, 2021. At 07:49:03, user grebfar created a thread titled "GME Megathread - Lemon Party 2: Electric Boogaloo." It received 67.84% of the comments, which is twice as many as the comments received by the daily discussion thread.

Figure A.13 shows further evidence on the relative influence of GME megathreads versus the daily discussion threads and how the relative influence evolves over time. The y-axis is the fraction of comments (on each day) received by a particular type of thread. The solid black line represents "GME Megathread," the dotted red line represents "Daily Discussion Thread" at market open, and the dash-dotted blue line represents "What Are Your Moves Tomorrow" at market close.[20] On January 20, 2021, the first GME megathread appeared and garnered as many comments as the daily discussion threads. It continued to be as influential as the daily discussion threads until mid-April of 2021, after which no new GME

20. To identify GME megathreads, I search for the keyword "GME Megathread" (case-insensitive) in the title of the threads. I identify "Daily Discussion Thread" and "What Are Your Moves Tomorrow" in a similar way. On a given day, there could be multiple threads of the same type, for example, multiple threads with "GME Megathread" in their titles. In that case, I take the total number of comments received by each type of thread and then compute the fraction of comments each type received, which is what I plot on the y-axis of Figure A.13.

megathreads were created.

Megathreads could facilitate "collective actions" in the following sense: they make users' views visible to each other at a designated place. A particular user is able to gain influence within a short period of time and his view can suddenly dominate the community, which then leads to the kind of "collective actions" that short sellers may fail to anticipate. Next, I explore the dynamics of the influence distribution among users and the dynamics of influencers' views.

Dynamics of the influence distribution across users

Figure A.14 plots the user network for GameStop discussion on January 14, 2021.[21] The red dots represent the top five most influential users. For each of these influencers, the percentage in parentheses is the fraction of users (on this network) that had commented on his posts within the past 30 days. Deep*******Value turns out to be the most influential user for GME discussion, and he attracted 20% of the users to comment on his posts.

Figure A.14 also reveals that the influence distribution is highly right-skewed, with a few influencers receiving a lot of attention. This is a common feature of many empirical social networks, and the heavy right tail of the influence distribution can be approximated by a power-law distribution (Newman, 2005; Rantala, 2019). If user influence $d^{in}_{j,t}$ (defined in equation (1.3)) is drawn from a power-law distribution, then it has PDF

$$f_{d^{in}_{j,t}}(x) = \frac{\xi - 1}{d_{\min}} \left(\frac{x}{d_{\min}} \right)^{-\xi}, \xi > 1 \tag{1.13}$$

with support $[d_{\min}, +\infty)$. The exponent ξ captures the right skewness of the influence distribution. Lower values of ξ correspond to heavier right tails and more right-skewed influence distribution. $d_{\min}$ is the lowest value at which the power law is obeyed (Newman,

21. Here I only use submissions and comments about GameStop to construct the network, and the rest of the construction follows Section 1.2.1.

2005).

$\xi = 3$ is the cutoff value under which the standard Central Limit Theorem holds. Specifically, as the number of users on the network (N) increases, the volatility of the aggregate retail sentiment decays at a rate of $\sqrt{N}$. A ξ value below 3 implies that the volatility of the aggregate retail sentiment decays at a slower rate. In this case, even with a large number of users on the network, idiosyncratic sentiment shocks do not average out and can still lead to large aggregate fluctuations in retail sentiment.

The power-law relationship implies that the log of influence $d^{in}_{j,t}$ and the log of the corresponding empirical frequencies (in the cross section of users) have a linear relationship. Figure A.15 plots this relationship for January 14, 2021. The x-axis is the log of user influence (or in-degree), and the y-axis is the log empirical frequency. The relationship is approximately linear, which is consistent with the power-law distribution.

I then fit the power-law distribution to the vector of user influence on each day. Following Rantala (2019), I estimate the exponent $\hat{\xi}_t$ and the cutoff value $\hat{d}_{\min,t}$ for each day t using the maximum likelihood method, and I compute the confidence bands using bootstrap methods. Appendix C.5 includes the computational details.

Figure A.16 plots the time series of the $\hat{\xi}_t$ estimates with the bootstrapped confidence intervals. $\hat{\xi}_t$ is below 3 throughout the sample. As discussed above, this means the influence distribution is highly right-skewed and the volatility of the aggregate retail sentiment decays at a rate slower than the standard Central Limit Theorem suggests. This right-skewed influence distribution is responsible for the aggregate fluctuations in retail sentiment. Moreover, from the beginning to the end of January 2021, $\hat{\xi}_t$ dropped by 10%, from 2.1 to 1.9. This suggests that the influence distribution became more right-skewed over time, which would allow influencers to spur more people.

Figure A.17 plots the time series of the cutoff value $\hat{d}_{\min,t}$, which remains relatively stable within the range $[5, 15]$. Furthermore, Figure C.12 plots the p-value of the Kolmogorov-

Smirnov test. A small p-value (less than 0.05) indicates that the test rejects the hypothesis that the original data could have been drawn from the fitted power-law distribution. For most of the dates, the test cannot reject the hypothesis that the original data are drawn from a power-law distribution.

Taken together, the influence distribution on WSB is right-skewed. This implies influencers' views would quickly become dominant. If they happen to be optimistic, the WSB community would quickly become optimistic as well. This could help explain the 15% increase in average retail sentiment from early to late January, 2021. In the next section, I document that influencers were indeed optimistic about GameStop.

Dynamics of influencers' views

In the previous section, I document that Deep*******Value was the most influential user in mid-January 2021. Figure A.18 plots some examples of his posts. The titles of his posts always started with "GME YOLO." "YOLO" is a jargon on WSB and is considered a positive word – it means "You Only Live Once." Hence, the influencer Deep*******Value was indeed optimistic about GameStop, and his influence would allow him to spur a large number of users in the community.

Figure A.3 shows the time variation of influencers' views. The dash-dotted green line is the influence-weighted sentiment for GameStop defined in equation (1.5), and the dotted red line is the equal-weighted sentiment in equation (1.4). From July to November 2020, the influence-weighted sentiment led the equal-weighted sentiment, which suggests influencers happened to be optimistic and they spurred other users on the network.

I collect the results from this section in the following fact.

Fact 4: The distribution of user influence on Reddit's WSB follows the power law with a heavy right tail, i.e., the influence distribution is highly right-skewed. Furthermore, the influencers on WSB happened to be optimistic in the period leading up to January 2021.

1.3.5 Proposed mechanism

Sections 1.3.1-1.3.4 provide a comprehensive analysis of the price, quantity, and retail sentiment dynamics before and after the GameStop frenzy. In this section, I propose a mechanism that can explain these facts. In Section 1.4, I will formalize the idea within a model.

At the beginning of 2020, short sellers like Melvin Capital were pessimistic about GameStop's future prospects and believed that GameStop was "over-valued." Hence, they maintained large short positions, hoping to profit from a future price drop.

In mid-2020, influencers on Reddit's WSB like Deep*******Value started to express their optimistic views about GameStop. Other users on WSB adopted the optimistic views and started to take long positions in GameStop. This resulted in a moderate price increase, which "drove out" price-elastic long-only institutions and attracted (more) short sellers to further increase their short positions, as they all thought the price was too high.

In January 2021, WSB went through a structural change – more users joined the network and the influence distribution remained highly right-skewed. This allowed influencers like Deep*******Value to be more influential and spur more people. Aggregate retail sentiment further increased, driving up the price and pushing short sellers towards their margin constraints. Short sellers did not expect this further sentiment increase, i.e., they were "surprised."

In late January of 2021, short sellers had to cover their short positions and suffered losses. The short covering led to an increase in the stock price, causing further losses for the short sellers. This ultimately led to the price surge on January 28, 2021. Some short sellers lost a large fraction of their capital and exited the market.

For the rest of 2021, retail investors and price-inelastic institutions like index funds remained in the market. Retail investors continued to be optimistic throughout 2021. Price-elastic long-only institutions and short sellers both dropped out of the market, and they no longer took the other side of the trade against the optimistic retail investors. Then a "small" retail sentiment shock would have a "large" price impact, due to a lack of price-elastic investors in this market.

Short sellers also adjusted their perceptions of retail sentiment risk, after observing a large influx of retail investors to the WSB forum in January 2021. They traded less aggressively in the latter half of 2021, being aware that the social network structure could change dramatically within a short period of time. This represents a new risk for them to adapt to.

1.4 The pricing of retail sentiment risk

In this section, I present a model to explain the price, quantity and retail sentiment dynamics documented in Section 1.3. In particular, I show a moderate increase in aggregate retail sentiment can have a large price impact, if it drives out price-elastic long-only institutions and squeezes short sellers. The price of retail sentiment risk depends on this shift in investor composition.

1.4.1 Setup

Time is discrete and is indexed by $t \in \{-1, 0, 1, 2\}$. There are $\bar{N} + 2$ investors who are divided into three groups: $\bar{N}$ retail investors indexed by j, a long institution (IL), and a short institution (IS).[22] Investors trade a risky asset and a risk-free asset. They differ in their beliefs about the risky asset's payoff, their risk aversion, and the portfolio constraints they face.

22. As will be clear in Section 1.4.2, I assume that all investors take price as given and they do not internalize that their trading affects prices. We can think of the long institution IL as representing a continuum of competitive long-only institutional investors with homogeneous beliefs, and each of them takes price as given. We can interpret the price-taking assumption for the short institution IS in a similar way.

Assets Assets are traded at time $t \in \{0, 1\}$. The risk-free asset is in zero net supply. Since there is no interim consumption (see footnote 24), it is without loss of generality to set the risk-free rate to be 1, i.e., the raw return of the risk-free asset is assumed to be $R_{f,t} = 1$.[23]

The risky asset has a constant supply of $\bar{S}$ shares, and it pays a one-time dividend $\tilde{D}$ at time 2. Let $\tilde{d} \equiv \log \tilde{D}$ denote its log payoff. The dividend payment is unobserved at time $t \in \{-1, 0, 1\}$. The time-t conditional distribution of $\tilde{d}$ is truncated normal on the interval $\left[\underline{d}, \bar{d}\right]$ with post-truncation mean μ_d and variance σ_d^2. I assume a bounded support for the dividend so that for any investor, given his portfolio choice in Section 1.4.2 below, there is zero probability of going bankrupt in the next period. In Section 1.4.3 below, I assume a bounded support for the aggregate retail sentiment for the same reason. Footnote 24 and Remark 1 further discusses the issue of bankruptcy in this discrete-time setting.

Let P_t and $p_t \equiv \log P_t$ denote the price and log price of the risky asset at time t, and let $\log X_t$ denote its log payoff at time t. Then

$$\log X_0 = p_0, \log X_1 = p_1, \log X_2 = p_2 = \tilde{d}.$$

Further define $\mathbb{E}_t \left[\log X_{t+1}\right]$ and $\sigma_t^2 \equiv \mathrm{Var}_t \left(\log X_{t+1}\right)$ as the time-t conditional mean and variance of next period's log payoff, respectively. Note $\sigma_1^2 = \sigma_d^2$.

Then the risky asset has one-period raw return $R_{t+1} \equiv \frac{X_{t+1}}{P_t}$ from time t to $t+1$. Define $r_{t+1} \equiv \log R_{t+1}$ as the one-period log return of the risky asset and $r_{f,t} \equiv \log R_{f,t} = 0$ as the one-period log return of the risk-free asset.

Investors' subjective beliefs Investors have subjective beliefs about the risky asset's payoff. Specifically, at time $t \in \{0, 1\}$, investor i believes the log payoff of the risky asset at

23. In general, zero net supply of the risk-free asset should determine the endogenous risk-free rate. In my model, however, the endogenous risk-free rate is indeterminate because there is no interim consumption. So I can impose an exogenous risk-free rate $R_{f,t} = 1$, and it does not violate the market clearing condition for the risk-free asset.

time $t+1$ has mean $\mathbb{E}_t^i[\log X_{t+1}]$ and variance $\text{Var}_t^i(\log X_{t+1})$. I assume investors know the true variance of the log payoff

$$\text{Var}_t^i(\log X_{t+1}) = \sigma_t^2, \forall i. \tag{1.14}$$

Investors disagree about the mean of the log payoff. First consider the institutional investors. At time 0, the two institutions IL and IS have subjective beliefs (about the mean)

$$\mathbb{E}_0^{IL}[\log X_1] = \mathbb{E}_0[p_1] + \delta_0^{IL}, \tag{1.15}$$

$$\mathbb{E}_0^{IS}[\log X_1] = \mathbb{E}_0[p_1] + \delta_0^{IS}, \tag{1.16}$$

where $\mathbb{E}_0[p_1]$ is the objective (conditional) mean of time-1 log price, which is an equilibrium outcome. δ_0^{IL} and δ_0^{IS} capture the wedges between the subjective beliefs and the objective beliefs, and they are exogenously given. At time 1, the two institutions have subjective beliefs that are consistent with the objective mean, i.e.,

$$\mathbb{E}_1^{IL}[\log X_2] = \mathbb{E}_1^{IS}[\log X_2] = \mathbb{E}_1[p_2] = \mu_d. \tag{1.17}$$

Hence, at time 0, institutions disagree about the mean, while at time 1 they know the "true" mean.

There are two types of retail investors: at time t, the first N_t retail investors (labeled as "type 1") have subjective beliefs that deviate from the objective beliefs, while the rest $\bar{N} - N_t$ retail investors (labeled as "type 2") have subjective beliefs that conform with the objective ones. In particular, at time $t \in \{0, 1\}$, the subjective belief of type-1 retail investor

$j \in \{1, 2, \cdots, N_t\}$ is

$$\mathbb{E}_t^j \left[\log X_{t+1}\right] = \mathbb{E}_t \left[p_{t+1}\right] + y_t^j, \tag{1.18}$$

where y_t^j is the deviation of j's belief from the objective expectation. I call y_t^j the "sentiment" of retail investor j.

Type-1 retail investors communicate on a social network. They form subjective beliefs (and thus sentiment) by "listening to" other people on the network. In Section 1.5, I microfound their sentiment dynamics using a model of naive learning on networks. The model yields the conditional distribution of retail investor sentiment $\left\{y_t^j\right\}_{j=1}^{N_t}$.

Note the number of type-1 retail investors, N_t, is time-varying. I assume $0 \leq N_t \leq \bar{N}$, and I define the fraction of type-1 retail investors at time t as

$$\theta\left(N_t\right) \equiv \frac{N_t}{\bar{N}} \in [0, 1]. \tag{1.19}$$

Investors' preferences, budget constraint, and wealth share dynamics Investor i solves the following myopic portfolio choice problem[24]

$$\begin{aligned}\max_{w_t^i} \quad & w_t^i \left(\mathbb{E}_t^i\left[r_{t+1}\right] - r_{f,t}\right) + \frac{1}{2} w_t^i \left(1 - w_t^i\right) \operatorname{Var}_t^i\left(r_{t+1}\right) \\ & + \frac{1}{2}\left(1-\gamma^i\right)\left(w_t^i\right)^2 \operatorname{Var}_t^i\left(r_{t+1}\right),\end{aligned} \tag{1.21}$$

where γ^i is his constant relative risk aversion, and w_t^i is the fraction of end-of-period wealth invested in the risky asset, i.e., the portfolio weight on the risky asset. Define risk tolerance $\tau^i \equiv \frac{1}{\gamma^i}$. I assume institutional investors (IL and IS) have the same relative risk tolerance $\tau^I = \frac{1}{\gamma^I}$. The $\bar{N}$ retail investors have the same risk tolerance $\tau^R = \frac{1}{\gamma^R}$.

The budget constraint for investor i is

$$A_{t+1}^i \quad = \quad A_t^i \left(w_t^i \exp\left(r_{t+1}\right) + \left(1 - w_t^i\right) \exp\left(r_{f,t}\right)\right), \tag{1.22}$$

where A_t^i is the investor's wealth entering period t.

Since the risk-free asset is in zero net supply, the aggregate wealth is equal to the market

24. The objective in equation (1.21) is consistent with the portfolio choice problem of an investor with power utility over his next period's wealth, i.e.,

$$\max_{w_t^i} \mathbb{E}_t^i \left[\frac{\left(A_{t+1}^i\right)^{1-\gamma^i}}{1-\gamma^i}\right], \tag{1.20}$$

subject to the budget constraint in equation (1.22) and using the Campbell and Viceira (2002) approximation of the portfolio return. The Campbell and Viceira (2002) approximation works under the assumption that the log return of the risky asset is normally distributed and the time interval is short. In my model, however, I assume the log return of the risky asset is truncated normal. Moreover, the three dates −1, 0, and 1 in the model correspond to early 2020, late 2020, and January 2021 in the data, and thus the time interval is not short. This means I cannot use the Campbell and Viceira (2002) approximation to derive the objective (1.21) from the utility maximization problem (1.20). I instead assume that investors solve the mean-variance portfolio choice problem in (1.21). To rule out the possibility of bankruptcy (when an investor takes a levered position or a short position in equilibrium), I assume a bounded support for the dividend payment of the risky asset and also a bounded support for the retail sentiment. Remark 1 discusses the issue of bankruptcy in detail. Note that the objective (1.21) and the budget constraint (1.22) imply that there is no interim consumption and the investor only cares about the mean and variance of his portfolio return.

value of the risky asset. Hence, the time-1 wealth share of investor i is

$$\alpha_t^i \equiv \frac{A_t^i}{P_t \bar{S}}. \tag{1.23}$$

Appendix C.1.1 shows that the budget constraint (1.22) implies the following wealth share dynamics

$$\alpha_{t+1}^i = \alpha_t^i \left(\left(1 - w_t^i\right) \exp\left(p_t - p_{t+1}\right) + w_t^i \right). \tag{1.24}$$

Non-negative wealth constraint All investors are subject to the non-negative wealth constraint

$$A_t^i \geq 0, \forall t.$$

If an investor loses all his wealth, he cannot invest and has to exit the market.

Portfolio constraints Institutional investors face portfolio constraints. The long institution IL faces short-sale constraint of the form

$$w_t^{IL} \geq 0. \tag{1.25}$$

The short institution IS faces margin constraint on short selling. Following Gârleanu and Pedersen (2011), I assume the margin constraint limits the "leverage" the short seller can take, i.e.,

$$w_t^{IS} \geq -\frac{1}{m}, \tag{1.26}$$

where $m \in (0, 1)$.

Market clearing Following Caballero and Simsek (2021), I show in Appendix C.1.2 that the market clearing conditions for the risky asset and the risk-free asset are equivalent to the set of conditions

$$\sum_i A_t^i = \sum_i w_t^i A_t^i = P_t \bar{S}. \tag{1.27}$$

Equation (1.27) says aggregate wealth is equal to the market value of the risky asset, both before and after investors make portfolio decisions. The conditions in equation (1.27) are also equivalent to

$$\sum_i \alpha_t^i w_t^i = 1, \tag{1.28}$$

where the wealth share α_t^i is defined in equation (1.23). This condition says the wealth-share-weighted sum of portfolio weights is equal to 1.

Endowment and implicit price at time -1 At time -1, investor i is endowed with wealth share α_{-1}^i and portfolio weight w_{-1}^i. I assume that at time -1, investors do not anticipate future sentiment shocks. They all believe the prices at time 0 and 1 will reflect the present value of the final dividend payment. In Appendix C.1.10, I derive the implicit price p_{-1} that is consistent with this belief. Under this price, investors do not want to trade at time -1 and they enter time 0 with their initial endowment.

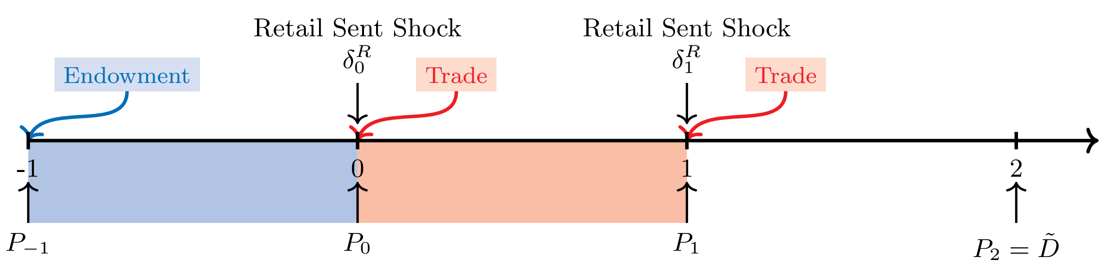


Figure 1.2. Timeline of the model.

Timeline Figure 1.2 shows the timeline of the model. At time -1, investors receive their endowment. At time 0 and 1, investors form subjective beliefs about next period's asset payoff and trade according to their beliefs. At time 2, the risky asset pays dividend.

In addition, I impose the following assumption.

Assumption 1. *At time $t \in \{0, 1\}$ before trading, retail investors first split their time $t-1$ end-of-period wealth equally among themselves. In particular, they split their aggregate stock position as well as aggregate bond position equally. Then they make portfolio choices based on their wealth after the splitting.*

Assumption 1 says that retail investors split their wealth equally before trading. This assumption together with linear demand implies that there exists an aggregate retail investor whose sentiment matters for asset prices. Lemma 1 in Section 1.4.2 formalizes this argument.

1.4.2 Investor demand

In this section, I first derive the asset demand of individual investors. Then I show that there exists an aggregate retail investor whose sentiment matters for asset prices.

Retail investors Type-1 retail investor j solves the portfolio problem in (1.21). His subjective expectation deviates from the objective expectation by y_t^j. Appendix C.1.3 shows that his optimal portfolio weights on the risky asset are

$$w_0^j = \tau^R \left(\frac{\mathbb{E}_0 [p_1] + y_0^j - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{1.29}$$

$$w_1^j = \tau^R \left(\frac{\mu_d + y_1^j - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{1.30}$$

Type-2 retail investors' subjective beliefs conform with the objective beliefs. Hence, a type-2 retail investor j' chooses portfolio weights

$$w_0^{j'} = \tau^R \left(\frac{\mathbb{E}_0 [p_1] - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{1.31}$$

$$w_1^{j'} = \tau^R \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{1.32}$$

Long institution The long institution solves the portfolio problem in (1.21), subject to the short-sale constraint in (1.25). Appendix C.1.3 shows that his optimal portfolio weights on the risky asset are

$$w_0^{IL} = \max \left\{ 0, \tau^I \left(\frac{\mathbb{E}_0 [p_1] + \delta_0^{IL} - p_0}{\sigma_0^2} + \frac{1}{2} \right) \right\}, \tag{1.33}$$

$$w_1^{IL} = \max \left\{ 0, \tau^I \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right) \right\}. \tag{1.34}$$

Short institution The short institution solves the portfolio problem in (1.21), subject to the margin constraint in (1.26). Appendix C.1.3 shows that his optimal portfolio weights on the risky asset are

$$w_0^{IS} = \max \left\{ -\frac{1}{m}, \tau^I \left(\frac{\mathbb{E}_0 [p_1] + \delta_0^{IS} - p_0}{\sigma_0^2} + \frac{1}{2} \right) \right\}, \tag{1.35}$$

$$w_1^{IS} = \max \left\{ -\frac{1}{m}, \tau^I \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right) \right\}. \tag{1.36}$$

For the rest of the paper, I focus on scenarios where in equilibrium, the portfolio constraints for institutions do not bind at time 0, while they may bind at time 1 depending on the retail sentiment realization $\left\{ y_1^j \right\}_{j=1}^{N_1}$.

Before characterizing the equilibrium, I first show that there exists an aggregate retail investor, whose sentiment drives asset price fluctuations.

Lemma 1 (Existence of an aggregate retail investor). *Under Assumption 1, the aggregate demand of the $\bar{N}$ retail investors is equal to the demand of an aggregate retail investor (R).*

- *The aggregate retail investor has subjective beliefs*

$$\mathbb{E}_0^R [p_1] = \mathbb{E}_0 [p_1] + \delta_0^R, \mathrm{Var}_0^R (p_1) = \sigma_0^2,$$
$$\mathbb{E}_1^R \left[\tilde{d}\right] = \mu_d + \delta_1^R, \mathrm{Var}_1^R \left(\tilde{d}\right) = \sigma_d^2.$$

His time-t sentiment δ_t^R $(t \in \{0,1\})$ aggregates individual retail investors' sentiment in the following way

$$\delta_t^R = \theta(N_t)\, y_t^R, \tag{1.37}$$
$$y_t^R \equiv \frac{1}{N_t}\sum_{j=1}^{N_t} y_t^j, \tag{1.38}$$

where N_t is the number of type-1 retail investors at time t, and $\theta(N_t)$ is the fraction of type-1 retail investors defined in equation (1.19).

- *The aggregate retail investor's demand for the risky asset (in terms of portfolio weights) takes the form*

$$w_0^R = \tau^R \left(\frac{\mathbb{E}_0 [p_1] + \delta_0^R - p_0}{\sigma_0^2} + \frac{1}{2}\right), \tag{1.39}$$
$$w_1^R = \tau^R \left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right). \tag{1.40}$$

- *The aggregate retail investor's time-t wealth aggregates individual retail investors'* *wealth*

$$A_t^R = \sum_{j=1}^{\bar{N}} A_t^j, \alpha_t^R = \sum_{j=1}^{\bar{N}} \alpha_t^j,$$

where A_t^R *and* α_t^R *are his dollar wealth and wealth share, respectively. His wealth share evolves according to*

$$\alpha_{t+1}^R = \alpha_t^R \left(\left(1 - w_t^R\right) \exp\left(p_t - p_{t+1}\right) + w_t^R \right). \tag{1.41}$$

- *The time-t equilibrium price of the risky asset is determined by the market clearing condition*

$$\alpha_t^R w_t^R + \alpha_t^{IL} w_t^{IL} + \alpha_t^{IS} w_t^{IS} = 1. \tag{1.42}$$

Proof. See Appendix C.1.4. □

This existence result comes from Assumption 1 and the linearity of investors' demand. From equations (1.29) and (1.30), an individual investor's demand is linear in his own sentiment. After retail investors split their wealth equally, their aggregate demand will be linear in the aggregate retail sentiment δ_t^R. Lemma 1 allows me to first study the pricing of aggregate retail sentiment shock in Sections 1.4.3 and 1.4.4, without characterizing individual retail investors' sentiment shocks.

The aggregate retail sentiment δ_t^R depends on the fraction of type-1 investors in the retail investor population ($\theta\left(N_t\right)$) and also the average sentiment among the type-1 investors (y_t^R). In Section 1.5, I show that the average sentiment y_t^R depends on the network geometry, in particular, the right-skewness of influence distribution on the network.

Remark 1 (Bankruptcy in discrete time). As pointed out in footnote 24, the objective in equation (1.21) is consistent with the portfolio choice problem of an investor with power utility over his next period's wealth, assuming that the risky asset's return is log-normally distributed and the time interval is short. For an investor with power utility, he would not take a short position (or a levered position) in this period if it yields a non-zero probability of going bankrupt next period. In this case, the portfolio rules in equations (1.29)-(1.36) would be suboptimal. To rule out the possibility of bankruptcy in this discrete-time setting, I assume a bounded support for the time-2 dividend of the risky asset (see Section 1.4.1) and also for the time-1 retail sentiment (see Section 1.4.3 below). Then in the numerical examples of Section 1.5.3 and Section 1.6, I set the parameters such that for any investor $i \in \{R, IL, IS\}$ and at any time $t \in \{0, 1\}$, in equilibrium, the conditional probability of going bankrupt in the next period is zero.

1.4.3 Equilibrium at time 1

At time 1, the aggregate retail sentiment δ_1^R drives the fluctuations in the price of the risky asset. The time-1 equilibrium (log) price $p_1\left(\delta_1^R\right)$ is a function of retail sentiment. I assume the time-0 conditional distribution of δ_1^R is truncated normal on the interval $\left[\underline{\delta}_1, \bar{\delta}_1\right]$[25] with CDF $\Psi(\cdot)$.

Under certain realizations of the retail sentiment shock, the portfolio constraints will be binding for the institutional investors, and there will be multiple equilibria. I focus on the class of monotone equilibria defined below.

Definition 1 (Monotone equilibrium at time 1**).** *A monotone equilibrium at time* 1 *is an equilibrium where the log price of the risky asset is strictly increasing in the retail sentiment realization, i.e.,* $p_1\left(\delta_1^R\right)$ *is strictly increasing in* δ_1^R.

To characterize the time-1 equilibrium, I first derive two cutoff prices p_1^m and p_1^h such

25. I assume a bounded support for the time-1 aggregate retail sentiment to rule out the possibility of bankruptcy. Remark 1 discusses the issue of bankruptcy in this discrete-time setting.

that: if $p_1 < p_1^m$, none of the institutions are constrained; if $p_1 \in \left[p_1^m, p_1^h\right)$, the long institution is constrained, while the short institution is unconstrained; if $p_1 \geq p_1^h$, both the long institution and the short institution are constrained. p_1^m is the cutoff price at which the short-sale constraint exactly binds for the long institution. I calculate p_1^m by setting the long institution's unconstrained demand to zero, which yields

$$p_1^m \equiv \mu_d + \frac{1}{2}\sigma_d^2. \tag{1.43}$$

p_1^h is the cutoff price at which the margin constraint exactly binds for the short institution, then

$$p_1^h \equiv \mu_d + \left(\frac{1}{2} + \frac{1}{m\tau^I}\right)\sigma_d^2. \tag{1.44}$$

Importantly, $p_1^m < p_1^h$. This immediately follows from comparing (1.43) with (1.44). The intuition is as follows. The two institutions have the same beliefs (recall from equation (1.17)) and only differ in their financial constraints – the long institution cannot short and thus faces a "tighter" constraint than the short institution. As retail sentiment increases and drives up the price, the long institution would first hit the short-sale constraint at a price of p_1^m. If retail sentiment continues to increase, then the price would rise further and the short institution would ultimately hit the margin constraint at a price of p_1^h.

In the type of monotone equilibrium of Definition 1, the two cutoff prices p_1^m and p_1^h correspond to two cutoff sentiment shocks $\delta_1^m = (p_1)^{-1}\left(p_1^m\right)$ and $\delta_1^h = (p_1)^{-1}\left(p_1^h\right)$,[26] Then $p_1^m < p_1^h$ implies that $\delta_1^m < \delta_1^h$. I impose the market clearing condition (1.28) to derive these

26. $(p_1)^{-1}(\cdot)$ denotes the inverse function of $p_1(\cdot)$. The price function $p_1(\cdot)$ is invertible in a monotone equilibrium of Definition 1.

cutoffs

$$\delta_1^m \equiv \frac{\sigma_d^2}{\alpha_1^R\left(p_1^m\right)\tau^R}, \tag{1.45}$$

$$\delta_1^h \equiv \frac{\frac{1}{m\tau^I}\hat{\tau}_1\left(p_1^h\right)+1}{\alpha_1^R\left(p_1^h\right)\tau^R}\sigma_d^2, \tag{1.46}$$

where $\hat{\tau}_1\left(p_1^h\right) \equiv \alpha_1^R\left(p_1^h\right)\tau^R + \alpha_1^{IS}\left(p_1^h\right)\tau^I$.

For a low retail sentiment shock realization, $\delta_1^R < \delta_1^m$, none of the investors are constrained. For an intermediate shock realization $\delta_1^R \in \left(\delta_1^m, \delta_1^h\right)$, the long institution is constrained while the short institution is unconstrained. For a high shock realization $\delta_1^R > \delta_1^h$, both the long institution and the short institution are constrained. If $\underline{\delta}_1 < \delta_1^m$ and $\delta_1^h < \bar{\delta}_1$, then as sentiment increases from $\underline{\delta}_1$ to $\bar{\delta}_1$, the long institution first hits the short-sale constraint, and then the short institution hits the margin constraint. Table 1.1 below summarizes the features of each sentiment region.

Table 1.1
Sentiment regions and binding constraints

Sentiment	Shock	Constrained		
region	realization	Agg. Retail	Long Inst.	Short Inst.
Low	$\delta_1^R \in \left[\underline{\delta}_1, \delta_1^m\right)$	No	No	No
Medium	$\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$	No	Yes	No
High	$\delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$	No	Yes	Yes

For the rest of the paper, I focus on equilibria where the three sentiment regions are non-empty, i.e., $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$.

Proposition 1 (Time-1 price). *Suppose a monotone equilibrium of Definition 1 exists at time 1 and $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$. Taking time-0 portfolios $\left\{w_0^i\right\}$ and wealth shares $\left\{\alpha_0^i\right\}$ as given, the time-1 equilibrium price function $p_1\left(\delta_1^R\right)$ is determined as follows:*

- *For* $\delta_1^R \in \left[\underline{\delta}_1, \delta_1^m\right)$, *the equilibrium features a price* $p_1 < p_1^m$ *that solves*

$$J\left(p_1, \delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R\left(p_1\right)\tau^R\delta_1^R - \sigma_d^2}{\tau_1\left(p_1\right)}\right) - p_1 = 0, \tag{1.47}$$

where $\tau_1\left(p_1\right)$ *is the aggregate risk tolerance of unconstrained investors, which is defined as*

$$\tau_1\left(p_1\right) \equiv \alpha_1^R\left(p_1\right)\tau^R + \left(1 - \alpha_1^R\left(p_1\right)\right)\tau^I. \tag{1.48}$$

- *For* $\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, *the equilibrium features a price* $p_1 \in \left[p_1^m, p_1^h\right)$ *that solves*

$$H\left(p_1, \delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R\left(p_1\right)\tau^R\delta_1^R - \sigma_d^2}{\hat{\tau}_1\left(p_1\right)}\right) - p_1 = 0, \tag{1.49}$$

where $\hat{\tau}_1\left(p_1\right)$ *is the aggregate risk tolerance of unconstrained investors, which is defined as*

$$\hat{\tau}_1\left(p_1\right) \equiv \alpha_1^R\left(p_1\right)\tau^R + \alpha_1^{IS}\left(p_1\right)\tau^I. \tag{1.50}$$

- *For* $\delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, *the equilibrium features a price* $p_1 > p_1^h$ *that solves*

$$G\left(p_1, \delta_1^R\right) \equiv \mu_d + \delta_1^R + \left(\frac{1}{2} - \frac{1 + \alpha_1^{IS}\left(p_1\right)\frac{1}{m}}{\alpha_1^R\left(p_1\right)\tau^R}\right)\sigma_d^2 - p_1 = 0. \tag{1.51}$$

The cutoff prices p_1^m *and* p_1^h *are defined in equations* (1.43) *and* (1.44), *and the cutoff sentiment shocks* δ_1^m *and* δ_1^h *are defined in equations* (1.45) *and* (1.46).

Proof. See Appendix C.1.5. □

Proposition 1 shows that in each of the sentiment region, the equilibrium price solves an implicit function. This is because the equilibrium price not only enters investors' demand but also determines their wealth shares. These implicit functions may have multiple solutions, which means there could be multiple equilibria. As retail sentiment realization δ_1^R increases, certain class of equilibria may disappear, giving rise to endogenous discontinuity in equilibrium price. Proposition 2 below presents the formal argument.

Proposition 2 (Endogenous discontinuity in time-1 price). *Consider an equilibrium with the following properties:*

- *Investors' time-0 optimal portfolios satisfy:* $w_0^R > 1$, $w_0^{IS} < 0 < w_0^{IL} < w_0^R$.
- *For any sentiment shock realization* $\delta_1^R \in \left(\underline{\delta}_1, \bar{\delta}_1\right)$*, the equilibrium price* $p_1\left(\delta_1^R\right)$ *is such that all investors have strictly positive wealth at time* 1.
- *The time-*1 *equilibrium is a monotone equilibrium of Definition 1.*

If $p_1\left(\delta_1^R\right)$ *is continuous on* $\left[\underline{\delta}_1, \delta_1^h\right)$ *and* $\left.\frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1}\right|_{p_1 = p_1^h} > 0$*, then* $p_1\left(\delta_1^R\right)$ *jumps discontinuously at* $\delta_1^R = \delta_1^h$*, i.e.,*

$$\lim_{\delta_1^R \to \left(\delta_1^h\right)^-} p_1\left(\delta_1^R\right) < \lim_{\delta_1^R \to \left(\delta_1^h\right)^+} p_1\left(\delta_1^R\right).$$

Proof. See Appendix C.1.7. □

To understand the endogenous discontinuity, I provide a numerical example with parameters given in Section 1.5.3. Figure A.19 plots the time-1 equilibrium price $P_1\left(\delta_1^R\right)$ as a function of the sentiment shock δ_1^R.[27] There is an endogenous jump at the cutoff δ_1^h, at which the margin constraint exactly binds for the short institution. Figure A.20 plots all the time-1 equilibria in this numerical example. Generically, for a given sentiment shock realization δ_1^R, there are one or three equilibria. In the knife-edge cases, there are two

27. Recall that $p_1\left(\delta_1^R\right)$ denotes the log price, and $P_1\left(\delta_1^R\right)$ denotes the price.

equilibria. In particular, there are two equilibrium prices at $\delta_1^R = \delta_1^h$ with p_1^h being the lower price.[28] As sentiment increases further above δ_1^h, the low-price equilibrium disappears and the high-price equilibrium becomes the unique equilibrium, giving rise to the endogenous jump. Moreover, under this set of parameter values, we cannot find a price path $p_1\left(\delta_1^R\right)$ that is continuous in the sentiment realization δ_1^R. Hence, we can pick any other class of equilibrium (i.e., not necessarily the low-price equilibrium), and there will still be a price jump at certain sentiment shock realization.

Hence, the endogenous jump in price is a result of multiple equilibria. Next, I show that the margin constraint and the wealth effect are responsible for multiple equilibria. I first analyze demand and supply around the cutoff sentiment δ_1^h from the short institution's perspective. The demand curve of the short institution can be written as

$$\frac{Q_1}{\bar{S}} = \begin{cases} \alpha_1^{IS}(p_1)\,\tau^I\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right), & p_1 \in \left[p_1^m, p_1^h\right] \\ -\frac{1}{m}\alpha_1^{IS}(p_1), & p_1 > p_1^h \end{cases}. \tag{1.52}$$

Around the cutoff δ_1^h, the long institution demands zero shares due to the binding short-sale constraint (recall from Table 1.1). Hence, the "residual supply curve" faced by the short institution is 1 minus the demand of the aggregate retail investor, which is

$$\frac{Q_1}{\bar{S}} = 1 - \alpha_1^R(p_1)\,\tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right). \tag{1.53}$$

Figure A.21 panel (a) plots the inverse demand curve (solid black line) and the inverse supply curves (blue lines) under different sentiment shock realizations. The demand curve is downward-sloping for $p_1 \leq p_1^h$, but is upward-sloping for $p_1 < p_1^h$. For a price higher

28. At the cutoff sentiment δ_1^h, both institutions hit their portfolio constraints, and the aggregate retail investor is the only marginal investor. There are two equilibria due to the wealth effect. A similar phenomenon arises in Caballero and Simsek (2021), where they assume that the investors have constant relative risk aversion and thus the wealth effect is present.

than p_1^h, the margin constraint binds for the short institution and he can only allocate a constant fraction $-\frac{1}{m}$ of his wealth to the risky asset. As price increases, he loses wealth on the short position. This wealth effect together with the margin constraint limits the number of shares he can short, resulting in an upward-sloping demand curve. The supply curves are upward-sloping for $p_1 > p_1^h$, but they are downward-sloping for $p_1 < p_1^h$ due to the wealth effect. In this numerical example, the aggregate retail investor has a levered position in the risky asset. As price decreases below p_1^h, he loses wealth and demands less shares, which effectively "increases" the number of shares supplied to the short institution.

The yellow dots represent the three equilibria under a sentiment shock that is slightly below δ_1^h. As sentiment increases to δ_1^h, the lower and middle equilibria collapse into one, so there are two equilibria represented by the two green dots. As sentiment increases further above δ_1^h, the low-price equilibrium disappears, and price jumps discontinuously to the red dot (high-price equilibrium).

Intuitively, when sentiment increases further above δ_1^h, an unconstrained short seller would increase his short position and there will still be a low-price equilibrium. With the margin constraint, short seller would short less than in the unconstrained case, and the low-price equilibrium no longer clears the market and price has to rise further. As price rises further, the short seller loses wealth and has to short even less, this again drives up the price. This feedback loop implies that the market (for the risky asset) only clears at a very high price, which is the high-price equilibrium.

This phenomenon ties to Gennotte and Leland (1990), who analyze an endogenous price drop due to multiple equilibria. To see this, I define the short institution's "excess demand" as his demand minus the "supply" from the aggregate retail investor, i.e.,

$$\frac{Q_1^{IS}}{\bar{S}} + \frac{Q_1^R}{\bar{S}} = \begin{cases} \left(\alpha_1^{IS}(p_1)\tau^I + \alpha_1^R(p_1)\tau^R\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) + \alpha_1^R(p_1)\tau^R\frac{\delta_1^R}{\sigma_d^2}, & p_1 \in \left[p_1^m, p_1^h\right] \\ -\frac{1}{m}\alpha_1^{IS}(p_1) + \alpha_1^R(p_1)\tau^R\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) + \alpha_1^R(p_1)\tau^R\frac{\delta_1^R}{\sigma_d^2}, & p_1 > p_1^h \end{cases}. \tag{1.54}$$

Then market clearing implies the "excess supply" is equal to 1. Figure A.21 panel (b) plots the "excess demand" and "excess supply," which is a mirror image of the scenario in Gennotte and Leland (1990).

A similar phenomenon also arises in Van Wesep and Waters (2021). They assume that there is a group of "all-in" investors whose demand curve is upward-sloping. Then they show that there could be endogenous discontinuity in the equilibrium price, due to multiple equilibria. In my model, the demand curve is upward-sloping because I allow for wealth effect.

Proposition 2 shows the price can jump discontinuously at certain sentiment cutoff, and the jump is one reason why a moderate sentiment shock can have a large price impact. Proposition 3 then characterizes the price impact within each sentiment region.

Proposition 3 (Price impact of time-1 aggregate retail sentiment shock). *Consider an equilibrium where $p_1\left(\delta_1^R\right)$ is continuous and differentiable in the interior of the three sentiment regions. The price impact of an aggregate retail sentiment shock, $\frac{dp_1(\delta_1^R)}{d\delta_1^R}$, can be decomposed into two components – the direct effect and the redistribution effect.*

- *Low sentiment region $\delta_1 \in \left(\underline{\delta}_1, \delta_1^m\right)$:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{\frac{\alpha_1^R(p_1)\tau^R}{\tau_1(p_1)}}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1 - \frac{1}{\tau_1(p_1)}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R + \frac{d\tau_1(p_1)}{dp_1}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)\right)}}_{\text{redistribution effect}}.$$

- *Medium sentiment region* $\delta_1 \in \left(\delta_1^m, \delta_1^h\right)$*:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{\frac{\alpha_1^R(p_1)\tau^R}{\hat{\tau}_1(p_1)}}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1-\frac{1}{\hat{\tau}_1(p_1)}\left(\frac{d\left(\alpha_1^R(p_1)\right)}{dp_1}\tau^R\delta_1^R+\frac{d\hat{\tau}_1(p_1)}{dp_1}\left(\mu_d+\frac{1}{2}\sigma_d^2-p_1\right)\right)}}_{\text{redistribution effect}}.$$

- *High sentiment region* $\delta_1 \in \left(\delta_1^h, \bar{\delta}_1\right)$*:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{1}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1-\frac{1}{\alpha_1^R(p_1)\tau^R}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R+\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\left(\mu_d+\frac{1}{2}\sigma_d^2-p_1\right)-\frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{1}{m}\sigma_d^2\right)}}_{\text{redistribution effect}}.$$

Proof. See Appendix C.1.8. □

Within each sentiment region, the price impact of an aggregate retail sentiment shock can be decomposed into the direct effect and the redistribution effect.

The direct effect depends on the aggregate demand elasticity in the market of the risky asset, keeping the wealth distribution fixed. The aggregate demand elasticity is a wealth-weighted average of individual investors' demand elasticities. In the low sentiment region, all three investors are marginal. Then the aggregate demand elasticity is determined by the aggregate risk tolerance $\tau_1(p_1)$ defined in equation (1.48), which is a wealth-weighted average of individual investors' risk tolerance. In the medium sentiment region, the long institution is constrained and is no longer marginal. Then the aggregate demand elasticity depends on the aggregate risk tolerance of the two marginal investors (i.e., the aggregate retail investor and the short institution), which is the $\hat{\tau}_1(p_1)$ defined in equation (1.50). Finally, in the high sentiment region, the aggregate retail investor is the only marginal investor, so the aggregate demand elasticity depends on his risk tolerance.

The redistribution effect reflects the fact that wealth is redistributed across investors in response to a retail sentiment shock. Recall from equation (1.17) and Lemma 1 that the aggregate retail investor has different beliefs from the institutional investors, and thus he "bets against" the institutional investors. In the presence of disagreement and wealth effect, those investors who happen to have made the "right" bets gain wealth at the expense of others. In particular, if the equilibrium price at time 1 is higher than that at time 0, then those investors who have shorted the risky asset at time 0 would lose wealth, while those who have taken a levered long position at time 0 would gain wealth. Then the aggregate demand elasticity (and thus the price impact of the retail sentiment shock) would change in response to this wealth redistribution.

1.4.4 Equilibrium at time 0

Proposition 4 characterizes the time-0 equilibrium.

Proposition 4 (Equilibrium at time 0**).** *Consider an equilibrium where the short-sale constraint for the long institution and the margin constraint for the short institution are not binding at time* 0 *(under the equilibrium price* p_0*), and the time*-1 *equilibrium is a monotone equilibrium of Definition 1. Then the time*-0 *price is determined as follows:*

1. *Investors' time*-0 *beliefs about time*-1 *price distribution are consistent with the time*-1 *pricing function* $p_1\left(\delta_1^R\right)$ *and the shock distribution* $\Psi\left(\delta_1^R\right)$*, i.e.,*

$$\mathbb{E}_0^i\left[p_1\left(\delta_1^R\right)\right] = \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] + \delta_0^i = \int_{\underline{\delta}_1}^{\bar{\delta}_1} p_1\left(\delta_1^R\right) d\Psi\left(\delta_1^R\right) + \delta_0^i,$$

$$\mathrm{Var}_0^i\left(p_1\left(\delta_1^R\right)\right) = \sigma_0^2 = \int_{\underline{\delta}_1}^{\bar{\delta}_1} \left(p_1\left(\delta_1\right) - \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right]\right)^2 d\Psi\left(\delta_1^R\right).$$

2. *Given the time-1 pricing function* $p_1\left(\delta_1^R\right)$, *the time-0 equilibrium price* p_0 *clears the market, i.e.,*

$$p_0 = \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] + \left(\frac{1}{2}\sigma_0^2 + \frac{\sum_i \alpha_0^i\left(p_0\right)\tau^i\delta_0^i - \sigma_0^2}{\tau_0\left(p_0\right)}\right),$$

where $\tau_0\left(p_0\right)$ *is the aggregate risk tolerance at time* 0 *defined as*

$$\tau_0\left(p_0\right) \equiv \alpha_0^R\left(p_0\right)\tau^R + \left(1 - \alpha_0^R\left(p_0\right)\right)\tau^I.$$

Hence, the equilibrium is a fixed-point problem. The time-0 price p_0 depends on the shape of the time-1 pricing function $p_1\left(\delta_1^R\right)$ through investors' beliefs, while the time-1 pricing function depends on the time-0 price through the wealth shares.

1.5 The network origins of aggregate retail sentiment fluctuations

Section 1.4 shows how investor composition matters for the pricing of aggregate retail sentiment shock. In this section, I microfound the distribution of the aggregate retail sentiment shock. I assume the type-1 retail investors communicate on a social network and update their beliefs by "listening to" others on the network. The influence distribution on the network is right-skewed, which means the influencers' views will carry disproportionately high weights in the aggregate view of retail investors. Then idiosyncratic shocks to retail investors' sentiment would not cancel out and would instead translate into an aggregate retail sentiment shock.

This microfoundation allows me to study two counterfactual scenarios in Section 1.6. These two counterfactuals shed light on why short sellers got squeezed in January 2021 and why they exited the market afterwards.

1.5.1 Naive learning on a growing network

At time $t = 1$, type-1 retail investor j draws a noisy signal

$$x_t^j = \rho y_{t-1}^R + \varepsilon_t^j,$$

where y_{t-1}^R is the average retail sentiment at time $t-1$, and ε_t^j is an error term that is i.i.d. across investors and time. I assume ε_t^j follows a truncated normal distribution on $[-\bar{\varepsilon}, \bar{\varepsilon}]$[29] with post-truncation mean 0 and variance σ_ε^2. $\rho \in (0, 1]$ is a parameter that captures the persistence of the sentiment.

Type-1 retail investors communicate on a social network and reveal their signals to others. Then each investor on the network updates his belief by "listening to" other people on the network. I use the adjacency matrix $\boldsymbol{A}_t = \left(a_{jk,t}\right)$ to capture the relationship between pairs of investors. If investor j "listens to" or "attends to" investor k at time t, then $a_{jk,t} = 1$, otherwise $a_{jk,t} = 0$. Investor j assigns weight $\omega_{jk,t}$ to investor k's signal, and $\omega_{jk,t}$ is defined as

$$\omega_{jk,t} \equiv \frac{a_{jk,t}}{\sum_{k=1}^{N_t} a_{jk,t}}.$$

Hence, each investor on the network assigns equal weights to people he listens to. Note $\sum_{k=1}^{N_t} \omega_{jk,t} = 1$.

After the updating, investor j's view becomes

$$y_t^j = \sum_{k=1}^{N_t} \omega_{jk,t} x_t^k = \sum_{k=1}^{N_t} \omega_{jk,t} \left(\rho y_{t-1}^R + \varepsilon_t^k\right) = \rho y_{t-1}^R + \sum_{j=1}^{N_t} \omega_{jk,t} \varepsilon_t^k.$$

29. I assume a bounded support for ε_t^j so that the time-1 aggregate retail sentiment will also have a bounded support (see equations (1.56) and (1.57)). Then I can rule out the possibility of bankruptcy under specific parameter choices. Remark 1 discusses the issue of bankruptcy in this discrete-time setting.

y_t^j is the sentiment of investor j in equation (1.18).

Dynamics of aggregate retail sentiment Using the definition in equation (1.37), the time-t aggregate retail sentiment is

$$\delta_t^R \equiv \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R + \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} d_{j,t}^{in} \varepsilon_t^j, \tag{1.55}$$

where $d_{j,t}^{in}$ is the time-t influence or in-degree of retail investor j defined as

$$d_{j,t}^{in} \equiv \sum_{i=1}^{N_t} \omega_{ij,t}.$$

This is the same definition of influence as in equation (1.3). δ_t^R has support $\left[\underline{\delta}_t, \bar{\delta}_t\right]$, where

$$\underline{\delta}_t = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R - \theta\left(N_t\right) \bar{\varepsilon}, \tag{1.56}$$

$$\bar{\delta}_t = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R + \theta\left(N_t\right) \bar{\varepsilon}. \tag{1.57}$$

Motivated by the findings in Section 1.3.4, I assume $d_{j,t}^{in}$ is drawn from a power-law distribution and is i.i.d. in the cross section of the N_t retail investors on the social network. The PDF of $d_{j,t}^{in}$ is

$$f_{d_{j,t}^{in}}(x) = \frac{\xi - 1}{d_{\min}} \left(\frac{x}{d_{\min}}\right)^{-\xi}, \xi > 1, \tag{1.58}$$

with support $[d_{\min}, d_{\max}(N_t)]$. The exponent ξ captures the right-skewness of the influence distribution. Lower values of ξ correspond to heavier right tails and more right-skewed influence distribution. The upper bound $d_{\max}(N_t) = d_{\min} \cdot N_t^{\frac{1}{\xi-1}}$.[30] Lemma 2 computes the moments of the influence distribution.

Lemma 2 (Moments of the influence distribution). *In the cross section of the N_t (type-1) retail investors, the m-th moment of influence $d_{j,t}^{in}$ is*

$$\mathbb{E}^{CS}\left[\left(d_{j,t}^{in}\right)^m\right] = \frac{\xi-1}{\xi-m-1}\frac{1}{d_{\min}^{1-\xi}}\left(d_{\min}^{m+1-\xi} - \left(d_{\max}(N_t)\right)^{m+1-\xi}\right).$$

The cross-sectional variance of $d_{j,t}^{in}$ is

$$\begin{aligned} \mathrm{Var}^{CS}\left(d_{j,t}^{in}\right) = & \frac{\xi-1}{3-\xi}\frac{1}{d_{\min}^{1-\xi}}\left(\left(d_{\max}(N_t)\right)^{3-\xi} - d_{\min}^{3-\xi}\right) \\ & - \left(\frac{\xi-1}{\xi-2}\right)^2\frac{1}{d_{\min}^{2-2\xi}}\left(d_{\min}^{2-\xi} - \left(d_{\max}(N_t)\right)^{2-\xi}\right)^2, \end{aligned} \tag{1.59}$$

which implies that $\mathrm{Var}^{CS}\left(d_{j,t}^{in}\right) = O\left(N_t^{\frac{3-\xi}{\xi-1}}\right)$ for $\xi > 1$.

Proof. See Appendix C.1.11. □

1.5.2 Aggregate fluctuations in retail sentiment

Proposition 5 below relates the volatility of the aggregate sentiment shock to the volatility of idiosyncratic shocks σ_ε and the network parameters. This proposition is a direct application of Acemoglu et al. (2012) Theorem 2 and Corollary 1.

30. Following Newman (2005), the upper bound can be computed in a heuristic way. First note

$$\Pr\left(d_j^{in} > x\right) = \int_x^{+\infty} \frac{\xi-1}{d_{\min}}\left(\frac{y}{d_{\min}}\right)^{-\xi} dy = -\int_x^{+\infty} d\left(\frac{y}{d_{\min}}\right)^{1-\xi} = \left(\frac{x}{d_{\min}}\right)^{1-\xi}.$$

The probability of observing a value greater than $d_{\max}(N)$ is approximately $\frac{1}{N}$. Hence, $d_{\max}(N)$ can be computed from

$$\Pr\left(d_j^{in} > d_{\max}(N)\right) = \frac{1}{N} \implies d_{\max}(N) = d_{\min} \cdot N^{\frac{1}{\xi-1}}.$$

Acemoglu et al. (2012) also impose this upper bound.

Proposition 5 (Moments of the aggregate retail sentiment). *Suppose the network size N_t evolves deterministically over time. Then at time $t-1$, the conditional mean and conditional variance of the aggregate retail sentiment δ_t^R are*

$$\mathbb{E}_{t-1}\left[\delta_t^R\right] = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)}\rho\delta_{t-1}^R, \tag{1.60}$$

$$\mathrm{Var}_{t-1}\left(\delta_t^R\right) = \left(\theta\left(N_t\right)\right)^2 \frac{2d_{\min}^{\xi-1}}{N_t}\frac{1}{3-\xi}\left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi} - d_{\min}^{3-\xi}\right)\sigma_\varepsilon^2. \tag{1.61}$$

Furthermore, the conditional volatility satisfies

$$\sqrt{\mathrm{Var}_{t-1}\left(\delta_t^R\right)} = O\left(N_t^{\frac{2-\xi}{\xi-1}}\right).$$

Proof. See Appendix C.1.12. □

Proposition 5 shows that the volatility of the aggregate retail sentiment shock decreases with ξ. Intuitively, a smaller ξ corresponds to a more right-skewed influence distribution. Then idiosyncratic shocks to influencers' views will carry higher weights in the aggregate view, thereby amplifying the aggregate fluctuations.

$\xi = 3$ corresponds to the standard Central Limit Theorem, which says the aggregate volatility decreases at a rate of $\sqrt{N_t}$. Section 1.3.4 shows that for the Reddit's WSB social network, $\xi < 3$. Hence, the volatility decreases at a rate that is much lower than $\sqrt{N_t}$. Even with a large number of users on the network, idiosyncratic sentiment shocks may still lead to large aggregate sentiment fluctuations. The 15% increase in average sentiment of GameStop in January 2021 is thus a result of influencers' idiosyncratic sentiment shocks and a small ξ.

1.5.3 Numerical example

I present a numerical example that matches the price and quantity patterns of GameStop observed in the data. Table 1.2 shows the parameters.

I assume the network size remains constant over time with $N_0 = N_1 = N_L$. When investors form their subjective expectations, they also perceive the network size as constant. When drawing time-1 sentiment shocks, I assume the aggregate sentiment shock δ_1^R follows a truncated normal distribution with post-truncation mean and variance given by equations (1.60) and (1.61) and with support $\left[\underline{\delta}_1, \bar{\delta}_1\right]$ given by (1.56) and (1.57). Appendix C.1.13 shows the true distribution of δ_1^R (by aggregating the y_1^j's) can be approximated by this truncated normal distribution, if the influence distribution is skewed.

Figure A.19 panel (a) plots the time-1 price as a function of the aggregate sentiment shock realization. Figure A.19 panel (b) plots the pricing function together with the PDF of the aggregate retail sentiment shock. As shown in Section 1.4.3, the price impact within each sentiment region is determined by the direct effect and the redistribution effect. At the cutoff sentiment δ_1^h, there is an endogenous jump in the price, due to the margin constraint and the wealth effect.

In this example, investors' time-0 portfolio weights are $w_0^R = 1.90$, $w_0^{IL} = 1.76$, and $w_0^{IS} = -0.25$. Both the aggregate retail investor and the long institution take a levered position in the risky asset. Hence, as retail sentiment drives up the price, wealth redistributes from the short institution to retail investors and the long institution (Figure A.22 panel (c)).

Figure A.23 shows the time series predictions from the model. The time-1 values correspond to an aggregate retail sentiment realization $\delta_1^R = 2.18$. The model can match the price and quantity patterns documented in Sections 1.3.1-1.3.3. In particular, panel (a) shows that short sellers increase their short positions following the first retail sentiment shock δ_0^R, but then they significantly reduce their short positions after the second sentiment shock δ_1^R hits.

Table 1.2
Model parameters

Description	Parameter	Value
Risky Asset		
Mean of log dividend	μ_d	4
Volatility of log dividend	σ_d^2	0.1
Lower bound of log dividend	$\underline{d}$	-2.5
Upper bound of log dividend	$\bar{d}$	10.5
Supply of shares	$\bar{S}$	100
Endowment		
Retail investors	α_{-1}^R	0.3
	w_{-1}^R	1.194
Long institution	α_{-1}^{IL}	0.14
	w_{-1}^{IL}	4.800
Short institution	α_{-1}^{IS}	0.56
	w_{-1}^{IS}	-0.054
Risk Aversion		
Retail investors	γ^R	2
Institutions	γ^I	1
Constraints		
Margin constraint	m	0.5

Description	Parameter	Value
Sentiment Shocks		
Retail investors	δ_0^R	1.028
	$\bar{\varepsilon}$	2.872
	σ_ε^2	0.081
Long institution	δ_0^{IL}	0.256
Short institution	δ_0^{IS}	-0.505
Network		
Population of type-1 retail investors	N_L	80000
	N_H	140000
Population of retail investors	$\bar{N}$	200000
Exponent of power-law distribution	ξ	2.1
Cutoff value of power-law distribution	$d_{\min}$	10
Persistence of agg. retail sent shock	ρ	1

1.6 Counterfactuals

I conduct two counterfactuals, which shed light on why short sellers got squeezed in January 2021 and why they exited the market afterwards.

1.6.1 Why were short sellers squeezed in January 2021?

In Section 1.3.1, I document the average retail sentiment on GME had been steadily increasing from mid-2020 to January 2021, while the WSB discussion volume on GME spiked in January 2021. Both forces would contribute to a large positive realization of aggregate retail sentiment, as is shown in equation (1.37). This realized retail sentiment not only drove out the price-sensitive long-only investors but also squeezed the short sellers.

I formalize this idea through the lens of the model using the parameters for the numerical example in Section 1.5.3. In particular, an increase in discussion volume in the data corresponds to an unexpected increase in the network size in the model, i.e., an "MIT shock" to network size. Given the right-skewness of the influence distribution and the optimism of the influencers, the growth of the network translates into a large sentiment realization, which exceeds the short squeeze cutoff δ_1^h in equation (1.46). The long institution liquidates his position and the short institution gets squeezed under this sentiment shock realization.

I consider a counterfactual scenario where the discussion volume did not spike in January 2021, i.e., the network size does not change in the model. I show that in this case, the average retail sentiment still remains positive, but the counterfactual aggregate (retail) sentiment is lower than the realized aggregate (retail) sentiment, and short sellers would not get squeezed.

I begin by analyzing the factors that contribute to the large positive realization of aggregate retail sentiment: network size, network geometry (or influence distribution), and the optimism of individual retail investors on the network. I assume the network size grows from time 0 to time 1 with $N_0 = N_L < N_H = N_1$, and the values of N_L and N_H are given in Table 1.2. Substitute into equation (1.55) to get the realized aggregate retail sentiment

$$\delta_1^R = \underbrace{\frac{\theta(N_H)}{\theta(N_L)}\rho\delta_0^R}_{\text{persistence}} + \underbrace{\theta(N_H)\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j}_{\text{aggregation of idio. shocks}} . \tag{1.62}$$

The first component captures the persistence of aggregate retail sentiment ($\rho\delta_0^R$) and the amplification effect through a growing network ($\frac{\theta(N_H)}{\theta(N_L)}$). $\rho > 0$ is the persistence of average retail sentiment, and $\frac{\theta(N_H)}{\theta(N_L)} = \frac{N_H}{N_L} > 1$ reflects the growth of the social network from time 0 to time 1. Suppose $\delta_0^R > 0$, i.e., at time 0, retail investors are optimistic in aggregate. Then retail investors who newly join the network will adopt the optimistic views from existing investors, and the average optimism of existing investors will get amplified and be reflected in the aggregate retail sentiment.

The second component ($\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j$) captures the aggregation of idiosyncratic sentiment shocks to investors on the network. Since the influence distribution is right-skewed, idiosyncratic sentiment shocks do not average out across investors, and influencers' sentiment shocks will carry higher weights in the aggregate sentiment, amplifying the fluctuations in aggregate sentiment. If influencers happen to draw positive sentiment shocks, aggregate sentiment will also be positive. Importantly, on the intensive margin, the aggregate optimism will depend on the right-skewness of the influence distribution and the network size. To see this, if we have a large number of retail investors on the network, i.e., $N_H \to +\infty$, we first apply the Law of Large Numbers in the cross section of retail investors

$$\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j \xrightarrow{p} \mathbb{E}\left[d_{j,1}^{in}\varepsilon_1^j\right] = \text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)\sqrt{\text{Var}\left(d_{j,1}^{in}; N_H\right)}\sigma_\varepsilon, \tag{1.63}$$

where $\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)$ is the cross-sectional correlation between users' influence and the idiosyncratic shocks they draw. If $\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) > 0$, it means influencers are optimistic.

The weight that influencers' views carry in the aggregate view depends on the cross-sectional dispersion in user influence, which is captured by $\sqrt{\text{Var}\left(d_{j,1}^{in}; N_H\right)}$. In Section 1.3.4, I estimate that the power-law exponent $\hat{\xi}_t \in (1, 3)$. Then it immediately follows from Lemma 2 that, as the network grows, the influence distribution is more dispersed in the cross section of retail investors, and influencers' views will get amplified more and carry a higher weight.

Next, I consider a counterfactual scenario where the network size remains constant from time 0 to time 1, i.e., $N_0 = N_1 = N_L$. Using (1.63) to approximate the aggregation of idiosyncratic sentiment shocks, the realized aggregate sentiment in (1.62) can be approximated by

$$\delta_1^R \approx \frac{\theta (N_H)}{\theta (N_L)} \rho \delta_0^R + \theta (N_H) \,\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) \sqrt{\text{Var}\left(d_{j,1}^{in}; N_H, \xi\right)} \sigma_\varepsilon. \tag{1.64}$$

The counterfactual aggregate retail sentiment is

$$\hat{\delta}_1^R \approx \rho \delta_0^R + \theta (N_L) \,\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) \sqrt{\text{Var}\left(d_{j,1}^{in}; N_L, \xi\right)} \sigma_\varepsilon. \tag{1.65}$$

In this counterfactual scenario, influencers remain as optimistic as they are in the realized scenario, i.e., $\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)$ remains the same. But due to a smaller network size, the counterfactual aggregate retail sentiment is smaller, i.e., $\hat{\delta}_1^R < \delta_1^R$.

The model in Section 1.4 allows me to quantify the price impact of the counterfactual sentiment shock. From the pricing function $P_1\left(\delta_1^R\right)$ in Figure A.19 panel (a) and the price of GameStop observed from the data ($P_1 = 349.73$ in January 2021), I can back out the realized aggregate retail sentiment $\delta_1^R = 2.18$. Given the network parameters $\left(N_L, N_H, \bar{N}, d_{\min}, \xi\right)$ in Table 1.2 and using equation (1.64), I then back out how optimistic the influencers are, which is $\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) = 0.00135$. Now fix the optimism of influencers, I can calculate the counterfactual retail sentiment from equation (1.65), which yields $\hat{\delta}_1^R = 1.20$. Finally, using the pricing function $P_1\left(\delta_1^R\right)$ in Figure A.19 panel (a), the counterfactual price is thus

$P_1\left(\hat{\delta}_1^R\right) = 65.63.$

Figure A.24 panel (a) plots the equilibrium price under the realized sentiment $\delta_1^R = 2.18$ versus that under the counterfactual sentiment $\hat{\delta}_1^R = 1.20$. In the latter case, short sellers do not get squeezed, since the counterfactual sentiment is below the short squeeze cutoff δ_1^h.

As discussed above, we can decompose the gap between the realized sentiment and the counterfactual sentiment into two parts: one captures the persistence of aggregate retail sentiment and the amplification through a growing network, while the other captures the aggregation of idiosyncratic shocks on a network with right-skewed influence distribution. Formally, compare equation (1.64) with equation (1.65) and compute the difference

$$\begin{aligned} &\delta_1^R - \hat{\delta}_1^R \\ &= \underbrace{\left(\frac{\theta\left(N_H\right)}{\theta\left(N_L\right)} - 1\right)\rho\delta_0^R}_{\Delta_1} \\ &\quad + \underbrace{\operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)\left(\theta\left(N_H\right)\sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_H, \xi\right)} - \theta\left(N_L\right)\sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_L, \xi\right)}\right)\sigma_\varepsilon}_{\Delta_2}. \end{aligned} \tag{1.66}$$

$\Delta_1 = 0.771$ is the component due to the persistence of aggregate retail sentiment, and $\Delta_2 = 0.206$ is the component due to the aggregation of idiosyncratic sentiment shocks. Figure A.24 panel (b) plots the two components. The second component alone would be sufficient to squeeze the short sellers, which suggests that the right-skewed influence distribution on the social network has an economically large impact on asset prices.

1.6.2 Why did short sellers exit the market after January 2021?

In the model, there are three mechanisms that can help explain why short sellers stayed out of the market and price remained high after January 2021: (1) short sellers updated their perceptions about retail sentiment risk after the GameStop frenzy; (2) the market for GameStop became price inelastic due to binding financial constraints and wealth redistribution; (3) short sellers lost wealth and were forced to exit the market.

Change in short sellers' risk perceptions After observing a large influx of retail investors to Reddit's WSB in January 2021, short sellers may have updated their perceptions about the retail sentiment risk and may become more cautious in taking large short positions. The changing risk perception can explain why price stayed high and short interest stayed low after January 2021. I use the model to formalize this idea.

In the model, the short institution's risk perception depends on his perception about the growth of the social network. In the numerical example of Section 1.5.3, the short institution believes that the network size will remain constant from time 0 to time 1. Let $\tilde{N}_1$ denote the short institution's time-0 perception about the network size at time 1, then $\tilde{N}_1 = N_L$. His perception of the retail sentiment distribution under $\tilde{N}_1$ is plotted as the solid blue line in Figure A.25.

Now consider a counterfactual scenario where the short institution perfectly anticipates the growth of the network from time 0 to time 1, i.e., their perception about time-1 network size is $\tilde{N}_1 = N_H$. This perceived network size corresponds to a different perceived sentiment distribution, which is plotted as the dashed red line in Figure A.25. I solve the time-0 equilibrium under this counterfactual risk perception. Table 1.3 compares the time-0 price under these two different risk perceptions and decomposes the equilibrium price into three components – the expected payoff, the price of time-0 realized sentiment, and the price of time-1 risk. Table C.2 compares other equilibrium outcomes at time 0.

Table 1.3 shows that the time-0 price under risk perception $\tilde{N}_1 = N_H$ (column 4) is higher than that under $\tilde{N}_1 = N_L$ (column 3). This is primarily because the expected payoff of the risky asset is higher under the new risk perception, which is reflected in the first component $\mathbb{E}_0 [p_1] + \frac{1}{2}\sigma_0^2$ in Table 1.3. Under the new risk perception $\tilde{N}_1 = N_H$, the short institution would rather take a long position at time 0 (see Table C.2). This is the sense in which the short institution becomes more "conservative" in taking large short positions.

Importantly, the conditional variance of the log return, σ_0^2, is higher under the new risk perception $\tilde{N}_1 = N_H$. This implies that the asset's payoff is now perceived as being more risky, and thus all investors would trade less aggressively. On the one hand, the relatively more optimistic retail investor would be more conservative in taking long positions. This puts downward pressure on the time-0 price. On the other hand, the relatively more pessimistic short institution would also be more conservative in taking large short positions, which would put upward pressure on the time-0 price. In this numerical example, the former effect outweighs the latter. This implies that under the new risk perception, the net trading by investors puts downward pressure on the time-0 price, which is reflected in the third component $-\frac{1}{\tau_0(p_0)}\sigma_0^2$ in Table 1.3. Moreover, Figure A.26 plots the distribution of the time-1 price given different risk perceptions of the investors. Panel (a) plots the distribution under the original risk perception $\tilde{N}_1 = N_L$, while panel (b) plots the distribution under the updated risk perception $\tilde{N}_1 = N_H$. In the latter case, the time-1 price has more extreme realizations, which is consistent with a higher perceived risk.

To the extent that retail investors may not be as "sophisticated" as the short sellers, their demand may not respond to the change in perceived risk. Short sellers, however, would be more conservative in taking large short positions due to the updated risk perception, which would then put upward pressure on the equilibrium price. This can help explain why short sellers exited the market of GameStop after January 2021 and why the price of GameStop remained high throughout 2021.

Table 1.3
Time-0 equilibrium price under different risk perceptions

This table compares the time-0 equilibrium prices when changing investors' time-0 perceptions of risk. Column 3 shows the equilibrium outcomes when all investors believe the size of the network will remain constant from time 0 to time 1, i.e., $\tilde{N}_1 = N_L = N_0$. Column 4 shows the equilibrium outcomes when all investors believe the the size of the network will grow (deterministically) from time 0 to time 1, i.e., $\tilde{N}_1 = N_H > N_L = N_0$. The time-0 equilibrium price in each scenario is decomposed into three components: the expected log payoff after risk adjustment, the price of time-0 realized retail sentiment, and the price of time-1 retail sentiment risk. The parameter values are given in Table 1.2.

		Value	
Description	Notation	$\tilde{N}_1 = N_L$	$\tilde{N}_1 = N_H$
(1)	(2)	(3)	(4)
Expected log payoff (risk-adjusted)	$\mathbb{E}_0[p_1] + \frac{1}{2}\sigma_0^2$	4.658	5.664
Price of time-0 realized retail sentiment	$\frac{\sum_i \alpha_0^i(p_0)\tau^i\delta_0^i}{\tau_0(p_0)}$	0.039	0.163
Price of time-1 risk	$-\frac{1}{\tau_0(p_0)}\sigma_0^2$	-0.449	-1.215
Sum	p_0	4.249	4.612

Change in aggregate demand elasticity in the market for GameStop After the January 2021 short squeeze, the market for GameStop may have become price-inelastic for two reasons. First, price-elastic long-only institutions hit their constraints and effectively became price-inelastic. Second, retail investors' wealth share increased and they are less price-elastic than (unconstrained) institutions. Since the aggregate demand elasticity is a weighted average of individual elasticities, the market for GameStop may have become price-inelastic after January 2021.

Section 1.3.1 documents that the average retail sentiment has been positive and stable throughout 2021. Given the drop in aggregate demand elasticity, a moderate positive sentiment shock can have a large price impact and sustain a high price. This helps explain why the price of GameStop remained high after January 2021.

Capital loss Short sellers like Melvin Capital lost a large fraction of their wealth and shut down.[31] Since short sellers exited the market, short interest remained low and price remained high after January 2021.

1.7 Discussions

The baseline model in Section 1.4 makes a few simplifying assumptions. This section discusses the empirical evidence underlying these assumptions and the related model extensions. First, I assume the cost of short-selling is exogenous in the sense that the institutions face exogenous portfolio constraints on taking short positions. This implies that in the securities borrowing and lending market, the price (i.e., borrowing fee) and quantity (i.e., short interest) movements are driven by the changing demand for short selling, rather than the changing supply of lendable shares. Section 1.7.1 discusses the empirical evidence consistent with this demand-side story. Second, I assume that the expansion of the social network is exogenous. Section 1.7.2 discusses the empirical evidence and the potential feedback from asset prices to social network expansion.

1.7.1 Securities borrowing and lending market

In Section 1.3.3, I document a new puzzle – short interest remained low after the January 2021 short squeeze episode. This could be driven by low demand for short selling (by the short sellers) or a contraction in the supply of lendable shares (by the long investors). To shed light on the demand v.s. supply stories, I examine the price (i.e., borrowing fee) and quantity (i.e., short interest) in the securities borrowing and lending market.

Figure A.9 plots the borrowing fee (solid blue line) and the short interest (dotted red line) from 2020-2021. After January 2021, we see a significant drop in both price (i.e., borrowing

31. Chung, J. (2022, May 19). Melvin Capital to Close Funds, Return Cash to Investors. *WSJ.* `https://www.wsj.com/articles/melvin-capital-to-close-funds-return-cash-to-investors-11652910350`.

fee) and quantity (i.e., short interest). This is consistent with a demand-driven story: the demand for short selling has dropped after January 2021. This fact cannot be explained by an exogenous contraction in the supply of lendable shares.

1.7.2 Feedback from asset prices to social dynamics

Section 1.3.1 documents a spike in the discussion volume in January 2021, which coincided with an expansion of Reddit's WSB social network. Hu et al. (2021) document that the discussion volume on Reddit's WSB can predict stock returns, using a large cross section of stocks. This suggests that the expansion of the social network indeed affects equilibrium asset prices.

However, the expansion of the social network may in turn be driven by movements in asset prices. For example, more volatile asset prices may draw retail investors' attention. It would be interesting to incorporate this feedback from asset prices to social dynamics into the model and quantify the contribution of this feedback to asset price swings.

1.8 Conclusion

This paper demonstrates how social media has fundamentally changed the nature of retail trading. The growth and concentration of social network can lead to extreme realizations of retail sentiment and amplify the fluctuations in asset prices. Additionally, after a "disastrous" realization, short sellers may update their perceptions of retail sentiment risk and be more conservative in taking large short positions. Social-media-fueled retail trading becomes a new risk to institutional investors, and social network dynamics shape the distribution of retail sentiment.

This paper also argues that social-media-fueled retail trading can induce a shift in investor composition, which determines the price of this new risk. In particular, positive retail sentiment can drive out price-sensitive long-only institutions, causing a decline in the aggregate demand elasticity in the market for an individual stock. Then a moderate retail sentiment shock can drive up the stock price and put short sellers at risk. From short sellers' perspective, price-sensitive long-only institutions act like a "buffer" against retail sentiment fluctuations. However, over the past two decades, this "buffer" has been shrinking due to the rise of passive investing, implying that short sellers are now more "vulnerable" to retail sentiment risk. Hence, this change in investor composition is also a new risk for short sellers to heed.

APPENDIX A

FIGURES

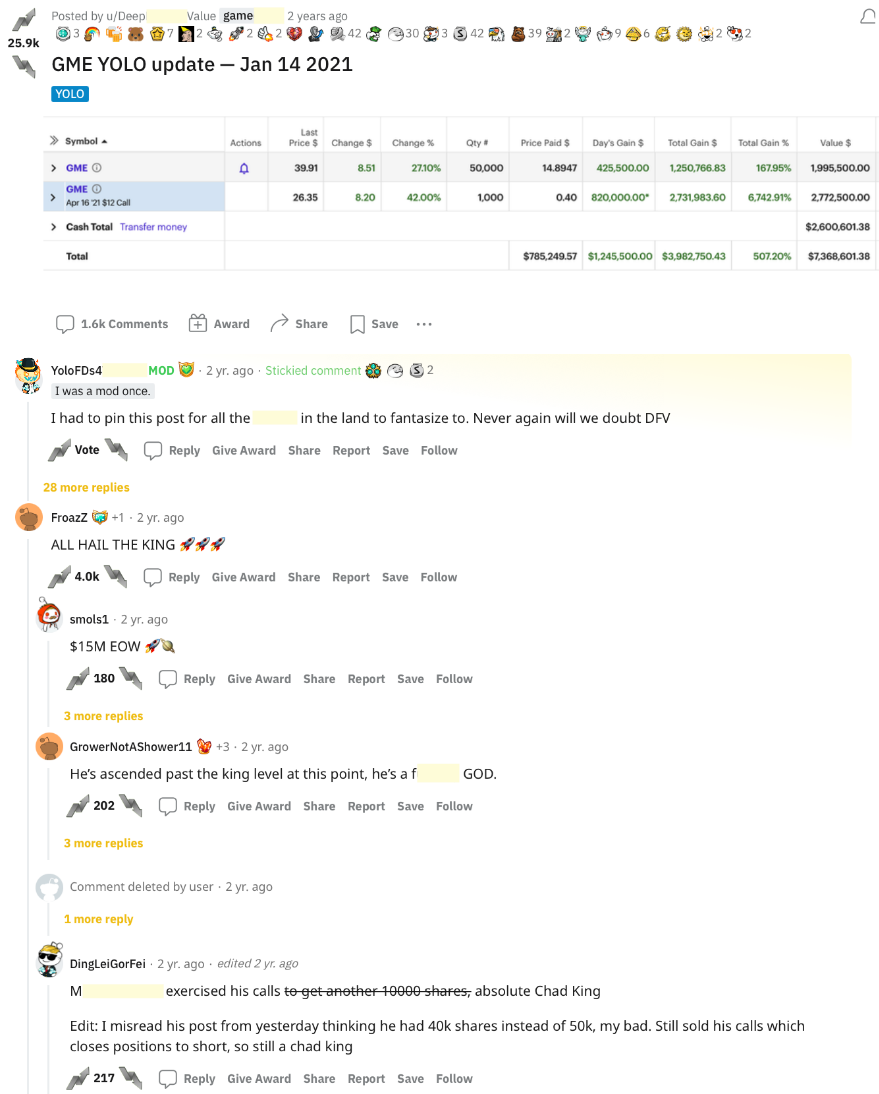
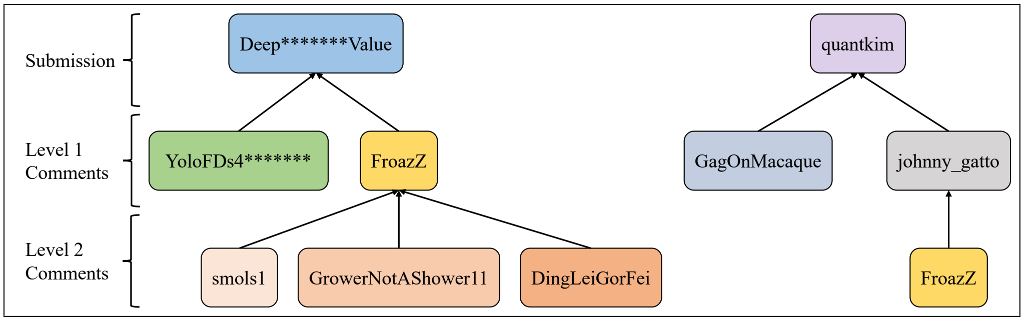
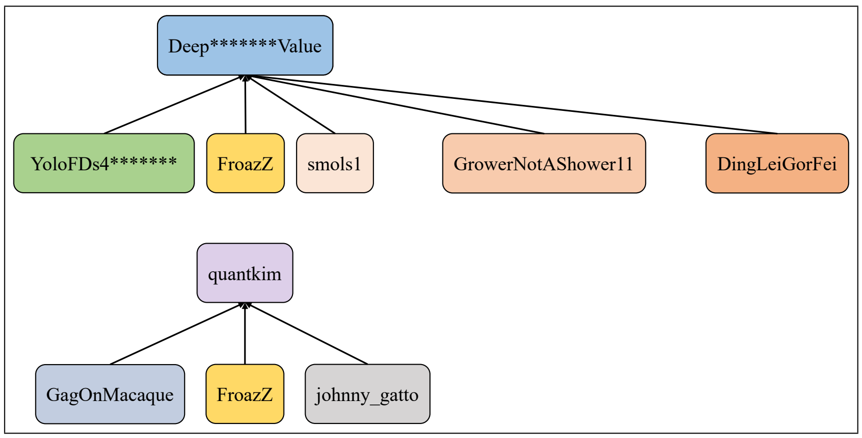


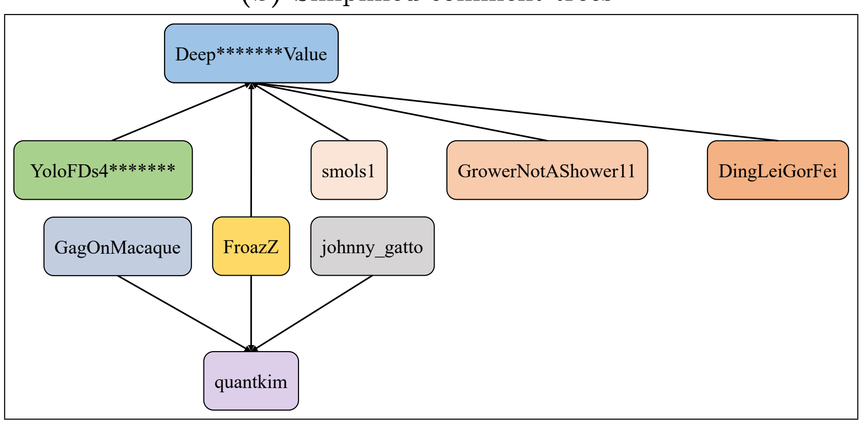
Figure A.1. Example of a conversation tree. This figure shows an example of a conversation tree on Reddit's WallStreetBets (WSB) forum. The conversation is retrieved from `https://www.reddit.com/r/wallstreetbets/comments/kxeq23/gme_yolo_update_jan_14_2021/`.



(a) Comment trees



(b) Simplified comment trees



(c) User network

Figure A.2. Generic representations of comment trees and the user network. This figure shows an example of two comment trees from WSB and the corresponding user network. Panel (a) plots two trees, and the left one corresponds to the conversation shown in Figure A.1. Panel (b) plots the simplified trees that correspond to the original ones in panel (a). Panel (c) plots the user network constructed from these two simplified trees.

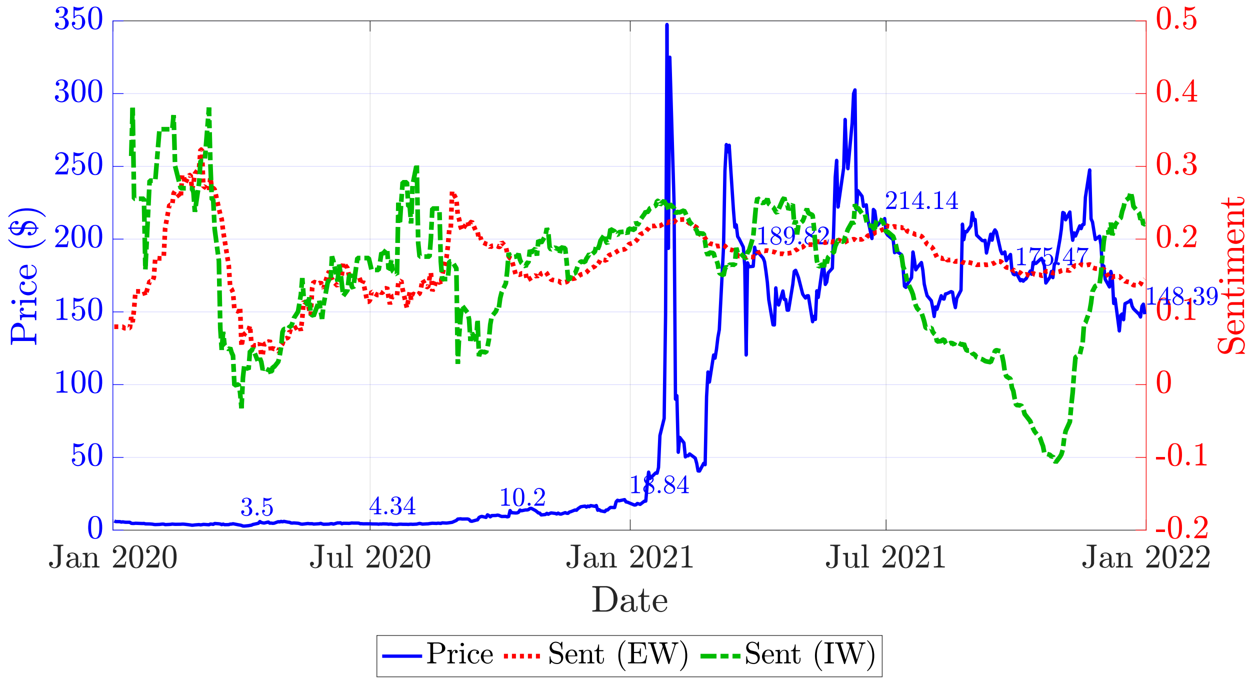
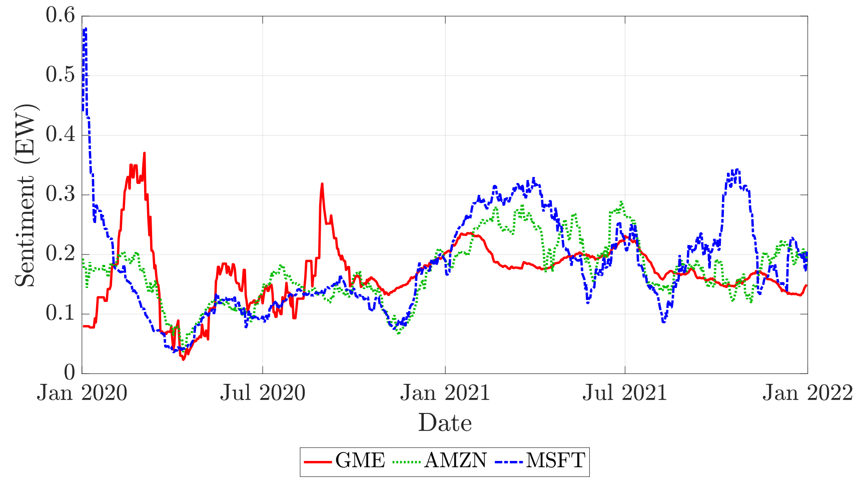
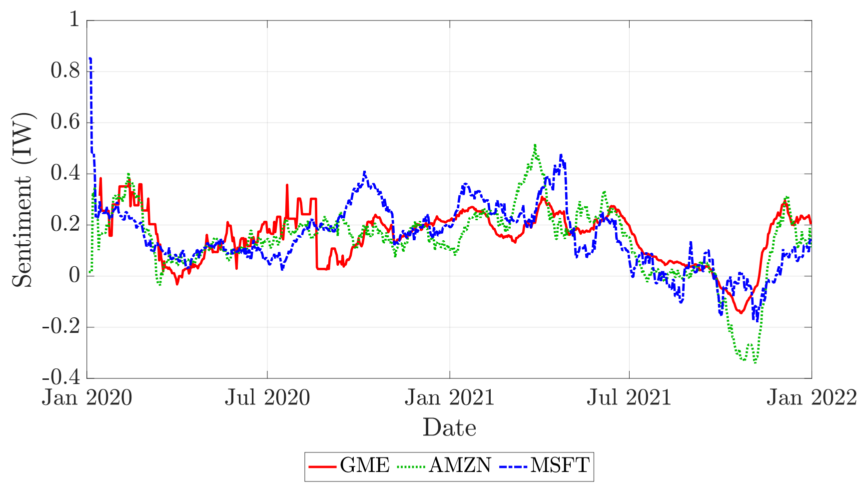


Figure A.3. Price and sentiment of GameStop. This figure shows the daily close price (left y-axis) and the daily WSB sentiment measures (right y-axis) of GameStop, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price, the dotted red line plots the equal-weighted sentiment defined in equation (1.4), and the dash-dotted green line plots the influence-weighted sentiment defined in equation (1.5). The sentiment series are 30-day moving averages.

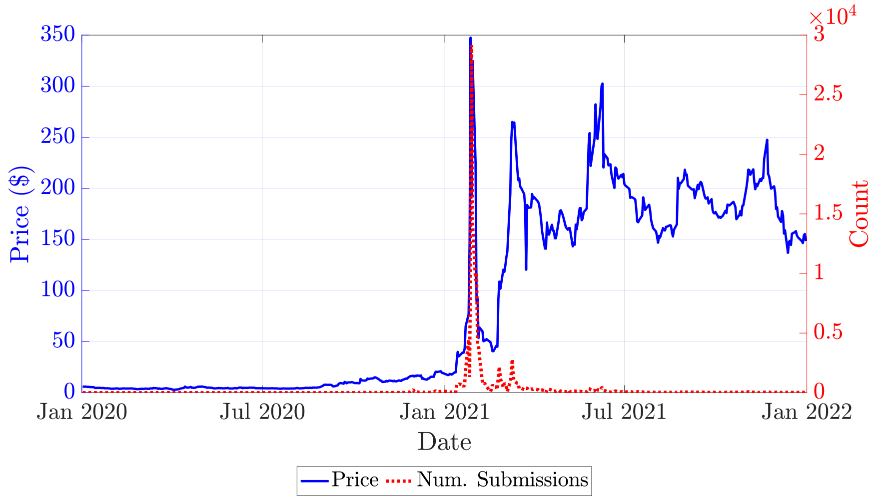


(a) Equal-weighted sentiment

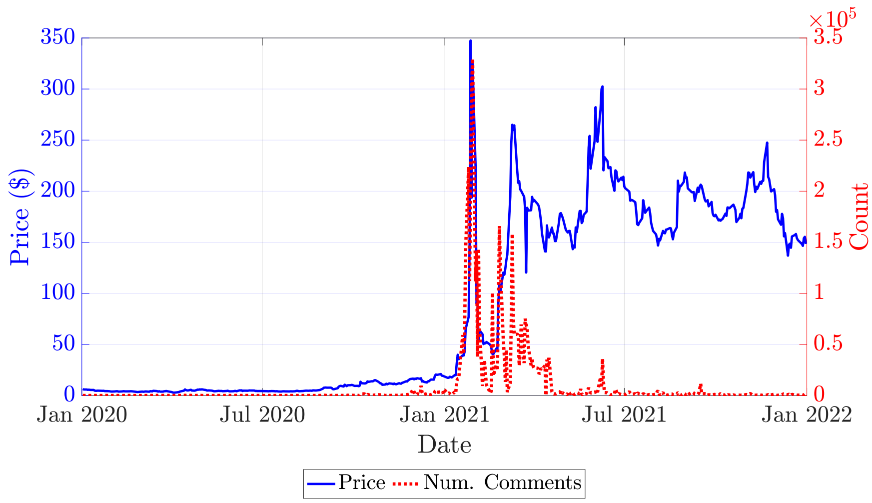


(b) Influence-weighted sentiment

Figure A.4. Sentiment of GameStop versus technology stocks. This figure plots the daily WSB sentiment of GameStop versus two technology stocks, Amazon and Microsoft, for the period from January 1, 2020 to December 31, 2021. Panel (a) plots the equal-weighted sentiment defined in equation (1.4). Panel (b) plots the influence-weighted sentiment defined in equation (1.5). In each panel, the solid red line represents GameStop, the dotted green line represents Amazon, and the dash-dotted blue line represents Microsoft. The sentiment series are 30-day moving averages.



(a) Number of submissions



(b) Number of comments

Figure A.5. Price and discussion volume of GameStop. This figure shows the daily close price (left y-axis) and the daily WSB discussion volume (right y-axis) of GameStop, for the period from January 1, 2020 to December 31, 2021. Panel (a) plots the close price of GameStop (solid blue line) and the daily number of new submissions about GameStop on WSB (dotted red line). Panel (b) plots the close price of GameStop (solid blue line) and the daily number of new comments about GameStop on WSB (dotted red line).

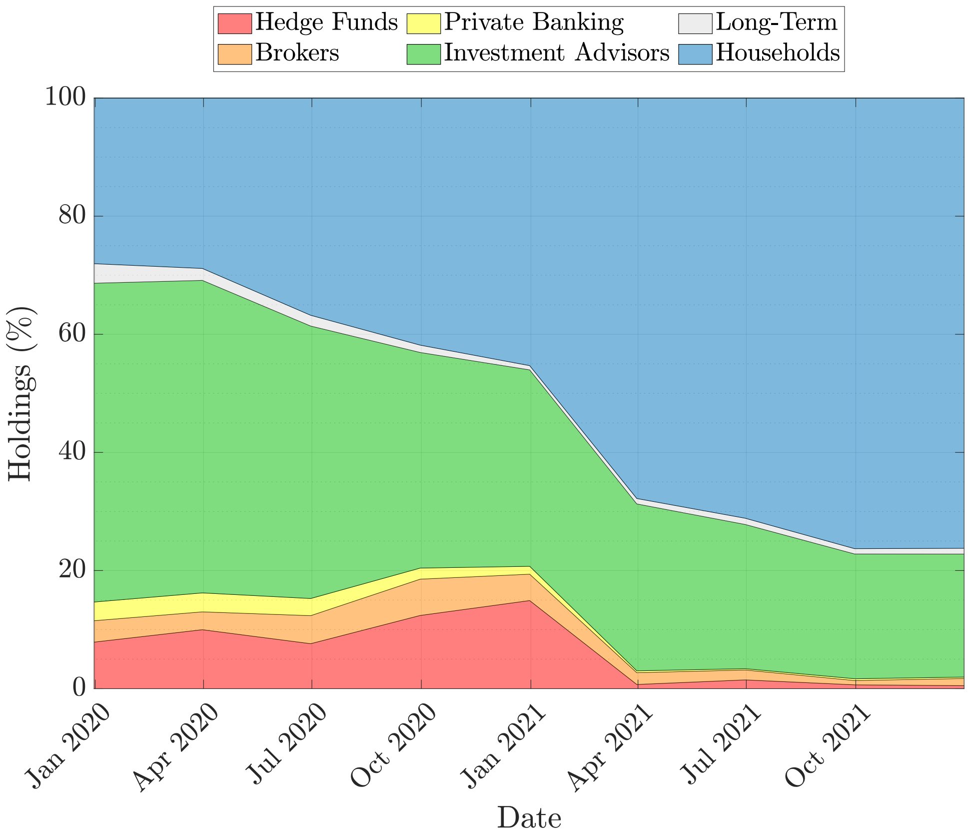
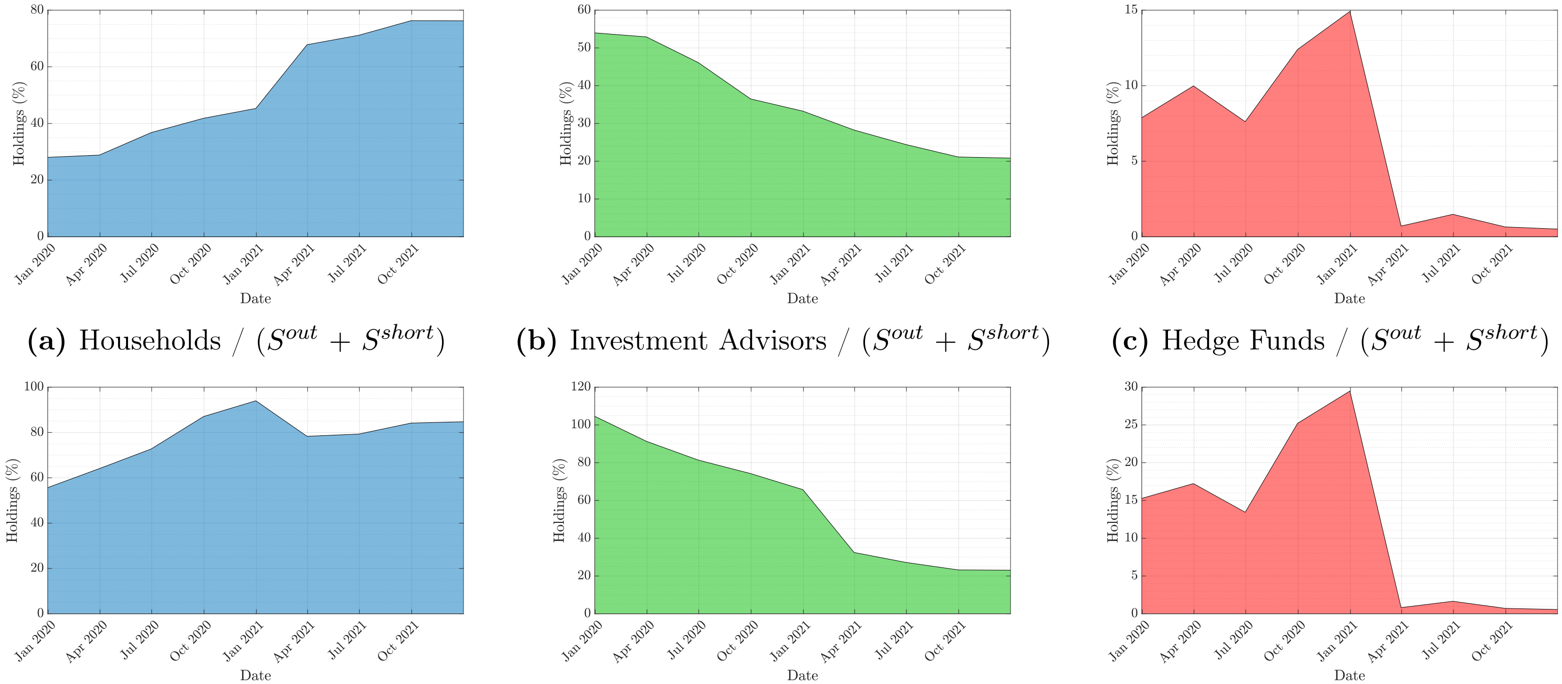


Figure A.6. Ownership of GameStop by investor type. This figure plots the end-of-quarter holdings of GameStop by 13F institutions and households, for the period from 2019 Q4 to 2021 Q4. 13F holdings data are from FactSet. I aggregate 13F institutional holdings to investor-type level using the method in Appendix C.3. The five institutional investor types are: Hedge Funds (red area), Brokers (orange aread), Private Banking (yellow area), Investment Advisors (green area), and Long-Term Investors (gray area). I calculate household holdings from equation (1.8) using data on the number of shares sold short from Compustat. The blue area represents households. The y-axis is the percentage holdings defined in equation (1.10), which is the number of shares held by each type of investor divided by the sum of the number of shares outstanding and the number of shares sold short.



(a) Households / $(S^{out} + S^{short})$ **(b)** Investment Advisors / $(S^{out} + S^{short})$ **(c)** Hedge Funds / $(S^{out} + S^{short})$

(d) Households / S^{out} **(e)** Investment Advisors / S^{out} **(f)** Hedge Funds / S^{out}

Figure A.7. Ownership of GameStop by Households, Investment Advisors, and Hedge Funds. This figure plots the end-of-quarter holdings of GameStop by Households (panels (a) and (d)), Investment Advisors (panels (b) and (e)), and Hedge Funds (panels (c) and (f)), for the period from 2019 Q4 to 2021 Q4. 13F institutional investors are classified into Investment Advisors and Hedge Funds according to Appendix C.3, and the 13F holdings data are from FactSet. Household holdings are calculated from equation (1.8). In panels (a), (b), and (c), the y-axis is the number of shares held by the investor group, divided by the sum of the number of shares outstanding and the number of shares sold short (equation (1.10)). Data on the number of shares sold short is from Compustat. In panels (d), (e), and (f), the y-axis is the number of shares held by the investor group, divided by the number of shares outstanding (equation (1.9)).

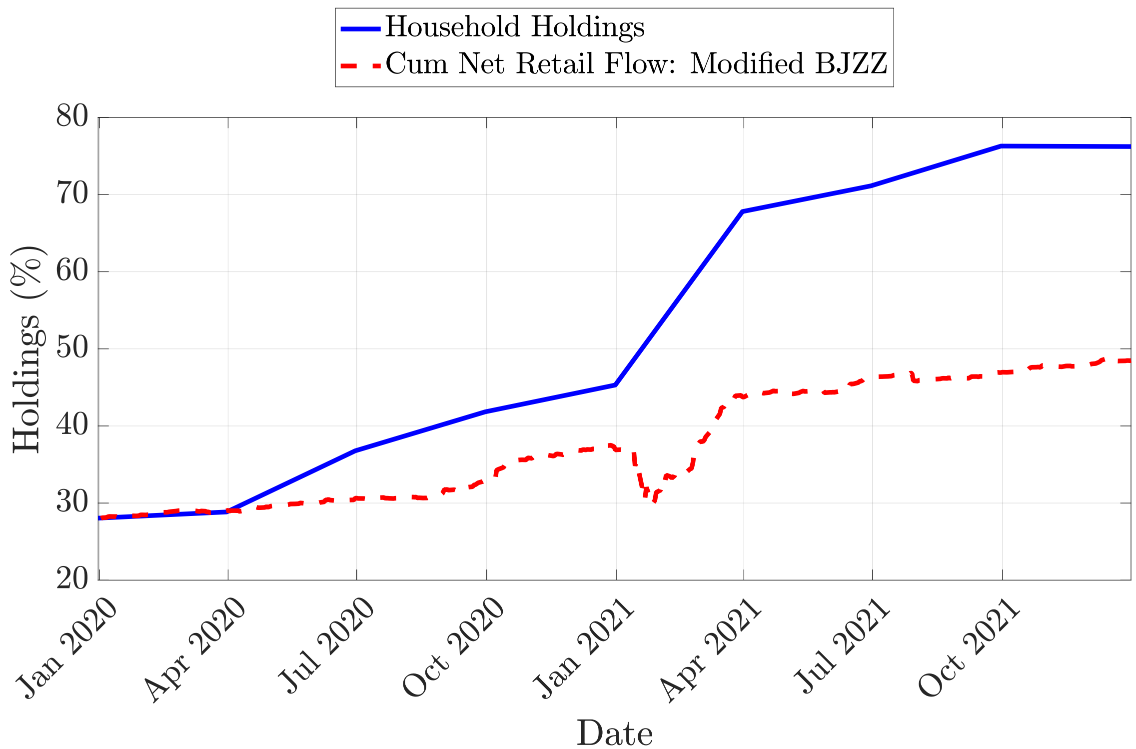


Figure A.8. Ownership by households versus cumulative net retail flow of GameStop. This figure plots the end-of-quarter percentage holdings of GameStop by Households (solid blue line) and the daily cumulative net retail flow (dashed red line), for the period from January 1, 2020 to December 31, 2021. Percentage holdings by households is defined in equation (1.10), which is the number of shares held by households (equation (1.8)) divided by the sum of the number of shares outstanding and the number of shares sold short. Cumulative net retail flow is defined in equation (1.12), which is the cumulative net retail buy volume (equation (1.11)) divided by the sum of the number of shares outstanding and the number of shares sold short. Data on the number of shares sold short is from Compustat. The initial value of the cumulative net retail flow (on Dec 31, 2019) is set to be the percentage holdings by households at the end of 2019 Q4. I apply the modified BJZZ algorithm in Appendix C.4 to identify retail trades from the TAQ data.

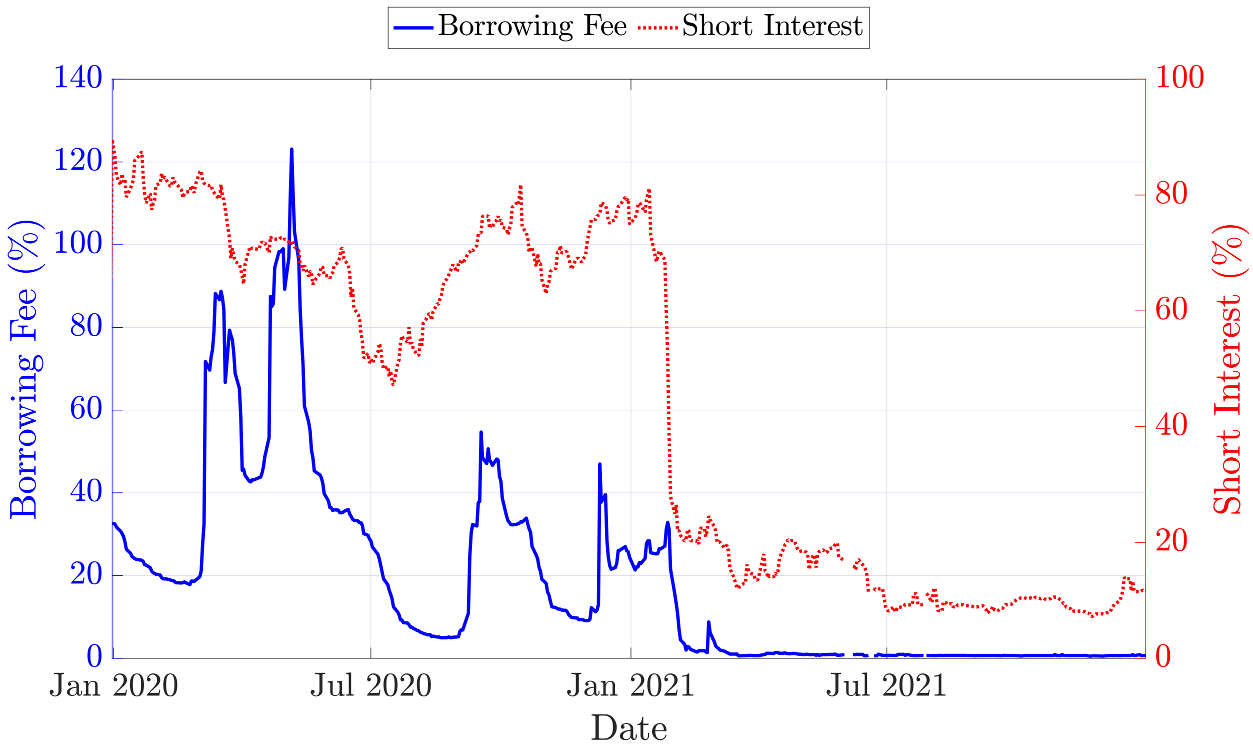
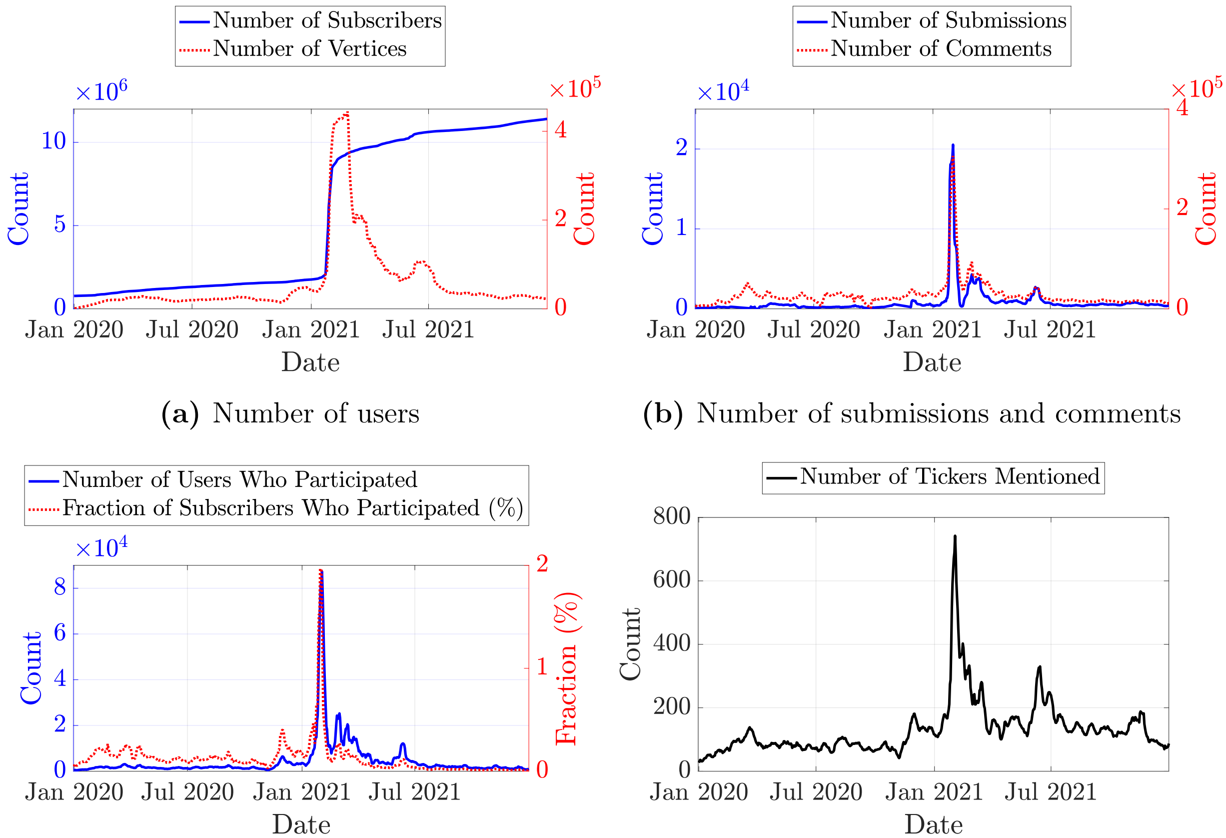


Figure A.9. Borrowing fee and short interest of GameStop. This figure shows the daily borrowing fee (left y-axis) and the daily short interest (right y-axis) of GameStop, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the borrowing fee. The dotted red line plots the short interest, which is defined as the ratio of the number of shares sold short to the number of shares outstanding (equation (1.6)). Data on the borrowing fee and the number of shares sold short is from IHS Markit.



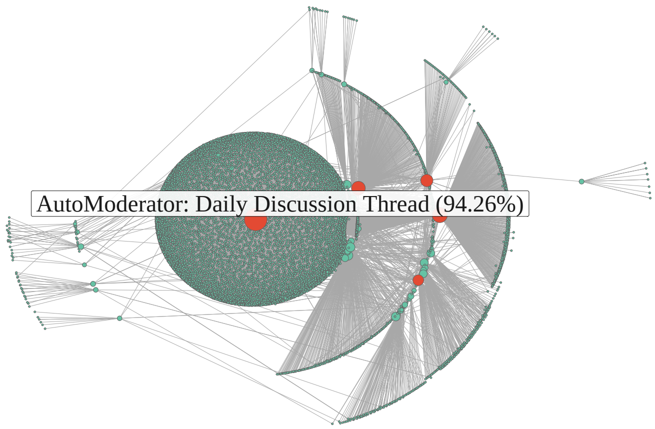
(a) Number of users

(b) Number of submissions and comments

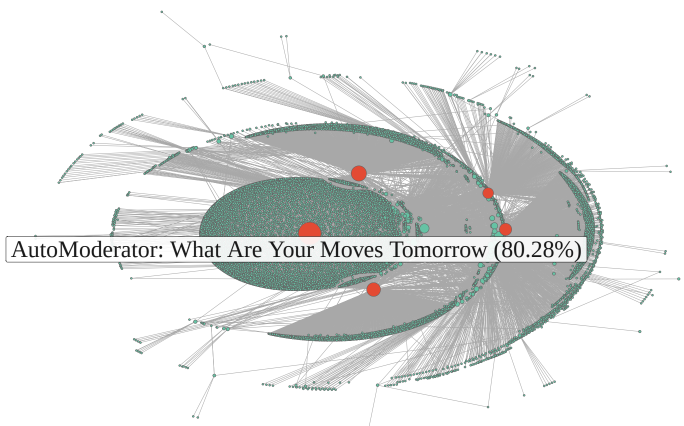
(c) Number of users who participated

(d) Number of tickers mentioned

Figure A.10. WSB statistics. This figure shows the time variation in WSB statistics, during the period from January 1, 2020 to December 31, 2021. Each line is a daily time series. Panel (a) plots the total number of subscribers to WSB (solid blue line) and the number of vertices (i.e., nodes) of the constructed network (dotted red line) on each day. When calculating the number of vertices, I use the network constructed from the sample of submissions and comments about CRSP common stocks over a 30-day rolling window (see Section 1.2.1 for details). Panel (b) plots the number of new submissions (solid blue line) and the number of new comments (dotted red line) on WSB forum on each day. Panel (c) plots the number of users who participated in the discussion of CRSP common stocks (solid blue line) and the fraction of WSB subscribers who participated in these discussions (dotted red line) on each day. Panel (d) plots the number of stock tickers mentioned on WSB on each day. The series in panels (b)-(d) are 7-day moving averages.



(a) 6-9am



(b) 4-7pm

Figure A.11. WSB user communications on January 14, 2022. This figure shows WSB user communications on January 14, 2022. Panel (a) plots the user communications from 6-9am, and panel (b) plots the user communications from 4-7pm. Each dot represents a unique user who made a new submission or new comment within this three-hour window. For any two users i and j in this plot, if i commented on j's submission within the three-hour window, then I draw a directed edge from i to j. For example, the largest red dot represents the AutoModerator, and the dots clustered around it represent the users who commented on AutoModerator's submission. The number in the parentheses is the number of comments received by the corresponding user, as a fraction of the total number comments received by the new submissions that came out within the three-hour window. The five red dots represent the top five users by the fraction of comments they received.

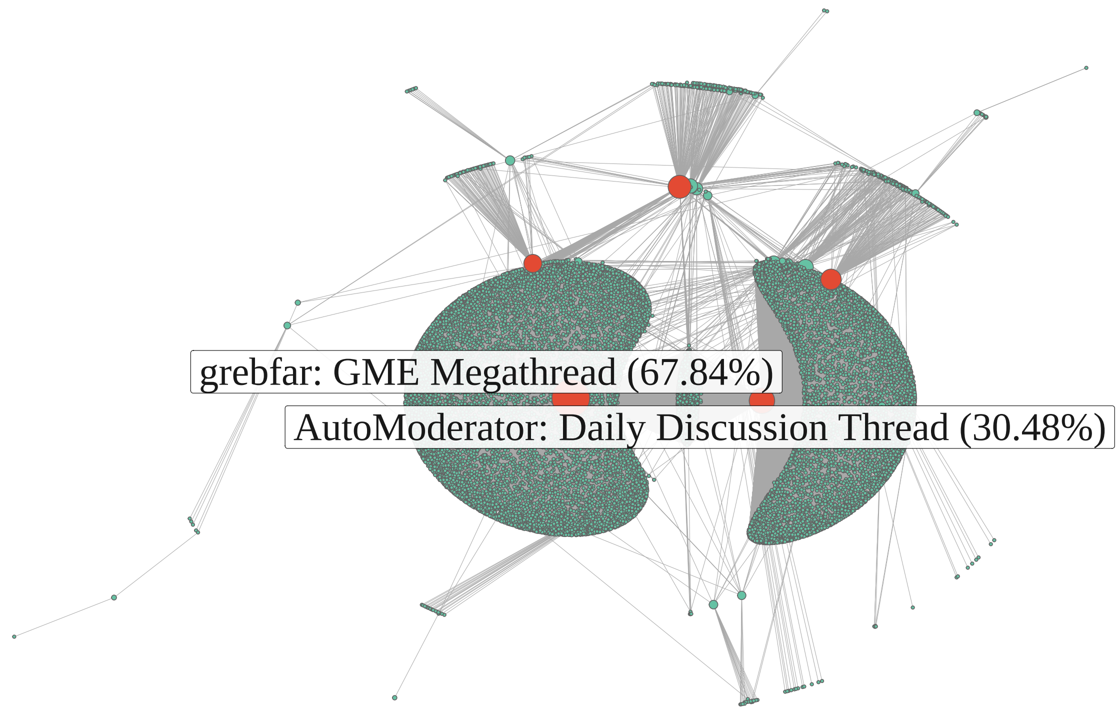


Figure A.12. WSB user communications on January 21, 2021. This figure shows WSB user communications from 6-8am on January 21, 2021. Each dot represents a unique user who made a new submission or new comment within this two-hour window. For any two users i and j in this plot, if i commented on j's submission within the two-hour window, then I draw a directed edge from i to j. For example, the largest red dot represents the user grebfar, and the dots clustered around it represent the users who commented on grebfar's submission titled "GME Megathread." The number in the parentheses is the number of comments received by the corresponding user, as a fraction of the total number of comments received by new submissions that came out within the two-hour window. The five red dots represent the top five users by the fraction of comments they received.

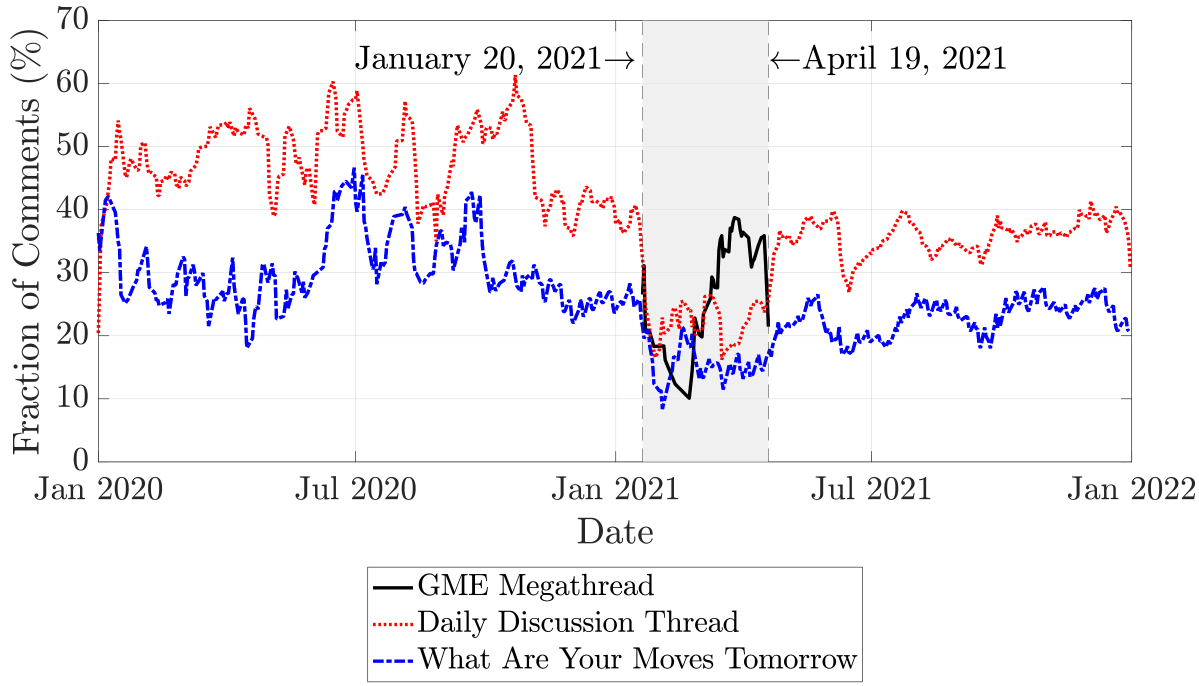


Figure A.13. Fraction of comments received by different types of megathreads. This figure plots the fraction of comments received by three types of megathreads: GME Megathreads (solid black line), Daily Discussion Threads (dotted red line), and What Are Your Moves Tomorrow (dash-dotted blue line). On a given day, there could be multiple threads of the same type, e.g., multiple threads with "GME Megathread" in their titles. In that case, the fraction of comments received by each type of thread is the total number of comments received by all threads of the type divided by the total number of new comments that came out on that day. In this figure, each line is a daily time series, and I plot the 7-day moving average of each daily series. The sample period is from January 1, 2020 to December 31, 2021.

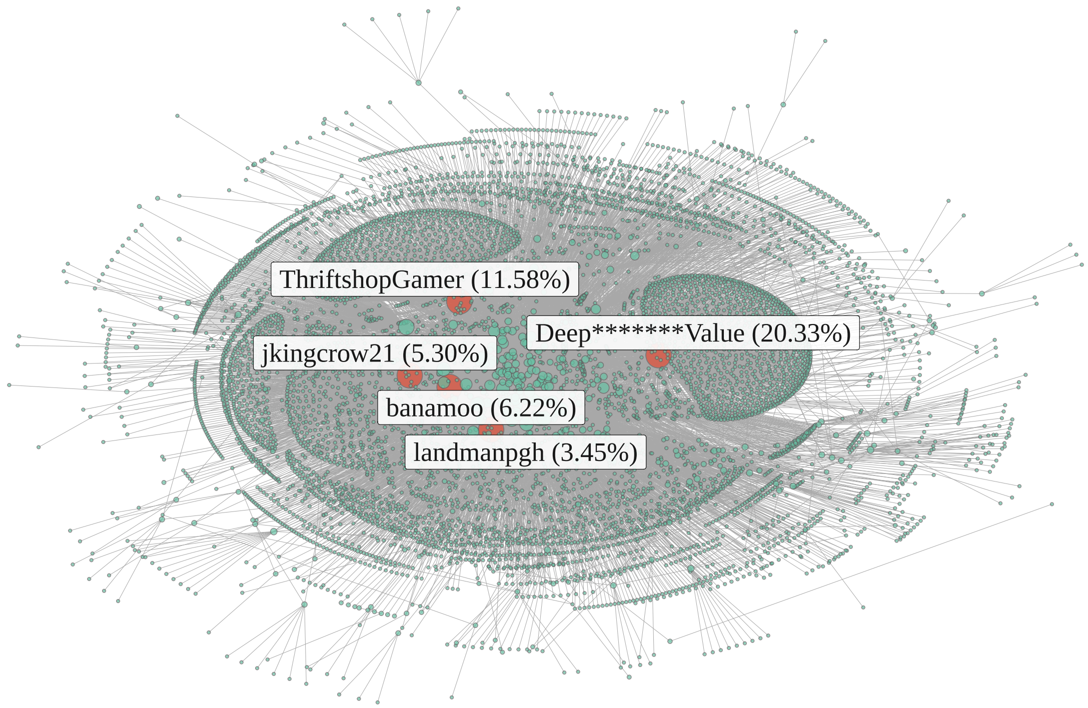


Figure A.14. WSB user network on January 14, 2021 constructed from GameStop discussions. This figure shows the WSB user network on January 14, 2021, constructed from the submissions and comments about GameStop during the 30-day window from December 15, 2020 to January 13, 2021. Each dot represents a unique user who authored at least one of the submissions or comments. For any two users i and j in this plot, if i commented on j's submission, then i "listened to" j, and I draw a directed edge from i to j. The five red dots represent the top five users by the fraction of users (on the network) who "listened to" them, and the numbers in the parentheses correspond to this fraction.

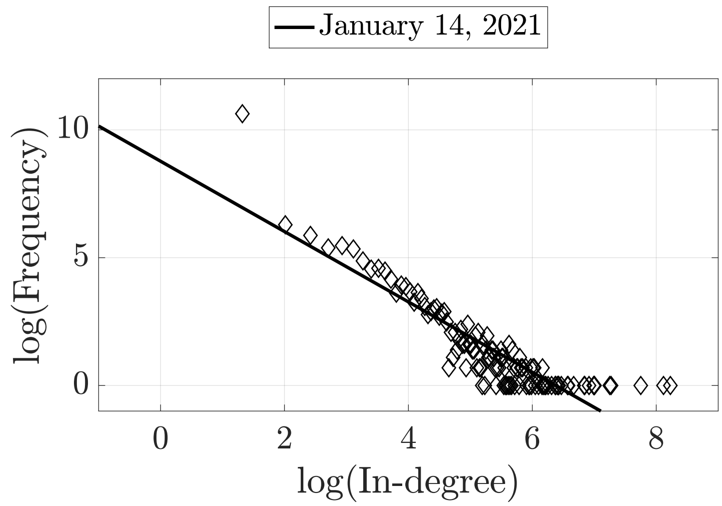


Figure A.15. Log-log plot of the influence distribution on January 14, 2021. This figure plots the cross-sectional distribution of user influence for the user network on January 14, 2021. The network is constructed according to Section 1.2.1. User influence (or in-degree) is defined in equation (1.3). The x-axis is the log of in-degree, and the y-axis is the log empirical frequency. The solid black line is a fitted linear regression line.

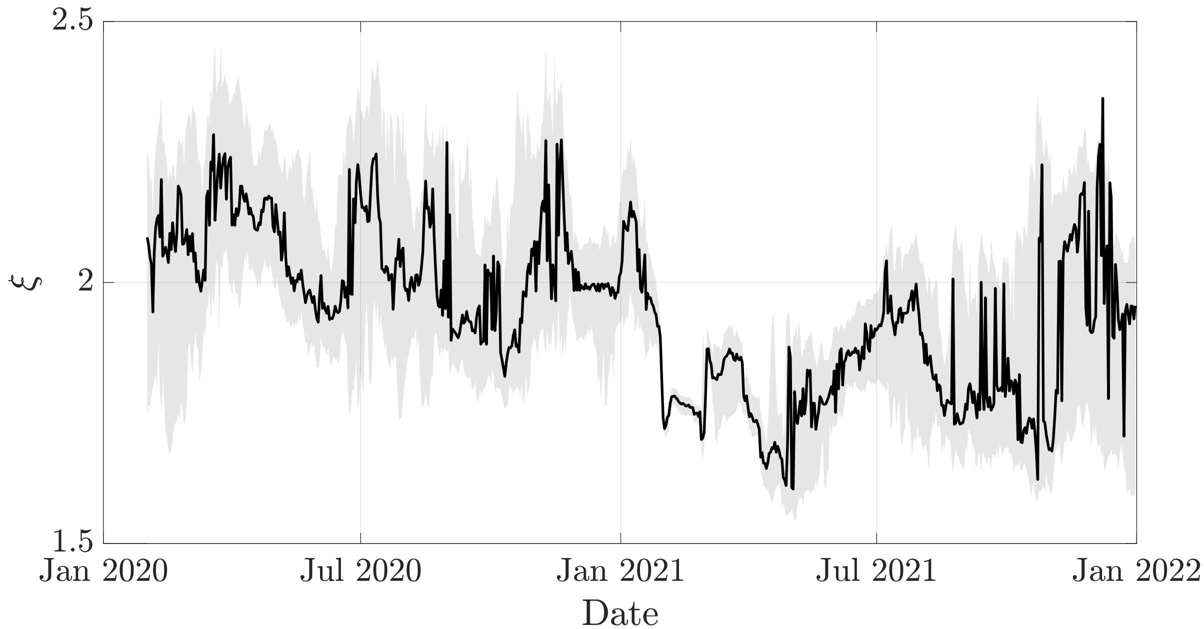


Figure A.16. Estimates of the power-law exponent $\hat{\xi}_t$. This figure plots the daily estimate of the power-law exponent $\hat{\xi}_t$, for the period from January 1, 2020 to December 31, 2021. On each day t, I fit a power-law distribution to the vector of user influence (defined in equation (1.3)) and estimate the exponent ξ in equation (1.13). The solid black line plots the $\hat{\xi}_t$ estimates from the maximum likelihood method as in Rantala (2019). The gray area shows the 95% confidence interval for the estimates, computed from the bootstrap method in Appendix C.5.

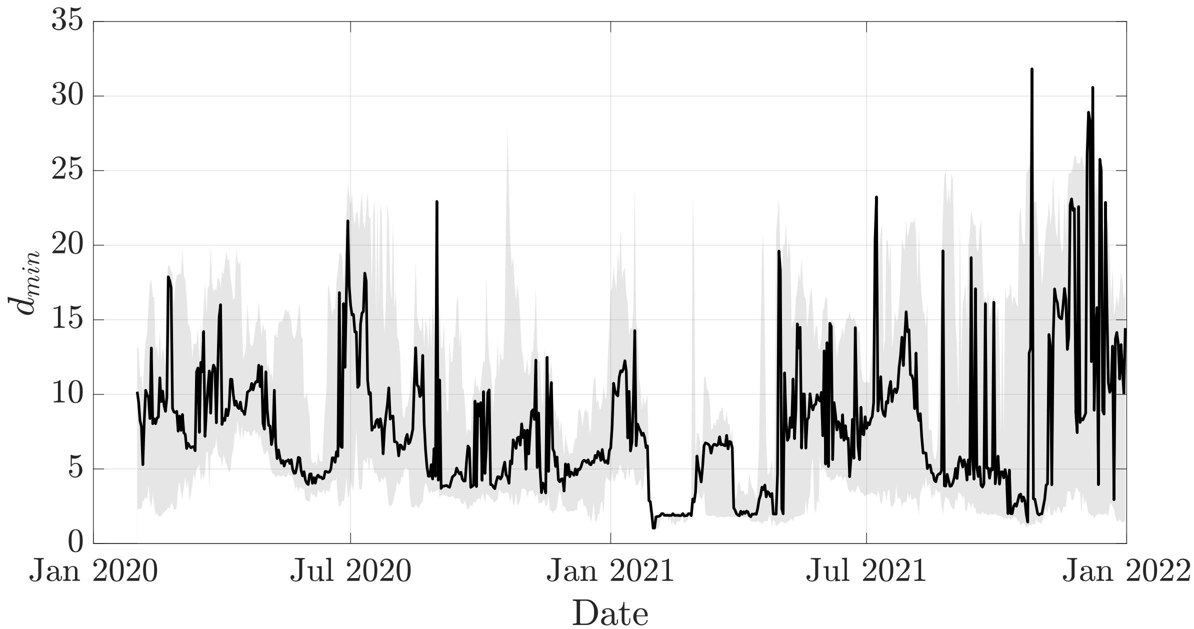


Figure A.17. Estimates of the power-law cutoff $\hat{d}_{\text{min},t}$. This figure plots the daily estimate of the power-law cutoff $\hat{d}_{\text{min},t}$, for the period from January 1, 2020 to December 31, 2021. On each day t, I fit a power-law distribution to the vector of user influence (defined in equation (1.3)) and estimate the cutoff value d_{min} in equation (1.13). The solid black line plots the $\hat{d}_{\text{min},t}$ estimates from the maximum likelihood method as in Rantala (2019). The gray area shows the 95% confidence interval for the estimates, computed from the bootstrap method in Appendix C.5.

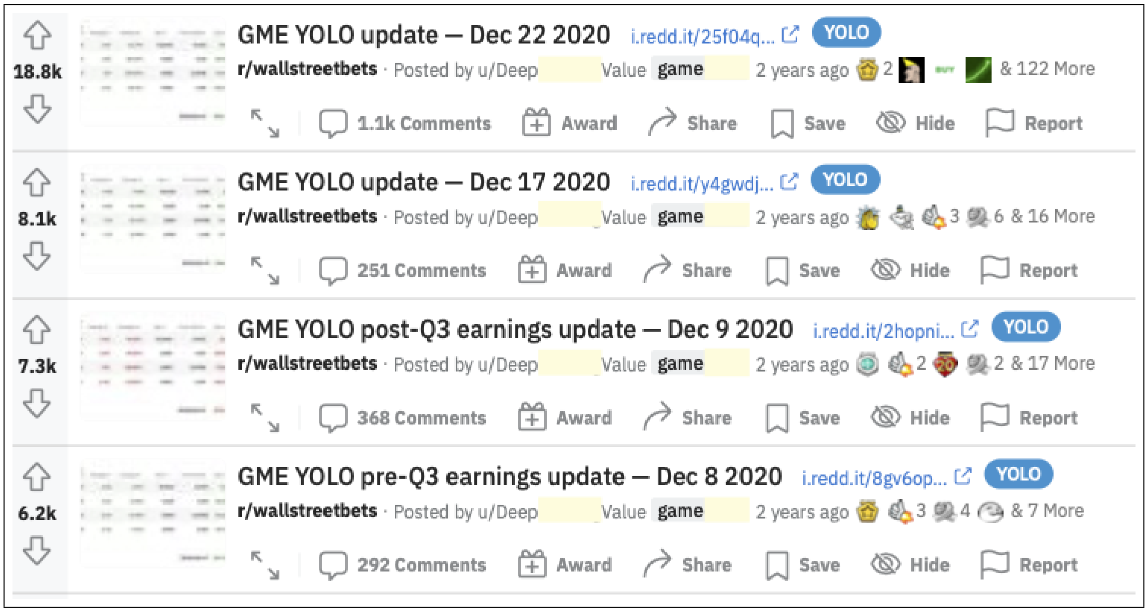
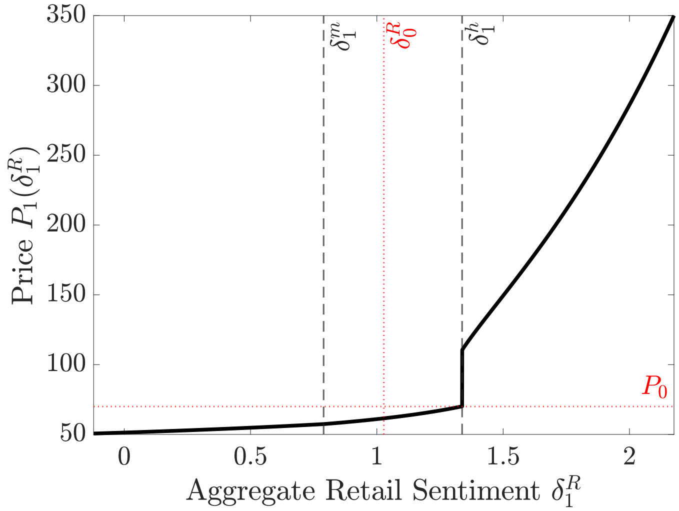
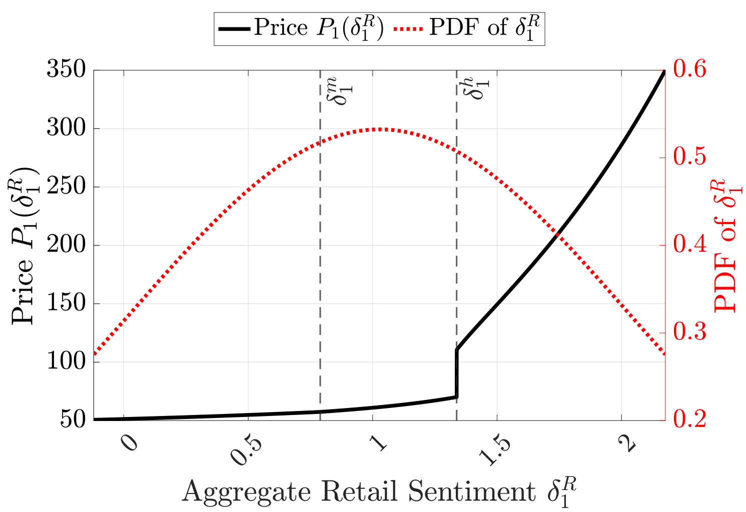


Figure A.18. Examples of Deep*****Value's submissions.** This figure shows examples of WSB submissions made by the user Deep*******Value in December 2020.



(a) Price function



(b) Price function and PDF of aggregate retail sentiment

Figure A.19. Price impact of aggregate retail sentiment at time 1. This figure shows the time-1 equilibrium price $P_1\left(\delta_1^R\right)$ as a function of the aggregate retail sentiment realization δ_1^R. In panel (a), the solid black line is the price function $P_1\left(\delta_1^R\right)$. The two vertical dashed lines represent the two sentiment cutoffs δ_1^m and δ_1^h defined in equations (1.45) and (1.46), respectively. The vertical dotted line represents the time-0 aggregate retail sentiment, and the horizontal dotted line corresponds to the time-0 equilibrium price. In panel (b), the solid black line is the price function $P_1\left(\delta_1^R\right)$, and the dotted red line is the PDF of the aggregate retail sentiment δ_1^R perceived by investors. In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The parameter values are given in Table 1.2.

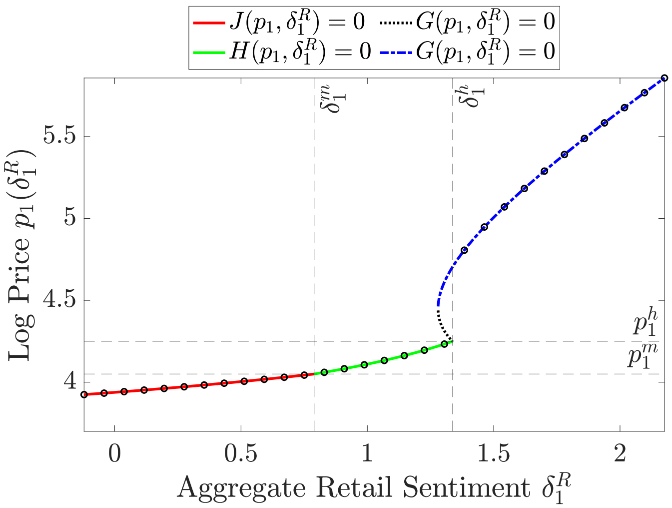
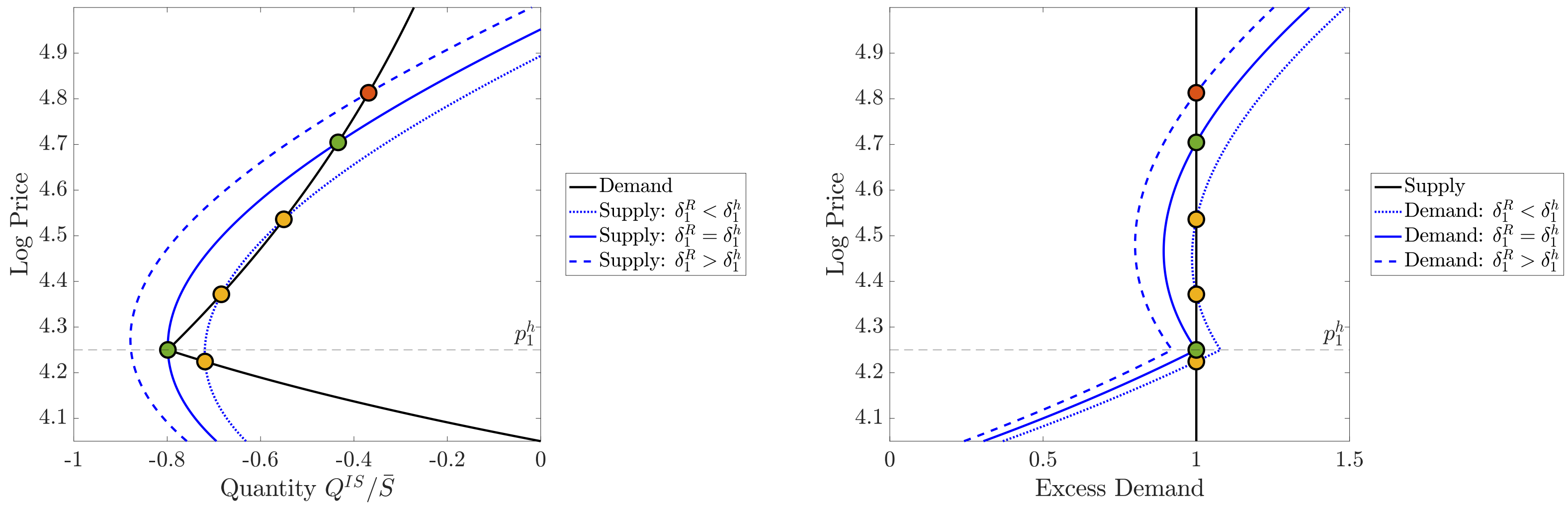


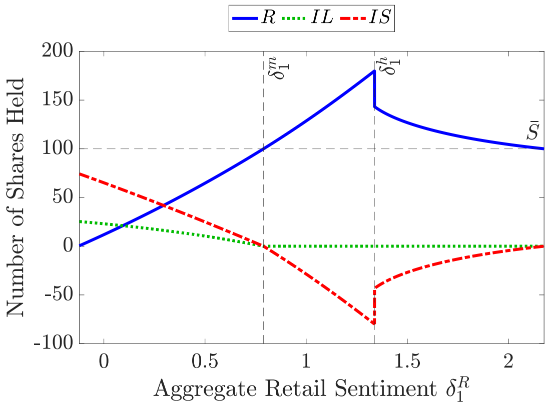
Figure A.20. Multiple equilibria. This figure shows all the equilibria at time 1. The x-axis is the aggregate retail sentiment at time 1, and the y-axis is the log price at time 1. There are three classes of equilibria: the low-price equilibria (solid red line and solid green line), the medium-price equilibria (dotted black line), and the high-price equilibria (dash-dotted blue line). In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The parameter values are given in Table 1.2.



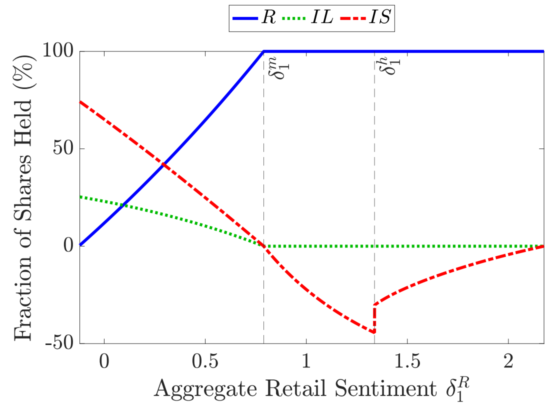
(a) Demand and supply

(b) Excess demand and supply

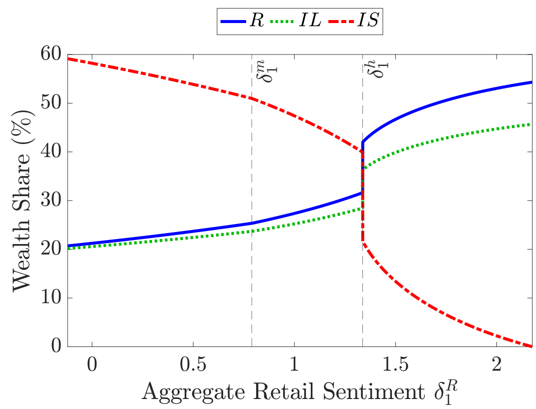
Figure A.21. Demand and supply at time 1 from the short institution's perspective. This figure shows the demand and supply curves from the short institution's perspective. Panel (a) plots the demand curve of the short institution (solid black line) defined in equation (1.52), together with three supply curves (blue lines) defined in equation (1.53) which correspond to different aggregate retail sentiment shock realizations. Panel (b) plots three excess demand curves (blue lines) that correspond to different aggregate retail sentiment shock realizations, together with the excess supply (solid black line). Excess demand is defined in equation (1.54). In each panel, the horizontal dashed black line represents the cutoff price p_1^h defined in equation (1.44), and each dot represents an equilibrium. In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The parameter values are given in Table 1.2.



(a) Holdings (shares)

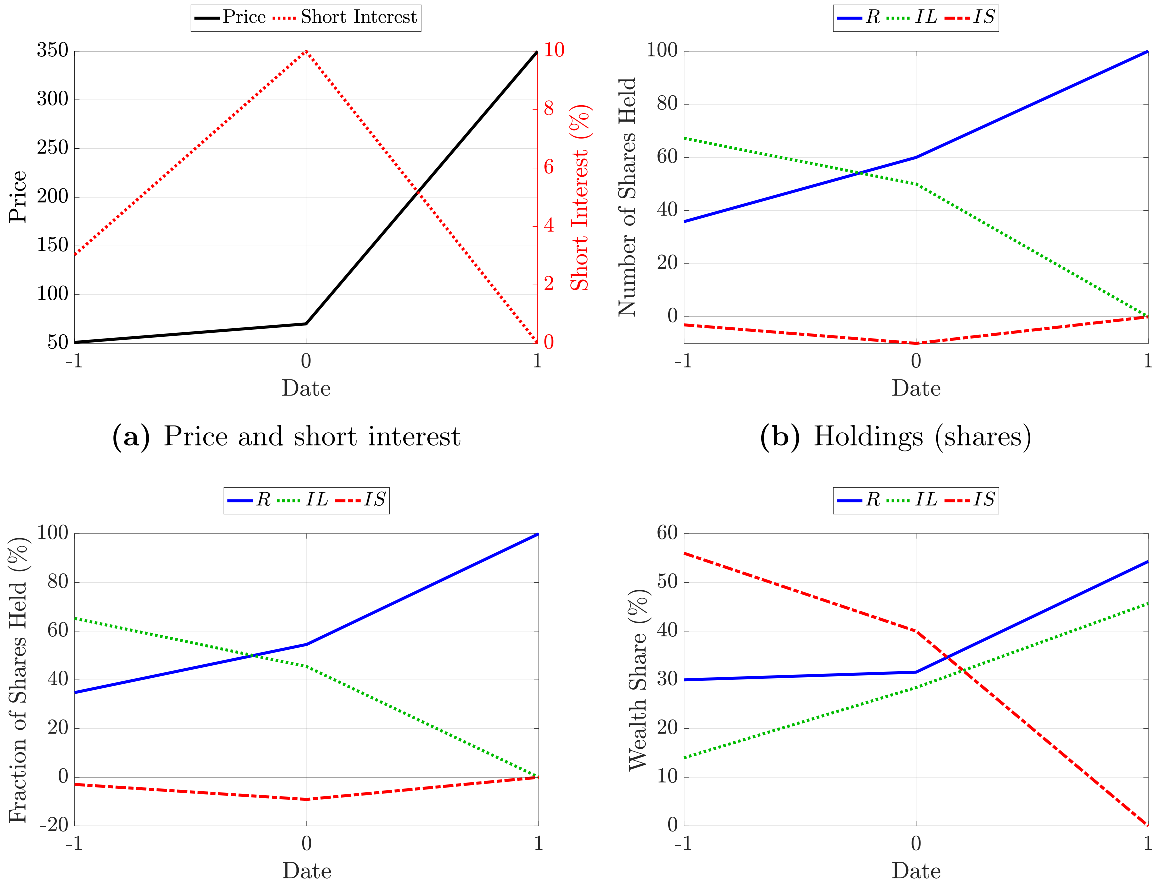


(b) Holdings (%)



(c) Wealth shares

Figure A.22. Holdings and wealth shares at time 1. This figure shows the time-1 holdings and wealth shares of different investors as functions of the aggregate retail sentiment realization δ_1^R. Panel (a) plots the number of shares held by the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). The horizontal dashed black line represents the number of shares outstanding. Panel (b) plots the percentage holdings by the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). Percentage holdings is defined as the number of shares held divided by the sum of the number of shares outstanding and the number of shares sold short. Panel (c) plots the wealth shares of the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). In each panel, the two vertical dashed black lines represent the two sentiment cutoffs δ_1^m and δ_1^h defined in equations (1.45) and (1.46). In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The parameter values are given in Table 1.2.



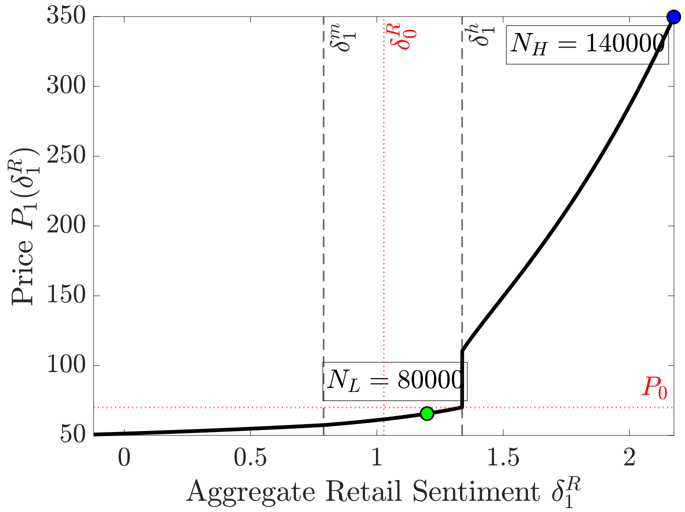
(a) Price and short interest

(b) Holdings (shares)

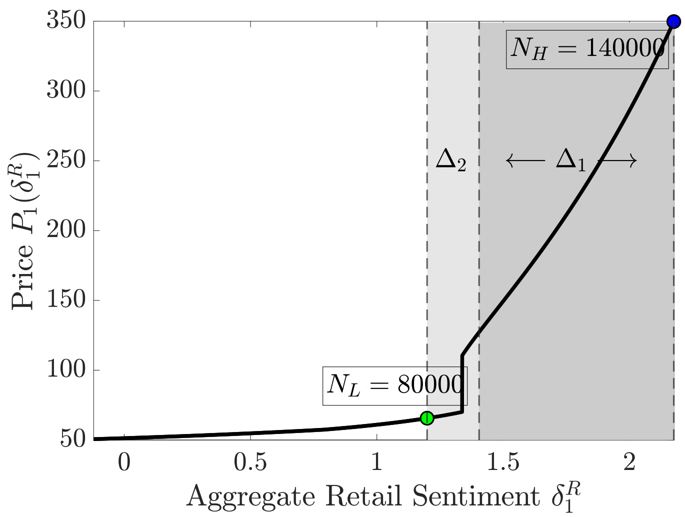
(c) Holdings (%)

(d) Wealth shares

Figure A.23. Time series predictions from the model. Panel (a) plots the equilibrium price (solid black line) and short interest (dotted red line). Short interest is defined as the number of shares shorted divided by the number of shares outstanding. Panel (b) plots the number of shares held by the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). Panel (c) plots the percentage holdings by the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). Percentage holdings is defined as the number of shares held divided by the sum of the number of shares outstanding and the number of shares sold short. Panel (d) plots the wealth shares of the aggregate retail investor (solid blue line), the long institution (dotted green line), and the short institution (dash-dotted red line). In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The time-1 equilibrium outcomes correspond to an aggregate retail sentiment realization $\delta_1^R = 2.18$. The parameter values are given in Table 1.2.



(a) Prices under different network sizes



(b) Decomposing the price difference

Figure A.24. Time-1 aggregate retail sentiment realizations under different network sizes. This figure shows the time-1 aggregate retail sentiment realizations under different network sizes. In each panel, the blue dot represents the realized aggregate retail sentiment under network size $N_1 = N_H = 140000$, while the green dot represents the realized aggregate retail sentiment under $N_1 = N_L = 80000$. Panel (b) decomposes the difference between the two sentiment realizations according to equation (1.66). In this numerical example, investors believe the size of the network will remain constant from time 0 to time 1 with $\tilde{N}_1 = N_0 = N_L$. The parameter values are given in Table 1.2.

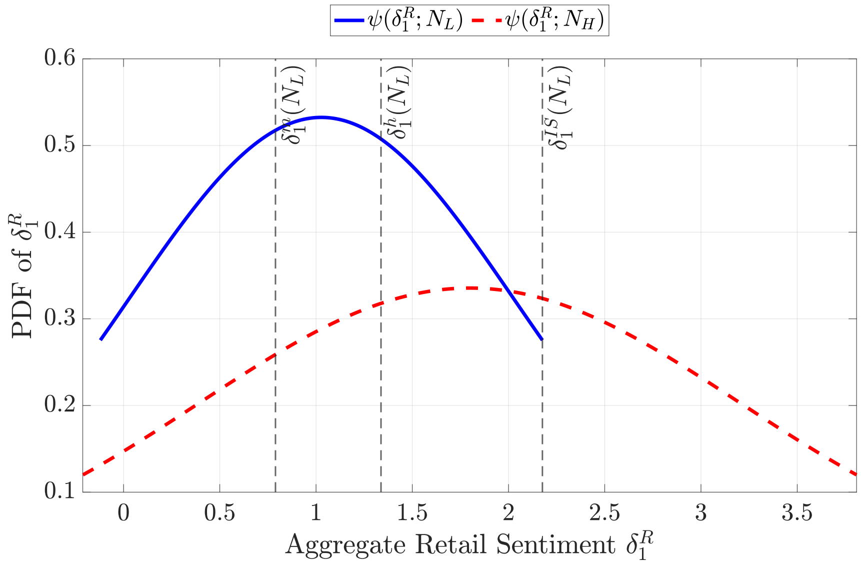
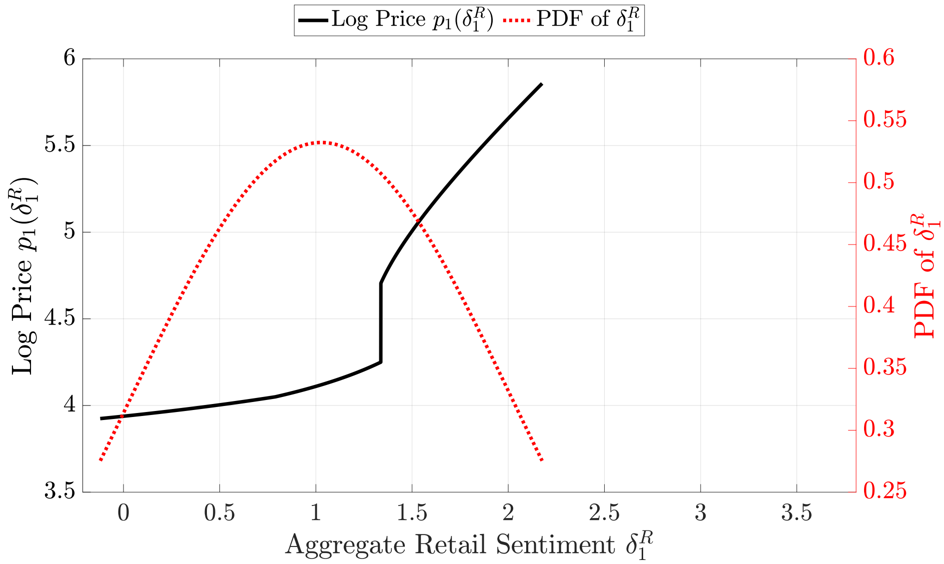
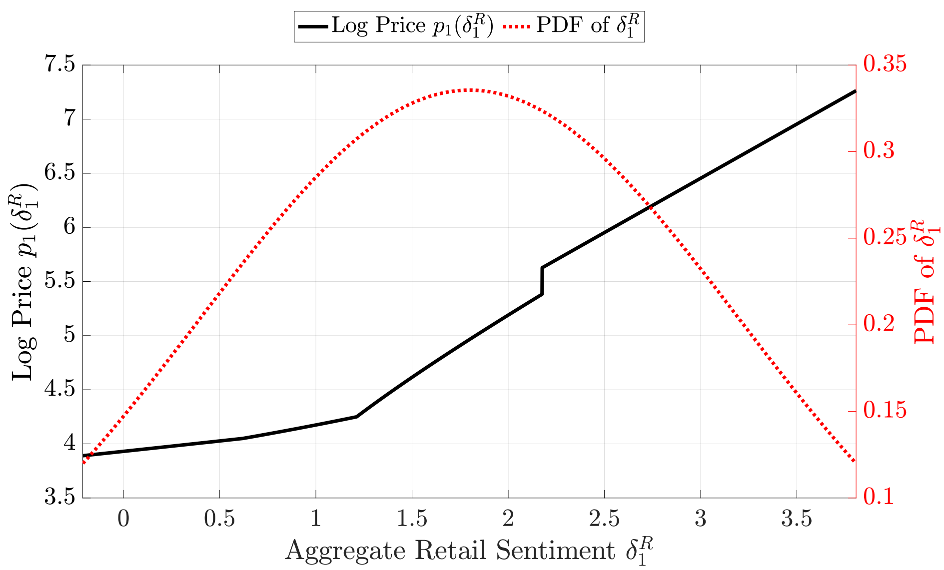


Figure A.25. Time-1 retail sentiment distributions under different network sizes. This figure plots the distribution of time-1 aggregate retail sentiment (δ_1^R) under different network sizes. The solid blue line is the PDF of δ_1^R under $N_1 = N_L$. The dashed red line is the PDF of δ_1^R under $N_1 = N_H$. The parameter values are given in Table 1.2.



(a) Risk perception $\tilde{N}_1 = N_L$



(b) Risk perception $\tilde{N}_1 = N_H$

Figure A.26. Time-1 equilibria under different risk perceptions. This figure shows the time-1 price function when changing investors' time-0 perceptions of risk. In panel (a), investors believe the size of the network will remain constant from time 0 to time 1, i.e., $\tilde{N}_1 = N_L = N_0$. In panel (b), investors believe the the size of the network will grow (deterministically) from time 0 to time 1, i.e., $\tilde{N}_1 = N_H > N_L = N_0$. The parameter values are given in Table 1.2.

APPENDIX B

TABLES

Table B.1
Modified VADER lexicon

This table shows the modification to the VADER lexicon.

Positive			Negative		
Word	Emoji	Score	Word	Emoji	Score
rocket	🚀	4.0	bear(s)		-2.0
moon(ing)		4.0	paper		-4.0
diamond		4.0			
gem (stone)	💎	4.0			
hold(ing)		4.0			
tendie(s)		4.0			
yolo		4.0			
retard(s-ed)		2.0			
autist(s)		2.0			
degenerate(s)		2.0			
ape(s)		2.0			
gorilla(s)	🦍	2.0			

Table B.2
Top 13F institutions by long positions in GameStop in 2020 Q4

This table shows the top two 13F institutions within each institution type, ranked by their long positions in GameStop in 2020 Q4. 13F holdings data are from FactSet. I classify 13F institutions into five types using the method in Appendix C.3. The five investor types are: Hedge Funds, Brokers, Private Banking, Investment Advisors, and Long-Term Investors.

Hedge Funds	Maverick Capital Ltd.
	Senvest Management LLC
Brokers	Goldman Sachs & Co. LLC
	Morgan Stanley & Co. LLC
Private Banking	Aperio Group LLC
	Permit Capital LLC (Private Equity)
Investment Advisors	Fidelity Management & Research Co. LLC
	BlackRock Fund Advisors
Long-Term Investors	The Public Sector Pension Investment Board
	The California Public Employees Retirement System

APPENDIX C

SUPPORTING ANALYSES

C.1 Omitted derivations and proofs

C.1.1 Dynamics of wealth shares

Since the risk-free asset is in zero net supply, the time-t aggregate wealth is equal to the market value of the risky asset, $P_t\bar{S}$.

Investor i's wealth share at time $t+1$ is thus

$$
\begin{aligned}
\alpha_{t+1}^{i} &\equiv \frac{A_{t+1}^{i}}{P_{t+1}\bar{S}} \\
&= \frac{A_t^i\left(w_t^i\frac{P_{t+1}}{P_t}+1-w_t^i\right)}{P_{t+1}\bar{S}} \\
&= \frac{\alpha_t^i P_t\bar{S}\left(w_t^i\frac{P_{t+1}}{P_t}+1-w_t^i\right)}{P_{t+1}\bar{S}} \\
&= \alpha_t^i\left(\left(1-w_t^i\right)\exp\left(p_t-p_{t+1}\right)+w_t^i\right),
\end{aligned}
$$

where the second line uses the budget constraint (1.22) together with the assumption of constant risk-free rate $R_{f,t}=1$.

C.1.2 Market clearing

Market clearing for the risk-free asset holds if and only if the aggregate wealth is equal to the market value of the risky asset, i.e.,

$$\sum_i A_t^i = P_t\bar{S}.$$

Market clearing condition for the risky asset is

$$\sum_i Q_t^i = \bar{S} \iff \sum_i \frac{w_t^i A_t^i}{P_t} = \bar{S} \iff \sum_i w_t^i A_t^i = P_t \bar{S}.$$

Hence, the two market clearing conditions reduce to

$$\sum_i A_t^i = \sum_i w_t^i A_t^i = P_t \bar{S}.$$

This is equivalent to the following condition

$$\sum_i \alpha_t^i w_t^i = 1, \alpha_t^i = \frac{A_t^i}{P_t \bar{S}}. \tag{C.1}$$

From equation (C.1), we can solve for the equilibrium price.

C.1.3 Optimal portfolio choice

Retail investors

Retail investor j solves the following problem

$$\begin{aligned} U_t^j\left(A_t^j\right) &= \max_{w_t^j} w_t^j \left(\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t}\right) + \frac{1}{2} w_t^j \left(1 - w_t^j\right) \mathrm{Var}_t^j\left(r_{t+1}\right) \\ &\quad + \frac{1}{2}\left(1 - \gamma^R\right)\left(w_t^j\right)^2 \mathrm{Var}_t^i\left(r_{t+1}\right). \end{aligned}$$

The F.O.C. is

$$\begin{aligned} & \mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right) - \gamma^R w_t^j \mathrm{Var}_t^j\left(r_{t+1}\right) = 0 \\ \implies \quad & w_t^j = \frac{\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right)}{\gamma^R \mathrm{Var}_t^j\left(r_{t+1}\right)} = \tau^R \frac{\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right)}{\mathrm{Var}_t^j\left(r_{t+1}\right)}. \end{aligned}$$

Substitute retail investors' subjective beliefs into the above expression, we get their demands for the risky asset.

- For a type-1 retail investor j, his time-0 and time-1 demands for the risky asset are

$$w_0^j = \tau^R \left(\frac{\mathbb{E}_0 [p_1] + y_0^j - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{C.2}$$

$$w_1^j = \tau^R \left(\frac{\mu_d + y_1^j - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{C.3}$$

- For a type-2 retail investor j', his time-0 and time-1 demands for the risky asset are

$$w_0^{j'} = \tau^R \left(\frac{\mathbb{E}_0 [p_1] - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{C.4}$$

$$w_1^{j'} = \tau^R \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{C.5}$$

Long institution

The long institution IL solves the following problem

$$\begin{aligned} U_t^{IL}\left(A_t^{IL}\right) &= \max_{w_t^{IL}} w_t^{IL}\left(\mathbb{E}_t^{IL}[r_{t+1}] - r_{f,t}\right) + \frac{1}{2} w_t^{IL}\left(1 - w_t^{IL}\right) \text{Var}_t^{IL}(r_{t+1}) \\ &\quad + \frac{1}{2}\left(1 - \gamma^I\right)\left(w_t^{IL}\right)^2 \text{Var}_t^{IL}(r_{t+1}) \\ \text{s.t.} &\quad w_t^{IL} \geq 0. \end{aligned}$$

Since the objective function is quadratic in portfolio weight w_t^{IL} and has a global maximum, the solution to this constrained problem is

$$w_t^{IL} = \max\left\{0, \tau^I \frac{\mathbb{E}_t^{IL}[r_{t+1}] - r_{f,t} + \frac{1}{2}\text{Var}_t^{IL}(r_{t+1})}{\text{Var}_t^{IL}(r_{t+1})}\right\}.$$

Substitute IL's beliefs (equations (1.14), (1.15), and (1.17)) into the above expression, we get his time-0 and time-1 demands for the risky asset

$$\begin{aligned} w_0^{IL} &= \max\left\{0, \tau^I\left(\frac{\mathbb{E}_0\left[p_1\right] + \delta_0^{IL} - p_0}{\sigma_0^2} + \frac{1}{2}\right)\right\}, \\ w_1^{IL} &= \max\left\{0, \tau^I\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right)\right\}. \end{aligned}$$

Short institution

The short institution IS solves the following problem

$$\begin{aligned} U_t^{IS}\left(A_t^{IS}\right) &= \max_{w_t^{IS}} w_t^{IS}\left(\mathbb{E}_t^{IS}\left[r_{t+1}\right] - r_{f,t}\right) + \frac{1}{2}w_t^{IS}\left(1 - w_t^{IS}\right)\mathrm{Var}_t^{IS}\left(r_{t+1}\right) \\ &\quad + \frac{1}{2}\left(1 - \gamma^I\right)\left(w_t^{IS}\right)^2 \mathrm{Var}_t^{IS}\left(r_{t+1}\right) \\ \text{s.t.} &\quad w_t^{IS} \geq -\frac{1}{m}. \end{aligned}$$

The solution is

$$w_t^{IS} = \max\left\{-\frac{1}{m}, \tau^I \frac{\mathbb{E}_t^{IS}\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^{IS}\left(r_{t+1}\right)}{\mathrm{Var}_t^{IS}\left(r_{t+1}\right)}\right\}.$$

Substitute IS's beliefs (equations (1.14), (1.16), and (1.17)) into the above expression, we get his time-0 and time-1 demands for the risky asset

$$\begin{aligned} w_0^{IS} &= \max\left\{-\frac{1}{m}, \tau^I\left(\frac{\mathbb{E}_0\left[p_1\right] + \delta_0^{IS} - p_0}{\sigma_0^2} + \frac{1}{2}\right)\right\}, \\ w_1^{IS} &= \max\left\{-\frac{1}{m}, \tau^I\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right)\right\}. \end{aligned}$$

C.1.4 Proof of Lemma 1

Proof. I prove the existence result in two steps. First, I show that the aggregate demand of the $\bar{N}$ retail investors is equal to the demand of the aggregate retail investor specified in equations (1.39) and (1.40), and thus the equilibrium price can be solved from the market clearing condition (1.42). Then I derive the wealth share dynamics of the aggregate retail investor in equation (1.41).

I begin by restating the timeline and the wealth share dynamics of individual retail investors. At time $t-1$ after trading, retail investor j has dollar wealth A_t^j and wealth share α_t^j. At time t before trading, the $\bar{N}$ retail investors first split their aggregate wealth $\sum_{k=1}^{\bar{N}} A_t^k$ equally. In particular, they split their aggregate stock position and aggregate bond position equally. After that, retail investor j has wealth $\hat{A}_t^j = \frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} A_t^k$ and wealth share

$$\hat{\alpha}_t^j \equiv \frac{\hat{A}_t^j}{A_t} = \frac{\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} A_t^k}{A_t} = \frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} \alpha_t^k. \tag{C.6}$$

Retail investors then trade with each other. Specifically, retail investor j allocates his wealth $\hat{A}_t^j$ into the risky asset and the risk-free asset. His demand for the risky asset (in terms of the number of shares) is $Q_t^j = \frac{w_t^j \hat{A}_t^j}{P_t}$, where w_t^j is his optimal portfolio weight. After trading, his end-of-period wealth share becomes

$$\alpha_{t+1}^j = \hat{\alpha}_t^j \left(\left(1 - w_t^j\right) \exp\left(p_t - p_{t+1}\right) + w_t^j \right). \tag{C.7}$$

Next, I show that the equilibrium price of the risky asset is the same as that in an economy with three investors: an aggregate retail investor, the long institution, and the short institution. Furthermore, the demand of the aggregate retail investor is the sum of the demand of the $\bar{N}$ retail investors.

At time t, market clearing for the risky asset implies that

$$
\begin{aligned}
& \sum_{j=1}^{\bar{N}} Q_t^j + Q_t^{IL} + Q_t^{IS} = \bar{S} \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} \frac{w_t^j \hat{A}_t^j}{P_t} + \frac{w_t^{IL} A_t^{IL}}{P_t} + \frac{w_t^{IS} A_t^{IS}}{P_t} = \bar{S} \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} w_t^j \hat{\alpha}_t^j + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} w_t^j \left(\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} \alpha_t^k \right) + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \frac{1}{\bar{N}} \sum_{j=1}^{N_t} \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1} \right] + y_t^j - p_t}{\sigma_t^2} + \frac{1}{2} \right) \\
& + \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \frac{1}{\bar{N}} \sum_{j=N_t+1}^{\bar{N}} \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1} \right] - p_t}{\sigma_t^2} + \frac{1}{2} \right) \\
& + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1} \right] - p_t}{\sigma_t^2} + \frac{1}{2} \right) + \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \theta \left(N_t \right) \frac{1}{N_t} \sum_{j=1}^{N_t} \tau^R \frac{y_t^j}{\sigma_t^2} \\
& + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1} \right] + \theta \left(N_t \right) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j - p_t}{\sigma_t^2} + \frac{1}{2} \right) + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1,
\end{aligned}
$$

where the fourth line uses the definition of $\hat{\alpha}_t^j$ in equation (C.6), and the fifth line uses the optimal portfolio weights of retail investors in equations (1.29)-(1.32).

Define

$$A_t^R \equiv \sum_{j=1}^{\bar{N}} A_t^j, \alpha_t^R \equiv \sum_{j=1}^{\bar{N}} \alpha_t^j, \tag{C.8}$$

$$\delta_t^R \equiv \theta(N_t) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j, \tag{C.9}$$

$$w_t^R \equiv \tau^R \left(\frac{\mathbb{E}_t[p_{t+1}] + \delta_t^R - p_t}{\sigma_t^2} + \frac{1}{2} \right) = \frac{1}{\bar{N}} \sum_{j=1}^{\bar{N}} w_t^j. \tag{C.10}$$

Then the market clearing condition can be written as

$$w_t^R \alpha_t^R + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1$$

with $\alpha_t^R + \alpha_t^{IL} + \alpha_t^{IS} = \sum_{j=1}^{\bar{N}} \alpha_t^j + \alpha_t^{IL} + \alpha_t^{IS} = 1$.

Hence, the equilibrium price of the risky asset is the same as that in an economy with three investors: an aggregate retail investor R, the long institution IL, and the short institution IS. The three investors have demands $\left(w_t^R, w_t^{IL}, w_t^{IS}\right)$ and wealth shares $\left(\alpha_t^R, \alpha_t^{IL}, \alpha_t^{IS}\right)$. In other words, there exists an aggregate retail investor whose demand for the risky asset is given by equation (C.10). The aggregate retail investor has constant relative risk tolerance $\tau^R = \frac{1}{\gamma^R}$ and subjective beliefs

$$\mathbb{E}_0^R[p_1] = \mathbb{E}_0[p_1] + \delta_0^R, \mathrm{Var}_0^R(p_1) = \sigma_0^2,$$
$$\mathbb{E}_1^R\left[\tilde{d}\right] = \mu_d + \delta_1^R, \mathrm{Var}_1^R\left(\tilde{d}\right) = \sigma_d^2.$$

Finally, I derive the wealth share dynamics of the aggregate retail investor. From the definition of α_{t+1}^R in (C.8),

$$\begin{aligned}
\alpha_{t+1}^R &\equiv \sum_{j=1}^{\bar{N}} \alpha_{t+1}^j \\
&= \sum_{j=1}^{\bar{N}} \hat{\alpha}_t^j \left(\left(1 - w_t^j\right) \exp\left(p_t - p_{t+1}\right) + w_t^j\right) \\
&= \left(\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} \alpha_t^k\right) \sum_{j=1}^{\bar{N}} \left(\left(1 - w_t^j\right) \exp\left(p_t - p_{t+1}\right) + w_t^j\right) \\
&= \alpha_t^R \left(\left(1 - \frac{1}{\bar{N}} \sum_{j=1}^{\bar{N}} w_t^j\right) \exp\left(p_t - p_{t+1}\right) + \frac{1}{\bar{N}} \sum_{j=1}^{\bar{N}} w_t^j\right) \\
&= \alpha_t^R \left(\left(1 - w_t^R\right) \exp\left(p_t - p_{t+1}\right) + w_t^R\right),
\end{aligned}$$

where the second line uses investor j's wealth share dynamics in equation (C.7), and the last line uses the aggregate retail investor's demand in equation (C.10). □

C.1.5 Proof of Proposition 1

Proof. I focus on monotone equilibrium of Definition 1 with sentiment cutoffs δ_1^m, δ_1^h satisfying $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$. Hence, $\forall \delta_1^R \in \left[\underline{\delta}_1, \delta_1^m\right)$, the equilibrium price $p_1\left(\delta_1^R\right) < p_1^m$. Similarly, $\forall \delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, the price $p_1\left(\delta_1^R\right) \in \left[p_1^m, p_1^h\right)$. $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the price $p_1\left(\delta_1^R\right) \geq p_1^h$.

Next, I solve the equilibrium price from the market clearing condition in equation (1.42).

- For $\delta_1^R \in \left[\underline{\delta}_1, \delta_1^m\right)$, I look for an equilibrium price $p_1 < p_1^m$. Substituting the optimal portfolio choices of the three investors, (1.40), (1.34), and (1.36), into the market clearing condition (1.42), I get

$$
\begin{aligned}
& \frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \sum_i \alpha_1^i(p_1)\,\tau^i\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & p_1 = \mu_d + \left(\frac{\frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R - 1}{\sum_i \alpha_1^i(p_1)\,\tau^i} + \frac{1}{2}\right)\sigma_d^2 \\
\Longrightarrow \quad & p_1 = \mu_d + \left(\frac{1}{2} + \frac{\frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R - 1}{\tau_1(p_1)}\right)\sigma_d^2,
\end{aligned}
$$

where

$$
\tau_1(p_1) \equiv \sum_i \alpha_1^i(p_1)\,\tau^i = \alpha_1^R(p_1)\,\tau^R + \left(1 - \alpha_1^R(p_1)\right)\tau^I.
$$

Define the function

$$
J\left(p_1, \delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R(p_1)\,\tau^R\delta_1^R - \sigma_d^2}{\tau_1(p_1)}\right) - p_1.
$$

Then the equilibrium price p_1 solves $J\left(p_1, \delta_1^R\right) = 0$.

The cutoff sentiment shock δ_1^m solves $J\left(p_1^m, \delta_1^m\right) = 0$, which yields

$$
\delta_1^m = \frac{\left(p_1^m - \mu_d - \frac{1}{2}\sigma_d^2\right)\tau_1\left(p_1^m\right) + \sigma_d^2}{\alpha_1^R\left(p_1^m\right)\tau^R} = \frac{\sigma_d^2}{\alpha_1^R\left(p_1^m\right)\tau^R}.
$$

- For $\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, I look for an equilibrium price $p_1 \in \left[p_1^m, p_1^h\right)$. Substituting the optimal portfolio choices of the three investors, (1.40), (1.34), and (1.36), into the market clearing condition (1.42), I get

$$
\begin{aligned}
& \alpha_1^R(p_1)\,\tau^R\left(\frac{\mu_d+\delta_1^R-p_1}{\sigma_d^2}+\frac{1}{2}\right)+\alpha_1^{IS}(p_1)\,\tau^I\left(\frac{\mu_d-p_1}{\sigma_d^2}+\frac{1}{2}\right)=1 \\
\Longrightarrow\quad & \frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R+\left(\alpha_1^R(p_1)\,\tau^R+\alpha_1^{IS}(p_1)\,\tau^I\right)\left(\frac{\mu_d-p_1}{\sigma_d^2}+\frac{1}{2}\right)=1 \\
\Longrightarrow\quad & p_1=\mu_d+\left(\frac{1}{2}+\frac{\frac{1}{\sigma_d^2}\alpha_1^R(p_1)\,\tau^R\delta_1^R-1}{\alpha_1^R(p_1)\,\tau^R+\alpha_1^{IS}(p_1)\,\tau^I}\right)\sigma_d^2 \\
\Longrightarrow\quad & p_1=\mu_d+\left(\frac{1}{2}+\frac{\frac{1}{\sigma_d^2}\alpha_1^R(p_1)\,\tau^R\delta_1^R-1}{\hat{\tau}_1(p_1)}\right)\sigma_d^2,
\end{aligned}
$$

where

$$
\hat{\tau}_1(p_1) \;\equiv\; \alpha_1^R(p_1)\,\tau^R+\alpha_1^{IS}(p_1)\,\tau^I.
$$

Define the function

$$
H\left(p_1,\delta_1^R\right) \;\equiv\; \mu_d+\left(\frac{1}{2}\sigma_d^2+\frac{\alpha_1^R(p_1)\,\tau^R\delta_1^R-\sigma_d^2}{\hat{\tau}_1(p_1)}\right)-p_1.
$$

Then the equilibrium price p_1 solves $H\left(p_1,\delta_1^R\right)=0$.

The cutoff sentiment shock δ_1^h solves $H\left(p_1^h,\delta_1^h\right)=0$, which yields

$$
\delta_1^h=\frac{\left(p_1^h-\mu_d-\frac{1}{2}\sigma_d^2\right)\hat{\tau}_1\left(p_1^h\right)+\sigma_d^2}{\alpha_1^R\left(p_1^h\right)\tau^R}=\frac{\frac{1}{m\tau^I}\hat{\tau}_1\left(p_1^h\right)+1}{\alpha_1^R\left(p_1^h\right)\tau^R}\sigma_d^2.
$$

- For $\delta_1^R\in\left[\delta_1^h,\bar{\delta}_1\right]$, I look for an equilibrium price $p_1\geq p_1^h$. Substituting the optimal portfolio choices of the three investors, (1.40), (1.34), and (1.36), into the market clearing condition (1.42), I get

$$\alpha_1^R(p_1)\,\tau^R\left(\frac{\mu_d+\delta_1^R-p_1}{\sigma_d^2}+\frac{1}{2}\right)-\alpha_1^{IS}(p_1)\,\frac{1}{m}=1$$

$$\implies\quad p_1=\mu_d+\delta_1^R+\left(\frac{1}{2}-\frac{1+\alpha_1^{IS}(p_1)\,\frac{1}{m}}{\alpha_1^R(p_1)\,\tau^R}\right)\sigma_d^2.$$

Define the function

$$G\left(p_1,\delta_1^R\right)\;=\;\mu_d+\delta_1^R+\left(\frac{1}{2}-\frac{1+\alpha_1^{IS}(p_1)\,\frac{1}{m}}{\alpha_1^R(p_1)\,\tau^R}\right)\sigma_d^2-p_1.$$

Then the equilibrium price p_1 solves $G\left(p_1,\delta_1^R\right)=0$.

□

C.1.6 Lemma 3 and proof

Lemma 3 (Properties of the implicit function $G\left(p_1,\delta_1^R\right)$). *Consider a monotone equilibrium of Definition 1, where the time-0 portfolios satisfy $w_0^R>1$, $w_0^{IS}<0$, $w_0^R>w_0^{IL}>w_0^{IS}$, and investors always have strictly positive wealth $\forall\delta_1\in\left(\underline{\delta}_1,\bar{\delta}_1\right)$. Let p_1^R denote the price at which the retail investor's time-1 wealth is zero*

$$p_1^R\;\equiv\;p_0+\log\left(1-\frac{1}{w_0^R}\right).$$

Then the implicit function $G\left(p_1,\delta_1^R\right)$ has the following properties on $p_1\in\left(p_1^R,+\infty\right)$:

1. *$G\left(p_1,\delta_1^R\right)$ is continuous and strictly increasing in δ_1^R: $\frac{\partial G(p_1,\delta_1^R)}{\partial\delta_1}=1>0$.*
2. *$G\left(p_1,\delta_1^R\right)$ is continuous and strictly concave in p_1: $\frac{\partial^2 G(p_1,\delta_1^R)}{\partial p_1^2}<0$.*
3. *$\frac{\partial G(p_1,\delta_1^R)}{\partial p_1}$ does not depend on δ_1^R: $\frac{\partial^2 G(p_1,\delta_1^R)}{\partial p_1\partial\delta_1^R}=0$.*

4. $G\left(p_1, \delta_1^R\right)$*, as a function of* p_1*, has at most two distinct roots on* $p_1 \in \left(p_1^R, +\infty\right)$*.*

Proof. First, I derive p_1^R from

$$\begin{aligned} & \alpha_1^R\left(p_1^R\right) = 0 \\ \implies \quad & 0 = \alpha_0^R\left(\left(1 - w_0^R\right)\exp\left(p_0 - p_1^R\right) + w_0^R\right) \\ \implies \quad & p_1^R = p_0 + \log\left(1 - \frac{1}{w_0^R}\right). \end{aligned}$$

Then $\forall p_1 > p_1^R$, $\alpha_1\left(p_1\right) > 0$, and thus $G\left(p_1, \delta_1^R\right)$ is continuous and twice differentiable, $\forall p_1 > p_1^R$, $\forall \delta_1^R$.

To show Properties 1-3, compute the following derivatives

$$
\begin{aligned}
\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial \delta_1^R} &= 1, \\
\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} &= -\left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2} \\
&\quad \cdot\left(\frac{d\alpha_1^{IS}\left(p_1\right)}{dp_1}\frac{1}{m}\alpha_1^R\left(p_1\right)\tau^R-\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\tau^R\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)\right)\sigma_d^2-1 \\
&= \left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\exp\left(p_0-p_1\right) \\
&\quad \cdot\tau^R\left(\alpha_0^{IS}\left(1-w_0^{IS}\right)\frac{1}{m}\alpha_1^R\left(p_1\right)-\alpha_0^R\left(1-w_0^R\right)\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)\right)\sigma_d^2 \\
&\quad -1 \\
&= \left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\exp\left(p_0-p_1\right) \\
&\quad \cdot\alpha_0^R\tau^R\left(w_0^R-1+\frac{1}{m}\alpha_0^{IS}\left(w_0^R-w_0^{IS}\right)\right)\sigma_d^2-1, \\
\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1\partial \delta_1^R} &= 0, \\
\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1^2} &= -\left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\sigma_d^2 \\
&\quad \cdot\left(\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\tau^R\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)-\frac{d\alpha_1^{IS}\left(p_1\right)}{dp_1}\frac{1}{m}\alpha_1^R\left(p_1\right)\tau^R\right) \\
&\quad \cdot\left(1+\frac{2}{\alpha_1^R\left(p_1\right)}\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\right).
\end{aligned}
$$

From the wealth share dynamics, we get

$$
\begin{aligned}
& \alpha_{t+1}^i\left(p_{t+1}\right)=\alpha_t^i\left(\left(1-w_t^i\right)\left(p_t-p_{t+1}\right)+w_t^i\right) \\
\Longrightarrow \quad & \frac{d\alpha_{t+1}^i\left(p_{t+1}\right)}{dp_{t+1}}=-\alpha_t^i\left(1-w_t^i\right)\exp\left(p_t-p_{t+1}\right).
\end{aligned}
$$

Since $w_0^R > 1$ and $w_0^{IS} < 0$, we have

$$\frac{d\alpha_1^R(p_1)}{dp_1} > 0, \frac{d\alpha_1^{IS}(p_1)}{dp_1} < 0.$$

Hence, $\frac{\partial^2 G(p_1,\delta_1^R)}{\partial p_1^2} < 0$, i.e., $G\left(p_1, \delta_1^R\right)$ is strictly concave in p_1, $\forall p_1 \in \left(\delta_1^R, +\infty\right)$.

Next, I show Property 4. For a given δ_1^R, suppose $G\left(p_1, \delta_1^R\right)$ has more than two roots. Let x_1, x_2, x_3 denote three of the roots with $x_1 < x_2 < x_3$. Then $\exists\lambda \in (0,1)$ such that $x_2 = \lambda x_1 + (1-\lambda)x_3$. Since $G\left(p_1, \delta_1^R\right)$ is continuous and strictly concave in p_1,

$$0 = \lambda G\left(x_1, \delta_1^R\right) + (1-\lambda) G\left(x_3, \delta_1^R\right) = G\left(\lambda x_1 + (1-\lambda)x_3, \delta_1^R\right) < G\left(x_2, \delta_1^R\right) = 0,$$

which is a contradiction. Hence, $\forall p_1 \in \left(p_1^R, +\infty\right)$, $G\left(p_1, \delta_1^R\right)$ as a function of p_1 has at most two distinct roots. □

C.1.7 Proof of Proposition 2

Proof. I first show that $\forall\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1, \delta_1^R\right) = 0$ has exactly one root that satisfies $p_1 > p_1^h$. Suppose otherwise, then from Lemma 3, there are two roots x_1 and x_2 which satisfy $p_1^h < x_1 < x_2$, and $G\left(x_1, \delta_1^R\right) = G\left(x_2, \delta_1^R\right) = 0$. Since $G\left(p_1^h, \delta_1^h\right) = 0$ and $\frac{\partial G(p_1,\delta_1^R)}{\partial \delta_1^R} = 1 > 0$, then $G\left(p_1^h, \delta_1^R\right) > G\left(p_1^h, \delta_1^h\right) = 0$, $\forall\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$. $p_1^h < x_1 < x_2 \rightarrow \exists\lambda \in (0,1)$ such that $x_1 = \lambda p_1^h + (1-\lambda)x_2$. Since $G\left(p_1, \delta_1^R\right)$ is strictly concave in p_1, we have

$$0 < \lambda G\left(p_1^h, \delta_1^R\right) + (1-\lambda) G\left(x_2, \delta_1^R\right) < G\left(\lambda p_1^h + (1-\lambda)x_2, \delta_1^R\right) = G\left(x_1, \delta_1^R\right) = 0,$$

which is a contradiction. Hence, $\forall\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1, \delta_1^R\right)$ has exactly one root that satisfies $p_1 > p_1^h$. In a monotone equilibrium of Definition 1, this is the unique equilibrium price in the high sentiment region $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$.

Next, I derive the conditions for discontinuity in price. Consider the following two cases:

- Case 1: $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0.$

  From the strict concavity of $G\left(p_1,\delta_1^R\right)$ in Lemma 3, $\forall p_1 > p_1^h$, $\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1} < \left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0$. This implies that $G\left(p_1,\delta_1^h\right) < G\left(p_1^h,\delta_1^h\right) = 0, \forall p_1 > p_1^h$. Hence, p_1^h is the largest root of $G\left(p_1,\delta_1^h\right) = 0$.

  From Lemma 3, $\frac{\partial G(p_1,\delta_1^R)}{\partial p_1 \partial \delta_1^R} = 0$ and $\frac{\partial^2 G(p_1,\delta_1^R)}{\partial p_1^2} < 0$. Then

$$
\begin{aligned}
& \left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0 \\
\Longrightarrow \quad & \left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0, \forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right] \\
\Longrightarrow \quad & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} < 0, \forall p_1 > p_1^h, \forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right].
\end{aligned}
$$

  Moreover, if $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} = 0$, then $\left.\frac{\partial G(p_1,\delta_1^R)}{\partial p_1}\right|_{p_1=p_1^h} = 0$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$. Otherwise, $\left.\frac{\partial G(p_1,\delta_1^R)}{\partial p_1}\right|_{p_1=p_1^h} < 0$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$.

  Using the implicit function theorem, $\forall p_1 > p_1^h$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$,

$$
\begin{aligned}
& \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial \delta_1^R} = 0 \\
\Longrightarrow \quad & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + 1 = 0 \\
\Longrightarrow \quad & \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = -\frac{1}{\frac{\partial G(p_1,\delta_1^R)}{\partial p_1}} > 0.
\end{aligned}
$$

Hence, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the equilibrium price $p_1\left(\delta_1^R\right)$ is strictly increasing in δ_1^R. Furthermore, $p_1\left(\delta_1^R\right)$ is continuous in δ_1^R on $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$ and is right-continuous at $\delta_1^R = \delta_1^h$.

- Case 2: $\left.\frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1}\right|_{p_1 = p_1^h} > 0$.

  First, I prove that $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1, \delta_1^R\right) = 0$ has two distinct roots, denoted as $x_1\left(\delta_1^R\right)$ and $x_2\left(\delta_1^R\right)$, with $x_1\left(\delta_1^R\right) \leq p_1^h < x_2\left(\delta_1^R\right)$. Furthermore, $x_1\left(\delta_1^R\right) = p_1^h$ if and only if $\delta_1^R = \delta_1^h$.

  - $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, we have $G\left(p_1^h, \delta_1^R\right) > G\left(p_1^h, \delta_1^h\right) = 0$ and $G\left(+\infty, \delta_1^R\right) = -\infty$. Let p_1^R denote the price at which the retail investor's time-1 wealth share is exactly zero, then p_1^R satisfies

$$
\begin{aligned}
& \alpha_1^R\left(p_1^R\right) = 0 \\
\Longrightarrow \quad & 0 = \alpha_0^R\left(\left(1 - w_0^R\right)\exp\left(p_0 - p_1^R\right) + w_0^R\right) \\
\Longrightarrow \quad & p_1^R = p_0 + \log\left(1 - \frac{1}{w_0^R}\right),
\end{aligned}
$$

    and we have $G\left(p_1^R, \delta_1^R\right) = -\infty$. Then $G\left(p_1^R, \delta_1^R\right) = G\left(+\infty, \delta_1^R\right) = -\infty < 0 < G\left(p_1^h, \delta_1^R\right)$. By the intermediate value theorem, $G\left(p_1, \delta_1^R\right) = 0$ has two distinct roots $x_1\left(\delta_1^R\right)$ and $x_2\left(\delta_1^R\right)$ such that $p_1^R < x_1\left(\delta_1^R\right) < p_1^h < x_2\left(\delta_1^R\right)$, $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$. In a monotone equilibrium of Definition 1, $x_2\left(\delta_1^R\right)$ is the unique equilibrium price.

    Next, I show that $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, $\left.\frac{\partial G(p_1, \delta_1^R)}{\partial p_1}\right|_{p_1 = x_2(\delta_1^R)} < 0$. Suppose otherwise, $\left.\frac{\partial G(p_1, \delta_1^R)}{\partial p_1}\right|_{p_1 = x_2(\delta_1^R)} \geq 0 \implies \frac{\partial G(p_1, \delta_1^R)}{\partial p_1} > 0, \forall p_1 < x_2\left(\delta_1^R\right)$. This implies $0 = G\left(p_1^h, \delta_1^h\right) < G\left(p_1^h, \delta_1^R\right) < G\left(x_2\left(\delta_1^R\right), \delta_1^R\right) = 0$, a contradiction.

– At the cutoff $\delta_1^R = \delta_1^h$, $\left.\frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1}\right|_{p_1 = p_1^h} > 0$ implies that, $\exists \varepsilon > 0$ and small, $G\left(p_1^h + \varepsilon, \delta_1^h\right) > G\left(p_1^h, \delta_1^h\right) = 0$. Together with $G\left(+\infty, \delta_1^h\right) = -\infty < 0$, this implies that $G\left(p_1, \delta_1^h\right)$ has two distinct roots $x_1\left(\delta_1^h\right)$ and $x_2\left(\delta_1^h\right)$ such that $x_1\left(\delta_1^h\right) = p_1^h < x_2\left(\delta_1^h\right)$.

  Next, I show that $\left.\frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1}\right|_{p_1 = x_2\left(\delta_1^h\right)} < 0$. Suppose otherwise, $\left.\frac{\partial G\left(p_1, \delta_1^R\right)}{\partial p_1}\right|_{p_1 = x_2\left(\delta_1^h\right)} \geq 0 \implies \frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1} > 0, \forall p_1 < x_2\left(\delta_1^h\right)$. This implies $0 = G\left(p_1^h, \delta_1^h\right) < G\left(x_2\left(\delta_1^h\right), \delta_1^h\right) = 0$, a contradiction.

In a monotone equilibrium of Definition 1, $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, the equilibrium price has to be greater than p_1^h. Hence, $x_2\left(\delta_1^R\right)$ is the unique equilibrium price on $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$. Since $p_1^h < x_2\left(\delta_1^h\right)$, the pricing function $p_1\left(\delta_1^R\right)$ is discontinuous at $\delta_1^R = \delta_1^h$.

Using the implicit function theorem, $\forall p_1 > x_2\left(\delta_1^h\right)$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$,

$$
\begin{aligned}
& \frac{\partial G\left(p_1, \delta_1^R\right)}{\partial p_1} \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + \frac{\partial G\left(p_1, \delta_1^R\right)}{\partial \delta_1^R} = 0 \\
\implies \quad & \frac{\partial G\left(p_1, \delta_1^R\right)}{\partial p_1} \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + 1 = 0 \\
\implies \quad & \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = -\frac{1}{\frac{\partial G\left(p_1, \delta_1^R\right)}{\partial p_1}} > 0.
\end{aligned}
$$

Hence, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the equilibrium price $p_1\left(\delta_1^R\right)$ is strictly increasing in δ_1^R. Furthermore, $p_1\left(\delta_1^R\right)$ is continuous in δ_1^R on $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$ and is discontinuous at $\delta_1^R = \delta_1^h$.

□

C.1.8 Proof of Proposition 3

Proof. Consider the three sentiment regions:

- Low sentiment $\delta_1^R \in \left(\underline{\delta}_1, \delta_1^m\right)$: from the optimal portfolio choices of the three investors, (1.40), (1.34), (1.36), and the market clearing condition (1.42), we get

$$\begin{aligned}
&\frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \sum_i \alpha_1^i(p_1)\,\tau^i\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad &\frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \tau_1(p_1)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad &\alpha_1^R(p_1)\,\tau^R\delta_1^R + \tau_1(p_1)\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) = \sigma_d^2.
\end{aligned}$$

Using the implicit function theorem,

$$\begin{aligned}
&\alpha_1^R(p_1)\,\tau^R + \frac{d\left(\alpha_1^R(p_1)\,\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\tau_1(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) \\
&-\tau_1(p_1)\frac{dp_1}{d\delta_1^R} = 0 \\
\Longrightarrow \quad &\frac{dp_1}{d\delta_1^R} = \frac{\frac{\alpha_1^R(p_1)\tau^R}{\tau_1(p_1)}}{1 - \frac{1}{\tau_1(p_1)}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R + \frac{d\tau_1(p_1)}{dp_1}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)\right)}.
\end{aligned}$$

- Medium sentiment $\delta_1^R \in \left(\delta_1^m, \delta_1^h\right)$: from the optimal portfolio choices of the three investors, (1.40), (1.34), (1.36), and the market clearing condition (1.42), we get

$$\begin{aligned}
&\frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \left(\alpha_1^R(p_1)\,\tau^R + \alpha_1^{IS}(p_1)\,\tau^I\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad &\frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \hat{\tau}_1(p_1)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad &\alpha_1^R(p_1)\,\tau^R\delta_1^R + \hat{\tau}_1(p_1)\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) = \sigma_d^2.
\end{aligned}$$

Using the implicit function theorem,

$$\alpha_1^R(p_1)\tau^R + \frac{d\left(\alpha_1^R(p_1)\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\hat{\tau}_1(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)$$
$$-\hat{\tau}_1(p_1)\frac{dp_1}{d\delta_1^R} = 0$$
$$\implies \frac{dp_1}{d\delta_1^R} = \frac{\frac{\alpha_1^R(p_1)\tau^R}{\hat{\tau}_1(p_1)}}{1 - \frac{1}{\hat{\tau}_1(p_1)}\left(\frac{d\left(\alpha_1^R(p_1)\right)}{dp_1}\tau^R\delta_1^R + \frac{d\hat{\tau}_1(p_1)}{dp_1}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)\right)}.$$

- High sentiment $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right)$: from the optimal portfolio choices of the three investors, (1.40), (1.34), (1.36), and the market clearing condition (1.42), we get

$$\alpha_1^R(p_1)\tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right) - \alpha_1^{IS}(p_1)\frac{1}{m} = 1$$
$$\implies \frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R + \alpha_1^R(p_1)\tau^R\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) - \alpha_1^{IS}(p_1)\frac{1}{m} = 1$$
$$\implies \alpha_1^R(p_1)\tau^R\delta_1^R + \alpha_1^R(p_1)\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \alpha_1^{IS}(p_1)\frac{1}{m}\sigma_d^2 = \sigma_d^2.$$

Using the implicit function theorem,

$$\alpha_1^R(p_1)\tau^R + \frac{d\left(\alpha_1^R(p_1)\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\alpha_1^R(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)$$
$$-\alpha_1^R(p_1)\tau^R\frac{dp_1}{d\delta_1^R} - \frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\frac{1}{m}\sigma_d^2 = 0$$
$$\implies \frac{dp_1}{d\delta_1^R} =$$
$$\frac{1}{1 - \frac{1}{\alpha_1^R(p_1)\tau^R}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R + \frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{1}{m}\sigma_d^2\right)}.$$

□

C.1.9 Proof of Proposition 4

Proof. To derive the time-0 equilibrium price, we substitute the optimal portfolio choices of the three investors, (1.39), (1.33), and (1.35), into the market clearing condition (1.42)

$$\begin{aligned}
&\left(\alpha_0^R(p_0)\tau^R + \left(1-\alpha_0^R(p_0)\right)\tau^I\right)\left(\frac{\mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right]-p_0}{\sigma_0^2}+\frac{1}{2}\right)+\sum_i \frac{\alpha_0^i(p_0)\tau^i\delta_0^i}{\sigma_0^2} \\
&= 1 \\
\Longrightarrow\quad & \tau_0(p_0)\left(\mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right]-p_0+\frac{1}{2}\sigma_0^2\right)+\sum_i \alpha_0^i(p_0)\tau^i\delta_0^i = \sigma_0^2 \\
\Longrightarrow\quad & p_0 = \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right]+\left(\frac{1}{2}\sigma_0^2+\frac{\sum_i \alpha_0^i(p_0)\tau^i\delta_0^i-\sigma_0^2}{\tau_0(p_0)}\right),
\end{aligned}$$

where

$$\tau_0(p_0) \equiv \sum_i \alpha_0^i(p_0)\tau^i = \alpha_0^R(p_0)\tau^R + \left(1-\alpha_0^R(p_0)\right)\tau^I.$$

The rest of the proof follows Proposition 1. □

C.1.10 Implicit price at time −1

I assume that at time −1, investors do not anticipate future sentiment shocks. They believe the prices at time 0 and 1 will reflect the present value of the terminal dividend, and the prices are deterministic. Hence, from time −1 to 0 and from time 0 to 1, the risky asset should have the same one-period return as the risky-free asset. This implies that $p_{-1} = \tilde{p}_0 = \tilde{p}_1$, where $\tilde{p}_0$ and $\tilde{p}_1$ denote investors' beliefs about time-0 and time-1 prices, respectively.

The implicit price p_{-1} is such that investors do not want to trade at time −1. Since $p_{-1} = \tilde{p}_0 = \tilde{p}_1$, investors believe they will not have incentives to trade at time 0 and 1, and thus they believe their asset positions and wealth shares remain constant from time −1 to time 1. In this case, the aggregate risk tolerance remains constant from time −1 to time 1 and is equal to

$$\tau_{-1} = \alpha_{-1}^{R}\tau^{R} + \left(1 - \alpha_{-1}^{R}\right)\tau^{I}.$$

Imposing the market clearing condition in equation (1.42), we can solve for the implicit price

$$p_{-1} = \tilde{p}_0 = \tilde{p}_1 = \mu_d + \left(\frac{1}{2} - \frac{1}{\tau_{-1}}\right)\sigma_d^2.$$

Note that at time -1, investors do not want to trade, because they believe the risky asset has the same return as the risk-free asset.

C.1.11 Proof of Lemma 2

Proof. I first compute the m-th moment of $d_{j,t}^{in}$ in the cross section of retail investors using the PDF specified in equation (1.58) with support $[d_{\min}, d_{\max}(N_t)]$.

$$\begin{aligned}
\mathbb{E}^{CS}\left[\left(d_{j,t}^{in}\right)^{m}\right] &= \int_{d_{\min}}^{d_{\max}(N_t)} x^{m}\frac{\xi-1}{d_{\min}}\left(\frac{x}{d_{\min}}\right)^{-\xi} dx \\
&= \frac{\xi-1}{d_{\min}^{1-\xi}}\int_{d_{\min}}^{d_{\max}(N_t)} x^{m-\xi} dx \\
&= \frac{\xi-1}{d_{\min}^{1-\xi}}\frac{1}{m+1-\xi}x^{m+1-\xi}\Big|_{d_{\min}}^{d_{\max}(N_t)} \\
&= \frac{\xi-1}{\xi-m-1}\frac{1}{d_{\min}^{1-\xi}}\left(d_{\min}^{m+1-\xi} - \left(d_{\max}(N_t)\right)^{m+1-\xi}\right).
\end{aligned}$$

The cross-sectional variance of $d_{j,t}^{in}$ is thus

$$\begin{aligned}
\mathrm{Var}^{CS}\left(d_{j,t}^{in}\right) &= \mathbb{E}\left[\left(d_{j,t}^{in}\right)^2\right] - \left(\mathbb{E}\left[d_{j,t}^{in}\right]\right)^2 \\
&= \frac{\xi-1}{3-\xi}\frac{1}{d_{\min}^{1-\xi}}\left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi} - d_{\min}^{3-\xi}\right) \\
&\quad - \left(\frac{\xi-1}{\xi-2}\right)^2 \frac{1}{d_{\min}^{2-2\xi}}\left(d_{\min}^{2-\xi} - \left(d_{\max}\left(N_t\right)\right)^{2-\xi}\right)^2.
\end{aligned}$$

□

C.1.12 Proof of Proposition 5

Proof. The proof follows Acemoglu et al. (2012). Using the PDF of $d_{j,t}^{in}$ in equation (1.58), I first derive the counter-CDF

$$P_{N_t}\left(x\right) \equiv \Pr\left(d_{j,t}^{in} > x\right) = \int_x^{+\infty} \frac{\xi-1}{d_{\min}}\left(\frac{y}{d_{\min}}\right)^{-\xi} dy = \left(\frac{x}{d_{\min}}\right)^{1-\xi}. \tag{C.11}$$

Define the empirical counterpart as

$$\hat{P}_{N_t}\left(x\right) = \frac{1}{N_t}\left|\left\{j \in \mathcal{I}_{N_t} : d_{j,t}^{in} > x\right\}\right| = \frac{1}{N_t}\sum_{j=1}^{N_t} \mathbf{1}\left\{d_{j,t}^{in} > x\right\}.$$

Let $\boldsymbol{B}_t = \left\{b_{1,t}, b_{2,t}, \cdots, b_{m_t,t}\right\}$ denote the set of values $d_{j,t}^{in}$ takes, with $b_{1,t} < b_{2,t} < \cdots < b_{m_t,t}$, and the convention that $b_{0,t} = 0$. Then

$$
\begin{aligned}
&\sum_{j=1}^{N_t} \left(d_{j,t}^{in}\right)^2 = N_t \sum_{k=1}^{m_t} \left(b_{k,t}\right)^2 \left(\hat{P}_{N_t}\left(b_{k-1,t}\right) - \hat{P}_{N_t}\left(b_{k,t}\right)\right) \\
= \;& N_t \left(b_{1,t}^2 \left(\hat{P}_{N_t}\left(b_{0,t}\right) - \hat{P}_{N_t}\left(b_{1,t}\right)\right) + \cdots + b_{m_t}^2 \left(\hat{P}_{N_t}\left(b_{m_t-1,t}\right) - \hat{P}_{N_t}\left(b_{m_t,t}\right)\right)\right) \\
= \;& N_t \left(\left(b_{1,t}^2 - b_{0,t}^2\right) \hat{P}_{N_t}\left(b_{0,t}\right) + \cdots + \left(b_{m_t,t}^2 - b_{m_t-1,t}^2\right) \hat{P}_{N_t}\left(b_{m_t-1,t}\right) - b_{m_t,t}^2 \hat{P}_{N_t}\left(b_{m_t,t}\right)\right) \\
= \;& N_t \sum_{k=0}^{m_t-1} \left(b_{k+1,t}^2 - b_{k,t}^2\right) \hat{P}_{N_t}\left(b_{k,t}\right) \\
= \;& N_t \sum_{k=0}^{m_t-1} \left(b_{k+1,t} + b_{k,t}\right)\left(b_{k+1,t} - b_{k,t}\right) \hat{P}_{N_t}\left(b_{k,t}\right) \\
= \;& 2N_t \sum_{k=0}^{m_t-1} \left(\frac{b_{k,t} + b_{k+1,t}}{2}\right)\left(b_{k+1,t} - b_{k,t}\right) \hat{P}_{N_t}\left(b_{k,t}\right).
\end{aligned}
$$

Replacing the empirical counter-CDF $\hat{P}_{N_t}\left(b_{k,t}\right)$ with the continuous function in (C.11), we get

$$
\begin{aligned}
\sum_{j=1}^{N_t}\left(d_{j,t}^{in}\right)^2 &= 2N_t\int_{d_{\min}}^{d_{\max}(N_t)} x\left(\frac{x}{d_{\min}}\right)^{1-\xi} dx \\
&= 2N_t\int_{d_{\min}}^{d_{\max}(N_t)} x\frac{d_{\min}}{2-\xi}d\left(\frac{x}{d_{\min}}\right)^{2-\xi} \\
&= 2N_t\frac{d_{\min}}{2-\xi}\left(x\left(\frac{x}{d_{\min}}\right)^{2-\xi}\Bigg|_{d_{\min}}^{d_{\max}(N_t)} - \int_{d_{\min}}^{d_{\max}(N_t)}\left(\frac{x}{d_{\min}}\right)^{2-\xi} dx\right) \\
&= 2N_t\frac{d_{\min}}{2-\xi}\left(x\left(\frac{x}{d_{\min}}\right)^{2-\xi}\Bigg|_{d_{\min}}^{d_{\max}(N_t)} - \frac{d_{\min}}{3-\xi}\left(\frac{x}{d_{\min}}\right)^{3-\xi}\Bigg|_{d_{\min}}^{d_{\max}(N_t)}\right) \\
&= 2N_t\frac{d_{\min}}{2-\xi} \\
&\quad\cdot\left(d_{\max}\left(N_t\right)\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} - d_{\min} - \frac{d_{\min}}{3-\xi}\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{3-\xi} + \frac{d_{\min}}{3-\xi}\right) \\
&= 2N_t\frac{d_{\min}}{2-\xi} \\
&\quad\cdot\left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi}\left(d_{\max}\left(N_t\right) - \frac{1}{3-\xi}d_{\max}\left(N_t\right)\right) - \left(d_{\min} - \frac{d_{\min}}{3-\xi}\right)\right) \\
&= 2N_t\frac{d_{\min}}{2-\xi}\left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi}\frac{2-\xi}{3-\xi}d_{\max}\left(N_t\right) - \frac{2-\xi}{3-\xi}d_{\min}\right).
\end{aligned}
$$

Using the dynamics of aggregate retail sentiment δ_t^R in equation (1.55), we can compute the conditional mean of δ_t^R

$$
\mathbb{E}_{t-1}\left[\delta_t^R\right] = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)}\rho\delta_{t-1}^R
$$

and the conditional variance

$$\begin{aligned}
\mathrm{Var}_{t-1}\left(\delta_t^R\right) &= \left(\theta\left(N_t\right)\right)^2 \frac{1}{N_t^2}\sum_{j=1}^{N_t}\left(d_{j,t}^{in}\right)^2\sigma_\varepsilon^2 \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{1}{N_t}\frac{2d_{\min}}{2-\xi}\sigma_\varepsilon^2\left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi}\frac{2-\xi}{3-\xi}d_{\max}\left(N_t\right)-\frac{2-\xi}{3-\xi}d_{\min}\right) \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{2d_{\min}}{N_t}\frac{1}{3-\xi}\left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi}d_{\max}\left(N_t\right)-d_{\min}\right)\sigma_\varepsilon^2 \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{2d_{\min}^{\xi-1}}{N_t}\frac{1}{3-\xi}\left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi}-d_{\min}^{3-\xi}\right)\sigma_\varepsilon^2 \\
&= O\left(N_t^{\frac{4-2\xi}{\xi-1}}\right).
\end{aligned}$$

The last equality uses $d_{\max}\left(N_t\right) = O\left(N_t^{\frac{1}{\xi-1}}\right)$. Hence, the conditional volatility of aggregate retail sentiment is

$$\sqrt{\mathrm{Var}_{t-1}\left(\delta_t^R\right)} = O\left(N_t^{\frac{2-\xi}{\xi-1}}\right).$$

□

C.1.13 Distribution of time-1 aggregate retail sentiment shock

Define $c_j \equiv \frac{1}{N}d_j^{in}$ and the random variable $X_j = \mu + \varepsilon_1^j$, where $\mu = \delta_0^R$. Let σ^2 denote the pre-truncation variance of ε_1^j, then X_j follows a truncated normal distribution on $[-\bar{\varepsilon}, \bar{\varepsilon}]$ with pre-truncation mean μ and variance σ^2, and X_j is i.i.d. in the cross section. Further define $\rho \equiv \frac{\bar{\varepsilon}}{\sigma}$, $a = \mu - \rho\sigma$, and $b = \mu + \rho\sigma$. Then the PDF of X_j is

$$f_{X_j}\left(x\right) = \frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)} = \frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi\left(\rho\right)-1},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the PDF and CDF of a standard normal random variable, respectively.

The time-1 aggregate retail sentiment shock δ_1^R can be written as

$$\delta_1^R = \sum_{j=1}^{N} c_j X_j.$$

Hence, the characteristic function of δ_1^R is

$$\begin{aligned}
\varphi_{\delta_1^R}(t) &= \varphi_{X_1}(c_1 t)\,\varphi_{X_2}(c_2 t)\cdots\varphi_{X_N}(c_N t) \\
&= \prod_{j=1}^{N} \varphi_{X_j}(c_j t) = \prod_{j=1}^{N} \mathbb{E}\left[e^{itc_j X_j}\right] \\
&= \prod_{j=1}^{N} \left[\int_a^b e^{itc_j x} \frac{1}{\sigma} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1} dx\right].
\end{aligned}$$

Note

$$\begin{aligned}
& \int_a^b e^{itc_j x} \frac{1}{\sigma_j} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1} dx \\
= & \frac{1}{2\Phi(\rho)-1} \int_a^b \frac{1}{\sqrt{2\pi}\sigma} \exp\left(itc_j x - \frac{(x-\mu)^2}{2\sigma^2}\right) dx \\
= & \frac{1}{2\Phi(\rho)-1} \int_a^b \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2 - 2\mu x + \mu^2 - 2itc_j x\sigma^2}{2\sigma^2}\right) dx \\
= & \frac{1}{2\Phi(\rho)-1} \exp\left(\frac{\left(\mu + itc_j\sigma^2\right)^2 - \mu^2}{2\sigma^2}\right) \int_a^b \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(x - \left(\mu + itc_j\sigma^2\right)\right)^2}{2\sigma^2}\right) dx \\
= & \frac{1}{2\Phi(\rho)-1} \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \int_a^b \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(x - \left(\mu + c_j\sigma^2 it\right)\right)^2}{2\sigma^2}\right) dx.
\end{aligned}$$

Define $y \equiv \frac{x-(\mu+c_j\sigma^2 it)}{\sigma} \implies x = \sigma y + \left(\mu + c_j\sigma^2 it\right) \implies dx = \sigma dy$, and note

$\frac{a-(\mu+c_j\sigma^2 it)}{\sigma} = -\rho - c_j\sigma it$, $\frac{b-(\mu+c_j\sigma^2 it)}{\sigma} = \rho - c_j\sigma it$. Then

$$\begin{aligned}
& \int_a^b e^{itc_j x} \frac{1}{\sigma} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1} dx \\
= \; & \frac{1}{2\Phi(\rho)-1} \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \int_a^b \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(x-\left(\mu+c_j\sigma^2 it\right)\right)^2}{2\sigma^2}\right) dx \\
= \; & \frac{1}{2\Phi(\rho)-1} \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \int_{\frac{a-\left(\mu+c_j\sigma^2 it\right)}{\sigma}}^{\frac{b-\left(\mu+c_j\sigma^2 it\right)}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy \\
= \; & \frac{1}{2\Phi(\rho)-1} \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \int_{-\rho-c_j\sigma it}^{\rho-c_j\sigma it} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy \\
= \; & \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \frac{\Phi\left(\rho-c_j\sigma it\right) - \Phi\left(-\rho-c_j\sigma it\right)}{2\Phi(\rho)-1} \\
= \; & \exp\left(c_j\mu it - \frac{1}{2}c_j^2\sigma^2 t^2\right) \frac{\Phi\left(\rho-c_j\sigma it\right) + \Phi\left(\rho+c_j\sigma it\right) - 1}{2\Phi(\rho)-1}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\varphi_{\delta_1^R}(t) &= \prod_{j=1}^N \left[\int_a^b e^{itc_j x} \frac{1}{\sigma} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1} dx\right] \\
&= \exp\left(\left(\sum_{j=1}^N c_j\mu\right) it - \frac{1}{2}\left(\sum_{j=1}^N c_j^2\sigma^2\right) t^2\right) \\
&\quad \cdot \prod_{j=1}^N \frac{\Phi\left(\rho-c_j\sigma it\right) + \Phi\left(\rho+c_j\sigma it\right) - 1}{2\Phi(\rho)-1} \\
&= \exp\left(\mu it - \frac{1}{2}\left(\sum_{j=1}^N c_j^2\right) \sigma^2 t^2\right) \\
&\quad \cdot \prod_{j=1}^N \frac{\Phi\left(\frac{\bar{\varepsilon}}{\sigma}-c_j\sigma it\right) + \Phi\left(\frac{\bar{\varepsilon}}{\sigma}+c_j\sigma it\right) - 1}{2\Phi\left(\frac{\bar{\varepsilon}}{\sigma}\right)-1}.
\end{aligned}$$

Consider another random variable $\tilde{\delta}_1$, which follows a truncated normal distribution on $[\mu-\bar{\varepsilon}, \mu+\bar{\varepsilon}]$ with mean $\sum_{j=1}^N c_j\mu = \mu$ and variance $\sum_{j=1}^N c_j^2\sigma^2$. The characteristic function of $\tilde{\delta}_1$ is

$$\varphi_{\tilde{\delta}_1}(t) = \exp\left(\mu it - \frac{1}{2}\left(\sum_{j=1}^{N} c_j^2\right)\sigma^2 t^2\right) \cdot \frac{\Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N} c_j^2}\sigma} - \sqrt{\sum_{j=1}^{N} c_j^2}\sigma it\right) + \Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N} c_j^2}\sigma} + \sqrt{\sum_{j=1}^{N} c_j^2}\sigma it\right) - 1}{2\Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N} c_j^2}\sigma}\right) - 1}.$$

Comparing the characteristic function of δ_1^R with that of $\tilde{\delta}_1$, we see that the distribution of δ_1^R can be approximated by a truncated normal distribution, if the cross-sectional distribution of c_j is highly right-skewed.

C.2 Reddit data

C.2.1 Variable definitions

I construct two data frames following the steps in Section 1.2.1: one includes all the submissions, and the other includes all the comments.

In the data frame of submissions, each row is a unique submission with the following fields:

- `id`: the unique id of the submission, e.g., eifjq5. I add the prefix t3_ to the submission `id` to facilitate the mapping between the submission and its associated comments.
- `author`: the name of the author of the submission, e.g., Ituglobal.
- `author_fullname`: the unique user id of the author of the submission prefixed by t2_, e.g., t2_6rjw5.
- `created_utc`: the UTC date and time at which the submission was created.

- `title`: the textual content of the title of the submission.
- `selftext`: the textual content of the body text of the submission.

In the data frame of comments, each row is a unique comment with the following fields:

- `id`: the unique id of the comment, e.g., fctzgly. I add the prefix t1_ to the `id` to facilitate the mapping between the comment in question and its parent comment.
- `link_id`: the unique id of the submission that the comment in question replies to, e.g., t3_eiwx9h.
- `parent_id`: the unique id of the parent comment (or submission) of the comment in question. If the comment is a reply to another comment, then it is prefixed by t1_. Otherwise, it is a reply to a submission, and it is prefixed by t3_.
- `created_utc`: the UTC date and time at which the comment was created.
- `author`: the name of the author of the comment, e.g., urfriendosvendo.
- `author_fullname`: the unique user id of the author of the comment prefixed by t2_, e.g., t2_12ol3k.
- `body`: the textual content of the comment.

C.2.2 Constructing the sample of submissions and comments

I first run the following algorithm to tag submissions and comments with stock tickers, and then I select samples of submissions and comments.

1. Retrieve the list of tickers of CRSP common stocks.
2. Search for stock tickers in the text of the submission.[1]

1. For GameStop, I search for both its ticker GME and the company name GameStop.

(a) First pass search: search for CRSP stock tickers in the augmented body text.[2]

i. Preprocess the augmented body text in the following order:

- Replace ‘’ / - with space.
- Replace & with space if it appears between words.
- Replace . with space.
- Remove all other punctuation marks.
- Tokenize augmented body text and only keep non-empty tokens.

ii. Search for CRSP stock tickers in the augmented body text in a case-insensitive way. A submission is tagged with a ticker if the ticker can be found in the list of tokens.

(b) Manually go over the matched tickers, add $ sign in front of those tickers that are common words, and use this updated list of tickers in the second pass search.

(c) Second pass search: repeat the procedures in the first pass search, but using the updated list of tickers from the previous step.

3. Drop submissions where `author_fullname` is empty, or [deleted], or [removed]. I also drop those where `id` is empty, or [deleted], or [removed].

4. Drop submissions where `author` is one of the bots in Table C.1.

5. Only keep submissions tagged with at least one CRSP common stock ticker and only keep the comments associated with these selected submissions (see Appendix C.2.3 below for the procedure of matching submissions with comments).

   If a submission is tagged with a ticker, the associated comments are also tagged with the same ticker. A submission or comment can be tagged with multiple stock tickers.

2. A submission has its title and body text. I obtain the augmented body text by appending the body text to the title, separated by a white space.

Finally, I construct the following two samples of submissions and comments:

- Sample of submissions and comments for CRSP common stocks by performing steps 1-5 above.
- Sample of all submissions and comments by performing steps 1-4 above.

For each of the sample, I keep one data frame for submissions and another data frame for comments with the structure described in Appendix C.2.1. I construct the network using these two data frames.

C.2.3 Constructing the network

As is described in Appendix C.2.1, the submission data frame and the comment data frame have a common field: the field `id` in the submission data frame corresponds to the `link_id` in the comment data frame. This allows me to recover the comment tree described in Section 1.2.1.

For each of the sample described in Appendix C.2.2, I merge the submission data frame and comment data frame by the common field described above, and I only keep submissions with at least one comment. In the merged dataset, each row corresponds to a comment with information on (1) the author of the comment and (2) the author of the submission that the comment replies to. This allows me to construct the network of users from the commenting relationship.

C.3 FactSet data

I follow the procedure in Gabaix and Koijen (2022) and Koijen et al. (2022):

1. Merge the holdings data (`[own_v5].[own_inst_eq_v5].[own_inst_13f_detail]`) with the entity sub type data

([own_v5].[own_hub_ent_v5].[own_ent_institutions]),

by factset_entity_id.

Each record in this merged dataset corresponds to a filer entity (with unique id factset_entity_id).

2. For those filer entities with missing entity sub type (from the previous step), find the corresponding roll-up entity
(from [own_v5].[own_hub_ent_v5].[own_ent_13f_combined_inst]) and assign the sub type of the roll-up entity to the filer entity.

    - To identify the sub type of the roll-up entity, merge the roll-up entity data
([own_v5].[own_hub_ent_v5].[own_ent_13f_combined_inst])
with the entity sub type data
([own_v5].[own_hub_ent_v5].[own_ent_institutions]),
by factset_rollup_entity_id in the former (factset_entity_id in the latter).
$\Longrightarrow$ 12,276 out of the 12,295 roll-up entities have non-missing entity sub type.

3. Classify institutions into five types using entity_sub_type:

    - Hedge Funds: entity_sub_type = AR, FH, FF, FU, FS, HF.
    - Brokers: entity_sub_type = BM, IB, ST, MM, BR.
    - Private Banking: entity_sub_type = CP, FY, VC, PB.
    - Investment Advisors: entity_sub_type = IC, RE, PP, SB, MF, IA.
    - Long-Term Investors: entity_sub_type = FO, SV, IN, PF.

C.4 Modified BJZZ algorithm to identify retail trades

I follow Boehmer et al. (2021) and Barber et al. (2022) to identify retail buy trades and sell trades:

1. Start with any trade with price not at the midpoint of the bid and the ask.
2. Match the NBBO to the timestamp of the trade and then compute bid-ask spread quoted before the trade.
3. If the spread quoted before the trade is one cent, use the original BJZZ algorithm to sign the trade.
4. If the trade price is outside the bid-ask spread, use the original BJZZ algorithm to sign the trade.
5. Otherwise, if the trade is below the midpoint, label the trade as a sell. If the trade is above the midpoint, label the trade as a buy. I also implement the $[0.4, 0.6]$ "donut" as in the original BJZZ algorithm.

C.5 Fitting power-law distribution

For each calendar day t, I fit a power-law distribution to the vector of user influence $\left(d_{1,t}^{in}, d_{2,t}^{in}, \cdots, d_{N_t,t}^{in}\right)^\intercal$ computed in Section 1.2.1 and estimate the exponent $\hat{\xi}_t$ and the threshold value $\hat{d}_{\min,t}^{in}$. Following Rantala (2019), I use the maximum likelihood method to estimate these parameters. Specifically, I use the power.law.fit function with the plfit implementation in R igraph package.

I use the bootstrap methods to compute the confidence intervals:

1. Generate a bootstrap sample $\left\{d_{k,t}^{in}(b)\right\}_{k=1}^{N_t}$ by sampling the original data $\left(d_{1,t}^{in}, d_{2,t}^{in}, \cdots, d_{N_t,t}^{in}\right)^\intercal$ randomly with replacement.
2. Estimate the parameters $\xi_t(b)$ and $d_{\min,t}(b)$ for this bootstrapped sample using the maximum likelihood method described above.
3. Repeat steps 1 and 2 for $B = 5000$ times and obtain the vector of estimates $\{\xi_t(b)\}_{b=1}^{B}$ and $\left\{d_{\min,t}(b)\right\}_{b=1}^{B}$.

4. For the $\hat{\xi}_t$ estimate, the lower (upper) bound of the 95% confidence interval is the 2.5th (97.5th) percentile of the empirical distribution $\{\xi_t(b)\}_{b=1}^{B}$. Similarly, for the $\hat{d}_{min,t}$ estimate, the lower (upper) bound of the 95% confidence interval is the 2.5th (97.5th) percentile of the empirical distribution $\left\{d_{\min,t}(b)\right\}_{b=1}^{B}$.

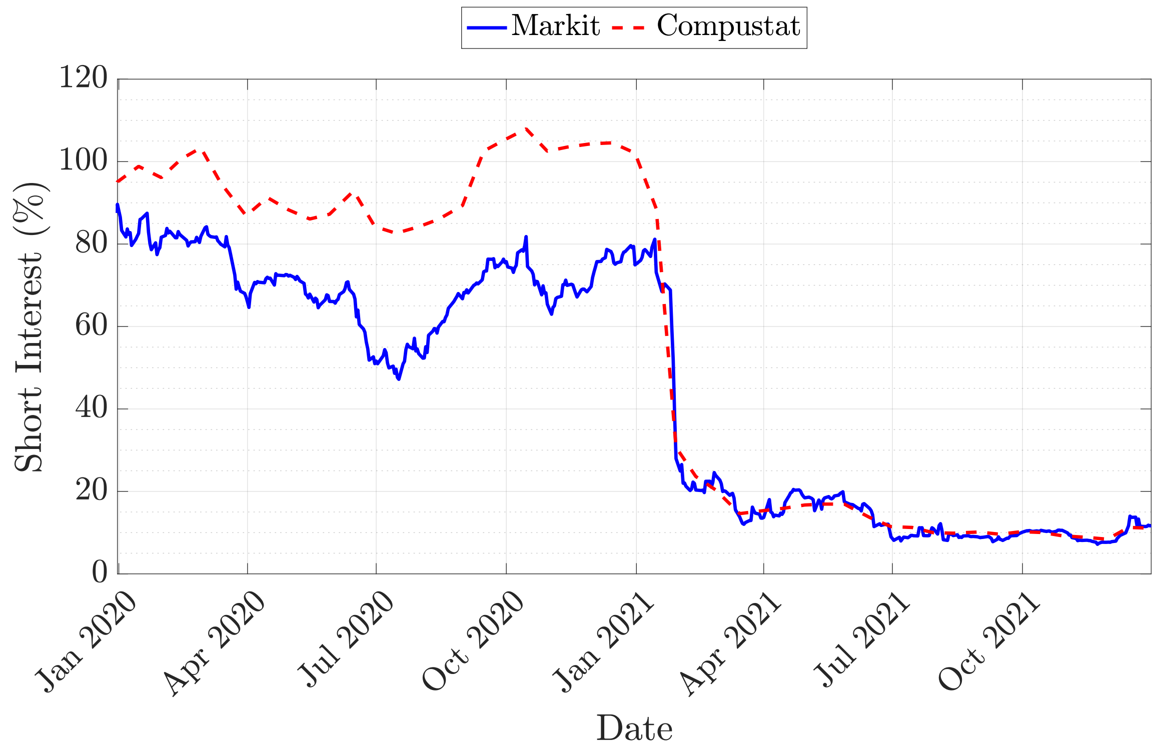


Figure C.1. Short interest of GameStop from IHS Markit versus Compustat. This figure compares the short interest of GameStop computed using IHS Markit data versus that using Compustat data, for the period from January 1, 2020 to December 31, 2021. Short interest is defined as the ratio of the number of shares sold short to the number of shares outstanding (equation (1.6)). The solid blue line is the short interest computed using daily data on the number of shares sold short from IHS Markit. The dashed red line is the short interest computed using the mid-month and month-end number of shares sold short from Compustat.

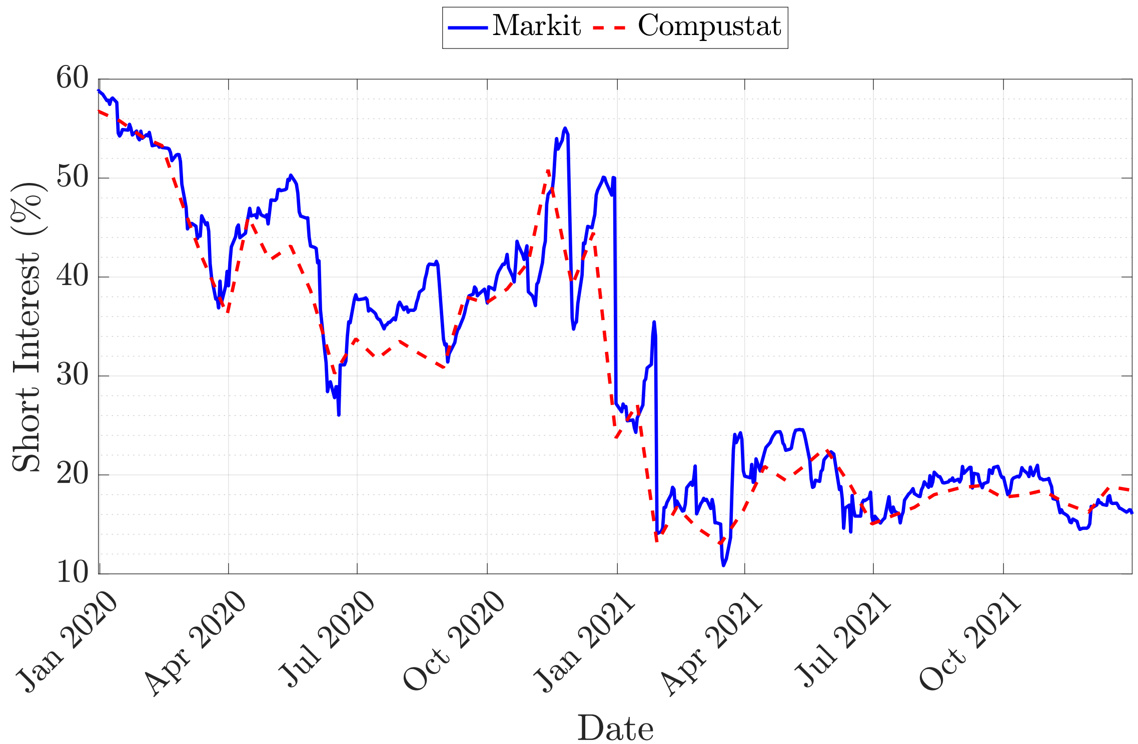


Figure C.2. Short interest of AMC from IHS Markit versus Compustat. This figure compares the short interest of AMC computed using IHS Markit data versus that using Compustat data, for the period from January 1, 2020 to December 31, 2021. Short interest is defined as the ratio of the number of shares sold short to the number of shares outstanding (equation (1.6)). The solid blue line is the short interest computed using daily data on the number of shares sold short from IHS Markit. The dashed red line is the short interest computed using the mid-month and month-end number of shares sold short from Compustat.

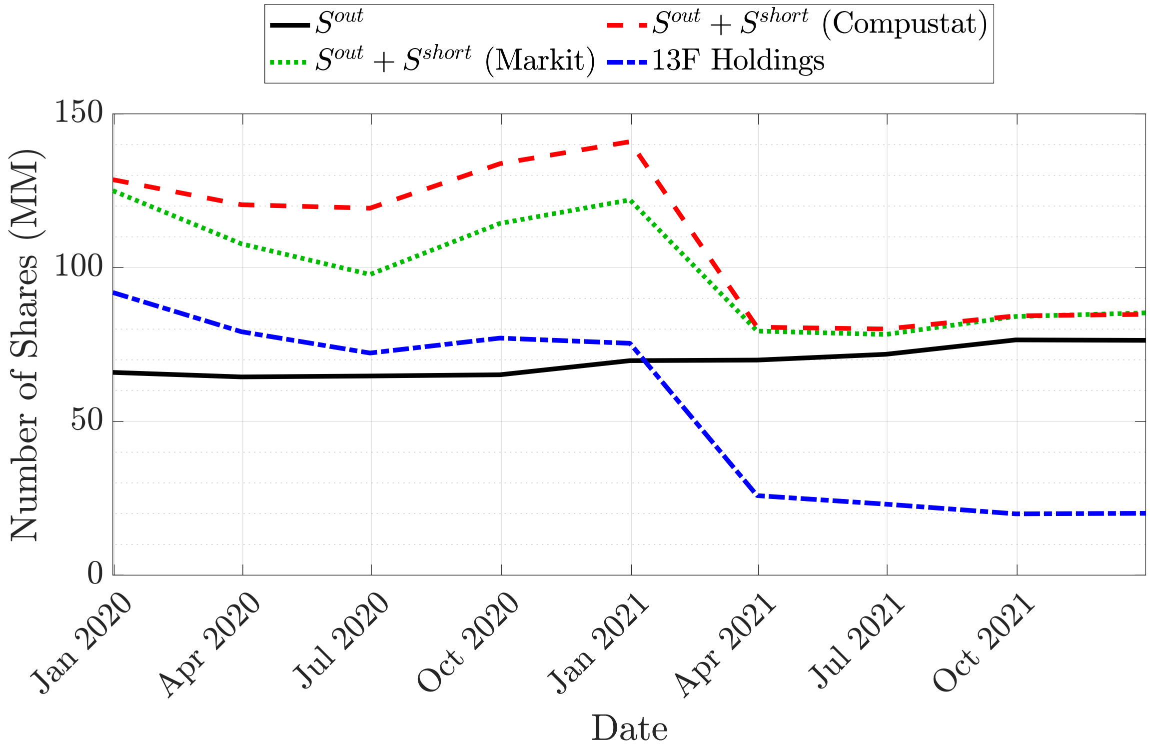


Figure C.3. Shares outstanding and 13F institutional ownership of GameStop. This figure compares the number of shares outstanding of GameStop with 13F institutional ownership of GameStop, for the period from January 1, 2020 to December 31, 2021. The solid black line is the number of shares outstanding. The dashed red line is the number of shares outstanding plus the number of shares sold short from Compustat. The dotted green line is the number of shares outstanding plus the number of shares sold short from IHS Markit. The dash-dotted blue line is the number of shares held by 13F institutions.

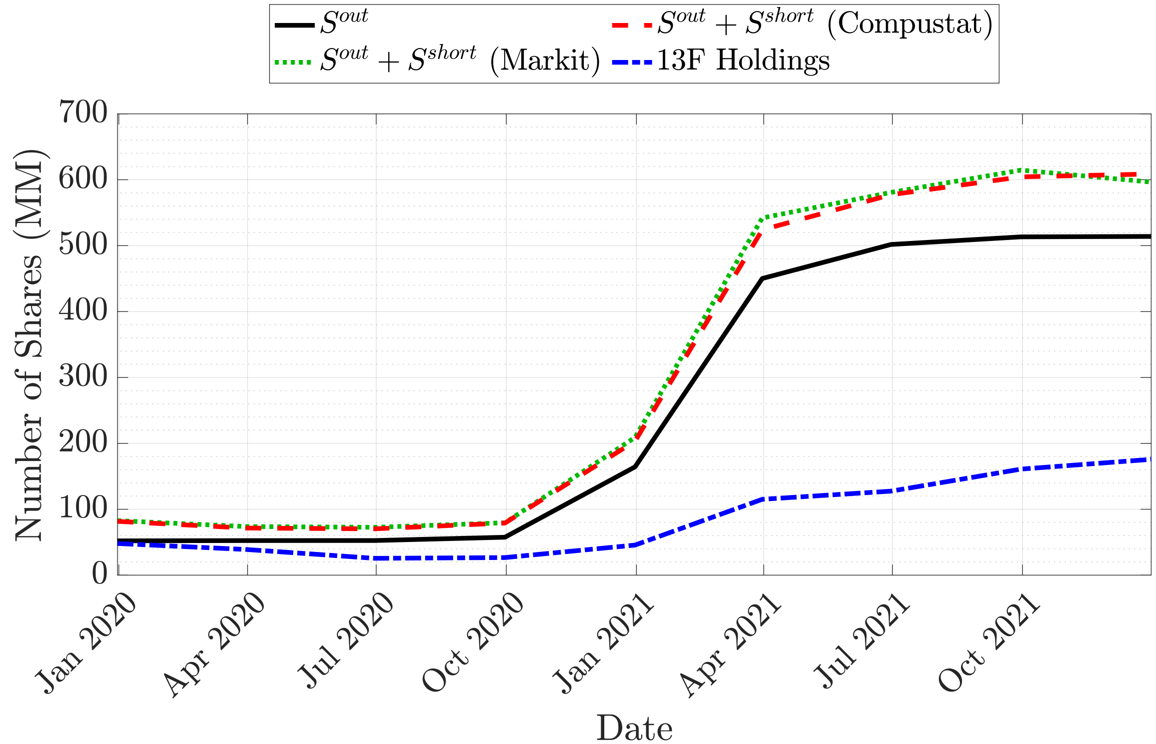


Figure C.4. Shares outstanding and 13F institutional ownership of AMC. This figure compares the number of shares outstanding of AMC with 13F institutional ownership of AMC, for the period from January 1, 2020 to December 31, 2021. The solid black line is the number of shares outstanding. The dashed red line is the number of shares outstanding plus the number of shares sold short from Compustat. The dotted green line is the number of shares outstanding plus the number of shares sold short from IHS Markit. The dash-dotted blue line is the number of shares held by 13F institutions.

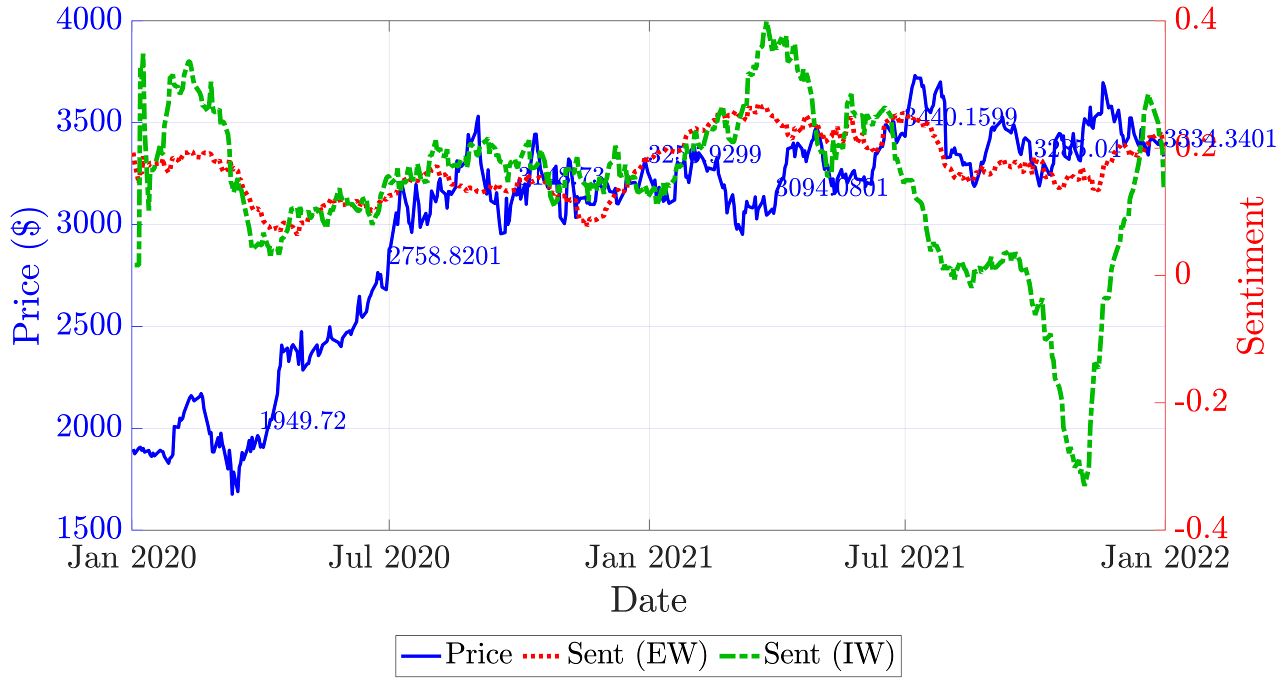


Figure C.5. Price and sentiment of Amazon. This figure shows the daily close price (left y-axis) and the daily WSB sentiment measures (right y-axis) of Amazon, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price, the dotted red line plots the equal-weighted sentiment defined in equation (1.4), and the dash-dotted green line plots the influence-weighted sentiment defined in equation (1.5). The sentiment series are 30-day moving averages.

Figure C.6. Price and sentiment of Microsoft. This figure shows the daily close price (left y-axis) and the daily WSB sentiment measures (right y-axis) of Microsoft, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price, the dotted red line plots the equal-weighted sentiment defined in equation (1.4), and the dash-dotted green line plots the influence-weighted sentiment defined in equation (1.5). The sentiment series are 30-day moving averages.

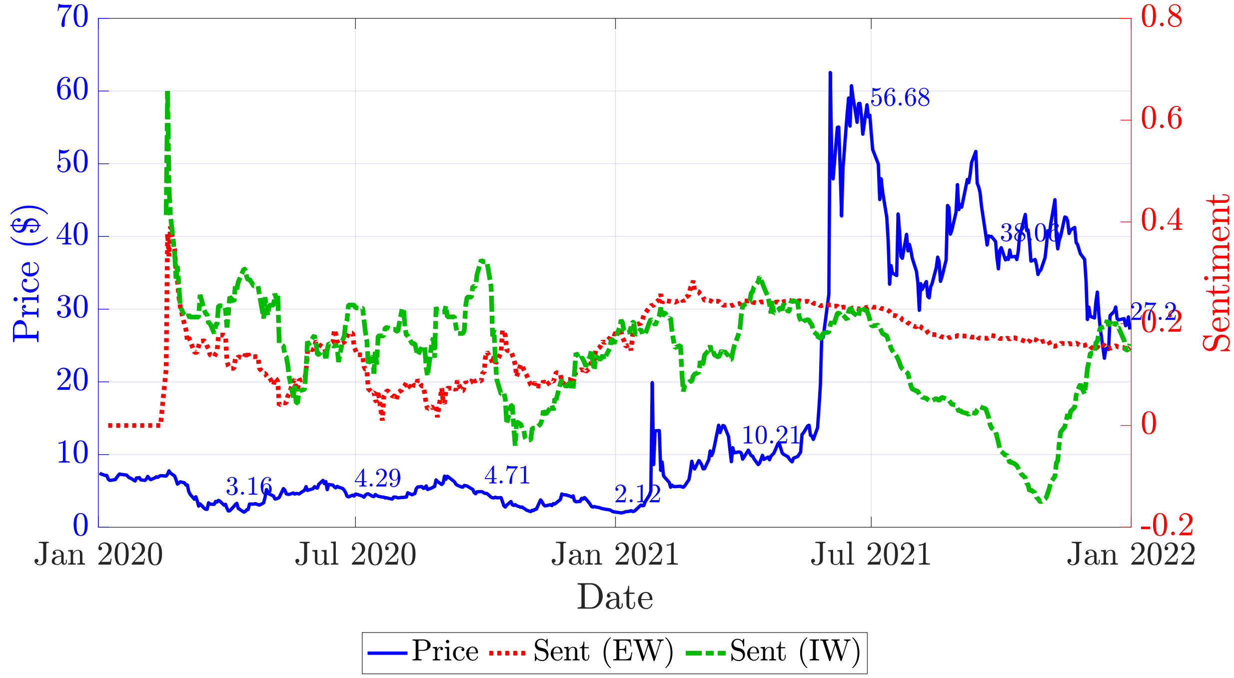


Figure C.7. Price and sentiment of AMC. This figure shows the daily close price (left y-axis) and the daily WSB sentiment measures (right y-axis) of AMC, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price, the dotted red line plots the equal-weighted sentiment defined in equation (1.4), and the dash-dotted green line plots the influence-weighted sentiment defined in equation (1.5). The sentiment series are 30-day moving averages.

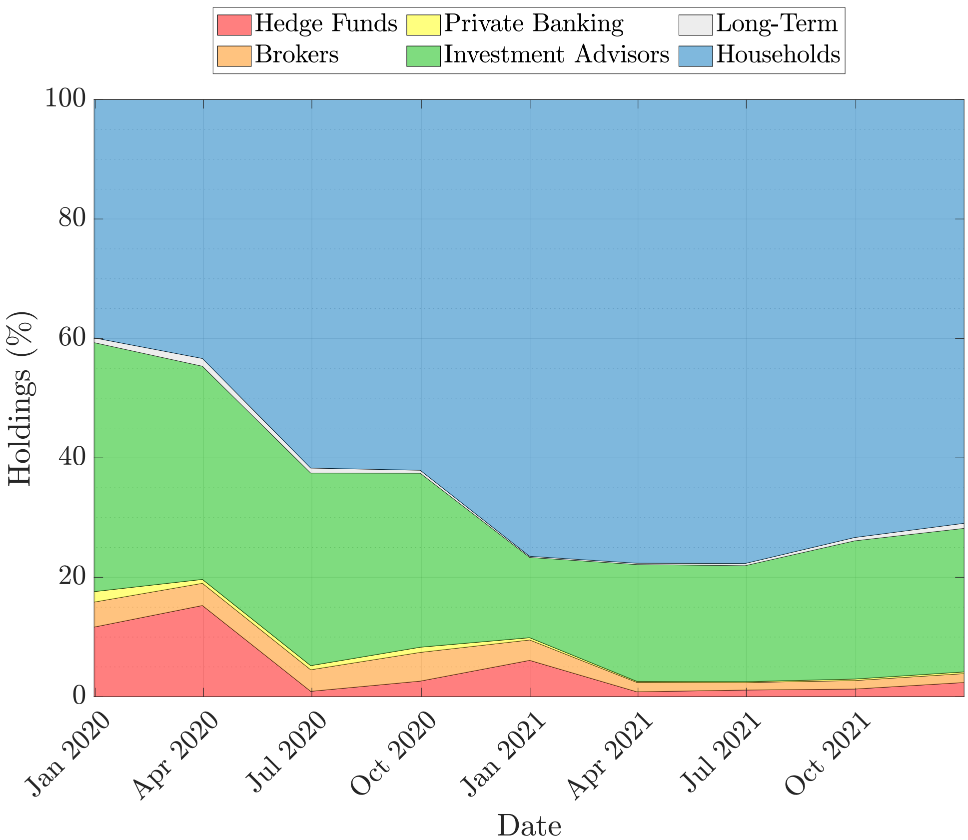
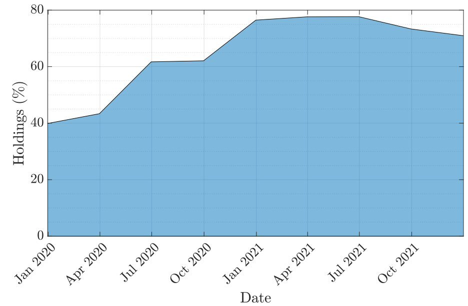
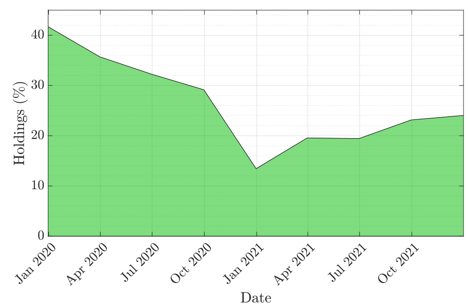


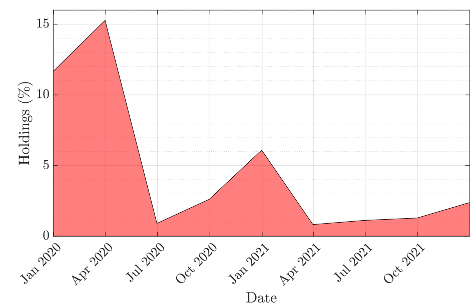
Figure C.8. Ownership of AMC by investor type. This figure plots the end-of-quarter holdings of AMC by 13F institutions and households, for the period from 2019 Q4 to 2021 Q4. 13F holdings data are from FactSet. I aggregate 13F institutional holdings to investor-type level using the method in Appendix C.3. The five institutional investor types are: Hedge Funds (red area), Brokers (orange aread), Private Banking (yellow area), Investment Advisors (green area), and Long-Term Investors (gray area). I calculate household holdings from equation (1.8) using data on the number of shares sold short from Compustat. The blue area represents households. The y-axis is the percentage holdings defined in equation (1.10), which is the number of shares held by each type of investor divided by the sum of the number of shares outstanding and the number of shares sold short.



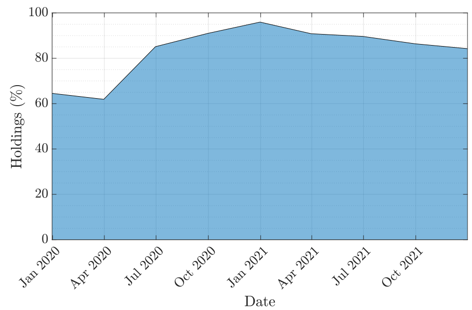
(a) Households / $(S^{out} + S^{short})$



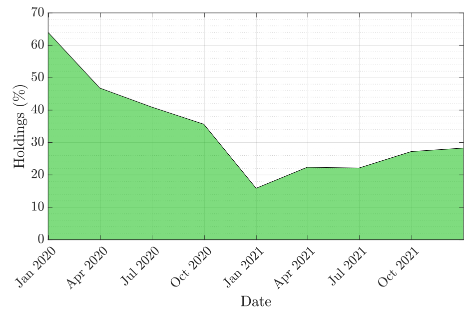
(b) Investment Advisors / $(S^{out} + S^{short})$



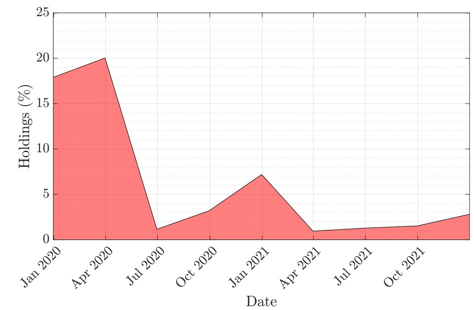
(c) Hedge Funds / $(S^{out} + S^{short})$



(d) Households / S^{out}



(e) Investment Advisors / S^{out}



(f) Hedge Funds / S^{out}

Figure C.9. Ownership of AMC by Households, Investment Advisors, and Hedge Funds. This figure plots the end-of-quarter holdings of AMC by Households (panels (a) and (d)), Investment Advisors (panels (b) and (e)), and Hedge Funds (panels (c) and (f)), for the period from 2019 Q4 to 2021 Q4. 13F institutional investors are classified into Investment Advisors and Hedge Funds according to Appendix C.3, and the 13F holdings data are from FactSet. Household holdings are calculated from equation (1.8). In panels (a), (b), and (c), the y-axis is the number of shares held by the investor group, divided by the sum of the number of shares outstanding and the number of shares sold short (equation (1.10)). Data on the number of shares sold short is from Compustat. In panels (d), (e), and (f), the y-axis is the number of shares held by the investor group, divided by the number of shares outstanding (equation (1.9)).

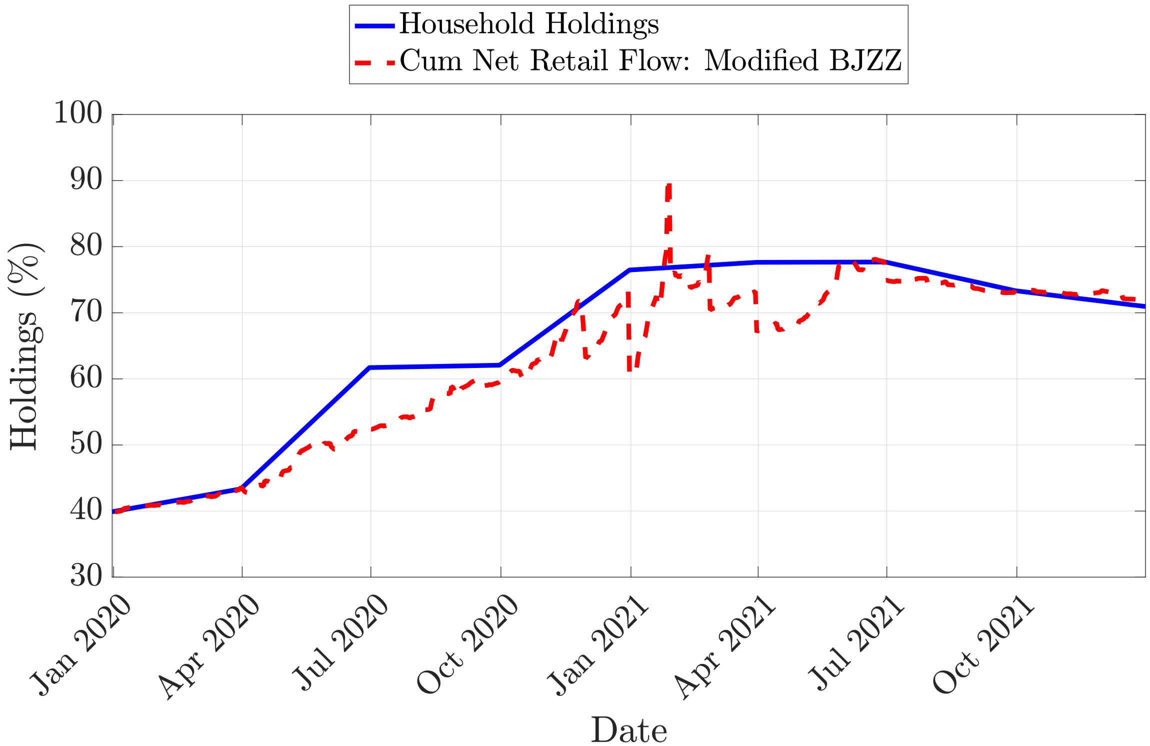


Figure C.10. Ownership by households versus cumulative net retail flow of AMC. This figure plots the end-of-quarter percentage holdings of AMC by Households (solid blue line) and the daily cumulative net retail flow (dashed red line), for the period from January 1, 2020 to December 31, 2021. Percentage holdings by households is defined in equation (1.10), which is the number of shares held by households (equation (1.8)) divided by the sum of the number of shares outstanding and the number of shares sold short. Cumulative net retail flow is defined in equation (1.12), which is the cumulative net retail buy volume (equation (1.11)) divided by the sum of the number of shares outstanding and the number of shares sold short. Data on the number of shares sold short is from Compustat. The initial value of the cumulative net retail flow (on Dec 31, 2019) is set to be the percentage holdings by households at the end of 2019 Q4. I apply the modified BJZZ algorithm in Appendix C.4 to identify retail trades from the TAQ data.

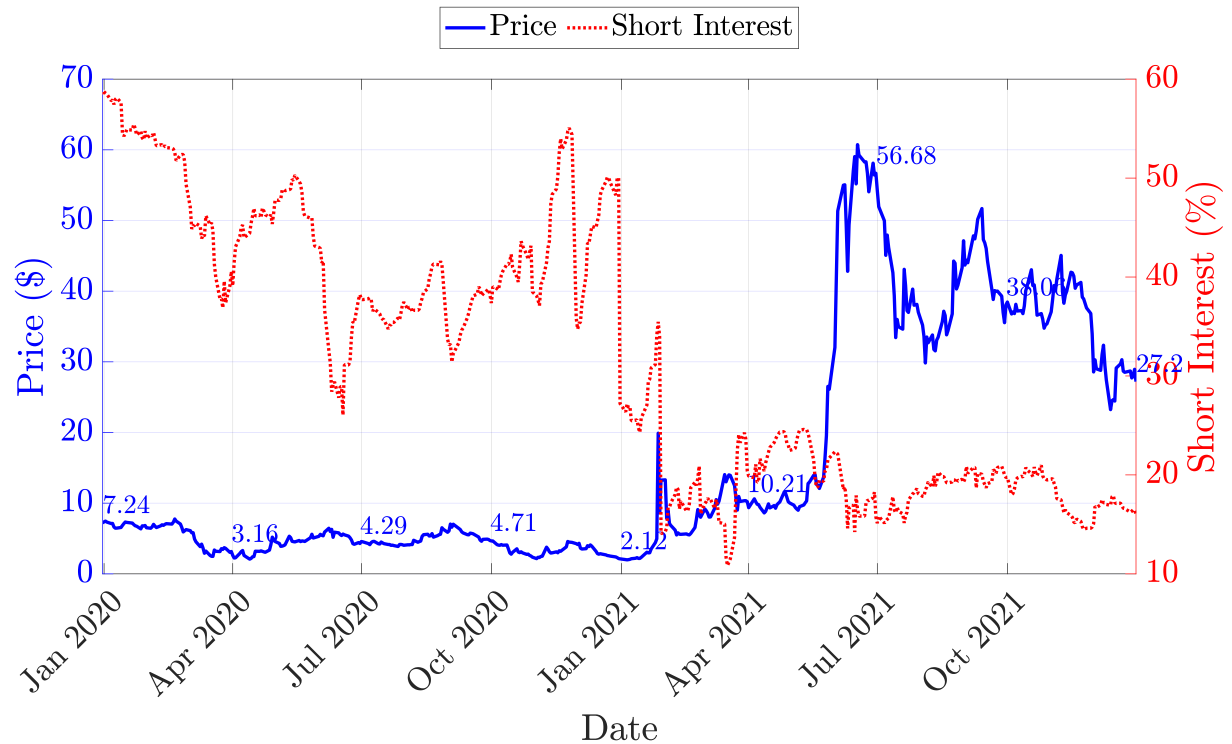


Figure C.11. Price and short interest of AMC. This figure shows the daily close price (left y-axis) and the daily short interest (right y-axis) of AMC, for the period from January 1, 2020 to December 31, 2021. The solid blue line plots the close price. The dotted red line plots the short interest, which is defined as the ratio of the number of shares sold short to the number of shares outstanding (equation (1.6)). Data on the number of shares sold short is from IHS Markit.

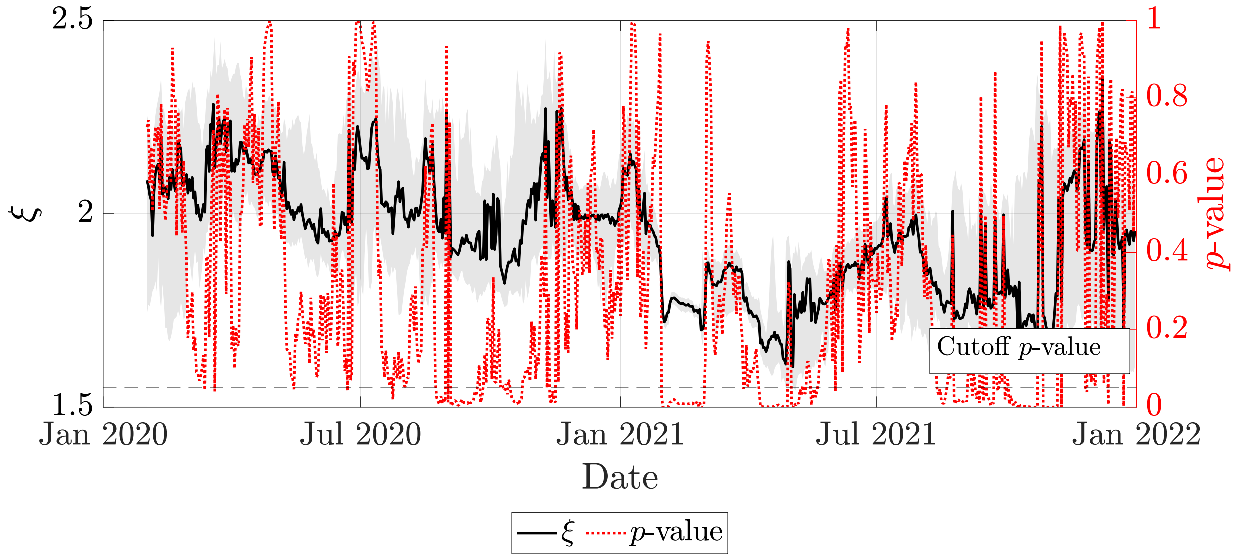


Figure C.12. ***p*-value for fitting the power-law distribution.** This figure plots the daily estimate of the power-law exponent $\hat{\xi}_t$ (left y-axis) and the p-value (right y-axis) of the Kolmogorov-Smirnov test, for the period from January 1, 2020 to December 31, 2021. On each day t, I fit a power-law distribution to the vector of user influence (defined in equation (1.3)) and estimate the exponent ξ in equation (1.13). The solid black line plots the $\hat{\xi}_t$ estimates from the maximum likelihood method as in Rantala (2019). The gray area shows the 95% confidence interval for the estimates computed from the bootstrap method in Appendix C.5. The dotted red line plots the p-value of the Kolmogorov-Smirnov test. The cutoff p-value is 0.05 (dashed horizontal line). Small p-values (less than 0.05) indicate that the test rejected the hypothesis that the original data could have been drawn from the fitted power-law distribution.

Table C.1
Reddit bots removed from the sample

This table shows the Reddit bots whose submissions are removed from the sample.

Bot Name
WSBVoteBot
RemindMeBot
Generic_Reddit_Bot
ReverseCaptioningBot
LimbRetrieval-Bot
NoGoogleAMPBot
RepostSleuthBot
GetVideoBot
CouldWouldShouldBot

Table C.2
Time-0 equilibrium outcomes under different risk perceptions

This table compares the time-0 equilibrium outcomes when changing investors' time-0 perceptions of risk. Column 3 shows the equilibrium outcomes when all investors believe the size of the network at time 1 will remain the same as that at time 0, i.e., $\tilde{N}_1 = N_L = N_0$. Column 4 shows the equilibrium outcomes when all investors believe the the size of the network will grow (deterministically) from time 0 to time 1, i.e., $\tilde{N}_1 = N_H > N_L = N_0$. The parameter values are given in Table 1.2.

		Value	
Description	Notation	$\tilde{N}_1 = N_L$	$\tilde{N}_1 = N_H$
(1)	(2)	(3)	(4)
Log price	p_0	4.249	4.612
	w_0^R	1.900	1.024
Portfolio weights	w_0^{IL}	1.759	1.288
	w_0^{IS}	−0.250	0.539
	Q_0^R	60	34
Num. shares held	Q_0^{IL}	50	52
	Q_0^{IS}	−10	14
	α_0^R	0.316	0.329
Wealth shares	α_0^{IL}	0.284	0.403
	α_0^{IS}	0.400	0.269
Expected log payoff	$\mathbb{E}_0[p_1]$	4.469	5.157
Variance of log return	σ_0^2	0.378	1.015

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