Evolutionary diversification of prey and predator species facilitated by asymmetric interactions

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S4 Appendix. Global asymptotical stability of $(N^*(\mathbf{x}), P_1^*(\mathbf{x}), P_2^*(\mathbf{x}))$.

In this appendix, we use the method of Lyapunov function to prove that if c_i $(i = 1, 2, 3)$ in (26) of main text are positive, then the ecological equilibrium $(N^*(\mathbf{x}), P_1^*(\mathbf{x}), P_2^*(\mathbf{x}))$ of model (25) of main text is globally asymptotically stable in $\mathbf{R}^3_{\dagger} = \{N > 0, P_1 > 0, P_2 > 0\}$. For simplicity of notation, (N^*, P_1^*, P_2^*) is used to instead of $(N^*(\mathbf{x}), P_1^*(\mathbf{x}), P_2^*(\mathbf{x}))$. The Lyapunov function is given by

$$V_3 = b\left(N - N^* - N^* \ln \frac{N}{N^*}\right) + \left(P_1 - P_1^* - P_1^* \ln \frac{P_1}{P_1^*}\right) + \left(P_2 - P_2^* - P_2^* \ln \frac{P_2}{P_2^*}\right). \tag{1}$$

It is easy to show that $V_3 \geq 0$ and the equality holds only if $(N, P_1, P_2) = (N^*, P_1^*, P_2^*)$. The time derivative of V_3 along the solutions of model (25) of main text is given by

$$\begin{aligned}
\frac{dV_3}{dt} &= b\,(N - N^*)\,\frac{1}{N}\frac{dN}{dt} + (P_1 - P_1^*)\,\frac{1}{P_1}\frac{dP_1}{dt} + (P_2 - P_2^*)\,\frac{1}{P_2}\frac{dP_2}{dt} \\
&= b\,(N - N^*)\,(r(x_1) - kN - a(x_1 - x_{21})P_1 - a(x_1 - x_{22})P_2) \\
&\quad + (P_1 - P_1^*)\,(ba(x_1 - x_{21})N - m(x_{21}) - c(P_1 + P_2)) \\
&\quad + (P_2 - P_2^*)\,(ba(x_1 - x_{22})N - m(x_{22}) - c(P_1 + P_2)) \\
&= b\,(N - N^*)\,(-k(N - N^*) - a(x_1 - x_{21})(P_1 - P_1^*) - a(x_1 - x_{22})(P_2 - P_2^*)) \\
&\quad + (P_1 - P_1^*)\,(ba(x_1 - x_{21})(N - N^*) - c(P_1 - P_1^*) - c(P_2 - P_2^*)) \\
&\quad + (P_2 - P_2^*)\,(ba(x_1 - x_{22})(N - N^*) - c(P_1 - P_1^*) - c(P_2 - P_2^*)) \\
&= -bk(N - N^*)^2 - c\left((P_1 - P_1^*) + (P_2 - P_2^*)\right)^2.
\end{aligned} \tag{2}$$

From (2), we can see that if there is a positive ecological equilibrium $(N^*(\mathbf{x}), P_1^*(\mathbf{x}), P_2^*(\mathbf{x}))$, then $dV_3/dt \leq 0$ in $\mathbf{R}^3_{\dagger}$. In addition, it can be seen that $dV_3/dt = 0$ if and only if $(N, P_1, P_2) = (N^*, P_1^*, P_2^*)$. Therefore, by the Lyapunov-LaSalle's invariance principle, we can see that if c_i $(i = 1, 2, 3)$ in (26) of main text are positive, then the ecological equilibrium $(N^*(\mathbf{x}), P_1^*(\mathbf{x}), P_2^*(\mathbf{x}))$ is globally asymptotically stable.