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Harder Course, Higher Score?

Exploring the Heterogeneous Effects of Eighth-grade Algebra Enrollment on Average Math Achievement

By

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Abstract

Many empirical studies have provided evidence both supporting and opposing the practice of detracking. While some researchers have shown that high-achieving students from disadvantaged backgrounds can be disadvantaged by detracking, there has been limited research on how this within-school revolution may impact the achievement gap between schools. To address this gap, this study applies a propensity score-based method to hierarchical models and utilizes data from the Early Childhood Longitudinal Study—Kindergarten class of 1998–1999 (ECLS-K) in order to explore the causal inference of expanding algebra access. More specifically, it examines the average and heterogeneous effects of enrolling more eighth-grade students in algebra on school average achievement. The study finds, in conclusion, that enrolling all students in algebra has insignificant effect on school average test scores due to the heterogeneity of the effect. Only high-achieving schools benefit from this comprehensive detracking process, resulting in an increase in the achievement gap between schools. However, for schools that do not offer algebra, enrolling some students in algebra increases their average achievement without significant differences in the effects among different schools. These findings provide insights into how course offering policies should be implemented among schools.

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Introduction

Scholars have long debated the reproduction of inequality within the education system. There are diverging opinions regarding the extent to which inequality is reproduced through the separation of students into different schools (i.e., school segregation) (Massey & Denton, 1993; Reardon, 2016), as opposed to differentiating students into different courses within schools through tracking (Gamoran, 1987; Oakes, 2005; Bowles & Gintis, 2011). These two viewpoints have led to efforts for desegregation and detracking in schools, with the aim of achieving educational equity among students of different racial backgrounds and socioeconomic statuses. In this paper, I focus on the impact of detracking on schools with varying average math achievement and explore how educational reforms within schools could affect the achievement gap between them.

Students are not provided with equal opportunities vis-à-vis tracking in the education system. In the 20th century, students were often assigned to an academic track, vocational track, or a general track according to their family background (Bowles & Gintis, 2011). By placing students in completely separate tracks, students were taught disparate skills and had different identities imposed on them: vocational tracks for obedient workers, general tracks for housewives, and academic tracks for elites. This unequal tracking mechanism has been replaced by enabling all students the opportunity to enroll in the university system.

For clarification, in this paper, tracking is defined as the practice of placing students in different classes based on their tested ability, rather than tracking them for different occupations as was done in the 20th century. Detracking, on the other hand, is defined as teaching students with heterogeneous skills in one classroom. Specifically, enrolling a fraction of eighth-grade students in algebra is considered a tracking practice, while enrolling all eighth-grade students in algebra is considered a detracking practice.

The legacy of the previous tracking mechanism, however, has not been entirely dismantled, as its unequal structure is still present in tracked schools today. Although the current intention of tracking is to provide tailored instruction based on students' skills, the reality paints a more complex picture: White and Asian American students are more frequently placed in higher tracks and receive better preparation for further education, while Black and Hispanic students are overrepresented in lower tracks and often face challenges in competitiveness after graduation. Considering the correlation between race and class, many scholars argue that tracking perpetuates existing social inequalities (Gamoran, 2010; Oakes, 2005).

Given the disproportionate harm that tracking may have on students from less-privileged backgrounds, many scholars view detracking as a potential solution (Gamoran, 2010; Oakes, 2005). In line with the potential benefits of detracking, numerous states have implemented policies aimed at expanding rigorous courses to more students, providing equal learning opportunities, and reducing the courses previously considered lower tracks. (Nomi, 2012; Domina et al., 2016). However, other scholars argue that placing students with diverse skill levels in the same course is not a panacea for educational inequality (Rosenbaum, 1999; Rubin, 2008; Nomi, 2012).

This paper focuses on the impact of expanding algebra to all eighth-grade students. Specifically, it explores the average effect and heterogenous effect across schools on math score of (a) expanding algebra enrollment to all eighth-graders versus enrolling no student in algebra, (b) expanding algebra enrollment to all eighth-graders versus enrolling a fraction of eighth-graders in algebra, and (c) expanding algebra enrollment to a fraction of eighth-graders versus enrolling no student in algebra. While algebra is considered an advanced math course for eighth-graders, this research argues that schools are not equally benefited from enrolling all eighth-grade students in algebra. Schools with varied average achievement experience this process differently and students in the most disadvantaged schools are handicapped in this process. However, enrolling some students in algebra does not enlarge between-school achievement gap. Rather, making algebra available but optional to eighth-grade students is helpful in improving average achievement within a school.

Theoretical Background

1. Reproduction of Inequality Within School by Tracking

As Bowles and Gintis suggest in their seminal work, tracking system in high school reproduced the hierarchy in the labor market by assigning students to specific tracks (Bowles & Gintis, 2011). Although the obsolete universal tracking systems may be currently dismantled and all the students in high school can apply to university theoretically, Oakes argues that the reproduction mechanism still exists yet is veiled by meritocracy: the students who are not performing well in school are conceptualized as "not working hard" and placed in lower-tracks, yet they are also experiencing less instruction time and less qualified teaching (Oakes, 2005). Moreover, the low-achieving students are disproportionately from low-SES families and Black and Hispanic students (Lewis & Diamond, 2015). Since nominal segregation between different tracks no longer exists, the current tracking system fails the disadvantaged students more imperceptibly.

Not only do different tracks differ in the quality of instruction, but also indicate different possibilities of getting into college (Lucas, 2001). By combining previous tracking research and educational transitions research, Lucas formulates the effectively maintained inequality theory to explain how tracking functions for students from advantaged backgrounds to maintain educational privilege in high school even though a high school degree has become universal (Lucas, 2001). This theory demonstrates that when a level of schooling is universally attained, the differences in course taking contribute to disparate educational transition path. By examining students' family backgrounds in four separate tracks, Lucas illustrates that the proportion of students from high-income families is disproportionately higher in the college prep track, and thus, they are also more likely to get into college and get a more competitive degree.

Tracking in high school reflects the achievement gaps and disparities in course selection among students at earlier ages (Tyson, 2011), and tracking in mathematics is particularly significant in terms of differentiating students and perpetuating inequality (Schmidt, 2009). Compared to eighth-grade general math and pre-algebra, algebra covers significantly different content. Due to the hierarchical nature of mathematics, the math course taken by students in eighth-grade plays a crucial role in determining their eligibility for more advanced 9th-grade math (Stevenson et al., 1994; Schmidt, 2009), and thus, deciding how far they can go in math before the end of high school, which affects college admissions (Spielhagen, 2006). Therefore, the eighth-grade math course predicts students' math development in high school and even educational trajectory to some extent.

While early exposure to algebra offers many benefits, there exists a racial gap in terms of the proportion of students taking algebra in eighth-grade. Researchers using data from the Early Childhood Longitudinal Study—Kindergarten class of 1998–1999 (ECLS-K) demonstrate that 37% of White students enrolled in algebra in eighth-grade, while only 34% of Hispanic students and 17% of Black students did so (Walston & McCarroll, 2010). Furthermore, the racial gap in course selection persists even when controlling for previous achievement and prerequisite courses(Kelly, 2009; Faulkner et al., 2014). These studies reveal a pattern of inequality in schools regarding the placement of students in different math courses during middle school.

2. Detracking as a Potential Approach to Improve Educational Equity

Given the problems associated with tracking, detracking is widely suggested by many scholars. For example, Gamoran and Hannigan propose that providing algebra to all students can benefit students with varied previous math skills, although the benefit for students with lower skills may be minimal (Gamoran & Hannigan, 2000). During a time when not everyone has access to algebra, expanding the course offering appears to be a promising suggestion.

In addition to offering more advanced courses to all students, some scholars also provide optimistic evidence regarding placing students with diverse skill levels in the same classroom. Burris et al. examine the impact on math scores after implementing heterogeneous grouping in a high school in New York (Burris et al., 2006). The results indicate that bottom-performing students show improvement after having an accelerated mathematics curriculum, without negatively affecting the performance of top-performing students. However, it is important to note that the study also involved providing mathematics workshops to meet the different needs of various student groups. Additionally, the accelerated mathematics curriculum in this particular school was well-designed and of high quality.

It is important to note that the success of detracking is not guaranteed and there are certain conditions that need to be met. Gamoran highlights three requirements for successful mixed-ability settings: 1) teachers must be able to effectively respond to the diverse range of skills among students, 2) teachers must be able to adapt their instruction to meet the needs of different students, and 3) teachers require adequate resources and support to enhance their instruction and cater to students' needs (Gamoran, 2010). However, these requirements may vary in terms of their demands among different types of high schools.

The stringent conditions for success in detracking make it challenging to fully achieve the goal of educational equity that many scholars aspire to. While detracking has the potential to address inequality and promote inclusivity, it requires significant effort and resources to ensure that all students receive the support and tailored instruction they need. The varying levels of resources and capacities among different schools can pose challenges to the successful implementation of detracking and may limit its ability to meet the expectations of achieving educational equity.

3. The Reality of Detracking Practice

As indicated by the demanding requirements for successful detracking, instructing students with diverse skill levels in a single classroom can be challenging for teachers and may inadvertently harm certain students (Rosenbaum, 1999). Research by Nomi suggests that the performance of high-achieving students from modest backgrounds was unintentionally affected negatively after the implementation of Algebra-for-All programs, which aimed to benefit low-achieving students (Nomi, 2012). Schools in Chicago opted to create mixed-ability classrooms after eliminating remedial math classes in response to the policy. However, they did not provide additional instruction to struggling students. Consequently, teachers responded by reducing the curriculum's level of difficulty for high-achieving students.

Furthermore, evidence from a public high school in California also questions the effectiveness of detracking. The study articulates five distinct dimensions of the cross-classroom tracking system within the school and concludes that tracking has only a modest impact on overall student performance. However, it finds that students perform better in highly exclusive tracking systems, where instruction can be more tailored to their individual skills (Domina et al., 2019). Despite arguments from some scholars that heterogeneous classes can benefit all students more, the reality is that teachers' instruction may not be sensitive enough to effectively address the diverse range of student skills and achieve the ideal situation.

Indeed, it is important to consider the specific context and conditions in which detracking is implemented. The study conducted by Burris et al. highlighting the benefits of detracking was conducted in a high school with small class sizes and predominantly middle-class students(Nomi et al., 2021). Additionally, this high school provided math workshops to support students who were struggling in regular classes. However, meeting such requirements may be challenging for schools with more disadvantaged students, making it difficult to replicate the same positive outcomes. Therefore, the impact of undifferentiated detracking practices may disproportionately disadvantage students in these schools.

The effects of detracking can vary significantly across schools with different socioeconomic backgrounds. Rubin's research examines detracking practices in three different high schools: one serving low-income and predominantly Black and Latino students, one serving high-income and predominantly White students, and one serving a socioeconomically diverse population of predominantly Black and White students. (Rubin, 2008) The results reveal unequal benefits for students in these schools. In the school with the most disadvantaged students, where low expectations were held for all students, the curriculum was adjusted to accommodate the skills of all students, resulting in a less challenging environment for top students and compromising their achievements. In contrast, the high-income school provided college preparatory learning for all students after detracking, enabling more students to benefit from the change.

Furthermore, schools adopt various strategies to address the pressure of expanding algebra offerings. Under the algebra-for-all policy in California, disadvantaged schools detracking by enrolling all students in algebra courses, while advantaged schools implemented "tracking up" by creating more advanced geometry opportunities for students. This strategy resulted in disadvantaged students being less likely to enroll in the most advanced math class in advantaged schools, effectively perpetuating inequality (Domina et al., 2016). When combined with the research by Rubin and Rosenbaum, it suggests that expanding eighth-grade algebra enrollment may be implemented with varying levels of quality and can be harmful to disadvantaged students.

Study Aims

While there have been numerous studies regarding the impact of detracking, specifically requiring all eighth-graders to take algebra, on students' academic performance, there are still some gaps that can be filled. Although some studies have explored the heterogeneous effect of detracking (Nomi, 2012; Domina et al., 2016), they have not answered how the within-school detracking process impacts the between-school achievement gap. In this paper, my objective is to examine (a) the effect on the average math test scores within a school of expanding algebra enrollment to eighth-grade students and (b) the differential effect on average math test scores among schools with varied average math achievement.

I expect that although offering algebra to students who currently lack access to it can be helpful for their improvement, enforcing all students to take algebra is not the appropriate path to achieve this goal for some schools. According to the existing evidence, I hypothesize:

Hypothesis I: There is no statistically significant average effect on math test scores in schools when all eighth-grade students are enrolled in algebra compared to enrolling no student in algebra and enrolling some students in algebra.

Hypothesis II: Schools with higher average achievement benefit from enrolling all students in algebra, while schools with lower average achievement suffer losses from this process. Therefore, the average effect is null. In other words, the effect differs among schools with different average achievement rather than among different counterfactual scenarios.

Hypothesis III: Enrolling some students in algebra generally improves math test scores for schools that do not offer algebra. There is no statistically significant heterogeneous effect across schools.

Method of the Current Study

Sample

This study utilizes data from the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K), which was conducted by the National Center for Educational Statistics. The ECLS-K dataset is nationally representative and consists of approximately 20,000 students. It includes detailed information on school policies, demographics, course offerings, staffing, and student achievement from kindergarten to eighth-grade, collected in seven waves. For this research, data from the final wave are primarily utilized, which focuses on eighth-grade students and covers a subset of the initial population.

The analysis in this study focuses solely on the effect of enrolling more students in algebra in public schools. This is due to the availability of specific measures such as school mean socioeconomic status (SES) and school average achievement, which are only provided for public schools in the dataset. After excluding private schools and schools that lack information about their sector, the dataset includes 1,658 schools and 7,808 students. Further filtering for missing cases using listwise deletion reduces the dataset to 1,015 schools and 4,141 students.

Methods

This paper employs a propensity score-based method combined with a hierarchical linear model to investigate the causal effect of requiring all eighth-graders to take algebra. The use of a propensity score-based model allows for the elimination of selection bias and facilitates causal inference (Rosenbaum & Rubin, 1983). The causal model in this research is under the stable unit treatment value assumption (SUTVA) that the potential result a unit can achieve from the treatments is not influenced by how the treatments are assigned or what treatments other units received (Rubin, 1986). This assumption might not be satisfied in education scenarios, but some researchers have shown that SUTVA can be relaxed when using multilevel data in education and identify at which a treatment is implemented (Hong & Raudenbush, 2005). In this research, the outcome is student achievement measured at the student level while the treatment is the enrollment of algebra measured at the school level. A two-level hierarchical linear model is utilized to account for the clustering of students within schools and control for any associated biases (Raudenbush & Bryk, 2002).

Table 1a presents summary statistics for the school-level variables utilized in the subsequent model, while table 1b provides summary statistics for the student-level variables employed in the model.

Table 1a

Descriptive Statistics for Main Study Variables (School-level)

Variables	N	Mean	SD	Minimum	Maximum
% free lunch	1,657	36.222	30.025	0	95
Average achievement	1,320	61.932	23.663	0	100
Offer Algebra to eighth-grader					
Not offering	1,361	0.173	0.379	0	1
Enrolling some students	1,361	0.786	0.410	0	1
Enrolling all students	1,361	0.040	0.197	0	1
% Black Students					
Less than 1%	1,454	0.058	0.235	0	1
1% to less than 5%	1,454	0.384	0.487	0	1
5% to less than 10%	1,454	0.130	0.336	0	1
10% to less than 25%	1,454	0.191	0.393	0	1
25% or more	1,454	0.236	0.425	0	1
% Hispanic Students					
Less than 1%	1,455	0.071	0.257	0	1
1% to less than 5%	1,455	0.355	0.479	0	1
5% to less than 10%	1,455	0.143	0.350	0	1
10% to less than 25%	1,455	0.162	0.369	0	1
25% or more	1,455	0.269	0.443	0	1
School Size (measured by number of eighth-grader)					
Large (more than 180 students)	1,531	0.683	0.465	0	1
Medium (81 - 180 students)	1,531	0.182	0.386	0	1
Small (0 - 80 students)	1,531	0.135	0.341	0	1
Urbanicity					
Large and mid-size city	1,423	0.356	0.479	0	1
Large and mid-size suburb and large town	1,423	0.450	0.498	0	1
Small town and rural	1,423	0.193	0.395	0	1
Instruction Method					
provide differentiated core class	1,444	0.807	0.395	0	1
provide undifferentiated core class	1,444	0.193	0.395	0	1

%free lunch: The percentage of the students who are eligible for free lunch in the school, reported by school administrators. The value of this variable ranges from 0 to 95, with mean = 36.222, S.D. = 30.025.

Average achievement: The percentage of eighth-grade students in the school who are tested at or above grade level on mathematics State tests, as reported by school administrators. The value of this variable ranges from 0 to 100, with mean = 61.932, S.D. = 23.663. For analysis purposes, the "Average achievement" variable has been recoded into an ordinal variable based on quartiles. This allows for the examination of the heterogeneous effect on the highest quartile and lowest quartile in terms of enrolling all eighth-grade students in algebra.

% Black Students: The percentage of Black students in the school reported by school administrators, recoded into an ordinal variable by ECLS-K. The medium of the percentage of Black students lies in 5% to less than 10%.

% Hispanic Students: The percentage of Hispanic students in the school reported by school administrators, recoded into an ordinal variable by ECLS-K. The medium of the percentage of Hispanic students lies in 5% to less than 10%.

School Size: The school size is measured by the number of enrolled eighth-grader reported by school administrators. The reported number was recoded into ordinal variable by ECLS-K. I recode the variable into three categories, where the medium size of a school is more than 180 students.

Urbanicity: Schools administrators' reported school location. Most students are in schools where locate in large and mid-size suburb and large town.

Instruction Method: Measures the approach taken by schools in providing instruction in their core curriculum to students with different skills, learning rates, interests, or motivations. School administrators provided the response for this item, which consists of four options: (1) the school offers differentiated courses but students have open access provided they have taken the required prerequisite(s); (2) the school offers differentiated courses and differentiated grouping;

(3) the school offer undifferentiated courses and students have open access provided they have taken the required prerequisite(s); (4) other. For the purpose of this paper, the variable was recoded as follows: Option (1), (2), and (4) were recoded as “differentiated core curriculum”; Option (3) was recoded as “undifferentiated core curriculum”. This recoding allows for a distinction between schools that provide differentiated instruction and those that offer undifferentiated instruction in their core curriculum.

Offer Algebra to eighth-grader: recoded from the percentage of eighth-grade students at the school enrolled in algebra, reported by school administrators. 0 is recoded as no-algebra schools to eighth-grader, mean = 0.173, SD = 0.379; 1-99 is recoded as school enrolling a fraction of eighth-grade students in algebra, mean = 0.786, SD = 0.410; 100 is recoded as school enrolling a fraction of eighth-grade students in algebra, mean = 0.040, SD = 0.197.

Table 1b

Descriptive Statistics for Main Study Variables (Student-level)

Variables	N	Mean	SD	Minimum	Maximum
eighth-grade math theta scores	7,655	1.478	0.440	0.2338	2.5447

eighth-grade math theta scores: Theta scores measure child’s ability at each round along a single continuous scale, making it ideally suited for measuring changes in achievement across different waves (Tourangeau et al., 2009). For all the eighth-grader respondents, mean = 1.48, S.D. = 0.44. eighth-grade math theta scores are used as outcome variable in the final model.

Table 2 summarizes the mean differences of each predictor among three groups. As shown by Table 2, no-algebra schools are mainly different from other groups in terms of school size and urbanicity. Small schools in small town and rural area are more likely to be no-algebra schools.

Some-algebra schools have less Black and Hispanic students and higher mean SES (less students eligible for free lunch), and the school average achievement is higher than the other groups. Surprisingly, all-algebra schools have similar feature to no-algebra schools in terms of racial composition, mean SES and average achievement, and mainly different in terms of school size and urbanicity.

Table 2

Observed Mean of Predictors Among Three Algebra-enrollment Groups

	No Algebra	Some Algebra	All Algebra
% Black Students			
Less than 1%	0.141	0.041	0.118
1% to less than 5%	0.319	0.425	0.353
5% to less than 10%	0.052	0.158	0.029
10% to less than 25%	0.156	0.182	0.176
25% or more	0.333	0.194	0.324
% Hispanic Students			
Less than 1%	0.170	0.042	0.059
1% to less than 5%	0.363	0.383	0.147
1.5% to less than 10%	0.096	0.153	0.235
10% to less than 25%	0.111	0.174	0.059
25% or more	0.259	0.248	0.500
School Size			
Large	0.422	0.760	0.706
Medium	0.222	0.168	0.206
Small	0.356	0.073	0.088
Urbanicity			
Large and mid-size city	0.400	0.318	0.500
Large and mid-size suburb and large town	0.274	0.511	0.471
Small town and rural	0.326	0.172	0.029
Instruction Method			
Provide undifferentiated core class	0.244	0.165	0.353

%free lunch	45.200	31.212	45.477
Average Achievement	54.089	64.675	53.618

Data Analysis

The central aim of this article is to explore the impacts of expanding the enrollment of eighth-grade algebra on schools' average achievement. To establish a causal inference, it is important to clarify the counterfactual outcomes. Based on the percentage of eighth-grade students enrolled in algebra at each school, the public schools in the sample are divided into three groups: those enrolling no student in algebra (no-algebra schools), those enrolling some students in algebra (some-algebra schools), and those enrolling all students in algebra (all-algebra schools). Consequently, there are three counterfactual scenarios to consider: (1) no-algebra versus all-algebra, (2) some-algebra versus all-algebra, and (3) no-algebra versus some-algebra. Thus, there will be three sets of propensity score analysis in this paper.

I will analyze these three sets of comparisons separately. For Part A, the treatment effect is defined as $E[\mathrm{Y}(Z_1 = 1) - \mathrm{Y}(Z_1 = 0)]$, where $Z_1 = 1$ represents enrolling all students in algebra, and $Z_1 = 0$ represents enrolling no students in algebra. Therefore, for the analysis in Part A, I will only include data from no-algebra and all-algebra schools to estimate the counterfactual outcome that would have been observed under my models had the no-algebra schools enrolled all students in algebra.

For the analysis in Part B, the treatment effect is defined as $E[\mathrm{Y}(Z_2 = 1) - \mathrm{Y}(Z_2 = 0)]$, where $Z_2 = 1$ represents enrolling all students in algebra, and $Z_2 = 0$ represents enrolling some students in algebra. Therefore, I will only include data from some algebra and all-algebra schools to measure the counterfactual outcome that would have been observed under my models had the some-algebra schools enrolled all students in algebra.

For the analysis in Part C, the treatment effect is defined as $E[\mathrm{Y}(Z_3 = 1) - \mathrm{Y}(Z_3 = 0)]$, where $Z_3 = 1$ represents enrolling some students in algebra, and $Z_3 = 0$ represents enrolling no students in algebra. Thus, I will only include data from no-algebra and some-algebra schools to estimate the counterfactual outcome that would have been observed had the no-algebra schools enrolled some students in algebra.

Each part will consist of three sections. In the first section, I will present the propensity score model and the balance of propensity. This model will include school-level variables to control for selection bias. The variables used for controlling selection bias will include the percentage of Black students, percentage of Hispanic students, school size, urbanicity, school sector, percentage of students eligible for free lunch, instruction method, and average achievement.

Next, I will measure the average treatment effect of enrolling all eighth-grade students in algebra. This will be done using a two-level random intercept model, where the treatment variable is at the school level and the outcome variable is at the student level.

In the final step, I will measure the heterogeneous treatment effect among schools with different average achievement. To do this, I will introduce interaction terms of the treatments and average achievement into the model to examine how the treatment effect varies based on the average achievement of the school. This will provide insights into whether the impact of enrolling more students in algebra differs across schools with varying levels of achievement.

Part A Results

Propensity Score Model

In Part A, I discuss the treatment effect in terms of enrolling all students in algebra in no-algebra schools. As shown by Table 2, compared with no-algebra schools, schools that the percentage of Hispanic students is 5% to less than 10% or more than 25% is more likely to enroll all students in algebra in eighth-grade; small schools and schools in small town and rural area are less likely to enroll all students in algebra in eighth grade. These covariates suggest that the smaller rural schools with more Hispanic students are more likely to enrolling no student in algebra to students, making the students in these schools lose their opportunities to learn algebra at eighth-grade. Thus, the propensity score model is formulated as below to control the selection bias caused by these differences:

$$\eta_j = \ln\left(\frac{P_j}{1-P_j}\right) = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \cdots + \beta_8 X_{8j}$$

P_j = Probability of school j enrolling all eighth-grade students in algebra.

η_j = Log odds of school j enrolling all eighth-grade students in algebra.

β_0 = Log odds of being treated for school j with a set of covariates $X_{kj} = 0$.

β_1 ... β_8= Changes in log odds for one unit change in one of the eight predictors for the percentage of Hispanic students, school size, and urbanicity.

15 no-algebra schools that are least likely to enroll all eighth-grade students in algebra laid outside the region of common support and 154 schools left for stratification. The balance in logit of the propensity score for enrolling all eighth-grade students in algebra is shown in Table 3. Distribution balance is achieved in five strata. Neither the predictors nor the propensity scores are significantly correlated to the treatment after stratification.

Table 3

Balance in Logit of the Propensity Score for Algebra Enrollment (Part A)

	Enrolling all students in algebra (Z = 1)			Enrolling no student in algebra (Z = 0)		
Stratum	**N**	**Mean**	**S.D.**	**N**	**Mean**	**S.D.**
1	1	-4.206	*NA*	32	-3.683	0.583
2	4	-1.979	0.239	28	-1.962	0.273
3	7	-1.369	0.179	28	-1.34	0.165
4	8	-0.601	0.034	16	-0.686	0.226
5	14	-0.184	0.205	16	-0.249	0.175

Outcome Model

1. Average Effect

Hierarchical linear models test the average treatment effect of enrolling all eighth-grade student in algebra versus enrolling no students in algebra. I use the students' eighth-grade math theta scores as the outcome and put the treatment and strata on the second level. Model 1 is formulated as below to measure the average treatment effect:

Model 1

Level-1 Model

$MATH8_{ij} = \beta_{0j} + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01}Z_{1j} + \sum_{s=2}^{5} \gamma_{0s}L_{sj} + u_{0j}$

Mixed Model

$MATH8_{ij} = \gamma_{00} + \gamma_{01}Z_{1j} + \sum_{s=2}^{5} \gamma_{0s}L_{sj} + u_{0j} + r_{ij}$

In this model,

β_{0j} = Average math theta score at school j.

γ_{00} = Average math theta score of all no-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

γ_{0s} = Effect of being in each stratum.

u_{0j} = School-specific variation in $\beta_{0j.}$

r_{ij} = Student-specific variation in $MATH8_{ij.}$

Results of Model 1 are summarized in Table 4. As shown by Table 4, the expected average math score of all no-algebra schools is 1.276 (S.E. = 0.060, p<0.001). Enrolling all students in algebra versus enrolling no students in algebra increases the school average math score by 0.134 (S.E.= 0.072, p<0.1). Given the S.D. of math theta score is 0.44, the treatment effect is big in this case. However, the treatment effect is not statistically significant on the conventional level so this result cannot be extrapolated to the whole population. Thus, it is difficult to draw a definitive conclusion that enrolling all students in algebra has a positive effect in general. This result aligns with the hypothesis put forward in the paper that the average effect is null.

Table 4

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling no student in algebra School

Fixed Effect			coefficient	S.E.	
Intercept of enrolling no student in algebra school, γ_{00}			1.276***	0.060	
effect of enrolling all students in algebra, γ_{01}			0.134.	0.072	
Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.28320	0. 08020	149	362.33186	<0.001
Student-specific variatio, r_{ij}	0.36416	0.13262			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

2. Heterogeneous Effect

To test heterogeneous effects of enrolling all eighth-grade students in algebra, I introduce interaction terms of average achievement and treatment based on Model 1 in Model 2a and 2b. In these two models, I treat Average Achievement as continuous variable and categorical variable respectively. In other words, in Model 2a, Average Achievement represent the percentage of eighth-grade students in the school who are tested at or above grade level on mathematics State tests; in Model 2b, Average Achievement is recoded into an ordinal variable based on quartiles, allowing for the examination of the heterogeneous effect on the highest quartile and lowest quartile in terms of enrolling all eighth-grade students in algebra. In this way, I demonstrate the treatment effect on no-algebra schools in terms of (1) the whole picture of heterogeneous effect among schools with different average achievement and (2) the achievement gap caused by this heterogeneous effect.

Model 2a

Level-1 Model

$MATH8_{ij} = \beta_{0j} + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \sum_{s=2}^{5} \gamma_{0s} L_{sj} + \gamma_{06}(A_j - \overline{A_j}) + \gamma_{07}(A_j - \overline{A_j}) * Z_{1j} + u_{0j}$

Mixed Model

$MATH8_{ij} = \gamma_{00} + \gamma_{01} Z_{1j} + \sum_{s=2}^{5} \gamma_{0s} L_{sj} + \gamma_{06}(A_j - \overline{A_j}) + \gamma_{07}(A_j - \overline{A_j}) * Z_{1j} + u_{0j} + r_{ij}$

In Model 2a, A_j represents the percentage of students performing at or above grade level and centers around grand mean.

γ_{00} = Average math theta score of all no-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

γ_{06} = Expected difference in average math score for each one percentage point increase in students who are tested at or above grade level on mathematics on state tests in untreated schools

γ_{07} = Additional expected difference in average math score for each one percentage point increase in students who are tested at or above grade level on mathematics on state tests in treated schools.

Results for the Model 2a are summarized in Table 5a.

Table 5a

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling No Student in Algebra (Part 1)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no student in algebra school, γ_{00}	1.317***	0.055
Effect of enrolling all students in algebra, γ_{01}	0.157*	0.074
Effect of 1% increase on average achievement, γ_{06}	0.00397**	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{07}	0.00467*	0.002

Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.2033	0.04132	146	358.52884	<0.001
Student-specific variation, r_{0j}	0. 3714	0.13797			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

The results from Table 5a demonstrate significant heterogeneity in the effects of enrolling all students in algebra versus enrolling no students in algebra. When treating school average achievement as a continuous variable, enrolling all students in algebra increases the school average math score by 0.157 (S.E. = 0.074, $p<0.05$). Moreover, for every one percentage increase in students tested at or above grade level in mathematics State tests, the school average math score increases by 0.00467 (S.E. = 0.002, $p<0.05$) when treated. This suggests that the treatment effect shows a positive correlation with school average achievement, with higher-achieving schools experiencing a larger positive effect.

Although the average treatment effect for no-algebra schools is positive, high-achieving schools and low-achieving schools could experience the treatment differently. In order to give a straightforward understanding in terms of the aftermath of the heterogeneity, I will present the average achievement of each quartile after treatment. I use Model 2b to measure the heterogeneous effect across different groups and report the results in Table 5b.

Model 2b

Level-1 Model

$$MATH8_{ij} = \beta_{0j} + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \sum_{s=2}^{5} \gamma_{0s} L_{sj} + \sum_{k=1}^{3} \gamma_{5+k} A_{kj} + \sum_{m=1}^{3} \gamma_{8+m} A_{mj} Z_{1j} + u_{0j}$$

Mixed Model

$$MATH8_{ij} = \gamma_{00} + \gamma_{01} Z_{1j} + \sum_{s=2}^{5} \gamma_{0s} L_{sj} + \sum_{k=1}^{3} \gamma_{5+k} A_{kj} + \sum_{m=1}^{3} \gamma_{8+m} A_{mj} Z_{1j} + u_{0j} + r_{ij}$$

In Model 2b,

γ_{00} = Average math theta score for no-algebra schools to eighth-grade students and being in the fourth quartile of average achievement (i.e. the reference group).

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

γ_{5+k} = Expected difference in average math score between each quartile and the fourth quartile in average achievement in untreated schools.

γ_{8+m} = Additional expected difference in average math score between each quartile and the fourth quartile in average achievement in treated schools.

Table 5b

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling No Student in Algebra (Part 2)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no student in algebra &4th quartile school, γ_{00}	1.353***	0.089
effect of enrolling all students in algebra, γ_{01}	0.555***	0.147
effect of being in schools in the 1st quartile of average achievement, γ_{06}	-0.234*	0.096
effect of being in schools in the 2nd quartile of average achievement, γ_{07}	-0.052	0.097
effect of being in schools in the 3rd quartile of average achievement, γ_{08}	0.022	0.108
Difference of treatment effect on schools in the 1st quartile, γ_{09}	-0.561**	0.174
Difference of treatment effect on schools in the 2nd quartile, γ_{10}	-0.473*	0.190
Difference of treatment effect on schools in the 3rd quartile, γ_{11}	-0.623**	0.213

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.28934	0. 08372	142	352.48917	<0.001
Student-specific variation, r_{0j}	0.36337	0.13204			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

The positive correlation between the treatment effect and school average achievement contributes to widening the achievement gap between high-achieving schools and low-achieving schools. As shown in Table 5b, compared to the most high-achieving group, the positive treatment effect is almost neutralized for other groups. For schools in the 4th quartile of average achievement, being treated would have resulted in a mean math test score increase of 0.555 (S.E. = 0.147, p<0.001) points. In contrast, the treatment effect would have decreased by 0.561 (S.E. = 0.174, p<0.01) for schools in the 1st quartile, by 0.473 (S.E. = 0.190, p<0.05) for schools in the 2nd quartile, and by 0.623 (S.E. = 0.213, p<0.01) for schools in the 3rd quartile. Therefore, enrolling all students in algebra widens the achievement gap between schools in the 4th quartile and schools in other quartiles.

Figure 1

Observed and Counterfactual Math Growth Trajectory of Schools in the 1st and 4th Quartile (No Algebra versus All Algebra)

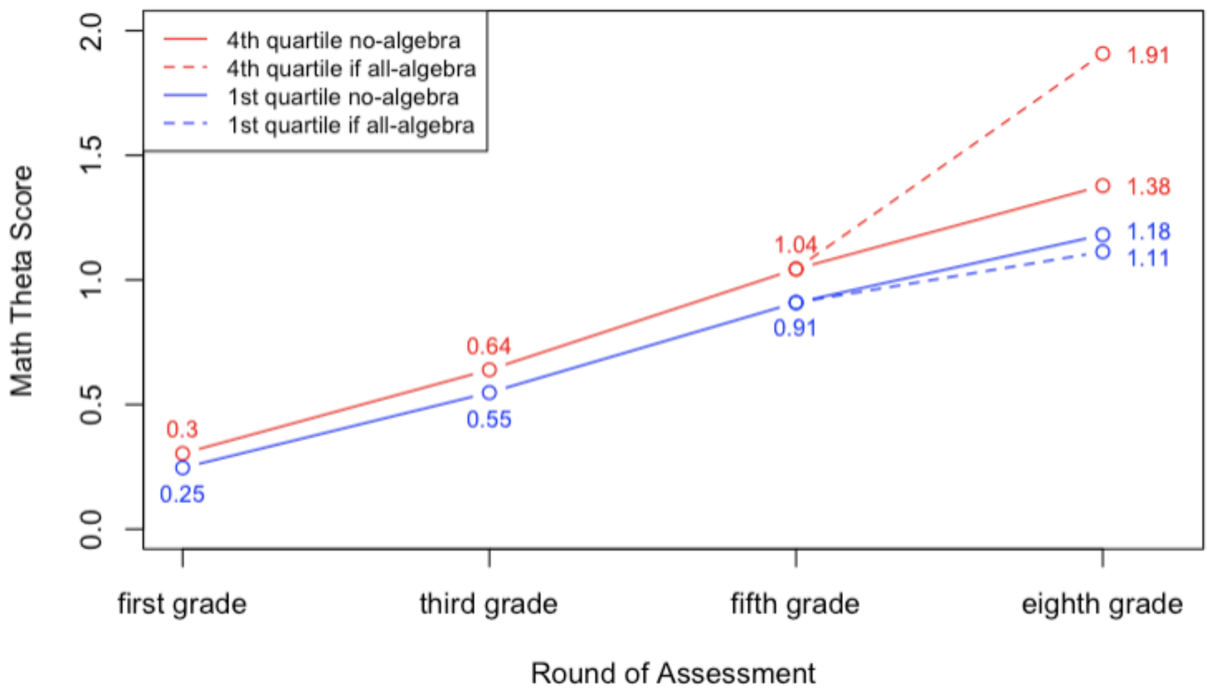

Figure 1 illustrates the observed math growth trajectories of the no-algebra schools in the first quartile and fourth quartile of average achievement, as well as the counterfactual outcome if treated. On the basis of the above estimation results, the red lines represent the trajectory of schools in the fourth quartile and the blue lines represent the trajectory of schools in the first quartile; the dashed line represents the counterfactual growth trajectories that would have been observed under Model 2b had all students enrolled in algebra. According to the observed math growth trajectories, the achievement gap between the schools in the first quartile and fourth quartile slowly enlarge from the first grade to the eighth grade. However, if the no-algebra schools enrolled all students in algebra, the achievement gap in the eighth grade would have suddenly enlarged. Since the average math score of schools in the fourth quartile would have increased from 1.38 to 1.91 if treated, while the schools in the first quartile would have decreased from 1.18 to 1.11. The tremendous growth in the fourth quartile and the small loss in the first quartile collectively contribute to an enlarging gap.

Part B Results

Propensity Score Model

As shown by Table 2, compared with some-algebra schools, schools with 1% to less than 10% Black students, small town and rural schools, schools with higher average achievement are less likely to enroll all students in algebra; schools provide undifferentiated core classes, schools with more students eligible for free lunch are more likely to enroll all students in algebra. Surprisingly, all-algebra schools seem to be more disadvantaged than some-algebra schools.

To address potential selection bias and achieve balance, a propensity score model is employed, following the same formulation as Part A. Table 6 presents the balance achieved in the logit of the propensity score with seven strata for enrolling all eighth-grade students in algebra. 67 schools laid outside the region of common support, resulting in 881 schools left for stratification.

After stratification, none of the predictors listed previously show a significant correlation with the treatment, and the propensity score is not related to the treatment either.

Table 6

Balance in Logit of the Propensity Score for Algebra Enrollment (Part B)

	Enrolling all students in algebra (Z = 1)			Enrolling some students in algebra (Z = 0)		
Stratum	**N**	**Mean**	**S.D.**	**N**	**Mean**	**S.D.**
1	1	-6.133	*NA*	50	-5.74	0.261
2	1	-4.652	*NA*	154	-4.746	0.21
3	3	-3.89	0.4	120	-4.123	0.25
4	6	-3.19	0.379	193	-3.196	0.229
5	7	-2.359	0.247	88	-2.423	0.221
6	7	-1.423	0.211	26	-1.622	0.221
7	4	-1.053	0.006	10	-0.765	0.258

Outcome Model

1. Average Effect

I use the essentially same model as Model 1 with different interpretations of some coefficients to measure the average treatment effect in Part B. Interpretations of the covariates in this model are identical to Part A except:

γ_{00} = Average math theta score of all some-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

The results of the average effect of enrolling all students in algebra are summarized in Table 7.

Table 7

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling Some Students in Algebra

Fixed Effect			coefficient	S.E.	
Intercept of some-algebra schools, γ_{00}			1.484***	0.042	
Effect of enrolling all students in algebra, γ_{01}			0.037	0.064	
Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.22691	0.05149	660	1288.55534	<0.001
Student-specific variation, r_{0j}	0.37958	0.14408			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1
Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

As shown by Table 7, the expected average math score of all some-algebra schools is 1.484 (S.E. = 0.042, p<0.001), which is higher than the expected average math score of all no-algebra schools. However, after controlling for other covariates, enrolling all students in algebra versus enrolling some students in algebra does not have a significant impact on school average math score, which is in line with the findings in Part A. This suggests that the average effect of enrolling all eighth-grade students in algebra in both scenarios are insignificant, indicating that enrolling all students in a more advanced math does not significantly impact their math test score on average.

2. Heterogeneous Effect

I use essentially the same models as Model 2a to measure the overall heterogeneity of enrolling all students in algebra versus enrolling some students in algebra among schools with varied average achievement. Interpretations of the covariates are identical to Part A except:

γ_{00} = Average math theta score of all some-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

The average achievement in this model also centers around grand mean. The results of overall heterogeneous effect are reported in Table 8a.

Table 8a

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling Some Students in Algebra (Part 1)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling some students in algebra, γ_{00}	1.466***	0.038
Effect of enrolling all students in algebra, γ_{01}	0.078	0.063
Remaining effect of being in schools with 1% increase on average achievement, γ_{08}	0.00394***	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{09}	0.00496*	0.002

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.1672	0. 02796	658	1276.57747	<0.001
Student-specific variation, r_{0j}	0. 3927	0. 15423			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1
Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

Similar results in terms of the heterogeneous effect can also be found in Part B. Table 8a demonstrates substantial heterogeneity within the effect of enrolling all students in algebra among schools. When treating school average achievement as a continuous variable, there is no statistically significant average treatment effect on all some-algebra schools. However, with a one-unit increase in the percentage of students being tested at or above grade level on mathematics State tests, the school average math score increases by 0.00496 (S.E. = 0.002, p<0.05) for those being treated. This indicates a positive correlation between the treatment effect and school average achievement, consistent with the findings in Part A.

To measure the achievement gap between the some-algebra schools in the highest quartile and the lowest quartile, I use essentially the same model as Model 2b to measure the heterogeneous effect and report the results in Table 8b.

In this model,

γ_{00} = Average math theta score for some-algebra schools to eighth-grade students and being in the fourth quartile of average achievement (i.e. the reference group).

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and all-algebra schools.

The interpretations of γ_{08}… γ_{13} are the same as those presented in Part A.

Table 8b

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling Some Students School (Part 2)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling some student in algebra &4th quartile school, γ_{00}	1.573***	0.044
effect of enrolling some students in algebra, γ_{01}	0.370**	0.130
effect of being in schools in the 1st quartile of average achievement, γ_{08}	-0.231***	0.037
effect of being in schools in the 2nd quartile of average achievement, γ_{09}	-0.103***	0.033
effect of being in schools in the 3rd quartile of average achievement, γ_{10}	-0.104**	0.034
Difference of treatment effect on schools in the 1st quartile, γ_{11}	-0.512***	0.154
Difference of treatment effect on schools in the 2nd quartile, γ_{12}	-0.294	0.172
Difference of treatment effect on schools in the 3rd quartile, γ_{13}	-0.461*	0.204

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
INTRCPT1, u0	0.22600	0. 05108	654	1268.46478	<0.001
level-1, *r*	0. 37976	0. 14422			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

The pattern of widening achievement gaps becomes more evident when examining the heterogeneous effect based on different achievement groups. Table 8b shows that compared to the most high-achieving group, the positive effect of being treated is almost neutralized for other groups. For schools in the 4th quartile of average achievement, their mean math test score would have increased by 0.370 (S.E. = 0.127, $p<0.001$) points if treated, while the treatment effect would have decrease by 0.512 (S.E. = 0.174, $p<0.001$) for schools in the 1st quartile, and by 0.461 (S.E. = 0.213, $p<0.05$) for schools in the 3rd quartile. Therefore, enrolling all students in algebra widens the achievement gap between schools in the 4th quartile and those in other groups.

Figure 2

Observed and Counterfactual Math Growth Trajectory of Schools in the 1st and 4th Quartile (Some Algebra versus All Algebra)

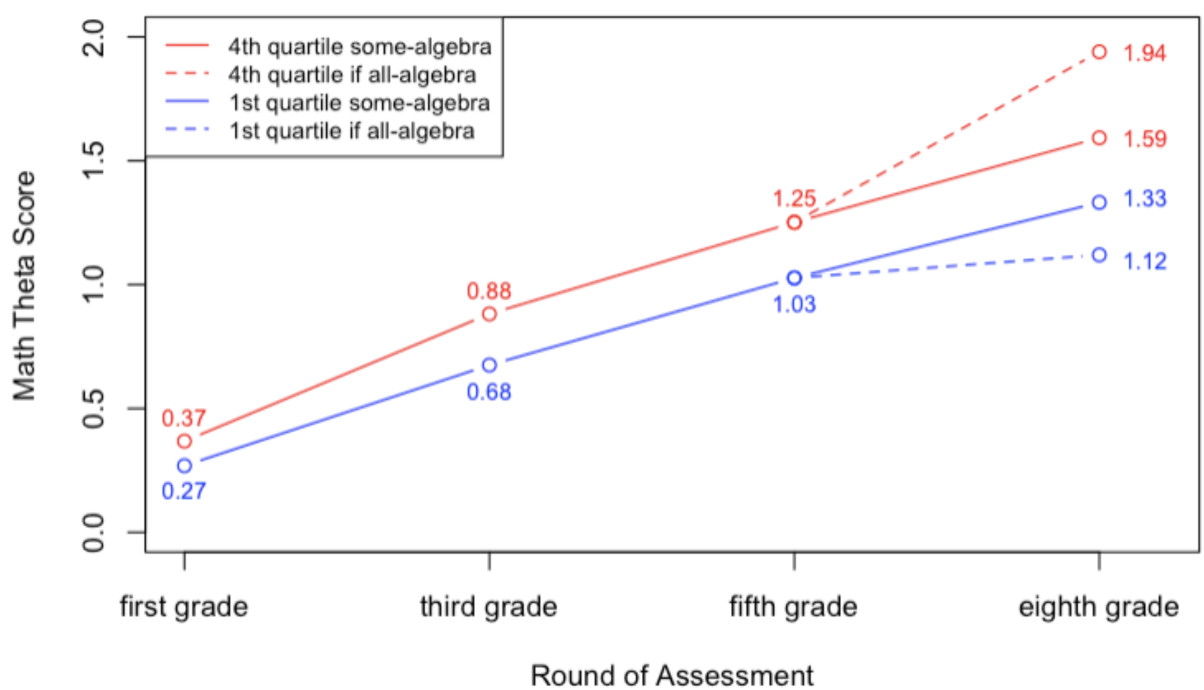

Figure 2 illustrates the observed math growth trajectories of the some-algebra schools in the first quartile and fourth quartile of average achievement, as well as the counterfactual outcome if treated. On the basis of the above estimation results, the red lines represent the trajectory of schools in the fourth quartile and the blue lines represent the trajectory of schools in the first quartile; the dashed line represents the counterfactual growth trajectories that would have been observed under Model 2b had all students enrolled in algebra. Essentially similar to the observed math growth trajectories in Figure 1, the achievement gap between the schools in the first quartile and fourth quartile slowly enlarge from the first grade to the eighth grade, but the achievement gap in the eighth grade would have suddenly enlarged if the some-algebra schools enrolled all students in algebra. The average math score of schools in the fourth quartile would have increased from 1.59 to 1.94 if treated, while the schools in the first quartile would have decreased from 1.33 to 1.2. The counterfactual achievement gap becomes even larger compared to the one in Part A. The tremendous growth in the fourth quartile and the loss in the first quartile collectively contribute to an enlarging gap.

Part C Results

The results in both Part A and Part B indicate that enrolling all students in algebra is primarily beneficial for high-achieving schools and can lead to the widening of achievement gaps between schools. However, it does not suggest that expanding the availability of algebra for eighth-grade students is problematic in any scenario. Making algebra available for schools that do not currently offer it can still contribute to increased average achievement.

Propensity Score Model

In Table 2, it is observed that compared to no-algebra schools, schools with higher proportions of Black and Hispanic students are more likely to enroll some students in eighth-grade algebra. Additionally, larger schools located in mid-size suburbs and large towns are more likely to offer algebra to some students. Schools providing undifferentiated instructions and schools with a higher percentage of students eligible for free lunch are less likely to enroll some students in eighth-grade algebra. Furthermore, schools with higher average achievement are more likely to enroll some students in algebra. These differences are controlled in the propensity score model.

Table 9 presents the balance in the logit of the propensity score for enrolling some eighth-grade students in algebra versus enrolling no student in algebra. I use the same propensity score model in Part A to control selection bias. 12 schools laid outside the region of common support, resulting in 881 schools left for stratification. Distribution balance was achieved in ten strata. After stratification, none of the predictors listed earlier are significantly correlated with the treatment, and the propensity score is not related to the treatment either. This indicates that the propensity score model successfully achieves balance in the covariates related to enrolling some students in algebra.

Table 9

Balance in Logit of the Propensity Score for Algebra Enrollment (Part C)

	Enrolling some students in algebra (Z = 1)			Enrolling no student in algebra (Z = 0)		
Stratum	**N**	**Mean**	**S.D.**	**N**	**Mean**	**S.D.**
1	42	-0.026	0.426	47	-0.144	0.557
2	58	0.987	0.199	30	0.983	0.185
3	77	1.511	0.101	11	1.484	0.127
4	81	1.833	0.082	7	1.816	0.075
5	79	2.086	0.07	9	2.047	0.042
6	78	2.333	0.083	10	2.354	0.092
7	81	2.606	0.073	7	2.587	0.049
8	83	2.867	0.079	5	2.891	0.042
9	85	3.138	0.069	3	3.138	0.096
10	84	3.464	0.156	4	3.514	0.252

Outcome Model

1. Average Effect

I use the essentially similar model as Model 1 with different interpretations of some coefficient to measure the average treatment effect in Part C. The interpretations of most covariates are the same as those presented in Part A except:

γ_{00} = Average math theta score of all no-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and some-algebra schools.

The results of the average effect of enrolling some students in algebra are summarized in Table 10.

Table 10

Model-based Estimation of the Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra

Fixed Effect			coefficient	S.E.	
Intercept of enrolling no student in algebra school, γ_{00}			1.273***	0.035	
Effect of enrolling some students in algebra, γ_{01}			0.086**	0.031	
Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.21445	0.04599	867	1650.18564	<0.001
Student-specific variation, r_{0j}	0.37583	0. 14125			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

Different from the insignificant average treatment effect in Part A and Part B, enrolling some students in algebra increase average math test scores for no-algebra schools by 0.086 (S.E. = 0.031, p<0.01), with a 95% confident interval of (0.025, 0.147). This indicates that all no-algebra schools would have benefitted if enrolled some students in algebra on average. According to the analytical results, had the algebra been accessible to students, some students in no-algebra schools would have benefitted from more advanced math course. Enrolling some students in algebra facilitates some students in these schools and increase the average math score of the schools.

2. Heterogeneous Effect

I use essentially the same models as Model 2a to measure the overall heterogeneity of enrolling some students in algebra versus enrolling no students in algebra among schools with varied average achievement. The interpretations of most covariates are the same as those presented in Part A except:

γ_{00} = Average math theta score of all no-algebra schools.

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and some-algebra schools.

The average achievement in this model also centers around grand mean. The results of overall heterogeneous effect are reported in Table 11a.

Table 11a

Model-based Estimation of the Heterogeneous Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra (Part 1)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no students in algebra, γ_{00}	1.319***	0.038
Effect of enrolling some students in algebra, γ_{01}	0.092**	0.063
Remaining effect of being in schools with 1% increase on average achievement, γ_{11}	0.00193	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{12}	0.000999	0.001

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.21435	0. 04595	865	1628.74011	<0.001
Student-specific variation, r_{0j}	0. 37595	0. 14134			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

Different from the results in Part A and Part B, there is no statistically significant heterogeneous effect of enrolling some students in algebra in the schools currently enrolling no student in algebra. In the other word, we cannot expect different schools experience the treatment differently in population. Instead, enrolling some students in algebra would have increased the average math test score by 0.092 (S.E. = 0.063, p<0.01) among all schools. These results are consistent with the measurement of average effect.

I use essentially the similar model as model 2b to further detect the achievement gap between the schools in the lowest quartile and highest quartile. The interpretations of most covariates are identical to those presented in Part A except:

γ_{00} = Average math theta score for no-algebra schools to eighth-grade students and being in the fourth quartile of average achievement (i.e. the reference group).

γ_{01} = Treatment effect in terms of the average change in the math theta scores between the reference group and some-algebra schools.

The results are reported in Table 11b.

Table 11b

Model-based Estimation of the Heterogeneous Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra (Part 2)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no student in algebra &4th quartile school, γ_{00}	1.329***	0.071
effect of enrolling some students in algebra, γ_{01}	0.142*	0.072
effect of being in schools in the 1st quartile of average achievement, γ_{11}	-0.111	0.081
effect of being in schools in the 2nd quartile of average achievement, γ_{12}	0.016	0.081
effect of being in schools in the 3rd quartile of average achievement, γ_{13}	-0.002	0.090
Difference of treatment effect on schools in the 1st quartile, γ_{14}	-0.045	0.087
Difference of treatment effect on schools in the 2nd quartile, γ_{15}	-0.081	0.087
Difference of treatment effect on schools in the 3rd quartile, γ_{16}	-0.062	0.095

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.21583	0. 04658	861	1643.27670	<0.001
Student-specific variation, r_{0j}	0.37560	0. 14108			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Note. For brevity, I have omitted the coefficient estimates for the propensity strata in this table.

As the heterogeneous effect shown by Table 11a, instead of observing a negative treatment effect among schools in lower achievement groups, no statistically significant difference is detected in terms of the treatment effect among the groups in Table 11b. This model reveals that for no-algebra schools and in the 4th quartile of achievement, enrolling some eighth-grade students in algebra increases their average math test scores by 0.142 (S.E. = 0.072, $p<0.05$). Importantly, this effect does not statistically differ among different achievement groups. In other words, all no-algebra schools can benefit from offering algebra to some eighth-grade students. The observed gap and counterfactual scenario between the first quartile and the fourth quartile are visualized in Figure 3.

Figure 3

Observed and Counterfactual Math Growth Trajectory of Schools in the 1st and 4th Quartile (No Algebra versus Some Algebra)

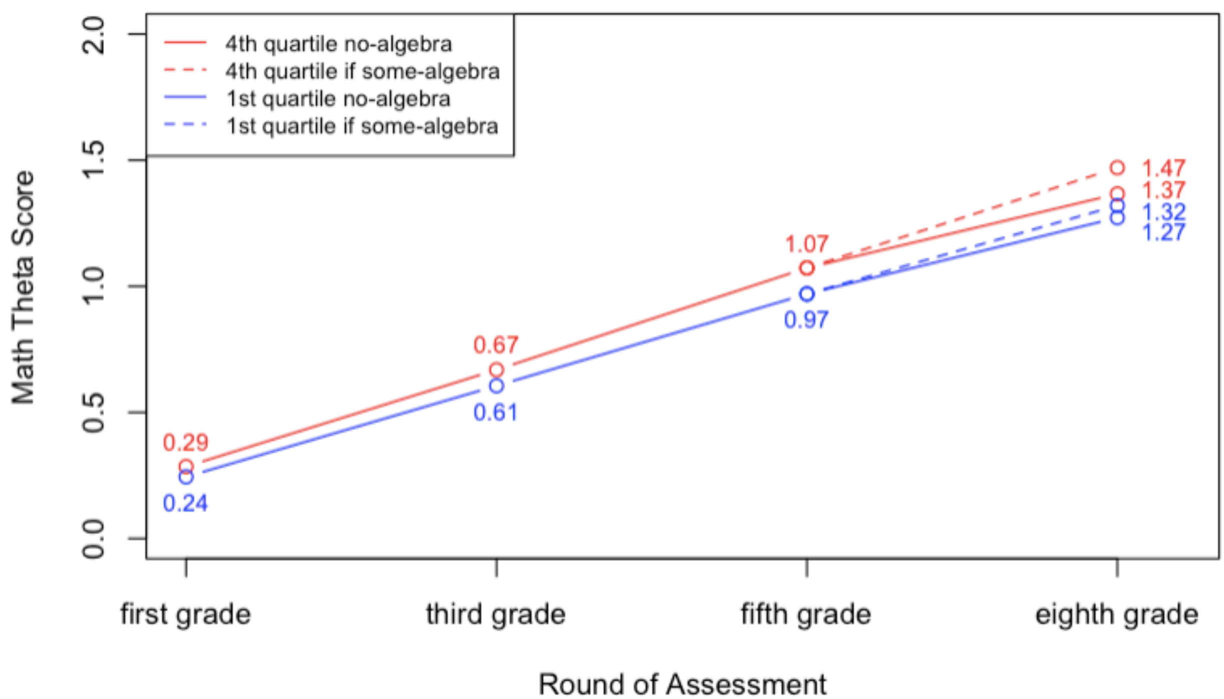

Figure 3 illustrates the observed math growth trajectories of the no-algebra schools in the first quartile and fourth quartile of average achievement, as well as the counterfactual outcome if treated. On the basis of the above estimation results, the red lines represent the trajectory of schools in the fourth quartile and the blue lines represent the trajectory of schools in the first quartile; the dashed line represents the counterfactual growth trajectories that would have been observed under Model 2b had some students enrolled in algebra. Different from the observed math growth trajectories in Figure 1 and Figure 2, the treatment effect on the schools in the first quartile and the schools in the fourth quartile are not contrary. Instead, in this sample, the average math scores of schools in the first quartile and the fourth quartile both increase if treated. The average math score of schools in the fourth quartile would have increased from 1.37 to 1.47 if treated, and the schools in the first quartile from 1.27 to 1.32.[1] Indeed, in this sample, the schools in the fourth quartile benefit more than schools in the first quartile and the achievement gap slightly enlarges, but the treatment effect is still positive on average.

Conclusion and Discussion

The current studies have sparked a debate on whether enrolling disadvantaged students in equally challenging courses or instructing all students in homogeneous classrooms is more beneficial. As addressed above, some scholars view this detracking process as a solution to educational inequality within schools (Oakes, 2005; Gamoran, 2010). However, other scholars have raised concerns about the technical challenges and unintended consequences of handicapping students from disadvantaged backgrounds (Rosenbaum, 1999; Domina et al., 2016; Nomi, 2012). These studies argue that high-achieving students from disadvantaged backgrounds may be hindered due to limited instructional resources in their schools, making it less likely for them to access more rigorous courses.

[1] Since γ_{06} and γ_{09} are not statistically significant, we cannot extrapolate this result to population.

In sum, the findings from my research align with the drawbacks of detracking and contribute further to the discussion. Based on the analytical results, the average effect of enrolling all eighth-grade students in algebra is null due to the heterogeneity of the treatment effect. Taking algebra in eighth-grade can be overwhelming and detrimental for students who are not adequately prepared in terms of their learning skills and prerequisite courses. Enforcing algebra for all eighth-grade students primarily benefits schools with high average achievement, while exacerbating the performance of low-achieving schools. This selective process inadvertently widens the achievement gap between schools.

However, it is important to note that enrolling more eighth-grade students in algebra is not inherently undesirable in terms of educational equity. Unlike enforcing all students in algebra, giving students an option to enroll in algebra is beneficial for their growth. Some high-achieving students in schools currently not offering algebra can benefit from taking algebra if it is available in their schools. Several scholars have suggested that starting to learn algebra in eighth-grade is beneficial for students' academic development in both the short term and long term (Spielhagen, 2006; Schmidt, 2009; Schneider, 2018). This research also supports the idea that enrolling some students in algebra has a positive effect on average achievement for schools that do not offer algebra, across all achievement groups. This indicates that some eighth-grade students in schools without algebra courses can benefit from taking algebra classes, and accessibility to algebra courses is crucial for their academic progress. However, it is clear that algebra may not be appropriate for all eighth-grade students at present. The optimal approach would be to make algebra available to all eighth-grade students but not make it a requirement for everyone.

Consequently, the findings in this research have implications for school principals and policymakers in terms of expanding the offering of eighth-grade algebra. For principals in high-achieving schools, it is reasonable to enroll all eighth-grade students in algebra, as students are skilled enough benefit from more advanced math on average. Nevertheless, for principals in other schools, enrolling only a fraction of students in algebra is more desirable since algebra could be too challenging for some students. Moreover, the findings also remind policymakers that enrolling all students in algebra, due to its heterogeneity, is an inappropriate policy for all schools. Instead of mandating algebra for all students, the focus should be on making algebra accessible to all eighth-graders.

Surely, it is important to acknowledge the limitations of this research. First, although I have achieved balance in three propensity score models, I assumed that there are no additional confounding variables uncontrolled by propensity scores. Additionally, by using stratification to achieve balance, I assumed that the treatment effects across strata are homogenous. These assumptions could be problematic and may cause bias in my analysis.

Second, the use of math theta score as the outcome variable to measure the treatment effect suggests potential issues with algebra for all eighth-graders. Qualitative research has shown that enrolling in more advanced classes can have positive implications for students' learner identity and increase their educational expectations (Lofton, 2021). In this research, the treatment effect was not measured beyond standardized test scores, and using different indicators may yield different results. While it is possible that some students may not exhibit improvement in terms of their standardized test scores, they might still increase their educational expectations and enhance their non-cognitive skills through other means.

Moreover, in this research, I treated enrolling some students in algebra as a homogeneous group. However, this group actually ranges from enrolling 1% to 99%, and there must be a huge heterogeneity within this group. The simplification might cause bias in my analysis and reduce the precision of my estimation. Therefore, I cannot draw any conclusions about what percentage of algebra enrollment is ideal for schools that currently do not offer algebra.

Last but not least, I did not measure student-level covariates in the analysis of school-level policy. Students from diverse backgrounds might react differently when encountering identical school policies. It is possible that although offering algebra to some students is optimal for average achievement within a school, it also broadens the achievement gap within the school and disproportionately hinders some students. These questions need further scrutiny to provide answers.

Appendix

The complete tables including effects of each stratum are listed below in sequence.

Table 4

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling no student in algebra School

Fixed Effect			coefficient	S.E.	
Intercept of enrolling no student in algebra school, γ_{00}			1.276***	0.060	
effect of enrolling all students in algebra, γ_{01}			0.134.	0.072	
effect of stratum 2, γ_{02}			0.067	0.090	
effect of stratum 3, γ_{03}			0.021	0.086	
effect of stratum 4, γ_{04}			-0.004	0.100	
effect of stratum 5, γ_{05}			0.078	0.099	
Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
School-specific variation, u_{0j}	0.2832	0. 08020	149	362.33186	<0.001
Student-specific variatio, r_{ij}	0.36416	0.13262			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 5a

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling No Student in Algebra (Part 1)

Fixed Effect						Coefficient	S.E.
Intercept of enrolling no student in algebra school, γ_{00}						1.317***	0.055
Effect of enrolling all students in algebra, γ_{01}						0.157*	0.074
effect of stratum 2, γ_{02}						0.028	0.081
effect of stratum 3, γ_{03}						0.021	0.077
effect of stratum 4, γ_{04}						0.058	0.091
effect of stratum 5, γ_{05}						0.090	0.089
Effect of 1% increase on average achievement, γ_{06}						0.00397**	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{07}						0.00467*	0.002
Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value		
School-specific variation, u_{0j}	0.2033	0.04132	146	358.52884	<0.001		

Student-specific variation, r_{0j}	0. 3714	0.13797

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 5b

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling No Student in Algebra (Part 2)

Fixed Effect					Coefficient	S.E.
Intercept of enrolling no student in algebra &4th quartile school, γ_{00}					1.353***	0.089
effect of enrolling all students in algebra, γ_{01}					0.555***	0.147
effect of stratum 2, γ_{02}					0.047	0.078
effect of stratum 3, γ_{03}					0.030	0.076
effect of stratum 4, γ_{04}					0.084	0.089
effect of stratum 5, γ_{05}					0.104	0.087
effect of being in schools in the 1st quartile of average achievement, γ_{06}					-0.234*	0.096
effect of being in schools in the 2nd quartile of average achievement, γ_{07}					-0.052	0.097
effect of being in schools in the 3rd quartile of average achievement, γ_{08}					0.022	0.108
Difference of treatment effect on schools in the 1st quartile, γ_{09}					-0.561**	0.174
Difference of treatment effect on schools in the 2nd quartile, γ_{10}					-0.473*	0.19
Difference of treatment effect on schools in the 3rd quartile, γ_{11}					-0.623**	0.213
Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value	
School-specific variation, u_{0j}	0.28934	0. 08372	142	352.48917	<0.001	
Student-specific variation, r_{0j}	0.36337	0.13204				

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 7

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling Some Students in Algebra

Fixed Effect	coefficient	S.E.
Intercept of enrolling some students in algebra school, γ_{00}	1.484***	0.042
Effect of enrolling all students in algebra, γ_{01}	0.037	0.064
effect of stratum 2, γ_{02}	0.066	0.049

effect of stratum 3, γ_{03}			-0.036	0.051	
effect of stratum 4, γ_{04}			0.001	0.048	
effect of stratum 5, γ_{05}			-0.131*	0.053	
effect of stratum 6, γ_{06}			-0.025	0.069	
effect of stratum 7, γ_{07}			-0.228*	0.104	
Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.22691	0.05149	660	1288.55534	<0.001
Student-specific variation, r_{0j}	0.37958	0.14408			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 8a

Model-based Estimation of the Heterogeneous Effect of Enrolling All Students in Algebra versus Enrolling Some Students in Algebra (Part 1)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling some students in algebra, γ_{00}	1.466***	0.038
Effect of enrolling all students in algebra, γ_{01}	0.078	0.063
effect of stratum 2, γ_{02}	0.052	0.045
effect of stratum 3, γ_{03}	-0.025	0.047
effect of stratum 4, γ_{04}	0.016	0.044
effect of stratum 5, γ_{05}	-0.036	0.051
effect of stratum 6, γ_{06}	0.026	0.064
effect of stratum 7, γ_{07}	-0.078	0.098
Remaining effect of being in schools with 1% increase on average achievement, γ_{08}	0.00394***	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{09}	0.00496*	0.002

Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.1672	0. 02796	658	1276.57747	<0.001
Student-specific variation, r_{0j}	0.3927	0.15423			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 8b

Model-based Estimation of the Effect of Enrolling All Students in Algebra versus Enrolling Some Students School (Part 2)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling some student in algebra &4th quartile school, γ_{00}	1.573***	0.044
effect of enrolling some students in algebra, γ_{01}	0.370**	0.13
effect of stratum 2, γ_{02}	0.061	0.045
effect of stratum 3, γ_{03}	-0.019	0.047
effect of stratum 4, γ_{04}	0.021	0.044
effect of stratum 5, γ_{05}	-0.045	0.051
effect of stratum 6, γ_{06}	0.015	0.065
effect of stratum 7, γ_{07}	-0.088	0.099
effect of being in schools in the 1st quartile of average achievement, γ_{08}	-0.231***	0.037
effect of being in schools in the 2nd quartile of average achievement, γ_{09}	-0.103***	0.033
effect of being in schools in the 3rd quartile of average achievement, γ_{10}	-0.104**	0.034
Difference of treatment effect on schools in the 1st quartile, γ_{11}	-0.512***	0.154
Difference of treatment effect on schools in the 2nd quartile, γ_{12}	-0.294	0.172
Difference of treatment effect on schools in the 3rd quartile, γ_{13}	-0.461	0.204

Random Effect	S.D.	Variance	*d.f.*	$\chi 2$	p-value
INTRCPT1, u0	0.226	0. 05108	654	1268.46478	<0.001
level-1, *r*	0. 37976	0. 14422			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 10

Model-based Estimation of the Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra

Fixed Effect	coefficient	S.E.
Intercept of enrolling no student in algebra school, γ_{00}	1.273***	0.035
Effect of enrolling some students in algebra, γ_{01}	0.086**	0.031
effect of stratum 2, γ_{02}	-0.022	0.045
effect of stratum 3, γ_{03}	0.024	0.046
effect of stratum 4, γ_{04}	0.057	0.046
effect of stratum 5, γ_{05}	0.084.	0.046
effect of stratum 6, γ_{06}	0.156***	0.046
effect of stratum 7, γ_{07}	0.146***	0.045
effect of stratum 8, γ_{08}	0.215***	0.045

effect of stratum 9, γ_{09}			0.228***	0.046	
effect of stratum 10, γ_{10}			0.199***	0.048	
Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.21445	0.04599	867	1650.18564	<0.001
Student-specific variation, r_{0j}	0.37583	0. 14125			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 11a

Model-based Estimation of the Heterogeneous Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra (Part 1)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no students in algebra, γ_{00}	1.319***	0.038
Effect of enrolling some students in algebra, γ_{01}	0.092**	0.063
effect of stratum 2, γ_{02}	-0.047	0.044
effect of stratum 3, γ_{03}	0.003	0.045
effect of stratum 4, γ_{04}	0.011	0.046
effect of stratum 5, γ_{05}	0.034	0.045
effect of stratum 6, γ_{06}	0.100*	0.046
effect of stratum 7, γ_{07}	0.087.	0.045
effect of stratum 8, γ_{08}	0.148***	0.046
effect of stratum 9, γ_{09}	0.139***	0.047
effect of stratum 10, γ_{10}	0.099*	0.050
Remaining effect of being in schools with 1% increase on average achievement, γ_{11}	0.00193	0.001
Difference in treatment effect with 1% increase on average achievement, γ_{12}	0.000999	0.001

Random Effect	S.D.	Variance	*d.f.*	χ2	p-value
School-specific variation, u_{0j}	0.21435	0. 04595	865	1628.74011	<0.001
Student-specific variation, r_{0j}	0. 37595	0. 14134			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

Table 11b

Model-based Estimation of the Heterogeneous Effect of Enrolling Some Students in Algebra versus Enrolling No Students in Algebra (Part 2)

Fixed Effect	Coefficient	S.E.
Intercept of enrolling no student in algebra &4th quartile school, γ_{00}	1.329***	0.071
effect of enrolling some students in algebra, γ_{01}	0.142*	0.072
effect of stratum 2, γ_{02}	-0.034	0.044
effect of stratum 3, γ_{03}	0.013	0.046
effect of stratum 4, γ_{04}	0.025	0.046
effect of stratum 5, γ_{05}	0.042	0.046
effect of stratum 6, γ_{06}	0.107*	0.047
effect of stratum 7, γ_{07}	0.096*	0.046
effect of stratum 8, γ_{08}	0.164***	0.047
effect of stratum 9, γ_{09}	0.161***	0.048
effect of stratum 10, γ_{10}	0.121*	0.050
effect of being in schools in the 1st quartile of average achievement, γ_{11}	-0.111	0.081
effect of being in schools in the 2nd quartile of average achievement, γ_{12}	0.016	0.081
effect of being in schools in the 3rd quartile of average achievement, γ_{13}	-0.002	0.09
Difference of treatment effect on schools in the 1st quartile, γ_{14}	-0.045	0.087
Difference of treatment effect on schools in the 2nd quartile, γ_{15}	-0.081	0.087
Difference of treatment effect on schools in the 3rd quartile, γ_{16}	-0.062	0.095

Random Effect	S.D.	Variance	*d.f.*	$\chi2$	p-value
School-specific variation, u_{0j}	0.21583	0. 04658	861	1643.2767	<0.001
Student-specific variation, r_{0j}	0.3756	0. 14108			

*** p<0.001, ** p<0.01, * p<0.05, . p<0.1

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