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To my parents, Horst and Lynn Fedderke

In memoriam

Waltraut Lieselotte Fedderke (née Dedekind)

1926–2016

Poets say science takes away from the beauty of the stars—mere globs of gas atoms. Nothing is ‘mere’. I too can see the stars on a desert night, and feel them. But do I see less or more? The vastness of the heavens stretches my imagination. . . . What is the pattern, or the meaning, or the *why*? It does not do harm to the mystery to know a little about it. For far more marvelous is the truth than any artists of the past imagined! Why do the poets of the present not speak of it? What men are poets who can speak of Jupiter if he were like a man, but if he is an immense spinning sphere of methane and ammonia must be silent?

Richard Feynman

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ABSTRACT

Open questions related to the mechanism of electroweak symmetry breaking, the phenomenology and nature of the recently discovered Higgs boson, and the identity of dark matter are leading motivations for the study of physics beyond the Standard Model. Advances in these areas are often made even more compelling by their important implications for our understanding of the cosmology of the early universe, just as ideas from early universe cosmology broaden and enrich our attempts to answer those questions.

This dissertation presents five related studies of particle physics phenomenology which exploit the fruitful interplay of these fields of research: (1) a detailed investigation of how dark matter annihilating to electroweak gauge and/or Higgs bosons, as described in a bottom-up approach via effective operators, can be constrained by indirect detection experiments that bound γ -ray fluxes from the Galactic Centre and from nearby dwarf galaxies; (2) an examination of the direct detection and collider constraints that can be placed on the dimension-5 effective operators coupling fermionic dark-matter WIMPs to the Higgs Portal; (3) a study of the precision electroweak and Higgsstrahlung cross section constraints that a variety of proposed high-energy electron-positron colliders operating in the few-hundred GeV range can place on a simplified singlet-doublet model which UV completes the dimension-5 fermionic Higgs Portal; (4) a study of how the recently proposed cosmological relaxation mechanism can be used to generate the Little Hierarchy in a concrete composite Higgs model, and possible signatures of this framework; and finally, (5) a detailed numerical and analytical investigation of the irruptive production of late-time supermassive particles—a possible candidate for dark matter—during the inflationary epoch of the early universe, in models where those particles become momentarily light or tachyonic during inflation.

CHAPTER 1

INTRODUCTION AND REVIEW

The construction of the Standard Model (SM) of particle physics, pioneered variously by theoretical insight and experimental discovery, is one of the crowning achievements of late twentieth- and early twenty-first-century physics. Yet despite its success, there are many outstanding questions, both intrinsic and extrinsic, which the Standard Model does not address. These include the nature of dark matter (DM), and numerous questions related to the physics of the Higgs sector.

We live at an exciting time for the investigation of these outstanding issues. Dynamic and multifaceted experimental programmes currently underway at both terrestrial and space-based facilities afford hope that new empirical handles may soon have an impact on our understanding of the identity and nature of dark matter. In parallel, the recent success of the current generation of collider experiments in discovering the Higgs boson has supplied us with an entirely new probe of nature. As more data is obtained in the near future, there will undoubtedly be much more to learn about, and discover with, this probe. The Higgs is also fortuitously light enough to be accessible to a variety of proposed colliders which are currently in their planning stages, enabling future high-precision experimental programmes to thoroughly test for signs of new physics in the Higgs sector.

The five studies we present in this dissertation [4–8] explore the interface of these areas of particle physics phenomenology, and their interaction with concepts from early universe cosmology, with the dual focus of understanding how current and future experimental programmes will inform and constrain theory, and how the exchange of ideas among these diverse disciplines leads to novel and intrinsically interesting avenues for research.

This Introduction and Review aims to provide a brief introduction to, and overview of the current status of, the broad research fields which are the subject of the studies presented in later chapters of this dissertation. In Section 1.1, we review the basic construction of the Standard Model, by way of introduction to our discussion in Section 1.2 of some of the

outstanding ‘big questions’ associated with it. The evidence for dark matter, some interesting dark-matter candidates, and how they may be produced and detected, are surveyed in Section 1.3. A detailed discussion of outstanding issues in the Higgs sector, and motivation for their study, is given in Section 1.4. Some elementary background theory related to inflationary cosmology is surveyed briefly in Section 1.5. Finally, in Section 1.6, we outline the remaining content and structure of this dissertation, giving brief summaries of the studies and results which are presented in each of the later chapters.

1.1 The Standard Model

The Standard Model is a renormalizable¹ Yang-Mills [13] quantum field theory constructed around the spontaneously broken [14–19] gauged symmetry group $SU(3)_C \times SU(2)_W \times U(1)_Y$ [20–25]. It describes the interactions of three ‘generations’ of spin- $\frac{1}{2}$ chiral matter fermions with the spin-1 vector bosons which mediate the strong and electroweak forces, as well as the interaction of these particles with the spin-0 Higgs boson [26–30]. In order to establish conventions and introduce the SM before discussing its various shortcomings, we will in this section demonstrate the construction of the SM Lagrangian at tree-level and in unitary gauge.

Our exposition draws heavily from Refs. [31–37]. We follow the conventions of Ref. [35] with respect to two-component fermion notation and matrix index placement, and we will assume in this section that all fermion fields are two-component Weyl fermions which transform in the left-handed $(\frac{1}{2}, 0)$ representation of the Lorentz group [35, 36].²

Each of the three generations of chiral SM fermions is assigned to representations of the

1. From a Wilsonian viewpoint, this is not a deep mystery: only the renormalizable and super-renormalizable operators are, respectively, marginal or relevant [9–12]. Non-renormalizable operators are irrelevant in the Wilsonian sense: they do not impact on the low-energy physics [9–12].

2. A four-component Dirac fermion $\mathbb{F}$ is in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group [35, 36] and, in the Weyl basis, is constructed out of two independent $(\frac{1}{2}, 0)$ two-component fermions ξ and χ^c [35]: $\mathbb{F} \equiv \begin{pmatrix} \xi \\ (\chi^c)^\dagger \end{pmatrix}$.

Table 1.1: Gauge charges and representations for one generation of SM chiral fermion matter content. SM generation indices i run $i = 1, 2, 3$. A superscript c is considered to be an integral part of the field name and merely denotes that the field transforms in the anti-fundamental of the relevant gauge groups. The ν^c states are SM-singlets, which may or may not exist (see discussion in text).

Description	Field Name	$(SU(3)_C, SU(2)_W)_{U(1)_Y}$
Quark doublet	$Q_i \equiv \begin{pmatrix} U_i \\ D_i \end{pmatrix}$	$(\mathbf{3}, \mathbf{2})_{+1/6}$
Down-type quark singlet	$(D^c)^i$	$(\bar{\mathbf{3}}, \mathbf{1})_{+1/3}$
Up-type quark singlet	$(U^c)^i$	$(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$
Lepton doublet	$L_i \equiv \begin{pmatrix} \nu_i \\ E_i \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$
Electron-type singlet	$(E^c)^i$	$(\mathbf{1}, \mathbf{1})_{+1}$
(Neutrino-type singlet)	$(\nu^c)^i$	$(\mathbf{1}, \mathbf{1})_0$

Table 1.2: Names, electromagnetic charges, and masses of the SM fermion and gauge boson mass eigenstates, and the Higgs boson. Data are from Ref. [38]. Per Ref. [38], the light ($q = u, d, s$) quark masses shown here are the current-quark masses in the $\overline{\text{MS}}$ scheme at $\mu \approx 2 \text{ GeV}$ [38]; the $Q = c, b$ quark masses are the running masses evaluated at $\mu = m_Q$ [38]. The quoted top mass is ‘from direct measurements’ [38] and the uncertainties are statistical and systematic, respectively [38]. Apart from the Z , γ , and h , the only particles that are possibly their own anti-particles are the neutrinos (see discussion in text).

Gen.	Particle Name	Symbol	EM Charge	Mass
1	Electron	e^-	-1	$510.9989461(31) \text{ keV}$
	Electron neutrino	ν_e	0	sub-eV
	Up quark	u	$+2/3$	$2.2^{+0.6}_{-0.4} \text{ MeV}$
	Down quark	d	$-1/3$	$4.7^{+0.5}_{-0.4} \text{ MeV}$
2	Muon	μ^-	-1	$105.6583745(24) \text{ MeV}$
	Muon neutrino	ν_μ	0	sub-eV
	Charm quark	c	$+2/3$	$1.27(3) \text{ GeV}$
	Strange quark	s	$-1/3$	96^{+8}_{-4} MeV
3	Tau	τ^-	-1	$1.77686(12) \text{ GeV}$
	Tau neutrino	ν_τ	0	sub-eV
	Top quark	t	$+2/3$	$173.21 \pm 0.51 \pm 0.71 \text{ GeV}$
	Bottom quark	b	$-1/3$	$4.18^{+0.04}_{-0.03} \text{ MeV}$
—	Gluon	g	0	0
	Photon	γ	0	0
	W boson	$W^\pm$	± 1	$80.385(15) \text{ GeV}$
	Z boson	Z	0	$91.1876(21) \text{ GeV}$
	Higgs boson	h	0	$125.09(24) \text{ GeV}$

SM gauge group as shown in Table 1.1. The specific names, electromagnetic charges, and masses of the physical SM fermions in each generation are shown in Table 1.2. The spin-1 vector boson force carriers are the eight gluons G_μ^a ($a = 1, \dots, 8$) of $SU(3)_C$, the three weak bosons $W_\mu^{\tilde{a}}$ ($\tilde{a} = 1, 2, 3$) of $SU(2)_W$, and the hypercharge boson B_μ of $U(1)_Y$. The W^3 and B bosons mix after electroweak symmetry breaking (EWSB) to form the Z boson and photon; the masses and charges of the mass-eigenstate vector bosons are also listed in Table 1.2.

We define Y to be the hypercharge operator, the fundamental-representation matrices for $SU(2)_W$ to be $\tau^{\tilde{a}} \equiv \frac{1}{2}\sigma^{\tilde{a}}$ where $\sigma^{\tilde{a}}$ are the Pauli σ -matrices [34, 39], and the fundamental-representation matrices for $SU(3)_C$ to be $T^a \equiv \frac{1}{2}\lambda^a$ where λ^a are the Gell-Mann matrices [34, 39]. We write the matrix-valued gauge fields $G_\mu \equiv G_\mu^a T^a$, and $W_\mu \equiv W_\mu^{\tilde{a}} \tau^{\tilde{a}}$, define the fundamental-representation gauge-covariant derivative as

$$D_\mu \equiv \partial_\mu - ig_s G_\mu - ig_2 W_\mu - ig_1 B_\mu Y, \quad (1.1)$$

and define the three matrix-valued field strength tensors as³

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig_s [G_\mu, G_\nu], \quad (1.2)$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig_2 [W_\mu, W_\nu], \quad (1.3)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (1.4)$$

Under ‘forward’ $SU(3)_C$, $SU(2)_W$, and $U(1)_Y$ gauge transformations parametrized respectively by α^a , $\beta^{\tilde{a}}$, and γ , a field ψ with hypercharge Y and in the fundamental representation of the simple non-Abelian factors of the SM gauge group, and a field A in the adjoint rep-

3. With only one simple gauge group factor with gauge coupling g and $D_\mu \equiv \partial_\mu - igA_\mu$, we have the field strength tensor $F_{\mu\nu} \equiv \frac{i}{g} [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu]$.

representation of those simple non-Abelian factors (e.g., a field-strength tensor), transform as

$$SU(3)_C : \quad U_3 \equiv \exp[i\alpha^a T^a] \quad \psi \rightarrow U_3 \psi, \quad A \rightarrow U_3 A U_3^\dagger, \quad (1.5)$$

$$SU(2)_W : \quad U_2 \equiv \exp[i\beta \tilde{a} \tau \tilde{a}] \quad \psi \rightarrow U_2 \psi, \quad A \rightarrow U_2 A U_2^\dagger, \quad (1.6)$$

$$U(1)_Y : \quad U_Y \equiv \exp[i\gamma Y] \quad \psi \rightarrow U_Y \psi, \quad A \rightarrow A. \quad (1.7)$$

The matrix-valued gauge fields transform under the same forward rotation as

$$G_\mu \rightarrow U_3 G_\mu U_3^\dagger + \frac{i}{g_s} U_3 \partial_\mu U_3^\dagger, \quad (1.8)$$

$$W_\mu \rightarrow U_2 W_\mu U_2^\dagger + \frac{i}{g_2} U_2 \partial_\mu U_2^\dagger, \quad (1.9)$$

$$B_\mu \rightarrow B_\mu + \frac{1}{g_1} \partial_\mu \gamma. \quad (1.10)$$

The manifestly locally gauge invariant kinetic and gauge-interaction part of the SM Lagrangian for the gauge and fermion fields is then

$$\begin{aligned} \mathcal{L}_{\text{SM}} \supset & -\frac{1}{2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}] - \frac{1}{2} \text{Tr} [W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{(g_s)^2}{16\pi^2} \theta_{\text{QCD}}^0 \text{Tr} [G^{\mu\nu} \tilde{G}_{\mu\nu}] \\ & + \sum_{f=Q,L} i f^\dagger (\bar{\sigma} \cdot D) f + \sum_{f^c=U^c,D^c,E^c} i f^c (\sigma \cdot D) (f^c)^\dagger, \end{aligned} \quad (1.11)$$

where we have suppressed fermion generation indices. We have here also included the QCD θ -term for completeness; $\tilde{G}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$ is the (matrix-valued) QCD dual field strength tensor. The topologically non-trivial structure of the QCD vacuum gives rise to physical effects via this term [40–48].

In this section, we will assume the simplest possible spontaneous electroweak symmetry breaking sector for the theory by adding a doublet of Higgs fields $H \sim (\mathbf{1}, \mathbf{2})_{+1/2}$ on which the $SU(2)_W$ symmetry is realized linearly: $H \rightarrow U_2 H$. Then the most general renormalizable

kinetic and potential terms we can add to the SM Lagrangian for the H field are

$$\mathcal{L}_{\text{SM}} \supset |D_\mu H|^2 + \mu^2 |H|^2 - \lambda |H|^4. \quad (1.12)$$

The potential for H implied here supports a non-zero vacuum expectation value (vev) if $\mu^2, \lambda > 0$.

We will also need to write Yukawa couplings of the SM fermion fields to the H field to achieve fermion masses when the H gets a vev. In order to do this, we must additionally define the opposite-hypercharge field $\widetilde{H} \sim (\mathbf{1}, \mathbf{2})_{-1/2}$:

$$\widetilde{H} \equiv i\sigma^2 (H^\dagger)^T, \quad (1.13)$$

where ^{T} denotes transpose in $SU(2)$ index space; in component form, $\widetilde{H}_i \equiv \epsilon_{ij} (H^\dagger)^j$, where $\epsilon_{12} = \epsilon^{12} = -\epsilon^{21} = -\epsilon_{21} = +1$ is the anti-symmetric symbol.⁴

We add to the SM Lagrangian the following Yukawa couplings to give masses to the quarks and charged leptons upon EWSB:

$$\mathcal{L}_{\text{SM}} \supset -D^c Y_D H^\dagger Q - U^c Y_U \widetilde{H}^\dagger Q - E^c Y_E H^\dagger L + \text{h.c.}, \quad (1.14)$$

where we have suppressed indices: $Y_{D,U,E}$ are generic complex 3×3 matrices in generation space.

Observations of neutrino oscillations [49–51] tell us that neutrinos have non-zero mass [38].⁵ The mechanism of the generation of these masses is however not presently understood, and depends on the nature of the neutrino (see, e.g., ch. 14 of Ref. [38], or Refs. [53–55] for reviews). From a phenomenological perspective, we can fairly straightforwardly accommo-

4. This is distinct from, and should not be confused with, the Lorentz-indexed invariant symbol $\epsilon^{\alpha\beta}$ ($\epsilon^{\dot{\alpha}\dot{\beta}}$) which plays a role in the fermion field algebra [35].

5. Strictly speaking, experimental observations only indicate that there are three light neutrino mass eigenstates with distinct masses [38]; there is at present no lower bound on the lightest neutrino mass. So more precisely stated, observations tell us that at least two neutrino masses are non-zero [52].

date neutrino masses in the SM:⁶ if neutrinos are Dirac (which requires that we assume that the sterile neutrino states ν^c exist), then neutrino masses are added to the SM by simply extending Eq. (1.14) to include also $\mathcal{L}_{\text{SM}} \supset -\nu^c Y_\nu \widetilde{H}^\dagger L + \text{h.c.}$, and taking the Yukawa couplings to be exceedingly small (of order 10^{-12}). If neutrinos are Majorana (so we do not have to assume that the sterile ν^c states exist), then from a low-energy phenomenological point of view, one can simply add a non-renormalizable dimension-5 operator [56] which will give rise to (Majorana) neutrino masses upon EWSB [35]: $\mathcal{L}_{\text{SM}} \supset -\Lambda^{-1}(\widetilde{H}^\dagger L)^T \mathcal{Y}(\widetilde{H}^\dagger L) + \text{h.c.}$, where $\mathcal{Y}$ is a 3×3 complex symmetric matrix in generation space, ^{T} denotes transpose in that space, and Λ is some large dimensionful parameter that characterizes the scale at which the mechanism (here unspecified) giving rise to this dimension-5 effective operator is operative. Possible UV-completions (e.g., the seesaw mechanism [57]) and further discussion of this scenario lie beyond the scope of this dissertation (see, e.g., ch. 14 of Ref. [38], or Refs. [53–55, 57]).

As our interest in this section is merely to demonstrate the basic structure and some elementary results of the theory, we will omit discussion of the general R_ξ -gauge regularization and renormalization procedures for the Standard Model (see, e.g., Refs. [31–33, 48, 58–61]), and instead make the dramatically simplifying unitary gauge choice directly in the Lagrangian. Let us write without any loss of generality

$$H(x) \equiv \frac{1}{\sqrt{2}} \exp \left[-\frac{i}{v} \xi^{\tilde{a}}(x) \tau^{\tilde{a}} \right] \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \quad (1.15)$$

where $h(x)$ is a spin-0 neutral quantum field and $v \approx 246 \text{ GeV}$ is the vacuum expectation value of the neutral part of the doublet. Note that $|H|^2 = \frac{1}{2}(v + h)^2$ is then independent of the $\xi^{\tilde{a}}$: the latter characterize field excursions along flat directions in the potential and are thus the would-be Nambu–Goldstone bosons (NGBs) [14–19] of the spontaneous breaking of

6. Although the distinction is definitional, it is sometimes stated that non-zero neutrino masses are physics beyond the (canonical) SM. While this is certainly true for Majorana neutrinos as a mechanism must exist to generate the dimension-5 mass term discussed in the text, the extension for Dirac neutrinos is somewhat trivial.

the $SU(2)_W \times U(1)_Y$ local gauge symmetry. Making a forward $SU(2)_W$ local gauge transformation with gauge parameters $\xi^{\tilde{a}}(x)/v$ eliminates the $\xi^{\tilde{a}}(x)$ fields from the Lagrangian: they are thus gauged away into (‘eaten by’) the electroweak gauge bosons in order to supply the third (longitudinal) polarizations required for three linear combinations of the four initially massless electroweak vector bosons to gain masses [26–30]. We will make this gauge choice explicit below.

Our penultimate manipulation is to rotate the basis for the quarks and leptons via the usual CKM [62, 63] and PMNS [64–66] manipulations. Following the exposition of Ref. [32], and letting $X \in \{U, D\}$ for the remainder in this paragraph (with X always to be read consistently as either U or D in every formula), it is always possible to write an arbitrary square complex matrix as $Y_X \equiv P_X Q_X y_X Q_X^\dagger$ where P_X and Q_X are unitary matrices in generation space and y_X are diagonal matrices whose entries are the real, non-negative square roots of the eigenvalues of the Hermitian matrix $Y_X^\dagger Y_X$. The necessary chiral quark rotations to bring the fields to the mass-eigenstate basis are then given by $X^c \rightarrow X^c Q_X^\dagger P_X^\dagger$ and $X \rightarrow Q_X X$, which, owing to the $U(1)_A$ anomaly [48, 67–70], generate a shift in the QCD θ -angle:

$$\theta_{\text{QCD}}^0 \rightarrow \theta_{\text{QCD}}^0 - \arg \det Y_U Y_D \equiv \theta_{\text{QCD}}. \quad (1.16)$$

We define the CKM matrix $V_{\text{CKM}} \equiv Q_U^\dagger Q_D$; this matrix will appear in the weak interaction terms of the mass-eigenstate U and D quarks with the $W^\pm$ bosons (and, naturally, with the would-be NGBs in the H doublet, in non-unitary gauges) to characterize the misalignment of the mass and weak-interaction eigenbases for the quarks (see below).

In the lepton sector, the conventional manipulations are to diagonalize the charged lepton mass terms in Eq. (1.14), and to rotate to the weak interaction eigenbasis for the neutrino sector, because only the ν states (and not any possible ν^c states) couple to the SM gauge group [32]. If $Y_E \equiv P_E Q_E y_E Q_E^\dagger$ (with obvious parallel notation to above) we send $E^c \rightarrow E^c Q_E^\dagger P_E^\dagger$

and $E \rightarrow Q_E E$, which diagonalizes the electron-type lepton mass terms in Eq. (1.14). Sending $\nu \rightarrow Q_E \nu$ then rotates to the eigenbasis for the weak interactions. If neutrinos are Dirac, then the complex 3×3 neutrino Yukawa matrix is $Y_\nu \equiv P_\nu Q_\nu y_\nu Q_\nu^\dagger$ (again with obvious parallel notation to above), and we also send $\nu^c \rightarrow \nu^c Q_\nu^\dagger P_\nu^\dagger$. If neutrinos are Majorana, the situation is different [35]: $\mathcal{Y}$ is a complex symmetric matrix, so it is always possible to write $\mathcal{Y} \equiv (Q_\nu^\dagger)^T y_\nu Q_\nu^\dagger$ where ^{T} denotes matrix transpose in generation space, Q_ν is a unitary matrix, and y_ν is a diagonal matrix whose entries are the real, non-negative square roots of the eigenvalues of the Hermitian matrix $\mathcal{Y}^\dagger \mathcal{Y}$; in this case there are of course no ν^c states to rotate. In either case, the PMNS matrix is defined as $V_{\text{PMNS}} \equiv Q_\nu^\dagger Q_E$, and characterizes the misalignment of the weak-interaction and mass eigenbases for the neutrinos (see below).

We also finally rotate to the mass-eigenstate basis for the vector bosons:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \quad (1.17)$$

where $s_W \equiv g_1/\sqrt{g_1^2 + g_2^2}$, $c_W \equiv g_2/\sqrt{g_1^2 + g_2^2}$, and we will for convenience define $e \equiv g_1 g_2 / \sqrt{g_1^2 + g_2^2}$ and $g_Z \equiv e/c_W s_W = \sqrt{g_1^2 + g_2^2}$. Furthermore, we define

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad \Leftrightarrow \quad \begin{cases} W_\mu^1 \equiv \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \\ W_\mu^2 \equiv \frac{i}{\sqrt{2}} (W_\mu^+ - W_\mu^-) \end{cases}. \quad (1.18)$$

In terms of the mass-eigenstate vector bosons, the covariant derivative Eq. (1.1) can be expressed as

$$D_\mu \equiv \partial_\mu - i g_s G_\mu - i \frac{g_2}{\sqrt{2}} (W_\mu^+ \tau^- + W_\mu^- \tau^+) - i g_Z Z_\mu (\tau^3 - s_W^2 Q) - i e A_\mu Q, \quad (1.19)$$

where $\tau^\pm \equiv \tau^1 \mp i \tau^2$, and $Q \equiv \tau^3 + Y$ is the charge operator.

After these manipulations, the unitary-gauge SM Lagrangian is complete (see also Ref. [37]):

$$\begin{aligned}\mathcal{L}_{\text{SM}} = & \mathcal{L}_{\text{gauge kinetic}} + \mathcal{L}_{\text{gauge cubic}} + \mathcal{L}_{\text{gauge quartic}} \\ & + \mathcal{L}_{\text{fermion kinetic}} + \mathcal{L}_{\text{quark-gauge}} + \mathcal{L}_{\text{lepton-gauge}} + \mathcal{L}_{\text{Higgs}},\end{aligned}\quad (1.20)$$

where, if we define $\bar{V}_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ for any vector boson V (although, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$), we have the gauge kinetic terms (and QCD θ -angle term):

$$\begin{aligned}\mathcal{L}_{\text{gauge kinetic}} \equiv & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\bar{Z}_{\mu\nu}\bar{Z}^{\mu\nu} - \frac{1}{2}\bar{W}_{\mu\nu}^+\bar{W}^{-,\mu\nu} - \frac{1}{4}\bar{G}_{\mu\nu}^a\bar{G}^{a,\mu\nu} \\ & - \frac{(g_s)^2}{32\pi^2}\theta_{\text{QCD}}G^{a,\mu\nu}\tilde{G}_{\mu\nu}^a;\end{aligned}\quad (1.21)$$

the cubic and quartic gauge-boson self-interaction Lagrangians:

$$\begin{aligned}\mathcal{L}_{\text{gauge cubic}} = & ie \left[F^{\mu\nu}W_\mu^+W_\nu^- + A^\mu \left(\bar{W}_{\mu\nu}^-W^{+\nu} - \bar{W}_{\mu\nu}^+W^{-\nu} \right) \right] \\ & + ig_Z c_W^2 \left[\bar{Z}^{\mu\nu}W_\mu^+W_\nu^- + Z^\mu \left(\bar{W}_{\mu\nu}^-W^{+\nu} - \bar{W}_{\mu\nu}^+W^{-\nu} \right) \right] \\ & - \frac{g_s}{2}f^{abc}\bar{G}^{a,\mu\nu}G_\mu^bG_\nu^c,\end{aligned}\quad (1.22)$$

$$\begin{aligned}\mathcal{L}_{\text{gauge quartic}} = & \frac{g_2^2}{2} \left[(W^+ \cdot W^+)(W^- \cdot W^-) - (W^+ \cdot W^-)^2 \right] \\ & - g_2^2 c_W^2 \left[(Z \cdot Z)(W^+ \cdot W^-) - (Z \cdot W^+)(Z \cdot W^-) \right] \\ & - e^2 \left[(A \cdot A)(W^+ \cdot W^-) - (A \cdot W^+)(A \cdot W^-) \right] \\ & - eg_Z c_W^2 \left[2(A \cdot Z)(W^+ \cdot W^-) - (A \cdot W^+)(Z \cdot W^-) - (Z \cdot W^+)(A \cdot W^-) \right] \\ & - \frac{g_s^2}{4}f^{abc}f^{ade}G_\mu^bG_\nu^cG^{\mu,d}G^{\nu,e},\end{aligned}\quad (1.23)$$

where f^{abc} is the $\mathfrak{su}(3)$ Lie algebra structure factor: $[T^a, T^b] \equiv if^{abc}T^c$; the fermion kinetic terms:

$$\mathcal{L}_{\text{fermion kinetic}} = \sum_{f=U,D,E,\nu} if^\dagger(\bar{\sigma} \cdot \partial)f + \sum_{f^c=U^c,D^c,E^c,(\nu^c)} if^c(\sigma \cdot \partial)(f^c)^\dagger; \quad (1.24)$$

the gauge boson interaction terms with the quarks:

$$\begin{aligned}
\mathcal{L}_{\text{quark-gauge}} = & \sum_{f=U,D} g_s f^\dagger (\bar{\sigma} \cdot G^a) T^a f + \sum_{f=U^c,D^c} g_s f^c (\sigma \cdot G^a) T^a (f^c)^\dagger \\
& + \frac{2e}{3} U^\dagger (\bar{\sigma} \cdot A) U + \frac{2e}{3} U^c (\sigma \cdot A) (U^c)^\dagger - \frac{e}{3} D^\dagger (\bar{\sigma} \cdot A) D - \frac{e}{3} D^c (\sigma \cdot A) (D^c)^\dagger \\
& + g_Z \left(\frac{1}{2} - \frac{2}{3} s_W^2 \right) U^\dagger (\bar{\sigma} \cdot Z) U - \frac{2}{3} g_Z s_W^2 U^c (\sigma \cdot Z) (U^c)^\dagger \\
& + g_Z \left(-\frac{1}{2} + \frac{1}{3} s_W^2 \right) D^\dagger (\bar{\sigma} \cdot Z) D + \frac{1}{3} g_Z s_W^2 D^c (\sigma \cdot Z) (D^c)^\dagger \\
& + \frac{g_2}{\sqrt{2}} U^\dagger V_{\text{CKM}} (\bar{\sigma} \cdot W^+) D + \frac{g_2}{\sqrt{2}} D^\dagger V_{\text{CKM}}^\dagger (\bar{\sigma} \cdot W^-) U; \tag{1.25}
\end{aligned}$$

the gauge boson interaction terms with the leptons:

$$\begin{aligned}
\mathcal{L}_{\text{lepton-gauge}} = & -e E^\dagger (\bar{\sigma} \cdot A) E - e E^c (\sigma \cdot A) (E^c)^\dagger + \frac{g_Z}{2} \nu^\dagger (\bar{\sigma} \cdot Z) \nu \\
& + g_Z \left(-\frac{1}{2} + s_W^2 \right) E^\dagger (\bar{\sigma} \cdot Z) E + g_Z s_W^2 E^c (\sigma \cdot Z) (E^c)^\dagger \\
& + \frac{g_2}{\sqrt{2}} \nu^\dagger (\bar{\sigma} \cdot W^+) E + \frac{g_2}{\sqrt{2}} E^\dagger (\bar{\sigma} \cdot W^-) \nu; \tag{1.26}
\end{aligned}$$

and finally, the Higgs-dependent part of the Lagrangian, which includes also the vector boson and fermion mass terms:

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} = & + \frac{1}{2} (\partial_\mu h)^2 + \frac{\mu^2}{2} (v+h)^2 - \frac{\lambda}{4} (v+h)^4 \\
& + \frac{1}{2} \frac{g_Z^2}{4} (v+h)^2 Z^2 + \frac{g_Z^2}{4} (v+h)^2 W_\mu^+ W^{-,\mu} \\
& - \frac{1}{\sqrt{2}} (D^c y_D D + \text{h.c.}) (v+h) - \frac{1}{\sqrt{2}} (U^c y_U U + \text{h.c.}) (v+h) \\
& - \frac{1}{\sqrt{2}} (E^c y_E E + \text{h.c.}) (v+h) + \mathcal{L}_{\text{neutrino}}, \tag{1.27}
\end{aligned}$$

where

$$\mathcal{L}_{\text{neutrino}} = \begin{cases} -\frac{1}{\sqrt{2}} (\nu^c y_\nu V_{\text{PMNS}} \nu + \text{h.c.}) (v + h) & \text{Dirac neutrinos} \\ -\frac{1}{2\Lambda} (\nu^T V_{\text{PMNS}}^T y_\nu V_{\text{PMNS}} \nu + \text{h.c.}) (v + h)^2 & \text{Majorana neutrinos} \end{cases}. \quad (1.28)$$

The minimization condition for the Higgs potential $-\mathcal{L}_{\text{Higgs}} \supset V(h) \equiv -\frac{1}{2}\mu^2(v+h)^2 + \frac{\lambda}{4}(v+h)^4$ is $\partial_h V|_{h=0} = 0$ which yields the vev $v^2 = \mu^2/\lambda$. We also have the tree-level mass relationships:

$$m_h^2 = 2\mu^2 = 2\lambda v^2, \quad m_f = \frac{y_f}{\sqrt{2}}v, \quad m_W^2 = \frac{g_W^2}{4}v^2, \quad m_Z^2 = \frac{g_Z^2}{4}v^2, \quad (1.29)$$

where the subscript f in Eq. (1.29) refers to any fermion species if neutrinos are Dirac, but excludes the neutrinos if they are Majorana; in this latter case, the neutrino mass-eigenstates are

$$\hat{\nu} \equiv \begin{pmatrix} \hat{\nu}_1 \\ \hat{\nu}_2 \\ \hat{\nu}_3 \end{pmatrix} \equiv V_{\text{PMNS}} \nu \quad \text{and the masses are} \quad m_{\hat{\nu}} = \frac{v^2}{\Lambda} y_\nu. \quad (1.30)$$

Using the above relationships, the tree-level Higgs potential takes the form

$$V(h) = V_0 + \frac{1}{2}m_h^2 h^2 + \frac{1}{3!} \left(\frac{3m_h^2}{v} \right) h^3 + \frac{1}{4!} \left(\frac{3m_h^2}{v^2} \right) h^4. \quad (1.31)$$

The SM is a theory with either 26 or 28 free parameters: nine quark and charged-lepton Yukawa couplings; three (phenomenological) neutrino masses;⁷ the Higgs mass and vacuum

7. This refers to the three diagonal entries in the matrices y_ν for either case of Dirac or Majorana neutrinos. Note that in the Majorana case, Λ is not an independent physically meaningful parameter from the viewpoint of the low-energy physics: there is an invariance in the dimension-5 Majorana mass term under the rescaling $\Lambda \rightarrow c\Lambda$ and $y_\nu \rightarrow y_\nu/c$ that reduces the number of physically meaningful parameters in the low-energy theory from four (Λ , and the three diagonal entries of y_ν) to three (the three physical masses). Of course, in this latter case, new physics beyond the SM must exist to explain the origin of the dimension-5 neutrino mass term at the scale Λ , so this parameter counting may not be a strictly accurate reflection of the number of underlying free parameters, which is why we have referred to the free parameters as phenomenological neutrino masses within the context of the SM.

expectation value; three gauge couplings; three angles and a phase for the CKM mixing matrix;⁸ three angles, one Dirac phase and possibly 2 Majorana phases for the PMNS mixing matrix;⁹ and the QCD θ -angle. Of these parameters, the ones we currently have the least information about are the PMNS phase(s), and the absolute sizes of the neutrino masses. Global fits to all available precision electroweak data (when including multi-loop corrections to the exposition given above) indicate that the SM is consistent with such measurements with a p -value of around 0.21 [76]. Currently, there are no significant discoveries ($\geq 5\sigma$) of new physics beyond the Standard Model at high-energy collider experiments, although a variety of anomalies at lower statistical significance do exist.

1.2 The Substance of Things Hoped For: Outstanding Questions

Although the Standard Model is an extremely successful description of available high-energy collider experimental data, there are a number of outstanding ‘big questions’ which it does not solve. There are also number of essentially aesthetic problems with its construction which lead us to believe that there should be physics beyond the Standard Model. In this section we aim to overview some of the outstanding issues with the SM; we will expand at length on the two issues which form main themes of this dissertation, dark matter and Higgs physics, in Sections 1.3 and 1.4, respectively.

8. The CKM matrix is a 3×3 complex unitary matrix, parametrized by three angles and six phases. Individual quark field rephasings $X_i \rightarrow e^{i\alpha_i} X_i$ and $(X^c)^i \rightarrow e^{-i\alpha_i} (X^c)^i$ for $X \in \{U, D\}$ make five of the CKM phases unphysical as they can be absorbed into quark-field phase redefinitions. (An overall phase rotation on all quark fields does not effect any change on the CKM matrix as the action of such a rotation on the CKM matrix is conjugation by an identity matrix times a phase.) There are thus four degrees of freedom left in the CKM matrix: three angles and a phase. The phase violates CP in the quark sector. See, e.g., Refs. [71–74] and ch. 14 of Ref. [38] for detailed discussion of the points in this footnote.

9. The PMNS matrix has four degrees of freedom if neutrinos are Dirac (three angles and a phase), by exactly the same argument as for the CKM matrix; see footnote 8. If neutrinos are Majorana, two additional phases are possible [71–74]: the PMNS matrix initially has nine degrees of freedom (three angles, six phases), but rephasings $\nu_i \rightarrow e^{i\alpha_i} \nu_i$ and $E_i \rightarrow e^{i\alpha_i} E_i$ make three phases unphysical by absorbing them into the neutrino and electron field phases. The PMNS matrix ends up with six degrees of freedom: three angles and three phases. One of these phases is analogous to the Dirac PMNS matrix phase; the other two are called ‘Majorana phases’. The phase(s) violate CP in the lepton sector [75]. See, e.g., Refs. [71–74] and ch. 14 of Ref. [38] for detailed discussion of the points in this footnote.

Gravity. The SM as described in Section 1.1 most obviously omits gravity. The reconciliation of the principles of quantum mechanics and General Relativity (GR) is a major open research question. In practice, this means that the SM is a poor description of scenarios where energy densities are sufficiently large that gravitational effects are no longer negligible on the length and time scales of relevance for particle physics; i.e., there is a natural UV cutoff of the SM at the Planck Scale, $M_{\text{Pl.}} = 1/\sqrt{G_{\text{N}}} \approx 1.22 \times 10^{19} \text{ GeV}$ [38], beyond which we know not to trust its predictions.

Flavour and Neutrinos. As is readily apparent from Section 1.1, we do not as yet have a good understanding of the flavour structure of the SM, or of the underlying structure of the neutrino sector. Of the 26 or 28 free parameters in the SM, all but six are related to these two questions. The hope is that there are underlying organizing principles that provide a reason for these parameters to take their values, and which clarify some of the bigger questions here:

- We do not understand a deeper reason for why there are three generations of fermions; this is simply an empirical fact must be accommodated in the SM.
- The Majorana [77] or Dirac [78] fermion nature of the neutrinos is unsettled. Observation of neutrinoless double β -decay would point decisively to the former [79–81]; see, e.g., Ref. [82] for a recent review of experimental progress in this area.
- Observations of three-flavour neutrino mixing [49–51] imply that neutrinos have non-zero mass (see footnote 5, and also ch. 14 of Ref. [38] and references therein). Direct laboratory experiments [83, 84] and cosmological data [85, 86] thus far only provide upper bounds on the absolute size of the neutrino masses: for the effective¹⁰ electron anti-neutrino mass, the effective muon neutrino mass, and the effective tau neutrino

10. The effective masses are defined as $m_l^{\text{eff.}} \equiv \left[\sum_i |V_{\text{PMNS}}^{l,i}|^2 m_i^2 \right]^{1/2}$, where $m_l^{\text{eff.}}$ is the effective mass of the (anti-)neutrino of flavour l and m_i is the mass of the i th mass-eigenstate neutrino [38].

mass, the current direct upper bounds are around 2 eV, 190 keV, and 18 MeV, respectively [38]; the cosmological bounds are much more stringent, restricting the sum of the three SM neutrino masses to be less than around 200 meV [38]. Oscillations are only sensitive to differences in squared neutrino masses; from oscillation data, we know that one pair of the neutrino mass eigenstates have very similar masses, while the mass of the third eigenstate is either significantly higher (the ‘normal hierarchy’) or significantly lower (the ‘inverted hierarchy’) than that of the nearby pair [38]. We do not know which hierarchy is realized in Nature [38]. The mechanism that gives rise to neutrino masses is not known; if neutrinos are Majorana, the seesaw mechanism [57] may provide a solution here, and to the related question of why the masses are so small.

- It is unknown if there are only three neutrino species, or if additional massive, sterile (SM-singlet) neutrinos exist [87].¹¹ If they do, they can potentially mix with the ‘active’ SM neutrino species, in which case the PMNS matrix is no longer unitary (see also ch. 14 of Ref. [38]). Such sterile neutrinos could be a dark-matter candidate (see Section 1.3.3).
- The mass spectrum in the quark and charged-lepton sector is not understood. While the Higgs mechanism allows those particles to take non-zero masses, their sizes are individually controlled by the free Yukawa couplings; we do not understand why the top quark Yukawa is $\mathcal{O}(1)$, while the electron Yukawa is $\mathcal{O}(10^{-6})$.
- The values of the CKM and PMNS angles are measured [38], as is the CP-violating phase in the CKM matrix [38]; effort is under way to measure the CP-violating Dirac phase of the PMNS matrix (see, e.g., Refs. [88–91]).¹² All these parameters must however simply be accommodated in the SM; we do not understand the reason for the

11. It is known that the number of light SM-like neutrino species is three [38].

12. The possible Majorana phases are much harder to measure, as their only impact on oscillations is on highly suppressed ($\propto m_\nu/E_\nu \ll 1$) lepton-number-violating oscillations [71–75, 92]. They are also present in the effective mass m_{ee} which governs the amplitude of neutrinoless double- β decay amplitudes [75, 81, 93, 94], but are extremely hard to extract [95].

values they take, nor why in the quark sector the CKM matrix appears hierarchical (diagonal elements largest, with off-diagonal elements smaller) [38], while in the lepton sector the PMNS matrix appears to have fairly random entries [38].

Strong CP. The QCD θ -angle is experimentally constrained to be $\theta_{\text{QCD}} \lesssim 10^{-10}$ from bounds on the neutron electric dipole moment [96]; θ_{QCD}^0 is however a free parameter in the SM Lagrangian that should naturally be $\mathcal{O}(1)$, and is not expected to be fine-tuned against the Yukawa phases [see Eq. (1.16)] to 1 part in 10^{10} . Although the mechanism of an axion [97, 98] associated with the spontaneous breaking of a global $U(1)_{\text{PQ}}$ Peccei–Quinn symmetry [99, 100] at a high scale [101–104] can cause the effective QCD θ -angle to dynamically relax to zero, it is not known if this is the solution to this problem, partly because no such axion has yet been detected.

Gauge Unification. The SM gauge group is a semisimple Lie group consisting of three simple factors; this requires the independent specification of three gauge couplings g_1 , g_2 , and g_s . ‘Grand unification’ efforts to embed the SM gauge group in some larger group (spontaneously broken at some high scale) with a single unified coupling constant have a long history [105, 106]. Without additional field content, in the SM the gauge couplings do not unify as one runs them up to higher energy scales via the renormalization group; however, in supersymmetric (SUSY) extensions of the SM, all three gauge couplings can be made to run very close to the same value at an energy scale around 10^{16} GeV [107–110], at which point it may be natural to suppose that some larger gauge structure with a single coupling manifests itself. It is an open question whether or not any such unified structure is actually realized in Nature.

Baryon Asymmetry. The baryon abundance inferred from cosmological measurements (see Section 1.3 for discussion and references) implies an asymmetry between baryons and anti-baryons whose magnitude cannot be explained within the SM. The origin of the asym-

metry is a major open question. Mechanisms which can satisfy the Sakharov conditions [111] and generate the requisite baryon asymmetry through physics beyond the SM include electroweak baryogenesis, and leptogenesis; see Refs. [112] and [113], respectively, for reviews.

Dark Matter and Dark Energy. A wide variety of cosmological and astronomical observations indicate that the overwhelming majority—95.1% [114]—of the energy density in the universe does not consist of ordinary baryonic matter. We will discuss this issue in detail in Section 1.3.

The Higgs Sector, Electroweak Symmetry Breaking, and the Hierarchy Problem. There are a number of outstanding issues related to the Higgs sector and the mechanism of EWSB; we will discuss these issues in detail in Section 1.4.

1.3 The Evidence of Things Not Seen: The Dark Universe

Current evidence from the Planck satellite’s observations [114] of the temperature anisotropies [115] in the relic Cosmic Microwave Background (CMB) radiation [116, 117] indicates that the kind of ‘baryonic’ matter described by the Standard Model comprises 4.9% [114] of the total energy density of the universe today. A ‘dark energy’ component with an equation of state $w = -1.006(45)$ [114] (i.e., an effective cosmological constant) comprises 69.2% [114] and is responsible for causing the accelerating expansion of the universe [118, 119], while a non-relativistic ‘dark matter’ component which interacts extremely weakly with baryonic matter and photons comprises the remaining 25.9% [114].¹³ These two dark components, totaling 95.1% of our universe in the large, are unambiguous evidence for physics which lies beyond the SM. The dark-matter component will be one of the main topics of this dissertation; we will not comment further on the question of dark energy.

13. To be precise, dark matter is defined as a component of the total energy density of the universe which redshifts like matter, i.e., $\rho \propto a^{-3}$ where a is the FLRW scale factor [120], but which does not couple strongly to SM particles, and is not ordinary or ‘baryonic’ matter. A cosmological constant is equivalent to a component of the total energy density of the universe which does not redshift, $\rho \propto a^0$.

In this section, we aim to survey the wealth of evidence we currently have for the existence of dark matter, some of the vast number of ideas about what dark matter may be, how such dark-matter candidates are produced, and how current experiments attempt to detect their interactions with the SM, if they exist.

1.3.1 Observational Evidence for Dark Matter

Among the earliest evidence for what we now refer to as dark matter arose from Zwicky’s studies in the 1930s of the velocity dispersion of the constituent galaxies of the Coma Cluster [121, 122]. Making use of the virial theorem, Zwicky was able to use the observed dispersion to determine the total cluster mass and thereby infer a mass-to-light ratio for the Coma Cluster many times larger than expected [121, 122]; this strongly indicated that a significant amount of non-luminous matter was present in the cluster. It was not however until the 1970s and the work of Freeman [123], Rubin and Ford [124], and others¹⁴ on galactic rotation curves (see below) that the idea of dark matter as we currently understand the term gained widespread traction. With the advent in the 1990s of observational cosmology as a precision science, modern measurements have placed the evidence for dark matter on a firm footing.

In this subsection, we will briefly survey and summarize the present enormous body of observational evidence for dark matter that exists over a wide range of scales, from the size of (dwarf) galaxies, up to the scale of the observable universe (see Refs. [38, 127–136] for recent reviews and pedagogical introductions, many of which guide our discussion in this subsection).

Dynamics of Galaxies. On the smallest scales, study of stellar and gas dynamics over a wide range of galaxy classes yields strong evidence for a significant dark-matter contribution to galactic total masses [127]. In spiral galaxies, stars and gas are observed to orbit the galactic centre with orbital speeds that rise to a plateau that remains roughly constant

14. See Refs. [125, 126] for detailed historical review and additional references.

out to very large galactocentric distance r (see, e.g., Ref. [137] for a review of these ‘flat galactic rotation curves’), in stark contrast with the expectation based on the inferred mass distribution of the luminous matter: that the orbital speeds should instead fall as $v \propto r^{-1/2}$ for stars and gas on the outskirts of such galaxies. For non-rotating systems (e.g., elliptical or dwarf galaxies), velocity dispersions are measured to be higher than would be expected on the basis of the mass inferred from luminous matter alone [127, 138], in much the same fashion as Zwicky’s original analysis for galaxy clusters. These galactic-scale discrepancies between measured velocities and the expectations inferred from the amount of luminous matter present are resolved if galaxies lie within large, extended halos of dark matter which supply the additional required mass.

Dynamics of Galaxy Clusters. Modern studies of galaxy clusters confirm Zwicky’s observation [121, 122] that the velocity dispersions of their constituent galaxies greatly exceed expectations based on the observed amount of luminous matter alone [127]. Indeed, recent measurements of the Coma Cluster indicate that only about 15% of the mass required to account for the observed velocity dispersion is present in the (sub-dominant) galaxy and (dominant) gas components; the remaining 85% of the mass is inferred to be dark matter [127, 139].

Strong and Weak Lensing. Galaxies and galaxy clusters can strongly lens single objects which lie behind them (as viewed from Earth), yielding multiple images of the more distant object [140, 141]. Both clusters and galaxies can also weakly lens fields of more distant objects, imparting small but characteristic distortions to the images of the more distant field of objects [142]. In all cases, the lensing effect is sensitive to the total mass of the object causing the lensing. Significant amounts of dark matter are required to account for the observed lensing potentials of a broad range of objects [143].

The Bullet Cluster, and Similar Cluster Mergers. A dramatic piece of evidence for dark matter that it is often cited in a class of its own is the Bullet Cluster (1E 0657-56) [144–146]. This cluster of galaxies consists of two smaller subclusters that collided with each other about 10^8 years ago [145]. Visible observations show that the individual point-like galaxies in the subclusters simply passed through each other [144, 145]. X-ray observations indicate that the fluid-like gas clouds in the subclusters were slowed as they interacted [145]; they thus trail the visible subclusters, and there is a clearly visible bow shock structure in the gas, indicative of the merger [144]. Lensing measurements provide the key additional observation: while the gas component in each subcluster is much more massive than the galaxy component, the majority of the total mass of each subcluster is still centred on the visible galaxies, instead of on the gas [145]. This is easily explained if each subcluster also contains a significant non-luminous component of dark matter that did not interact (except gravitationally) during the merger [145] (see Refs. [147, 148] for self-interaction limits). There are almost no other plausible explanations for these observations. While the Bullet Cluster is an especially dramatic and early example, other similar cluster mergers have since been discovered, and similar conclusions can be drawn (see, e.g., Refs. [149–151]).

X-ray Emission in Galaxies and Clusters. As already mentioned, galaxy clusters (as well as large elliptical galaxies such as M87) contain (or are enveloped by) clouds of hot plasma that radiate in the X-ray region. Assuming hydrostatic equilibrium, measurements of the temperature and density profiles of the gas allow the total mass of the cluster or galaxy to be inferred [127, 152]. The results of this independent estimate of the amount of dark matter in these systems agree well with other estimates indicating that a significant amount of dark matter is present [127, 152].

Structure Formation. The ‘bottom-up’ formation of structure in the universe via hierarchical mergers (i.e., smaller structures form first, then merge to form larger ones) would be incapable of explaining the degree of structure observed today unless the universe contains,

in addition to the known luminous matter, a much greater mass of non-luminous dark matter [127, 135, 153, 154]. (The matter power spectrum, our probe of structure in the universe, can be inferred from CMB measurements, galaxy cluster abundance, galaxy surveys, weak lensing, and measurements of the Lyman- α Forest [155].) Moreover, observed structure in the universe leads us to conclude that dark matter is cold (or, at least, not too warm), as dark-matter free-streaming would have washed out the small-scale density perturbations in the early universe which are required to seed small-scale structure observed via Lyman- α measurements [156–159]. Direct large-scale N -body simulations of cold dark matter in a Λ CDM universe¹⁵ yield evolution and large-scale structure in statistical agreement with observations [135, 160]. There are however a number of potential anomalies in observed dwarf-galactic-scale structure as compared to numerical predictions, such as the ‘core vs. cusp’ [161], ‘missing satellites’ [162], and ‘too big to fail’ [163] problems, which may be hints that modifications [164] of the standard cold-dark-matter paradigm at these small length scales may be required [165] (although this is not a settled issue [166]).

The Cosmic Microwave Background. The Cosmic Microwave Background radiation [116, 117] carries imprints of acoustic oscillations of the baryon-photon plasma which existed in the early universe before matter-radiation decoupling [153, 154], by way of small $\mathcal{O}(10^{-5})$ anisotropies [115] in the 2.7255(6)K [167] temperature, and in the polarization, of the surface of last scattering. The angular power spectra of these anisotropies show a series of peaks and troughs, with the peaks corresponding to modes at compression or rarefaction extrema of the plasma oscillations [153, 154]. The TT power spectrum is now well measured up to $l \sim 3000$ [168, 169] or beyond [170], clearly showing a low- l Sachs-Wolfe [171] plateau, nine full acoustic peaks (and part of the tenth), and a high- l Silk [172] diffusion damping tail [153, 154, 168, 169, 173, 174] (the EE polarization power spectrum is measured over a similar range of l , while the BB polarization power spectrum is less well measured [175]).

15. That is, a universe including the effects of matter, radiation, cold dark matter and a cosmological constant.

The relative amplitudes and positions of the peaks, and the strength and location of the damping tail, allow a great deal of information to be derived about the dynamics of the acoustic oscillations, and as a result the energy budget of the universe (see, e.g., Refs. [153, 154, 173, 174]). A coherent picture emerges from fitting a Λ CDM cosmology to the full power spectrum: a flat universe with 4.9% baryonic matter, 25.9% cold dark matter, and 69.2% dark energy [114]. This picture is also remarkably consistent [114] with other probes of these quantities, such as Type Ia supernova measurements [118, 119, 176], Baryon Acoustic Oscillations (BAO) [177, 178] and other structure observables. The cosmological 5.3-to-1 ratio of dark to baryonic matter also agrees well with the dark-to-baryonic mass ratio of about 5.7-to-1 inferred for the Coma Cluster [114, 139].

Big Bang Nucleosynthesis (BBN). The formation of the light elements in the hot and dense early universe [179] is an exquisitely sensitive probe of conditions long before CMB last scattering, and provides a stringent constraint on the amount of baryonic matter in the universe [38, 120, 180]. This knowledge can be combined with the amount of total cold matter inferred from other sources to strengthen the conclusion (now also clear from CMB data alone [114]) that there is a large non-baryonic component to the matter mass budget of the universe [133], and that the dark matter cannot simply be unobserved baryonic matter [181].

1.3.2 Properties of Dark Matter

The wealth of observations which we have just highlighted also allow us to infer some broad properties of the dark matter, whatever its nature may be:

Massive. The dark matter must be massive. The allowed mass range is however extremely broad: from as low as 10^{-22} eV [182–184] up to above 10^{13} GeV [185–190].

Stable. Dark matter must be stable¹⁶ on cosmological timescales in order to have both impacted the dynamics of early-universe structure formation and of the baryon-photon plasma as observed from the CMB surface of last scattering, and to still be present in the universe today impacting the dynamics of objects which are as nearby as dwarf galaxies. Moreover, if dark matter were not sufficiently stable, we would generically be able to observe its decay products; see, e.g., Ref. [191]. These considerations place limits on the lifetime of the dark matter to be much longer than the age of the universe; see, e.g., Ref. [192].

Non-baryonic. CMB, BBN, and other structure observables indicate that dark matter must be dominantly non-baryonic.

Not too warm (‘cold’). The dark matter must be sufficiently non-relativistic early in the history of the universe so as not to wash out observed small-scale Lyman- α structure [156–159].

Neutral and uncoloured. The dark matter must be electrically and colour neutral in order for it to be sufficiently ‘dark’; fractional EM-millicharge may however be allowed (see, e.g., Ref. [193]).

Cannot self-interact too strongly. If the dark matter self-interacted too strongly, the dynamics of structure formation, and collisions such as that which occurred in the Bullet Cluster would be modified [147, 148]. By particle physics standards, the limits on the self-interaction cross section are actually quite weak: $\sigma/m \lesssim 1 \text{ cm}^2/\text{g} \sim 5 \times 10^3 \text{ GeV}^{-3}$ [148].

1.3.3 Dark Matter Candidates

Having summarized the evidence for the existence of the dark matter and the basic properties which it must have, we now turn to the question of what the dark matter may be.

16. We do not mean to say that all particles in any possible DM sector are stable, just that the lightest such state is stable against decay to Standard Model particles.

We take the commonly held position that dark matter is an as-yet undiscovered species of elementary particle (or a sector of such particles). Perhaps the most serious alternative to the assumption of particle dark matter is Modified Newtonian Dynamics (MOND) [194–196] which aims to explain away the ‘missing mass’ issues in galaxies by the agency of low-acceleration modifications to the laws of Newtonian mechanics (see, e.g., Refs. [197, 198] for reviews). While it can be successful in achieving this goal [197, 198], MOND faces a host of other theoretical and observational problems at the cluster and cosmological scales; perhaps most dramatically, explaining the CMB power spectrum, or the observations of the Bullet Cluster and other similar systems constitutes a severe challenge for most MOND models (although some contrary claims have appeared in the literature [199–201]). We will not consider such models further in this dissertation.

In the remainder of this section we will summarize some of the canonical and well-motivated candidates for particle dark matter, commenting on production and detection mechanisms where appropriate; we will mention some of the more esoteric candidates. The DM literature is vast and our survey is not intended to be exhaustive; our focus will be on WIMP and, to some extent, axion candidates. For reviews see, e.g., Refs. [131, 136].

Weakly Interacting Massive Particles (WIMPs). WIMPs are a class of cold thermal-relic dark-matter particles with weak-scale masses and weak-scale non-gravitational interaction cross sections with SM particles (for a recent review, see Ref. [202]). They are produced in thermodynamic equilibrium with the SM in the early universe by means of $2 \rightarrow 2$ interactions of the WIMPs with the SM [120, 202]. Their present-day abundance is fixed by the thermal freeze-out of the $2 \rightarrow 2$ interactions as the universe expands and cools; higher WIMP–SM interaction cross sections yield lower final abundances, as thermodynamic equilibrium can be maintained longer and the equilibrium abundance of non-relativistic species is suppressed by $e^{-m/T}$ [120, 202]. Thermal freeze-out of weak-scale particles with weak-scale cross sections happens to accidentally yield approximately the observed cosmological

DM abundance [120, 202], which is sometimes referred to as the ‘WIMP miracle’ [202]. Thermal WIMPs are usually considered to lie in the approximate mass range from a few GeV to about a little over a hundred TeV, where the upper end of the range is restricted by the requirement of perturbative unitarity in the annihilation cross section needed to obtain the measured DM abundance [203]. The lower bound is somewhat conventional; for light fermionic DM with weak-scale annihilation cross sections, taking the mass too light results in too large a relic abundance (the Lee–Weinberg bound [204, 205]), but the same considerations do not necessarily apply for scalar WIMP DM [206].

Well-motivated extensions of the SM whose *raison d’être* is not the solution of the DM problem nevertheless provide natural WIMP DM candidates: for example, in R -parity conserving SUSY, the lightest supersymmetric particle (LSP) is stable against decay [207, 208], and can easily have the correct mass and cross section. Similarly, the lightest Kaluza-Klein resonance (LKP) associated with the existence of universal extra dimensions [209] is a natural WIMP candidate, and is stable owing to KK-parity (see, e.g., Ref. [210] for a review). Generic expectations of weak-scale new physics owing to Naturalness arguments (see Section 1.4), and the fact that current colliders operate at this scale, give additional motivation to study WIMPs even in more simplified or bottom-up approaches, which will be the context in which they appear in this dissertation.

The WIMP–SM couplings which are required to fix the relic abundance via thermal freeze-out generically imply observable present-day signatures of WIMP dark-matter candidates; this is a key feature of the WIMP paradigm.

Direct detection [211] of WIMP dark matter aims to observe the recoil energy deposition in a detector from the elastic scattering of a Milky Way-halo DM particle off a SM particle (see, e.g., Refs. [212, 213] for recent reviews). Ultra-low background experiments searching for WIMP-induced nuclear recoils include large noble-liquid/gas dual-phase time-projection chambers [214–221], which can also search for WIMP-induced electron recoil [222]. Cryogenic crystalline detectors are also sensitive to nuclear recoils, and typically have lower recoil-energy

thresholds, but operate with smaller active detector masses [223–226]. Bubble chambers are also used, primarily as low-threshold detectors for spin-dependent scattering off protons [227].

Indirect detection of WIMP dark matter aims to observe annihilation and/or decay signals of DM from nearby regions of the cosmos where large present-day abundances of DM are to be found: e.g., the centre of our galaxy, nearby DM-dominated dwarf galaxies, and also possibly our Sun, or even the Earth itself, if DM is efficiently captured¹⁷ (see, e.g., Ref. [230] for a recent review). Signals vary depending on the annihilation and/or decay modes of the DM, and include primary or secondary γ -ray fluxes, which can be observed by ground- or space-based telescopes [231–236]; neutrinos, which can be observed by detectors which monitor large volumes of water or ice for Cherenkov light [229, 237–250], and charged particles (cosmic rays), which in this context are observed by satellite-based detectors [251, 252].

Collider signals for WIMP DM at the Large Hadron Collider (LHC) are based on detecting excess missing transverse energy (MET), as DM produced at colliders would escape the detector without interacting. Typical strategies include ‘mono- X ’ searches (see, e.g., Refs. [253–256]) which look for the signal of MET recoiling against a single photon [257–259], W boson [260, 261], Z boson [260–263], Higgs boson [264, 265], or jet [261, 266]; searches which look for MET in association with, for example, a pair of b jets [267]; or Vector Boson Fusion (VBF) topologies with large MET (and forward jets) [268].

WIMP DM candidates will be the exclusive focus of the chapters of this dissertation which touch on the dark-matter problem. Further discussion and more detailed exposition of production and detection mechanisms can be found in Chapters 2 and 3.

Sub-GeV Dark Matter. Dark matter lighter than about a few GeV is relatively unconstrained by the usual WIMP direct detection experiments owing to detector recoil-energy

17. Note that although the decay products from DM captured in the Sun or Earth are detected by indirect detection experiments, the resulting limits are actually on the direct detection cross section, as the latter controls the DM capture rate in the Sun or Earth; see, e.g., Refs. [228, 229].

threshold limitations in the experiments with the largest active detection volumes [214–216], although some of the dedicated low-threshold experiments are sensitive to DM masses around a GeV [224–226]. On the other hand, Lyman- α limits only dictate that thermally produced DM must be heavier than a few keV to be sufficiently cold [156–159, 269]. It is therefore entirely plausible that DM could be a particle in the few keV to few GeV mass range [269], below the canonical WIMP region, and have thus far evaded detection. Candidates in this mass range usually have their abundance set by processes other than the thermal freeze-out of $2 \rightarrow 2$ DM–SM interactions [270, 271].

One such example is the Strongly Interacting Massive Particle (SIMP) [270, 272], a class of sub-GeV DM candidates whose abundance is set by $3 \rightarrow 2$ annihilation processes in a secluded SIMP sector [270, 272], in which there are strong SIMP self-interactions. Weak SIMP–SM interactions allow the heat generated by the $3 \rightarrow 2$ decays in the SIMP sector to be dumped into the SM sector to avoid the warm DM bounds [270, 272], and also provide the avenue for present-day non-gravitational detection [270, 272]. Strong SIMP self-interactions provide one way to alleviate [164] the ‘core vs. cusp’ and ‘too big to fail’ problems [270, 272].

Another GeV-scale candidate is asymmetric dark matter (ADM) [271] (see, e.g., Refs. [273, 274] for reviews). In ADM models, the present-day DM abundance is fixed by a small asymmetry between DM particles and their anti-particles in the early universe, in much the same way as the baryon asymmetry in standard cosmology [271, 273, 274]. Moreover, ADM models link the DM and baryon asymmetries in such a way as to explain the apparent approximate coincidence (within a factor of about 5) of the dark and baryonic matter-densities in the universe [271, 273, 274]. This implies a mass range for such candidates on the order of a few GeV [269, 271, 273, 274].

A variety of novel ideas have recently been proposed for direct detection experiments with ultra-low thresholds, to allow the minuscule recoil energy deposition of sub-GeV DM to be detected [269, 275–281].

Axions and Axion-Like Particles (ALPs). At the extreme low end of the DM mass spectrum lie axions and axion-like particles: scalar particles with masses $\lesssim 10^{-3}$ eV (see, e.g., Refs. [282–285] for reviews). The originally proposed axion [97, 98] associated with the Peccei–Quinn solution [99, 100] to the strong CP problem has been ruled out [282, 285], but ‘invisible’ QCD axions [101–104] are still excellent non-thermal cold dark-matter candidates. Non-QCD axions are also allowed as a non-thermal cold dark-matter candidate, and are a generic feature of string theory–inspired models [282, 285]. Such particles can even be super-light Fuzzy Dark Matter (FDM) [182–184], with a mass $m_a \sim 10^{-22}$ eV; this implies a de Broglie wavelength approaching galactic length scales, which may alleviate some of the small-scale structure anomalies [182–184].

A model-independent non-thermal production mechanism for axions is ‘vacuum misalignment’ [120, 282, 285]: when PQ symmetry spontaneously breaks in the early universe the axion field will locally (i.e., in each Hubble patch) take a random value in its compact field range. Initially, the large Hubble constant and thermally suppressed axion mass [286] dictate that the axion field value is constant in time. However, as the universe cools through the QCD phase-transition temperature, non-perturbative QCD instanton effects [40–48] increase in importance, leading to an increase in the axion mass. At the same time, the Hubble parameter is decreasing; when the axion mass and Hubble parameter become comparable,¹⁸ the axion field begins to execute coherent oscillations, which redshift as a cold, non-relativistic dark-matter component [120, 282, 285].

18. The homogeneous axion field equation of motion is $\ddot{a} + 3H(T)\dot{a} + m_a^2(T)a = 0$. The thermal axion squared mass is $m_a^2(T) \approx c m_a^2(0)(T/100 \text{ MeV})^{-n}$ (for $T \gg 100 \text{ MeV}$) where $m_a^2(0) = (m_\pi^2 f_\pi^2 / f_a^2) z / (1 + z)^2$ (see, e.g., Ref. [287]) is the zero-temperature axion squared mass with f_a the axion decay constant, $m_\pi \approx 135 \text{ MeV}$ [38] the neutral pion mass, $f_\pi \approx 93 \text{ MeV}$ [38] the pion decay constant, and $z \equiv m_u/m_d \approx 0.5$ [38] the ratio of the up and down quark masses. The scaling power $n \approx 8$ in the dilute instanton gas model [120, 282, 285, 286]; more precise recent computations give a somewhat lower value $n \approx 6.9$ [282, 287, 288], and a coefficient of proportionality $c \approx 1.8$ [282, 287, 288]. The Hubble parameter is $H^2(T) = (8\pi/3M_{\text{Pl}}^2)\rho(T)$ where $\rho(T) \propto T^4$ is given by Eq. (B.4) (see also Section 1.5 and Appendix B.1.1). The axion field starts to oscillate (at a frequency roughly equal to its mass) when $m_a(T_{\text{osc.}}) \sim 3H(T_{\text{osc.}})$ [120, 282, 285]. The initial axion energy density is then $\rho_a^i \approx \frac{1}{2}m_a^2(T_{\text{osc.}})a_i^2$, where $a_i = f_a\theta_i$ is the initial axion field value, and θ_i the ‘initial misalignment angle’. This initial abundance is easily translated into the present-day abundance; see, e.g., Refs. [120, 282, 285].

Estimates for the production abundance depend on the axion decay constant f_a —or, equivalently, its mass $m_a \sim 6 \times 10^{-6} \text{ eV} \times (10^{12} \text{ GeV}/f_a)$ [282, 285]—and initial misalignment angle θ_i [282, 285]. If PQ symmetry was broken during inflation, the initial value of the axion field is uniform in our observable universe owing to inflationary homogenization [282, 285] (see Section 1.5) and can take any random value; in this case, the correct abundance of DM is obtained for QCD-axion masses $m_a \sim (2 \times 10^{-5} \text{ eV}) \times (\theta_i)^{12/7} \lesssim 8 \times 10^{-4} \text{ eV}$ [282]; equivalently, $f_a \sim (3.3 \times 10^{11} \text{ GeV}) \times (\theta_i)^{-12/7} \gtrsim 8 \times 10^9 \text{ GeV}$ [282] (in both cases, the estimate holds for $\theta_i \lesssim 1$, while the bound comes from taking $|\theta_i| \sim \pi$ and accounting for anharmonic effects [282]). If PQ symmetry was broken after inflation, causality demands that the average initial misalignment angle in our presently observable universe obeys $\langle \theta_i^2 \rangle \sim \pi^2/3$ [282, 285], and topological defect decays contribute to the axion abundance [282, 285]. The axion mass range is slightly different in this case: $7 \times 10^{-5} \text{ eV} \lesssim m_a \lesssim 1.2 \times 10^{-2} \text{ eV}$ [282, 289, 290]; equivalently, $5 \times 10^8 \text{ GeV} \lesssim f_a \lesssim 8.5 \times 10^{10} \text{ GeV}$ [282, 289, 290]. Here, the upper (lower) bound on m_a (f_a) is from the SN 1987A neutrino burst duration [289], and the lower (upper) bound on m_a (f_a) is from demanding the axion abundance is not larger than the observed DM abundance [282].

Axion detection is a broad subject, owing to the wide range of possible masses and couplings which both QCD and non-QCD axions can possess (for a recent review, see Ref. [290]). Canonical experiments rely on the axion-photon-photon coupling to convert an axion into a photon in the presence of a strong electromagnetic field via the Primakov effect [291]. They include high- Q resonant microwave cavities permeated by strong magnetic fields, in which the interaction of an axion with the magnetic field would excite a cavity mode [292, 293]; helioscopes, which utilize strong magnetic fields to trigger the conversion of an axion produced in the Sun into an X-ray photon which can be detected [294–297]; and laboratory-based photon regeneration (‘light shining through walls’) experiments, which generically aim to convert photons to axions, then back to photons on the other side of an electromagnetically opaque barrier [290]. Recent novel proposals for axion detection include looking for ax-

ion interactions with nucleon spins using NMR technology [298], ‘molecular interferometry’ experiments [299], and other techniques [300, 301].

Supermassive DM (‘WIMPzillas’). At the extreme upper end of the mass spectrum of dark-matter candidates are supermassive DM particles, the ‘WIMPzillas’ [185–190, 302, 303]. These particles can have a mass in excess of 10^{13} GeV, and as a result must be produced non-thermally in the early universe; a typical mechanism is gravitational particle production [304]. Because WIMPzillas can consistently have no non-gravitational couplings to the SM, their detection at the present epoch is highly model dependent; if they annihilate or decay to any of a variety of highly energetic SM particles, those signatures may be observable [230, 305], but extremely low particle densities (due to large individual particle masses) suppress many signals. Our work in Chapter 6 deals with the gravitational production of superheavy particles which could be WIMPzilla DM candidates.

keV Sterile Neutrinos. Sterile (i.e., SM singlet) neutrinos in the keV to tens of keV range are another possible dark-matter candidate (see, e.g., Ref. [81] for a comprehensive review). The existence of sterile neutrinos is well motivated in many scenarios that aim to resolve the mystery of neutrino mass generation [81]. The lower end of the currently allowed mass range is a hard limit dictated by phase-space packing arguments [81, 306] (as well as the avoidance of Lyman- α structure limits [81]), while the upper limit is experimental and depends on the amount of mixing of the sterile species with the SM. Production in the early universe can, for example, be via oscillation of ‘active’ SM neutrino states into the sterile states, either non-resonantly [81, 307] or via resonant effects [81, 308]; or by decays of scalar singlets [81, 309]. Mixing of the sterile species with the active neutrinos also allows for their astrophysical indirect detection via X-rays emitted in their decay [81, 310–312], and in terrestrial experiments that search for deviations in the shape and endpoint of the electron spectrum in tritium β -decay [81, 313].

Other Candidates and Exotics. Many other candidates for particle dark matter exist. To list just a few other examples: secluded WIMP dark matter, which is essentially thermal-relic WIMP dark matter with suppressed direct detection and collider signals [314]; Feebly Interacting Massive Particles (FIMPs), which are a class of massive (weak-scale or higher) DM candidates that couple so weakly to the SM that they never attain thermal equilibrium in the early universe, and whose abundance is fixed by ‘freeze-in’ [315]; superWIMPS, which are extremely weakly interacting with the SM, and are produced by the late-time decays of frozen-out thermal-relic WIMPs [316, 317]; the spin- $\frac{3}{2}$ gravitino [318, 319], which may be produced in the superWIMP scenario, thermally, or otherwise [320]; atomic dark matter, which consists of bound states of dark-sector particles formed by the agency of a long-range force in a dark sector [321]; millicharged dark matter, with a small fractional electric charge arising from a kinetic mixing of a dark $U(1)$ with $U(1)_{\text{EM}}$ [322–324]; Q -balls, or other non-topological solitonic field configurations [325–327]; MAssive Compact Halo Objects (MACHOs) [328], which are various forms of collapsed baryonic matter which could potentially account for some fraction of galactic dark matter [329] (although cannot possibly account for the non-baryonic cosmological abundance); and CHArged Massive Particles (CHAMPs) [330], which are charged dark-matter candidates severely restricted to be a subdominant component of the dark matter [331]. See, e.g., Refs. [131, 136] for review of other candidates, and further references.

1.3.4 *Open Questions*

As we have seen in the previous subsections, there are a number of open questions related to dark-matter physics, which we can broadly summarize as follows:

- What is the basic nature and identity of dark matter? Is it a particle, or something else? If it is a particle, which is the position this dissertation takes, is it only a single particle species, or is there a whole dark sector?

- What is the dark-matter mass? Or if there is a DM sector, what is the spectrum?
- What is the spin of the DM particle, or particles if there is a dark sector?
- How does the DM couple to the SM other than gravitationally?
- What is the mechanism for its production in the early universe (which is a question tied to its nature and identity)?
- Is it stable, or does it decay? If it does decay, what is its lifetime?

These open questions are among those that the ongoing and broad DM experimental programme aims to answer.

1.4 The Higgs Sector: Issues and Motivation

The recent 2012 discovery of the 125.09 GeV [38] Higgs boson [332, 333] has given us a qualitatively new probe of Nature. Measurements thus far indicate that the gross properties of this particle are as expected in the SM: a state with $J^{PC} = 0^{++}$ [334–338], with couplings to other particles that are broadly in agreement with the predictions of the SM over a wide range of channels and topologies at the LHC [339–341]. Nevertheless, many open questions remain about the Higgs, and the EWSB sector more broadly. This section aims to survey some of these issues, and some well-motivated directions for study in the Higgs sector.

The Hierarchy Problem. Perhaps the biggest conceptual issue of the SM is deeply tied to the EWSB / Higgs sector of the theory: the Hierarchy Problem. At its core, the Hierarchy Problem is that we have no convincing explanation for why the electroweak scale $\sim 10^2$ GeV is so much smaller than the Planck scale $\sim 10^{19}$ GeV, if no new physics lies between the two.

The problem in the SM can be understood as follows: while the fermion and gauge boson mass terms in the SM Lagrangian are protected from large quantum corrections by virtue of the symmetry of the theory being enhanced when the such terms are set to zero (i.e., they are

technically natural),¹⁹ no such protection of the Higgs squared-mass parameter μ^2 exists. As such, μ^2 is subject to large quantum corrections. Indeed, supposing that the Higgs couples to some new-physics particle of mass M , the one-loop corrections to the Higgs squared-mass parameter μ^2 are quadratically sensitive to the mass scale of the new physics: they are of order $M^2/16\pi^2$. If we take the Wilsonian view of the SM as an effective field theory which is a valid description below the scale $M \sim M_{\text{GUT}}$ or $M \sim M_{\text{Pl}}$, then the generic expectation is that the impact of the new physics at that scale is to give quantum corrections to μ^2 of $\mathcal{O}(M_{\text{GUT}}^2)$ or $\mathcal{O}(M_{\text{Pl}}^2)$. Therefore, in order for the Higgs potential to yield the correct weak-scale Higgs vev $\sim 10^2 \text{ GeV}$, a cancellation of the $\mathcal{O}(M_{\text{GUT}}^2)$ or $\mathcal{O}(M_{\text{Pl}}^2)$ correction against the tree-level value of μ^2 must be arranged to an accuracy of roughly 1 part in 10^{28} or 10^{34} . This constitutes an extreme fine-tuning of the fundamental parameters of the theory. While this is not an inconsistency, it is a breakdown of the principle of Naturalness:²⁰ i.e., the idea that the fundamental dimensionless parameters (or ratios of dimensionful parameters) of physical theories should not have to be finely tuned to obtain even gross agreement with observations. Historically, apparent breakdowns of Naturalness have hinted at the existence of an underlying mechanism (see, e.g., Ref. [343] for a recent discussion of the principle of Naturalness, which also discusses a number of examples of this point).

There are traditionally two broad schools of thought when it comes to ameliorating the Hierarchy Problem: (a) provide a mechanism that protects the Higgs squared-mass parameter from receiving large quantum corrections, explaining the separation of scales; or (b) lower the cutoff scale at which the SM ceases to be a good description, so that only milder tunings are required to still obtain the requisite weak-scale Higgs vev.

Supersymmetry is an example of type (a): an enlargement of the underlying spacetime symmetry beyond the Poincaré group—to include fermionic generators—is assumed [207,

19. Gauge boson mass terms break the SM gauge symmetry; SM fermion mass terms also break the SM gauge symmetry because the SM fermions are chiral. Fermion mass terms (even for vector-like—i.e., non-chiral—fermions) additionally break global chiral symmetry.

20. Although it is only the second most dramatic example known: the observed cosmological constant affords an unexplained 120-orders-of-magnitude tuning problem [342].

318, 344–347], which in turn implies the existence of a boson-fermion symmetry [207, 318] which serves to exactly cancel the quadratic sensitivity of the Higgs squared-mass parameter to quantum corrections, by tying μ^2 to the value of a fermion mass (which is protected by chiral symmetry) [207, 348]. Composite Higgs theories (discussed in more detail below) are an example of type (b).

In either case, the Hierarchy Problem and Naturalness arguments present good motivation to search for new physics around the weak scale: natural SUSY theories (e.g., the MSSM) must contain soft SUSY-breaking terms with characteristic scales not too far from the weak scale to avoid reintroducing a tuned hierarchy [207], and so the generic expectation is that (possibly many) superpartner states should lie not too far from the weak scale; in the case of theories which otherwise lower the effective SM cutoff scale, one can search directly for the signs of the new physics around the cutoff, or its effects on precision SM observables.

The Higgs Potential and Couplings. Precision characterization of the couplings and properties of any newly discovered SM particle has always constituted a basic step in testing our understanding of the theoretical mechanisms of the SM: the self-consistency of theory is powerfully probed by making precise measurements of multiple observables predicted by some smaller number of SM parameters. Absent dramatic indications from the LHC of new physics signatures beyond the SM, precision study of the couplings of the Higgs boson to itself, the SM fermions, and the SM vector bosons, is a compelling avenue for progress in the understanding of the EWSB sector.

While the gross properties of the Higgs are consistent with those expected in the SM [339–341, 349–361], couplings to the SM fermions and vector bosons have in the best cases only been constrained by direct measurements at the level of 10–20% [341] using the 7 and 8 TeV data at ATLAS and CMS; see Table 1.3. Direct measurements of the Higgs cubic coupling in the di-Higgs channel, to test for consistency of the SM predictions in the Higgs sector, are challenging [362–376]; by implication, the structure of the Higgs potential is at

Table 1.3: Current and projected percentage precisions for measurements of Higgs coupling constants at the LHC and HL-LHC upgrade. These results are for the usual κ -framework used to describe coupling constant modifications (see, e.g., Ref. [341]): $\kappa_{gZ} \equiv \kappa_g \kappa_Z / \kappa_H$, while $\lambda_{xy} \equiv \kappa_x / \kappa_y$. The current uncertainties for the combined ATLAS and CMS analysis of the 7 and 8 TeV data [341] have been converted to approximate percentage precisions here for illustrative purposes; see Ref. [341] for more detail. The ATLAS [377] and CMS [378] projections for the HL-LHC upgrade assume $\sqrt{s} = 14$ TeV and $\mathcal{L} = 3\text{ab}^{-1}$ of integrated luminosity; the range of percentage precisions shown for these projections correspond to varyingly aggressive assumptions about future improvement in theoretical and experimental systematics (see Refs. [377, 378] for further detail).

	ATLAS+CMS 7+8 TeV [341]	ATLAS HL-LHC [377]	CMS HL-LHC [378]
κ_{gZ}	10%	3–6%	2–5%
λ_{Zg}	17%	6–9%	3–5%
λ_{tg}	16%	6.5–9.5%	6–8%
λ_{WZ}	11%	2.5–3%	2–3%
$ \lambda_{\gamma Z} $	11%	2–2.5%	2–5%
$ \lambda_{\tau Z} $	15%	8–9%	2–4%
$ \lambda_{bZ} $	$\sim 30\%$	8–10%	3–5%
$\lambda_{\mu Z}$	—	6%	7–8%
$\lambda_{(Z\gamma)Z}$	—	14%	12%

present poorly probed.

Projections for the High Luminosity LHC upgrade (HL-LHC) [379] indicate significant improvements in the precision with which the Higgs couplings to fermions and vectors can be determined, with asymptotic projections for 3ab^{-1} of data at $\sqrt{s} = 14$ TeV indicating that the best-measured quantities will have uncertainties on the order of a few percent; see Table 1.3. Additionally, the HL-LHC with 3ab^{-1} of data at $\sqrt{s} = 14$ TeV will be able to make some initial progress in measuring the Higgs cubic coupling: if κ_3 is a multiplicative rescaling factor for the SM Higgs cubic coupling $3m_h^2/v$ [Eq. (1.31)], ATLAS projects that measurements in the $gg \rightarrow HH \rightarrow (b\bar{b})(b\bar{b})$ [368], $(b\bar{b})(\gamma\gamma)$ [369], and $(b\bar{b})(\tau\tau)$ [370] channels will be able to individually bound $\kappa_3 \in [0.2, 7.0]$, $[-0.8, 7.7]$, and $[-4, 12]$, respectively, at 95% confidence, where the first two results neglected systematic uncertainties [368, 369] and the latter did not [370]. A combined analysis of these channels would naturally improve the sensitivity. The ATLAS Collaboration projections in Ref. [369] are in line with recent theoretical work which indicates that the $b\bar{b}\gamma\gamma$ final state could perhaps constrain the cubic

couplings with 100–200% uncertainties (68% confidence) at the HL-LHC [367]²¹ (see also Ref. [381]).

By way of illustration of the potential improvements in the precision of the coupling measurements at future e^+e^- colliders, we consider the ‘model-independent’ ten-parameter fit for the coupling strength modifiers κ_Z , κ_W , κ_γ , κ_g , κ_b , κ_Z , κ_c , κ_τ , κ_μ , the invisible decay branching ratio, and the total Higgs width, which appears in Ref. [382] and allows comparison of available HL-LHC, CEPC [382] and ILC [383–387] projections. The HL-LHC is not sensitive to κ_c , κ_μ , or the total Higgs width [382], and the precisions predicted for all seven other parameters are in the 3.5–8.5% range. For the CEPC with 5 ab^{-1} of data, Ref. [382] predicts that κ_Z and the invisible decay width will both be measured to 2–3 parts per mille; that κ_W , κ_g , κ_b , κ_Z , κ_c , and κ_τ will be measured with a precision of 1–2%, the total width with a precision of 2.5%, κ_γ with a precision of 4.5%, and κ_μ with a precision of 8.5%. ILC projections are broadly similar, except that κ_Z is only measured at the percent level, and the invisible decay width at the level of 0.5% [382]. Within small factors, FCC-ee projections [388] for an operating point at $\sqrt{s} = 240\text{ GeV}$ are broadly similar to those for the CEPC [382], once differing integrated luminosities in the projections of Refs. [382] and [388] are accounted for. These future colliders will thus probe the Higgs couplings to the most massive SM particles at percent-level precision or better.

The proposed FCC-ee and CEPC e^+e^- colliders are only sensitive to the Higgs cubic coupling via loop effects on the Higgsstrahlung cross section σ_{Zh} [382], the latter of which can be measured to half-percent precision at those machines [382, 388]. With 3 ab^{-1} of data, Ref. [382] projects a 45% measurement of the Higgs cubic coupling at CEPC (a 50% precision projection exists for FCC-ee operating at $\sqrt{s} = 500\text{ GeV}$, with 1 ab^{-1} of data [388]), improving to 25% with 10 ab^{-1} of data. The ILC operating at $\sqrt{s} = 500\text{ GeV}$ with 0.5 ab^{-1} of data could make an 85% measurement, improving to 25% if operating at $\sqrt{s} = 1\text{ TeV}$

21. See Refs. [366, 367] for critical discussion of earlier theoretical studies [362, 364, 365, 380] that projected more optimistically a determination of the cubic coupling within 30%–50% at the HL-LHC.

with 1 ab^{-1} of data [382]. Recent estimates for a 100 TeV proton-proton collider (broadly similar to the SPPC [389–391] or FCC-hh [382] proposals) indicate that with 3 ab^{-1} of data, a 30–40% determination of the Higgs cubic coupling could be made [366, 367] in the $(b\bar{b})(\gamma\gamma)$ channel alone, improving to 12% with 30 ab^{-1} of data [366]. Ref. [392] indicates that better performance than this may actually be achievable: around 5–6% with 30 ab^{-1} , provided that current theoretical uncertainties are reduced [392]. This would allow some precision in testing for the self-consistency of the SM Higgs potential, or finding deviations from it. Direct measurement of the quartic coupling is considered to be extremely challenging, even at the next generation of 100 TeV-scale hadron colliders [392, 393]; although see Ref. [394].

Higgs Vacuum Stability. Given present state-of-the-art measurements of SM particle masses and couplings, there is some debate in the recent literature [395–398] as to whether the quartic coupling λ in the SM Higgs potential remains positive all the way to the Planck scale as it is run up via the renormalization group, or whether it changes sign at some point before the Planck scale is reached. That is, whether the SM vacuum is absolutely stable against tunnelling decay, or whether the vacuum is merely metastable with a lifetime many times the age of the universe [395–398]. At present, it appears that the SM vacuum lies just slightly into the region of metastability [395–398], although there is some disagreement in the literature as to how statistically significant this conclusion is, with older work claiming that absolute stability is ruled out at high significance (i.e., at around 98–99% confidence) [395, 397], and newer work claiming consistency with stability is still achieved at the level of 1.3σ [398]. In any event, improved measurements of the top quark mass (such as are possible at future high-precision e^+e^- colliders) appear to be required to resolve this question more definitively [396–398]. However, it should be emphasized that the investigations in Refs. [395–398] are all predicated on purely SM physics, and in particular on there being no new physics encountered as the SM is run up to the Planck scale via the renormalization group. New physics coupling to the Higgs at or around the weak scale could of course alter any profound

conclusions to be drawn as to the ultimate fate of the SM vacuum; this provides extra motivation for the exploration of the Higgs potential and couplings at current and future colliders (see, e.g., Ref. [392]).

Fundamental vs. Composite. Although the SM as formulated in Section 1.1 assumes a fundamental physical scalar Higgs, it is of course an experimental question as to whether or not the Higgs has substructure. Composite Higgs (CH) theories, in which the Higgs boson is a composite pseudo-Nambu–Goldstone boson of the spontaneous breaking of an approximate global flavour symmetry (which occurs when the gauge group responsible for confining the constituents of the composite Higgs state becomes strongly coupled), are one of the compelling solutions to the Hierarchy Problem (see, e.g., Ref. [399] for a recent review, or Ref. [400] for a pedagogical introduction). In CH models, one need only concern oneself with tunings (or a mechanism; see, e.g., Chapter 5) to arrange for the Higgs vev to be much less than the compositeness scale of the composite theory, while the hierarchy between the compositeness scale and the (Planck, or other high-energy) UV cutoff scale is explained by dimensional transmutation [399]. Such theories do however generically predict modifications of the Higgs (self-)couplings, in addition to new states near the confinement scale of the theory [399]. Precision studies of the Higgs (self-)couplings could reveal the former, while direct searches could discover the latter. Current ATLAS limits on two specific ‘minimal’ realizations [401] of the CH scenario place 95%-confidence lower limits on the compositeness scale in the 700–800 GeV range [402] for these realizations.

EWSB Sector. In many well-motivated extensions of the SM, there are more particles in the EWSB sector than just a single light $J^{PC} = 0^{++}$ Higgs state. By way of example: in the Minimal Supersymmetric Standard Model (MSSM), two Higgs doublets are required²² to give masses to all the SM fermions [207, 208]; i.e., the MSSM incorporates a type-II Two

22. The SUSY superpotential must be holomorphic, which forbids the usage of the $\tilde{H}$ field to generate the up quark masses; cf. Eq. (1.14) [207, 208, 318]

Higgs Doublet Model (2HDM) [403]. Such 2HDMs (whether in a SUSY context, or more generally), predict the existence of four additional heavy Higgs particles: a CP-even state H , a CP-odd state A , and a pair of charged Higgs states $H^\pm$ [403]; there are no current indications of the existence of such states [180, 404–406]. More generally, extensions of the minimal Higgs sector presented in Section 1.1 are possible (see, e.g., the Higgs Portal discussion in the next paragraph). Generically, they can effect modifications to the dynamics of EWSB, and are testable at colliders via their modifications to the Higgs potential and (self-)couplings, or by direct observation of the new states if the latter are light enough.

The Higgs Portal and Other Scalars. The Higgs bilinear $H^\dagger H$ is a dimension-2 super-renormalizable operator; together with $B_{\mu\nu}$, it is the lowest-dimension SM-singlet operator that can be built with SM fields. As such, (super-)renormalizable couplings of SM-singlet new physics to the ‘Higgs Portal’ [407], of the form $H^\dagger H \mathcal{O}_{\text{NP}}$, are well motivated; they also have rich phenomenology. In particular, they provide an avenue to weakly couple DM (sectors) to the SM (see, e.g., Chapter 3), or to couple additional SM-singlet scalars to the Higgs, which can lead to many effects, such as detectable modifications to the coupling strengths of the Higgs to the SM fermions and vector bosons (as discussed above; see also Ref. [408]), or significant modifications to EWSB dynamics [409]. Although we do not study the phenomenon in this dissertation, it is possible for these latter modifications to alter the order of the electroweak phase transition in the early universe [112, 409] (which in the SM is a cross-over [112]), which can have interesting cosmological implications [409]; for instance, if the transition is made (strongly) first order, it would allow electroweak baryogenesis as an explanation for the baryon asymmetry [112, 409]. Understanding the phenomenology of the Higgs Portal is thus a compelling direction for research, which often involves all of the major themes of this dissertation.

1.5 Particle Physics in the Early Universe: The Inflationary Epoch

The interface of particle physics with early universe cosmology touches on, and influences ideas toward the resolution of, many aspects of the open ‘big questions’. To reiterate just two examples that we have already mentioned: (a) regardless of the specific candidate in question, all particle DM production mechanisms rely on particle physics processes occurring in the early universe, requiring us to understand the relevant cosmological phenomena well in order to obtain, e.g., accurate and precise estimates of production abundances; and (b) generation of the baryon asymmetry via any of the proposed particle physics mechanisms proceeds of necessity in the hot and dense early universe before BBN, and detailed understanding of the relevant cosmology is thus required to track the evolution of the particle physics dynamics. There have also been a number of attempts, some recent, to invoke ideas from cosmology to address questions related to Naturalness [410–415], one of which [413] will occupy our attention in Chapter 5.

Although many applications of cosmological ideas to particle physics (and vice versa) involve later periods in the evolution of the universe, of most direct relevance to the work in this dissertation is the earliest phase in the standard history of the universe: the inflationary epoch. As such, in this section we aim to give a very brief introduction to the ideas and elementary background theory underlying inflation; our presentation draws heavily on Refs. [120, 153, 154, 416].

A number of observations of the universe at large are at first glance deeply perplexing. First, the universe is spatially flat to the percent-level today [85, 120, 153, 154]. Taken alone, that might be written off as coincidental, but by simple scaling²³ of the various energy-

23. The Friedmann equation is $H^2 + k/a^2 = (8\pi/3M_{\text{Pl}}^2)\rho$ where $\rho = \rho_R + \rho_M + \rho_\Lambda$ and $H \equiv \dot{a}/a$. It is conventional to define $\Omega_i \equiv (8\pi/3M_{\text{Pl}}^2 H^2)\rho_i \equiv \rho_i/\rho_c$ for $i \in \{R, M, \Lambda\}$, and to define $\Omega_K \equiv -k/a^2 H^2$. Then the Friedmann equation is $\Omega_R + \Omega_M + \Omega_\Lambda + \Omega_K = 1$. Moreover, $\Omega_K = -k/a^2 H^2 = -k[-k + (8\pi/3M_{\text{Pl}}^2)a^2 \sum_i \rho_i]^{-1} = \Omega_K^0 [\Omega_K^0 + \sum_i \Omega_i^0 \cdot (a^2 \rho_i/\rho_i^0)]^{-1} = \Omega_K^0 [1 - \sum_i \Omega_i^0 (1 - (a^2 \rho_i/\rho_i^0))]^{-1}$, where superscript ‘0’ indicates the present-day value. Recall also from the continuity equation that an energy component with equation of state $w \equiv P/\rho$ scales as $\rho \propto a^{-3(1+w)}$, where a is the FLRW scale

density components back in time (see, e.g., Ref. [120, 154]) it implies that at earlier times the universe must have had an exponentially small spatial curvature Ω_K . Indeed, at BBN ($T_{\text{BBN}} \sim \text{MeV}$ [180]), which is the earliest physical process for which we have unambiguous evidence, the curvature could be no larger than $\Omega_K \sim 10^{-17}$ [120, 154, 416]. This requires that the universe at early times had almost exactly the critical density $\rho_c = (3/8\pi)H^2 M_{\text{Pl}}^2$ [120] to 1 part in at least 10^{17} .

The second observation is that on large scales the universe is statistically homogeneous and isotropic [120, 153, 154]. In particular, the CMB temperature is (as we have already noted, and with the exception of a large dipole component due to the motion of the Local Group with respect to the CMB rest frame [417, 418]) isotropic to 1 part in 10^5 across the whole observable sky [114, 115]. Statistical homogeneity and isotropy seemingly defy notions of causality, as widely separated regions of the universe which naïvely appear to have never been in causal contact share the same properties to remarkable precision.

In a revolutionary insight, Guth [419], Linde [420], and Albrecht and Steinhardt [421] postulated that these problems can all be neatly solved if, extremely early in its history, the universe underwent an exponential expansion in size: a period of inflation. Provided it lasted sufficiently long, such an inflationary epoch would stretch and flatten initially $\mathcal{O}(1)$ spatially curved regions of space over volumes larger than the present-day observable universe, explaining why the universe was so nearly exactly spatially flat at early times, and is still approximately spatially flat today. Moreover, statistical homogeneity and isotropy are explained by virtue of the fact that a volume of spacetime in causal contact—and therefore able to equilibrate and homogenize—before or while it underwent inflation would be stretched by inflationary expansion to encompass a volume larger than our present-day ob-

factor [radiation scales as $\rho_{\text{R}} \propto a^{-4}$ ($w_{\text{R}} = 1/3$), matter scales as $\rho_{\text{M}} \propto a^{-3}$ ($w_{\text{M}} = 0$), and a cosmological constant scales as $\rho_{\Lambda} \propto a^0$ ($w_{\Lambda} = -1$)]. Moreover, $\rho \propto T^{3(1+w)}$ and $a \propto T^{-1}$. Therefore, $\Omega_K = \Omega_K^0 [1 - \sum_i \Omega_i^0 (1 - (T/T_0)^{1+3w_i})]^{-1}$. With $\Omega_{\Lambda}^0 = 0.6911(62)$ [114], $\Omega_{\text{M}}^0 = 0.3089(62)$ [114], $H_0 = 67.74(46) \text{ km/s/Mpc}$ [114], $T_0 = 2.7255(6) \text{ K}$ [167] and $N_{\text{eff}} = 3.04(18)$ [114] so that $\Omega_{\text{R}}^0 \approx 9.11(25) \times 10^{-5}$, we find $\Omega_K \approx 6.06(17) \times 10^{-16} \cdot \Omega_K^0 \cdot T[\text{MeV}]^{-2}$ for temperatures $T \gg T_{\text{eq}}$, with $T_{\text{eq}} \sim \text{eV}$ the temperature of matter-radiation equality. See, e.g., Refs. [120, 153, 154, 416].

servable universe. Regions of the universe which today are so widely separated so as not to have been in causal contact since, e.g., before CMB last scattering can thus nevertheless possess similar characteristics today by virtue of having been in causal contact at some point in the past, prior to or during inflation.

A mechanism must exist in order for inflation to occur, run for sufficiently many e -folds, end, and for the subsequent thermal history of the universe to then proceed. Particle physics is able to supply such a mechanism with remarkably alacrity: in the simplest incarnations, all that is required is a single scalar field slowly rolling down a potential [120, 153, 154, 416]. Suppose that in a flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe, $ds^2 = dt^2 - [a(t)]^2 d\mathbf{x} \cdot d\mathbf{x}$ [154, 422], where the FLRW scale factor a obeys the (flat space) Friedmann equation $(\dot{a}/a)^2 = H^2 = (8\pi/3M_{\text{Pl}}^2)\rho$, there exists a scalar field ϕ called the inflaton which is subject to a potential $V(\phi)$:

$$S \supset \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]. \quad (1.32)$$

This implies a classical equation of motion for the spatially homogeneous (i.e., $\mathbf{k} = 0$) mode of the ϕ field:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (1.33)$$

where $' \equiv \partial_\phi$. We suppose that the inflaton dominates the energy density of the universe, so that $\rho \approx \rho_\phi$. If the ‘slow-roll’ conditions²⁴ (see, e.g., Ref. [416]),

$$\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \ll 1 \quad \text{and} \quad \eta_V \equiv \frac{M_{\text{Pl}}^2}{8\pi} \frac{V''(\phi)}{V(\phi)} \ll 1, \quad (1.34)$$

are satisfied, then $\rho_\phi \approx V(\phi)$, and the inflaton simply rolls down its potential with approx-

24. The slow-roll conditions are also commonly given as $\epsilon \equiv -\dot{H}/H^2$ and $\eta \equiv -\ddot{\phi}/H\dot{\phi}$; see, e.g., Ref. [416]. To linear order, $\epsilon = \epsilon_V + \dots$ and $\eta = \eta_V - \epsilon_V + \dots$ [416]. In terms of ϵ , the following relations hold exactly [416]: $w_\phi = -1 + \frac{2}{3}\epsilon$ and $\ddot{a} = aH^2(1 - \epsilon)$, and inflation ends at $\epsilon = 1$, when $w_\phi = -1/3$.

imately constant velocity $\dot{\phi} \approx -V'(\phi)/3H$. During this epoch, ϕ has the equation of state $w_\phi \approx -1 + \frac{2}{3}\epsilon_V \approx -1$ which implies that the homogeneous ϕ field acts as a cosmological constant, driving a period of exponential expansion for the scale factor $a(t)$. Inflation ends at the point where $\ddot{a} = 0$, which happens at $\epsilon_V \approx 1$. For a potential to support a finite duration of inflation, it must thus naturally allow ϵ_V to become large as ϕ rolls: one simple example is a quadratic potential, $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$.

Under slow-roll conditions, the Hubble parameter $H \equiv \dot{a}/a$ is very nearly constant: $\dot{H} \approx -\epsilon_V H^2$. This implies that the comoving Hubble radius $R_H^c \equiv 1/(aH)$, which sets the scale for causal physical interactions, is shrinking exponentially with time t during inflation, implying a shrinking horizon for causal physical interactions for points stationary on the comoving grid [416], precisely as required to explain large scale statistical homogeneity and isotropy.²⁵ Moreover, $\Omega_K = -k(R_H^c)^2$ also shrinks exponentially during inflation, solving the spatial flatness problem.

Once inflation ends, the energy density of the universe resides entirely in the inflaton field, which usually ends up executing coherent matter-like oscillations about the minimum of its potential in the period immediately after the end of inflation [120, 153, 154, 416]. This energy density must then be transferred from the inflaton into SM particles as radiation:²⁶ this reheats the universe to some high temperature T_{RH} , which for BBN to be successful must be at least as large as $T_{\text{BBN}} \sim \text{MeV}$ [120, 153, 154, 416]. The patch of the universe corresponding to our present-day observable universe (and hence consisting of many BBN-epoch Hubble volumes) is then in the requisite hot, dense, statistically homogenous and isotropic (‘Hot Big Bang’) state required to explain our present-day observations. In order

25. That is, a larger comoving patch was initially able to participate in causal physical interactions at the start of inflation, than is able to do so at the end of inflation. As the comoving Hubble radius increases in size as $R_H^c \propto t^{1/2}$ or $t^{1/3}$ during radiation or matter domination, respectively, our current Hubble radius is expanding back into the larger, initially causally connected patch [120, 153, 154, 416].

26. Multiple mechanisms are possible: for example, vanilla inflaton decay to SM particles [120], or preheating (i.e., parametric resonance driven by the inflaton oscillations which efficiently converts inflaton energy into SM particles) [423]. Further discussion lies beyond the scope of this dissertation; see, e.g., Ref. [424] for a review.

for this picture to be consistent (and consistent with observations), inflation and reheating must have ended (at the latest) just before the epoch of BBN; since the comoving Hubble radius has expanded by about 17 e -folds since that time [154], this implies a lower bound of 17 e -folds on the duration of the inflationary period. If T_{RH} is higher (e.g., around the GUT or Planck scales), more e -folds are needed: typically, $N_e \sim 60$ [154] (although see Chapter 5 for a model in which, for other reasons, N_e is required to be exponentially large).

Moreover, ϕ is a quantum field, not just a classical expectation value. As such, it is subject to quantum fluctuations during inflation. Assuming that the inflaton is non-interacting, each co-moving momentum mode $\phi_{\mathbf{k}}$ is decoupled, and each is subject to Gaussian²⁷ quantum fluctuations. In an exact de Sitter universe with constant H , these fluctuations are characterized by the scale-invariant Harrison–Zel’dovich [428, 429] two-point power spectrum: if $\delta\phi_{\mathbf{k}}$ is the perturbation about ϕ with comoving momentum $\mathbf{k}$, then $\langle\delta\phi_{\mathbf{k}}(t)\delta\phi_{\mathbf{k}'}(t)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')(2\pi^2/k^3)\mathcal{P}_{\delta\phi}(k)$ with $\mathcal{P}_{\delta\phi}(k) = (H/2\pi)^2$ [120, 153, 154, 416]. As the co-moving Hubble radius R_{H}^c shrinks during inflation these fluctuations become frozen²⁸ in size as each mode crosses outside the comoving Hubble radius at $kR_{\text{H}}^c \sim 1$ [120, 153, 154, 416]. When the modes subsequently re-enter the expanding comoving Hubble radius after reheating, they seed the curvature perturbations ζ which eventually give rise to the temperature anisotropies of the CMB [120, 153, 154, 416]. As H is very nearly constant during inflation, the de Sitter result can be used with H evaluated at horizon crossing $k = aH$, leading the initial curvature perturbations to be nearly scale-invariant: $\mathcal{P}_{\zeta}(k) = (H/2\pi)^2(H/\dot{\phi})^2|_{k=aH}$ [120, 153, 154, 416]. The spectral tilt away from a scale invariant spectrum, $\mathcal{P}_{\zeta}(k) \propto k^{n_s-1}$, is given by $n_s \approx 1 + 2\eta_V - 6\epsilon_V|_{k=aH}$ [416]. Tensor modes (i.e., primordial gravitational waves, characterized by metric fluctuations h) are created by the same quantum process, and the tensor-to-scalar ratio is $r_k \equiv \mathcal{P}_h(k)/\mathcal{P}_{\zeta}(k) \approx 16\epsilon_V|_{k=aH}$ [416]. The latest measurements—

27. Interactions of the inflaton with other fields, or itself, during inflation can spoil Gaussianity [425, 426]. So far, no such non-Gaussianities have been observed; only limits have been placed on their size [427].

28. More precisely, a certain gauge-invariant variable, ζ , related to them is constant far outside the co-moving Hubble radius, assuming single-field inflation [120, 153, 154, 416].

$n_s = 0.968(6)$ [86], and $r_{0.05} < 0.07$ at 95% confidence [430]—have already begun to place non-trivial constraints on the underlying particle physics mechanism driving inflation; significant improvements on the constraint on r , by an order of magnitude or more, are expected with future-generation observations [175], which will severely circumscribe the set of viable inflationary particle physics models.

1.6 Outline of Dissertation

In this section, we outline the remaining structure of this dissertation by briefly summarizing the contents of each of the following chapters.

In Chapter 2 and Appendix A, we consider indirect detection limits on effective operators describing either scalar or fermionic WIMPs annihilating to electroweak gauge and/or Higgs bosons. Our analysis makes use of flux limits on gamma-rays from the Milky Way galactic centre and from MW-dwarf galaxies in order to individually constrain 34 such operators, up to and including mass-dimension-8, which were the focus of prior work in the literature (see Ref. [431]). Saturation of the dark-matter relic abundance via thermal freeze-out is assumed to be controlled by the same operator responsible for present-day annihilation. We identify a number of different classes into which the operators fall based on how the observational limits constrain them, and find this classification to be directly related to common field content and/or structure of the operators in each class. At the time the work [4] presented in Chapter 2 was completed (2013), there appeared to be a 130 GeV photon line in data from the Fermi-LAT experiment [432, 433]; we show that a subset of the operators are capable of accounting for this then-signal without running afoul of the other limits we consider. Independent of this conclusion, the results of our analysis provide a useful intermediate tool to understand the implications of the observational constraints we utilize, on UV-complete dark-matter models whose relevant low-energy description is given by only one (or one dominant) effective operator in the set. Chapter 2 and Appendix A are based on the following work of the Author [4]: M. A. Fedderke, E. W. Kolb, T. Lin, and

L.-T. Wang, *Gamma-ray constraints on dark-matter annihilation to electroweak gauge and Higgs bosons*, *JCAP* **1401** (2014) 001 [[arXiv:1310.6047](#)].

In Chapter 3 and Appendix B we consider direct detection and collider-based constraints on the two dimension-5 effective operators describing a fermionic thermal-relic WIMP coupled to the Higgs Portal: $\bar{\chi}\chi H^\dagger H$ and $\bar{\chi}i\gamma_5\chi H^\dagger H$. We map out in detail the regions of parameter space which remain viable in this scenario as a function of the dark-matter mass and the relative sizes of the coefficients of each operator, assuming again that the dark matter saturates the relic abundance. Largely in line with previous work in the literature (e.g., Ref. [434]), we find that the most stringent spin-independent direct detection limits eliminate a large fraction of the dark-matter mass parameter space in which the effective-operator analysis is valid, when the low-energy coupling of the physical Higgs field h is dominantly to $\bar{\chi}\chi$. Our analysis highlights a subtlety which requires an ill-motivated parameter coincidence in the electroweak-symmetric form of the effective theory in order to obtain a low-energy coupling of h purely to the bilinear $\bar{\chi}i\gamma_5\chi$, which is significant because direct detection constraints are much milder for this coupling. For dark matter lighter than approximately 55 GeV, we also find that collider constraints on the contribution to the width of the Higgs boson from invisible decays rule out both of these couplings, given our assumptions. Chapter 3 and Appendix B are based on the following work of the Author [5]: M. A. Fedderke, J.-Y. Chen, E. W. Kolb, and L.-T. Wang, *The Fermionic Dark Matter Higgs Portal: an effective field theory approach*, *JHEP* **1408** (2014) 122 [[arXiv:1404.2283](#)].

Continuing with the Higgs Portal theme, in Chapter 4 and Appendix C we investigate a simple singlet-doublet UV completion of the dimension-5 operator $\bar{\chi}\chi H^\dagger H$. In this work, we explore the constraints which precision physics programmes at a number of proposed electron-positron colliders could place on the parameter space for this model, independent of any dark-matter interpretation for the singlet field χ . As our model explicitly breaks the custodial symmetry of the Standard Model, we expect and obtain strong projected exclusion limits reaching into the TeV range on the singlet and doublet parti-

cle masses, driven mostly by forecasted improvements in the measurements of the precision electroweak S and T parameters. We also find complementary strong limits in the few-hundred-GeV to low-TeV range arise from the expected high-precision measurement of the $e^+e^- \rightarrow Zh$ Higgsstrahlung cross section which these colliders will be able to obtain. Although our model is UV complete, and part of the analysis makes use of direct one-loop computations of S and T , to extract the Higgsstrahlung cross section we also make extensive use of effective field theory tools: explicit matching of our UV model to the dimension-6 ‘Higgs Effective Field Theory’ operators, and the results of Ref. [435]. A summary version the work presented in Chapter 4 appeared in the Preliminary Conceptual Design Report for the CEPC-SPPC collider [382]. Chapter 4 and Appendix C are based on the following work of the Author [6]: M. A. Fedderke, T. Lin, and L.-T. Wang, *Probing the fermionic Higgs portal at lepton colliders*, *JHEP* **1604** (2016) 160 [arXiv:1506.05465].

Remaining with the Higgs sector, in Chapter 5 and Appendix D, we study how the recently proposed cosmological relaxation mechanism of Ref. [413] can be incorporated into a composite Higgs model in order to generate the Little Hierarchy in a technically natural fashion. We consider in detail the composite Higgs model based on the technicolour gauge group $SU(N)$, with three species of Dirac technifermions; this leads to the spontaneous global chiral symmetry breaking pattern $SU(3)_L \times SU(3)_R \times U(1)_V \rightarrow SU(3)_V \times U(1)_V$, with an $SU(2)_W$ -doublet of pseudo-Nambu–Goldstone bosons of the spontaneous global symmetry breaking serving as the Higgs. The pseudoscalar relaxion [413] is coupled to the technicolour and strong gauge groups as a KSVZ-axion via the agency of the clockwork mechanism [436, 437], and is given an additional very flat shift-symmetry-breaking potential down which to roll during an exponentially long period of low-scale inflation (to alleviate the severe strong-CP problem from which this class of models suffers, we employ the slope-drop mechanism of Ref. [413] to remove this potential after inflation). As the relaxion field rolls, it scans the techniquark masses, which are a source of explicit chiral symmetry

breaking. The Higgs potential is thus also scanned, eventually resulting in the large top-loop radiative corrections coming to dominate, thereby triggering electroweak symmetry breaking, at which point—following Ref. [413]—the non-trivial QCD potential triggers a back-reaction on the relaxion potential, stalling its rolling and dynamically locking in a Higgs vev and Higgs mass which are much smaller than the compositeness scale. Dimensional transmutation (of the technicolour gauge group) is assumed, as usual, to generate the larger SM Big Hierarchy. We find that we are able to generate Little Hierarchies as large as $\xi \equiv \langle h \rangle^2 / F_\pi^2 \sim \mathcal{O}(10^{-4})$ [i.e., $F_\pi \sim 20 \text{ TeV}$], while remaining within the region of parameter space in which the model is self-consistent, and without running afoul of QCD θ -angle constraints. Our detailed investigation of the phenomenology of this model shows that a precision physics programme at future electron-positron colliders would be able to explore the majority of its natural parameter space at the 95%-confidence exclusion level. We also highlight further phenomenological opportunities related to this, and other, composite Higgs models with large Little Hierarchies. Chapter 5 and Appendix D are based on the following work of the Author [7]: B. Batell, M. A. Fedderke, and L.-T. Wang, *Relaxation of the Composite Higgs Little Hierarchy*, [arXiv:1705.09666](#).

In Chapter 6 and Appendix E, we undertake a detailed study of the production of supermassive (i.e., Hubble-scale and above) scalar particles via gravitational effects in the inflationary epoch of the early universe. We consider two scenarios in which an otherwise minimally coupled scalar field is also given a direct coupling to the inflaton: either (i) an inflaton-field-dependent non-canonical kinetic term for the scalar field, or (ii) a direct interaction term between the scalar field and the inflaton in the potential. These couplings are engineered such that when the scalar field is canonically normalized, its effective mass becomes momentarily light or tachyonic when the inflaton field is near a singled-out value. This results in a large fraction of the particle production in either scenario occurring in a fairly short irruptive burst around this special value of the inflaton field. Our main results are a set of particle spectra and production abundances, computed numerically for

each coupling case as a function of the model parameters; limiting cases are also analytically accessible. In order to enable the robust numerical exploration of the models over a wide range of parameters while maintaining acceptable computational speed, we employ a novel synthesis of known numerical techniques in this investigation. We additionally develop comprehensive arguments to explain the remarkable range of non-trivial phenomena we observe in our results. In line with the large body of literature on this subject (e.g., Refs. [185–190, 302, 303]), we find in both scenarios that this production mechanism can create a non-thermal relic abundance of late-time supermassive particles easily large enough to yield the observed cosmological abundance of dark matter under reasonable parameter choices, allowing their interpretation as an alternative supermassive dark-matter candidate. Chapter 6 and Appendix E are based on the following work of the Author [8]: M. A. Fedderke, E. W. Kolb, and M. Wyman, *Irruption of massive particle species during inflation*, *Phys. Rev.* **D91** (2015) 063505 [[arXiv:1409.1584](#)].

We conclude briefly in Chapter 7.

CHAPTER 2

GAMMA-RAY CONSTRAINTS ON DARK-MATTER ANNIHILATION TO ELECTROWEAK GAUGE AND HIGGS BOSONS

This chapter contains material from Ref. [4]: M. A. Fedderke, E. W. Kolb, T. Lin, and L.-T. Wang, *Gamma-ray constraints on dark-matter annihilation to electroweak gauge and Higgs bosons*, *JCAP* **1401** (2014) 001 [[arXiv:1310.6047](#)].¹

Annihilation of dark matter (DM) into electroweak gauge and Higgs bosons results in prompt γ -ray emission, which may be observable at so-called ‘indirect detection’ experiments. In this chapter, we assume that the dark-matter species is a cold thermal-relic Weakly Interacting Massive Particle (WIMP) with a mass in the range 5 GeV to 20 TeV and with present mass-density equal to the measured average dark-matter density of the universe. The coupling of the WIMP to the Standard Model (SM) is assumed to be described in the low-energy limit by a single effective operator $\mathcal{O} \propto J_{\text{DM}} \cdot J_{\text{SM}}$, where J_{DM} is a Standard Model singlet consisting of dark-matter fields (assumed to be Dirac fermions or complex scalars), and J_{SM} is a Standard Model singlet consisting of electroweak gauge and/or Higgs bosons. Utilizing observational upper limits on the fluxes of γ -rays from the Milky Way Galactic Centre, and from MW-companion dwarf galaxies, we set exclusion limits on the allowed dark-matter masses under these assumptions. We consider, in turn, the 34 possible operators up to and including mass-dimension-8 with non-vanishing s -wave annihilation channels which satisfy the above assumptions; these operators were previously studied in Ref. [431]. Our limits can be grouped into 13 classes determined by the field content and structure of the operators, and are presented in a large number of figures: one for each of the 34 possible operators. We also identify three classes of operators (coupling to the Higgs and

1. Ref. [4] is copyright © 2014 by the Authors, and is under a Creative Commons Attribution (CC BY) 3.0 Licence [1].

$SU(2)_W$ gauge bosons) that could have supplied a 130 GeV line with the desired strength to fit the (c. 2013) hints of a line signal in the Fermi-LAT data,² while saturating the relic density and satisfying all other indirect constraints we consider.

2.1 Introduction

Until the nature of dark matter and dark energy is elucidated, the remarkable success of the standard Λ CDM model of cosmology in accounting for observations will be less than completely satisfying. The most popular explanation for dark matter is that it is an early-universe relic in the form of an undiscovered, electrically neutral, non-hadronic, stable species of elementary particle. Of the many possibilities for the origin and nature of this purported new elementary particle, one that is particularly amenable to experimental and observational tests is that the new species is a massive cold thermal relic.

The cold-thermal-relic scenario (see, e.g., Ref. [120]) assumes that the new particle species was established in local thermodynamic equilibrium when the temperature of the universe was larger than the mass of the particle. The processes establishing the initial equilibrium abundance are assumed to be the annihilation of the new species into Standard Model particles and the production of the new species from initial SM-particle states. As the universe cools to temperatures below the dark-matter mass, the abundance of the new species relative to the entropy density ‘freezes out’ as the rate of processes keeping the species in equilibrium falls below the expansion rate of the universe. The relative freeze-out abundance, and therefore the present mass-density, is thus determined by the cross section for dark-matter annihilation into SM particles.

Observationally, the ratio of the present average dark-matter mass-density to the critical density is well determined to be $\Omega_{\text{DM}} h^2 \simeq 0.12$ [85]. The requirement that this mass-density is saturated by the mass-density of the cold thermal relic determines the annihilation cross section (and, in some cases, also the mass) of the new species: both of order the weak scale.

2. See Appendix A.5.1 for an update on the status of this feature.

Such a massive cold thermal relic is thus known as a Weakly Interacting Massive Particle, or WIMP.

The existence of non-gravitational DM–SM couplings which are closely related to the present dark-matter mass-density is the key feature of WIMP paradigm that opens up the experimental and observational avenues that could lead to the discovery of the nature of the dark matter: detection of WIMPs through their present-day scattering with SM particles, known as ‘direct detection’ [211]; detection of WIMPs through the (astrophysical) observation of the present-day annihilation products of WIMPs into SM particles, known as ‘indirect detection’; and production and detection³ of WIMPs at particle colliders.

The common refrain of these DM–SM couplings in all WIMP scenarios notwithstanding, there are a tremendous number of variations on the theme. One possibility is that the WIMP is not the only new particle at the weak scale, and that additional new particles play a crucial role in the annihilation, production, or scattering of WIMPs (so-called ‘co-annihilations’). In the other limit that the masses of any additional new particles are much greater than momenta in processes of interest, we can integrate out the additional states and describe the WIMP–SM interaction in terms of an effective field theory (EFT). We work in this latter limit, sometimes known as the ‘Maverick’ limit [253, 438].

Even within this narrowed context there remain a large number of possibilities. The first study assumed that the WIMP couples to SM fermions [439]. In this case, direct detection [439] and collider searches [253–255, 440–448] offer the best possibility for testing the models.

Another class of possibilities which admits an EFT description is that dark matter couples to (SM) diboson final states of electroweak gauge and/or Higgs bosons [431]. Here, we consider DM–SM couplings of the form $\Lambda^{-n} J_{\text{DM}} \cdot J_{\text{SM}}$, where J_{DM} is a Standard Model singlet consisting of dark-matter fields (assumed to be Dirac fermions or complex scalars), J_{SM} is a Standard Model singlet consisting of electroweak gauge and Higgs bosons, and Λ is

3. Detection at colliders is typically by way of excess missing transverse energy/momentum, as the WIMPs leave the detector without interacting again.

the mass scale (‘suppression scale’) of the effective field theory. Moreover, we only consider the case that the dark-matter field is a singlet under $SU(2)_W \times U(1)_Y$.

Ref. [431] listed all such operators with SM-diboson final states, up to and including mass-dimension-8, and presented detailed calculations of dark-matter annihilation cross sections to all possible on-shell, two-body final states consistent with gauge symmetries: $\gamma\gamma$, γZ , γh , ZZ , Zh , W^+W^- , hh and $\bar{f}f$.⁴ In order to consistently describe this list of possible channels, the authors of Ref. [431] required J_{SM} to be a singlet under $SU(2)_W \times U(1)_Y$. Of the 50 such operators enumerated in Ref. [431], it was demonstrated there that 34 of them have at least one non-vanishing s -wave annihilation channel for at least some values of the dark-matter mass M , and hence could produce a potentially detectable indirect detection signal (p -wave annihilation is suppressed by $v^2 \sim 10^{-6}$ relative to s -wave annihilation at the present epoch).

DM annihilation to SM-diboson final states leads in all cases to prompt production of γ -rays, including possibly (quasi-)monochromatic photons. This leads naturally to the prospect that there could be non-negligible present-day DM-annihilation photon fluxes emanating from regions of high (present) dark-matter density: in particular, from the Galactic Centre (GC) of the Milky Way (MW), or from MW-companion dwarf galaxies. Our aim in this chapter is to utilize observational upper bounds on such photon fluxes to place indirect detection constraints on those EFT operators described above that have non-negligible s -wave annihilation cross sections.

More precisely, considering in turn each of the 34 EFT operators mentioned above, we will derive constraints on the allowed dark-matter mass M assuming that the WIMP annihilates only via the operator in question, and that it is a massive cold thermal relic which accounts for 100% of the measured DM density in the universe. We use the following analyses to

4. These cross sections are listed in Tables VI–XX of Ref. [431]. We will refer to an operator studied in Ref. [431] by a table number, and an operator number: for instance, operator VI-3 is the third operator of Table VI in Ref. [431], in this case, $\phi^\dagger \phi W_{\mu\nu}^a W^{a\mu\nu}$. Our notation for field operators will follow the notation in Ref. [431]. Note that our table numbering follows that of the `arXiv` preprint version of Ref. [431]; if consulting the published version, replace all Roman numeral table numbers by their Arabic numeral equivalents.

set limits: the GC Fermi-LAT line-search limits for photon energies 5–300 GeV presented in Ref. [449] based on their 3.7-year data set;⁵ the H.E.S.S. GC line-search limits for photon energies 500 GeV–20 TeV presented in Ref. [450] based on their 112hr data set;⁶ the analysis of the GC Fermi-LAT inclusive flux limits for photon energies 0.1–100 GeV presented in Ref. [451]; and the Fermi-LAT MW-companion dwarf limits presented in Ref. [452] based on the 2-year data set for photon energies 0.2–100 GeV.⁷ In addition, many of the operators have one or more annihilation modes with monochromatic photons; we investigate whether these operators can fit the (c. 2013) possible γ -ray line signal at $E_\gamma \approx 130$ GeV observed in the Fermi-LAT data, as first reported in Ref. [432], discussed at length in Ref. [433], and subject to detailed investigation in Ref. [449] (see also Appendix A.5.1 for a discussion of the subsequent disappearance of this signal after the work on which this chapter is based [4] was published). For operators that provide a reasonable fit to the photon line, we apply the limits presented in Ref. [453] for the ratio of continuum-to-line emission for DM masses of 125 to 150 GeV.

We present our limits in a large number of figures, one for each of the 34 possible operators, in Section 2.4. The limits on the operators depend on available annihilation channels, which are determined by both the field content and structure of the operator, and the dark-matter mass. Based on qualitative differences in their predicted indirect detection signals, we group the operators into 13 classes, which are summarized in Table 2.3 in Section 2.5. Overall, we observe that line searches give the most constraining limits for operators whose indirect detection signal has a significant contribution from one or more photon lines. Other limits, such as those from the continuum photons, are weaker but still useful for lighter dark-matter masses. Although some operators are quite severely constrained by the photon fluxes over a large region of mass parameter space, for the majority of the operators we find

5. See Appendix A.5.2 for an update on the Fermi-LAT line search limits.

6. See Appendix A.5.3 for an update on the H.E.S.S. line search limits.

7. See Appendix A.5.4 for an update on the Fermi-LAT MW-dwarf limits.

that most of the parameter space is still open, or is only weakly constrained. Among all the operators considered here, we also identify three classes that can simultaneously give the correct thermal relic abundance, account for the potential (c. 2013; see Appendix A.5.1) 130 GeV line signal, and be consistent with all the constraints from indirect searches which we consider. More importantly, our results demonstrate that it is possible to set interesting limits on dark-matter annihilations by combining various channels; moreover, by combining predictions for multiple channels, more discriminating power is obtained to learn about the nature of dark matter if a positive signal is eventually observed.

We also identify an interesting effect for fermionic DM coupling via pseudo-vector J_{DM} , where the p -wave annihilation cross section is enhanced (relative to the s -wave) at large M by coupling to the longitudinal mode of a massive vector boson in the final state. This p -wave enhancement has the effect, since saturation of $\Omega_{\text{DM}} h^2$ fixes the total cross section at freeze-out (when p -wave annihilation is not velocity-suppressed), of suppressing the present-day (i.e., s -wave) annihilation cross section, thereby alleviating constraints on these operators for large M .

As there have been many works that discuss indirect detection limits, it is appropriate for us to discuss why the analysis of Ref. [4], as presented in this chapter, is new. Our starting point is the assumption that low-energy WIMP annihilation processes can be described by an EFT with the assumptions discussed above. Working within that framework, for each possible operator, the full $SU(2)_{\text{W}} \times U(1)_{\text{Y}}$ gauge invariance of J_{SM} forces us to consider all possible final states.⁸ Moreover, the requirement of gauge invariance determines the relative strength of possible annihilation channels. We expect the existence of multiple annihilation channels to offer much more discriminating power in the analysis. A related earlier work is Ref. [454], where a more limited set of operators (without including the Higgs and without requiring full $SU(2)_{\text{W}}$ gauge invariance) was considered. In addition, some of these operators

8. For some operators, gauge invariance leads to the inclusion of SM fermions in the final state when s -channel SM vector boson exchange is involved.

have been discussed previously [455–457], with an emphasis on explaining the (c. 2013; see Appendix A.5.1) 130 GeV line, as well as a possible additional line at around 114 GeV [458].

We emphasize that, in general, the EFT approach must be taken with some caution; the case at hand is no exception. Every relevant process involving WIMPs has a characteristic scale for the momentum transfer, and the accuracy of the EFT approach can be measured by the comparison between this scale and the suppression scale of the effective operators [459–461]. The EFT approach becomes less accurate in describing the freeze-out and present-day DM annihilation if the suppression scale is close to the dark-matter mass. As it happens, in order to give the correct thermal relic abundances, some of the operators we study are forced to have $\Lambda \sim M$ and, in principle, one therefore needs to delve into possible UV completions in these cases. This caveat notwithstanding, given the large number of possible operators and final states we consider, we find it useful to use the effective operator approach ‘naïvely’ to give rough estimates for these cases. Considering the large systematic astrophysical uncertainties in the indirect detection limits we utilize, a more accurate description within a UV complete model should not change our conclusions qualitatively, unless our underlying assumptions are severely violated by such a model.

The remainder of this chapter is arranged as follows. In Section 2.2 we discuss some theoretical background on the WIMP relic density, DM halo profiles, annihilation photon fluxes, and continuum photon spectra from various DM annihilation channels. In Section 2.3 we discuss the individual experimental limits we have worked with, summarizing how they were obtained and how we have interpreted them. The results of our work are given in Figures 2.4–2.37, in Section 2.4. We discuss the results and conclude in Section 2.5. There are a number of appendices to this chapter: Appendix A.1 recapitulates some of the arguments leading to various important numerical factors in our analysis. Appendix A.2 discusses in some detail the contribution to the photon spectrum due to Inverse Compton Scattering, and motivates why we ignored it in the main analysis. Appendix A.3 summarizes the approximate magnitude of other systematic errors we did not take into account. Appendix A.4 discusses

some further analysis we have performed on one of the literature sources we have consulted [453] for the case where there are two partially resolved lines in the photon spectrum. Finally, Appendix A.5 gives an update on the current (2017) status of the 130 GeV Fermi-LAT photon line, and summaries of updates to the above-mentioned Fermi-LAT line, H.E.S.S line, and Fermi-LAT (and DES) MW-dwarf limits which have been published subsequent to the completion of Ref. [4], the work on which this chapter is based, and how they may impact on the results we present in this chapter.

2.2 Preliminaries

2.2.1 Relic Density

It is a well-known phenomenon (see, e.g., Ref. [120]) that a massive particle species capable of annihilation into less massive species, and which is initially in thermodynamic equilibrium in the early universe, cannot maintain an equilibrium abundance to arbitrarily low temperature as the universe expands. Eventually, annihilations become too infrequent⁹ owing to the dilution of the particles in the expanding volume of the universe, and creation of a pair of the new particle species becomes rare because it becomes exponentially unlikely to have collisions with sufficient centre-of-mass energy. This leads to a ‘freeze-out’ of the abundance of the new particle species to some relic abundance. We shall assume that the measured average dark-matter density of the universe, $\Omega_{\text{DM}} h^2 \simeq 0.12$ [85], is entirely ascribable to the WIMP, which we will assume to be either a complex scalar or a Dirac fermion.

The phenomenon of thermal freeze-out is well described by the Boltzmann equation [120]; given the necessary initial data and annihilation cross sections as a function of temperature, one can simply solve this equation numerically to find the relic abundance (see, e.g., Chapter 3 for an analysis following this procedure). There does however exist an approximate

9. If the annihilation rate is $\Gamma \sim n\sigma v$ where n is the DM number density and σ is the annihilation cross section, the condition is roughly that $\Gamma < H$, where H is the Hubble constant [120].

method which has the benefit of expressing the relic abundance in closed form.¹⁰ We shall assume that a non-relativistic approximation to the annihilation cross section can be written in the form $\langle\sigma v\rangle_{\text{NR}} = a + bv^2$, where a and b are constants, $v^2 = 6T/M$, and $\langle\cdots\rangle$ indicates the thermal average.¹¹ The DM relic density can then be given to c. 5% accuracy by [120, 431]¹²

$$\Omega_{\text{DM}}h^2 = \frac{1.04 \times 10^9 \text{GeV}^{-1}}{M_{\text{Pl.}}\sqrt{g_*(x_F)}} \frac{x_F}{(a + 3b/x_F)} \times \begin{cases} 1 & \text{for self-conjugate DM} \\ 2 & \text{for non-self-conjugate DM} \end{cases}, \quad (2.1)$$

$$x_F \equiv \frac{M}{T_F} = \ln \left[c(c+2)g\sqrt{\frac{45}{8}} \frac{MM_{\text{Pl.}}}{2\pi^3\sqrt{g_*(x_F)}} \frac{a + 6b/x_F}{\sqrt{x_F}} \right], \quad (2.2)$$

where $x_F \equiv M/T_F \sim \mathcal{O}(20)$ with T_F the freeze-out temperature [120], M is the WIMP mass, $M_{\text{Pl.}} = 1.22 \times 10^{19} \text{GeV}$ is the Planck mass, the numerical parameter c is chosen such that $c(c+2) = 1$, the number of relativistic degrees of freedom at freeze-out is taken to be $g_*(x_F) = 106$ [120, 431], and g is the number of spin degrees of freedom for the DM particle:¹³ for real or complex scalar DM, $g = 1$; for both Majorana and Dirac fermion DM, $g = 2$. For the results in this chapter we will assume non-self-conjugate DM, since some of the operators under consideration vanish for self-conjugate DM.

10. It is well known (see, e.g., Ref. [462]) that this expansion fails when the annihilation takes place (a) near a resonance (i.e., a pole in the cross section), (b) just below a kinematic threshold, or (c) when so-called ‘co-annihilations’ are possible. Although (a) and (b) are applicable concerns for some kinematic regions for some of the operators we consider, we will ignore these complications and merely utilize the ‘naïve’ NR expansion. Co-annihilations require states closely spaced in mass, and can never arise in an EFT framework in the ‘Maverick’ limit in which we work (i.e., one light DM particle only, with all other new states heavy enough to integrate out).

11. Although σ is by itself the annihilation cross section, we shall for reasons of brevity frequently also refer to the quantity σv as the annihilation cross section. Furthermore, we will only be interested in the cross section in the non-relativistic limit, so we will drop the subscript ‘NR’. Annihilations relevant for indirect detection occur at very low velocity, so when we write σv for indirect detection we mean the velocity-independent part of the non-relativistic cross section (i.e., a in the non-relativistic expansion $\langle\sigma v\rangle_{\text{NR}} = a + bv^2$).

12. See Appendix A.1 for a discussion of the factors of 2 in Eqs. (2.1) and (2.7).

13. We mean the number of degrees of freedom for the particle only, and not the sum of the particle and anti-particle degrees of freedom should the DM be non-self-conjugate.

Working in an EFT framework and assuming that the DM annihilation proceeds through only one EFT operator $\mathcal{O}$ of mass-dimension- d , the term in the Lagrangian responsible for the annihilation can be written $\mathcal{L} \supset \Lambda^{4-d}\mathcal{O}$. It then follows that $\langle\sigma v\rangle$ is proportional to $\Lambda^{2(4-d)}$. Thus, for a given operator, $\Omega_{\text{DM}}h^2$ is a function of two parameters: M and Λ . Since we assume the particle is a WIMP accounting for 100% of the observed DM in the universe, for a given mass M we shall numerically solve Eqs. (2.1) and (2.2) for the EFT scale parameter Λ to satisfy the relation $\Omega_{\text{DM}}h^2 = 0.12$. This requirement implies that, for each operator, the only free parameter in the low-energy annihilation cross section is the DM mass M , which we scan over.

2.2.2 DM Density Profiles

Although thermal freeze-out is predicated on the effective shut-off of annihilation in the early universe, this does not imply that WIMP annihilation remains negligible for the remainder of the evolution of the universe. Structure formation leads to large local densities of both baryonic and dark matter, and this obviously implies that the annihilation rate $\Gamma \sim n\sigma v$ may again become large enough for detectable levels of annihilation to occur in regions such as the Milky Way Galactic Centre, or in the dark-matter-rich Milky Way-companion dwarf galaxies.

Understanding the spatial distribution $\rho(r) = Mn(r)$ of the DM mass-density in the relevant regions of space is important in the interpretation of any putative annihilation signal. The N -body simulation community is actively studying this problem, but there is as yet no consensus in the literature for the exact form that this so-called halo profile takes in galaxies such as our own. Consequently, it is conventional to quote all results assuming a number of different profiles. For constraints from Milky Way GC observations, we will utilize the (generalized) Navarro–Frenk–White, the Einasto, and the Isothermal profiles [449–451, 463, 464].

Table 2.1: Milky Way DM halo profile normalizations. We utilize a band of normalizations ($\rho_{\odot} \pm 2\sigma$ -limits) consistent with microlensing and galactic rotational velocities given in Ref. [463]. For the generalized NFW and Einasto profiles, we assume $r_s = 20$ kpc and $r_{\odot} = 8$ kpc, and take $\rho_{\odot}$ from Ref. [463]; the uncertainty ranges indicated in the table are the 95%-confidence limits on $\rho_{\odot}$ from Ref. [463]. For the Isothermal profile, we assume $r_s = 5$ kpc and $r_{\odot} = 8$ kpc, and use a representative value for $\rho_{\odot}$; the indicated uncertainty range in the table is also representative.

	$\rho_{\odot}$ [GeV/cm ³]
Generalized NFW	
$\gamma = 1.0$	$0.40^{+0.14}_{-0.12}$
$\gamma = 1.2$	$0.36^{+0.12}_{-0.11}$
$\gamma = 1.3$	$0.34^{+0.11}_{-0.105}$
Einasto	
$\alpha = 0.17$	$0.36^{+0.21}_{-0.115}$
Isothermal	
—	$0.40^{+0.1}_{-0.1}$

The generalized Navarro–Frenk–White (NFW) DM density profile is given by [463]

$$\rho(r; \gamma) = \frac{\rho_0}{(r/r_s)^{\gamma} (1 + r/r_s)^{3-\gamma}}, \quad (2.3)$$

where the scale parameter is chosen to be $r_s = 20$ kpc.¹⁴ The parameter γ controls the central slope of the profile [$\rho \propto (r/r_s)^{-\gamma}$ for $r \ll r_s$]: $\gamma = 1$ for the canonical NFW profile, while $\gamma > 1$ defines a profile with a steeper central region, known as a contracted profile (NFWc), which may arise due to gravitational interactions between the dark and baryonic matter as the latter cools during galaxy formation [465]. In either case, the profile is peaked toward $r = 0$ and ‘cuspy’. The overall scale ρ_0 is chosen for galactic measurements such that the local (in the vicinity of the Sun) DM density $\rho(r_{\odot}) \equiv \rho_{\odot}$ takes some desired value (see Table 2.1), while for extragalactic (dwarf) measurements, ρ_0 is chosen to obtain the correct

14. $1 \text{ pc} \approx 3.09 \times 10^{16} \text{ m} \approx 3.26 \text{ lightyears}$ [38].

observed stellar velocity dispersions [452].

The Einasto profile is given by [463]

$$\rho(r) = \rho_0 \exp \left\{ -\frac{2}{\alpha} \left[\left(\frac{r}{r_s} \right)^\alpha - 1 \right] \right\} \quad (2.4)$$

where the scale parameter is chosen to be $r_s = 20$ kpc. In line with large-scale numerical simulations, the parameter $\alpha = 0.17$ is used. This profile is cuspy. Overall normalizations are chosen similarly to the NFW profile; see Table 2.1.

The Isothermal profile is given by [463]

$$\rho(r) = \frac{\rho_0}{1 + (r/r_s)^2} \quad (2.5)$$

where the scale-parameter $r_s = 5$ kpc is chosen. This profile is known as ‘cored’ since $\rho(r)$ flattens off to a constant value at small r . Overall normalizations are again chosen similarly to the NFW profile; see Table 2.1. Unless otherwise noted, we have scaled all the limits presented in this chapter to reflect the halo profile parameters presented in Table 2.1 (original parameters assumed in the literature are shown in Table 2.2; see Section 2.2.3 for more details on the rescaling).

2.2.3 *Annihilation Photon Flux*

For all the EFT operators under consideration, the detectable annihilation signal in γ -rays includes (a) one or more monochromatic prompt photon ‘lines’ arising from the direct annihilation $\text{DM DM} \rightarrow \gamma X$, where X can be any SM boson consistent with gauge symmetries ($X = \gamma, Z, h$), and/or (b) a prompt diffuse continuum photon flux arising from final-state radiation or radiative hadron decay arising from various non-photon primary DM annihilation products: $\text{DM DM} \rightarrow XY \rightarrow \gamma$ ’s where X and/or Y is not a photon.

For each particular operator there will be a characteristic per-annihilation differential

Table 2.2: Summary of the main literature references used to compute cross section limits, with corresponding choices of DM halo profile normalizations and J -factors where applicable. [The J -factor is defined in Eq. (2.8).] If no value for a parameter was explicitly stated in a given reference, we indicate in the table footnotes why we have assumed the given value. The region of interest (ROI) used in the analysis is indicated in parentheses, following the notational conventions of the sources referenced. Further details in each case can be found in the referenced section number. In the final column we give the rescaling factor K applied in order to map published limits onto the halo profiles and normalizations given in Table 2.1; further details on the rescaling are given in Section 2.2.3. For the most part, the rescaling comes from scaling J -factors with $\rho_\odot^2$. The only exception is the isothermal case used by Ref. [433], where there is a significant difference from our assumed value for r_s that affects the J -factor by a factor of 2.

Profile (ROI)	γ or α	r_s [kpc]	$r_\odot$ [kpc]	$\rho_\odot$ [GeV/cm ³]	J -factor [10 ²¹ GeV ² /cm ⁵]	K
Fermi-LAT line search [449] — Section 2.3.1						
NFWc (R3)	$\gamma = 1.3$	20.0	8.2 ^a	0.4	13.9	0.689
NFW (R41)	$\gamma = 1.0$	20.0	8.2 ^a	0.4	8.48	0.960
Einasto (R16)	$\alpha = 0.17$	20.0	8.2 ^a	0.4	8.53	0.770
Isothermal (R90)	—	5.0	8.2 ^a	0.4	6.94	0.962
H.E.S.S. line search [450] — Section 2.3.2						
NFWc ^b	$\gamma = 1.3$	20.0	8.0	0.42	22.1 ^c	0.655
NFW	$\gamma = 1.0$	20.0	8.0	0.42	4.37 ^c	0.907
Einasto	$\alpha = 0.17$	20.0	8.0	0.42	2.43 ^c	0.735
Isothermal	—	5.0	8.0	0.42	0.312 ^c	0.907

Table continued on next page ...

^a Not explicitly stated in Ref. [449], but using this value correctly reproduces their J_{R3} , for which no point-source masking was applied. Ref. [449] also cites Ref. [466], where $r_\odot = 8.2$ kpc is mentioned.

^b The ROI for all cases is a 1° radius circle centred on the GC, with $|b| < 0.3^\circ$ excluded and no point-source masking applied.

^c J -factors are not given explicitly in Ref. [450], but we have verified these reproduce their limits.

Table 2.2: ... continued from previous page.

Profile (ROI)	γ or α	r_s [kpc]	$r_\odot$ [kpc]	$\rho_\odot$ [GeV/cm ³]	J -factor [10 ²¹ GeV ² /cm ⁵]	K
Hooper et al. template-based analysis of Fermi-LAT data [451] — Section 2.3.3						
NFWc	$\gamma = 1.2$	20.0	8.0 ^d	0.25	—	2.07
NFW	$\gamma = 1.0$	20.0	8.0 ^d	0.28	—	2.04
Einasto	$\alpha = 0.17$	20.0	8.0 ^d	0.25	—	2.07
Weniger line search based on Fermi-LAT data [433] — Section 2.3.6						
NFWc (Reg4)	$\gamma = 1.3$	20.0	8.5	0.4	—	0.644
NFW (Reg4)	$\gamma = 1.0$	20.0	8.5	0.4	—	0.890
Einasto (Reg4)	$\alpha = 0.17$	20.0	8.5	0.4	—	0.715
Isothermal (Reg4)	—	3.5	8.5	0.4	—	0.443

^d Not explicitly stated in Ref. [451], but assumed as this is the value used in Ref. [463], from which Ref. [451] took all its normalizations.

prompt photon spectrum

$$\frac{dN_\gamma(E)}{dE} \equiv \sum_f \text{BR}_f \left(\frac{dN_\gamma^f(E)}{dE} \right) \quad (2.6)$$

where $\text{BR}_f \equiv \sigma(\text{DM DM} \rightarrow f) / \sigma_{\text{total}}$ is the branching ratio for the annihilation mode to final state f , $dN_\gamma^f(E)/dE$ is the per-annihilation differential prompt photon spectrum for that mode, and the sum runs over all kinematically allowed annihilation modes for the operator. Given this spectrum, the differential DM-annihilation prompt photon flux which an experiment would observe at photon energy E is given by [464]¹⁵

$$\frac{d\Phi(E)}{dE} = \frac{[\sigma v]_{\text{total}}}{16\pi M^2} J \frac{dN_\gamma(E)}{dE} \times \begin{cases} 2 & \text{for self-conjugate DM} \\ 1 & \text{for non-self-conjugate DM} \end{cases} \quad (2.7)$$

$$\text{where } J = \int_{\text{LOS-ROI}} \rho^2[r(s, l, b)] ds \cos b db dl. \quad (2.8)$$

Hereafter we will assume non-self-conjugate DM; i.e., complex scalars and Dirac fermions. In Eq. (2.8), $\rho(r)$ is the DM density profile at Galactocentric distance r . The astrophysical ‘ J -factor’ is computed by integrating the squared DM density along the line of sight (LOS) from the Earth, and over the relevant Galactic lat/long coordinates (l, b) ¹⁶ that define the search region of interest (ROI). A point observed at LOS-distance s at lat/long coordinates (l, b) has Galactocentric distance $r = (s^2 + r_\odot^2 - 2sr_\odot \cos l \cos b)^{1/2}$ where $r_\odot$ is the Sun-GC distance. We will always take $r_\odot = 8 \text{ kpc}$ [463]. Finally note that the polar angle θ from the GC satisfies $\cos \theta = \cos l \cos b$.

Profile Rescaling

The constraints derived in this chapter draw on published gamma-ray line searches and limits on diffuse emission from several sources, each of which employ different assumptions

15. See Appendix A.1 for a discussion of the factors of 2 in Eqs. (2.1) and (2.7).

16. The Galactic Centre is defined to be $l = b = 0$, where l is Galactic longitude and b Galactic latitude. We work in a convention where $b \in [-90^\circ, 90^\circ]$ and $l \in [-180^\circ, 180^\circ]$.

for Milky Way halo profiles. We give the parameters used by these references in Table 2.2. For consistency, however, we would like to derive limits for a single set of MW-halo profile parameters (given in Table 2.1).

The difference in halo profiles is encapsulated in the J -factor defined at Eq. (2.8). Typically we wish to take some given J -factor from the literature, J , computed using some values of $r_\odot$, $\rho_\odot$ and r_s , and rescale it to J' , computed using values of $r'_\odot$, $\rho'_\odot$ and r'_s that are our choices. All other things being equal, the limit for $\sigma v \propto 1/J$, so if $[\sigma v]$ is the cross section upper limit computed using the values $r_\odot$, $\rho_\odot$ and r_s , then the cross section limit $[\sigma v]'$ that would be set with the new set of halo parameters $r'_\odot$, $\rho'_\odot$ and r'_s is given by

$$[\sigma v]' = \frac{J}{J'} [\sigma v] \equiv \frac{[\sigma v]}{K} \quad \text{where} \quad K \equiv \frac{J'}{J}. \quad (2.9)$$

We thus rescale the published limits by the factor K (see Table 2.2); as is evident from Eq. (2.9), $K > 1$ implies the upper limit is rescaled to be more stringent, whereas $K < 1$ implies that the upper limit is rescaled to be weaker than the upper limit that would be set taking the literature reference at face value.

We find that scaling the J -factors with $\rho_\odot^2$ accounts for most of the difference in the halo profile normalizations. This is primarily because the variation in $\rho_\odot$ values used in the literature is large (up to 40%), while the variation in the $r_\odot$ values is less than 10%, and the r_s values are almost uniform across references. There is only one case where there is a large difference in r_s compared to our chosen value: for the isothermal profile for **Reg4** of Ref. [433] where $r_s = 3.5 \text{ kpc} \rightarrow r'_s = 5.0 \text{ kpc}$. In this case the rescaling factor K is roughly a factor of 2 smaller than one would have obtained from simply scaling J with $\rho_\odot^2$, accounting for the fact that the core is less dense when r_s is increased.

Note that for the Fermi-LAT ROIs, we have computed the rescaling factors K without considering the effects of point-source masking, which are estimated to impact the absolute magnitude of the J -factor by less than about 10%. Our justification for this simplification is

that since the halo profiles are smooth away from $r = 0$ and the masking is such a small effect on the ROIs, the ratios $K = J'/J$ computed either with or without point-source masking being considered should be very nearly equal (and certainly much closer than 10%).

We also point out that the rescaling of the **Reg4** values from Ref. [433] is appropriate even though the ROIs in that reference were chosen based on an expected signal/background analysis with a specific profile in mind. Ref. [433] reports cross section results from each such ROI choice assuming a number of different profiles in the conversion from flux to cross section while holding the ROI fixed, and it is only these flux-to-cross-section conversions we are rescaling. Similarly, the ROIs in Ref. [449] were chosen based on an expected signal/background analysis; again, we are simply holding their chosen ROI, and hence fluxes, fixed and are only rescaling the flux-to-cross-section conversion. We do however note that a more optimized ROI choice may possibly exist if one were to re-run the analyses of Refs. [433] and [449] using our chosen profiles from the start; this however is a question beyond the scope of our analysis.

2.2.4 Photon Spectra

In the models we study, dark-matter annihilation produces primary electroweak gauge bosons (γ , Z , $W^\pm$), Higgs bosons, or fermions. In addition to primary photons, the subsequent shower and hadronization of final states will produce, *inter alia*, abundant hadrons, giving rise to additional prompt photons dominantly through decay of neutral pions $\pi^0 \rightarrow \gamma\gamma$. To extract cross section upper limits from the measured flux upper limits requires knowledge of this per-annihilation prompt photon spectrum; see Eq. (2.7). Here we summarize the physics of prompt photon production from DM annihilation.

We begin our discussion with (quasi-)monochromatic photons. The $\gamma\gamma$ and γh final states give rise to monochromatic photon lines.¹⁷ The contribution to the photon spec-

17. We assume $m_h = 125$ GeV [332, 333] in this chapter. For a 125 GeV SM Higgs, $\Gamma_h \approx 4$ MeV [468], so the width of the photon line from the γh final state is completely negligible, and it can be treated as monochromatic.

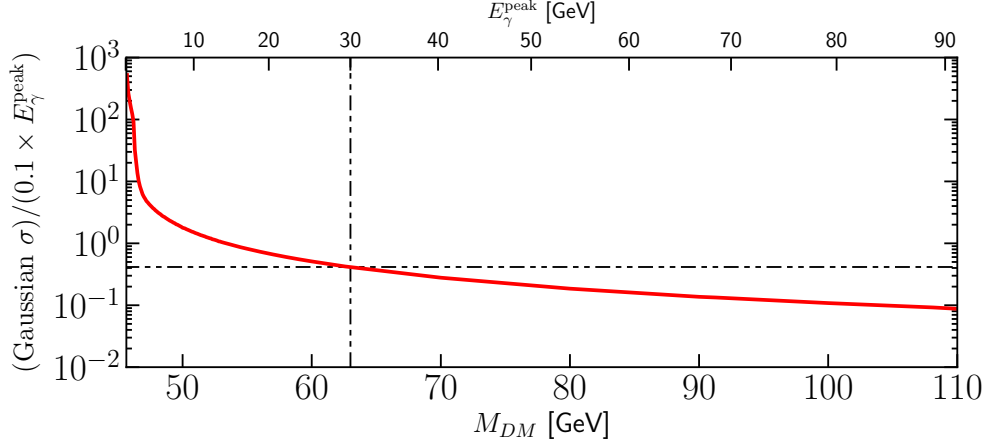


Figure 2.1: The Gaussian standard deviation of the photon line (i.e., the linewidth) arising from the γZ primary final state as a function of DM mass, expressed in units of $0.1 \times E_\gamma$, approximately the Fermi-LAT energy resolution (68%-containment half-width) [449, 467]. The linewidth is large with respect to the resolution for DM masses less than about 60 GeV; we only set limits on photon lines from γZ for $M > 63$ GeV ($E_\gamma > 30$ GeV), as discussed in the text (following Ref. [464]). The standard deviation here has been extracted by fitting a Gaussian to photon line shapes computed with Pythia (this provides a good fit to the central region of the peak but mismodels the tails).

trum is a delta function: $dN(E)/dE = N_\gamma \delta(E - \tilde{E}_\gamma)$, where simple kinematics gives $\tilde{E}_\gamma = M - m_X^2/4M$ for a γX final state, and N_γ counts the number of monochromatic photons per annihilation in the decay mode ($N_\gamma = 2$ for the $\gamma\gamma$ annihilation modes and $N_\gamma = 1$ otherwise). The photon line from a γZ annihilation mode is similarly peaked at $E_\gamma = M - m_Z^2/4M$, but obviously has a finite width due to the finite width of the Z . The width of the line will be relevant to our discussion of experimental constraints later and is shown in Figure 2.1.

For annihilation modes W^+W^- , ZZ , Zh , hh , $f\bar{f}$, γh , and γZ , there is also a continuum prompt photon spectrum. We employ PYTHIA v8.176 [469, 470] to perform the shower and hadronization (and hadron decay) to obtain the photon spectra. Following the procedures of Ref. [471], we create a fictitious spin-1 resonance with a mass of $2M$ and a width of 1 GeV, which we populate using fictitious e^+e^- colliding beams free of initial-state radiation at $\sqrt{s} = 2M$. We set by hand the resonance decay channels to be, in turn, 100% to each

of the desired primary final states, and the per-annihilation differential photon spectra are computed by correctly normalizing¹⁸ the histogrammed photon energies from 10^5 simulated annihilations per mass point per primary final state.

Good agreement is found between these computed PYTHIA spectra and a set of spectra from Ref. [471], for DM masses below 300 GeV. For larger DM masses, the effects of electroweak (EW) final-state radiation (FSR) of W 's and Z 's (which are not included in the PYTHIA shower in the version we utilized in this analysis) can significantly alter the spectra at low E_γ [471, 472]; these effects have been included in the spectra in Ref. [471]. However, we shall see below that at large DM mass we will be primarily interested in the continuum spectrum for $E_\gamma \in [10, 100]$ GeV. In this photon energy range, the EW corrections are unimportant for $M \lesssim 1$ TeV except for primary final states containing charged leptons, which in any event give rise to about an order of magnitude fewer photons per annihilation compared to other channels. Furthermore, the branching fraction for annihilation to leptonic final states in particular, and fermionic final states more generally, is subdominant at large M (see the result plots in Section 2.4), with the exception of operator XX-1, where all annihilations are mediated through an s -channel Z or γ . Therefore, since the error of neglecting the EW FSR is small, and since the necessary spectra are not all available in Ref. [471], we have chosen for the sake of uniformity to use the PYTHIA spectra which we have computed rather than attempting to use the EW FSR-corrected spectra from Ref. [471].

Qualitative Understanding of the Shape of the Photon Spectra

The shape, and in particular the DM-mass dependence, of the continuum photon spectrum differs qualitatively depending on whether the primary final states are (a) leptonic, (b) non-leptonic and colour neutral, or (c) coloured. We consider illustrative cases for the latter two: annihilation to W^+W^- and annihilation to $b\bar{b}$ (light $q\bar{q}$ modes are qualitatively similar), and

18. Since the spectra are normalized per annihilation, the appropriate normalization is $dN/dE = \text{counts per bin} / \text{bin width} / \text{total number of annihilations}$.

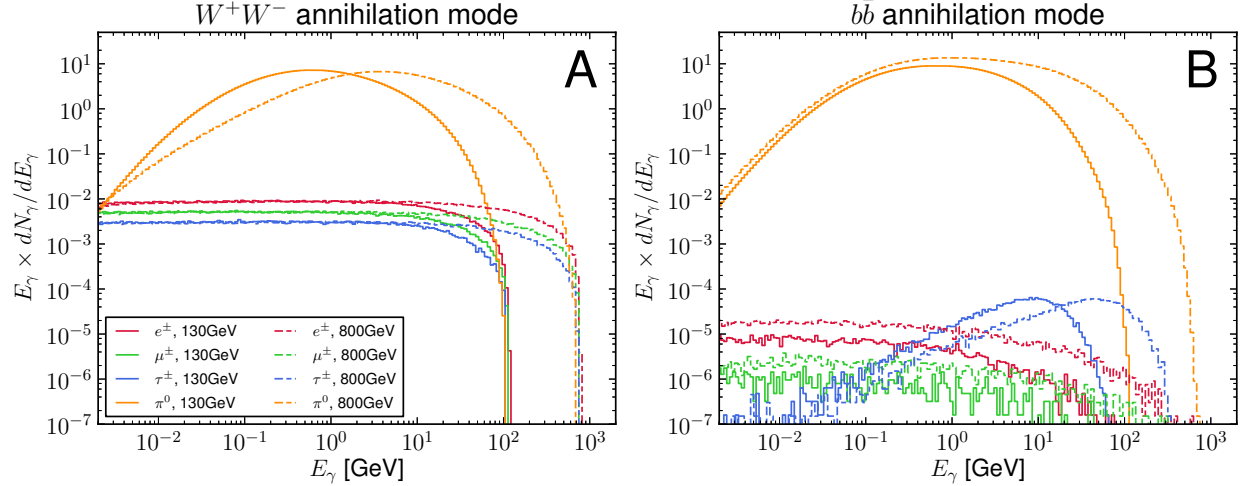


Figure 2.2: The photon spectrum arising from DM annihilation, decomposed according to the production mechanism of either neutral pion decay, or final state radiation of charged leptons, and computed using PYTHIA v8.176 [469, 470]. We show annihilation to W^+W^- (Panel A) and to $b\bar{b}$ (Panel B), for DM masses M of 130 GeV and 800 GeV. It is clear that for $E_\gamma \gtrsim 1$ MeV, π^0 decay dominates the spectrum. In the case of the W^+W^- final state, the hardening of the photon spectrum from pion decay with increasing M reflects the increasing boost of each W (and thus of its decay products). For the $b\bar{b}$ final state, the yield of pions increases with increasing M over the entire energy range of interest. We have omitted the spectra from other hadronic decay modes for clarity.

comment on the leptonic modes.

Consider first the WW case: in Figure 2.2 we plot the photon spectrum decomposed into photons from π^0 decay and those from final state radiation (FSR) of charged leptons (omitting for the sake of clarity the contributions from other hadronic decay processes). For $E_\gamma \gtrsim 1$ MeV, the photon production mechanism is dominantly due to neutral pion decay. The strongest continuum γ -ray constraints are for energies from 0.1 GeV to 100 GeV, so we will always be in the regime where the behaviour of the photon spectrum from diboson annihilation channels is dominated by photons from $\pi^0 \rightarrow \gamma\gamma$. Meanwhile, the FSR from leptons has a spectrum that goes like $dN_\gamma/dE_\gamma \sim E_\gamma^{-1}$ (until near the kinematic endpoint) and is subdominant for $E_\gamma \gtrsim 1$ MeV.

Since W is colour neutral, the spectrum and multiplicity of photons is determined by the independent decay and shower of each W , boosted into the centre of mass frame of the

annihilation. As M increases, the W 's are increasingly boosted, so the decay products also become more energetic, while the multiplicity (i.e., the pion yield) remains roughly constant at about 14 neutral pions per W decay; this is because the initial hard scale of the parton shower remains constant at m_W as M increases (see Ref. [469]). We can observe this in the hardening of the photon spectrum in Figure 2.2. Furthermore, this migration to higher energy decay products, while the yield remains constant, leads to a decrease in the number of photons produced at low energy as M increases. This can also be observed in, e.g., Panel B of Figure 2.6, where for $M > 100$ GeV—where the WW final state dominates—there is a decrease in the photon spectrum around $E_\gamma \sim 0.1$ GeV as M increases.

Since most of the operators considered in this chapter have annihilation modes into gauge bosons or Higgs, the mass dependence of the total γ -ray spectra has similar behaviour to the WW case, modulo the effect of changing branching fractions. Some operators which have only diboson annihilation modes also show first an increase in the low-energy photon spectrum with increasing M (below or around a hundred GeV), before the decrease discussed above sets in. This has less to do with the intrinsic photon spectrum from each annihilation mode and more to do with the branching fractions for each mode. If the $\gamma\gamma$ mode is present, then the branching fraction to γZ starts off small around $M \sim m_Z/2$ and rapidly increases with M , giving the observed increase in the continuum spectrum (e.g., Operator VI-1 in Figure 2.4 of Section 2.4). In this case, there may be a WW or ZZ mode entering once M is increased through $m_{W,Z}$, which results in a further increase in the number of photons since the spectra are normalized per-annihilation and twice as many W/Z 's per annihilation yields twice as many photons per annihilation (the W and Z intrinsic photon spectra are almost identical, modulo kinematic effects). For M above a (few) hundred GeV, the low-energy spectrum begins to decrease again as per the discussion in the previous paragraph. Explanations of a similar nature are possible for cases where other mixtures of diboson annihilation modes are present.

For annihilation to $b\bar{b}$, shown in Figure 2.2, the dominant photon-production mechanism

is still neutral pion decay, but the same qualitative picture does not obtain: there is a significant increase in the pion yield with increasing M because, in this case, the initial hard scale for the parton shower is set by M (see Ref. [469]) and so the number of FSR gluons and other strongly interacting partons produced in the shower increases significantly as the DM mass increases. Therefore, although the average pion energy does increase, the simultaneous increase in the overall pion yield means that the pion yield at low energies actually slowly increases as M increases; the photon yield thus shows a similar increasing trend. This is evident, e.g., in the slow overall monotonic increase in the spectrum as M increases for Operator XX-1, shown in Figure 2.37. (For the other operators considered here, this behaviour gets cut off for $M \gtrsim 100$ GeV since the branching fraction to quarks typically becomes subdominant once diboson annihilation modes become kinematically allowed.)

Finally, for annihilation to charged leptons, the dominant photon production mechanism is direct final-state photon radiation from the leptons. This spectrum falls roughly as E_γ^{-1} over most of the energy range with a high-energy kinematic cutoff for the photon spectrum at M ; the normalization of the photon spectrum grows slowly as M increases. Again however, with the exception of Operator XX-1 already noted above, fermionic final states tend to have rapidly falling branching fractions once on-shell diboson final states become kinematically allowed as M increases, so the increase in the spectrum at low $E_\gamma \sim 0.1$ GeV is cut off for $M \gtrsim 100$ GeV.

2.3 Limits

In this section we discuss the observational limits used in constraining the effective operators. We consider line searches in gamma-ray data from Fermi-LAT and H.E.S.S., as well as constraints on diffuse gamma-ray emission derived from Fermi-LAT observations of the GC and MW-dwarf galaxies, for which the continuum photon emission is relevant. We describe our methods of extracting limits for the operators, with an emphasis on how we have re-interpreted published results for the specific final states and branching ratios pertaining to

each operator. Our final results are shown in the figures in Section 2.4, along with a universal caption and a detailed summary of the constraints on the operators.

2.3.1 Low-Energy Line Limits: *Fermi-LAT*

In Ref. [449], the Fermi-LAT Collaboration presents 95% confidence level (CL) upper limits (UL) on the flux Φ of photons from regions on the sky centred on the Galactic Centre due to a strictly monochromatic underlying photon line in the photon energy range 5–300 GeV¹⁹ (see Appendix A.5.2 for a summary of the subsequent update [231] to this analysis). The underlying photon energy E_γ was scanned across this range, and for each energy the Collaboration obtained a flux limit by performing a maximum-likelihood analysis in which the Fermi-LAT line shape plus a power-law background was fitted²⁰ to the 3.7-year data set, over sliding energy ranges $E_\gamma \pm 6\sigma_E$, where σ_E is the Fermi-LAT energy resolution (68%-containment half-width) [449]. The flux limit Φ reported in Ref. [449] is the 95% CL UL of the normalization of the line in this fit, multiplied by a factor giving the effective exposure time of the experiment.

The Fermi-LAT search methodology selected a different signal region of interest (ROI) for each of four DM halo profiles (NFW, Einasto, Isothermal, NFWc _{$\gamma=1.3$}) considered in Ref. [449] in order to maximize the expected signal-to-background ratio²¹ as estimated through Monte Carlo simulations. Thus, four distinct flux limits are presented in Ref. [449]: one for each ROI. These ROIs are denoted R_α where α is the angular size of the circular ROI centred on the GC, with the region $|l| > 6^\circ$ and $|b| < 5^\circ$ masked; regions around known γ -ray point sources are also masked for all ROIs except **R3** [449].

We derive constraints on σv from the flux limits presented in Ref. [449], which are given

19. For the **R3** ROI in Ref. [449], the flux limits are only given for the energy range 30.93–300 GeV.

20. The position of the line in the fit was fixed; the line and background normalizations as well as the background spectral index were allowed to float [449].

21. This is the approach advocated in Refs. [432] and [433]. The methodology of the previous Fermi-LAT analysis [464] was to look at a single ROI for all profiles, which obviously leads to a single flux limit, which is then interpreted as a different σv limit for each profile.

at discrete E_γ in the energy range 5–300 GeV; we have interpolated (linearly in $\log \Phi$ vs. E_γ) these limits to intermediate energies where necessary. If the underlying photon line arising from a γX final-state mode is (quasi-)monochromatic, we could merely use the flux upper limit at $\tilde{E}_\gamma = M - M_X^2/4M$ combined with Eq. (2.8) and the J -factors in Ref. [449] (rescaled per Table 2.2) to derive limits on σv :

$$[\sigma v]^{95\% \text{ CL UL}} = \Phi^{95\% \text{ CL UL}}(\tilde{E}_\gamma) \times \frac{16\pi M^2}{JN_\gamma}, \quad (2.10)$$

where N_γ is the number of photons in the line per annihilation: 2 for $\gamma\gamma$ and 1 otherwise. This is always the case for the γh annihilation mode; complications however arise for the γZ and $\gamma\gamma$ modes for the operators we consider.

The photon line arising from γZ annihilation modes has an intrinsic width of order 1 GeV, which can be significant with respect to the c. 5–10% Fermi-LAT energy resolution at low energies [449, 467] (see Figure 2.1). However, even for a line with intrinsic width of 50% of the Fermi-LAT energy resolution, the resulting observed spectral feature is only expected to be broadened by around 11% after convolution with the Fermi-LAT energy dispersion function [449]. If such a feature were fitted with a line shape derived from the assumption of a purely monochromatic intrinsic line, the number of photons would be underestimated by about 7% (see Appendices D2–3 of Ref. [449]), which would set the flux limit correspondingly more stringent than it should. However, this effect is much smaller than both the expected statistical fluctuation (c. 50%) in the limits set by Ref. [449] and the uncertainty in the halo profile normalization (a factor of c. 10; see results), and it is roughly on the same order as other systematic effects (c. 10%) which are present in the Fermi-LAT analysis and ignored in their flux upper limits [449] (see also Appendix A.3). Provided the linewidth is expected to be less than about half the energy resolution, we therefore make no correction for the finite Z width, and merely use the Fermi-LAT flux limits derived under the assumption of a monochromatic intrinsic line in deriving limits on $[\sigma v]_{\gamma Z}$. To satisfy this linewidth

requirement, we follow the older Fermi-LAT line search analysis [464] and only present limits on σv from the γZ annihilation mode for $E_\gamma \geq 30$ GeV, which corresponds to $M \gtrsim 63$ GeV (see also Figure 2.1).

There is a significant complication [473] in interpreting the flux limits for all the operators with $\gamma\gamma$ annihilation modes: due to their gauge structure, all such operators under consideration also have a γZ annihilation mode which yields a second line at an energy $\Delta E = -m_Z^2/4M$ below the $\gamma\gamma$ line energy. Since the unbinned likelihood analysis in Ref. [449] is sensitive to both shape and normalization of the line spectral feature, this makes the interpretation of the single-line flux limits difficult in the region of DM masses where the two photon lines are separated by an amount on the order of the Fermi-LAT energy resolution. Depending on the WIMP mass, we have adopted a few different techniques to interpret the single-line flux limits for these operators. In all cases, we choose to use the flux limits to set a single limit on the quantity $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$.

$M < 63$ GeV

This mass range corresponds to $E_\gamma \leq 30$ GeV for the γZ channel (when kinematically allowed); as discussed above, we do not set a γZ line limit in this mass range. Our limits here are thus driven by the monochromatic line from the $\gamma\gamma$ final state; we find the 95% CL UL on $\sigma_{\gamma\gamma}v$ using Eq. (2.10) with the flux at $E_\gamma = M$, and use this to set, at each WIMP mass, the limit

$$\left[(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v \right]^{95\% \text{ CL UL}} = (\sigma_{\gamma\gamma}v)^{95\% \text{ CL UL}} \left[2 + \frac{\sigma_{\gamma Z}v(M)}{\sigma_{\gamma\gamma}v(M)} \right] \quad (2.11)$$

where the cross section ratio in the $[\dots]$ -brackets is computed using the analytic expressions in Ref. [431] and rescales the $\sigma_{\gamma\gamma}v$ limit to the quantity $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$.

$63 \text{ GeV} < M < 80 \text{ GeV}$

In this mass region, we have two lines which are relatively well separated²² assuming a 5 to 10% energy resolution for Fermi-LAT [467]. We thus use Eq. (2.10) to set independent 95% CL UL on $\sigma_{\gamma\gamma}v$ using the flux at $E_\gamma = M$, and on $\sigma_{\gamma Z}v$ using the flux at $E_\gamma = M - m_Z^2/4M$. We use these two quantities to set, at each WIMP mass, the limit

$$\begin{aligned} \left[(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v \right]^{95\% \text{ CL UL}} = \min \left\{ (\sigma_{\gamma\gamma}v)^{95\% \text{ CL UL}} \left[2 + \frac{\sigma_{\gamma Z}v(M)}{\sigma_{\gamma\gamma}v(M)} \right], \right. \\ \left. (\sigma_{\gamma Z}v)^{95\% \text{ CL UL}} \left[1 + \frac{2\sigma_{\gamma\gamma}v(M)}{\sigma_{\gamma Z}v(M)} \right] \right\} \quad (2.12) \end{aligned}$$

where the cross section ratios in the $[\dots]$ -brackets are computed using the analytic expressions in Ref. [431]. That is, we simply take the stronger of the limits on each individual line, which is a conservative way to set the limits.

$80 \text{ GeV} < M < 160 \text{ GeV}$

In this intermediate mass region, the merged spectral feature expected in the Fermi-LAT data corresponding to the two lines will be very broad with respect to the energy resolution, and may be a double-peaked structure depending on the relative strengths of the lines. Such a feature looks very different from the line shape assumed in the Fermi-LAT line search [449] and since, as noted above, that analysis was sensitive to the shape of the fitted feature as well as its normalization, we exercise an abundance of caution and do not set any limits on σv in this region. This caution notwithstanding, we do not expect the limits to differ from those at nearby energies by more than a factor of a few. A reproduction of the analysis in Ref. [449] with a different assumed line shape is a task best suited for the experimental

22. To be strictly correct, to use the limits independently the two lines must be separated enough that they do not lie in the same sliding-fit energy range used in the Fermi-LAT line search, which for a peak at E_γ was $E_\gamma \pm 6\sigma_E$ where σ_E is the 68%-containment half-width for the Fermi-LAT energy dispersion [449]. We have been a little more permissive in setting limits up to $M = 80 \text{ GeV}$, where the two lines are only about $3\sigma_E$ to $6\sigma_E$ apart (depending on whether one assumes a 10- or 5-percent energy resolution, respectively), but we do not expect this to be much of an issue.

collaboration, and is beyond the intended scope of our analysis.

$M > 160 \text{ GeV}$

In this mass region, the two lines are sufficiently close that the spectral feature expected in Fermi-LAT is merely a slightly broadened (width increased by $\lesssim 10\%$) line, so a single limit can be set on the combined quantity $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$. We do this by using the flux upper limit from Ref. [449] evaluated at the flux-weighted average energy

$$\bar{E}_\gamma = \frac{2\sigma_{\gamma\gamma}v(M) \times M + \sigma_{\gamma Z}v(M) \times (M - m_Z^2/4M)}{2\sigma_{\gamma\gamma}v(M) + \sigma_{\gamma Z}v(M)}, \quad (2.13)$$

where the cross sections here are computed using the analytic expressions in Ref. [431]. The limit is set as

$$\left[(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v\right]^{95\% \text{ CL UL}} = \Phi^{95\% \text{ CL UL}}(\bar{E}_\gamma) \times \frac{16\pi M^2}{J}. \quad (2.14)$$

2.3.2 High-Energy Line Limits: H.E.S.S.

In Ref. [450], the H.E.S.S. Collaboration presents 95% CL UL on the photon flux Φ from a strictly monochromatic photon line in the energy range 0.5–20 TeV (see Appendix A.5.3 for a summary of a subsequent supplement [232] to this analysis). The ROI is a 1° radius circle centred on the GC, with $|b| < 0.3^\circ$ excluded; no masking of point sources was performed [450]. The flux upper limits in Ref. [450] were extracted by performing a maximum-likelihood analysis in which a Gaussian peak (photon line) was fitted along with a parametrized background to 112 hours of data from the H.E.S.S. γ -ray experiment. The position of the Gaussian peak was scanned over the search energy range (with the standard deviation constrained to be equal to the H.E.S.S. energy resolution; see below), and the Gaussian normalization and background parameters were fitted at each search energy. The line flux limits reported in Ref. [450] are the 95% CL UL on the fitted Gaussian normalizations.

Flux limits are presented at discrete E_γ on the energy range 0.5–20 TeV in Ref. [450]; we have interpolated (linearly in $\log \Phi$ vs. $\log E_\gamma$) these limits to intermediate energies where necessary. Isothermal-profile limits here are very weak as a consequence of the cored profile and the ROI being a very small region near the GC. It was necessary for interpreting the H.E.S.S. limits to compute the J -factors defined in Eq. (2.8) for the halo profiles of our choice, given in Table 2.1. We did this by computing J -factors for the parameters used in Ref. [450] (see Table 2.2) and then rescaling the resulting J -factors as necessary, as discussed in Section 2.2.3.

Additionally, in computing cross section limits from the fluxes, we neglected the small shift in the photon line position for final states involving a massive particle (the largest this shift gets is about 7 GeV for the γh line at $M = 500$ GeV) and have simply evaluated all limits assuming $E_\gamma = M$. Furthermore, since the H.E.S.S. energy resolution is 17% at 500 GeV, dropping to 11% at 10 TeV [450], it is always a good approximation to combine the unresolved $\gamma\gamma$ and γZ lines into a single spectral feature to derive combined limits on $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$ for all the operators with both $\gamma\gamma$ and γZ annihilation modes. The width of the photon line from γZ has no significant effect here.

To summarize, we derived cross section limits from the H.E.S.S. flux upper limits using, as appropriate,

$$\left. \begin{aligned} & \left[(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z}) v \right]^{95\% \text{ CL UL}} \\ & \left[\sigma_{\gamma Z} v \right]^{95\% \text{ CL UL}} \\ & \left[\sigma_{\gamma h} v \right]^{95\% \text{ CL UL}} \end{aligned} \right\} = \Phi^{95\% \text{ CL UL}}(E_\gamma = M) \times \frac{16\pi M^2}{J} . \quad (2.15)$$

2.3.3 Inclusive Spectrum Limits: Hooper et al. Analysis

Ref. [451] presents 95% CL UL on the inclusive photon spectrum from the Galactic Centre region based on the 3.7-year Fermi-LAT data in a model-independent form as 95% CL UL

limits on the quantity $\mathcal{F}_{\text{bin } j} \equiv (\sigma v/M^2) \int_{\text{bin } j} (dN_{\gamma}^{\text{total}}/dE_{\gamma}) dE_{\gamma}$ for the four photon energy bins of 0.1–1 GeV, 1–3 GeV, 3–10 GeV, and 10–100 GeV. These limits were derived using a signal-template-based subtraction methodology with a photon-flux template proportional to $\int_{\text{LOS}} \rho^2(r[s, l, b]) ds$, rather than the ROI-integrated approach used in the other references we have used. As a result, we are unable to simply rescale the limits obtained with the choice of $\gamma = 1.2$ for the NFWc profile in Ref. [451] to the value $\gamma = 1.3$ that we used for presenting the results for the line searches, because this parameter change modifies the signal morphology in a way what would be uniquely impactful on this analysis; we do however still rescale to our choice for the normalization of $\rho_{\odot}$ for $\gamma = 1.2$ (see Table 2.2).

We employed these model-independent limits to set limits on $[\sigma v]_{\text{total}}$, always taking the most stringent of the limits from the four bins listed above. For each M , the prompt photon spectrum dN_{γ}/dE_{γ} (including all possible annihilation modes, including those giving photon lines were applicable) was numerically integrated over each of the four energy bins defined above. For each bin, the limit

$$[\sigma v]_{\text{bin } j}^{95\% \text{ CL UL}} = 2 \times \mathcal{F}_{\text{bin } j}^{95\% \text{ CL UL}} \times M^2 \times \left[\int_{\text{bin } j} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma} \right]^{-1} \quad (2.16)$$

is formed (the factor of 2 is a re-interpretation of a limit set for self-conjugate DM in Ref. [451] to our choice of non-self-conjugate DM [see Eq. (2.7)]), with the final limit on the total cross section taken as

$$[\sigma v]_{\text{total}}^{95\% \text{ CL UL}} = \min_{\text{bin } j} \left\{ [\sigma v]_{\text{bin } j}^{95\% \text{ CL UL}} \right\}. \quad (2.17)$$

For large M , the $E_{\gamma} \in [10, 100] \text{ GeV}$ bin typically gives the most stringent limit (cf. the comments in Section 2.2.4 on the regions of the continuum photon spectrum in which we are interested). For $M \lesssim 100 \text{ GeV}$ (the exact cutoff here depends on the identity of X for a γX annihilation mode), the inclusion of line photons makes the resulting limits much more aggressive, compared to the case if only continuum photons were present. We have made

no attempt to convolve the photon lines here with the Fermi-LAT experimental resolution; doing so would smooth some of the sharp transitions evident in our result plots at values of M where a line crosses one of the bin edges (see Section 2.4), but would make no other qualitative changes.

Ref. [451] also presents limits in a model-dependent form assuming 100% branching fraction to, in turn, WW , ZZ , $b\bar{b}$, $c\bar{c}$, $\mu^+\mu^-$, and $\tau^+\tau^-$ final states. We checked Ref. [451] for consistency between the limits on σv for each of these six final states obtained (a) directly from the model-dependent results of Ref. [451], and (b) by making use of our computed continuum photon spectra to convert the model-independent $\mathcal{F}_{\text{bin } j}^{95\% \text{ CL UL}}$ limits to limits on σv for each of these six final states. We found a discrepancy in the σv limits from (a) and (b), with the latter yielding the more aggressive limits on σv by a factor of c. 2.8. Following Ref. [474], we resolve this discrepancy in favour of setting more conservative limits.²³ To this end, we weakened the 95% CL UL on σv set for our operators using the model-independent $\mathcal{F}$ as described in the preceding paragraph, by multiplying them by a factor of 2.8.

2.3.4 Continuum Limits: Fermi-LAT

Assuming particular annihilation channels, the Fermi-LAT Collaboration [452] also presents 95% CL UL on the inclusive photon spectrum using observations of ten MW-companion dwarf galaxies (the ‘stacked dwarf’ limits). Their limits were obtained by performing a simultaneous maximum-likelihood analysis on all ten dwarfs in which the expected DM annihilation signal was fitted along with a diffuse galactic background model to two years of Fermi-LAT data for photons in the energy range 0.2–100 GeV [452]. The normalizations of the two components were floated in the fit and cross section limits were extracted from the normalization of the DM signal component assuming, in turn, that the annihilation was 100% via each of the modes $b\bar{b}$, WW (equivalent to ZZ for these purposes), $\mu^+\mu^-$, and $\tau^+\tau^-$ [452].

23. The Author would like to thank Dan Hooper for discussions about this point [474].

Unfortunately, since this analysis is sensitive to spectral shape, and since the continuum photon spectrum for annihilation modes have different shapes in general, it is not possible to simply reverse-engineer these limits to set the exact 95% CL UL considering all the continuum contributions from a combination of annihilation modes (which is the case for the operators we consider). We therefore extract approximate and rather conservative limits on $[\sigma v]_{\text{total}}$ by requiring that the individual WW/ZZ , $b\bar{b}$, $\mu^+\mu^-$, and $\tau^+\tau^-$ contributions to σv for any operator do not violate the individual experimental upper bounds. For a given final state, we take the limit

$$[\sigma v]_{\text{total, from } f}^{95\% \text{ CL UL}} = 2 \times \frac{[\sigma v]_f^{95\% \text{ CL UL}}}{\text{BR}_f}, \quad (2.18)$$

where $[\sigma v]_f^{95\% \text{ CL UL}}$ is the limit taken directly from Ref. [452] and BR_f is the branching fraction for the annihilation mode $f \in \{WW/ZZ, b\bar{b}, \mu^+\mu^-, \tau^+\tau^-\}$ computed using the analytic expressions in Ref. [431]. The factor of 2 is a re-interpretation of a limit set for self-conjugate DM in Ref. [452] to our choice of fermionic or complex scalar DM.

These constraints from dwarf galaxies as we have implemented them are conservative and approximate, and are not necessarily indicative of the full ability of such measurements to constrain these operators. Based on Figure 14 of Ref. [451], we expect the exclusion reach for an analysis which took into account the entire continuum spectrum at once—rather than only looking independently at each component of the spectrum from each final state and comparing to the relevant final state limit from Ref. [452] [thereby paying a cost of BR_f , per Eq. (2.18)]—would be similar to the inclusive Galactic Centre limits from Ref. [451], in cases where there are no lines.

Finally, we note that as the work on which this chapter is based [4] was being finalized, the Fermi-LAT Collaboration released updated constraints stacking 25 dwarf satellites based on four years of data [475]. These limits are weaker than expected, and in particular weaker than the limits we take from Ref. [452]. However, since we found that the stacked dwarf

limits from Ref. [452] are not constraining (as we implemented them), the updated analysis in Ref. [475] ultimately does not affect our results, and we thus have not updated our limits to account for the latter. See also Appendix A.5.4 for a summary of subsequent updates to the Fermi-LAT MW-dwarf limits.

2.3.5 *Continuum-to-Line Ratio Limits: Cohen et al. Analysis*

Ref. [453] finds 5.5σ (local) evidence for a photon line in the 3.7-year Fermi-LAT data at roughly 130 GeV (see also Refs. [432, 433], but bear in mind the discussion in Appendix A.5.1) and, based on the same data, presents limits on the ratio R of the annihilation cross section via modes which give rise to continuum photons, to the annihilation cross section giving lines ($\gamma\gamma$ and/or γZ). These limits are based on observations of photon fluxes from Fermi-LAT in the energy range 5–200 GeV in an annulus around the GC with inner and outer radii of 1° and 3° , respectively. We compare the two classes of R limits in Ref. [453]—shape and supersaturation—to the ratio $R_{\text{th}} = (\sigma_{WWv} + \sigma_{ZZv} + \sigma_{b\bar{b}v}) / (2\sigma_{\gamma\gamma v} + \sigma_{\gamma Zv})$, computed using the analytical expressions from Ref. [431].

The supersaturation limits on R are derived in Ref. [453] without assuming a background model, requiring only that the photons from DM annihilation do not supersaturate the observed photon spectrum. These limits are only presented for the case where 100% of the continuum photon flux comes from annihilations going to a W^+W^- primary final state, but due to the similarity of the spectra, these limits are also applicable if the annihilation is to ZZ or $b\bar{b}$. Although the limits are very conservative, they are more robust to details of astrophysical backgrounds or the DM model than are the shape constraints.

We briefly summarize the procedure the authors of Ref. [453] followed to obtain the supersaturation limits: for each M , the number of line photons was found by performing a maximum-likelihood analysis in which the photon lines for $\gamma\gamma$ and γZ were fitted to binned photon-count data, marginalizing over the relative normalization of the lines. The sum of the signals from the two lines was taken as the number of line photons. To find the

upper limit on the continuum normalization, the energy bin where the continuum photon spectrum is expected to peak relative to an assumed power-law astrophysical background with a spectral index of 2.8 was first found (this is the only point where spectral information was used); for a pure WW annihilation mode, this was determined to be the bin 10–20 GeV. The normalization for the continuum annihilation was then scaled up until the number of photons in the 10–20 GeV bin violated the 95% CL UL from data. Correcting for the effective area in Fermi-LAT, this results in a limit on the continuum-to-line cross section ratio.

The second (much stronger) class of limits presented in Ref. [453] are shape limits, derived there from a maximum-likelihood analysis including the two photon lines, the continuum photons, and a single power-law background over the energy range 5–200 GeV. At each M , the fit in Ref. [453] was performed for the ratio of continuum-to-line photons in that energy bin, marginalizing over all other parameters including the relative strengths of the two lines. This procedure effectively resulted in a two-dimensional likelihood profile in the M – R plane; the 2σ limit line in this M – R plane is taken as the 95% CL UL on R . We consider in turn the results of Ref. [453] for each of the continuum-photon-producing final states, 100% WW/ZZ and $b\bar{b}$; the $\mu^+\mu^-$ and $\tau^+\tau^-$ modes were also considered in Ref. [453], but for our operators annihilation to charged leptons is typically subdominant.

It should be borne in mind that the analysis of Ref. [453] marginalized over the $\sigma_{\gamma\gamma v}/\sigma_{\gamma Z v}$ ratio, while the operators we consider have fixed line ratio for a given M . Furthermore, the limits are most applicable if there is some region of M where a good fit to the photon line is possible; note that the line fits are discussed further in Section 2.3.7 and also indicated in panel C of our results figures. With more recent data [449], evidence for the line has weakened since Ref. [453] was published (see also Appendix A.5.1 discussing the subsequent complete disappearance of this signal), which requires fewer line photons and would lead to correspondingly weaker limits on the continuum-to-line ratio if this analysis were to be updated. Furthermore, Ref. [453] assumes 100% annihilation to each of the various continuum-photon-producing modes, while the operators we consider invariably have some

combination of multiple continuum-photon-producing annihilation modes. Note also that no Inverse Compton Scattering (ICS) contribution was included in the continuum spectrum used in Ref. [453], as it should be sub-dominant for WW/ZZ and $b\bar{b}$, nor was the contribution to the continuum spectrum of the Z in the γZ channel included. Finally, none of the limits presented is applicable to operators with γh final states as this annihilation mode introduces a new line which would dramatically improve the goodness-of-fit in the $M \sim 155$ GeV region (for which $E_\gamma \sim 130$ GeV for this final state).

Nevertheless, even given all these caveats and issues, the limits remain indicative for the case where a least one of the annihilation modes $\gamma\gamma$ or γZ is present, and the continuum photon spectrum for the operator is dominated by one of WW/ZZ or $b\bar{b}$. Overall, one should interpret the limits qualitatively, not quantitatively: for operators with continuum-to-photon ratios near the limit, one should be careful to re-perform the full likelihood fit with the correct, known branching ratios for the various modes for the specific operator in question in order to definitively settle the issue of whether or not the operator is excluded.

2.3.6 Line-like Feature: Weniger-like Analysis

It is interesting to examine whether it is possible for the photon lines arising from these operators to explain, with the correct normalization, the line-like feature near²⁴ 130 GeV first reported in Ref. [432] and determined to be present at 4.6σ local significance by Ref. [433] in the 3.7-year Fermi-LAT data, assuming for these purposes that we take this feature seriously as a DM signal. (However, see Appendix A.5.1: this feature now (2017) appears to have disappeared entirely, given more data and improved data analysis [231, 232].)

Assuming the annihilation is to $\gamma\gamma$, Ref. [433] reports values for σv required to account for this feature. For the operators we consider here, there may be annihilation modes to any of the final states γX where $X = \gamma, Z$, or h , where the DM mass must be $M = 130$ GeV,

²⁴. We shall assume here that this line is at exactly 130 GeV, although the analysis by the Fermi-LAT Collaboration [449] put the line energy at 133 GeV.

144 GeV, or 155 GeV, respectively, to explain the observed line. Eq. (2.7) indicates that we must account for this possible change in mass relative to the assumption of $M = 130$ GeV in Ref. [433] by performing a rescaling of the cross section result from Ref. [433]. To be explicit, where an operator has annihilation modes to γX , we rescale the $\gamma\gamma$ result $[\sigma v]_{\gamma\gamma}^{\text{Weniger}}$ from Ref. [433] as follows:

$$[\sigma v]_{\gamma X} = 2 \times \left(\frac{M}{130 \text{ GeV}} \right)^2 [\sigma v]_{\gamma\gamma}^{\text{Weniger}} \times \begin{cases} 2 & \text{for } X = Z \text{ or } h \\ 1 & \text{for } X = \gamma \end{cases}, \quad (2.19)$$

where for each choice of X , M is as given above; the leading factor of 2 is a re-interpretation of a limit set for self-conjugate DM in Ref. [433] to our choice of fermionic or complex scalar DM [see Eq. (2.7)]; and the other factor of 2 for the $X = Z$ or h final states compensates for the per-annihilation halving of the number of prompt (quasi-)monochromatic photons relative to the assumption of $\gamma\gamma$ in Ref. [433], necessitating a doubling of the cross section to obtain the same flux. In addition, we perform the rescaling necessary to account for differing halo profile normalizations, as discussed in Section 2.2.3; the rescaling factors we use are given in Table 2.2.

Owing once again to the only partial resolution in the Fermi-LAT data of the photon lines, we cannot naïvely use the required line cross section from Ref. [433] to give individual required cross sections for the $\gamma\gamma$ and γZ modes when both are present (see discussion in Section 2.3.1). Assuming M is either 130 GeV ($\gamma\gamma$) or 144 GeV (γZ) in order to explain the 130 GeV line in each case, the secondary line at either 114 GeV (γZ) or 144 GeV ($\gamma\gamma$), respectively, would give a significant photon contribution at the position of the line at 130 GeV and/or vice versa, depending on the relative branching fractions to each final state. Similarly, since the line separation here is still on the order of the Fermi-LAT energy resolution (even if assumed to be 10%), one also cannot strictly speaking assume the lines are completely unresolved to derive a combined required value for $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$.

Nevertheless, whilst we acknowledge this issue, for the purpose of giving a rough estimate of the annihilation cross section that would be required for those of our operators with both $\gamma\gamma$ and γZ annihilation modes to explain the line reported at 130 GeV in Ref. [433], we have performed the following approximate analysis. We note that it is usually the case that one or other of the lines from either the $\gamma\gamma$ or the γZ mode is dominant for M in the approximate range 120–150 GeV (for operators VI-1, IV-2, VIII-1, or VIII-2, it is the $\gamma\gamma$ mode which dominates by about a factor of 4 in terms of flux; for operators VI-3, VI-4, VIII-3, or VIII-4, it is the γZ mode which dominates by about a factor of 2.5 in flux). Therefore, we shall make the approximation that the line at 130 GeV reported in Ref. [433] is entirely due to the dominant of the γX ($X = \gamma, Z$) modes. As noted above, there will be a second line in either case, but we completely ignore this complication in our approximate treatment here (see the next section for a slightly more careful treatment of the same issue making use of results from Ref. [453]). We give results for the quantity $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$, requiring that the normalization of the line from the dominant annihilation mode can explain the line signal at 130 GeV. To be explicit, if the γX ($X = \gamma, Z$) annihilation mode explains the line signal at 130 GeV, we extend Eq. (2.19) to the two-line case as follows:²⁵

$$\begin{aligned} \left[(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v \right] &= 2 \times 2 \times \left(\frac{M}{130 \text{ GeV}} \right)^2 [\sigma v]_{\gamma\gamma}^{\text{Weniger}} \\ &\times \begin{cases} \left[1 + \frac{\sigma_{\gamma Z}(M)}{2\sigma_{\gamma\gamma}(M)} \right] & \text{for } M = 130 \text{ GeV}, X = \gamma \\ \left[1 + \frac{2\sigma_{\gamma\gamma}(M)}{\sigma_{\gamma Z}(M)} \right] & \text{for } M = 144 \text{ GeV}, X = Z. \end{cases} \end{aligned} \quad (2.20)$$

The factors in the brackets $[\dots]$ are computed using the analytical expressions in Ref. [431] and merely rescale the individual required $\sigma_{\gamma X}v$ ($X = \gamma, Z$) cross sections for the line at 130 GeV to correctly obtain the value for the quantity $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$.

²⁵. Both of the leading factors of 2 in Eq. (2.20) are correctly present for either case $X = \gamma$ or Z owing to the multiplication of $\sigma_{\gamma\gamma}$ by 2 on the LHS; cf. Eq. (2.19)

2.3.7 Line-like Features: Cohen et al. Analysis

In addition to presenting limits for the ratio of continuum-to-line cross sections, Ref. [453] also presents evidence for the existence for a line in the Fermi-LAT 3.7-year data in an annulus centred on the GC with inner and outer radii of 1° and 3° , respectively, with local significance of 5.5σ relative to the null hypothesis of no line (but, again, see Appendix A.5.1). To reach this conclusion, Ref. [453] performed a likelihood analysis fitting binned photon count data in the energy range 5–200 GeV to a single power-law background plus lines at $E_\gamma = M$ (with normalization $N_{\gamma\gamma}$) and at $E_\gamma = M - m_Z^2/4M$ (with normalization $N_{\gamma Z}$), both suitably convolved with the Fermi-LAT energy dispersion. No continuum photon contribution from DM annihilation was included in their analysis. In their fit, M and $\theta_{\gamma Z/\gamma\gamma} = \arctan(N_{\gamma Z}/N_{\gamma\gamma})$ were scanned over while the background normalization and spectral index, as well as the total line normalization $N_{\gamma\gamma} + N_{\gamma Z}$, were marginalized over. The best fit point for their analysis was found to be $M = 130$ GeV and $\theta_{\gamma Z/\gamma\gamma} = 0$ [453]. (An earlier analysis [458] fitting the data with $\gamma\gamma$ and γZ lines found very similar results; we have followed Ref. [453] since the latter work also gave limits on the continuum-to-line ratio.)

Ref. [453] presents 1σ , 2σ , and 3σ delta-log-likelihood contours for their fitting procedure in the M – $\theta_{\gamma Z/\gamma\gamma}$ plane, which we have reproduced in Figure 2.3, along with the theoretical values for $\theta_{\gamma Z/\gamma\gamma} = \arctan(\sigma_{\gamma Z}v/2\sigma_{\gamma\gamma}v)$ for the eight operators that have both $\gamma\gamma$ and γZ lines, as computed using the analytical expressions of Ref. [431]. We see that the relative normalizations of the lines for these operators are such there are regions around $M = 130$ GeV or $M = 144$ GeV where the sum of the two lines could fit the Fermi-LAT data very well (within 1σ in delta-log-likelihood, relative to the best-fit point).

Ref. [453] also gives contours displaying the total normalization (photon count) $N_{\text{total}} \equiv N_{\gamma\gamma} + N_{\gamma Z}$ of the lines in the M – $\theta_{\gamma Z/\gamma\gamma}$ plane which, knowing the ROI-averaged mission-time-integrated exposure-times-effective-area $\mathcal{E}_{\text{ROI}}$ relevant to the Fermi-LAT data they considered, would allow us to extract the cross sections required for the operators with both $\gamma\gamma$ and γZ modes to explain the fitted lines. As $\mathcal{E}_{\text{ROI}}$ was not relevant to the analysis presented

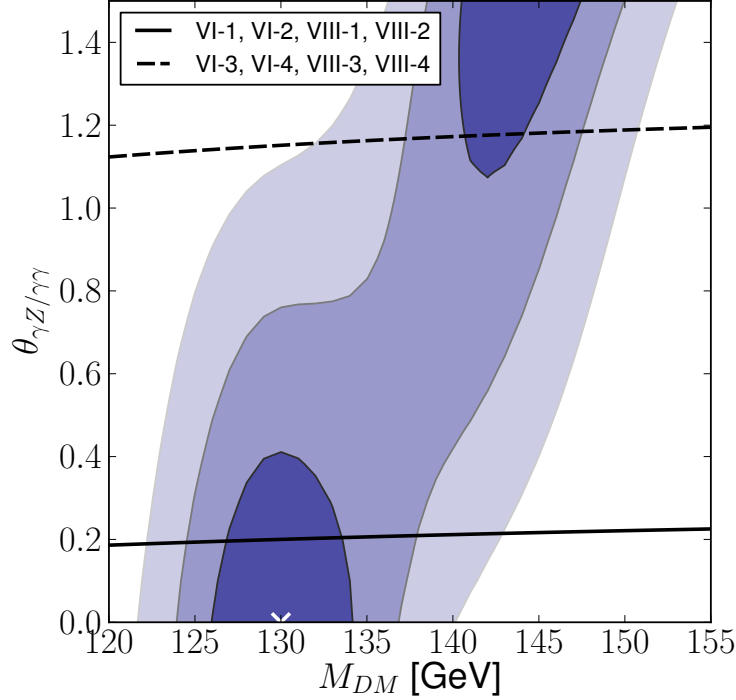


Figure 2.3: Line-strength ratios. The contours and coloured bands shown here are adapted^a from Figure 2 in Ref. [453] and, in order of decreasing darkness, are the 1σ , 2σ , and 3σ delta-log-likelihood contours for the line-existence analysis performed in Ref. [453], in which $\gamma\gamma$ and γZ lines plus a power-law background were fitted to the 3.7-year Fermi-LAT data; the white cross at $M = 130$ GeV and $\theta_{\gamma Z/\gamma\gamma} = \arctan(N_{\gamma Z}/N_{\gamma\gamma}) = 0$ is the best fit point found by Ref. [453], corresponding to a 5.5σ local significance relative to a null hypothesis of no line [453]. We have superimposed the solid and dashed black lines, which are the theoretical results for the relative normalization of the two lines $\theta_{\gamma Z/\gamma\gamma} = \arctan(\sigma_{\gamma Z}/2\sigma_{\gamma\gamma})$, as computed using the analytical cross section formulae in Ref. [431], for the eight operators (identified in the legend) that have both lines present. Although there are eight operators, the results fall onto only two curves, as indicated in the figure. There is clearly a range of DM masses for any of these operators where the relative line normalization is such that the double line provides a good fit (delta-log-likelihood $< 1\sigma$, relative to the best-fit point [453]) to the Fermi-LAT data.

^a Ref. [453] is © 2012 by SISSA. Adaptation and re-use permitted by the Creative Commons Attribution (CC BY) 3.0 Licence [1] under which Ref. [453] is distributed.

in Ref. [453], it was not given there. However, we have made use of the Fermi ScienceTools package [476] to reproduce a sufficient portion of the data-extraction and analysis of Ref. [453] to enable us to compute $\mathcal{E}_{\text{ROI}}$ ourselves (see Appendix A.4 for the full details of our analysis and the cross-checks we have performed). We find that the ROI-averaged expo-

sure factor for photon energies from c. 120–150 GeV (applicable to the line-fits presented in Ref. [453]) is $\mathcal{E}_{\text{ROI}} = 1.05 \times 10^{11} \text{ cm}^2 \text{ s}$. Knowing this factor, and extracting the fitted total line-normalization N_{total} from the contours in Figure 2 of Ref. [453], we have computed the photon flux corresponding to the fitted line normalization as

$$\Phi = \frac{N_{\text{total}}}{\mathcal{E}_{\text{ROI}}}. \quad (2.21)$$

Finally, making use of Eq. (2.7) we have converted this to the annihilation cross section required as a function of M to explain the line for our particular operators having both $\gamma\gamma$ and γZ modes,

$$(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v = \frac{16\pi M^2}{J}\Phi = \frac{16\pi M^2}{J}\frac{N_{\text{total}}}{\mathcal{E}_{\text{ROI}}}, \quad (2.22)$$

for the (operator-specific) range of M within the 3σ $|\Delta\ln\mathcal{L}|$ contours shown in Figure 2.3. In performing this conversion, we also required (and have computed) the J -factors for the ROI in Ref. [453], which we also give in Appendix A.4. We show these extracted cross sections in our results figures in Section 2.4.

2.4 Results

In Figures 2.4–2.37 we present the indirect detection limits for all operators that were studied in Ref. [431] and that have non-vanishing s -wave annihilation channels. The cross sections for various operators are listed in Tables VI–XX of Ref. [431]; see footnote 4 for a discussion of our notational conventions. Subsections 2.4.1–2.4.6 should be read in conjunction with Figures 2.4–2.37.

There is one figure for each operator, consisting of up to six panels. The captions for each panel are the same for each figure and described below. In addition, we make operator-specific comments in some individual figure captions.

2.4.1 Panel A

We show the present non-relativistic DM annihilation cross sections σv as a function of DM mass for each of the annihilation modes allowed for the particular EFT operator, assuming that (a) the DM annihilates only through that operator and (b) the DM particle is a massive, cold thermal relic with $\Omega_{\text{DM}} h^2 = 0.12$.

If all annihilation modes are pure s -wave, the total cross section for annihilation attains the canonical value of around $\sigma v \approx 3.6 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ for $M = 100 \text{ GeV}$ [showing logarithmic variation with M ; cf. Eqs. (2.1) and (2.2)]. However, p -wave components to some annihilation channels can cause the present-day total annihilation cross section to be suppressed, due to their non-negligible impact in the early universe when the relic density is set; these contributions naturally drive Λ larger and hence the present-day annihilation cross section smaller. This effect is particularly pronounced in operators where the s -wave annihilation is essentially independent of M but the p -wave annihilation contribution grows with M , e.g., the operator XVII-1, $\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi i \left(B_{\lambda \mu} Y_H H^\dagger D^\lambda H - \text{h.c.} \right)$.

Ignoring kinematic thresholds, for all final states involving fermions, except those in Table IX of Ref. [431], the fermion contributions separate into four distinct types: leptons with $T^3 = \pm 1/2$ and quarks with $T^3 = \pm 1/2$ (cf. the results for Operators XVIII–XX).

2.4.2 Panel B

The per-annihilation total differential spectrum $dN_\gamma/dE_\gamma = \sum_f \text{BR}_f \cdot dN_\gamma^f/dE_\gamma$ of prompt gamma rays as a function of E_γ is plotted for different values of DM mass, indicated by the colour scale. The mass dependence of the spectra varies based on whether the dominant final states are fermions, or if they are gauge and/or Higgs bosons. In the latter case, the spectrum becomes harder with increasing DM mass because the final states become more boosted. Further details on these spectra can be found in Section 2.2.4. No Inverse Compton Scattering (ICS) component is included. An estimate of the error introduced by neglect of ICS is given in Appendix A.2.

2.4.3 Panel C

If applicable, this panel shows the line-search limits for the DM mass region 5 GeV to approximately 300 GeV from the Fermi-LAT [449] line search using observed photon fluxes from the Galactic Centre (see also Appendix A.5.2). We show these limits in terms of the total annihilation rate to monochromatic photons (i.e., the annihilation rate to each final state weighted by the number of monochromatic photons in that final state). The solid black line shows the specific σv required for a thermal relic.

The solid coloured lines show 95% CL UL for various halo profiles and ROIs, assuming the central values for halo normalization $\rho_\odot$ from Ref. [463] (see Table 2.1). The like-coloured bands show the variation in the UL as $\rho_\odot$ is varied through its 2σ limits, also from Ref. [463], shown in Table 2.1. For operators with multiple possible lines (i.e., $\gamma\gamma$ and γZ annihilation modes) we do not set limits in the region of DM masses 80–160 GeV where we expect the Fermi-LAT line shape would become a very broad or double-peaked structure (see Section 2.3.1).

If present, the coloured squares with their error bars represent $(M, \sigma v)$ values where the operator in question could additionally supply the 130 GeV line (but see Appendix A.5.1) examined in Ref. [433], assuming the central values for $\rho_\odot$ for each halo profile (for this fixed halo profile normalization, the error bars indicate the uncertainty on the required flux to supply the line). As discussed in Section 2.3.6, for operators where there are multiple photon lines we make the assumption that the line at 130 GeV is due to the dominant photon line mode.

For operators with both $\gamma\gamma$ and γZ lines, we show coloured lines near 130–150 GeV, which indicate the cross sections required to explain a possible double-line feature in the Fermi-LAT data based on the analysis of Ref. [453] as discussed in Section 2.3.7, and extended by us in Appendix A.4. The opacity of the coloured band around each line indicates the delta-log-likelihood (computed in Ref. [453]) of the double-line fit as compared to the best-fit point found in Ref. [453] (but see Appendix A.5.1), with the darkest band being 1σ , the medium

band being 2σ and the lightest band being 3σ . The central line corresponds to the cross section obtained using the central value of the halo profile normalization, with the vertical shaded band giving the $\pm 2\sigma$ normalization uncertainties on the halo profiles.

2.4.4 *Panel D*

If applicable, this panel shows the line-search limits for the WIMP mass region from 0.5–20 TeV from the H.E.S.S. [450] line search discussed in Section 2.3.2 (see also Appendix A.5.3). The ROI is a circle of radius 1° centred on the GC with Galactic latitudes $|b| < 0.3^\circ$ excluded [450]. Note the change from Panel C to a logarithmic scale in M .

The black and solid coloured lines, and similarly coloured bands, are all as described in Panel C. Isothermal limits are very weak for this line search owing to the cored nature of the profile and very small ROI near the GC.

2.4.5 *Panel E*

This panel shows the inclusive (continuum and line) spectrum limits for DM mass from 5 GeV to 1 TeV based on the ‘model-independent’ limits given in Ref. [451] and described in Section 2.3.3. These are given as the solid coloured lines labelled ‘GC ...’; the coloured bands show the range of halo profile normalizations, as in Panel C. The solid black line is the result for the total σv for the operator.

The sharp features in the GC limits that can be seen in some cases at $M \sim 10$ GeV or 100 GeV are due to the photon lines migrating through the bins used in Ref. [451] as the DM mass changes. For operators with lines only from γZ or γh , we also see sharp features in the limits near threshold as the line energy migrates rapidly across the bins. Had we convolved the lines with the Fermi-LAT energy dispersion, all of these sharp cross-over regions would be smoothed out.

Also shown are the Fermi-LAT [452] stacked dwarf galaxy limits (see also Appendix

A.5.4), discussed in Section 2.3.4. These are given as a variety of grey lines and are a conservative estimate of the 95% CL UL limits; as discussed in Section 2.3.4, because the published limits in Ref. [452] were presented in terms of specific final states, there is not enough information to fully derive the limits for the continuum spectra here.

2.4.6 Panel F

If applicable, the limits presented here are on the ratio of selected continuum final state annihilation cross sections to the line final states $\gamma\gamma$ and γZ as described in Ref. [453] and summarized in Section 2.3.5; these limits are only applicable if the operator has no final state γh , and has dominant annihilation branching ratios to at least one of the final states WW/ZZ or $b\bar{b}$.

The solid black line is the ratio $R_{\text{th}} \equiv (\sigma_{WWv} + \sigma_{ZZv} + \sigma_{b\bar{b}v}) / (2\sigma_{\gamma\gamma v} + \sigma_{\gamma Zv})$ for this operator; the dashed coloured lines are individual annihilation mode contributions to this summed result. The various solid coloured lines give either the supersaturation 95% CL UL on R for fixed M , or the boundary of the ‘ 2σ ’ confidence region²⁶ in the M - R plane for the shape constraints from Ref. [453], assuming 100% annihilation to the indicated final state. Further discussion of the applicability of these limits is given in Section 2.3.5, and it should be borne in mind that operators which are only marginally excluded or allowed per the results given here merit further investigation to determine whether or not they are actually excluded. In particular, the continuum photon spectrum from the Z decay for the γZ final state is not accounted for. Furthermore, the ratio of $\gamma\gamma$ to γZ is not fixed at the correct ratio for each operator here considered, but is rather marginalized over in the analysis of Ref. [453]. Finally, we note that this analysis should be considered in light of the discussion in Appendix A.5.1.

26. That is, where the log-likelihood of the fit lies within 4.86 (4 d.o.f.) of that for the best fit point [453].

$\Lambda^{-2}\phi\phi^\dagger B_{\mu\nu}B^{\mu\nu}$:- Operator VI – 1

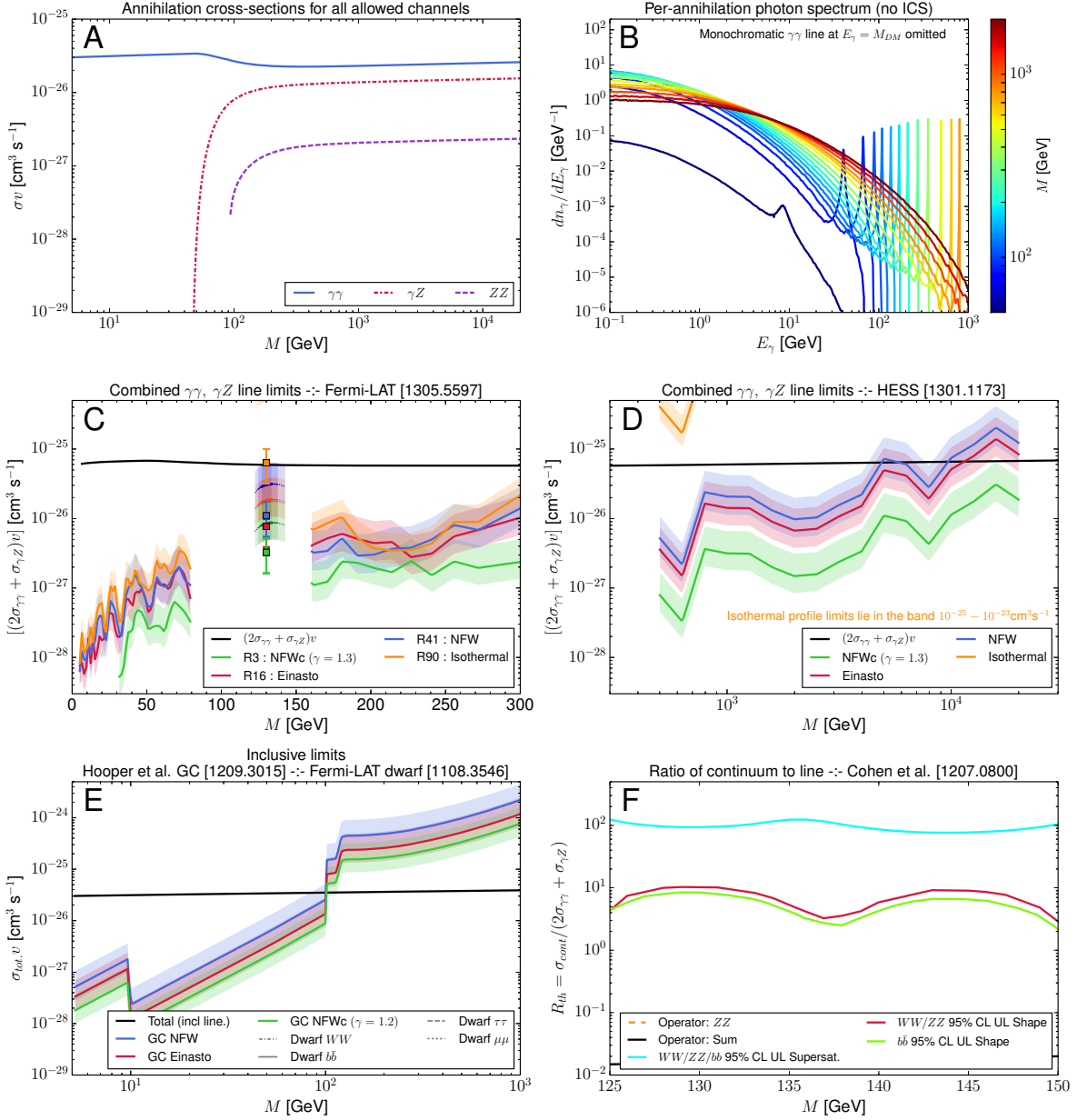


Figure 2.4: Limits for Operator VI-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$\Lambda^{-2}\phi\phi^\dagger B_{\mu\nu}\tilde{B}^{\mu\nu}$:- Operator VI – 2

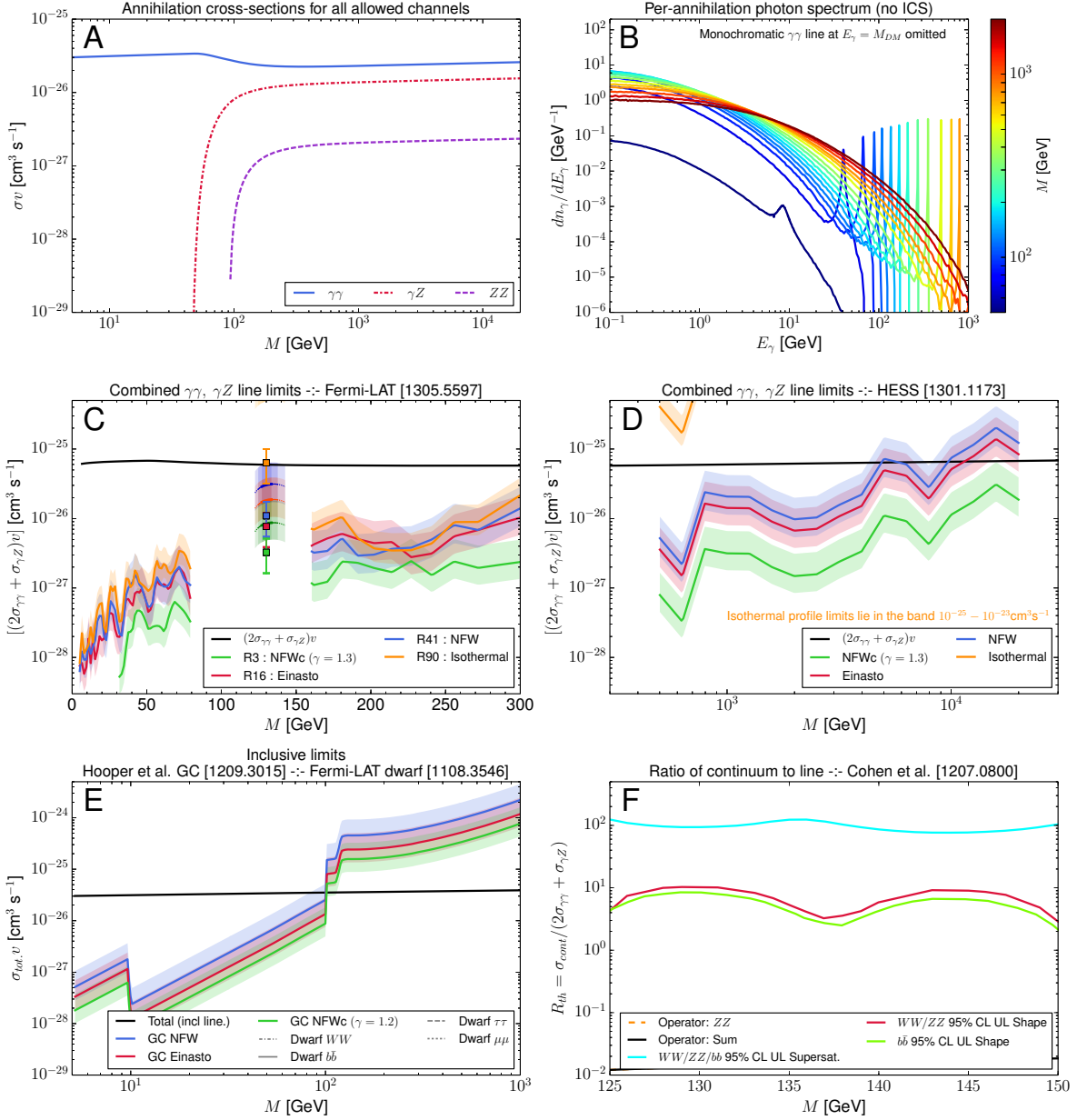


Figure 2.5: Limits for Operator VI-2. Sections 2.4.1–2.4.6.

Detailed figure captions are provided in

$\Lambda^{-2} \phi \phi^\dagger W_{\mu\nu}^a W^{a\mu\nu} \text{ :- Operator VI - 3}$

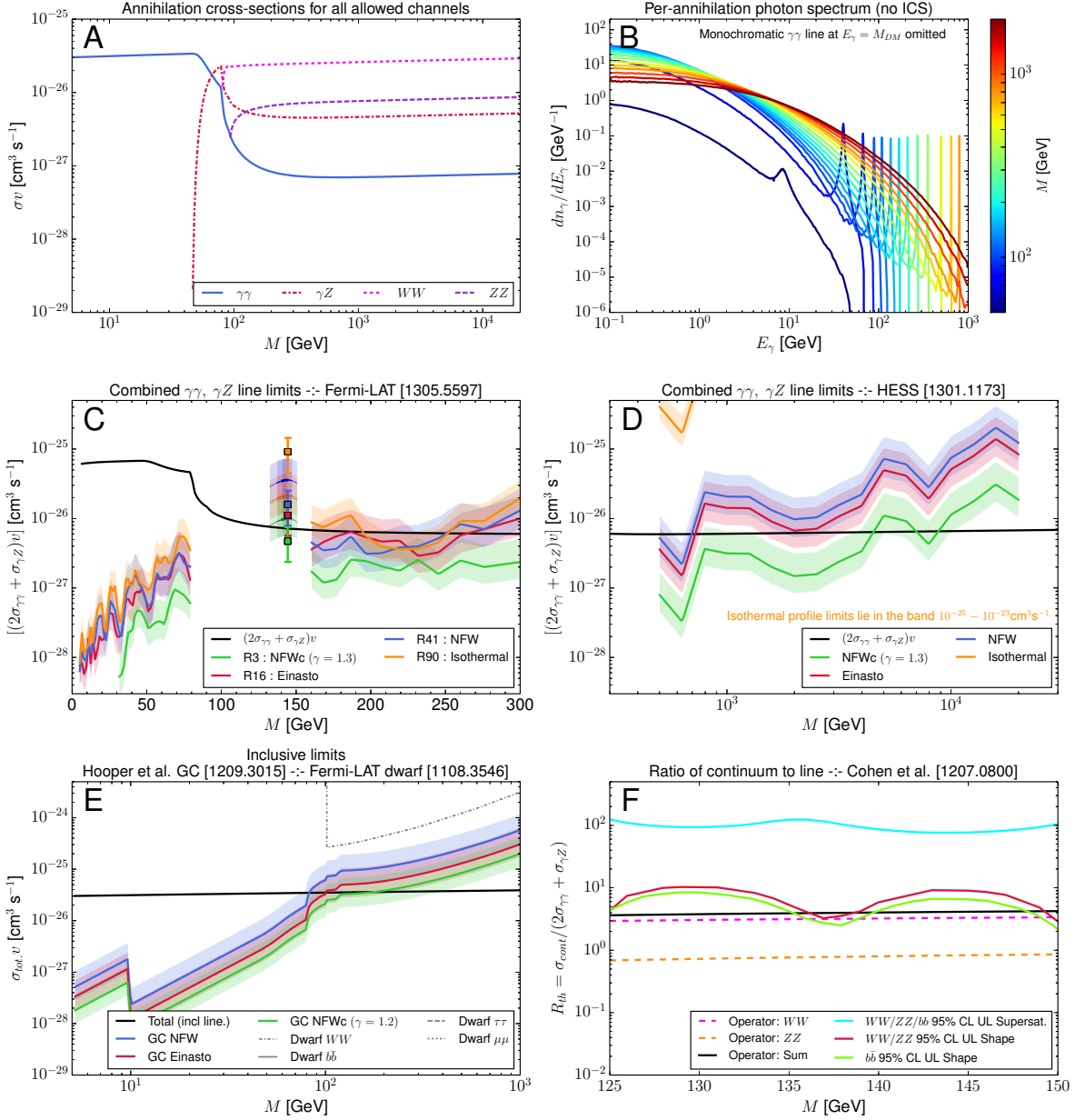


Figure 2.6: Limits for Operator VI-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$\Lambda^{-2} \phi \phi^\dagger W_{\mu\nu}^a \widetilde{W}^{a\mu\nu}$:- Operator VI – 4

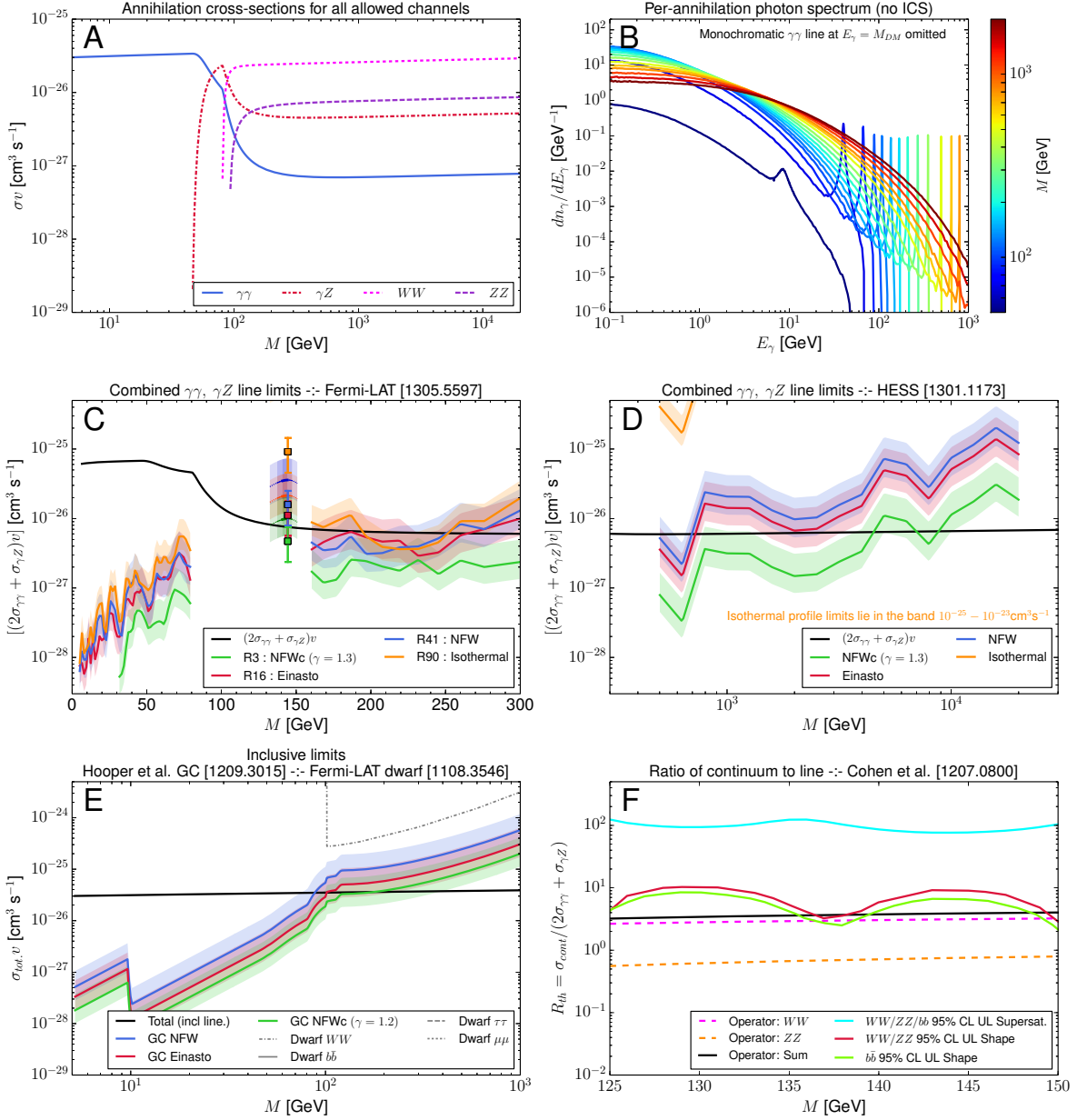


Figure 2.7: Limits for Operator VI-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$\Lambda^{-3} \bar{\chi} i \gamma^5 \chi B_{\mu\nu} B^{\mu\nu}$:- Operator VIII – 1

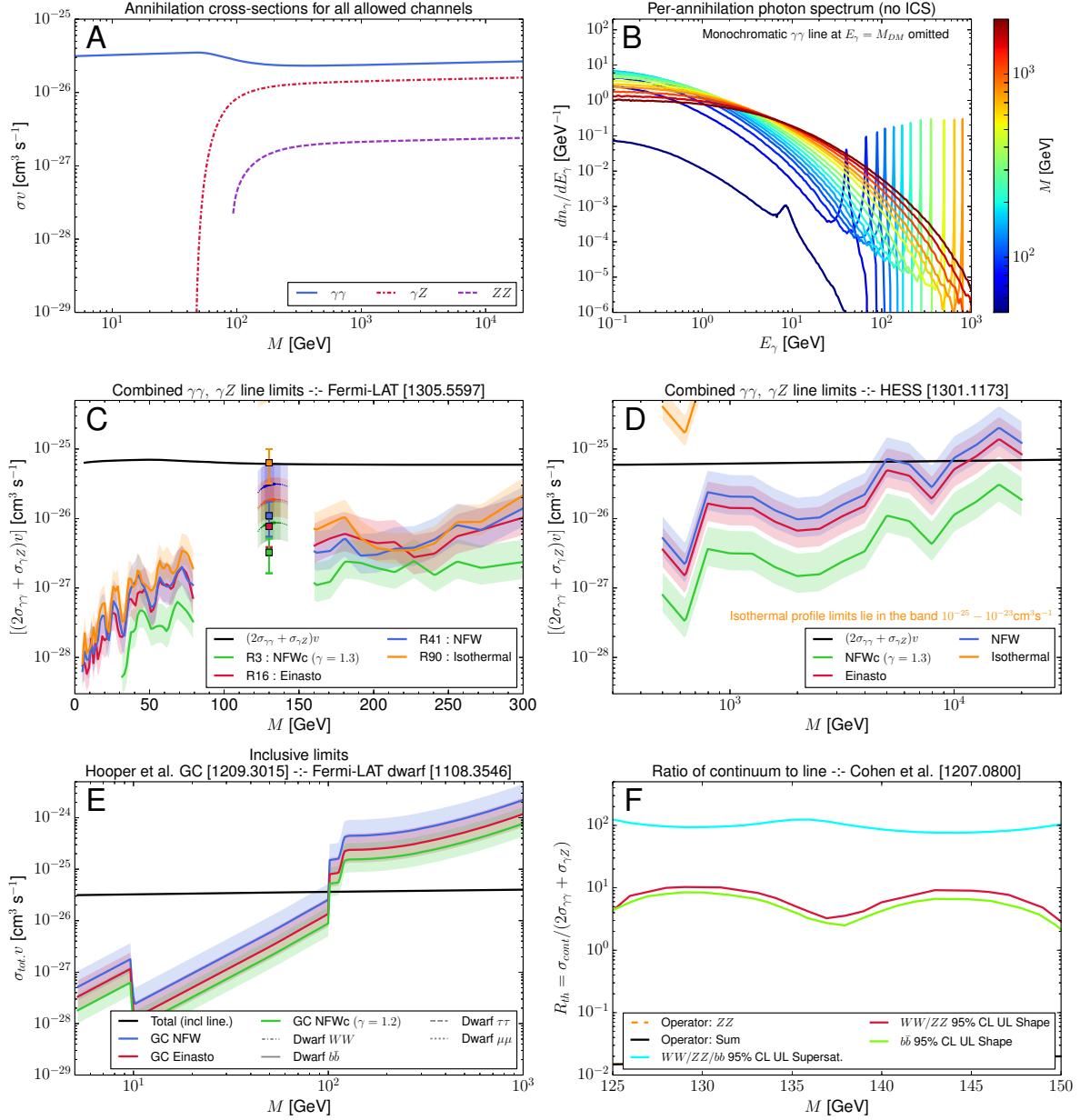


Figure 2.8: Limits for Operator VIII-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$\Lambda^{-3} \bar{\chi} i \gamma^5 \chi B_{\mu\nu} \tilde{B}^{\mu\nu}$:- Operator VIII – 2

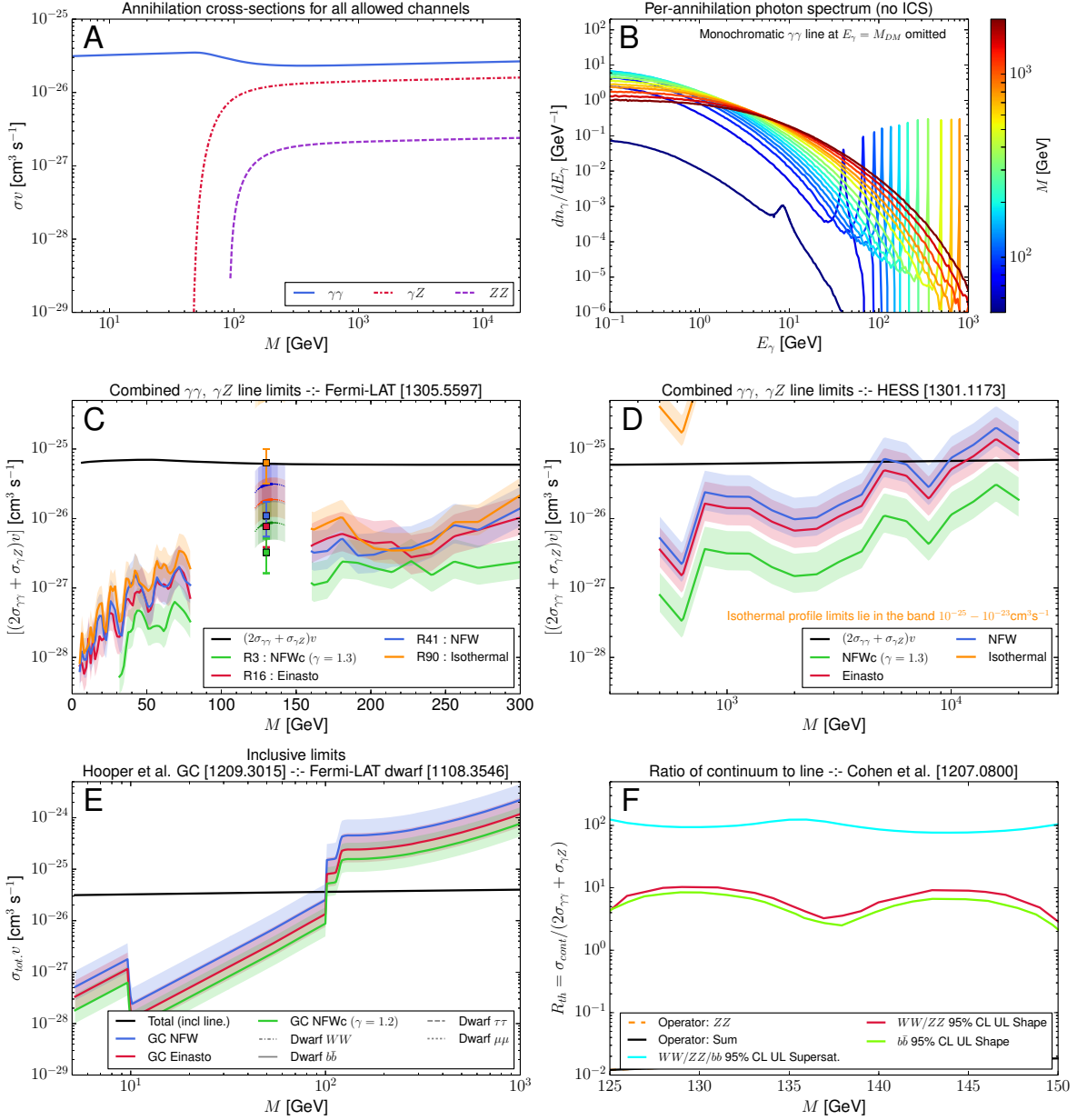


Figure 2.9: Limits for Operator VIII-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-3} \bar{\chi} i \gamma^5 \chi W_{\mu\nu}^a W^{a\mu\nu} \text{ :- Operator VIII - 3}$$

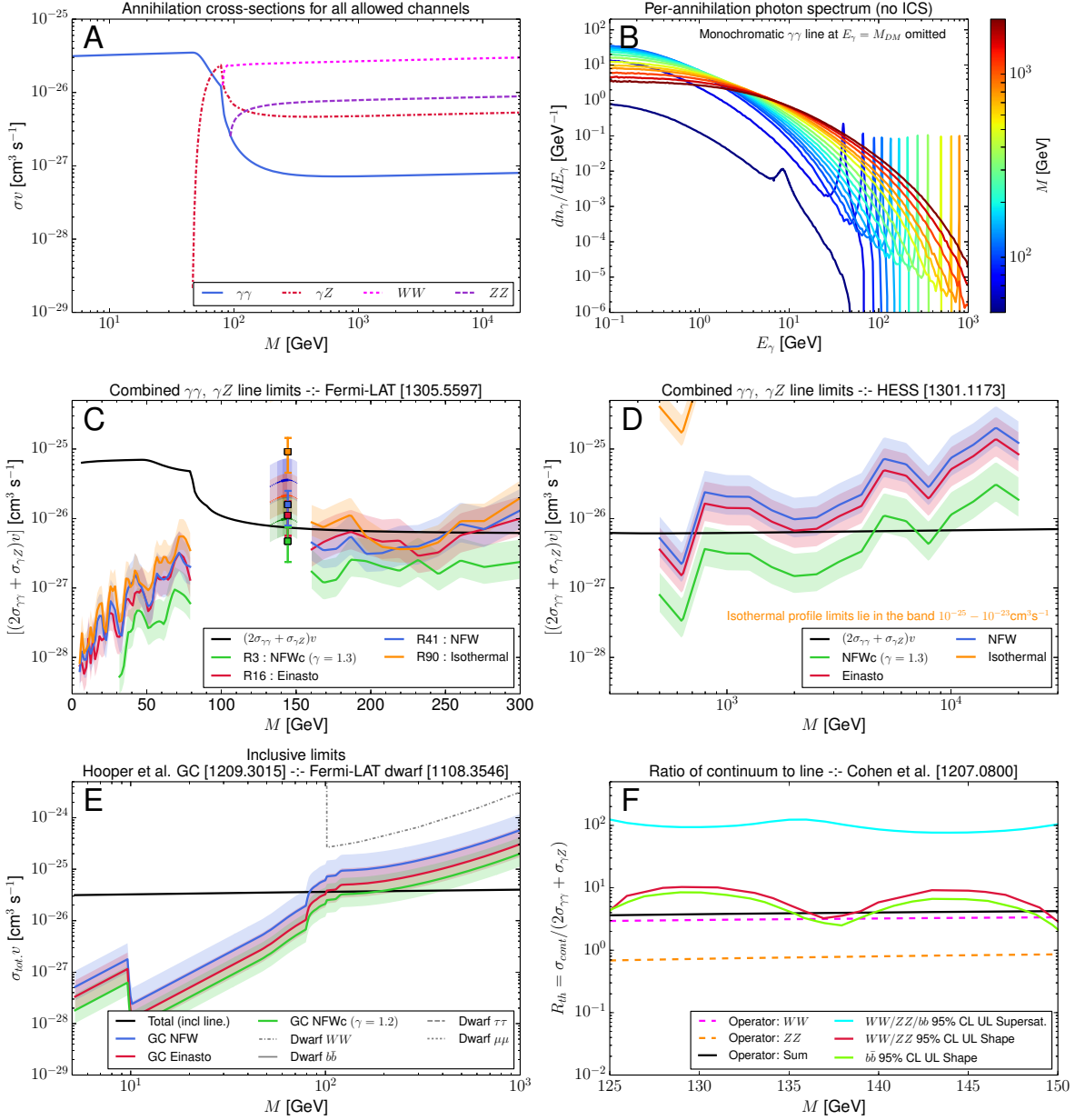


Figure 2.10: Limits for Operator VIII-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$$\Lambda^{-3} \bar{\chi} i \gamma^5 \chi W_{\mu\nu}^a \widetilde{W}^a{}_{\mu\nu} \text{ :- Operator VIII - 4}$$

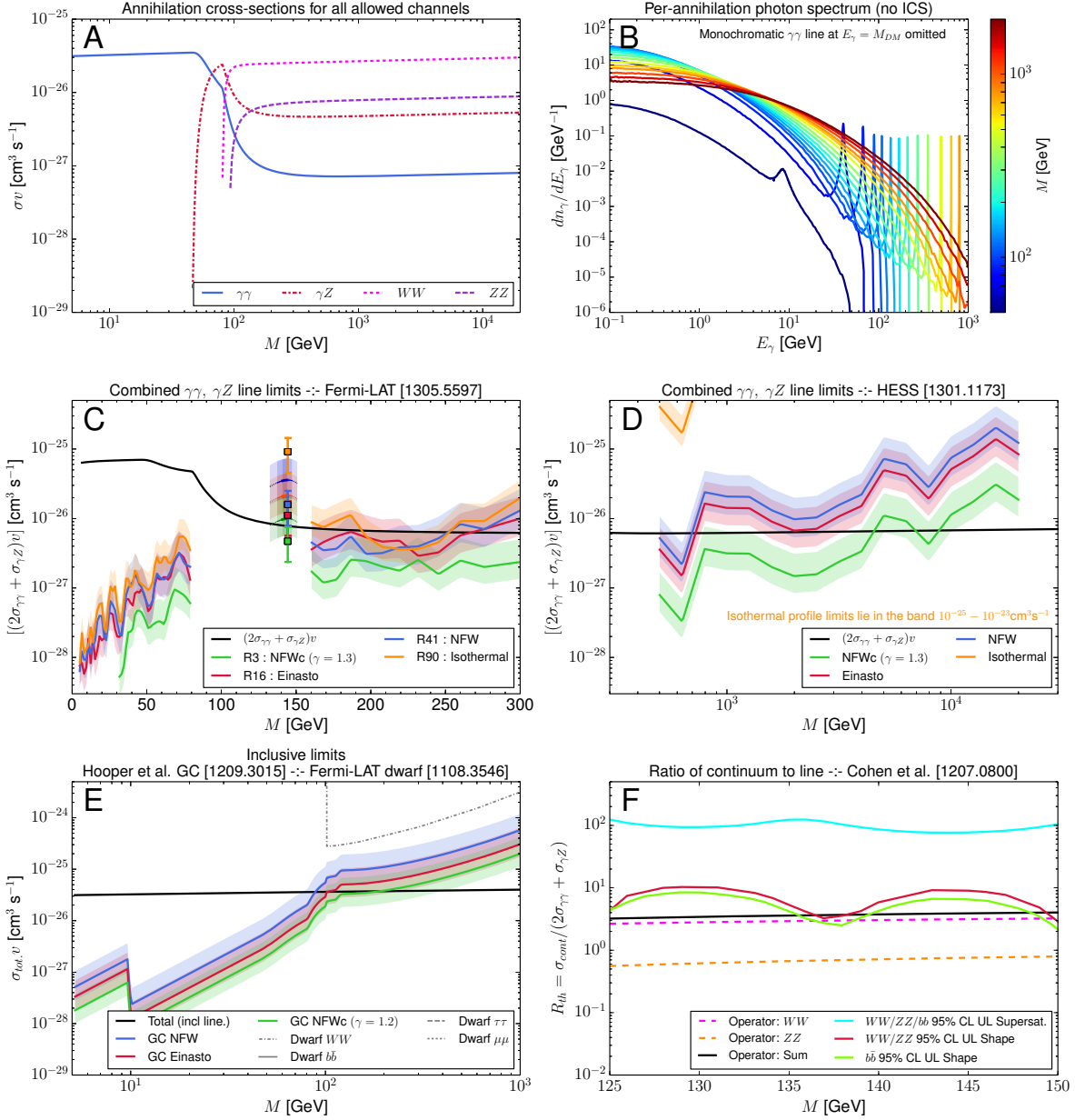


Figure 2.11: Limits for Operator VIII-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$\Lambda^{-1} \bar{\chi} i \gamma_5 \chi H^\dagger H$:- Operator IX – 2

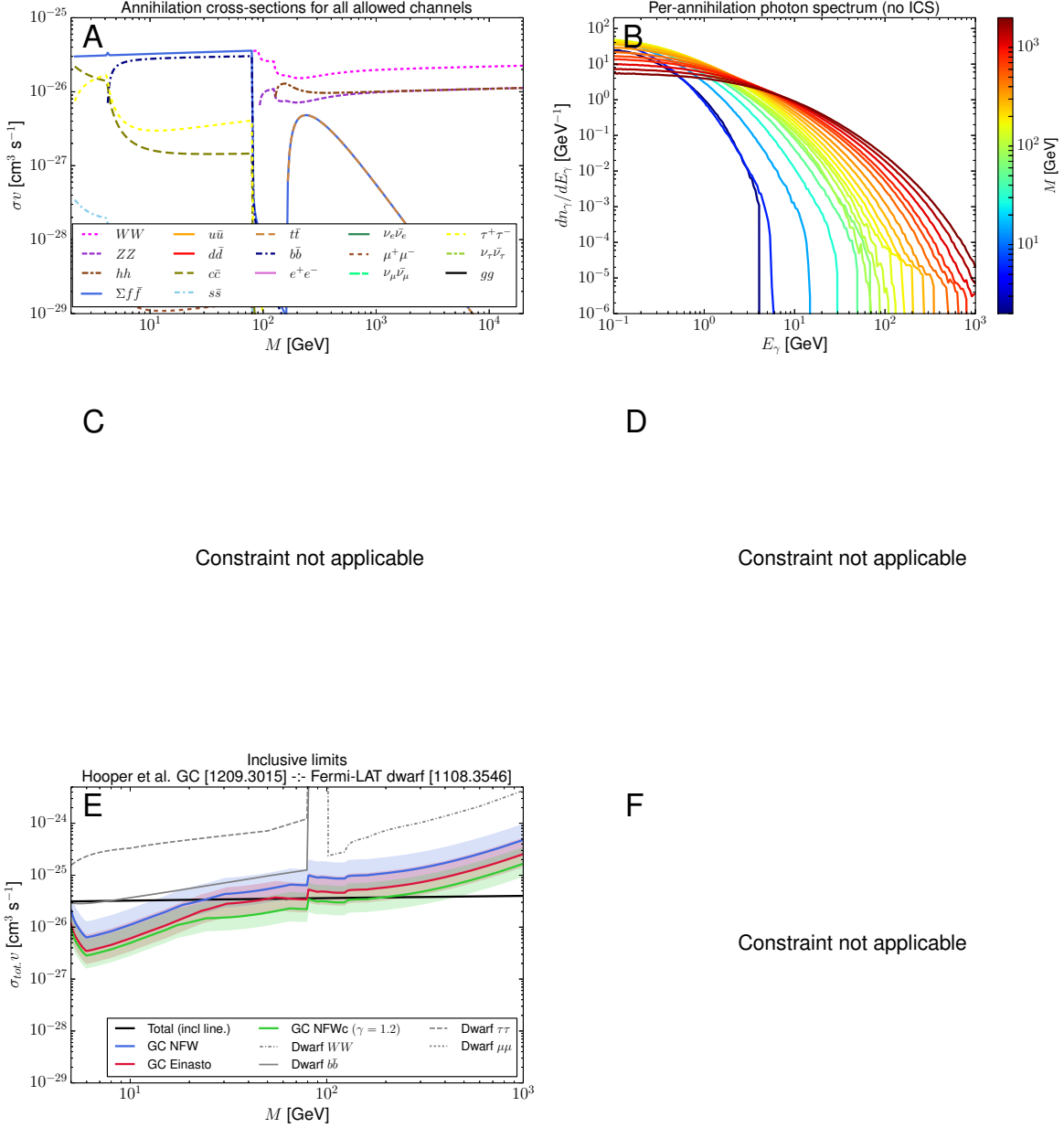
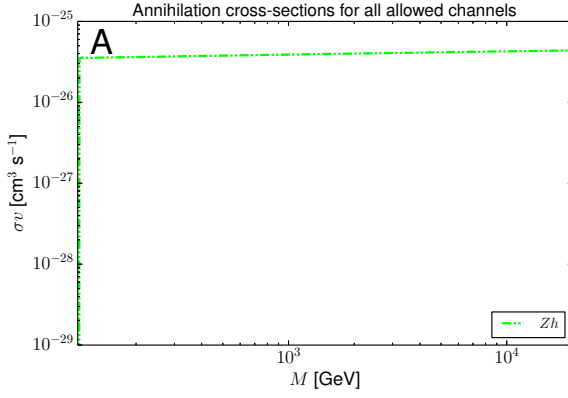


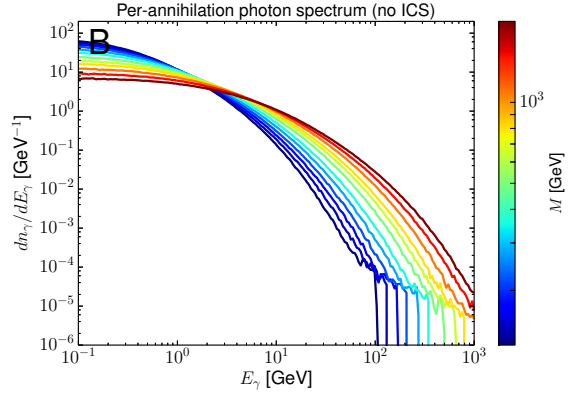
Figure 2.12: Limits for Operator IX-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6. The DM annihilation for this operator proceeds via s -channel Higgs exchange (except for the four-body contact contribution to the hh final state), and we have only utilized the cross sections for the two-body on-shell final states that couple at tree-level to the h ; we have however checked that including the loop-induced couplings to the $g\bar{g}$ final state makes only a negligible difference to the limits. We have not quantitatively estimated the effect of the three- or four-body decays involving intermediate off-shell W/Z bosons, but again we would not expect the limits to change too dramatically.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.)(B_{\lambda\mu} Y_H H^\dagger D^\lambda H + h.c.) \text{ :- Operator X - 1}$$



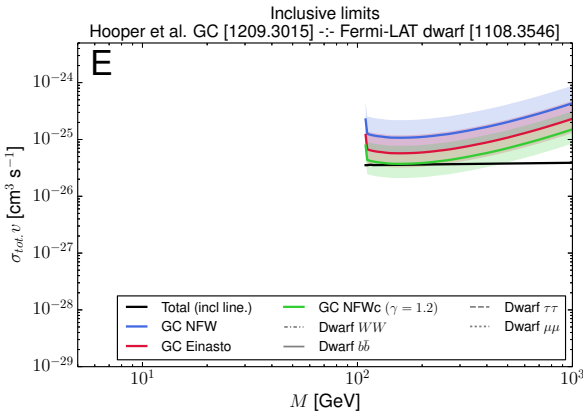
C

Constraint not applicable



D

Constraint not applicable

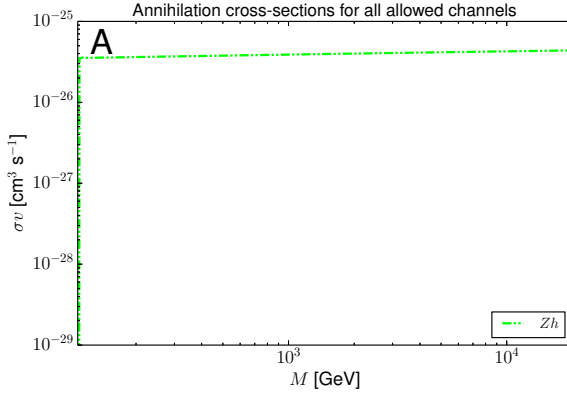


F

Constraint not applicable

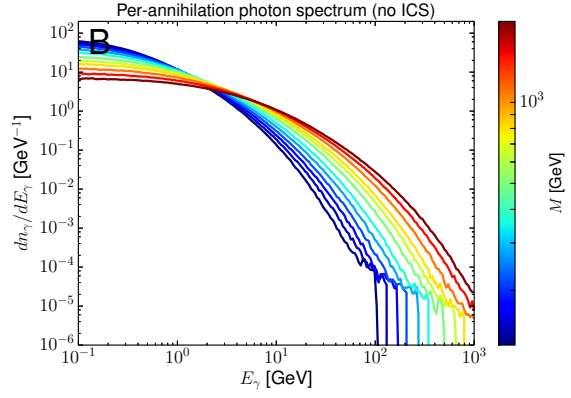
Figure 2.13: Limits for Operator X-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.)(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + h.c.) \text{ :- Operator X - 2}$$



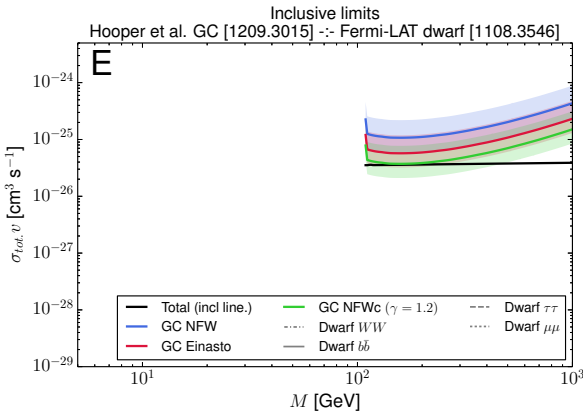
C

Constraint not applicable



D

Constraint not applicable



F

Constraint not applicable

Figure 2.14: Limits for Operator X-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.) i(B_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ :- Operator XI - 1}$$

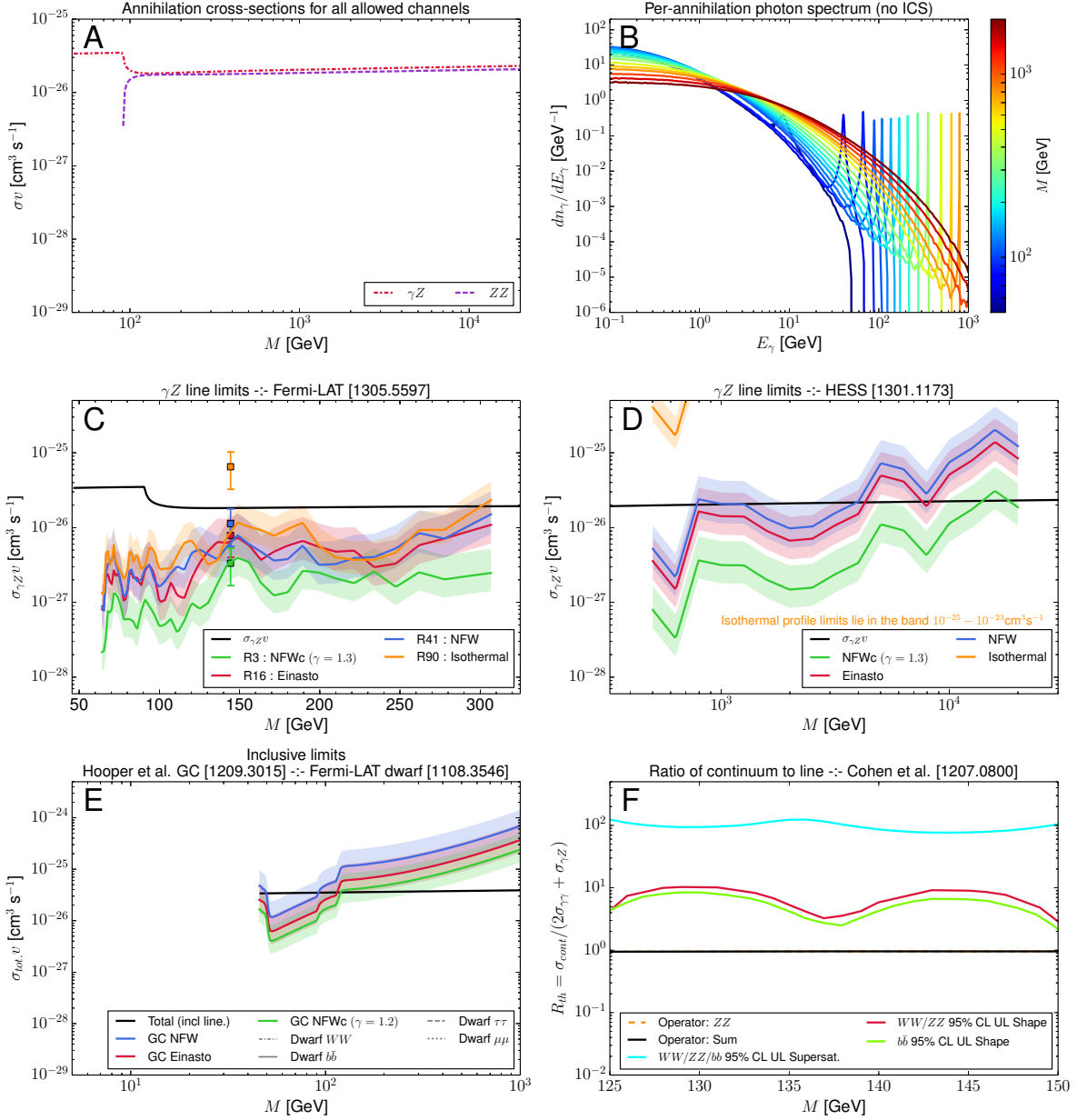


Figure 2.15: Limits for Operator XI-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.) i(\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ -- Operator XI -- 2}$$

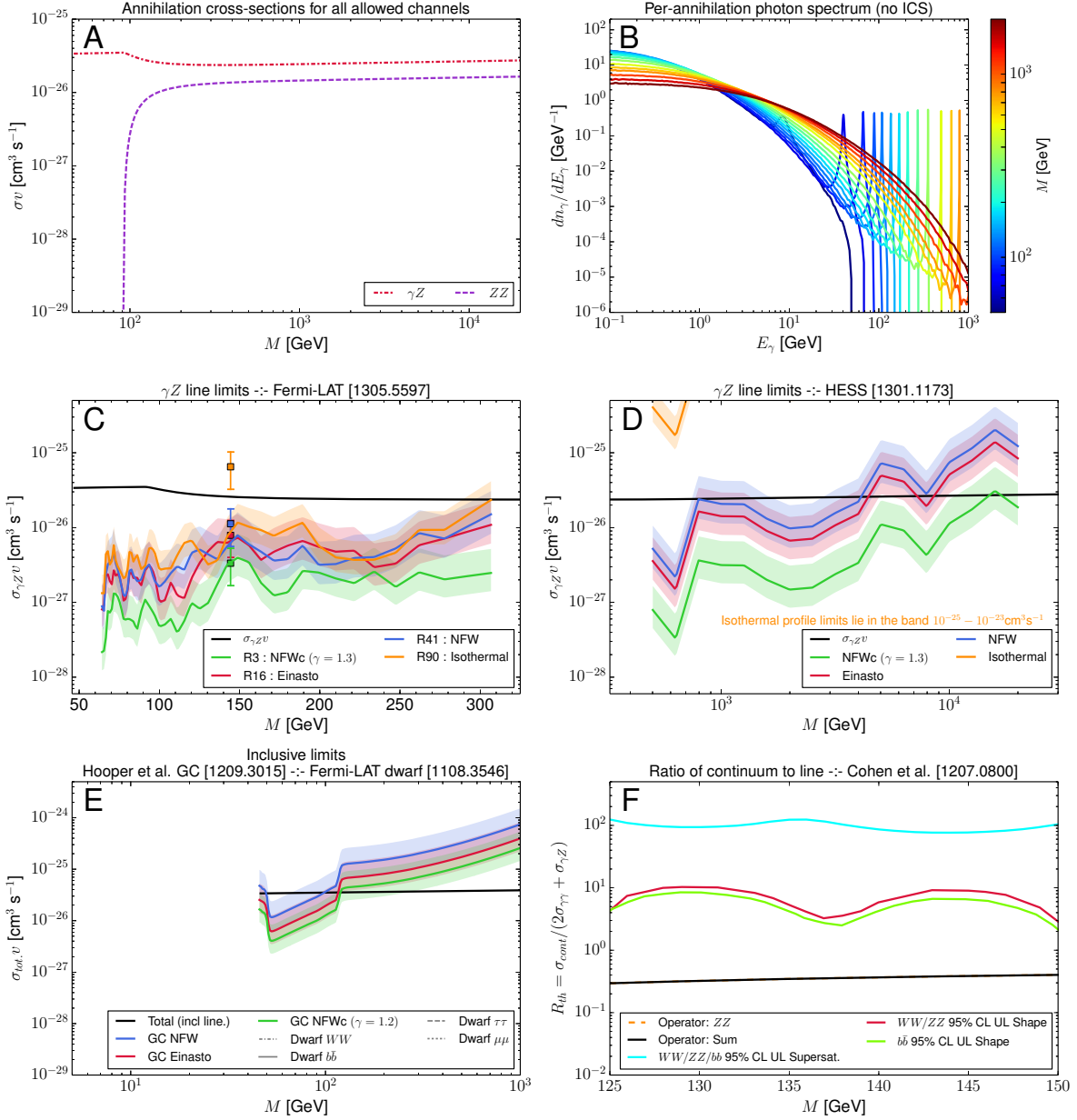


Figure 2.16: Limits for Operator XI-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.) i(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XI - 3}$$

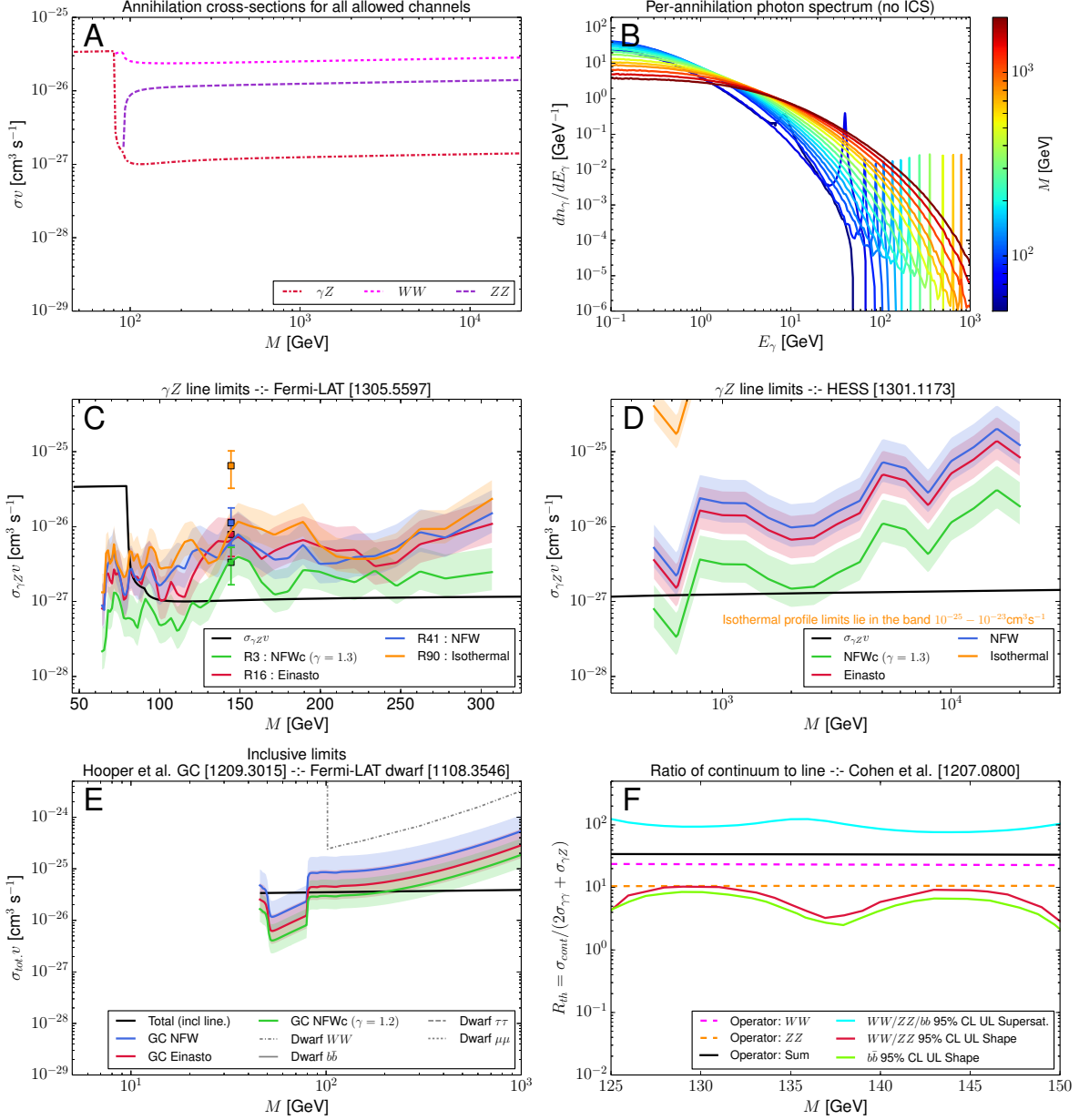


Figure 2.17: Limits for Operator XI-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices, although the line BR is somewhat small.

$$\Lambda^{-4}(\phi^\dagger \partial^\mu \phi + h.c.) i(\widetilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XI - 4}$$

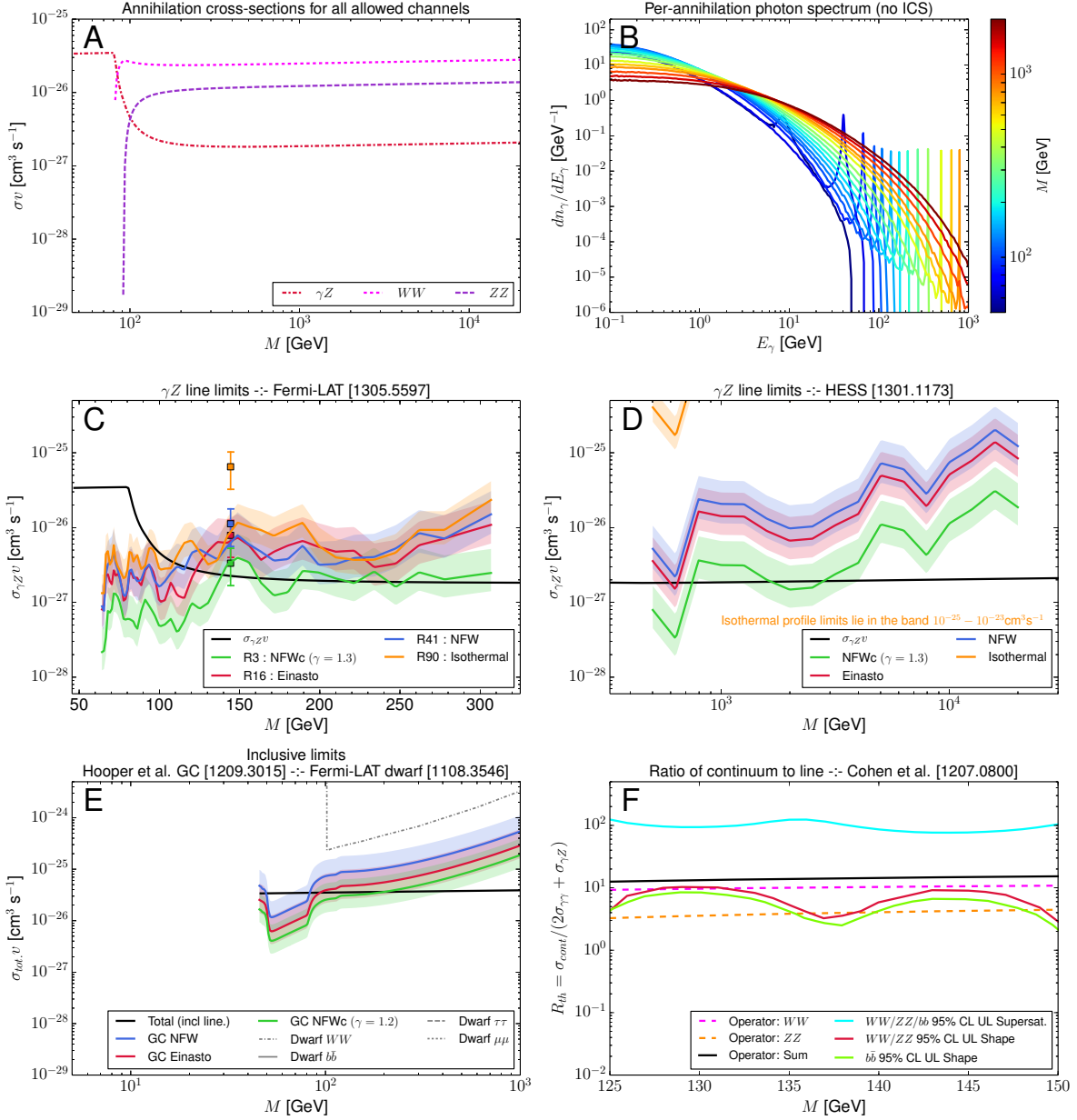


Figure 2.18: Limits for Operator XI-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi (B_{\lambda\mu} Y_H H^\dagger D^\lambda H + h.c.) \text{ :- Operator XIV - 1}$$

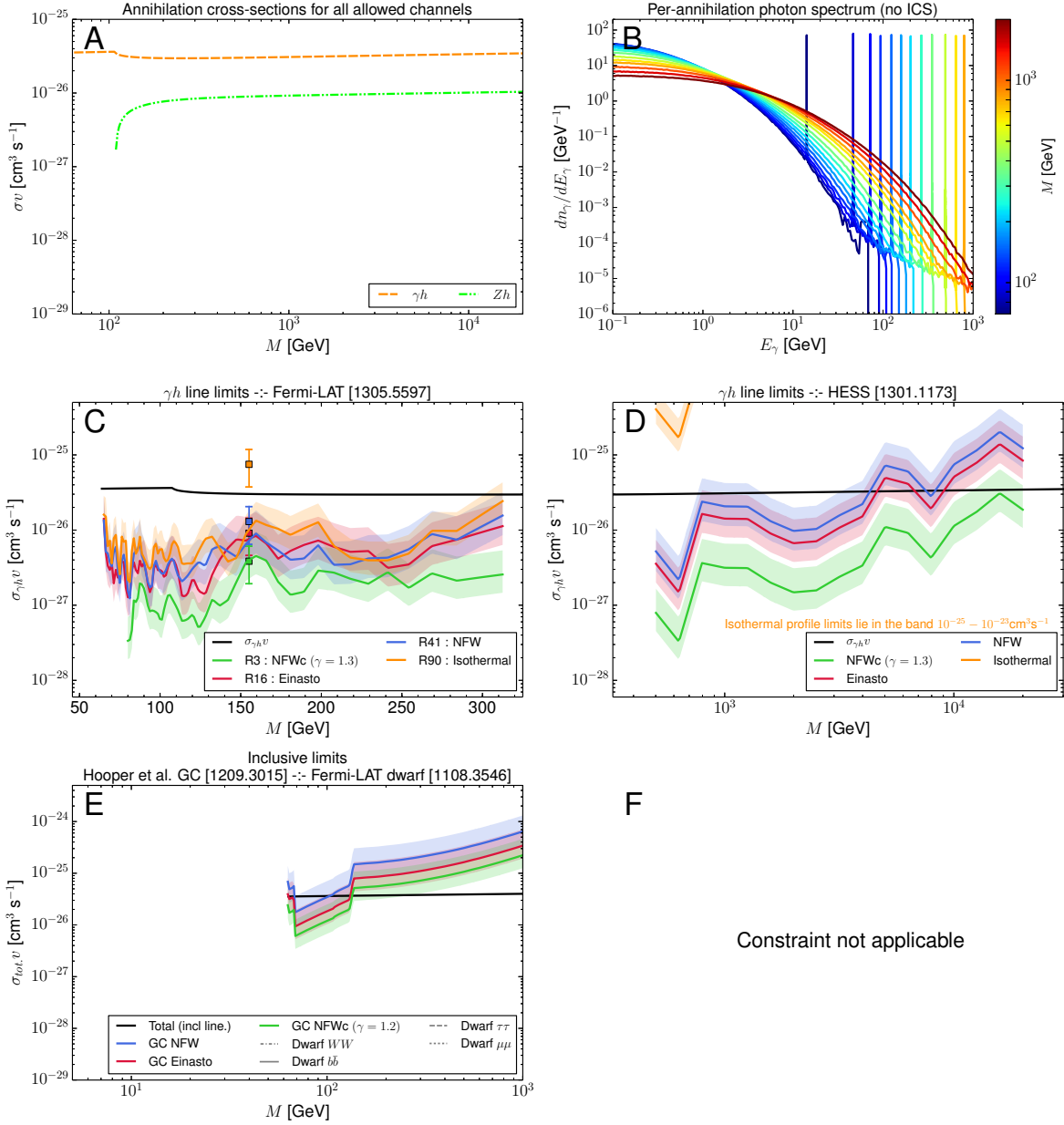


Figure 2.19: Limits for Operator XIV-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H + h.c.) \text{ --: Operator XIV -- 2}$$

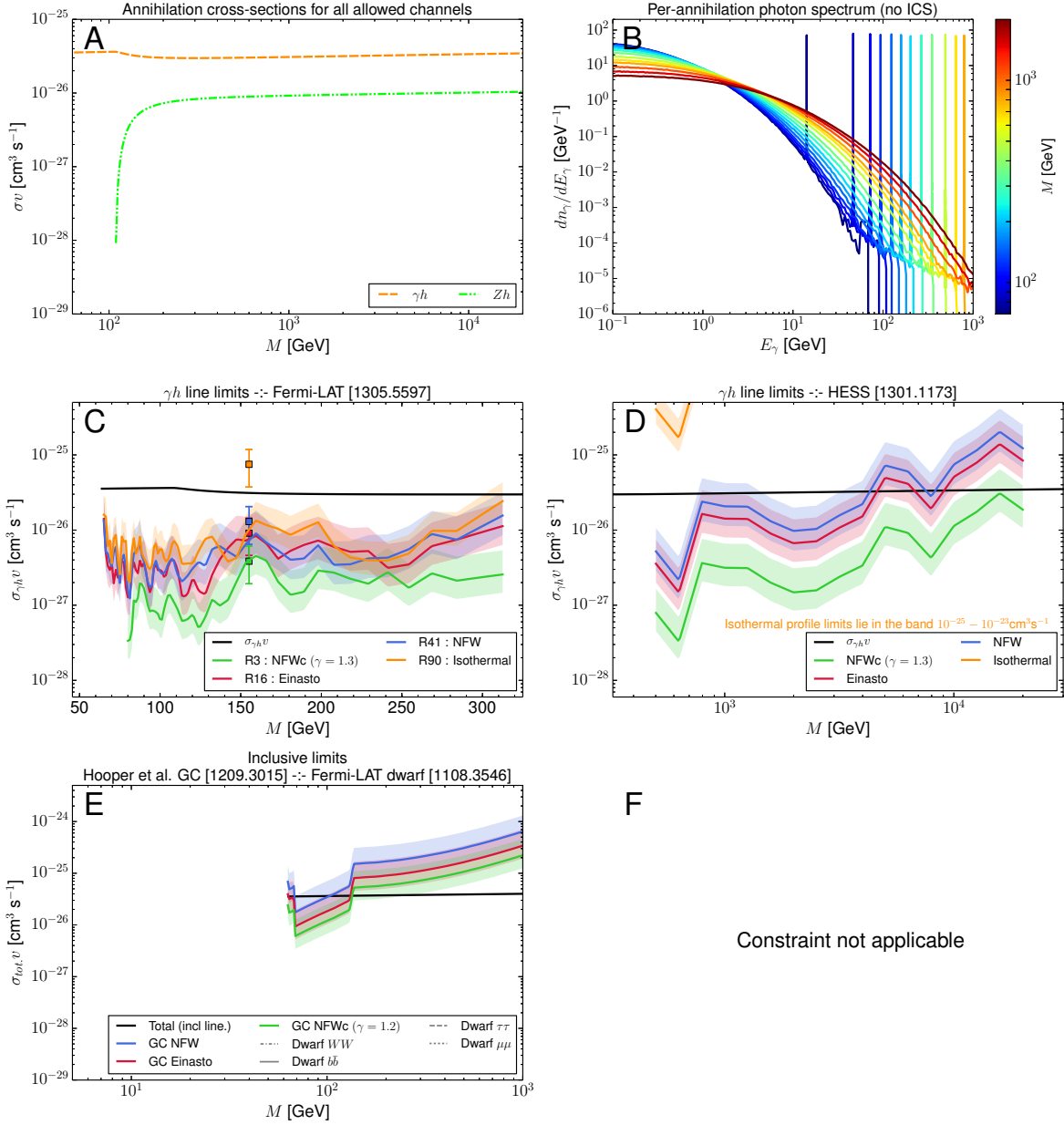


Figure 2.20: Limits for Operator XIV-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi (W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + h.c.) \text{ :- Operator XIV - 3}$$

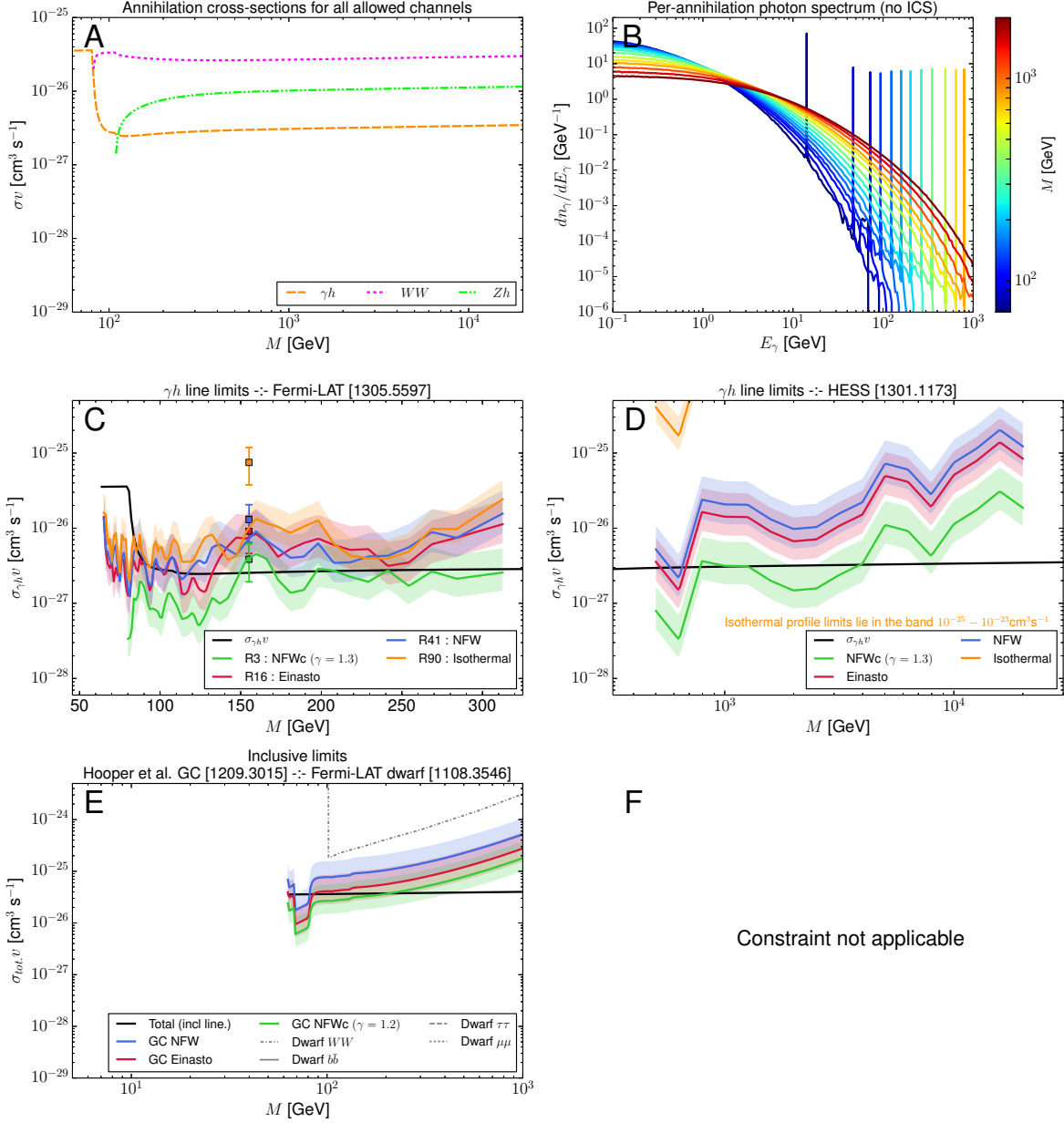


Figure 2.21: Limits for Operator XIV-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi (\widetilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H + h.c.) \text{ :- Operator XIV - 4}$$

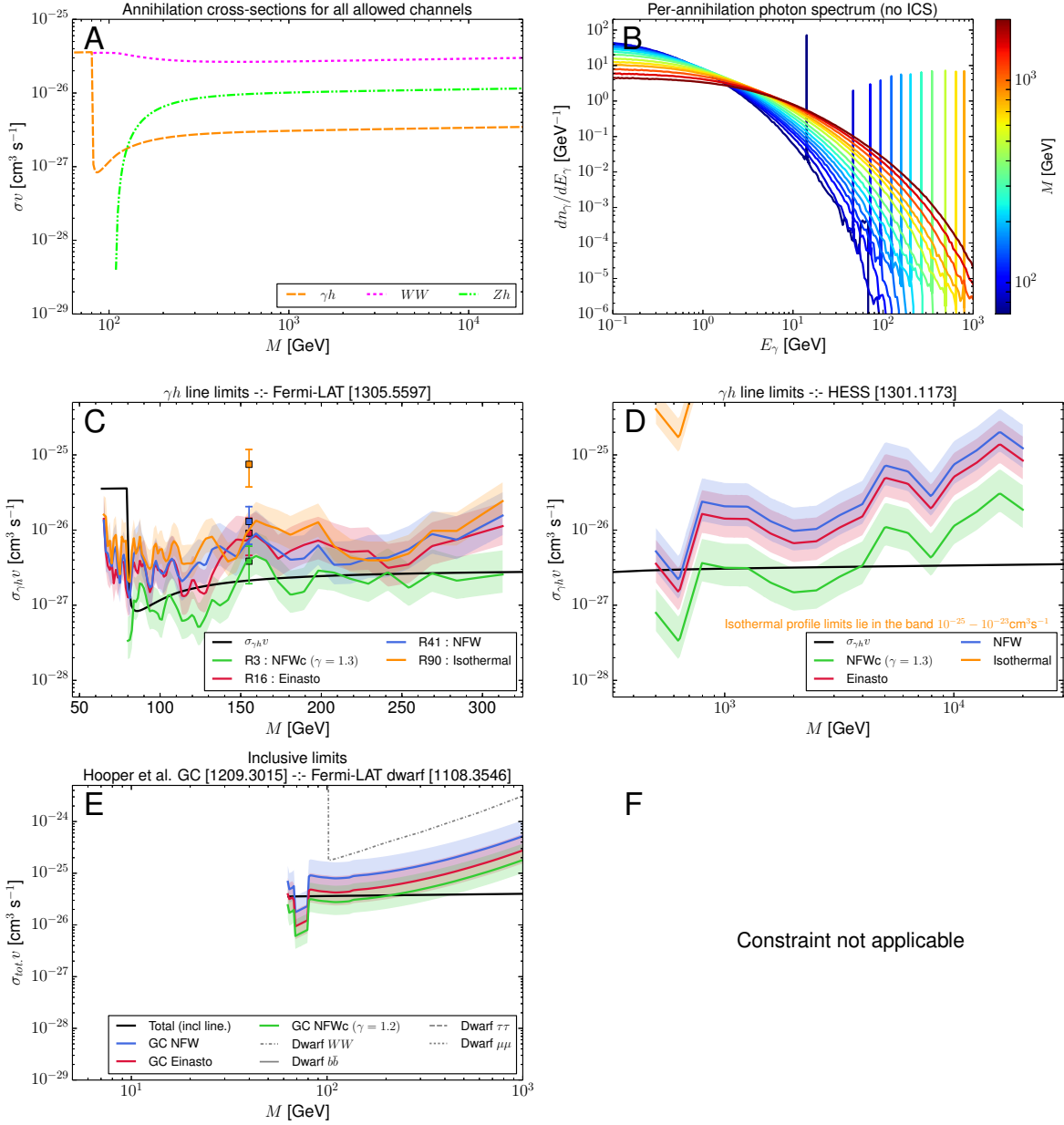


Figure 2.22: Limits for Operator XIV-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi i (B_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ :- Operator XV - 1}$$

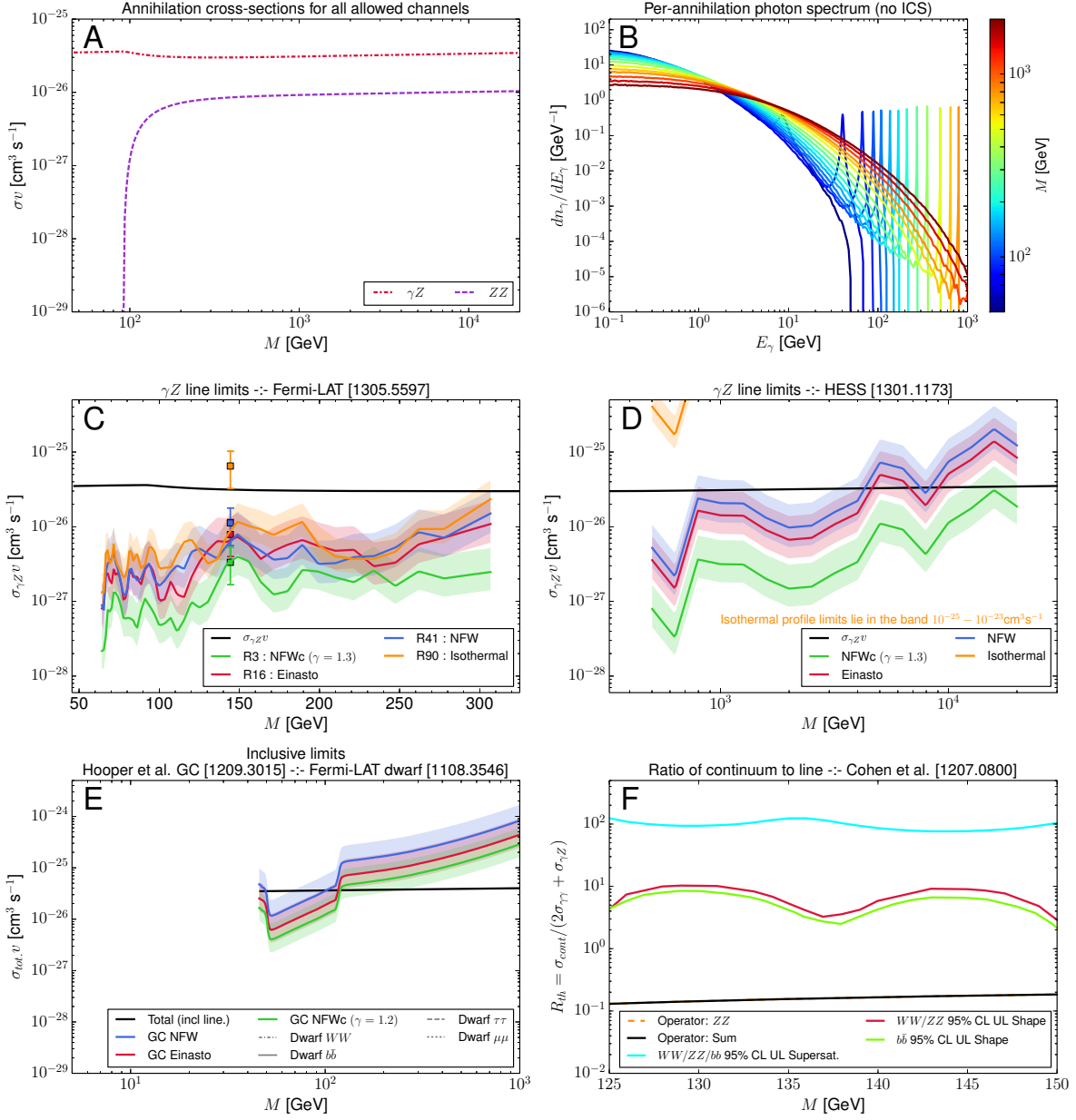


Figure 2.23: Limits for Operator XV-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi i (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ :- Operator XV - 2}$$

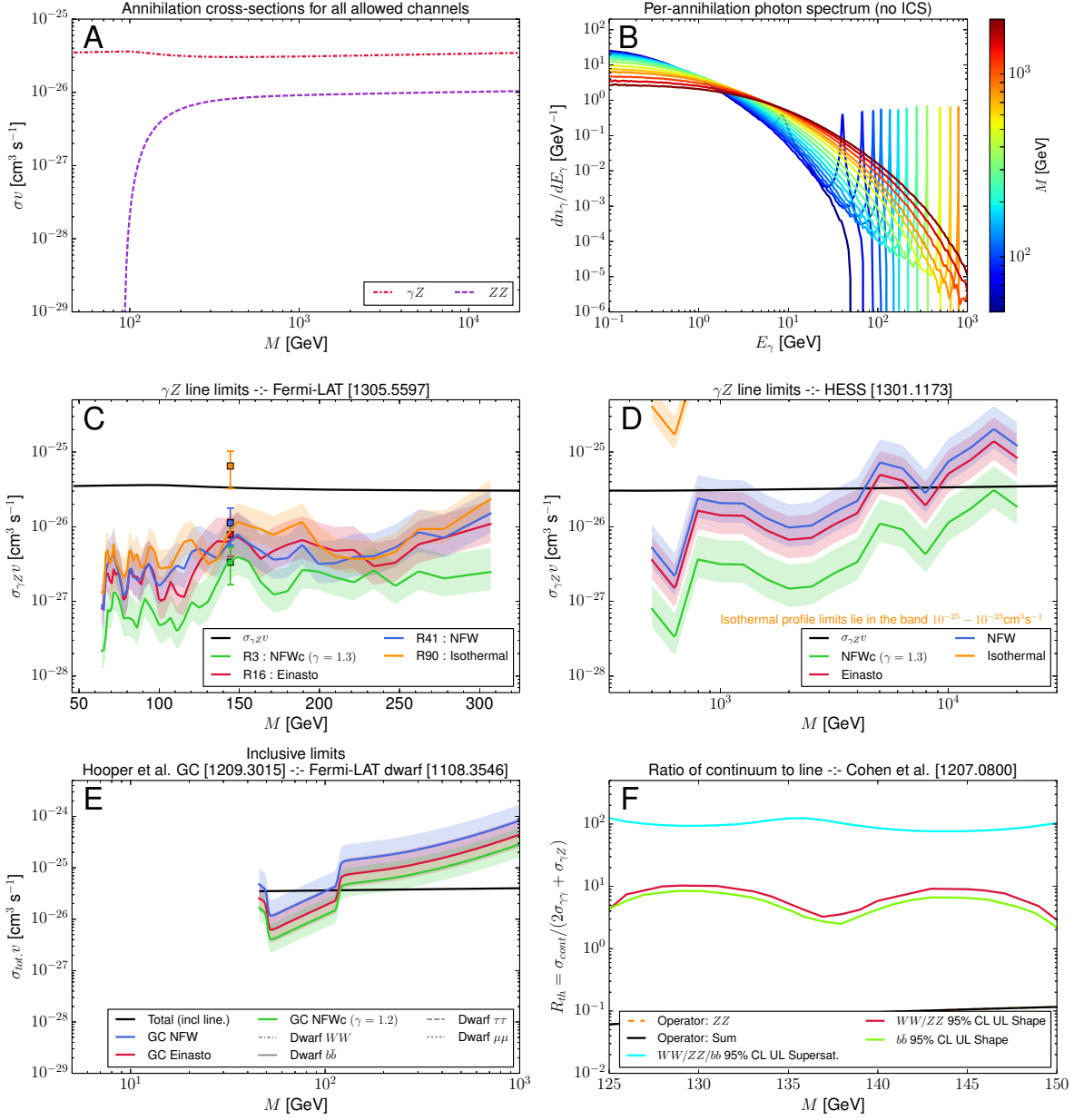


Figure 2.24: Limits for Operator XV-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi i(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XV - 3}$$

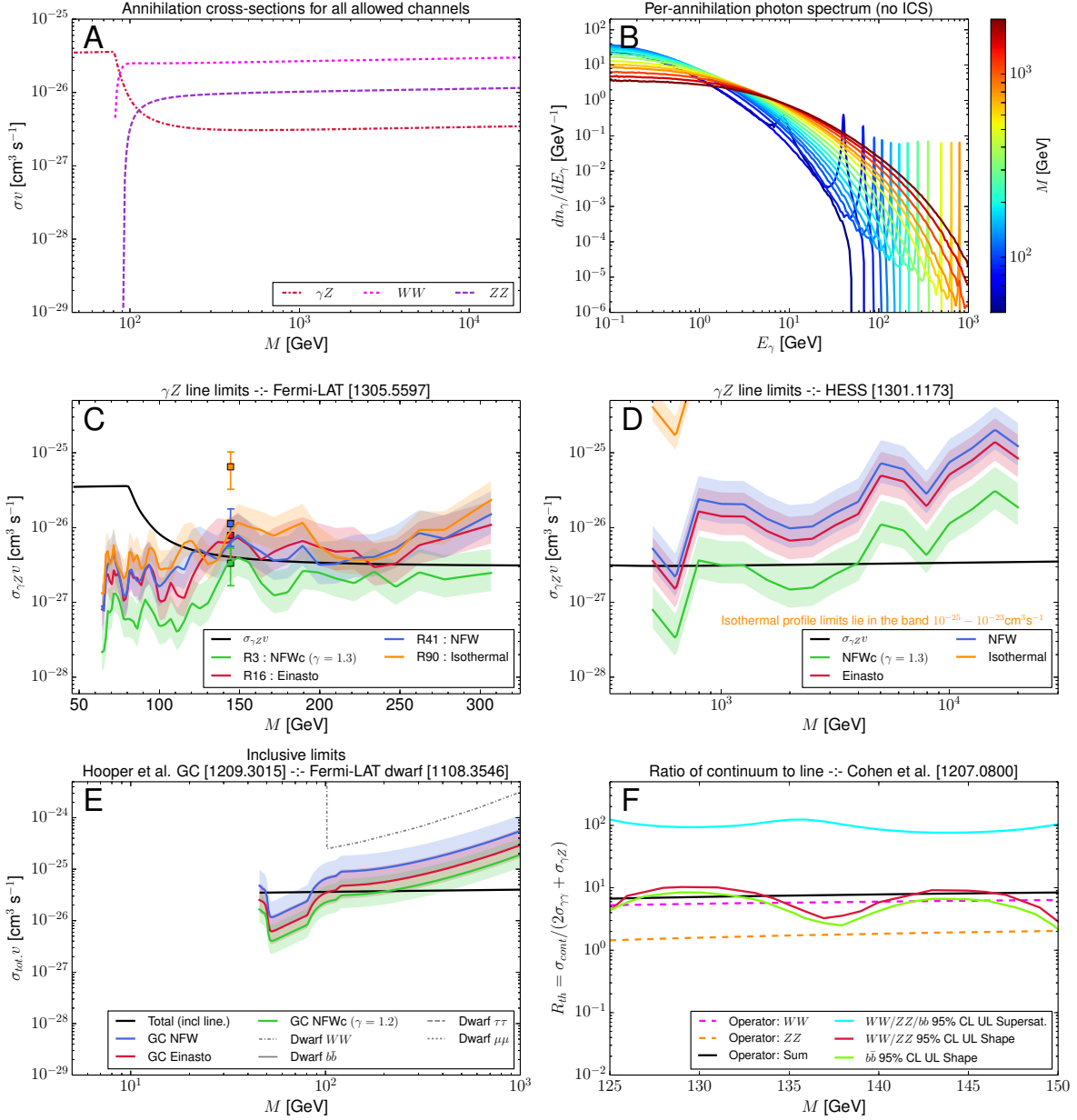


Figure 2.25: Limits for Operator XV-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and account well for the 130 GeV photon line for some profile choices.

$$\Lambda^{-4} \bar{\chi} \gamma^\mu \chi i (\widetilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XV - 4}$$

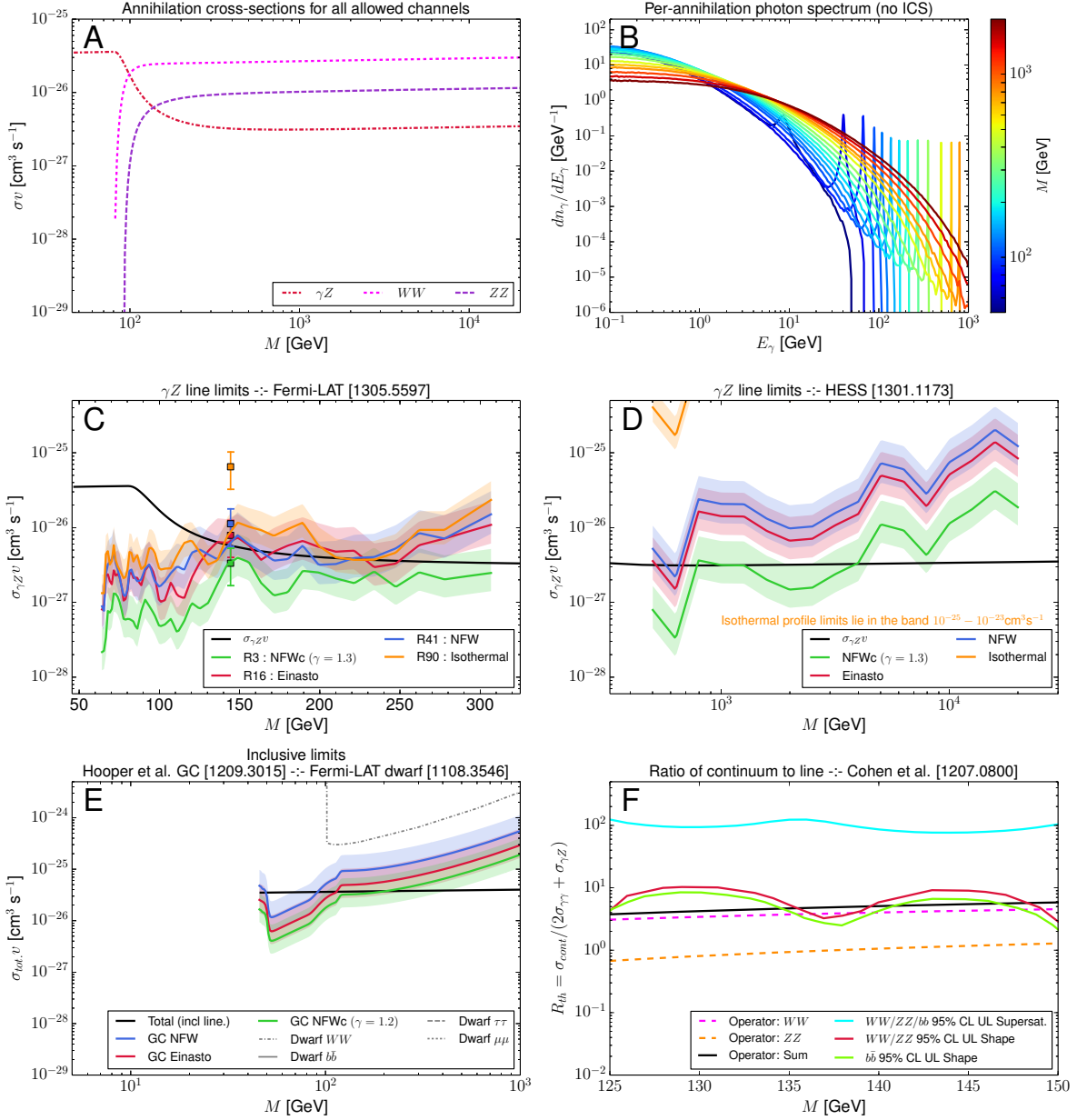


Figure 2.26: Limits for Operator XV-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and account well for the 130 GeV photon line for some profile choices.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi (B_{\lambda \mu 5} Y_H H^\dagger D^\lambda H + h.c.) \text{ :- Operator XVI - 1}$$

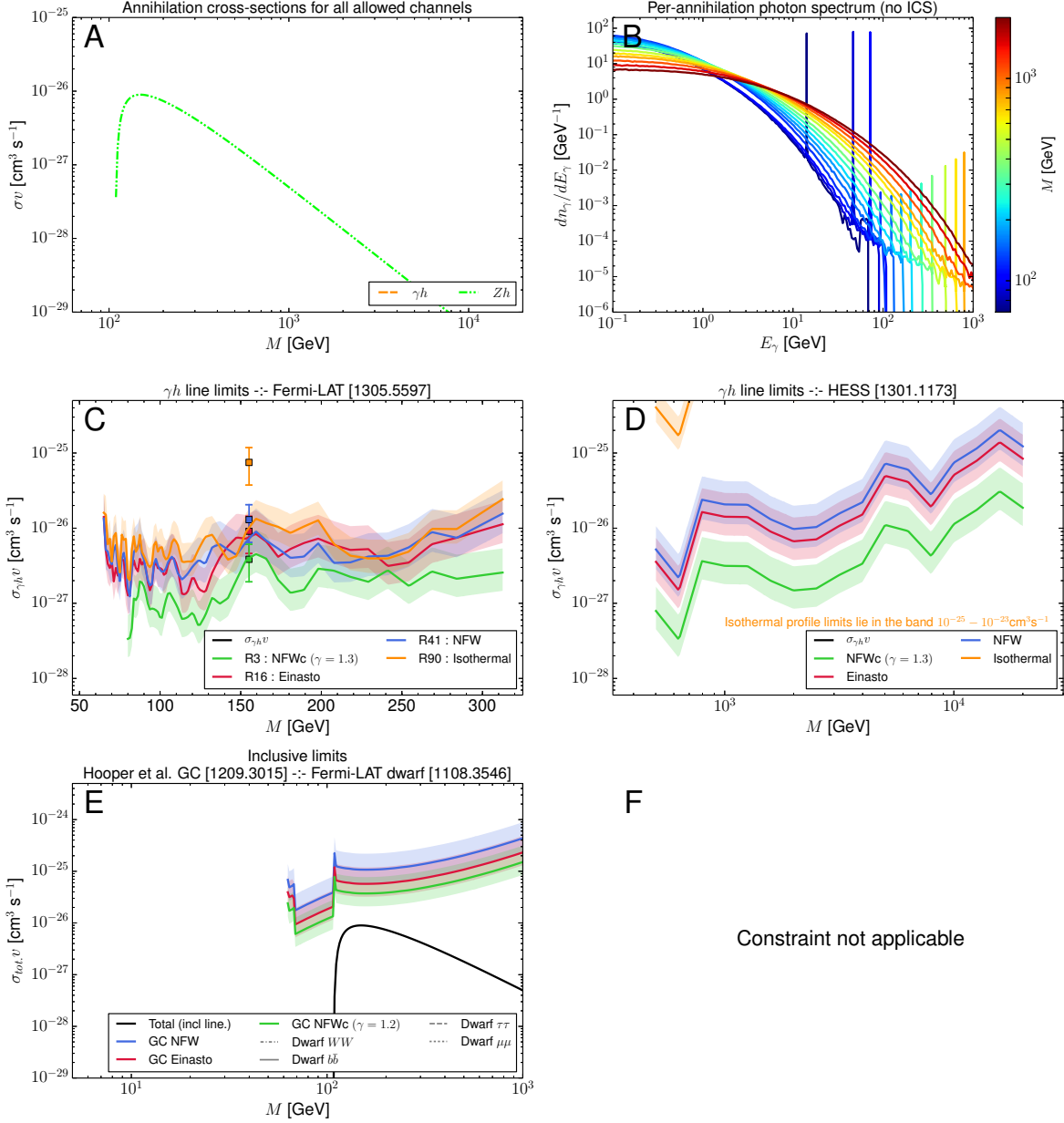


Figure 2.27: Limits for Operator XVI-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6. The γh mode is p -wave suppressed and so $\sigma_{\gamma h} v$ is orders of magnitude below the range of cross sections shown in panels C and D.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi (W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + h.c.) \text{ :- Operator XVI - 3}$$

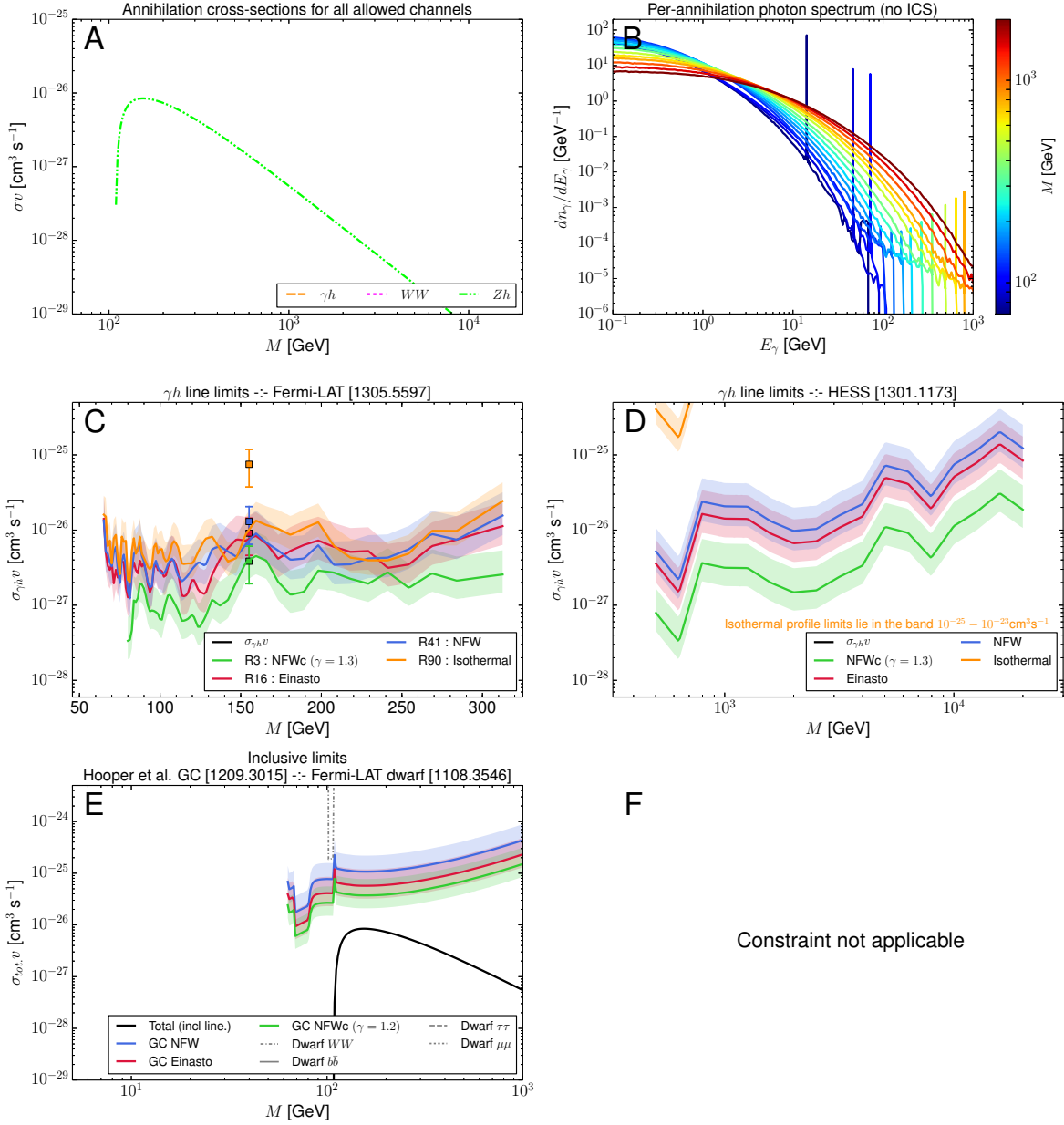


Figure 2.28: Limits for Operator XVI-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. The γh mode is p -wave suppressed and so $\sigma_{\gamma h} v$ is orders of magnitude below the range of cross sections shown in panels C and D.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi i (B_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ :- Operator XVII - 1}$$

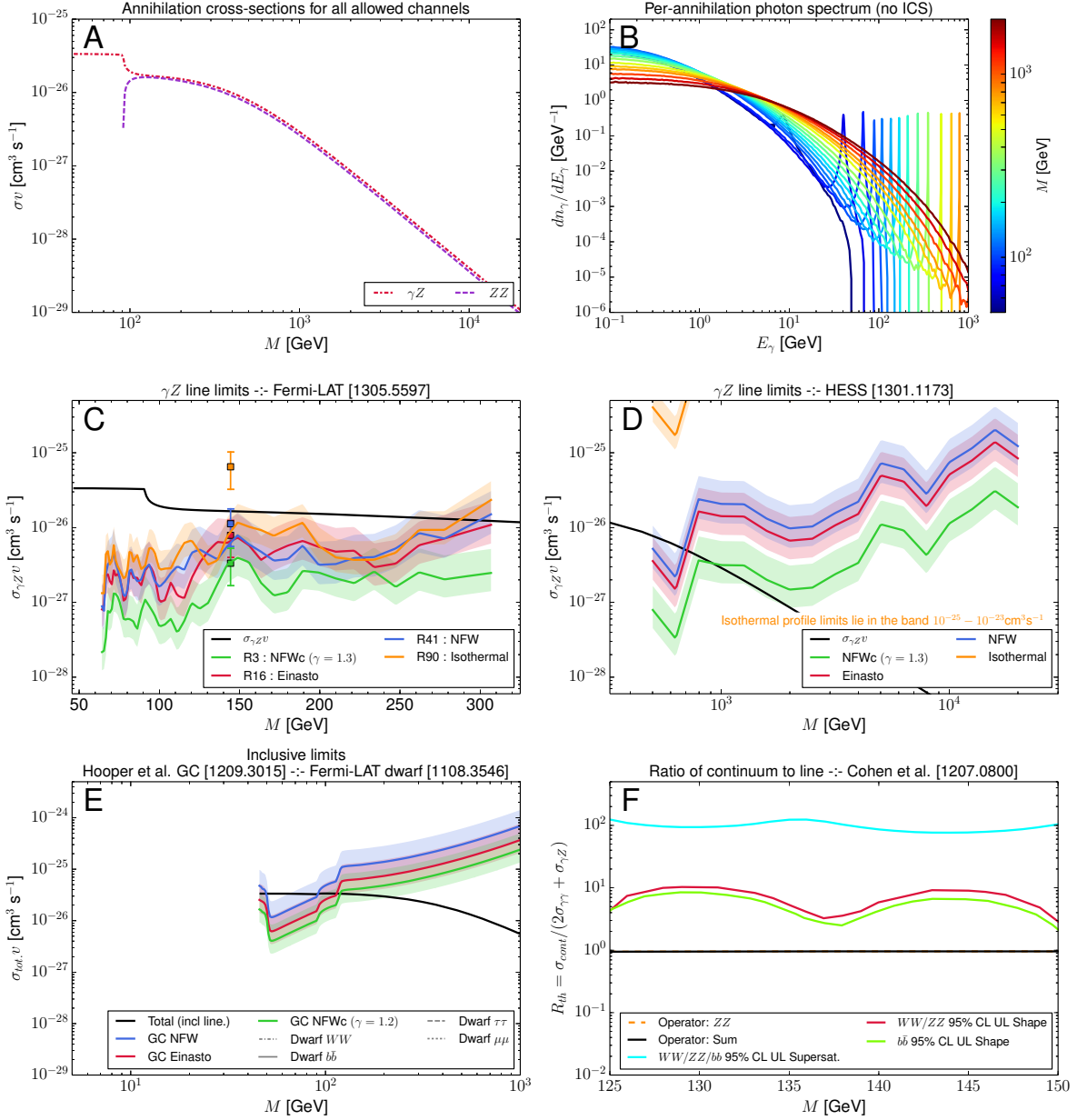


Figure 2.29: Limits for Operator XVII-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi i (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - h.c.) \text{ :- Operator XVII - 2}$$

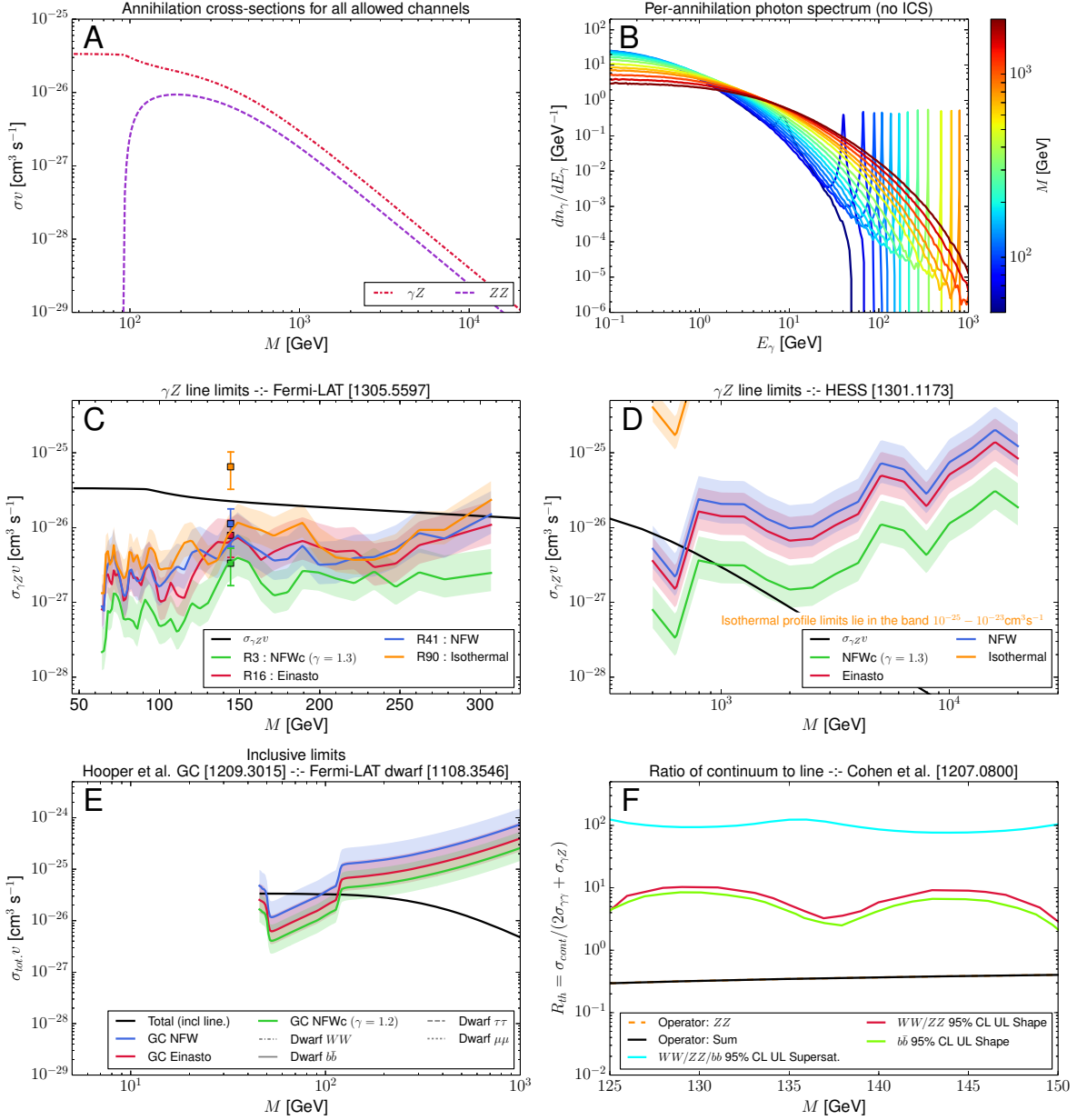


Figure 2.30: Limits for Operator XVII-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi i (W_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XVII - 3}$$

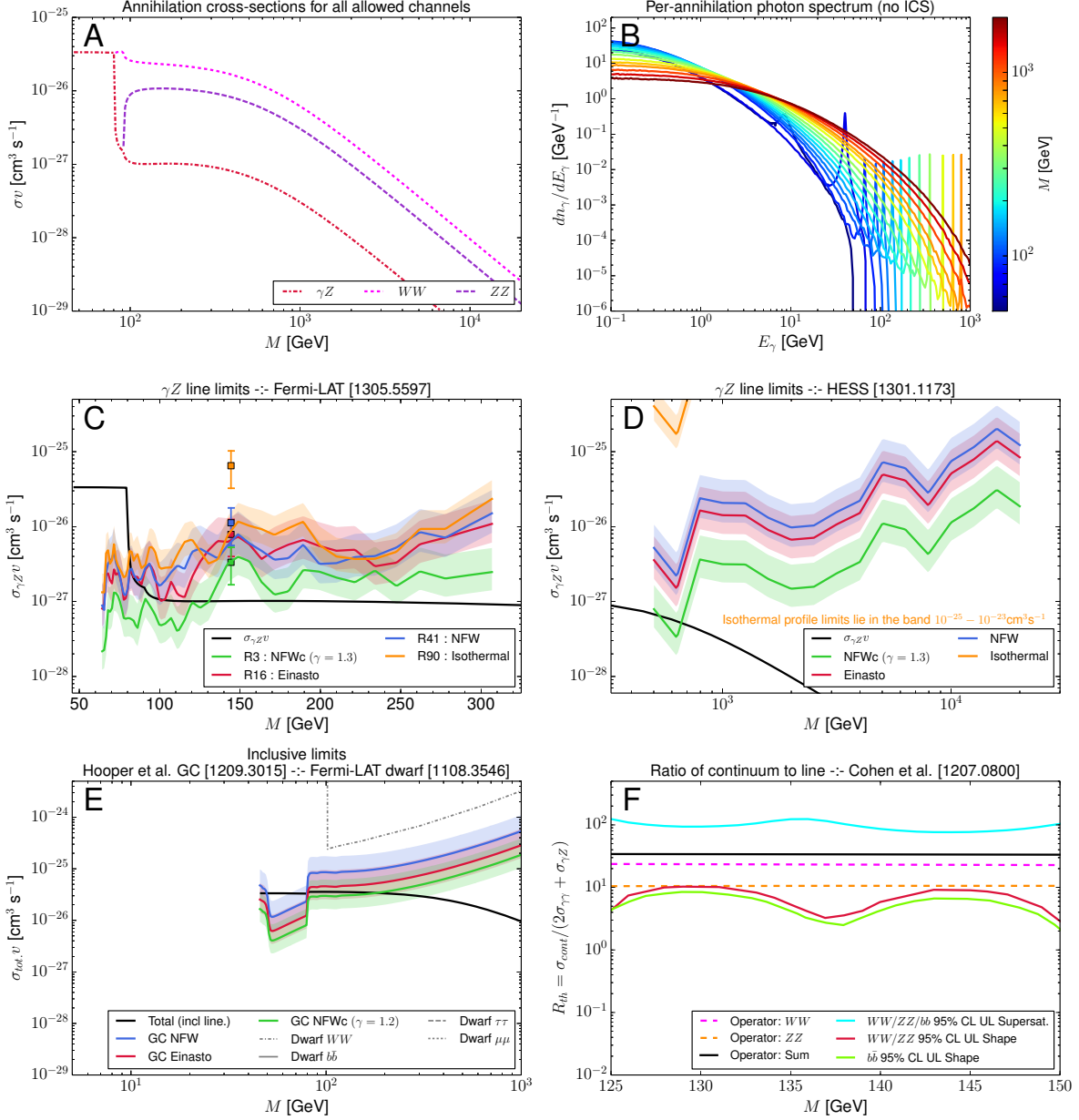


Figure 2.31: Limits for Operator XVII-3. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices, although the line BR is somewhat small.

$$\Lambda^{-4} \bar{\chi} \gamma^{\mu 5} \chi i (\widetilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - h.c.) \text{ :- Operator XVII - 4}$$

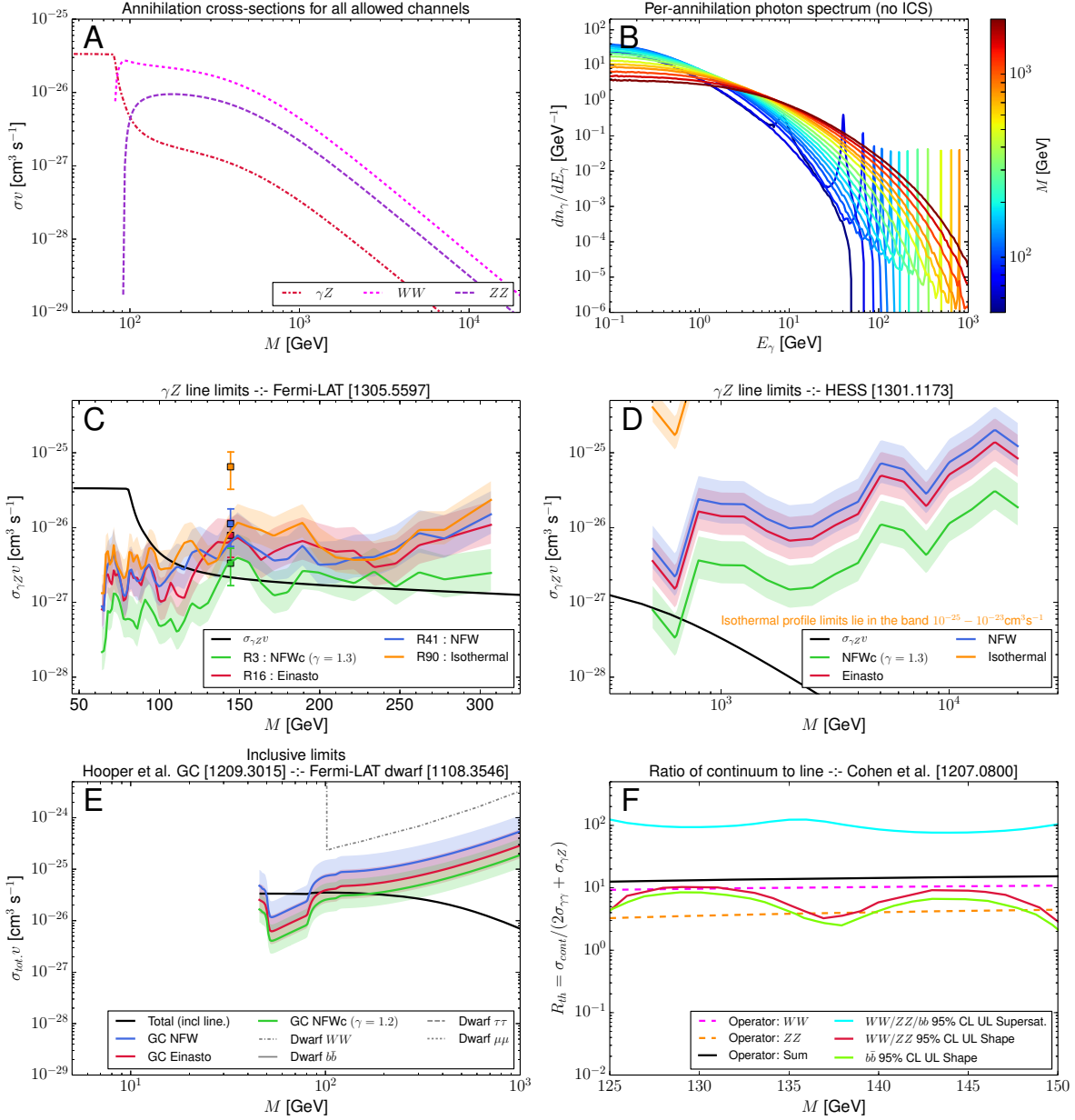


Figure 2.32: Limits for Operator XVII-4. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices.

$\Lambda^{-3} \bar{\chi} \gamma^{\mu\nu} \chi B_{\mu\nu} Y_H H^\dagger H$:- Operator XVIII – 1

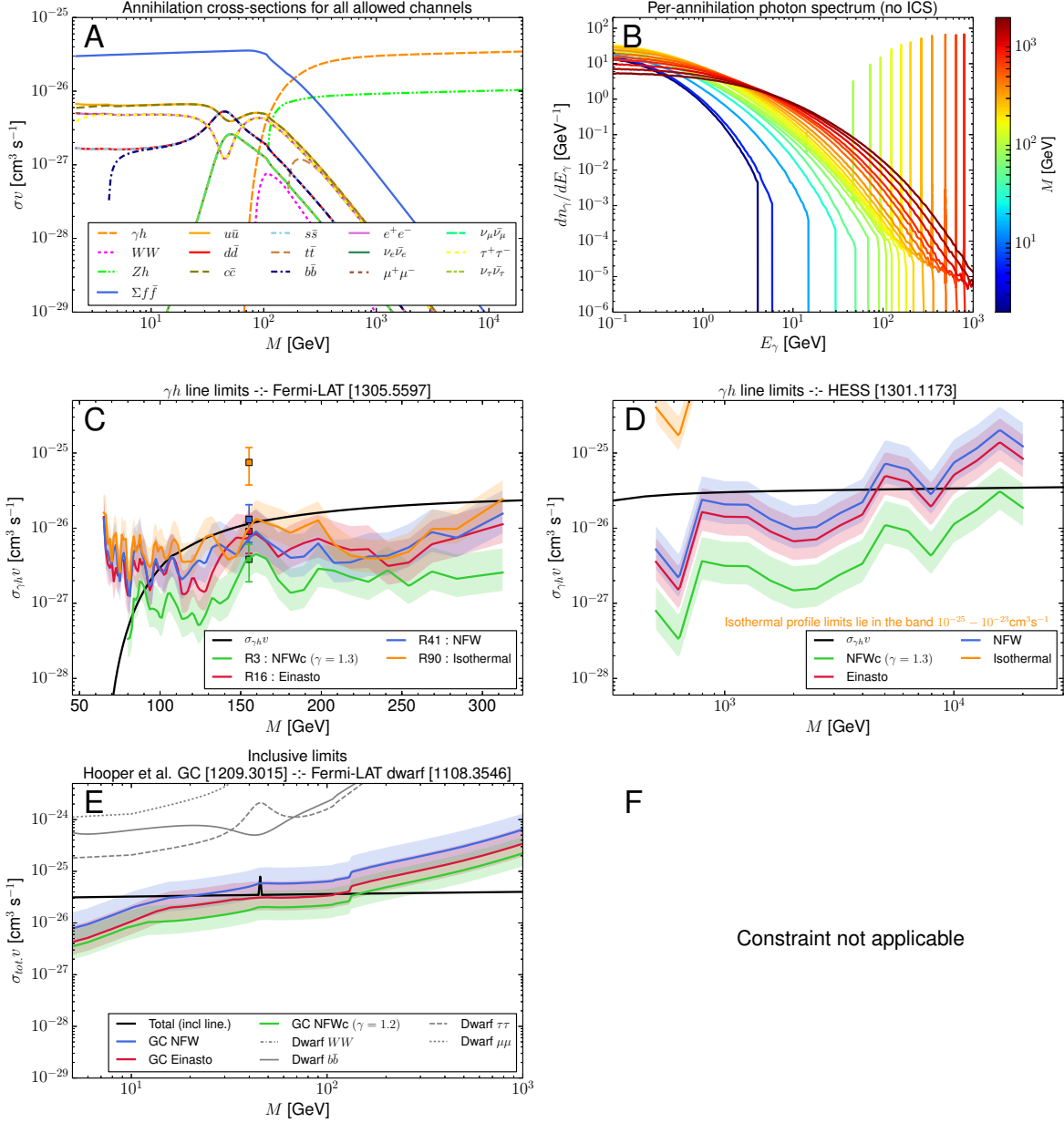


Figure 2.33: Limits for Operator XVIII-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and account well for the 130 GeV photon line for some profile choices; however, this operator gives rise to a magnetic dipole coupling of the DM and as such is excluded at 90% confidence by the CDMS and XENON direct detection experiments [477, 478] for M from 10 GeV to c. 1 TeV. The small upward blip in the black line in Panel E is an unphysical plotting artefact.

$\Lambda^{-3} \bar{\chi} \gamma^{\mu\nu} \chi \tilde{B}_{\mu\nu} Y_H H^\dagger H$:- Operator XVIII – 2

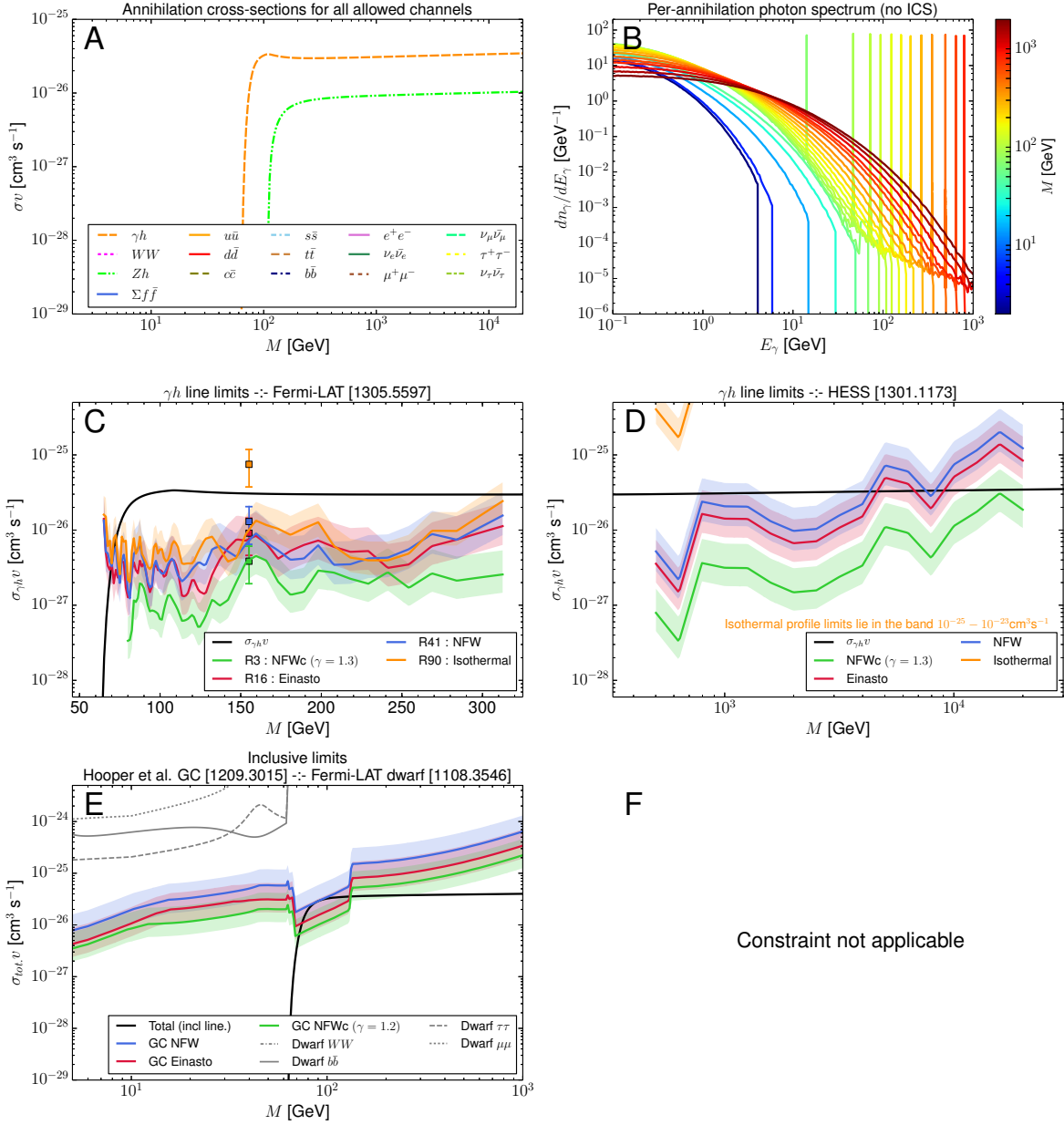


Figure 2.34: Limits for Operator XVIII-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator gives rise to an electric dipole coupling of the DM and as such is excluded at 90% confidence by the CDMS and XENON direct detection experiments [477, 478] for the entire range of M shown here.

$$\Lambda^{-3} \bar{\chi} \gamma^{\mu\nu} \chi W_{\mu\nu}^a H^\dagger t^a H \text{ :- Operator XIX - 1}$$

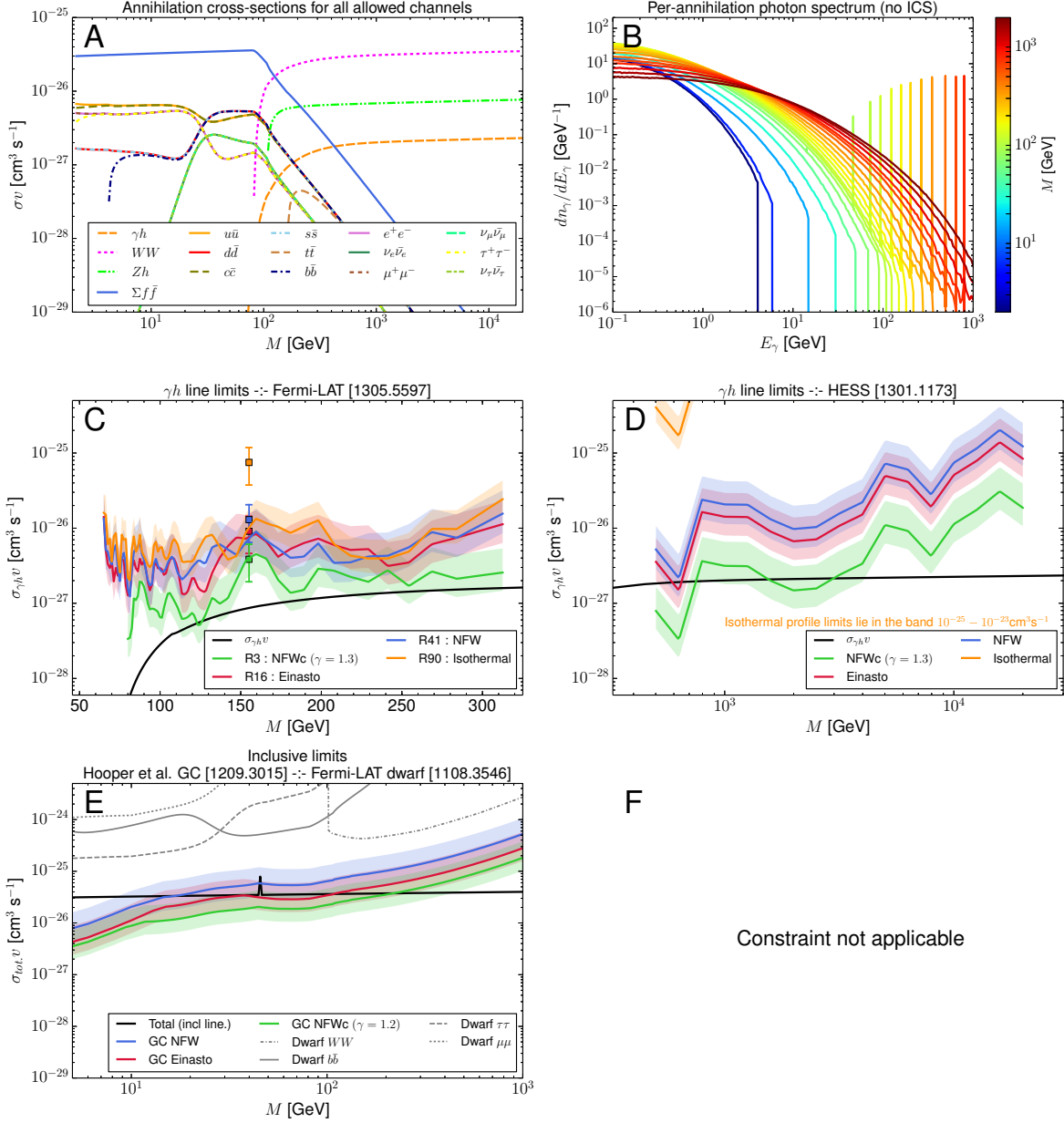


Figure 2.35: Limits for Operator XIX-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator gives rise to a magnetic dipole coupling of the DM and as such is excluded at 90% confidence by the CDMS and XENON direct detection experiments [477, 478] for M from 10 GeV to c. 400 GeV. The small upward blip in the black line in Panel E is an unphysical plotting artefact.

$$\Lambda^{-3} \bar{\chi} \gamma^{\mu\nu} \chi \widetilde{W}_{\mu\nu}^a H^\dagger t^a H \text{ :- Operator XIX - 2}$$

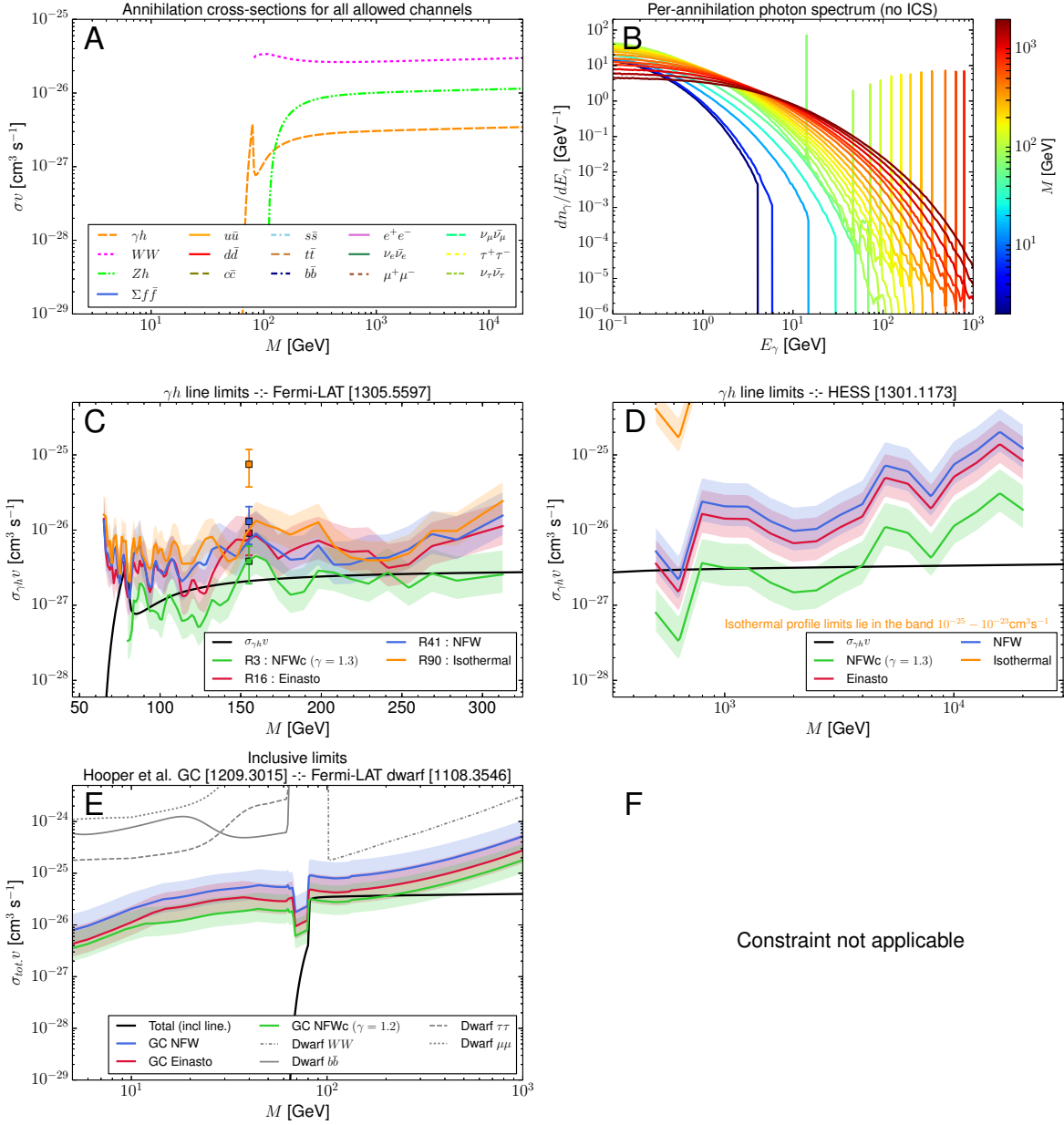


Figure 2.36: Limits for Operator XIX-2. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator may be compatible with present observational limits and capable of accounting for the 130 GeV photon line for some profile choices; however, this operator gives rise to an electric dipole coupling of the DM and as such is excluded at 90% confidence by the CDMS and XENON direct detection experiments [477, 478] for the entire range of M shown here.

$$\Lambda^{-1} \bar{\chi} \gamma^{\mu\nu} \chi B_{\mu\nu} \text{ :- Operator XX - 1}$$

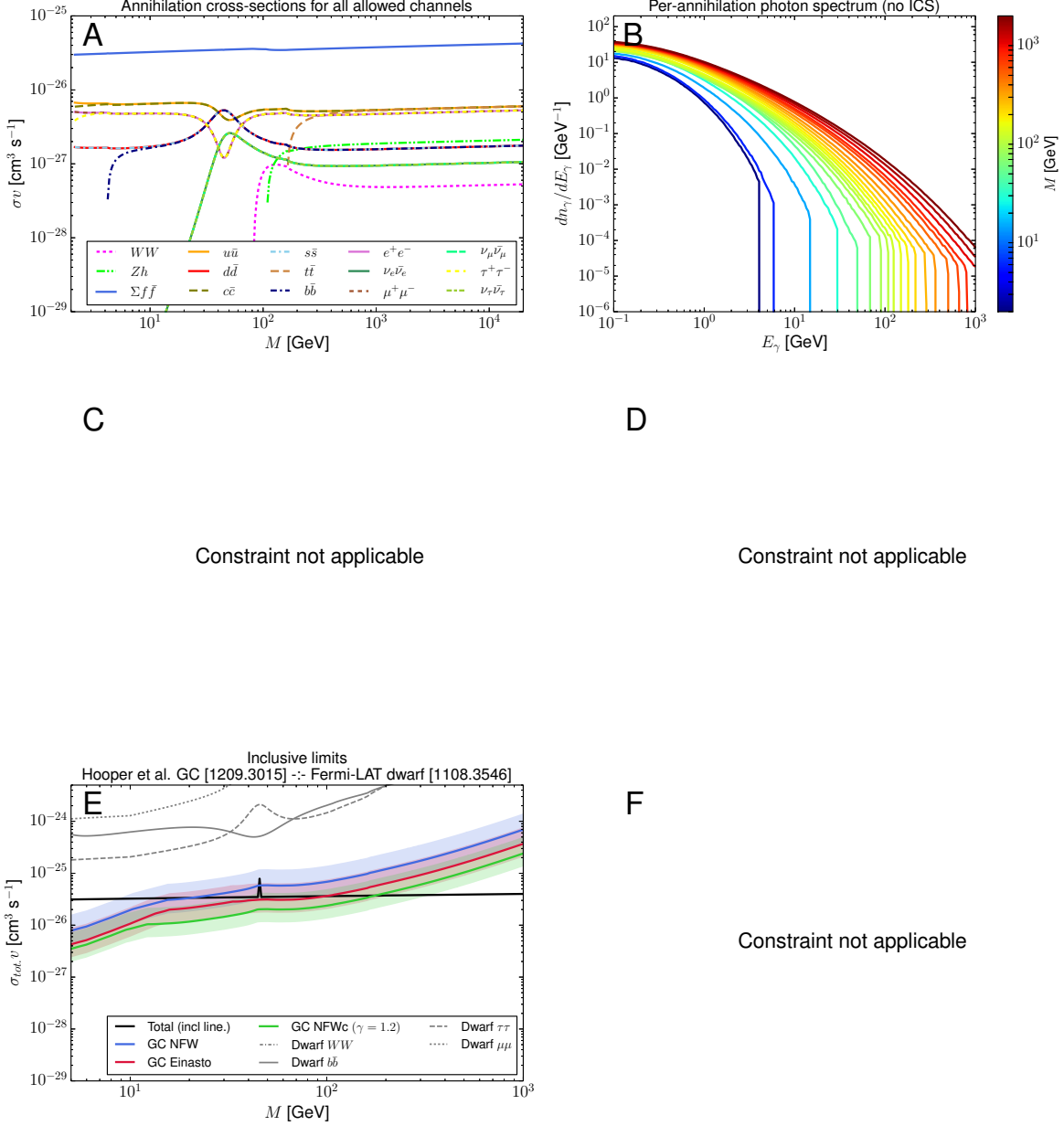


Figure 2.37: Limits for Operator XX-1. Detailed figure captions are provided in Sections 2.4.1–2.4.6. This operator gives rise to a magnetic dipole coupling of the DM and as such is excluded at 90% confidence by the CDMS and XENON direct detection experiments [477, 478] for the entire range of M shown here. The small upward blip in the black line in Panel E is an unphysical plotting artefact.

2.5 Discussion and Conclusions

Broadly speaking, we find that the 34 operators with non-vanishing s -wave annihilation channels can be classified into 13 qualitatively distinct classes on the basis of the limits which apply to each operator; we summarize these in Table 2.3. We discuss these categories in more detail in this subsection.

- **Operators coupling to hypercharge gauge bosons; photon lines present (categories 1–3)**

For operators in the first 3 categories in Table 2.3, there is a large branching fraction for annihilation to final states giving lines as soon as the dark-matter mass is larger than lowest kinematic threshold for such channels. This is a consequence of the fact that the B field strength tensor has a dominant photon component so that couplings to the Z are parametrically suppressed with respect to couplings to the γ by a factor of $\tan^2 \theta_W \sim 0.3$ (this suppression may be partially ameliorated by other large factors arising in the evaluation of the matrix element, but nevertheless serves as a useful rule of thumb). As a result, the operators in categories 1 and 2 are excluded at 95% confidence by the Fermi-LAT [449] and H.E.S.S. [450] line limits (see also Appendices A.5.2 and A.5.3) for most masses from the lowest kinematic threshold for a line final state up to a few TeV, except if (a) the very conservative isothermal profile choice is made for the H.E.S.S. limits and (b) the dark-matter mass is in the region 300 GeV and 500 GeV, which is not addressed by any of the line limits we consider here (but see Appendices A.5.2 and A.5.3). Depending on the profile choice, the required thermal cross section for these operators may also be in tension with the experimental upper limits from a few to 20 TeV.

For operators in category 3, the total s -wave cross section drops away from the canonical thermal value of approximately $\sigma v \sim 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ as M^{-2} for M above a few hundred GeV. This phenomenon occurs generically for operators with fermion DM

Table 2.3: Classification of operators by limit results. Starred (*) categories have operators that could fit the Fermi-LAT 130 GeV γ -ray line (but see Appendix A.5.1) and also match the correct relic abundance.

#	Labels	Operators	Summary of annihilation modes and limits
1	VI-1 VI-2 VIII-1 VIII-2	$\phi^\dagger \phi B_{\mu\nu} B^{\mu\nu}$ $\phi^\dagger \phi \tilde{B}_{\mu\nu} B^{\mu\nu}$ $\bar{\chi} i \gamma^5 \chi B_{\mu\nu} B^{\mu\nu}$ $\bar{\chi} i \gamma^5 \chi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\gamma\gamma$ is present and dominant for all masses, leading to strong line limits for masses up to 10 TeV; γZ and ZZ enter above kinematic thresholds but are sequentially more parametrically suppressed by powers of $(\sin \theta_W / \cos \theta_W)^2$.
2	XI-1 XI-2 XIV-1 XIV-2 XV-1 XV-2	$(\phi^\dagger \partial^\mu \phi + \text{h.c.}) i (B_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$ $(\phi^\dagger \partial^\mu \phi + \text{h.c.}) i (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$ $\bar{\chi} \gamma^\mu \chi (B_{\lambda\mu} Y_H H^\dagger D^\lambda H + \text{h.c.})$ $\bar{\chi} \gamma^\mu \chi (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H + \text{h.c.})$ $\bar{\chi} \gamma^\mu \chi i (B_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$ $\bar{\chi} \gamma^\mu \chi i (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$	γX and ZX where $X = Z$ (for ‘- h.c.’ operators) OR $X = h$ (for ‘+ h.c.’) are present above thresholds, with the latter in each case typically suppressed by $(\sin \theta_W / \cos \theta_W)^2$. Line limits are constraining up to a TeV.
3	XVII-1 XVII-2	$\bar{\chi} \gamma^{\mu 5} \chi i (B_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$ $\bar{\chi} \gamma^{\mu 5} \chi i (\tilde{B}_{\lambda\mu} Y_H H^\dagger D^\lambda H - \text{h.c.})$	γZ and ZZ are present above thresholds. Line limits are constraining up to a few hundred GeV, however p -wave components to the annihilation cross section during freeze-out become important at larger DM mass leading to suppression of the s-wave component and significant weakening of constraints.
4	XVIII-1	$\bar{\chi} \gamma^{\mu\nu} \chi B_{\mu\nu} Y_H H^\dagger H$	Annihilation to fermions is dominant and s-wave at low WIMP mass, with strong inclusive limits. The γh line annihilation mode becomes dominant above threshold, and the line limits are constraining for masses of 100 GeV up to a few TeV.

Table continued on next page ...

Table 2.3: ... continued from previous page.

#	Labels	Operators	Summary of annihilation modes and limits
5	XVIII-2	$\bar{\chi}\gamma^{\mu\nu}\chi\tilde{B}_{\mu\nu}Y_HH^\dagger H$	Similar to the operator above, but the continuum final states are p -wave, so inclusive limits are not constraining below 70 GeV.
6*	VI-3	$\phi^\dagger\phi W_{\mu\nu}^a W^{a\mu\nu}$	$\gamma\gamma$ is present for all masses; γZ , ZZ and WW enter above kinematic thresholds. Above the W^+W^- threshold, annihilation modes with lines are heavily parametrically suppressed by powers of $\sin^2\theta_W$. Line searches are constraining below the W^+W^- threshold.
	VI-4	$\phi^\dagger\phi\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$	
	VIII-3	$\bar{\chi}i\gamma^5\chi W_{\mu\nu}^a W^{a\mu\nu}$	
	VIII-4	$\bar{\chi}i\gamma^5\chi\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$	
7*	XI-3	$(\phi^\dagger\partial^\mu\phi + \text{h.c.})i(W_{\lambda\mu}H^\dagger t^a D^\lambda H - \text{h.c.})$	γX , ZX and W^+W^- where $X = Z$ ('-h.c.') or h ('+h.c.') are present above thresholds. Line limits are severely constraining for masses less than around 100 GeV; once the W^+W^- mode enters, the γX annihilation mode is heavily suppressed. Inclusive limits are weakly constraining for masses up to a few hundred GeV. Where applicable, the ratio limits are either marginal or quite constraining on 125–150 GeV.
	XI-4	$(\phi^\dagger\partial^\mu\phi + \text{h.c.})i(\tilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - \text{h.c.})$	
	XIV-3	$\bar{\chi}\gamma^\mu\chi(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + \text{h.c.})$	
	XIV-4	$\bar{\chi}\gamma^\mu\chi(\tilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H + \text{h.c.})$	
	XV-3	$\bar{\chi}\gamma^\mu\chi i(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H - \text{h.c.})$	
	XV-4	$\bar{\chi}\gamma^\mu\chi i(\tilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - \text{h.c.})$	
8*	XVII-3	$\bar{\chi}\gamma^{\mu 5}\chi i(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H - \text{h.c.})$	Similar to the category just above, but p -wave components to the cross section cause suppression of the s -wave cross section and weaken line constraints above a few hundred GeV.
	XVII-4	$\bar{\chi}\gamma^{\mu 5}\chi i(\tilde{W}_{\lambda\mu}^a H^\dagger t^a D^\lambda H - \text{h.c.})$	
9	XIX-1	$\bar{\chi}\gamma^{\mu\nu}\chi W_{\mu\nu}^a H^\dagger t^a H$	Annihilation to continuum modes is dominant, and inclusive limits are constraining up to a hundred GeV.
10	XIX-2	$\bar{\chi}\gamma^{\mu\nu}\chi\tilde{W}_{\mu\nu}^a H^\dagger t^a H$	Annihilation to continuum modes dominates, but are p -wave at low DM masses.

Table continued on next page ...

Table 2.3: ... continued from previous page.

#	Labels	Operators	Summary of annihilation modes and limits
11	IX-2	$\bar{\chi} i \gamma^5 \chi \ H^\dagger H$	Only continuum annihilation modes exist: one of either the $f\bar{f}$ or W^+W^- modes dominate at all masses.
	XX-1	$\bar{\chi} \gamma^{\mu\nu} \chi \ B_{\mu\nu}$	
12	X-1	$\left(\phi^\dagger \partial^\mu \phi + \text{h.c.} \right) \left(B_{\lambda\mu} Y_H H^\dagger D^\lambda H + \text{h.c.} \right)$	Only Zh s -wave modes are present.
	X-2	$\left(\phi^\dagger \partial^\mu \phi + \text{h.c.} \right) \left(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + \text{h.c.} \right)$	
13	XVI-1	$\bar{\chi} \gamma^{\mu 5} \chi \left(B_{\lambda\mu} Y_H H^\dagger D^\lambda H + \text{h.c.} \right)$	Only Zh s -wave modes are present but there are also significant p -wave contributions.
	XVI-3	$\bar{\chi} \gamma^{\mu 5} \chi \left(W_{\lambda\mu}^a H^\dagger t^a D^\lambda H + \text{h.c.} \right)$	

coupling via an axial-vector J_{DM} current, and is due to the presence of p -wave components to the annihilation cross section which are enhanced over the s -wave by a factor of $s/m_V^2 \sim M^2$. This enhancement can be traced to a coupling in the p -wave (but not the s -wave) to the longitudinal component $\epsilon_L^\mu \sim k^\mu/m_Z$ of the massive vector (cf. the Goldstone Boson Equivalence Theorem). Since both s - and p -wave components of σv can be important at freeze-out [cf. Eqs. (2.1) and (2.2)], but only s -wave is relevant for present-day annihilation, having $a/b \sim M^{-2}$ naturally suppresses the present-day annihilation cross section for large M by driving up the value of Λ required to saturate the present-day average DM density. As a result, a significantly smaller range of M is definitively excluded by, or put in tension with, the experimental limits compared to categories 1 and 2, especially for TeV-range DM. The constraints from the inclusive limits from Ref. [451] are less stringent: in most cases, the limits are in tension with the thermal-relic cross section for M below one or two hundred GeV only for some choices of halo profile, except for operators with $\gamma\gamma$ modes where, below c. 100 GeV, the inclusive limits exclude such operators even for conservative profile choices.

- **Operators coupling to $SU(2)_W$ gauge bosons and Higgs; photon lines present (categories 6–8)**

For operators coupling to the $SU(2)_W$ gauge bosons and Higgs (categories 6–8 in Table 2.3), the W^+W^- final state is always dominant for $M \gtrsim m_W$ since in this case couplings to the Z and γ are parametrically suppressed by $\cos^2 \theta_W$ and $\sin^2 \theta_W$, respectively. Consequently, for operators in categories 6 and 7, the line limits are only severely constraining between the lowest kinematic threshold for any line mode and $M \simeq m_W$; for M greater than about 100 GeV and less than a few TeV the experimental limits are typically either only in tension with the required thermal cross section for more aggressive choices of halo profile, or are not constraining. Operators in category 8 again contain fermion DM with coupling via an axial-vector current and thus suffer the same falling present-day σv at large M as the operators in category 3, and as a

result are even less constrained at large M than the operators in categories 6 and 7. The inclusive limits from Ref. [451] are somewhat stronger for all these operators than for the operators in categories 1–3, and may be in tension with the required thermal cross section up to a few hundred GeV, depending on the profile choice. Nevertheless, taken together with the line limits, these operators are only weakly constrained for heavy DM.

In addition, for most operators in categories 6–8, the shape-based ratio limits [453] are either very constraining (operators XI-3 and XVII-3), or are marginal in the sense that the current limit is very close to the predicted value from the operators. We however caution the reader that the limits in Ref. [453] were obtained by assuming the line signal would account for the Fermi-LAT signal for some choice of DM mass (in this regard, see Appendix A.5.1), and allowing the relative strengths of the lines to float to the optimal value. The first assumption may or may not be applicable to our analysis depending on the operator considered (or in light of the updated information in Appendix A.5.1); but the second is never applicable to our analysis since the relative strengths of all annihilation channels are fixed for any given operator. Therefore, marginal cases may or may not actually be constrained by these limits and a more careful analysis is needed here.

- **Operators with annihilation modes to fermions; photon lines present (categories 4,5,9,10)**

The analysis in Ref. [451] as applied to our operators indicates that if the only available s -wave annihilation mode for $M < 10$ GeV dark matter is to Standard Model fermions, there is a strong constraint which appears to rule out the annihilation cross section necessary for obtaining the thermal relic abundance in this mass range.

The operators listed in categories 4, 5, 9 and 10 of Table 2.3 all have annihilation modes to, *inter alia*, both fermions and line final states. Of these, the operators coupling to

the field strength tensor (rather than its dual) have s -wave annihilation to $f\bar{f}$ final states and so can be excluded for M below 10 GeV; they may also be in tension with the inclusive limits, depending on the profile choice, from 10 GeV to a few hundred GeV. Also, for these operators, the branching ratio to $f\bar{f}$ annihilation modes falls once the on-shell final-state diboson modes become kinematically allowed. This is because the annihilation to fermions goes through s -channel gauge boson exchange and so requires an additional Higgs vev insertion and coupling of fermions to γ/Z compared to the Zh and γh modes (see the next paragraph for a comment on the W^+W^- mode). On dimensional grounds this results in a suppression of the $f\bar{f}$ modes relative to the Zh and γh modes by a factor $\sim m_{Z,W}^2/s$ (where $s \sim 4M^2$). For the operators coupling instead to the dual field strength tensor, the fermion cross section is p -wave so the inclusive limits are less constraining: they may be in tension with the required cross section from the γh threshold to a few hundred GeV, but only for fairly aggressive profile choices.

For the operators in categories 4 and 5 with couplings to hypercharge and Higgs bosons, the photon line becomes strong above the kinematic threshold for the γh annihilation mode, leading to strongly excluding line constraints, whereas for the operators in categories 8 and 9 with couplings to the $SU(2)_W$ gauge bosons and Higgs bosons, the diboson annihilation modes are dominated by W^+W^- , which leads to line limits for these operators in tension with the required thermal cross section only if an aggressive profile choice is made. We note that this occurs notwithstanding the fact that for the W^+W^- mode (as for the $f\bar{f}$ mode), both Higgs fields must be replaced by their vevs; the reason that this does not suppress this final state is that there is a cancellation of the suppression factor $\sim m_W^2/s$ against an enhancement $\sim s/m_W^2$ from the longitudinal mode of the $W^\pm$ (à la Goldstone Boson Equivalence Theorem). The by-now-familiar suppression, relative to $W^\pm$, of couplings to Z and γ by factors of $\cos^2 \theta_W$ or $\sin^2 \theta_W$, respectively, accounts for the relative strength of the W^+W^- and

Zh or γh modes.

- **Operators without photon lines (categories 11–13)**

Finally, categories 11–13 in Table 2.3 are operators which couple only to SM final states without photon lines.²⁷ The two operators in category 11 have strong annihilation to $f\bar{f}$ either for all M (operator XX-1)²⁸ or at low M (operator IX-2),²⁹ but in either case this means that they are strongly constrained for $M \lesssim 10$ GeV even for conservative profile choices. Owing to the very flat (with changing M) nature of the inclusive upper limits on σv , the astrophysical uncertainty implies a rather broad range of M for which the operators may be in tension with the experimental limits depending on the profile choice: 10 GeV to a few hundred GeV. The operators in categories 12 and 13 have only Zh final states: the former may be in tension with the astrophysical limits for aggressive profile choices for M between 100 and 400 GeV, but they are unconstrained for more modest profile choices; the latter are unconstrained regardless of the profile chosen due to s -wave cross section suppression by the same enhanced- p -wave effect which was discussed at length above.

In addition to these indirect limits, we emphasize the point made in Ref. [431] that the ‘tensor’ fermion DM currents $\chi\gamma^{\mu\nu}\chi$ (e.g., operators XVIII-n, XIX-n and XX-n in categories 4–5 and 9–11) give rise to magnetic or electric dipole couplings (if necessary, by replacing both Higgs fields with their vevs) which causes them to be very strongly constrained by direct detection experiments [477, 478]. Given our assumptions about the DM being a cold thermal-relic WIMP saturating the measured average density, the operators giving electric-dipole couplings (coupling to the dual field tensor) are excluded [477, 478] at at least 90%

27. Annihilation to monochromatic photons is also possible at sub-leading order, for example through loops, or see Ref. [455]; we do not include these subdominant modes.

28. The annihilation modes here are all through s -channel gauge boson exchange which explains why the suppression of the $f\bar{f}$ modes does not occur for large M as it does for operators XVIII-1, XIX-1 and IX-2.

29. The explanation here for the suppression of the $f\bar{f}$ branching ratio for $M \in [m_W, m_t]$, or for $M \gg m_t$, is that this annihilation mode is via s -channel Higgs exchange which leads to a suppression from the hff coupling $\sim m_f^2/s$.

confidence by both CDMS and XENON across the entire mass range from 10 GeV to at least 20 TeV (probably somewhat larger), while the operators giving magnetic-dipole couplings (coupling to the field tensor, not its dual) are excluded at at least 90% confidence from $M = 10$ GeV to $M > 20$ TeV, $M \simeq 1$ TeV, or $M \simeq 400$ GeV for operators XX-1, XVIII-1, or XIX-1, respectively (with the exception of a very narrow mass window around $m_Z/2$ for the latter two of these, owing to the resonant enhancement of the early-universe annihilation cross section in this region).

We have presented all of our results under the assumption that fermionic DM is Dirac; however, the compensating factors of 2 in Eqs. (2.1) and (2.7) imply that there is a uniform shifting of all σv values down by a factor of 2 if Majorana fermion DM was assumed instead; therefore, the same thermal cross section relative to all of our 95% CL UL limits would obtain, at least for all the operators which do not vanish identically for Majorana fermion DM, and so the limits on the mass parameter space should be largely unchanged if fermionic DM is assumed instead to be Majorana.

γ -ray Line Signals

As discussed in Section 2.3.6 and Section 2.3.7, there was some evidence at the time (c. 2013) that Ref. [4] (on which this chapter is based) was published, first reported in Ref. [432] and examined in Ref. [433], for a photon line in the Fermi-LAT data near 130 GeV (updated to approximately 133 GeV in Ref. [449]) with a flux matching DM annihilation into monochromatic photons with a cross section $\sigma v \approx 10^{-27} \text{ cm}^3/\text{s}$. The original Fermi-LAT Collaboration analysis [449] of this feature found less significant evidence for this putative line-like feature, but it could not fully explain it as a systematic effect, and cautioned that more work was needed to understand it. Subsequent reanalysis and further data-taking by the Fermi-LAT collaboration [231] has indicated that this putative spectral feature was instead likely not an actual feature ‘on the sky’, and was most likely a systematic [231] (see also the discussion in Appendix A.5.1 for a more detailed update).

Nevertheless, it is worth noting that we find that there are a number of operators (indicated in Table 2.3 by a superscript star in the category number) which could have plausibly explained the then-putative line signal, which would not have been excluded by either the upper limits on the continuum emission or by the official Fermi-LAT limits on lines c. 2013.

The putative signal strength for the photon line was a factor of a few (up to an order of magnitude, depending on the halo profile) below the thermal-relic cross section. Operators with annihilation into $SU(2)_W$ gauge bosons (categories 6–8) naturally give a suppression of this size simply from the mixing angle $\sin^2 \theta_W$. Numerical factors are also important, and those operators with smaller branching ratios into lines do not produce enough monochromatic photons if the cross section is fixed by the matching onto the correct relic density. Stated differently, these operators may be in some tension with continuum limits or the constraint on the continuum-to-line ratio since the BR to such continuum modes is then larger.

Operators XV-3 and XV-4 were most promising as an explanation of the then-putative line signal, but for completeness we note that almost any of the other operators in categories 6–8 in Table 2.3 could also have worked, including Operators VI-3, VI-4, VIII-3, VIII-4, XI-4, XIV-3, XIV-4, and XVII-4. The operators XI-3 and XVII-3 have smaller branching ratios to lines (less than 10%) and so did not fit the line as well for the reasons discussed above. However, given the systematic uncertainties, it was not conclusive that these operators could not explain the Fermi gamma-ray line. (Finally, although the operators XVIII-1 and XIX-2 also have lines of roughly the correct strength, they are ruled out by direct detection experiments, as discussed above.)

Other Indirect Constraints and Outlook

Photon fluxes do not of course supply the only prospects for indirect detection of dark matter. The IceCube experiment [237, 479] reports (c. 2013) limits on σv from neutrino fluxes which are at best on the order of $\sigma v \sim 10^{-22}\text{--}10^{-23}\text{cm}^3\text{s}^{-1}$, and are thus much too weak to be

constraining. The absence of sharp spectral features in the AMS-02 positron fraction data provides a constraint [480] on annihilation modes giving a sharp edge in the positron fraction: we expect that these limits could be stronger than the GC inclusive limits for M less than a few tens of GeV for operators with sizeable BR for annihilation to e^+e^- and, to a lesser extent, $\mu^+\mu^-$ final states (operators XVIII-1, XIX-1, IX-2 and XX-1; the latter may be more constrained since $f\bar{f}$ dominates for all masses). A slightly different analysis [481] constraining only the total number of positrons from various DM annihilation modes using a combination of Fermi-LAT and AMS-02 data confirms the approximate equivalence of the limits from positron measurements and the GC inclusive photon flux limits for the $\mu^+\mu^-$ annihilation mode, but indicates that constraints arising from positron measurements for other channels ($b\bar{b}$, W^+W^- , ZZ) would be between one and two orders of magnitude weaker than the GC inclusive photon flux limits. Additionally, constraints from antiproton fluxes may be useful to consider. Ref. [454] found that the PAMELA antiproton limits are typically weaker than the inclusive GC photon flux limits for W^+W^- or $b\bar{b}$ annihilation modes and, for at least some of the operators we have considered, do not typically have sufficient reach to exclude the thermal WIMP scenario; however, Ref. [482] finds that antiproton bounds are roughly comparable with the GC inclusive photon limits from Ref. [451] for pure $b\bar{b}/t\bar{t}$ or W^+W^-/ZZ final states for $M \gtrsim 100$ GeV (the antiproton limits are much weaker for smaller M , or for leptonic annihilation modes), but have fairly large uncertainties arising from cosmic-ray propagation models. One would have to fully reconstruct the analysis of Ref. [482] to take advantage of these comparable limits though, since independently comparing the individual annihilation mode cross sections to their respective limits would again be subject to the cost of a factor of BR_f , which is usually fairly small for $b\bar{b}$ or $t\bar{t}$ modes for our operators.

Looking forward, Ref. [483] indicates that it would be appropriate to consider a factor of 3–5 improvement in the present best limits from the next generation of indirect detection experiments looking at photon fluxes: GAMMA-400, CTA, and an upgrade of H.E.S.S. (see, e.g., Ref. [232] for early H.E.S.S.-II results, which are also discussed in Appendix A.5.3);

Ref. [484] provides a more recent outlook on the projected future sensitivity of Fermi-LAT for DM searches. This magnitude of improvement would be able to constrain a fair amount more of the parameter space for the mass of the thermal-relic WIMP scenario even considering the astrophysical uncertainties, but would still not be able to completely rule out all DM masses for every EFT operator that we have considered which has s -wave annihilation channels.

CHAPTER 3

THE FERMIONIC DARK MATTER HIGGS PORTAL: AN EFFECTIVE FIELD THEORY APPROACH

This chapter contains material from Ref. [5]: M. A. Fedderke, J.-Y. Chen, E. W. Kolb, and L.-T. Wang, *The Fermionic Dark Matter Higgs Portal: an effective field theory approach*, *JHEP* **1408** (2014) 122 [[arXiv:1404.2283](#)].¹

In this chapter, we consider fermionic (Dirac or Majorana) cold thermal-relic dark matter coupling to Standard Model particles through the effective dimension-5 Higgs Portal operators $\Lambda^{-1} \mathcal{O}_{\text{DM}} \cdot H^\dagger H$, where $\mathcal{O}_{\text{DM}}$ is an admixture of scalar $\bar{\chi}\chi$ and pseudoscalar $\bar{\chi}i\gamma_5\chi$ DM operators. Utilizing the relic abundance requirement to fix the couplings, we consider direct detection and invisible Higgs width constraints, and map out the remaining allowed parameter space in terms of the dark-matter mass and the admixture of scalar and pseudoscalar couplings. We also emphasize a subtlety which has not previously been carefully studied in the context of the EFT approach, in which an effect arising due to electroweak symmetry breaking can cause a naïvely pure pseudoscalar coupling to induce a scalar coupling at higher order in the EFT power counting, which can have important implications for interpreting direct detection bounds. Finally, we provide some comments on indirect detection bounds and collider searches.

3.1 Introduction

As discussed at length in Chapter 2, among the menagerie of possible dark-matter candidates that have been proposed and explored in the literature, the Weakly Interacting Massive Particle (WIMP) scenario stands out as particularly compelling. To recap (see also Ref. [120]),

1. Ref. [5] is copyright © 2014 by the Authors, and is under a Creative Commons Attribution (CC BY) 4.0 Licence [2].

WIMPs are established in local thermodynamic equilibrium (LTE) in the early universe through the coupling of the WIMPs to SM particles. The present abundance of WIMPs is determined by the freeze-out from LTE of the WIMPs. If the correct relic abundance is attained, freeze-out occurs when the temperature of the universe drops a factor of 20 or so below the mass of the WIMP.

As explained in Chapter 2, within the WIMP paradigm there are typically multiple complementary experimental probes available owing to the requisite WIMP–SM coupling. There are two main approaches in specifying this WIMP–SM coupling. In a top-down approach, one imagines a complete enveloping (renormalizable) model or theory that contains a WIMP and a prescription for how the WIMP couples to SM particles. An example of this approach is the assumption of low-energy supersymmetry where the WIMP is the lightest supersymmetric particle [485]. The other approach is a bottom-up effective field theory (EFT) parameterization.² In the latter approach one usually assumes a DM–SM interaction of the form $\Lambda^{-n} \mathcal{O}_{\text{DM}} \cdot \mathcal{O}_{\text{SM}}$, where Λ is the EFT mass scale, and $\mathcal{O}_{\text{DM}}$ and $\mathcal{O}_{\text{SM}}$ are, respectively, DM and SM operators that are singlets under the Standard Model gauge groups [253, 439, 440]. Among the advantages of the bottom-up approach are that (a) it provides the simplest method to combine the different experimental avenues for WIMP discovery, and (b) limits derived on any one EFT operator can be quite broadly applicable to whole classes of UV-complete models whose low-energy behaviour is well approximated by that operator.

In the EFT approach it is necessary to make assumptions for the form of $\mathcal{O}_{\text{DM}}$ and $\mathcal{O}_{\text{SM}}$. The Higgs bilinear, $H^\dagger H$, is the lowest mass-dimension gauge-invariant operator consisting of SM matter fields. It is therefore natural to consider DM couplings to the SM via the so-called Higgs Portal operators, of the form $\mathcal{O}_{\text{DM}} \cdot H^\dagger H$. Such Higgs Portal couplings can have interesting cosmological implications [409]; and as the Higgs field plays the central role

2. There is also a third, intermediate approach: so-called ‘simplified models’. These (renormalizable) models aim to incorporate only a minimal amount of additional field content in order to specify just enough of the DM physics and the WIMP–SM couplings to obtain the relevant phenomenology; they are thus less all-enveloping or complete compared to, say, SUSY models, but they clearly provide a more UV-complete picture than the EFT approach.

in electroweak symmetry breaking (EWSB), the former will have important effects on the dark-matter mass and couplings in this scenario. There exists an extensive literature on Higgs Portal dark matter (e.g., Refs. [255, 407, 408, 434, 486–505]).

In this chapter, we present a complete study of the lowest-mass-dimension Higgs Portal coupling of fermionic dark matter. We perform a detailed study of the dark-matter masses and couplings which pass experimental bounds (the results presented in the main body of this chapter reflect the results of Ref. [5] and hence reflect the state-of-the-art limits c. 2014; where appropriate, we have made comments on subsequent improvements of these limits). Although previous work [434, 505] has explored some aspects of this scenario, we extend this work by considering in a systematic fashion simultaneous contributions from both the CP-conserving ($\mathcal{O}_{\text{DM}} \sim \bar{\chi}\chi$) and CP-violating ($\mathcal{O}_{\text{DM}} \sim \bar{\chi}i\gamma_5\chi$) Higgs Portal couplings. We also emphasize a subtlety which has not previously been carefully studied in the context of the EFT approach: although the CP-violating coupling only mediates highly (momentum-transfer) suppressed contributions to the direct detection process at leading order, effects arising due to EWSB can generate a significant CP-conserving coupling. Although this effect is higher order in the EFT suppression scale, the lifting of the momentum-transfer suppression can greatly enhance the direct detection cross section over the naïve expectation. We carefully take this effect into account by carrying out a consistent chiral rotation.

In our analysis, we compute the (tree-level) dark-matter annihilation cross section and use the cosmological dark-matter relic abundance to fix the EFT suppression scale. Numerical solution of the Boltzmann equation, including a full thermal averaging of the annihilation cross section during the freeze-out process, is carried out to accurately capture the sizeable resonance and threshold effects [462] near $M \sim m_h/2$ and $M \sim m_W$, respectively. With the EFT suppression scale thus fixed, we find that LUX direct detection bounds [214, 506], and—for $M < m_h/2$ —Higgs invisible decay [507] and total width [508] constraints arising from ATLAS and CMS data rule out significant portions of the parameter space. We combine these constraints and map out the remaining unconstrained parameter space. While

we do not investigate indirect detection bounds in depth in this chapter, considering the inherently continuum nature of the signals and being conservative with respect to the large astrophysical uncertainties, these limits are expected to be weak in comparison to the other probes, especially for the mostly CP-even case for which present-day annihilation is p -wave and thus highly velocity-suppressed [431] (see Figure 2.12 in Chapter 2, and also Ref. [4]).

In principle, the fermionic Higgs Portal couplings we consider can also contribute to signals of dark-matter production at high-energy colliders, although the validity of the EFT approach *per se* at high energy can be degraded by perturbative unitarity issues [460, 461, 509]. This issue notwithstanding, the dominant contribution probably arises from the $h\bar{\chi}\chi$ coupling induced by this operator. Although a detailed analysis of the reach is beyond the scope of the analysis we will present this chapter, we offer some brief comments here. One possible signal would be a weak vector boson fusion process in which a dark-matter pair is produced through an off-shell Higgs, giving rise to two forward tagging jets and missing transverse energy; see Figure 3.1. An off-shell Higgs could also be produced by gluon fusion, which when combined with an initial state radiation would lead to a mono-jet plus missing energy signal; see Figure 3.1. Given the sizeable SM backgrounds, we expect the reach in both of these channels to be fairly limited. Of course, the Higgs can be on-shell if $M < m_h/2$, but we will see that this scenario is already strongly constrained by limits to Higgs invisible decay signals; in the future, we expect the Higgs invisible decay limits to continue to provide stronger limits in this regime than the direct collider search.

The rest of this chapter is organized as follows. In Section 3.2, we carry out the chiral rotation necessary to obtain a canonical quadratic Lagrangian, and present our parameterization of the model parameters. In Section 3.3, we present our analytic calculation of the annihilation cross section, and examine the validity of our truncation of the EFT expansion. Our calculation of the limits from Higgs decay, relic abundance, and direct detection are presented in Section 3.4, Section 3.5, and Section 3.6, respectively. Finally, we combine all the constraints and present the remaining unconstrained parameter space in Section 3.7,



Figure 3.1: Two possible collider signals for the Higgs Portal EFT DM couplings considered in this chapter: weak vector boson fusion (VBF), left; and gluon fusion, right. All SM fermions run in the loop, but the dominant contribution is from the top quark, owing to its large Yukawa coupling.

before concluding in Section 3.8.

Appendix B.1 is a collection of results from the literature that are related to the relic density computation (i.e., solving the Boltzmann equation). Appendix B.2 contains discussion of some selected results presented in a fashion complementary to how they are presented in this chapter. Appendix B.3 contains a detailed discussion of our result for the direct detection cross section and aims to clarify some aspects of its validity and applicability. Finally, Appendix B.4 contains a detailed investigation of the impact on our results of the improvement in LUX direct detection limits between 2013 [506] and 2016 [214], subsequent to the publication of Ref. [5], on which this chapter is based; we present there versions of some of the figures in this chapter which are updated to account for the most recent (2016) LUX limits.

3.2 The Effective Field Theory

We consider a convenient parametrization³ of the effective pre-EWSB mass-eigenstate Lagrangian coupling an admixture scalar and pseudoscalar SM-singlet fermionic DM operators

3. The parametrization in terms of θ and Λ is convenient for a numerical scan of the parameter space, but we should caution the reader that the ‘EFT suppression scale’ Λ in this parametrization is only approximately the scale of new physics: the scalar (CP-conserving) and pseudoscalar (CP-violating) operators can logically have different new physics scales associated with them and this gets mixed up in our parametrization. This issue should be borne in mind when judging issues of perturbative unitarity.

to the SM via the Higgs Portal $H^\dagger H$:⁴

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\chi} (i\not{\partial} - M_0) \chi + \Lambda^{-1} \left(\cos \theta \bar{\chi} \chi + \sin \theta \bar{\chi} i \gamma_5 \chi \right) H^\dagger H . \quad (3.1)$$

Since we are assuming that the non-SM operators in Eq. (3.1) do not participate in EWSB, one expects M_0 and Λ are greater than the weak scale, although we will allow $M_0 < \langle v \rangle$ in this work.

After EWSB, the Higgs field develops a vacuum expectation value $\langle v \rangle$ and the Higgs field content becomes (in the unitary gauge, with $\langle v \rangle \approx 246 \text{ GeV}$)

$$H^\dagger H \longrightarrow \frac{\langle v \rangle^2}{2} + \langle v \rangle h + \frac{h^2}{2} . \quad (3.2)$$

The Lagrangian then becomes

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\chi} i \not{\partial} \chi - \left[M_0 \bar{\chi} \chi - \frac{\langle v \rangle^2}{2\Lambda} \left(\cos \theta \bar{\chi} \chi + \sin \theta \bar{\chi} i \gamma_5 \chi \right) \right] \\ + \Lambda^{-1} \left(\cos \theta \bar{\chi} \chi + \sin \theta \bar{\chi} i \gamma_5 \chi \right) \left(\langle v \rangle h + \frac{1}{2} h^2 \right) . \end{aligned} \quad (3.3)$$

If we were to assume instead that the DM is Majorana, we would insert the conventional factor of $1/2$ in front of every fermionic bilinear; the subsequent analysis of the Lagrangian is then unchanged from the Dirac case, modulo possible initial or final state symmetry factors in computing amplitudes.

If $\sin \theta \neq 0$, it is necessary after EWSB to perform a chiral rotation and field redefinition to have a properly defined χ field with a real mass (i.e., to obtain a canonical quadratic

4. Unless explicitly stated, we will consider the DM field χ to be a Dirac fermion and point out differences for the Majorana fermion case.

Lagrangian)⁵

$$\chi \rightarrow \exp(i\gamma_5 \alpha/2) \chi \quad \Rightarrow \quad \bar{\chi} \rightarrow \bar{\chi} \exp(i\gamma_5 \alpha/2). \quad (3.4)$$

Note that a chiral rotation by $\alpha = \pi$ would change the sign of the mass term in Eq. (3.3) and also change the sign of the interaction terms. We can thus without loss of generality take $M_0 > 0$, so long as we preserve the relative signs between the mass term and the interaction terms.⁶

After chiral rotation and field redefinition, we demand that the coefficient of $\bar{\chi}i\gamma_5\chi$ vanishes in order to go to the real mass basis; this determines the proper chiral rotation and gives the mass of the field after EWSB in terms of the Lagrangian parameters (the mass after EWSB, M , is the coefficient of $-\bar{\chi}\chi$ in the rotated field variables). The requisite rotation is

$$\tan \alpha = \left[\frac{\langle v \rangle^2}{2\Lambda} \sin \theta \right] \left[M_0 - \frac{\langle v \rangle^2}{2\Lambda} \cos \theta \right]^{-1}. \quad (3.5)$$

This of course determines $\sin^2 \alpha$ and $\cos^2 \alpha$, but not the (common) sign of $\cos \alpha$ and $\sin \alpha$:

$$\cos^2 \alpha = \frac{\left(M_0 - \frac{\langle v \rangle^2}{2\Lambda} \cos \theta \right)^2}{\left(M_0 - \frac{\langle v \rangle^2}{2\Lambda} \cos \theta \right)^2 + \left(\frac{\langle v \rangle^2}{2\Lambda} \right)^2 \sin^2 \theta} \quad \text{and} \quad (3.6)$$

$$\sin^2 \alpha = \frac{\left(\frac{\langle v \rangle^2}{2\Lambda} \right)^2 \sin^2 \theta}{\left(M_0 - \frac{\langle v \rangle^2}{2\Lambda} \cos \theta \right)^2 + \left(\frac{\langle v \rangle^2}{2\Lambda} \right)^2 \sin^2 \theta}. \quad (3.7)$$

5. Since χ is a SM-singlet, there is no anomalous shift (à la Fujikawa [69, 70]) to the Lagrangian from this redefinition.

6. In our parametrization this sign can be absorbed by a redefinition $\theta \rightarrow \theta + \pi$ leading back to the same form. Thus, by suitable choice of the quadrant in which θ lies, the form Eq. (3.3) is completely general with $M_0 > 0$.

Using this rotation angle, the mass becomes

$$M = \pm \sqrt{\left(M_0 - \frac{\langle v \rangle^2}{2\Lambda} \cos \theta\right)^2 + \left(\frac{\langle v \rangle^2}{2\Lambda}\right)^2 \sin^2 \theta} . \quad (3.8)$$

The signs of M , $\cos \alpha$, and $\sin \alpha$ are common; we choose⁷ the common sign to be ‘+’ for M , so $\cos \alpha = +\sqrt{\cos^2 \alpha}$, and $\sin \alpha = +\sqrt{\sin^2 \alpha}$. With this choice the Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\chi} i \not{\partial} \chi - \bar{\chi} M \chi + \Lambda^{-1} \left(\langle v \rangle h + \frac{1}{2} h^2 \right) \left[\cos \xi \bar{\chi} \chi + \sin \xi \bar{\chi} i \gamma_5 \chi \right], \quad (3.9)$$

where we have defined $\xi = \theta + \alpha$:

$$\cos \xi = \frac{M_0}{M} \left[\cos \theta - \frac{\langle v \rangle^2}{2\Lambda M_0} \right] \quad \text{and} \quad \sin \xi = \frac{M_0}{M} \sin \theta . \quad (3.10)$$

For a fixed value of Λ , we note that the mapping between (M_0, θ) and (M, ξ) is, given our sign conventions, bijective. However, as will be explained more fully below, our analysis scans over (M, ξ) and fixes Λ by requiring the correct DM relic density. In this way, $\Lambda = \Lambda(M, \xi)$, and the mapping back to (M_0, θ) from (M, ξ) with $\Lambda = \Lambda(M, \xi)$ may not be one-to-one (injective) in some regions of parameter space. Put another way, if one scans over (M_0, θ) and asks for the value of Λ required to give the correct relic density, there are regions of parameter space where two or more solutions may be possible, corresponding necessarily to physically distinct scenarios (different values of Λ , M and ξ) in the Lagrangian relevant below the electroweak phase transition. As we are never interested in the regime where we must work with (M_0, θ) [see below], this subtlety does not enter our work further (although, see Appendix B.2), but it should be borne in mind in when relating parameters of some UV completion to our results; of course, if Λ is fixed *a priori*, then this concern is not applicable.

Comparing Eqs. (3.9) and (3.1), it appears that the entire preceding discussion about

7. If we had chosen the opposite signs for M , $\cos \alpha$, and $\sin \alpha$, we could perform a further chiral rotation by $\alpha = \pi$ and field definition to recover the sign conventions in Eq. (3.9).

chiral rotations to have a proper mass term could have been avoided by just substituting⁸ $H^\dagger H \rightarrow H^\dagger H - \frac{1}{2}\langle v \rangle^2 = \langle v \rangle h + \frac{1}{2}h^2$ in Eq. (3.1). In the spirit of effective field theories, as we do not know the origin of the mass M_0 in the UV theory, one would naïvely expect that we should not care whether or not M in Eq. (3.9) has a contribution from EWSB. However, we have learned something important because, due to the pseudoscalar interaction term, making the substitution $H^\dagger H \rightarrow H^\dagger H - \frac{1}{2}\langle v \rangle^2$ in Eq. (3.1)—thereby avoiding the above discussion—is equivalent to requiring a carefully chosen phase⁹ of the χ mass term in the effective theory above the EWSB scale, which in turn would require some conspiracy in the UV-complete theory to arrange. The opposite side of the same coin is that if we do work with the form of the Lagrangian at Eq. (3.1), it is unnatural to have a pure pseudoscalar coupling after EWSB¹⁰ ($\cos \xi = 0$) because this requires $\Lambda M_0 \cos \theta = \langle v \rangle^2/2$, which is an ill-motivated coincidental relationship between parameters in the effective high-energy theory (and thereby, its UV completion) and the electroweak vacuum expectation value.¹¹

Although we perform a general parameter scan, there are a few limiting cases that are interesting to consider:

1. $\sin \theta = 0$, $\cos \theta = \pm 1$: This would be a pure scalar interaction before EWSB. After EWSB, the interaction term is $\pm \Lambda^{-1} \bar{\chi} \chi (\langle v \rangle h + h^2/2)$ and the mass is $M = |M_0 \mp \langle v \rangle^2/2\Lambda|$. Thus, a pure scalar interaction before EWSB will remain a pure scalar interaction with no admixture of pseudoscalar interactions, because this scenario is protected by the discrete CP symmetry of the Lagrangian. However, note that the mass M is in general different from M_0 .

8. This substitution preserves manifest $SU(2)_W \times U(1)_Y$ gauge invariance.

9. The presence of both normal ($\propto \bar{\chi} \chi$) and axial ($\propto \bar{\chi} i \gamma^5 \chi$) mass terms is equivalent a complex mass term ($\mathcal{L} \supset -M' \bar{\chi}_L \chi_R + \text{h.c.}$) with a non-zero phase for M' .

10. Note that it is already clear at the level of the original Lagrangian that a vanishing scalar coupling is a not naturally stabilized situation as it is not protected by any symmetry (cf. the case of vanishing pseudoscalar coupling, which is protected by the overall CP-symmetry of the Lagrangian). What we have really learned additionally is that EWSB itself causes changes to the pure-pseudoscalar nature of the original coupling, already at tree-level.

11. We would like to thank the authors of Ref. [510] for useful discussions on this point and for sharing an early version of their work with us.

2. $\cos \theta = 0$, $\sin \theta = \pm 1$: This would be a pure pseudoscalar interaction before EWSB.

After EWSB the interaction term is

$$\Lambda^{-1} \left[-\frac{\langle v \rangle^2}{2\Lambda M} \bar{\chi}\chi \pm \left[1 - \left(\frac{\langle v \rangle^2}{2\Lambda M} \right)^2 \right]^{\frac{1}{2}} \bar{\chi} i \gamma_5 \chi \right] (\langle v \rangle h + h^2/2),$$

and in both cases $M = \sqrt{M_0^2 + (\langle v \rangle^2/2\Lambda)^2}$. Even if the Higgs Portal coupling is purely pseudoscalar in the EW-symmetric Lagrangian, after EWSB a scalar term proportional to $\langle v \rangle^2/2\Lambda M$ is generated; note that this is one order higher in the EFT power counting.

3. $M_0 = 0$ (or more generally, $M_0 \ll \langle v \rangle^2/2\Lambda$): In this case $M = \langle v \rangle^2/2\Lambda$. If $M_0 = 0$, then $\cos \xi = -1$ and $\sin \xi = 0$, and the interaction term is purely scalar: $\mathcal{L} \supset -\Lambda^{-1} (vh + h^2/2) \bar{\chi}\chi$. The chiral rotation that results in a real mass term transforms the interaction into a purely scalar interaction irrespective of the value of θ . The only two parameters in this limit are M and Λ ; one of the parameters may be set by the requirement that freeze-out results in the correct relic abundance.

Whether scalar, pseudoscalar, or a combination of both, the nature of the interactions is of great importance: annihilation through a pure scalar interaction ($\sin \xi = 0$) is velocity suppressed, while elastic scattering of WIMPs with nucleons through a pure pseudoscalar interaction ($\cos \xi = 0$) is momentum-transfer suppressed.¹² If both interactions are present, then the (non-momentum-transfer-suppressed) interaction most important for direct detection (scalar) may not be the same as the (non-velocity-suppressed) interaction most important for determining the relic abundance (pseudoscalar), or for any potential present-day indirect detection signal (pseudoscalar; see Chapter 2 and Refs. [5, 431]).

We note finally that the form of the Lagrangian in terms of the chirally rotated field variables is only appropriate to use ‘below’ the electroweak phase transition. We restrict ourselves to considering DM lighter than 3 TeV where the direct detection constraints from

12. For elastic scattering, this leads to velocity suppression.

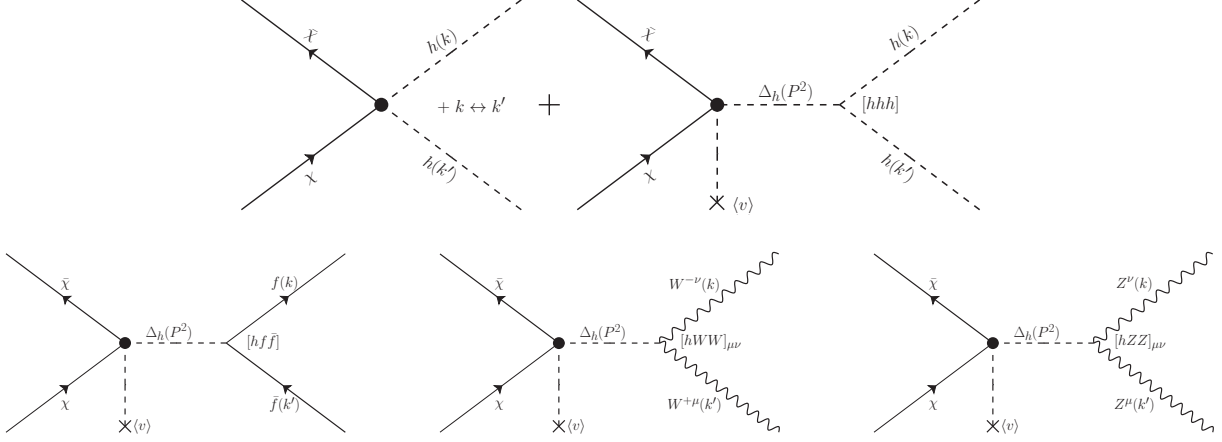


Figure 3.2: Lowest-order $[\mathcal{O}(\Lambda^{-1})]$ DM-annihilation diagrams resulting from the effective dimension-5 Higgs Portal coupling to DM.

LUX [506] which we consider are explicitly available, so this condition is always satisfied since such DM decouples at $T \lesssim \mathcal{O}(200)$ GeV (the freeze-out temperature $T_F \sim M/x_F$ with $x_F \sim 20 - 25$ [120]). ‘Above’ the phase transition, the unrotated form should be used in the freeze-out computation, while the rotated form would be relevant to compute all present-day low-energy observables: we do not explore this regime further in this chapter.

3.3 The Annihilation Cross Section

We now turn to the computation of the DM annihilation cross section relevant for finding the relic abundance. It is straightforward to calculate the tree-level cross section for the hh , WW , ZZ and $f\bar{f}$ on-shell 2-body final states. In diagrammatic form, the lowest order processes are illustrated in Figure 3.2. The vertex functions of Figure 3.2 are [31]

$$\begin{aligned}
 [hhh] &= -3i \frac{m_h^2}{\langle v \rangle}, & [h\bar{f}f] &= -i \frac{m_f}{\langle v \rangle}, \\
 [hWW]_{\mu\nu} &= 2i \frac{m_W^2}{\langle v \rangle} g_{\mu\nu}, & [hZZ]_{\mu\nu} &= 2i \frac{m_Z^2}{\langle v \rangle} g_{\mu\nu}.
 \end{aligned} \tag{3.11}$$

The cross section for final state f may be expressed as [431]

$$\sigma_f(s; M, m') = \frac{1}{32\pi M^2} \sqrt{\frac{4M^2}{s}} \sqrt{\frac{M^2}{s - 4M^2}} \sqrt{1 - \frac{4m'^2}{s}} \Sigma_f(s; M, m'), \quad (3.12)$$

where (see also Ref. [487])

$$\begin{aligned} \Sigma_f(s; M, m') &\equiv \frac{1}{4} \sum_{\text{spins}} \int \frac{d\Omega}{4\pi} |\mathcal{M}_f|^2 \\ &= \frac{1}{4} \frac{s}{\Lambda^2} \frac{\cos^2 \xi (1 - 4M^2/s) + \sin^2 \xi}{\left(1 - m_h^2/s\right)^2 + \left(m_h \Gamma_h/s\right)^2} \times \begin{cases} \left(1 - 4m_Z^2/s + 12m_Z^4/s^2\right) & ZZ \\ 2 \left(1 - 4m_W^2/s + 12m_W^4/s^2\right) & W^+W^- \\ \left(1 - 4m_f^2/s\right) \left(4m_f^2/s\right) & f\bar{f} \\ \left[\left(1 + 2m_h^2/s\right)^2 + \left(m_h \Gamma_h/s\right)^2\right] & hh. \end{cases} \end{aligned} \quad (3.13)$$

In Eq. (3.13), Γ_h is the total width of the Higgs (including the partial width for $h \rightarrow \bar{\chi}\chi$ when $M < m_h/2$) and the factors of m_f in the expression for $f\bar{f}$ are the running masses¹³ at the scale $q^2 = s$. Note that there are no interference terms between the CP-even and CP-odd contributions to the cross sections here; note also that we explicitly ignore the possible three- and four-body final states mediated by one or two off-shell W, Z , and/or h bosons, which would be important for a high-precision computation just below the thresholds for on-shell WW , ZZ , and/or hh final states (see, e.g., Ref. [511] for the same point in the context of the SM Higgs branching-ratio computations).

Away from resonances and thresholds [462], the non-relativistic (NR) cross section closely matches the thermally averaged cross section relevant for the early-universe freeze-out calculation; this limit is obtained by the substitution $s \rightarrow 4M^2$ unless s appears in the combination $s - 4M^2$, in which case one substitutes $s - 4M^2 \rightarrow v^2 M^2$, where in the NR limit v is the

13. To be explicit, we utilize the three-loop running masses from Ref. [511] and references therein (e.g., Refs. [512, 513]).

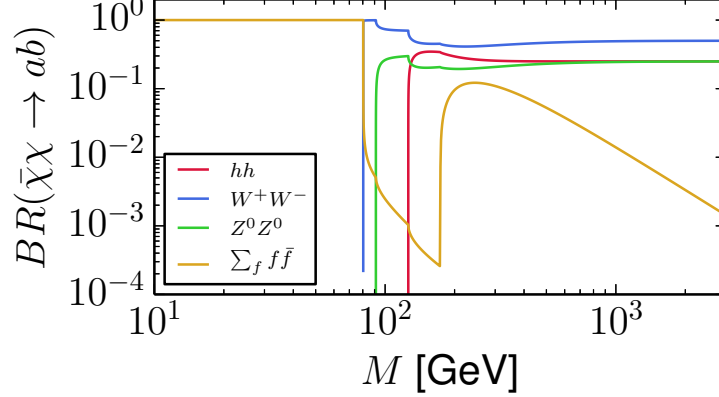


Figure 3.3: The branching fractions to the various two-body annihilation final states in the NR limit ($s \rightarrow 4M^2$, $s - 4M^2 \rightarrow M^2 v^2$) as a function of M .

Møller velocity that appears in the Boltzmann equation for the early-universe evolution of the DM density [120, 514] (see Section 3.5). With these substitutions, one can see from Eq. (3.13) that the term proportional to $\cos^2 \xi$ is proportional to v^2 , as expected from scalar interactions.

Although the annihilation cross sections for the various channels depend on Λ and ξ , the branching fractions only depend on s (equal to $4M^2$ in the NR limit);¹⁴ these are shown as a function of M in the NR limit in Figure 3.3. Above the W^+W^- threshold, the largest branching fraction is to W^+W^- , with the branching fractions to ZZ and hh —where kinematically allowed—smaller by a factor of a few.¹⁵ Below the W^+W^- threshold the only annihilation channel is to fermion pairs, predominately to the largest mass kinematically allowed because the annihilation to $f\bar{f}$ is through an s -channel Higgs and so the cross section is proportional to $y_f^2 = m_f^2/\langle v \rangle^2$.

Note that the cross sections depend only on $\cos^2 \xi$ and $\sin^2 \xi$; this will no longer be true

14. This is an exact statement only in the NR limit. The Γ_h -dependent term in the numerator of Σ_{hh} in Eq. (3.13) does not cancel in the ratio when we compute the BR, and generally depends on both Λ and ξ . However, the hh channel is only open in the NR limit for $M \geq m_h$, where the width is independent of any exotic contribution. More generally, there is a dependence on Λ, ξ in the BR for $M < m_h/2$ (so that there is an exotic width contribution), but only when the cross sections are considered at $s > 4m_h^2$ (so that the hh final state is kinematically allowed), which suppresses this dependence by at least $(\Gamma_h/m_h)^2 \ll 1$.

15. In the limit of large M , the ratios are $W^+W^- : ZZ : hh = 2 : 1 : 1$, as one would expect by the Goldstone Boson Equivalence Theorem.

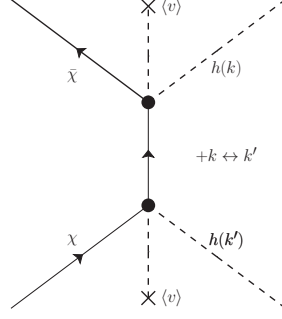


Figure 3.4: Next-order diagrams $[\mathcal{O}(\Lambda^{-2})]$ for the process $\bar{\chi}\chi \rightarrow hh$ resulting from the Higgs Portal operator $H^\dagger H$.

at higher order in Λ^{-1} . For instance, at higher order in Λ^{-1} there are the additional tree-level t - and u -channel diagrams for the hh final state, illustrated in Figure 3.4. If we include these diagrams, $\Sigma_{hh}(s; M, m_h)$ becomes significantly more complicated:¹⁶

$$\begin{aligned}
& \Sigma_{hh}(s; M, m_h) \\
&= \frac{1}{4} \frac{s}{\Lambda^2} \frac{\left(1 + 2m_h^2/s\right)^2 + \left(\Gamma_h m_h/s\right)^2}{\left(1 - m_h^2/s\right)^2 + \left(\Gamma_h m_h/s\right)^2} \left[\cos^2 \xi \left(1 - \frac{4M^2}{s}\right) + \sin^2 \xi \right] \\
&+ \frac{2M\langle v \rangle^2 \cos \xi}{\Lambda^3} \frac{\left(1 - m_h^2/s\right) \left(1 + 2m_h^2/s\right) + \left(\Gamma_h m_h/s\right)^2}{\left(1 - m_h^2/s\right)^2 + \left(\Gamma_h m_h/s\right)^2} \\
&\quad \times \left[1 + \frac{1}{\beta} \left(1 - \frac{8M^2}{s} \cos^2 \xi + \frac{2m_h^2}{s}\right) \tanh^{-1} \left(\frac{\beta}{1 - 2m_h^2/s} \right) \right] \\
&- \frac{\langle v \rangle^4}{2\Lambda^4} \left[\frac{M^2}{s} \left(1 - \frac{4m_h^2}{s}\right) + \frac{m_h^4}{s^2} \right]^{-1} \left[\frac{16M^4}{s^2} \cos^4 \xi + \frac{2M^2}{s} \left(1 - \frac{4m_h^2}{s} (1 + \cos^2 \xi)\right) + \frac{3m_h^4}{s^2} \right] \\
&+ \frac{\langle v \rangle^4}{\Lambda^4} \beta^{-1} \left(1 - \frac{2m_h^2}{s}\right)^{-1} \left[1 - \frac{4m_h^2}{s} + \frac{6m_h^4}{s^2} + \frac{16M^2}{s} \left(1 - \frac{m_h^2}{s}\right) \cos^2 \xi - \frac{32M^4}{s^2} \cos^4 \xi \right] \\
&\quad \times \tanh^{-1} \left(\frac{\beta}{1 - 2m_h^2/s} \right) \tag{3.14}
\end{aligned}$$

16. This result was derived with the aid of FEYNALC [515] and verified numerically at some sample parameter values using the FEYNRULES [516] and CALCHEP [517] pipeline.

where we have defined

$$\beta(s; M, m_h) \equiv \sqrt{\left(1 - 4M^2/s\right) \left(1 - 4m_h^2/s\right)}. \quad (3.15)$$

Note that the (contact + s) – (t, u) interference term in Eq. (3.14) is proportional to $\cos \xi$, while all other terms with ξ dependence are proportional to $\cos^2 \xi$ or $\sin^2 \xi$.

This is however just one example of how effects that are higher order in the EFT suppression scale Λ can arise. Within the context of any UV completion, the low-energy EFT will contain a tower of operators beginning at Λ^{-1} , with other operators in the tower suppressed by higher powers of Λ^{-1} (dimensionally compensated as appropriate by derivatives or additional fields). Some of these operators will of course be more important than others for a particular application but, generically, their presence implies that a result [such as Eq. (3.14)] computed beyond leading order in Λ^{-1} , but using only the couplings arising from the lowest-order effective operator, is not necessarily complete to that order in Λ^{-1} , but is merely indicative.

Absent motivation to the contrary then,¹⁷ we should really only keep the lowest order term, using the known (if incomplete) higher order terms as a way to gauge whether we trust the EFT in any given region of parameter space: if the numerical coefficients for the higher order terms are ‘too large’ we should be wary of trusting the lowest-order approximation and must be alert to the possibility that neglected contributions may actually be important.

In this spirit, at fixed (M, ξ) , we will mostly work to lowest order in Λ^{-1} , but occasionally we will present results using all the terms in Eq. (3.14) to illustrate the potential magnitude of terms higher order in Λ^{-1} .

17. A good example of where some higher-order-in- Λ^{-1} effects can be much more important than expected and thus should be retained, is furnished by the chiral rotation discussion in Section 3.2: for initially pure pseudoscalar coupling, the coefficient of the scalar coupling which is generated upon rotation is one order higher in Λ^{-1} than the leading-order coefficient of the pseudoscalar coupling. However, simply neglecting this effect can be a grave error when considering direct detection, where velocity suppression applies to the pseudoscalar coupling only. In order to neglect the higher-order-in- Λ^{-1} coefficient of the scalar coupling, it would have to be much smaller than $v^2 \sim 10^{-6}$. For the parameter space we consider though, this never occurs.

3.4 The Width of the Higgs

Before we compute the values of the EFT suppression scale Λ required to obtain the correct relic abundance (see Section 3.5) using the annihilation cross sections shown in Section 3.3, we must discuss the Higgs width.

For $M < m_h/2$, the width of the Higgs will differ significantly from the SM value because the process $h \rightarrow \chi\bar{\chi}$ is kinematically allowed. The presence of this exotic or ‘invisible’ contribution to the Higgs width implies a nontrivial constraint on light DM (see, e.g., Refs. [490, 493, 501, 505, 507, 518]). A simple tree-level computation of the partial decay width of the Higgs to a $\chi\bar{\chi}$ pair yields the result

$$\begin{aligned}\Gamma_{h \rightarrow \chi\bar{\chi}} &= \frac{m_h}{8\pi} \frac{\langle v \rangle^2}{\Lambda^2} \sqrt{1 - \frac{4M^2}{m_h^2}} \left[1 - \frac{4M^2}{m_h^2} \cos^2 \xi \right] \\ &= \left(3.034 \times 10^2 \text{ MeV} \right) \times \left(\frac{1 \text{ TeV}}{\Lambda} \right)^2 \sqrt{1 - \frac{4M^2}{m_h^2}} \left[1 - \frac{4M^2}{m_h^2} \cos^2 \xi \right],\end{aligned}\quad (3.16)$$

where we took $m_h = 126 \text{ GeV}$.

A recent CMS result [508] utilizes the comparison, first proposed in Ref. [519], of two different kinematic regions for the processes $pp \rightarrow ZZ \rightarrow 4l, 2l2\nu$ to limit the total Higgs width. As discussed at length in Ref. [519], two leading-order diagrams can contribute to the $pp \rightarrow ZZ$ process (one diagram proceeds by s -channel Higgs mediation; the other is a box diagram), and if one chooses the two different kinematic regions such that in one region the s -channel Higgs can be on-shell, while in the other region the s -channel Higgs is always far off-shell, a strong constraint on the Higgs width can be derived by comparing the number of events in each region [508, 519]. Indeed, Ref. [508] reports a limit of $\Gamma_{h, \text{total}}^{95\% \text{ CL UL}} \leq 17.4 \text{ MeV}$ at 95% confidence, which is only 4.2 times the expected SM width [508, 520].¹⁸

18. The constraint given here, $\Gamma_{h, \text{total}}^{95\% \text{ CL UL}} \leq 17.4 \text{ MeV}$ at 95% confidence, was taken from the CMS Physics Analysis Summary [508]; the published version of this analysis, Ref. [521], which appeared after Ref. [5] (the paper on which this chapter is based) was released as a preprint and as the latter was being finalized for submission for publication, found a slightly weaker limit: $\Gamma_{h, \text{total}}^{95\% \text{ CL UL}} \leq 22 \text{ MeV}$ at 95% confidence [521]. However this c. 25% change does not significantly modify the DM mass at which this limit becomes constraining owing to the strong dependence of Γ_h on M ; i.e., the fiducial value of the width in Eq. (3.16) is two

One can actually do better: the invisible branching fraction for Higgs induced by the $h \rightarrow \chi\bar{\chi}$ decay is defined by

$$\mathcal{B}_{\text{inv}} = \frac{\Gamma_{h \rightarrow \chi\bar{\chi}}}{\Gamma_{\text{SM}} + \Gamma_{h \rightarrow \chi\bar{\chi}}}, \quad (3.17)$$

where the theoretical value of the total width of a 126 GeV Higgs boson is $\Gamma_{\text{SM}} = 4.21 \text{ MeV}$ [520]. This is an extremely small value compared to the fiducial partial width to $\chi\bar{\chi}$ shown in Eq. (3.16), which implies that even for fairly modest limits on $\mathcal{B}_{\text{inv}}$ the resulting constraints will be very strong in the kinematically allowed region.

The best limits (c. 2014) on the invisible branching ratio come from a global fit to Higgs data and are $\mathcal{B}_{\text{inv}} < 0.19$ (0.38) [507] for the case where the Higgs couplings are fixed to their theoretical SM values (allowed to float freely in a global fit); these results were computed in Ref. [507] using $m_h = 125.5 \text{ GeV}$, but should be very similar for our choice of $m_h = 126 \text{ GeV}$. Although we do not utilize them further in our analysis, we also note that searches for invisible Higgs decay in the ZH associated production mode at ATLAS [522] and in the ZH associated production and vector boson fusion (VBF) modes at CMS [523] yield 95% confidence level upper limits on $\mathcal{B}_{\text{inv}}$ of 0.75 and 0.58, respectively, assuming in both cases SM production cross sections and Higgs masses of 125.5 GeV and 125 GeV, respectively.

The invisible width is halved for the Majorana case: while the conventional factor of 1/2 in the Lagrangian ensures the same amplitude for decay as for the Dirac case, the Majorana fermions are now both in the final state necessitating an additional factor of 1/2 to avoid double-counting the phase space.

Assuming the values of Λ required for the correct relic abundance (see Section 3.5), the resulting invisible-width limits from the global fit analysis of Ref. [507] on the allowed values

orders of magnitude larger than the SM Higgs width, so as the kinematic suppression factor $\sqrt{1 - 4M^2/m_h^2}$ moves away from zero with decreasing DM mass M , the total width rapidly increases to exceed the width limit, and the DM mass at which this occurs is somewhat insensitive to the exact numerical value of that width limit. We have therefore not updated our results to reflect the slightly more conservative Higgs width limit from Ref. [521].

of M are approximately independent of the value of ξ and are $M \gtrsim 56.8$ (56.2) GeV for the Dirac case and $M \gtrsim 55.3$ (54.6) GeV for the Majorana case for fixed (floating) couplings. The constraints from the CMS limit on the total width [508, 519] are only slightly weaker and also approximately independent of ξ , limiting the DM mass to be $M \gtrsim 55.7$ GeV for the Dirac case and $M \gtrsim 53.8$ GeV for the Majorana case.

3.5 The Relic Abundance

Armed with the annihilation cross section and Higgs width expressions, we can now make progress computing the values of the EFT suppression scale Λ required to saturate the observed DM relic abundance.

The familiar Boltzmann equation [120, 514, 524] for a single species of DM particle of number density¹⁹ n whose equilibrium abundance is n_{EQ} (see Appendix B.1 for a collection of relevant standard thermodynamics results), undergoing only annihilations with itself or its anti-particle is written as:²⁰

$$\dot{n} + 3Hn = -\langle \sigma v_{\text{Møller}} \rangle [n^2 - n_{\text{EQ}}^2], \quad (3.18)$$

where $\langle \sigma v_{\text{Møller}} \rangle$ is the thermal average of $\sigma v_{\text{Møller}}(s)$, given by [524]

$$\langle \sigma v_{\text{Møller}} \rangle = \left[8M^4 T K_2^2(M/T) \right]^{-1} \int_{4M^2}^{\infty} \sigma(s) (s - 4M^2) \sqrt{s} K_1(\sqrt{s}/T) ds, \quad (3.19)$$

and $K_{1,2}$ are modified Bessel functions (this expression assumes Boltzmann statistics for the

19. Note that n here is the particle number density only: $n \equiv n_\chi$. If χ is not self-conjugate, the total number density is $n_{\text{total}} = n_\chi + n_{\bar{\chi}} = 2n$ (we assume that non-self-conjugate DM is symmetric: $n_\chi = n_{\bar{\chi}}$). See Appendix A.1.

20. The same form obtains for both particle-particle (e.g., Majorana fermion), and particle-anti-particle (e.g., Dirac fermion) annihilations because while there is a factor of 2 on the RHS to account for the loss of two particles per annihilation in the former case, it cancels a factor of 1/2 to avoid double-counting the *initial* state phase space—that is, the factor of 1/2 which accounts for the combinatoric factor of $N(N-1)/2 \approx N^2/2$ possible pairs of interactions given N total particles undergoing annihilation among themselves [514]; see also Appendix A.1.

DM at freeze-out).²¹ With the usual definition $Y \equiv n/s$ (s is the entropy density), and Y_∞ denoting the post-freeze-out value of Y , the present ratio of the WIMP mass-density to the present critical density $\rho_c = 3H_0^2/8\pi G_N$ is [120]

$$\Omega = N_{\text{DM}} \frac{Ms_0}{\rho_c} Y_\infty, \quad (3.20)$$

where the present value of the entropy density is $s_0 = 2891 \text{ cm}^{-3}$ [38] and $N_{\text{DM}} = 1$ (2) for (non-)self-conjugate DM.²² Observationally, the DM relic abundance is determined to be $\Omega h^2 = 0.1186(31)$ [85], where $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. See Appendix B.1 for comments on the numerical solution of Eq. (3.18) following Refs. [120, 524].

There are three parameters in the EFT: Λ , M , and ξ . If we consider only the lowest non-vanishing order in Λ^{-1} , the cross section depends only upon $\cos^2 \xi$. The values of Λ required to give $\Omega h^2 = 0.1186$ [85] are shown as a function of M and $\cos^2 \xi$ in Figure 3.5. In agreement with previous literature (see, e.g., Ref. [434]), we find that for $M < m_h/2$ if there is at least one allowed value of Λ giving the correct relic density, then there are typically two such allowed values²³ for Λ since the annihilation cross sections scale parametrically as $1/\sigma \sim \Lambda^2 [A + B\Lambda^{-4}] \sim [A\Lambda^2 + B\Lambda^{-2}]$ for some A, B , due to the presence of the exotic contribution to the Higgs width which enters in the denominator of the s -channel resonance peak in Eq. (3.13). However, one solution for Λ is typically a factor of a few or more smaller than the other, and we always take the larger value of Λ . This is done for two reasons: (a)

21. We note that for the large M/T regime, the RHS of Eq. (3.19) can be numerically problematic to evaluate as it involves the ratio of two exponentially small numbers; a large-argument asymptotic-series expansion of the Bessel functions is necessary to avoid this issue:

$$K_n(z) \xrightarrow{|z| \rightarrow \infty} \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{r=0}^{\infty} \frac{(2z)^{-r}}{r!} \frac{\Gamma(n+r+\frac{1}{2})}{\Gamma(n-r+\frac{1}{2})}$$

for $-\pi/2 < \arg z < \pi/2$, $n \in \mathbb{Z}$ and $n \geq 0$. See, e.g., Eq. (11.126) in Ref. [525].

22. This is the only place where the difference between Majorana and Dirac fermions enters in this relic density computation [514].

23. Note that this is an independent concern from that mentioned above in the context of discussing the mapping between (M_0, θ) and (M, ξ) .

the smaller value of Λ can run far below $\langle v \rangle$, which is a region where we do not particularly trust the lowest-order EFT results due to (i) possible large corrections proportional to $(\langle v \rangle/\Lambda)^n$ from neglected higher order operators (see discussion below Eq. (3.14)—although the corrections from Eq. (3.14) are not themselves relevant in this mass range), (ii) possible perturbative unitarity issues, and, relatedly, (iii) possible issues with having implicitly integrated out physics at or below the weak scale (although, see footnote 3), while keeping other weak-scale physics in the theory explicitly; and (b) the largest value of Λ implies the loosest constraints on any given operator from present-day experimental data and this gives the most conservative approach to setting exclusion bounds.

There are also regions where, for a given M and $\cos^2 \xi$, there is no value of Λ that will give the correct relic abundance (the cross section has an upper bound as a function of Λ with all other parameters held fixed). This is illustrated by the doubly hatched regions of Figure 3.5. The singly hatched regions denote where the values of Λ to give the correct Ωh^2 are smaller than $\langle v \rangle$; for the variety of reasons already advanced above, we expect that our lowest-order EFT results are not entirely trustworthy in this region, and we thus conservatively choose not to present results here.

In Figure 3.5 one clearly sees the region where resonant annihilation occurs around $M \simeq m_h/2$. In this region a very large value of $\Lambda \gtrsim 10 \text{ TeV}$ is required. This is the ‘resonant Higgs Portal’ scenario of Ref. [434] (see also Ref. [526]).

So far we have ignored possible terms higher order in Λ^{-1} in presenting our results. As we have seen from one of the possible contributions considered in Eq. (3.14), higher-order terms may depend on $\cos \xi$ and not simply $\cos^2 \xi$. An illustration of the possible magnitude of these terms, we have calculated the values of Λ necessary to arrive at the correct relic density for various values of ξ and M , including only those higher-order terms shown in Eq. (3.14). The result is illustrated in Figure 3.6; the effect of higher-order terms is small, and we will henceforth ignore them.

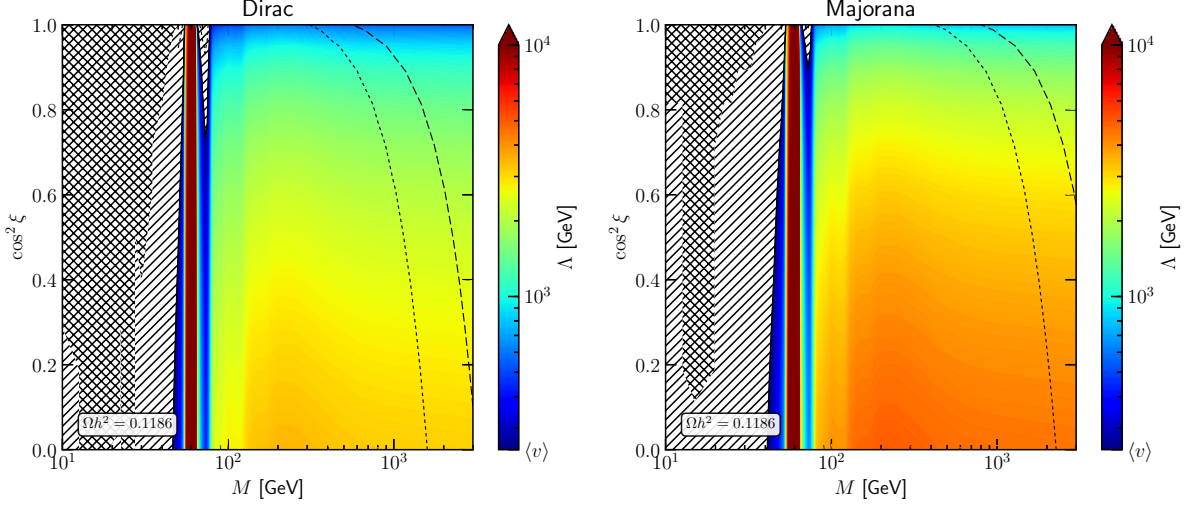


Figure 3.5: Colourmaps giving the (interpolated) values of the EFT suppression scale Λ required for the correct relic abundance, $\Omega h^2 = 0.1186$, for various values of $(\cos^2 \xi, M)$. Apart from a region near $M \sim m_h/2$ where the presence of the s -channel resonance [434] forces the value of Λ to exceed 10 TeV, and the troughs on either side of this resonance, the values of Λ are generically between a few hundred GeV and a few TeV, with smaller values required for more scalar cases ($\cos^2 \xi \rightarrow 1$) due to the mild velocity suppression ($v^2 \sim 0.3$ at freeze-out [120]) of the pure-scalar annihilation channel cross section. The dotted and dashed lines indicate, respectively, where $\Lambda \lesssim 2M$ and $\Lambda \lesssim M$ (for reference, the simplest perturbatively unitary UV completion requires the scale of new physics to be $\geq M/2\pi$ [509]; while Λ in our parametrization is not necessarily exactly this scale, it is of the same order of magnitude; see footnote 3). Note that there may be regions where Λ is too small for the EFT to be taken seriously: in the singly hatched region, $\Lambda \lesssim \langle v \rangle$ (see discussion in the text). The doubly hatched region is where no Λ can be found which gives the correct relic density for the chosen M and $\cos^2 \xi$; the boundaries of this region as shown are not entirely smooth due to sampling effects on the computation grid and should thus be taken as indicative only (as they always lie in the regions where $\Lambda < \langle v \rangle$, their validity is in any event open to question).

3.6 Direct Detection

Direct detection constraints are particularly important to consider for these Higgs Portal operators [434, 487, 490, 491, 501, 505]. The relevant process is the Higgs-mediated t -channel elastic scattering of WIMPs on nucleons (bound in target nuclei). Using the interaction Lagrangian of Eq. (3.9) together with the Higgs-quark coupling term from the SM Lagrangian

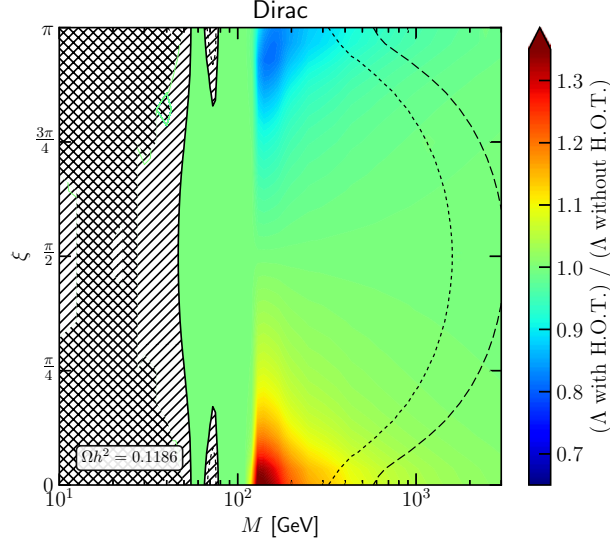


Figure 3.6: The ratio of the values of the EFT suppression scale Λ required for the correct relic abundance for various values of (M, ξ) assuming that $\Omega h^2 = 0.1186$ for the two cases of (1) when the higher-order terms (H.O.T.) in the cross section of Eq. (3.14) are included, and (2) when they are neglected. The differences are fairly small over most of the parameter space, and even in regions where they differ by $\mathcal{O}(30\%)$, we have explicitly checked that their ultimate impact on the exclusion bounds we set (see below) is very small. See the caption of Figure 3.5 for a description of the hatched regions and dotted/dashed lines. As this figure is mainly for illustrative purposes, we present results only for the Dirac case.

yields

$$\mathcal{L} \supset - \sum_q \frac{m_q}{\langle v \rangle} h \bar{q} q + \Lambda^{-1} [\cos \xi \bar{\chi} \chi + \sin \xi \bar{\chi} i \gamma_5 \chi] \langle v \rangle h. \quad (3.21)$$

Since the momentum transfer in the scattering process is typically less than an MeV—very much less than the Higgs mass (126 GeV)—the Higgs can be integrated out to obtain the effective operator connecting DM to quarks:

$$\mathcal{L}_{\text{eff}}^{\text{direct detection}} \supset - \sum_q \frac{1}{m_h^2} \frac{m_q}{\Lambda} \bar{q} q [\cos \xi \bar{\chi} \chi + \sin \xi \bar{\chi} i \gamma_5 \chi], \quad (3.22)$$

where the factor of m_h^{-2} comes from integrating out the h propagator. The sum runs over all quarks, with the heavier quarks entering the direct detection process through triangle

diagrams which induce effective couplings of the h to the gluons in the nucleon; this is usually accounted for by writing an effective matrix element for the heavy quarks in the nucleon (see, e.g., Ref. [527] and references therein, or Ref. [510]).

Following the well-known procedure to extract the single-nucleon matrix elements (see, e.g., Appendix B of Ref. [527], or Ref. [510]), the spin-independent (SI) direct detection cross section, normalized on a per-nucleon basis, can be written as²⁴ (see Appendix B.3 for a clarification of this result)

$$\sigma_{\text{SI}}^{\chi N} = \frac{1}{\pi} \left(\frac{\mu_{\chi N}}{m_h^2} \right)^2 \left(\frac{f_N}{\Lambda} \right)^2 \left[\cos^2 \xi + \frac{1}{2} \left(\frac{\mu_{\chi T}}{M} \right)^2 \nu_\chi^2 \right] \quad (3.23)$$

$$= 4.7 \times 10^{-38} \text{ cm}^2 \left(\frac{M}{\Lambda} \right)^2 \left(\frac{1 \text{ GeV}}{0.94 \text{ GeV} + M} \right)^2 \left[\cos^2 \xi + \frac{1}{2} \left(\frac{\mu_{\chi T}}{M} \right)^2 \nu_\chi^2 \right], \quad (3.24)$$

where $f_N \equiv M_N \left(\sum_{q=u,d,s} f_{Tq}^{(N)} + \frac{2}{9} f_{TG}^{(N)} \right) \approx 0.35 M_N \approx 0.33 \text{ GeV}$ [527, 528] is the nuclear matrix element accounting for the quark (and gluon, through heavy quark triangle diagrams) content of the nucleon to which the Higgs couples, $\mu_{\chi T} = MM_T/(M_T + M)$ is the reduced mass of the WIMP–target-nucleus system, $M_T \approx AM_N$ is the nuclear target mass, M_N is the nucleon mass, and ν_χ is the DM speed in the nuclear-target rest-frame. We will make the simplifying approximation that the DM speed is $\nu_\chi \sim 220 \text{ km/s}$ in the nuclear target rest-frame (i.e., the Earth rest-frame). Note that the WIMP–target-nucleus reduced mass is appropriate here in the $\mu_{\chi T}/M$ factor, as the scattering in direct detection is off quasi-stationary nuclei, not quasi-stationary individual nucleons; Appendix B.3 contains further details and discussion.²⁵ Ref. [497] contains a similar result, but note the appearance of $\mu_{\chi T}$ here, rather than $\mu_{\chi N}$ indicated in that reference.

We compare this cross section to the 2013 LUX upper limits [506] on the spin-independent

24. We have ignored corrections of order ν_χ^2 to the term proportional to $\cos^2 \xi$, as they are always subdominant by many orders of magnitude.

25. Note that in Eqs. (6.3)–(6.5) of Ref. [5] we erroneously wrote $(\mu_{\chi N}/M)^2$ in place of the factor of $(\mu_{\chi T}/M)^2$ which appears here in our Eqs. (3.23) and (3.24). However, in light of the discussion related to our treatment of the pseudoscalar coupling in Section 3.6.1, this minor error in Eqs. (6.3)–(6.5) of Ref. [5] has no impact on the parameter space limit obtained.

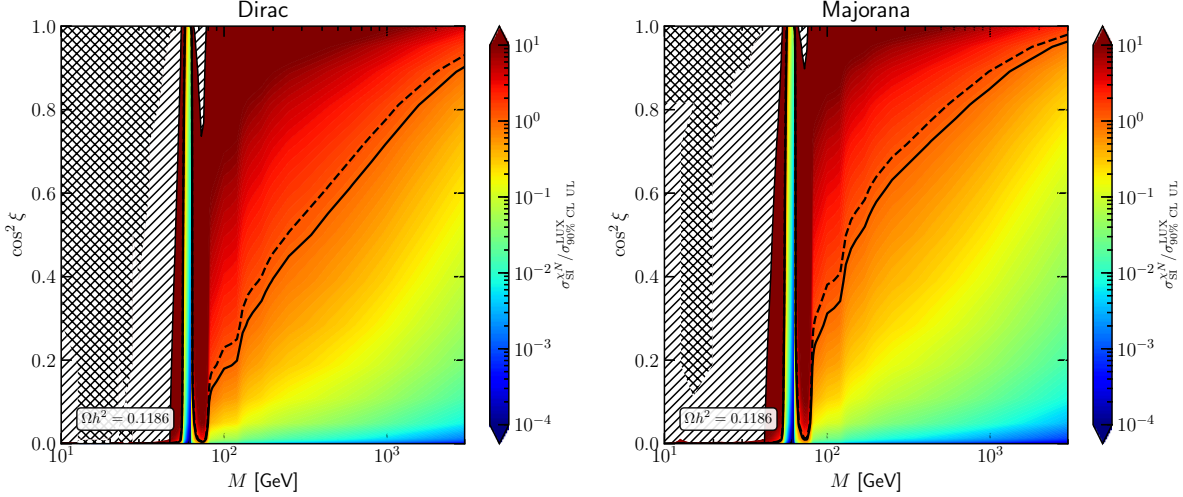


Figure 3.7: Colourmaps representing interpolated values of $\sigma_{\text{SI}}^{\chi^N} / \sigma_{90\% \text{ CL UL}}^{\text{LUX}}$, with the solid black line showing the equality of the computed cross section and the (2013) LUX limit from Ref. [506]. Redder points ‘above’ the black line are excluded, bluer points ‘below’ the black line are allowed. For reference, the dashed black line is the cognate of the solid black line, except for the 95% CL UL from LUX taken from DMTools [529] (note that 95% CL UL from LUX [506] are only available up to 2 TeV from DMTools [529]; we have extrapolated the limit linearly up to 3 TeV—this is justified since the limit $\sim 1/n_{\text{DM}} \sim M$ and since, in the data, the limit is already scaling approximately linearly in this region). Note that the mass region near $M \approx m_h/2$ is allowed for any value of ξ : this is the resonant Higgs Portal scenario [434]. The singly hatched region is where $\Lambda < \langle v \rangle$; the doubly hatched region at low mass is where no Λ value can be found to obtain the correct relic density. See also Appendix B.4, and in particular Figure B.3, for an updated version of this figure computed using the latest (2016) LUX limits [214].

WIMP–nucleon cross section as supplied in numerical form by DMTools [529]. These LUX limits were state-of-the-art when Ref. [5], the paper on which this chapter is based, was published. Results are shown in Figure 3.7 for both Dirac and Majorana DM. In Appendix B.4 we give a detailed discussion of the subsequent strengthening of the state-of-the-art SI direct detection limits, and how the latest (2016) LUX SI direct detection limits [214] would impact on the results shown in Figure 3.7.

3.6.1 Treatment of the Pseudoscalar Coupling for Direct Detection

We have been very approximate in our treatment of the pseudoscalar coupling in the direct detection computation. We assumed that we could treat the pseudoscalar case in the same manner as the scalar case by just finding the total SI nucleon-level cross section and then making the appropriate comparison to published SI LUX bounds [214, 506]. However, in the pseudoscalar case the nuclear cross section is suppressed by momentum transfer $Q^2 = 2m_T E_R \propto \mu_{\chi T}^2 \nu_\chi^2$ (see Appendix B.3). This E_R -suppression modifies the shape of the assumed differential nuclear recoil spectrum dR/dE_R that is used in extracting direct detection limits from underlying event data. A careful and correct treatment thus requires an extraction of bounds on the pseudoscalar interaction directly from the LUX event data with this different assumed recoil spectrum, rather than a comparison of the total cross section to the LUX cross section limit extracted assuming a normal SI recoil energy spectrum. Such an analysis is far beyond the intended scope of our work here. Moreover, our approximate approach led us to further assume that all the DM incident on the detector has speed $\nu_\chi \sim 220\text{km/s}$, rather than correctly treating the spread of DM velocities dictated by the underlying galactic-rest-frame DM velocity distribution boosted into the Earth rest frame. Appendix B.3 contains a more detailed discussion of these points. With these limitations in mind, the contribution to the SI direct detection cross section from the pseudoscalar part of the interaction as shown in Eq. (3.23) is probably only correct to around an order of magnitude.

Nevertheless, this is sufficient to capture the current direct detection limits on our parameter space sufficiently accurately owing to the following *a posteriori* argument. Consider that the ν_χ^2 term in Eq. (3.23) is at most $\mathcal{O}(10^{-6})$; in order for this term to drive where the direct detection limits bound our parameter space, $\cos^2 \xi$ would have to be no larger than $\mathcal{O}(10^{-6})$ at the point where we find the LUX SI cross section upper bounds are violated. However, the smallest $\cos^2 \xi$ values on the contour where the 2013 LUX [506] 90% CL upper limits on the SI cross section are violated are 5×10^{-3} to 10^{-2} (depending on the Dirac or

Majorana nature of the DM, respectively; see Figure 3.8 for an enlarged view of the region between 40–90 GeV in Figure 3.7). As a result, the careful treatment outlined above would need to result in a cross section limit between 3 and 4 orders of magnitude larger than our naïve estimate to effect any significant changes on the parameter space limits; we are of the opinion that such a large difference would be unlikely to result. Even with the updated limits shown in Appendix B.4 using the more recent 2016 LUX limits [214], our naïve estimate of the impact of the pseudoscalar coupling would still need to be between 2 and 3 orders of magnitude smaller than the result of a more careful treatment before we would need to be concerned about the inaccuracies introduced by our approximate treatment of this coupling; again, we think that unlikely to be the case. Stated somewhat differently, the LUX SI direct detection cross section limits (either the 2013 limits [506], or more recent 2016 limits [214]) are still a couple of orders of magnitude too large to be sensitive to the pure pseudoscalar coupling; they thus act merely to limit the amount of scalar coupling admixture that is allowed.

3.7 Combined Limits

Figure 3.8 shows the combined Higgs-width and direct detection limits on the $M\text{-}\cos^2\xi$ parameter space, for both Dirac and Majorana DM. The inserts are regions where $\cos^2\xi$ is very close to zero and the EFT DM–Higgs coupling is nearly completely pseudoscalar; as discussed above, such a pure pseudoscalar coupling is unnatural. Indirect limits are not included, but are not expected to significantly improve the exclusions shown: since the particles relevant for indirect detection have fairly featureless continuum spectra, the resulting indirect limits (c. 2014) are only constraining for $M \lesssim 30$ GeV, being conservative with respect to the large astrophysical uncertainties (see Figure 2.12 in Chapter 2, and Ref. [4]). This mass region is however already strongly excluded by invisible decays of the Higgs. Updating the indirect detection limits in Figure 2.12 to account for subsequently

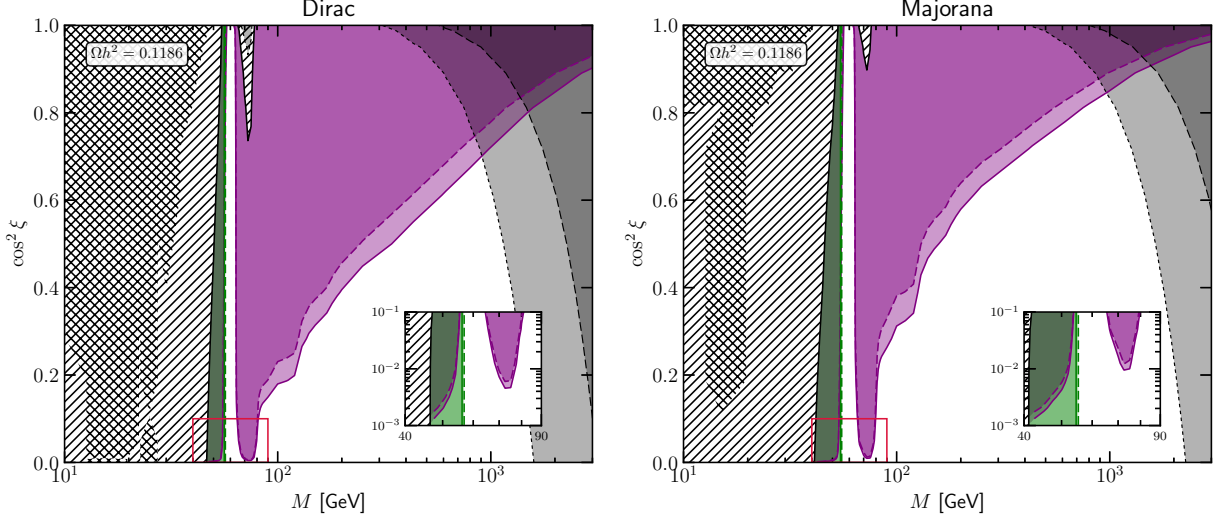


Figure 3.8: Combined limits keeping only Λ^{-2} terms in all cross sections. The grey regions bounded by black lines indicate the regions where $\Lambda \leq 2M$ (dotted line, light grey) and $\Lambda \leq M$ (dashed line, dark grey). The singly hatched region is where $\Lambda < v$. For reference, the simplest perturbative UV completion requires $\Lambda \gtrsim M/2\pi$. The doubly hatched region at low mass is where no Λ value can be found to obtain the correct relic density. The purple shaded regions bounded by purple lines are the regions excluded by the 2013 LUX [506] direct detection limits at 95% CL UL (dashed line; limit extrapolated between 2 and 3 TeV) and 90% CL UL (solid line). The green regions bounded by green lines indicate regions excluded by invisible width constraints arising from a global fit to data [507] with the SM-Higgs couplings floating (solid line) or fixed to SM values (dashed line); for clarity, we do not show the limits from the CMS direct width constraints [508] as they are slightly weaker. The insets show detail for small $\cos^2 \xi$ for masses $M \in [40, 90]$ GeV (i.e., the regions indicated by a red boxes on the main axes) and indicate that the coupling must be nearly pure pseudoscalar for masses around 70 GeV in order to not conflict with experimental limits. See also Appendix B.4, and in particular Figure B.4, for an updated version of this figure computed using more recent (2016) LUX limits [214].

released data (discussed in Appendix A.5.4) could potentially strengthen these limits enough that, for the mostly pseudoscalar case, interesting regions of mass parameter space could become marginally constrained; however, we expect that further improvements in the indirect detection limits (by a factor of a few) would be required before these limits became truly interesting. We also note that in the remaining allowed region, the EFT suppression scale necessary for this scenario to work is in the fairly narrow region $\Lambda \sim 1\text{--}5\text{ TeV}$ except near the resonance [434], and this may have interesting implications for collider searches.

Appendix B.4 contains a discussion of how the latest (2016) LUX limits [214] would impact the results shown in Figure 3.8.

3.8 Conclusions

In this chapter, we have examined in an effective field theory approach both Majorana and Dirac SM-singlet fermionic dark matter interacting with the SM via some combination of scalar ($\bar{\chi}\chi$) and pseudoscalar ($i\bar{\chi}\gamma_5\chi$) DM operators coupling to the Higgs Portal operator $H^\dagger H$. We have performed a systematic scan over the DM mass and the ratio of scalar to pseudoscalar coupling strengths, using cosmological measurements of the DM relic density to constraint the EFT suppression scale. We have observed that EWSB necessarily destabilizes a scenario in which the coupling is pure pseudoscalar before EWSB, concluding that this scenario is thus ill-motivated. We have constrained the post-EWSB DM mass and scalar-to-pseudoscalar-coupling ratio with a combination of direct detection bounds from the LUX experiment [214, 506] and with Higgs width constraints as measured by CMS [508], as well as those inferred from a global fit to available Higgs data [507]. We find in agreement with Ref. [434] that the ‘resonant Higgs Portal’ scenario in which $M \sim m_h/2$, is still (c. 2014) allowed for any admixture of scalar and pseudoscalar couplings, although the most recent (2016) LUX limits [214] are beginning to impinge on this region too (we also note that absent a compelling case from the UV theory for this particular DM mass, this scenario does not seem well motivated). We find that for $M \lesssim 54\text{--}56\text{ GeV}$, this Higgs Portal scenario is ruled

out by a combination of direct detection and invisible width constraints independent of the nature of the coupling or Majorana/Dirac nature of the fermionic DM, although in this regime the EFT validity becomes increasingly open to question as M decreases due both to neglected higher order terms and possible perturbative unitarity issues. For masses $m_h/2 \lesssim M \lesssim m_W$, the coupling must be almost pure pseudoscalar ($\cos^2 \xi \lesssim 1 \times 10^{-2}$ [5×10^{-3}] for the Dirac [Majorana] case) to be consistent with present data; this is an ill-motivated scenario due to the accidental relation between parameters $\Lambda M_0 \cos \theta \approx \langle v \rangle^2/2$ required to achieve it. However, for masses above the threshold for annihilation to W^+W^- (and presumably also for masses slightly below this threshold if we had accounted for three- and four-body decays through one or two off-shell W s) the interaction need not be so finely tuned to be pure pseudoscalar: it suffices for lower M that the coupling is predominantly pseudoscalar, and as the DM mass M is increased, the admixture of scalar coupling allowed increases due to the weakening of the LUX direct detection bounds. Other than in the resonant Portal mass region [434], we find that a pure-scalar Higgs Portal coupling is robustly ruled out at at least 90% confidence for M up to at least 3 TeV for both Majorana and Dirac fermion DM.

While we did not perform an in-depth indirect detection analysis of the Higgs Portal couplings in this chapter, recent analyses (see, e.g., Chapter 2 and Ref. [4]) indicate that they do not yet significantly strengthen the exclusion bounds on this scenario, although with future projected improvements in these limits, they may become of interest for the lower-mass region for the mostly pseudoscalar coupling. Direct searches at colliders in the mass region $M < m_h/2$ are expected to remain weaker than the invisible width limits, while collider signals for larger M cases (specifically, signals with two forward tagging jets and large missing energy [VBF MET], or mono- X and missing energy) may be interesting to examine, but we anticipate that sizeable SM backgrounds will make such searches fairly challenging.

CHAPTER 4

PROBING THE FERMIONIC HIGGS PORTAL AT LEPTON COLLIDERS

This chapter contains material from Ref. [6]: M. A. Fedderke, T. Lin, and L.-T. Wang, *Probing the fermionic Higgs portal at lepton colliders*, *JHEP* **1604** (2016) 160 [[arXiv:1506.05465](#)].¹

In this chapter, we continue our study of one of the fermionic Higgs Portal operators which we examined in Chapter 3, $H^\dagger H \bar{\chi} \chi$. In Chapters 2 and 3 our focus was on bottom-up—i.e., effective field theory (EFT)—analyses of various operators coupling a dark-matter (DM) Weakly Interacting Massive Particle (WIMP) to Standard Model (SM) particles. Our approach and aims in this chapter are somewhat different: rather than performing a bottom-up analysis, we study the sensitivity of future electron-positron colliders to UV completions of this operator quite generally, without necessarily imposing a DM interpretation on the singlet fermion χ . We consider the impact of projected improvements in the measurements of the precision electroweak S and T parameters and the $e^+e^- \rightarrow Zh$ Higgsstrahlung cross section at the proposed Circular Electron Positron Collider (CEPC), the proposed Future Circular Collider (FCC-ee), and the proposed International Linear Collider (ILC). The bulk of our analysis in this chapter is devoted to a so-called singlet-doublet fermion completion of the Higgs Portal operator $H^\dagger H \bar{\chi} \chi$. We assume that the doublet in this completion is heavy (because this is the region of the parameter space where the model supplies a UV completion of the operator $H^\dagger H \bar{\chi} \chi$), and explicitly match our UV completion to the low-energy effective field theory (EFT) at dimension-6 in order to compute contributions to the observables.² Additionally, we provide full one-loop results for S and T in the general mass parameter

1. Ref. [6] is copyright © 2016 by the Authors, and is under a Creative Commons Attribution (CC BY) 4.0 Licence [2].

2. Note an important distinction: we utilize EFT here as a computational tool only, whereas in Chapters 2 and 3 it served as the fundamental starting point for our analyses.

space for this UV completion. We also summarize existing results and make some comments on the so-called scalar completion of the fermionic Higgs Portal, although we argue that the phenomenology of this completion is closely related to that of the usual scalar Higgs Portal. In both completions, we find that future precision measurements can probe the new states at the (multi-)TeV scale, well beyond the direct reach of the LHC.

4.1 Introduction

Among the particularly compelling scenarios in which new physics couples to the Standard Model (SM) are the so-called vector, neutrino and Higgs ‘Portals’, which involve the three lowest-dimension SM-singlet operators which can couple to new physics (see, e.g., Ref. [530] and references therein). Of these, the Higgs Portal, which encompasses operators of the form $H^\dagger H \mathcal{O}_{\text{NP}}$ where $\mathcal{O}_{\text{NP}}$ is a SM-singlet operator built from new fields, has been the focus of much interest in the literature (e.g., Refs. [4, 5, 407, 408, 486, 489, 491, 494, 501, 503, 531–536]), especially in the wake of the 2012 discovery [332, 333] of a light Higgs boson with SM-like properties [339, 537]. As we have already mentioned in Chapter 3, there are a number of reasons for its appeal aside from its low dimensionality. The Higgs Portal provides a possible coupling of the dark matter (sector) to the SM, as we discuss in more detail below. Some well-studied models of this class—such as $H^\dagger H S^2$ with S a scalar singlet—have the ability to effect non-trivial modification of the dynamics of electroweak symmetry breaking (EWSB), rendering a first-order phase transition possible, with interesting implications for electroweak baryogenesis [409, 500, 538–542]. Additionally, such models may characterize the low-energy physics in theories which ameliorate the SM hierarchy problem [543].

While the simple picture of only considering the lowest-dimension fermionic Higgs Portal operator in a bottom-up analysis is of course instructive (as our work in Chapters 2 and 3 shows [4, 5]), much more can be said about particular possible UV completions of this model. In this chapter we study two such completions (see Figure 4.1) for the CP-even fermionic Higgs Portal operator, $H^\dagger H \bar{\chi} \chi$, along with their precision constraints. In the first model, a

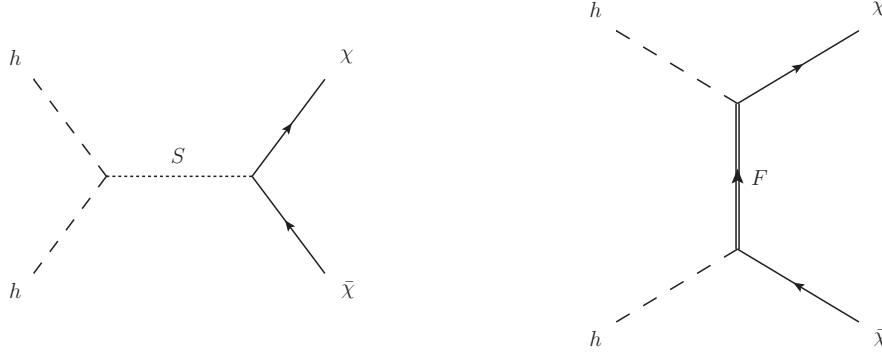


Figure 4.1: Two possible UV completions of the fermionic Higgs Portal operator, $H^\dagger H \bar{\chi} \chi$. In the ‘scalar completion’ (left diagram), a singlet scalar S has a super-renormalizable coupling to the H doublet and a Yukawa coupling to the singlet χ field; in the ‘fermionic completion’ (right diagram), a Yukawa-like coupling exists between a vector-like $SU(2)$ doublet F and the H and χ fields.

heavy singlet S acts as a mediator, with a Yukawa coupling to the fermion χ (which we take to be Dirac) and renormalizable couplings to the Higgs sector of the SM. This is essentially an extension of the singlet-scalar Higgs Portal scenario. In the second model, we introduce a new heavy vector-like $SU(2)_W$ fermion doublet F and couple both the singlet and doublet fermions to the SM Higgs: $-\kappa H \bar{F} \chi + \text{h.c.}$ Such singlet-doublet models have also been studied in the past [544–546].

Since the new states in either completion are coupled to the Higgs sector of the SM (and in the latter scenario, also have non-trivial charges under the SM gauge group), precision electroweak and Higgs physics will be important probes of these potential UV completions; for example, measurements of S , T and the Higgsstrahlung cross section σ_{Zh} are particularly sensitive. Although the present electroweak precision constraints have been known for a long time to be stringent, the current round of conceptual studies for future precision high-energy e^+e^- machines—e.g., the ILC, CEPC, and FCC-ee³—have resulted in significantly sharper projections for how the constraints could improve. This brings into even clearer focus the discovery and exclusion potential possible through a study of these probes. It is the major

3. The collider formerly known as TLEP.

goal of this chapter to more fully understand how the projected sensitivities of these machines will allow stringent constraints to be placed on the UV completions of the Higgs Portal which we consider.

Although we do not impose a dark-matter interpretation on the singlet χ in our models, it is worth noting this possibility has received much attention recently. For the dimension-five fermionic Higgs Portal, $H^\dagger H(a\bar{\chi}\chi + ib\bar{\chi}\gamma_5\chi)$, which comprises both CP-even and CP-odd coupling terms, a thermal-relic dark-matter interpretation is tightly constrained in the CP-even scenario by spin-independent direct detection constraints, whereas for the CP-odd coupling the direct detection constraints are alleviated and much parameter space remains open (see, e.g., Chapter 3, Refs. [5, 434, 491, 505, 536] and references therein). Turning to the UV completion of these operators, additional parameter space for thermal-relic dark matter is available when the new mediator mass (either the doublet fermion or singlet scalar) is similar to or below the mass scale of χ . For example, the relic density may be set by co-annihilation of the charged and neutral fermions [546, 547], or by dark-matter annihilation into lighter scalars (see Refs. [532, 548–551] for studies of the singlet scalar UV completion). These scenarios can have small couplings and thus evade electroweak precision and other constraints. As our focus is not on the dark-matter interpretation of χ , but rather on precision probes of the fermionic Higgs Portal coupling and its UV completions generally, we will usually restrict ourselves to the regime where the new mediator is heavy and the EFT is valid.

The remainder of this chapter is structured as follows: we begin in Section 4.2 summarizing the experimental sensitivities on measurements of S , T and σ_{Zh} which are currently projected to be attainable at future colliders. In Section 4.3 we introduce and briefly discuss the singlet-scalar UV completion, summarizing existing results in the literature. In Section 4.4 we introduce the fermionic singlet-doublet UV completion and discuss our general expectations. We then detail the construction of the EFT we used to analyse this model in Section 4.4.1, and give the EFT results for the precision electroweak and Higgsstrahlung

constraints in Sections 4.4.2 and 4.4.3, respectively. In Section 4.4.4 we discuss the one-loop computation of the precision electroweak limits in the general mass parameter space for this model. We conclude in Section 4.5. Appendix C.1 contains technical details on the one-loop computation of S and T , while Appendix C.2 contains a clarification on a technical point of the EFT matching.

4.2 Experimental Sensitivities

Several proposed e^+e^- collider experiments which can make advances in precision electroweak and Higgs physics measurements are currently under consideration: the International Linear Collider (ILC) [383–387], the Future Circular Collider (FCC-ee) [388] and the Circular Electron Positron Collider (CEPC) [382]. We summarize the sensitivities for various present and future measurements of the electroweak S and T parameters [552–554], and the Higgsstrahlung cross section σ_{Zh} , along with the references on which they are based, in Table 4.1. For each collider, we include several scenarios for the run parameters in order to compare the improvements in sensitivity that are possible with collider upgrades.

For the S and T electroweak precision observable (EWPO) limits, we present a parametrization of the covariance matrix used to construct the 68%-coverage likelihood contours presented graphically in the references. We made the simplifying assumption of exact Gaussian uncertainties centred at $(S, T) = (0, 0)$, and have parametrized the covariance matrix as

$$\Sigma = \begin{pmatrix} \sigma_S^2 & \rho_{ST} \sigma_S \sigma_T \\ \rho_{ST} \sigma_S \sigma_T & \sigma_T^2 \end{pmatrix}, \quad (4.1)$$

from which the likelihood function is given by $-2 \log[\mathcal{L}/\mathcal{L}_0] = \Delta^T \Sigma^{-1} \Delta$, where $\Delta^T = (S, T)$. The parameters $(\sigma_S, \sigma_T, \rho_{ST})$ were obtained by least-squares fitting of the constraint equation $-2 \log[\mathcal{L}/\mathcal{L}_0] = 2.30$ to a large number of (S, T) co-ordinates read from the relevant

Table 4.1: Experimental sensitivities for current and future measurements of (S, T) and σ_{Zh} assumed in Chapter 4. The σ_{Zh} sensitivities are quoted as 68%-confidence percentage uncertainties, while the (S, T) limits are shown as the parameters $(\sigma_S, \sigma_T, \rho_{ST})$ which define the covariance matrix in Eq. (4.1), and are extracted from the 68%-coverage likelihood contours ($-2\log[\mathcal{L}/\mathcal{L}_0] = 2.30$) in the indicated references.^a The individual ILC Higgsstrahlung constraints must be combined where necessary by summing the relevant $-2\log[\mathcal{L}/\mathcal{L}_0]$ values to obtain the constraints for the three scenarios we consider in this chapter: 250 fb^{-1} @ 250 GeV , 250 fb^{-1} @ $250\text{ GeV} + 500\text{ fb}^{-1}$ @ 500 GeV , and 1.15 ab^{-1} @ $250\text{ GeV} + 1.6\text{ ab}^{-1}$ @ 500 GeV .

Precision Electroweak (S, T)		
Scenario	$(\sigma_S, \sigma_T, \rho_{ST})$	Reference
Current	$(8.62 \times 10^{-2}, 7.37 \times 10^{-2}, 0.906)$	[555], figure 1
CEPC ‘Baseline’	$(2.39 \times 10^{-2}, 1.93 \times 10^{-2}, 0.844)$	[555], figure 4
CEPC ‘Improved $\Gamma_Z, \sin^2 \theta$ ’	$(1.14 \times 10^{-2}, 8.79 \times 10^{-3}, 0.518)$	[555], figure 4
CEPC ‘Improved $\Gamma_Z, \sin^2 \theta, m_t$ ’	$(1.12 \times 10^{-2}, 7.26 \times 10^{-3}, 0.779)$	[555], figure 4
ILC	$(1.71 \times 10^{-2}, 2.14 \times 10^{-2}, 0.891)$	[555], figure 1
FCC-ee-Z (aka TLEP-Z)	$(9.32 \times 10^{-3}, 8.70 \times 10^{-3}, 0.440)$	[555], figure 1
FCC-ee-t (aka TLEP-t)	$(9.24 \times 10^{-3}, 6.18 \times 10^{-3}, 0.794)$	[555], figure 1
Higgsstrahlung σ_{Zh}		
Scenario	$(\Delta\sigma_{Zh})/\sigma_{Zh}$ (%)	Reference
CEPC: 5 ab^{-1} @ 240 GeV	0.5%	[382]
ILC: 250 fb^{-1} @ 250 GeV	2.6%	[380, 556]
ILC: 500 fb^{-1} @ 500 GeV	3.0%	[380, 556]
ILC: 1.15 ab^{-1} @ 250 GeV	1.2%	[380, 556]
ILC: 1.6 ab^{-1} @ 500 GeV	1.7%	[380, 556]
FCC-ee: 10 ab^{-1} @ 240 GeV (4IP)	0.4%	[388]

^aIn this parametrization, the T -axis intercepts of the 68%-coverage likelihood contour for two parameters are at $S = 0$, $T = \pm \sigma_T [1 - \rho_{ST}^2]^{1/2} \cdot [(\Delta\chi^2)_2^{68\%}]^{1/2}$ where $(\Delta\chi^2)_2^{68\%} = 2.30$, and similarly for the S -axis intercepts. In particular, this means that for $S = 0$, the coverage in the T parameter is about a factor of 2 better when going from the FCC-ee-Z to the FCC-ee-t scenario, despite the more modest 30% decrease in σ_T .

graphically presented 68%-coverage ellipses in the references indicated in Table 4.1.

The precision electroweak sensitivity is based primarily on Z -pole measurements at the various colliders. For the CEPC, the ‘baseline’ is a Z -mass-threshold scan. The ‘improved’ scenarios consider the gains if the collider were to be upgraded to allow for improved measurements of Γ_Z (with the resonant depolarization method for energy calibration), $\sin^2 \theta$, or possibly m_t (from an ILC top-threshold scan). The FCC-ee- Z sensitivities are for Z -pole measurements with polarized beams (denoted ‘TLEP- Z ’ in Ref. [555]). The FCC-ee- t scenario includes Z -pole measurements with polarized beams along with threshold scans for WW and $t\bar{t}$ (denoted ‘TLEP- t ’ in Ref. [555]) and has improved sensitivity owing to more precise measurements of the W and top masses.

As has recently been emphasized in Refs. [543, 557] (and investigated further in, e.g., Refs. [435, 558–561]), the ‘Higgsstrahlung’ process $e^+e^- \rightarrow Zh$ can be measured at the percent or sub-percent level at future colliders and is a sensitive probe of new physics.⁴ Among the various ways a new-physics model modifies the Higgsstrahlung cross section, a modification to the wavefunction renormalization of the Higgs (i.e., a modification of the momentum-dependent part of the Higgs two-point function) may be induced, leading to a ‘Higgs oblique’ correction.

The limits on $\Delta\sigma_{Zh}/\sigma_{Zh}$ are quoted in Table 4.1 as the percentage 68%-confidence bounds given in the various indicated references. We consider a CEPC projection with 5 ab^{-1} of data at $\sqrt{s} = 240\text{ GeV}$ [382], and an FCC-ee projection with the assumption that there are four interaction points with 10 ab^{-1} of combined integrated luminosity collected at $\sqrt{s} = 240\text{ GeV}$ [388]. Various configurations of proposed runs for the ILC have been analysed in the literature [380, 556, 568]; we consider only the configurations where (a) 250 fb^{-1} of data are collected at $\sqrt{s} = 250\text{ GeV}$, (b) 500 fb^{-1} of data at $\sqrt{s} = 500\text{ GeV}$ are added to the 250 fb^{-1} of data collected at $\sqrt{s} = 250\text{ GeV}$, and (c) the integrated luminosity is increased

4. See also Refs. [562–567] for previous work on constraining anomalous Higgs couplings with the Higgsstrahlung cross section.

to a total of 1.15 ab^{-1} at $\sqrt{s} = 250 \text{ GeV}$ and 1.6 ab^{-1} at $\sqrt{s} = 500 \text{ GeV}$.

4.3 Scalar UV Completion

The simplest possible UV completion of the CP-even fermionic Higgs Portal is to take the Standard Model (SM) augmented by a vector-like Dirac fermion SM-singlet χ and a SM-singlet scalar S . These couple via the following Lagrangian

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SM}} &+ i\bar{\chi}\not{\partial}\chi - m_{\chi}\bar{\chi}\chi \\ &+ \frac{1}{2}(\partial_{\mu}S)^2 - \frac{1}{2}m_S^2S^2 - \frac{b}{3!}m_S S^3 - \frac{\lambda_S}{4!}S^4 + a m_S S|H|^2 + \frac{\epsilon_S}{2}S^2|H|^2 - \kappa_S S\bar{\chi}\chi, \end{aligned} \quad (4.2)$$

where $a, b, \epsilon_S, \kappa_S, m_S$ and m_{χ} are real parameters. Recall from Eq. (1.12) that our sign and normalization conventions for the SM Higgs sector are

$$\mathcal{L}_{\text{SM}} \supset |D_{\mu}H|^2 + \mu^2|H|^2 - \lambda|H|^4, \quad (4.3)$$

and our unitary-gauge normalization conventions are such that $H = (0, (v+h)/\sqrt{2})^T$, with $v \approx 246 \text{ GeV}$.

This model is just the renormalizable scalar Higgs Portal model (see, e.g., Ref. [560] for detailed discussion closely related to the present work), augmented with the singlet- χ Yukawa coupling. If we consider the limit where $bm_S, \epsilon_S \ll am_S, \kappa_S$, and further assume that $m_S \gg v, m_{\chi}$, the mediator S can be integrated out at tree-level to give rise to the following effective field theory:

$$\begin{aligned} \mathcal{L}_{\text{EFT}} \supset \mathcal{L}_{\text{SM}} &+ i\bar{\chi}\not{\partial}\chi - m_{\chi}\bar{\chi}\chi \\ &+ \frac{1}{2}a^2|H|^4 - \frac{a\kappa_S}{m_S}H^{\dagger}H\bar{\chi}\chi + \frac{a^2}{m_S^2}\frac{1}{2}(\partial_{\mu}|H|^2)^2 + \frac{1}{2}\frac{\kappa_S^2}{m_S^2}(\bar{\chi}\chi)^2 + \dots, \end{aligned} \quad (4.4)$$

where $\dots$ represents terms proportional to various powers of ϵ_S, b and neglected higher-

dimensional operators. Allowing terms proportional to $\epsilon_S, b m_S$ leads, at dimension-6, to only one additional operator, $|H|^6$, which taken together with the ability to change the sign of the $|H|^4$ operator in the SM Higgs Lagrangian and yet maintain a stable minimum to the Higgs potential, can have interesting implications for the order of the electroweak phase transition [560, 569].

For the purposes of the discussion here, the $|H|^4$ operator can simply be absorbed into an unobservable shift of the SM Higgs quartic coupling: $\lambda \rightarrow \lambda + \frac{1}{2}a^2$. The fermionic Higgs Portal operator $H^\dagger H \bar{\chi} \chi$ leads to a variety of effects (see, e.g., Chapter 3 and Ref. [5] for a detailed analysis): upon EWSB, it contributes to the mass of the χ field, and allows both $h \bar{\chi} \chi$ and $h^2 \bar{\chi} \chi$ couplings.

It is possible in this scenario to have the χ field play the role of the stable dark matter (DM), saturating the relic density (see, e.g., Chapter 3 and Ref. [5]). If the mediator S is assumed to be somewhat heavier than the χ , an EFT analysis is applicable [5, 434]: the relic abundance requirement fixes $m_S/(a\kappa_S) \sim 0.5$ TeV, while satisfying the LUX direct detection bounds [214, 506] requires that the χ be fairly massive, $m_\chi^{\text{physical}} \gtrsim 3$ TeV. These requirements are actually in tension with the assumptions of an EFT analysis as they demand a very large coupling $a\kappa_S \gtrsim 2\pi$ to avoid $m_\chi > m_S$ and the resulting issues with perturbative unitarity [460, 509, 570] for the EFT description of the non-relativistic DM freeze-out. This essentially means that, with the finely tuned exception of $m_\chi \sim m_h/2$ [5, 434], the heavy-mediator scenario is ruled out here. On the other hand, the scenario with a light mediator, $m_S < m_\chi$, is still viable as the direct detection constraints can be avoided while maintaining the correct relic abundance [434]. Of course, if we were to abandon the DM interpretation for χ , and allow it to decay sufficiently quickly via some suitable modification to this model, the couplings a and/or κ_S can be freely dialled down without overclosing the universe, thereby alleviating the direct detection constraints and allowing light m_χ .

In the EFT description for this model, the operator $\frac{1}{2}(\partial_\mu |H|^2)^2$ leads to a ‘Higgs oblique’ correction to the Higgsstrahlung process; i.e., the operator induces a Higgs wavefunction

renormalization $\delta Z_h = a^2 v^2 / m_S^2$, which modifies the cross section by $\Delta\sigma_{Zh}/\sigma_{Zh} = \delta Z_h$ [435, 543]. With a sensitivity of $\Delta\sigma_{Zh}/\sigma_{Zh} \sim 0.5\%$ (corresponding closely to the CEPC and FCC-ee sensitivities in Table 4.1), values of $m_S/a \lesssim 2.5$ TeV could be ruled out at 95% confidence [435], with 5σ discovery reach up to $m_S/a \sim 1.6$ TeV [435]. (The limits on m_S/a just quoted from Ref. [435] are slightly weaker than those in figure 1 of Ref. [560], wherein a combined fit to all projected ILC Higgs coupling measurements is considered.)

In this model, the electroweak precision observables S and T are generated by one-loop running of the operators between $\mu = m_S$ and $\mu = m_W$, leading to a mixing of the Higgs oblique operator with the operators responsible for giving the S and T parameters. This one-loop running has been computed in Ref. [560], which considers the UV completion above in the limit that $\kappa_S \rightarrow 0$ (and with no assumption on the sizes of ϵ_S, b). The leading contributions to the S and T parameters are found to be log-enhanced, linearly proportional to the high-scale ($\mu = m_S$) value of the Wilson coefficient of the Higgs oblique operator, and suppressed by a loop factor, as expected.

To summarize the results of Ref. [560], the electroweak precision constraints on the model are marginally weaker than those from the Higgsstrahlung measurement, if the most optimistic Higgsstrahlung precision (with FCC-ee or CEPC) is compared to the most optimistic electroweak precision constraints (with FCC-ee-t). Current constraints on S and T exclude $m_S/a \lesssim 1.2$ TeV at 95% confidence [560].

In the discussion above, we have neglected the impact of the neutral χ field that is still present in the EFT given by Eq. (4.4). This objection notwithstanding, provided that $m_\chi \gg E^*$, where E^* is the energy scale for some process of interest, one can also integrate the χ out at one-loop, in which case the Wilson coefficient of the $\frac{1}{2}(\partial_\mu |H|^2)^2$ operator will shift by a subdominant amount; here, the shift will be smaller by a factor of $\sim \kappa_S^2/16\pi^2$ than the leading contribution. Therefore turning on κ_S as a non-zero weak coupling, and integrating the χ out at the scale $\mu = m_\chi$, will give a subdominant one-loop shift to σ_{Zh} , and higher-loop contributions to S and T . The ratio $\sigma_{Zh} : S : T$ should not change significantly,

thus roughly preserving the relative strengths of the limits prevailing without the χ added to the theory. The foregoing limits on m_S/a are therefore expected to be fairly accurate also for our model, with the χ present. They arise as an effect caused by the mediator, and are fairly insensitive to the presence of the fermionic Higgs Portal coupling *per se*.

In summary, the precision electroweak and Higgsstrahlung constraints on this scenario are very similar to those of the model in which the χ is simply absent, and this case has already received extensive attention in the literature.

4.4 Singlet-Doublet UV Completion

We devote the remainder of this chapter to the singlet-doublet UV completion of the fermionic Higgs Portal operator. The model consists of the Standard Model augmented by the same vector-like Dirac fermion SM-singlet χ as before, as well as a vector-like Dirac fermion $SU(2)_W$ -doublet $F = (C, N)^T$ transforming under the SM gauge group as $(\mathbf{1}, \mathbf{2})_{+1/2}$. The coupling of these particles to the SM is taken to be

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{\chi}\not{\partial}\chi - m_\chi\bar{\chi}\chi + i\bar{F}\not{D}F - M_F\bar{F}F - \kappa\bar{F}H\chi - \kappa\bar{\chi}H^\dagger F, \quad (4.5)$$

where $D_\mu \equiv \partial_\mu - ig_2 W_\mu^{\tilde{a}} \tau^{\tilde{a}} - ig_1 B_\mu Y$, and $\tau^{\tilde{a}} = \frac{1}{2}\sigma^{\tilde{a}}$. Without loss of generality we may absorb the phase of κ into the definition of χ and/or F , and so throughout we take κ to be a non-negative real parameter. Without much modification (it would suffice, e.g., to replace $m_\chi\bar{\chi}\chi \rightarrow m_\chi\bar{\chi}_L\chi_R + \text{h.c.}$, with $\arg m_\chi \neq 0$) it would also be possible to generate the CP-odd Higgs Portal operator $H^\dagger H \bar{\chi} i\gamma_5 \chi$ which we studied in Chapter 3 and Ref. [5]; we do not, however, consider this case further here, and we take all parameters to be real.

In the parameter region where $M_F \gg m_\chi$, the heavy doublet F can be integrated out at the scale $\mu = M_F$, and the leading observable correction to the SM Lagrangian is the fermionic Higgs Portal operator. We will consider constraints on this UV completion both in the regime where $M_F \gg m_\chi$, constructing an EFT to analyse the low-energy effects,

as well as in the more general mass parameter space. For general masses, this model does not necessarily provide the UV completion to the fermionic Higgs Portal operator as it is defined here and our EFT is not valid, so we instead perform direct one-loop computations of relevant observables.

Before discussing the computations and results in detail, it is worth laying out our general expectations. The Yukawa-like coupling κ between the F , χ and H fields in Eq. (4.5) is a hard breaking of the accidental global $SU(2)_V$ custodial symmetry [571] of the SM, and as such we expect fairly large corrections to the precision electroweak T parameter. Additionally, the mass splitting (i.e., weak isospin breaking) in the F -doublet which arises from the mixing of the neutral fermions N and χ after electroweak symmetry breaking (EWSB), leads us to expect that there will additionally be contributions to the electroweak S parameter.

Note that, although we do not consider such a case in this chapter, custodial symmetry could be restored—or broken in a controlled fashion—by augmenting the field content with an additional positively charged vector-like fermion $\psi \sim (\mathbf{1}, \mathbf{1})_{+1}$ with the same Yukawa-like coupling to the H and F fields; the mass-splitting $|m_\psi - m_\chi|$ would then control the degree to which the symmetry is broken (being restored in the degenerate-mass limit). There would however be additional experimental handles in this case as the new direct coupling of the Higgs doublet to electrically charged fermions would lead to one-loop corrections to the $h \rightarrow \gamma\gamma$ rate.

We also expect that, in the model defined by Eq. (4.5), the Higgs oblique correction will be generated (by closed N - χ loops on the Higgs propagator, in the full-theory picture), so that even if we were to tune away the large corrections to T by, e.g., the method indicated in the previous paragraph, significant constraints would still remain on this model owing to the shift in the Higgsstrahlung cross section effected by the oblique correction. To be clear, however, the Higgs oblique correction is by no means the only contribution to the shift in the Higgsstrahlung cross section which is induced in our model; indeed, non-zero S and T parameters themselves will also generate a shift to σ_{Zh} (see Ref. [435]), and we take all such

shifts into account.

4.4.1 Effective Field Theory Analysis

It is instructive to begin our analysis by integrating out the new field content, assuming it is heavy, and considering the low-energy effects of the dimension-6 operators that are generated. In many popular bases of gauge-invariant dimension-6 operators which extend the SM (see, e.g., Refs. [572–575]), one can immediately read off the S and T parameters as the Wilson coefficients of certain operators. In the so-called ‘HISZ’ basis [574], S is simply proportional to the Wilson coefficient of the operator⁵ $H^\dagger \hat{B}^{\mu\nu} \hat{W}_{\mu\nu} H$, and T is likewise proportional to the Wilson coefficient of the operator $|H^\dagger D_\mu H|^2$; the observable corrections to the momentum-dependent part of the Higgs two-point function would arise from the operators $|H^\dagger D_\mu H|^2$ and $(\partial_\mu |H|^2)^2$ in this basis, but the full σ_{Zh} corrections require significantly more work to obtain [435].

In the following, we first integrate out the F doublet at the ‘high scale’ $\mu = M_F$, performing the one-loop matching onto an effective theory with the Standard Model field content plus the singlet χ . We then repeat the procedure, integrating out the χ field at the ‘low scale’ $\mu = m_\chi$ and matching at one-loop onto a new EFT with the SM field content only. We assume the mass hierarchy $M_F \gg m_\chi \gg v \sim m_W$, where $M_F \gg m_\chi$ is imposed to guarantee that the leading correction upon integrating out the F is the CP-even fermionic Higgs Portal operator; this is not a necessary assumption of the model itself.

In performing the matching, we find it convenient to match onto the basis of dimension-6 operators with SM fields shown in Table 4.2. This basis of operators is not one of the standard ones and in particular certain operators (e.g., $\mathcal{O}_{(2,4)}$) are redundant in the sense that they are related to other operators by SM classical equations of motion (EOM). The redundant operators can be eliminated by field redefinitions whose effect is exactly equivalent to making replacements using the classical EOM (see, e.g., Refs. [572, 576]); this is correct

5. Here, $\hat{W}_{\mu\nu} \equiv ig_2 W_{\mu\nu}^a \tau^a$ and $\hat{B}_{\mu\nu} \equiv ig_1 B_{\mu\nu} Y$.

even at the quantum level if we consistently neglect higher-dimension operator corrections. We note for now that the operators in Table 4.2 form a convenient and sufficient set to match onto; we will return below to the treatment of the various redundancies in order to transform into the operator bases most convenient for calculating the S and T parameters, and σ_{Zh} .

Upon integrating out F , we find the operators and Wilson coefficients shown in Tables 4.2 and 4.3. The only operators of dimension-4 or less appearing in Table 4.2— $\mathcal{O}_{(2,0)}$, $\mathcal{O}_{(4,0)}$, and $\mathcal{O}_{(2,2)}$ —already appear in the SM. Our convention for these operators is such that the Wilson coefficients $B_{(2,0)}$, $B_{(4,0)}$, and $B_{(2,2)}$ simply add positively to the coefficients already present for these operators in the SM: i.e., $\mathcal{L} \supset (1 + B_{(2,2)})\mathcal{O}_{(2,2)} + (\mu^2 + B_{(2,0)})\mathcal{O}_{(2,0)} - (\lambda + B_{(4,0)})\mathcal{O}_{(4,0)}$. On the other hand, all operators with mass-dimension greater than four are taken to appear in the Lagrangian as $\mathcal{L} \supset c_i M_F^{4-d_i} \mathcal{O}_i$ where c_i is the Wilson coefficient and d_i is the dimension of the operator $\mathcal{O}_i$.

We worked to tree-level for the Wilson coefficients for any operator containing a χ -field bilinear, and to one-loop in obtaining the Wilson coefficients for operators containing only SM field content. The rationale is that whenever the χ later appears in any diagram computed in a consistent one-loop matching to obtain a Wilson coefficient of a dimension-6 operator containing only SM content, it will appear as a closed χ loop, so only the tree-level χ -SM couplings are required.

The tree-level matching was performed by inverting the classical equation of motion for the heavy doublet to formally solve for F in terms of H and χ , and expanding in powers of D_μ/M_F (some lower-dimensional results were checked diagrammatically). The one-loop matching was performed diagrammatically utilizing dimensional regularization and the $\overline{\text{MS}}$ renormalization scheme. Calculations were done in the broken electroweak phase and, on the EFT-side of the one-loop matching computations, we consistently included diagrams with one or more tree-level χ -SM coupling(s) and a single closed χ loop. As a check, some one-loop computations were performed both in the non-mass-diagonal basis of new neutral

Table 4.2: Operators appearing in the effective theory below the scale M_F (see text for sign and normalization conventions). For operators with a χ bilinear, we have performed tree-level matching at the high scale M_F up to and including dimension-10 operators in order to be able to compute corrections to fairly high order in m_χ/M_F when later matching at $\mu = m_\chi$, but omit these lengthy results here as we used only selected terms from them as necessary (see text). Operators containing only SM content appear in the one-loop matching both at the high scale $\mu = M_F$ and at the low scale $\mu = m_\chi$. The naming convention for the operators with only n Higgs doublets H and m (covariant) derivatives is $\mathcal{O}_{(n,m)}$; other operators are named on an ad-hoc basis. We define $\hat{W}_{\mu\nu} \equiv ig_2 W_{\mu\nu}^{\tilde{a}} \tau^{\tilde{a}}$ and $\hat{B}_{\mu\nu} \equiv ig_1 B_{\mu\nu} Y$. Note that the operator $\mathcal{O}_{(2,4)}$ is non-standard in the literature and can be eliminated using Eq. (4.6).

Tree-level — operators containing a χ bilinear			
Name	Operator	Name	Operator
$\mathcal{O}_5$	$H^\dagger H \bar{\chi} \chi$	$\mathcal{O}_{7B}$	$ D_\mu H ^2 \bar{\chi} \chi$
$\mathcal{O}_{6A}$	$\frac{1}{2} \bar{\chi} \gamma^\mu \chi i(H^\dagger D_\mu H - \text{h.c.})$	$\mathcal{O}_{7C}$	$\frac{1}{2} i \left((D_\mu H)^\dagger D_\nu H - \text{h.c.} \right) \bar{\chi} \sigma^{\mu\nu} \chi$
$\mathcal{O}_{6B}$	$\frac{1}{2} H^\dagger H i(\bar{\chi} \not{\partial} \chi - \text{h.c.})$	$\mathcal{O}_{7D}$	$\frac{1}{2} \left(H^\dagger D_\mu H - \text{h.c.} \right) (\bar{\chi} \gamma_\mu \not{\partial} \chi - \text{h.c.})$
$\mathcal{O}_{7A}$	$\frac{1}{2} H^\dagger H (\bar{\chi} \square \chi + \text{h.c.})$		etc.
One-loop — operators with only SM content			
$\mathcal{O}_{(2,2)}$	$ D_\mu H ^2$	$\mathcal{O}_{(2,0)}$	$ H ^2$
$\mathcal{O}_{(4,0)}$	$ H ^4$	$\mathcal{O}_{(4,2),A}$	$\frac{1}{2} (\partial_\mu H ^2)^2$
$\mathcal{O}_{(4,2),B}$	$ H^\dagger D_\mu H ^2$	$\mathcal{O}_{(4,2),C}$	$ H ^2 D_\mu H ^2$
$\mathcal{O}_{(6,0)}$	$ H ^6$	$\mathcal{O}_{(2,4)}$	$H^\dagger D_\mu D^\nu D_\nu D^\mu H + \text{h.c.}$
$\mathcal{O}_{(WW)}$	$H^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} H$	$\mathcal{O}_{(BB)}$	$H^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} H$
$\mathcal{O}_{(BW)}$	$H^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} H$	$\mathcal{O}_{(DW)}$	$\text{Tr} [D_\mu, \hat{W}_{\nu\rho}] [D^\mu, \hat{W}^{\nu\rho}]$
$\mathcal{O}_{(DB)}$	$-\frac{g_1^2}{2} (\partial_\mu B_{\nu\rho}) (\partial^\mu B^{\nu\rho})$	$\mathcal{O}_{(W)}$	$(D_\mu H)^\dagger \hat{W}^{\mu\nu} (D_\nu H)$
$\mathcal{O}_{(B)}$	$(D_\mu H)^\dagger \hat{B}_{\mu\nu} (D_\nu H)$	$\mathcal{O}_{(WWW)}$	$\text{Tr} [\hat{W}_{\mu\nu} \hat{W}^{\nu\sigma} \hat{W}_\sigma^\mu]$

Table 4.3: Wilson coefficients of the operators appearing in Table 4.2, evaluated at $\mu = M_F$, the ‘high scale’. Our sign convention is that the Lagrangian term containing any operator of dimension greater than 4 which is shown in Table 4.2 appears multiplied by its corresponding Wilson coefficient and the appropriate inverse power of the EFT cutoff-scale M_F ; *however*, for operators of dimension 4 or less, the sign appearing in front of the operator follows the SM conventions and any multiplying power of M_F is absorbed into the Wilson coefficient (see discussion in text). These expressions assume a hierarchy of scales $M_F \gg m_\chi$, and are correct to tree-level for the c_i and to one-loop for the $B_{(j)}$. We present results to fairly high order in m_χ/M_F , although not all of these terms are numerically necessary to obtain our results.

Tree-level — operators containing a χ bilinear			
Coefficient	Value	Coefficient	Value
c_5	$+\kappa^2$	c_{6A}	$+\kappa^2$
c_{6B}	$+\kappa^2$	c_{7A}	$-\kappa^2$
c_{7B}	$+\kappa^2$	c_{7C}	$-\kappa^2$
c_{7D}	$-\kappa^2$	etc.	

One-loop — operators with only SM content	
Coefficient	Value
$B_{(2,0)}$	$-(\kappa^2/4\pi^2) M_F^2 \left[1 + m_\chi/M_F + m_\chi^2/M_F^2 + m_\chi^3/M_F^3 + m_\chi^4/M_F^4 + m_\chi^5/M_F^5 + m_\chi^6/M_F^6 + \dots \right]$
$B_{(2,2)}$	$+(\kappa^2/16\pi^2) \left[1 - 2m_\chi/M_F - 4m_\chi^2/M_F^2 - 10m_\chi^3/M_F^3 - 15m_\chi^4/M_F^4 - 24m_\chi^5/M_F^5 - 32m_\chi^6/M_F^6 + \dots \right]$
$B_{(4,0)}$	$-(\kappa^4/8\pi^2) \left[1 + 4m_\chi/M_F + 9m_\chi^2/M_F^2 + 16m_\chi^3/M_F^3 + 25m_\chi^4/M_F^4 + \dots \right]$
$B_{(4,2),A}$	$-(\kappa^4/24\pi^2) \left[5 + 4m_\chi/M_F - 9m_\chi^2/M_F^2 \right]$
$B_{(4,2),B}$	$-(\kappa^4/48\pi^2) \left[5 - 2m_\chi/M_F - 15m_\chi^2/M_F^2 + \dots \right]$
$B_{(4,2),C}$	$-(\kappa^4/48\pi^2) \left[1 - 22m_\chi/M_F - 123m_\chi^2/M_F^2 + \dots \right]$
$B_{(6,0)}$	$-(\kappa^6/12\pi^2) \left[1 + 12m_\chi/M_F + 48m_\chi^2/M_F^2 \dots \right]$
$B_{(2,4)}$	$+(\kappa^2/48\pi^2) \left[1 - m_\chi/M_F - 3m_\chi^2/M_F^2 - 20m_\chi^3/M_F^3 - 40m_\chi^4/M_F^4 + \dots \right]$
$B_{(WW)}$	$-(\kappa^2/144\pi^2) \left[1 - 4m_\chi/M_F + 3m_\chi^2/M_F^2 + 10m_\chi^3/M_F^3 + 68m_\chi^4/M_F^4 + \dots \right]$
$B_{(BB)}$	$-(\kappa^2/144\pi^2) \left[1 - 4m_\chi/M_F + 3m_\chi^2/M_F^2 + 10m_\chi^3/M_F^3 + 68m_\chi^4/M_F^4 + \dots \right]$
$B_{(BW)}$	$-(\kappa^2/72\pi^2) \left[1 - 4m_\chi/M_F + 3m_\chi^2/M_F^2 + 10m_\chi^3/M_F^3 + 68m_\chi^4/M_F^4 + \dots \right]$
$B_{(DW)}$	$+(1/240\pi^2)$
$B_{(DB)}$	$+(1/240\pi^2)$
$B_{(W)}$	$+(\kappa^2/72\pi^2) \left[7 - 4m_\chi/M_F - 12m_\chi^2/M_F^2 + \dots \right]$
$B_{(B)}$	$+(\kappa^2/72\pi^2) \left[7 - 4m_\chi/M_F - 12m_\chi^2/M_F^2 + \dots \right]$
$B_{(WWW)}$	Operator generated, but coefficient not computed.
	Irrelevant to the observables of interest.

fermions, N and χ , and in the mass-diagonalized basis.⁶ In the latter case, expansion of the result in powers of the Higgs vev v allows the comparison of these two methods and the extraction of the Wilson coefficients. Finally, we have checked that, to the order we work, the EFT and full theory one-loop computations reproduce the same IR-divergent⁷ logarithms ($\sim \log[m_\chi^2/M_F^2]$ at $\mu = M_F$), as is of course required [577].

The EFT power-counting we employed is such that we kept only leading terms in v/M_F , but retained terms at higher order in m_χ/M_F , although this was not always numerically necessary to obtain the results we present below. In order to compute these higher-order terms in the mass-ratio m_χ/M_F in our one-loop matching, we have systematically carried out the tree-level matching up to and including dimension-10 operators so as to capture the operators with higher numbers of derivatives acting on the χ field since, in dimensional regularization and $\overline{\text{MS}}$, such higher-derivative operators contribute positive powers of m_χ to the S -matrix element when appearing in conjunction with a closed χ -loop. This latter point is perhaps opaque, and is illuminated by a simple concrete example: the term in $B_{(WW)}$ suppressed by m_χ^4/M_F^4 relative to the leading term can be found from, e.g., the $v^2(p^2/M_F^2)(m_\chi^4/M_F^4)$ part of the ZZ two-point function at one-loop (along with other VV two-point functions so as to break the degeneracy between $B_{(WW)}$, $B_{(BW)}$, and $B_{(BB)}$). Simple power-counting then indicates that, owing to the M_F^{-6} suppression, this requires in turn knowledge of the subset of the operators at dimension-10 in the tree-level matching which contain two Z boson fields, two Higgs fields, one $\bar{\chi}$ field, one χ field, two derivatives acting on the Z fields, and—to be dimensionally correct—one derivative acting on either of the χ or $\bar{\chi}$ fields. When the χ loop is closed and the Higgs field replaced by its vev, such operators contribute to the ZZ two-point function in the EFT with a term proportional to

6. Mixing-angle effects are necessarily small in the range of EFT validity, being suppressed by at least $\kappa v/M_F$ for $M_F \gg m_\chi, \kappa v$ [see Eq. (C.7)]. It is well known (see, e.g., §3.3 [pp. 37–38] of Ref. [577], and references therein) that physical effects which in the full theory would be ascribed to these small mixing angles are instead captured in the EFT description through the inclusion of higher-dimension operators, whose Wilson coefficients have their genesis (in part) in the UV-theory mixing angles. Readers unfamiliar with this point may wish to consult Appendix C.2 for further comments.

7. In the sense that they diverge if $m_\chi \rightarrow 0$.

$(v^2 p^2 / M_F^6) \int d^d k k^2 / (k^2 - m_\chi^2) \sim (v^2 p^2 / M_F^6) m_\chi^4$, as desired.

Having obtained the mixed tree-level/one-loop matching conditions, one should by all rights run the theory down from the high scale $\mu = M_F$ to the ‘low scale’ $\mu = m_\chi$, integrate the χ out at this scale, and then run the theory to the scale required to consider any process of interest (i.e., the matching conditions supply only the initial conditions for the renormalization group equation (RGE) running). However, we will choose to not run the operators using the RGE. The errors one makes in ignoring the running between $\mu = M_F$ and $\mu = m_\chi$ are proportional to $\log [m_\chi^2 / M_F^2]$; provided we do not take too large a hierarchy, this factor is not necessarily very large. Furthermore, for the electroweak precision observables S and T , we expect the effect of the running is either higher-loop or power-suppressed in m_χ / M_F and therefore a subdominant modification to the coefficients generated at the high scale. This is in contrast to, e.g., the case considered in Ref. [560], where the operators responsible for S and T are not generated at the high scale, and arise purely from operator mixing upon running: in that case, it is important to find the contributions from running to the corresponding Wilson coefficients because S and T constraints are so strong. Our neglect of the RGE in this model is also numerically justified, at least for the S and T parameters, as we will show in comparing EFT and full-theory one-loop results.

We thus move on to the matching at the low scale $\mu = m_\chi$ where we integrate the χ out, working consistently correct to one-loop in the matching. The operators $\mathcal{O}_i$ for $i \in \{5, 6A, 6B, 7A, \text{etc.}\}$ are thereby removed from the Lagrangian and the Wilson coefficients B_i for the dimension-6 operators with only SM field content receive new contributions in the EFT valid below $\mu = m_\chi$. We call these shifted Wilson coefficients in the low-scale EFT C_i ; they are shown in Table 4.4 assuming no running or operator mixing between the scales $\mu = M_F$ and $\mu = m_\chi$.

While the basis of operators shown in Table 4.2 was convenient for the matching computation, it still contains the various redundancies we mentioned earlier. It also does not happen to be the most convenient for the computation of the S and T parameters, and σ_{Zh} .

The S and T parameters are each simply related to the Wilson coefficient of a single operator when these redundancies are removed. In addition, Ref. [435] contains a computation of σ_{Zh} in terms of a different basis of operators, and it is useful to transform to this basis to enable us to perform a cross-check on our own independent computation of σ_{Zh} .

As $\mathcal{O}_{(2,4)}$ is non-standard in the literature, we eliminate it via the identity

$$\mathcal{O}_{(2,4)} = 2|D^2 H|^2 - \mathcal{O}_{(BB)} - 2\mathcal{O}_{(BW)} - \mathcal{O}_{(WW)} - 2\mathcal{O}_{(B)} - 2\mathcal{O}_{(W)}, \quad (4.6)$$

plus irrelevant total-derivative terms. We are also free to make use of the SM EOM for H to trade out $|D^2 H|^2$ for corrections to the coefficients of the operators $|H|^2, |H|^4, |H|^6$ and the SM Yukawa Lagrangian $\mathcal{L}_{\text{SM}}^{\text{YUKAWA}}$, in addition to two new types of dimension-6 operators— $H^\dagger H \times \mathcal{L}_{\text{SM}}^{\text{YUKAWA}}$ and a number of four-fermion contact operators proportional to two powers of the Yukawa couplings—as well as dimension-8 corrections, the latter of which we consistently neglect. The corrections to the $|H|^2, |H|^4$ and $\mathcal{L}_{\text{SM}}^{\text{YUKAWA}}$ terms are absorbed into unobservable shifts of the SM parameters μ^2, λ and Γ_i (the SM Yukawa coupling matrices), but the remaining additional terms here lead in principle to observable effects.

Our last step is to rescale the H field to obtain a canonically normalized covariant derivative term (i.e., we absorb the coefficient of $|D_\mu H|^2, (1+C_{(2,2)})$, into the H field) and re-define the SM couplings [μ^2, λ , and Γ_i] to absorb both the shifts proportional to $C_{(2,2)}$, as well as the shifts to the SM parameters arising from $C_{(4,0)}$ and $C_{(2,0)}$. We note that in the coefficients of all dimension-6 operators, the rescaling of the H field can simply be ignored as the leading coefficients are already linear in the C_i : any correction resulting from the H rescaling is therefore proportional to $C_{(2,2)} \times C_i$ and thus formally counted as a two-loop effect, since a single C_i insertion is formally counted as a one-loop correction. In other words, the coefficients $C_{(2,2)}, C_{(4,0)}$ and $C_{(2,0)}$ can simply be ignored in an analysis correct to one-loop (this assumes a degree of tuning in the bare value of μ^2 such that we still obtain the correct measured Higgs mass).

Table 4.4: Wilson coefficients of the operators appearing in Table 4.2, evaluated at $\mu = m_\chi$, the ‘low scale’, after the χ has been integrated out. These expressions are correct under the assumption of no operator running or mixing between the scales $\mu = M_F$ and $\mu = m_\chi$. Our sign conventions are as detailed in Table 4.3. These expressions assume a hierarchy of scales $M_F \gg m_\chi$.

Coefficient	Value
$C_{(2,0)}$	$-(\kappa^2/4\pi^2) [M_F^2 + M_F m_\chi + m_\chi^2]$ (this appears to be exact)
$C_{(2,2)}$	$+(\kappa^2/16\pi^2) [1 - 2 m_\chi/M_F - 4 m_\chi^2/M_F^2 - 6 m_\chi^3/M_F^3 - 8 m_\chi^4/M_F^4 - 10 m_\chi^5/M_F^5 - 12 m_\chi^6/M_F^6 + \dots]$
$C_{(4,0)}$	$-(\kappa^4/8\pi^2) [1 + 4 m_\chi/M_F + 8 m_\chi^2/M_F^2 + 12 m_\chi^3/M_F^3 + 16 m_\chi^4/M_F^4 + \dots]$
$C_{(4,2),A}$	$-(\kappa^4/24\pi^2) [7 + 8 m_\chi/M_F - 3 m_\chi^2/M_F^2 + \dots]$
$C_{(4,2),B}$	$-(\kappa^4/48\pi^2) [5 - 2 m_\chi/M_F - 15 m_\chi^2/M_F^2 + \dots]$
$C_{(4,2),C}$	$-(\kappa^4/48\pi^2) [1 - 22 m_\chi/M_F - 111 m_\chi^2/M_F^2 + \dots]$
$C_{(6,0)}$	$-(\kappa^6/12\pi^2) [1 + 14 m_\chi/M_F + 51 m_\chi^2/M_F^2 + \dots]$
$C_{(WW)}$	$-(\kappa^2/144\pi^2) [1 - 4 m_\chi/M_F + 3 m_\chi^2/M_F^2 + 10 m_\chi^3/M_F^3 + 41 m_\chi^4/M_F^4 + \dots]$
$C_{(BB)}$	$-(\kappa^2/144\pi^2) [1 - 4 m_\chi/M_F + 3 m_\chi^2/M_F^2 + 10 m_\chi^3/M_F^3 + 41 m_\chi^4/M_F^4 + \dots]$
$C_{(BW)}$	$-(\kappa^2/72\pi^2) [1 - 4 m_\chi/M_F + 3 m_\chi^2/M_F^2 + 10 m_\chi^3/M_F^3 + 41 m_\chi^4/M_F^4 + \dots]$
$C_{(2,4)}$	$+(\kappa^2/48\pi^2) [1 - m_\chi/M_F - 3 m_\chi^2/M_F^2 - 14 m_\chi^3/M_F^3 - 25 m_\chi^4/M_F^4 + \dots]$
$C_{(DB)}$	$+(1/240\pi^2)$
$C_{(DW)}$	$+(1/240\pi^2)$
$C_{(W)}$	$+(\kappa^2/72\pi^2) [7 - 4 m_\chi/M_F - 12 m_\chi^2/M_F^2 + \dots]$
$C_{(B)}$	$+(\kappa^2/72\pi^2) [7 - 4 m_\chi/M_F - 12 m_\chi^2/M_F^2 + \dots]$
$C_{(WWW)}$	Operator generated, but coefficient not computed.
	Irrelevant to the observables of interest.

4.4.2 Electroweak Precision Observables

Having eliminated $\mathcal{O}_{(2,4)}$ using Eq. (4.6), the S and T parameters can be read off from the new coefficients of the operators $\mathcal{O}_{(BW)}$ and $\mathcal{O}_{(4,2),B}$, respectively.⁸ These operators modify the gauge boson vacuum polarization amplitudes $\Pi_{VV}^{\mu\nu}(q^2) = ig^{\mu\nu}\Pi_{VV}(q^2) + \dots$. Correct to linear order in the Wilson coefficients C_i , we find that the new physics contributions⁹ to

8. Again, we do not run the operator coefficients between m_χ and the Z -pole.

9. The operators $\mathcal{O}_{(DB),(DW)}$ contribute only terms $\sim p^4$ to the gauge-boson two-point functions and hence do not contribute to the S , and T parameters [at least if S is defined per Eq. (4.8)]; instead, these operators give rise to the $\hat{Y}$ and $\hat{W}$ operators of Ref. [578]. However, we find the corresponding limits are not competitive with those from S and T and do not pursue this further.

S and T are:

$$S \equiv \frac{4c_W^2 s_W^2}{\alpha_e} \left[\frac{\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0)}{M_Z^2} - \frac{c_W^2 - s_W^2}{c_W s_W} \frac{\Pi_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2} \right] \quad (4.7)$$

$$\approx \frac{4c_W^2 s_W^2}{\alpha_e} \left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{c_W s_W} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right] \quad (4.8)$$

$$= -\frac{4c_W s_W}{\alpha_e} \Pi'_{W^3 B}(0) \quad (4.9)$$

$$\approx -4\pi \frac{v^2}{M_F^2} \left[C_{(BW)} - 2C_{(2,4)} \right], \quad (4.10)$$

and

$$T = \frac{\rho - 1}{\alpha_e} \quad (4.11)$$

$$\equiv \frac{1}{\alpha_e M_W^2} \left[\Pi_{W^+ W^-}(0) - \Pi_{W^3 W^3}(0) \right] \quad (4.12)$$

$$= \frac{1}{\alpha_e M_W^2} \left[\Pi_{W^+ W^-}(0) - c_W^2 \Pi_{ZZ}(0) \right] \quad (4.13)$$

$$\approx -\frac{1}{2\alpha_e} \frac{v^2}{M_F^2} C_{(4,2),B}, \quad (4.14)$$

where in arriving at Eqs. (4.8) and (4.13), we have made use of $\Pi_{Z\gamma}(0) = \Pi_{\gamma\gamma}(0) = 0$.

The U parameter is identically zero at dimension-6; the first non-zero contribution occurs at dimension-8 [579].

Substituting the explicit forms of the Wilson coefficients, we have

$$S \approx \frac{2\kappa^2}{9\pi} \frac{v^2}{M_F^2} \left[1 - \frac{7}{4} \frac{m_\chi}{M_F} - \frac{3}{2} \frac{m_\chi^2}{M_F^2} - 8 \frac{m_\chi^3}{M_F^3} - \frac{17}{2} \frac{m_\chi^4}{M_F^4} + \dots \right] \quad (4.15)$$

$$T \approx \frac{\alpha_\kappa}{\alpha_e} \frac{5\kappa^2}{24\pi} \frac{v^2}{M_F^2} \left[1 - \frac{2}{5} \frac{m_\chi}{M_F} - 3 \frac{m_\chi^2}{M_F^2} + \dots \right] \quad \text{where} \quad \alpha_\kappa \equiv \frac{\kappa^2}{4\pi} \quad (4.16)$$

$$U = 0 + (\text{dim-8}) \quad (4.17)$$

The parametric enhancement (or suppression, depending on the relative sizes of α_κ and α_e) of T compared to S can be understood as follows. For $M_F \gg m_\chi$, $\Pi_{W^+W^-}(0) - c_W^2 \Pi_{ZZ}(0) \sim g_2^2(\kappa^2 v^2)(\kappa^2 v^2/M_F^2)$, while $\Pi'_{ZZ}(0) \sim (g_1^2 + g_2^2)(\kappa^2 v^2/M_F^2)$. This is determined by the mass dimension of the quantity concerned and the fact that, in our model, a diagrammatic new-physics contribution proportional to the Higgs vev v must always actually be proportional to κv owing to the structure of the H - F - χ coupling. Therefore, $T \sim (g_2^2 v^2/M_W^2)(\kappa^2/\alpha_e)(\kappa^2 v^2/M_F^2) \sim (\alpha_\kappa/\alpha_e)(\kappa^2 v^2/M_F^2)$ while $S \sim (c_W^2 s_W^2/\alpha_e)\Pi'_{ZZ}(0) \sim \kappa^2 v^2/M_F^2$ since $\alpha_e/c_W^2 s_W^2 \sim g_1^2 + g_2^2$. The parametric dependence on κ can also be read off from the form of the dimension-6 operators giving rise to S and T . Upon EWSB, the S parameter arises from $\mathcal{O}_{(BW)} = H^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} H$. As a result, diagrammatic contributions to S come with two powers of the Higgs vev v and hence two powers of κv , since the two derivatives in the field strengths are removed in the definition of S as a momentum derivative of the two-point function. The T parameter arises upon EWSB from $\mathcal{O}_{(4,2),B} = |H^\dagger D_\mu H|^2$; the diagrammatic contributions to T thus come with four powers of the Higgs vev v and hence four powers of κv (two of the Higgs vev factors are later cancelled by the M_W^{-2} in the definition of T). Therefore, $T \sim \kappa^4$ and $S \sim \kappa^2$.

Note that an indirect effect of the dimension-6 operators may be to imply shifts to the Lagrangian coupling constants (e.g., the gauge couplings) away from their SM reference values [435, 580], as we will discuss when we turn to the Higgsstrahlung process in the following section. However, since the leading results for S and T are already proportional to C_i and hence formally one-loop, any coupling shift can be dropped as a higher-order correction ($S = T = 0$ in the SM at tree-level). Moreover, the coupling constant shifts will imply a shift in the numerical values of the one-loop computations of the SM contributions to the S and T parameters, but these are again formally at least two-loop effects. Since we work to one-loop accuracy, we neglect these contributions and the value of α_e appearing in T is taken to be its SM value (we take it at the Z pole).

In Figure 4.2, we show the 95%-confidence exclusion regions in the mass parameter space

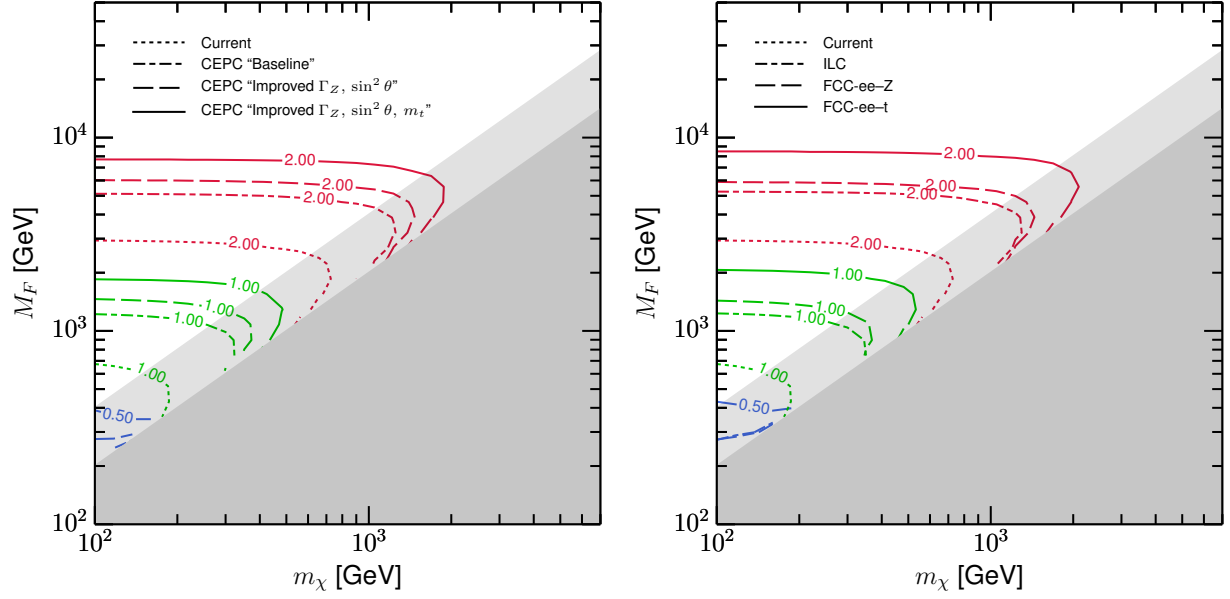


Figure 4.2: Two-parameter 95%-confidence exclusion regions ($-2\log[\mathcal{L}/\mathcal{L}_0] \gtrsim 5.99$) from measurement of the precision electroweak variables (S, T) , using results computed in the EFT. We present these as boundaries in the allowed mass parameter space for fixed representative values of κ , as annotated on each line; the unshaded region to the lower-left of these lines is excluded for the given value of κ . To compare EFT results with the full-theory calculation, see Figure 4.4. The light shaded region, $M_F/4 \lesssim m_\chi \lesssim M_F/2$, denotes the region where the EFT begins to break down: the error in the EFT result for T compared to the v^2/M_F^2 piece of the full result (i.e., the ‘dimension-6 part of’ the full result) is $\sim 20\%$ at $m_\chi \sim M_F/4$, reaches $\sim 50\%$ at $m_\chi \sim M_F/3$, and becomes $> 100\%$ before $m_\chi \sim M_F/2$. In the dark-shaded region, $m_\chi \gtrsim M_F/2$, the results have consequently been masked as they are invalid. As indicated in the legends, the various line styles correspond to the current constraints and various projected constraints on (S, T) for the proposed CEPC collider (left plot) and the proposed ILC and FCC-ee colliders (right plot); see Table 4.1.

from measurements of the electroweak precision variables (S, T) , for fixed representative values of κ . These results are computed in the EFT, and are shown for both current limits (LEP+SLD) and for the proposed CEPC, ILC and FCC-ee colliders with the sensitivities given in Table 4.1.

In the region of parameter space under consideration in Figure 4.2, the limits are driven almost exclusively by the size of the T parameter, owing to its parametric enhancement $\sim \alpha_\kappa/\alpha_e$ over the S parameter. In the regime where the EFT is valid, $m_\chi \lesssim \frac{1}{4}M_F$, the exclusion reach is also largely insensitive to the value of m_χ , as one might expect. Already

with current (LEP+SLD) constraints on S, T , masses M_F below about 675 GeV can be ruled out for $\kappa \sim 1$; due to the κ^4 dependence of T , this lower limit increases to around 2.9 TeV for a coupling $\kappa \sim 2$, but there is essentially no limit in the regime where the EFT is valid if $\kappa \sim 0.5$.

With even the ‘baseline’ proposal for the CEPC, these lower limits increase by a factor of approximately 1.75 owing to the ~ 3.5 times stronger limits on S and T in this scenario as compared to the current bounds [555]. For the CEPC scenario with improved measurements of Γ_Z and $\sin^2 \theta$ only, the lower limits on M_F increase by a factor of ~ 2 compared to the present limits, rising to ~ 1.5 TeV [6 TeV] for $\kappa \sim 1.0$ [2.0], in the limit where $m_\chi \ll M_F$. This results from the tightening in the limits on T (and S) by factor of ~ 4 compared to the present constraints [555]. For the best-case scenario for the CEPC, with improved measurements of Γ_Z , $\sin^2 \theta$, and m_t , the lower limits on M_F increase by a factor of ~ 2.7 compared to the present limits (i.e., about 30% over the previous scenario), rising to ~ 1.8 TeV [7.7 TeV] for $\kappa \sim 1.0$ [2.0], in the limit where $m_\chi \ll M_F$. This results from the tightening in the limit on T by factor of ~ 7 , and the limit in S by a factor of ~ 5 , compared to the present limits (i.e., the m_t measurement roughly halves again the measurement uncertainty on T compared to the previous scenario, and also improves the S constraint) [555].

The projected sensitivities of the ILC and FCC-ee show improvements in the lower limits on M_F which are broadly similar to the various CEPC scenarios. The ILC projection and CEPC ‘baseline’ projection are very similar, as are the FCC-ee-Z limits and the CEPC limits with improved measurements of Γ_Z and $\sin^2 \theta$ only. Finally, the FCC-ee-t lower-limits on M_F (for $\kappa \sim 1.0$ and $\kappa \sim 2.0$, these are 2.1 TeV and 8.5 TeV, respectively) are slightly stronger than the best-case CEPC scenario, but are nevertheless still broadly similar.

4.4.3 Higgsstrahlung

The computation of shifts to the Higgsstrahlung cross section σ_{Zh} is more involved than that required to obtain the S and T parameters. There are a large number of operators

which directly contribute to a shift in the cross section, both by way of contributions to wavefunction renormalization of the h and Z fields in the $hZ^\mu Z_\mu$ coupling, and by way of the introduction of additional diagrams. Furthermore, σ_{Zh} is non-zero at tree-level in the SM and so the one-loop new-physics shifts to the relationships between SM input parameters and SM Lagrangian parameter values must be accounted for here to maintain a result consistently correct to one-loop order: the standard set of SM input parameters for high-precision work is (G_F, m_Z, α_e) and the dimension-6 operators impact the processes (e.g., muon decay at $q^2 = 0$) used to relate the numerical values of these parameters to the values of Lagrangian parameters, leading to shifts in the Lagrangian coupling constants away from their SM reference values [580].

The requisite computation of σ_{Zh} has actually recently appeared in the literature in Ref. [435]. We have nevertheless independently repeated the computation of σ_{Zh} in the EFT as a cross-check; we find complete agreement with the results of Ref. [435].¹⁰ In order for us to make reference to the results of Ref. [435], we transform to the same basis of operators used there. In addition to the elimination of $\mathcal{O}_{(2,4)}$ described above, four further sets of manipulations on our basis of operators are required: (a) we re-write the operator

$$\mathcal{O}_{(4,2),B} = |H^\dagger D_\mu H|^2 = -\frac{1}{4}(H^\dagger \overleftrightarrow{D}_\mu H)^2 + \frac{1}{4}(\partial_\mu |H|^2)^2; \quad (4.18)$$

(b) we rewrite the operators $\mathcal{O}_{(DB)}$ and $\mathcal{O}_{(DW)}$, up to total derivative terms, as

$$\mathcal{O}_{(DB)} = -\frac{g_1^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}) = -g_1^2(\partial_\mu B^{\mu\nu})^2, \quad \text{and} \quad (4.19)$$

$$\mathcal{O}_{(DW)} = \text{Tr} \left[[D_\mu, \hat{W}_{\nu\rho}] [D^\mu, \hat{W}^{\nu\rho}] \right] = 2 \text{Tr} \left[[D_\mu, \hat{W}^{\mu\rho}] [D_\nu, \hat{W}^\nu_\rho] \right] + 4 \text{Tr} \left[\hat{W}_\rho^\nu \hat{W}_\nu^\mu \hat{W}_\mu^\rho \right], \quad (4.20)$$

and utilize the SM EOM for $\hat{B}_{\mu\nu}$ and $\hat{W}_{\mu\nu}$ to rewrite the first term on the RHS of each of

10. The Author thanks Nathaniel Craig for useful correspondence in resolving an error we discovered in an early preprint version (arXiv v2) of Ref. [435].

these relationships; and (c) we make two manipulations to remove $\mathcal{O}_{(4,2),C}$:

$$H^\dagger H |D_\mu H|^2 = -\frac{1}{2}(\partial_\mu |H|^2)^2 - \frac{1}{2}H^\dagger H \left[H^\dagger (D^2 H) + (D^2 H)^\dagger H \right], \quad (4.21)$$

followed by the substitution of the SM EOM to rewrite $D^2 H$. Finally, (d) we rewrite the operators $\mathcal{O}_{(B)}$ and $\mathcal{O}_{(W)}$, up to total-derivative terms, as

$$\mathcal{O}_{(B)} = \frac{1}{2}\mathcal{O}_{(BB)} + \frac{1}{2}\mathcal{O}_{(BW)} - \frac{g_1}{4} \left[H^\dagger i \overleftrightarrow{D}_\nu H \right] (\partial_\mu B^{\mu\nu}), \quad \text{and} \quad (4.22)$$

$$\mathcal{O}_{(W)} = \frac{1}{2}\mathcal{O}_{(WW)} + \frac{1}{2}\mathcal{O}_{(BW)} - \frac{g_2}{4} \left[H^\dagger i \sigma^a \overleftrightarrow{D}_\nu H \right] (D_\mu W^{a,\mu\nu}), \quad (4.23)$$

and then utilize the SM EOM for $\hat{B}_{\mu\nu}$ and $\hat{W}_{\mu\nu}$ to re-write the last term on the RHS in each line, and the SM EOM for H to re-write some factors of $D^2 H$ which appear upon doing so. This is followed by a final redefinition of the SM Higgs Lagrangian parameters, to absorb some unobservable shifts to the SM parameters which occur during this process.

When the dust settles, we are left with

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_j \frac{C_j}{M_F^2} \mathcal{O}_j^{(6)}, \quad (4.24)$$

where the dimension-6 operators $\mathcal{O}_j^{(6)}$ and Wilson coefficients C_j , correct to linear order in the original $C(\dots)$, are listed in Table 4.5, and $\mathcal{L}_{\text{SM}}$ takes the same form as the SM Lagrangian, except that all the parameters are now understood to be defined so as to absorb any of the unobservable shifts which occurred as a result of the manipulations just described.¹¹ As far as the $e^+e^- \rightarrow Zh$ process is concerned, all of the operators listed on the first page of Table 4.5 contribute, either through a shift in the SM input parameters or through the

11. These unobservable shifts can be consistently ignored; they are not the same as the physically relevant shifts which need to be accounted for in relating the input parameters to the values of the Lagrangian parameters.

Table 4.5: Operators $\mathcal{O}_j^{(6)}$ and corresponding Wilson coefficients C_j appearing in the effective Lagrangian which we utilize to compute σ_{Zh} [see Eq. (4.24)]. Only the operators listed above the horizontal line (i.e., those on the first page of this Table) contribute a shift at leading order to $e^+e^- \rightarrow Zh$, either through a shift in the relationships between the SM input parameters and the Lagrangian parameters, or through the addition of a new term in the amplitude itself. λ, μ and the Yukawa matrices Γ_i are understood to take their unobservably shifted values. We define $\hat{W}_{\mu\nu} \equiv ig_2 W_{\mu\nu}^{\tilde{a}} \tau^{\tilde{a}}$, $\hat{B}_{\mu\nu} \equiv ig_1 B_{\mu\nu} Y$, $D_{\mu}^{\leftrightarrow} \equiv D_{\mu} - \overleftarrow{D}_{\mu}$ and $\sigma^a D_{\mu}^{\leftrightarrow} \equiv \sigma^a D_{\mu} - \overleftarrow{D}_{\mu} \sigma^a$. Superscript p, q are generation indices; i, j are $SU(2)_W$ -fundamental indices; and a is an $SU(2)_W$ -adjoint index—all are summed over if repeated.

Operator $\mathcal{O}_j^{(6)}$	Wilson coefficient C_j
$\frac{1}{2}(\partial_{\mu} H ^2)^2$	$-C_{(4,2),C} + \frac{1}{2}C_{(4,2),B} + C_{(4,2),A} - \frac{3}{4}g_2^2 C_{(W)} + \frac{3}{2}g_2^4 C_{(DW)} + \frac{3}{2}g_2^2 C_{(2,4)}$
$\frac{1}{4}(H^{\dagger} \overleftrightarrow{D}_{\mu} H)^2$	$-C_{(4,2),B} - \frac{1}{2}g_1^2 C_{(B)} + g_1^4 C_{(DB)} + g_1^2 C_{(2,4)}$
$H^{\dagger} \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} H$	$C_{(WW)} + \frac{1}{2}C_{(W)} - 2C_{(2,4)}$
$H^{\dagger} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} H$	$C_{(BB)} + \frac{1}{2}C_{(B)} - 2C_{(2,4)}$
$H^{\dagger} \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} H$	$C_{(BW)} + \frac{1}{2}(C_{(W)} + C_{(B)}) - 4C_{(2,4)}$
$(H^{\dagger} \overleftrightarrow{D}_{\mu} H) (\sum_{f_L, f_R} Y_f \bar{f} \gamma^{\mu} f)$	$-g_1^4 C_{(DB)} + \frac{1}{4}g_1^2 (C_{(B)} - 2C_{(2,4)})$
$(H^{\dagger} \sigma^a \overleftrightarrow{D}_{\mu} H) (\bar{L}_L^p \gamma^{\mu} \sigma^a L_L^p)$	$-\frac{1}{2}g_2^4 C_{(DW)} + \frac{1}{8}g_2^2 (C_{(W)} - 2C_{(2,4)})$
$(\bar{L}_L^p \gamma_{\mu} \sigma^a L_L^p)^2$	$-\frac{g_2^4}{4} C_{(DW)}$

Table continued on next page ...

Table 4.5: ... continued from previous page.

Operator $\mathcal{O}_j^{(6)}$	Wilson coefficient C_j
$ H ^6$	$+C_{(6,0)} + \lambda \left(8C_{(2,4)} + 2C_{(4,2),C} - 2g_2^4 C_{(DW)} + g_2^2 C_{(W)} - 2g_2^2 C_{(2,4)} \right)$
$\text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\nu\sigma} \hat{W}_\sigma^\mu \right]$	$C_{(WWW)} + 4C_{(DW)}$
$(H^\dagger H) \left[\begin{array}{l} (\bar{L}^j \Gamma_e e) H^j + (\bar{Q}^j \Gamma_d d) H^j \\ + (\bar{Q}^j \Gamma_u u) \epsilon_{jk} (H^\dagger)^k \end{array} \right] H^j$	$+ \text{h.c.} \quad \frac{1}{2} C_{(4,2),C} - \frac{1}{2} g_2^4 C_{(DW)} + \frac{1}{4} g_2^2 C_{(W)} + \frac{1}{2} (8\lambda - g_2^2) C_{(2,4)}$
$(H^\dagger \sigma^a i \overleftrightarrow{D}_\mu H) (\bar{Q}_L^p \gamma^\mu \sigma^a Q_L^p)$	$-\frac{1}{2} g_2^4 C_{(DW)} + \frac{1}{8} g_2^2 (C_{(W)} - 2C_{(2,4)})$
$(\sum_{f_L, f_R} Y_f \bar{f} \gamma_\mu f)^2$	$-g_1^4 C_{(DB)}$
$(\bar{Q}_L^p \gamma_\mu \sigma^a Q_L^p)^2$	$-\frac{1}{4} g_2^4 C_{(DW)}$
$(\bar{L}_L^p \gamma_\mu \sigma^a L_L^p) (\bar{Q}_L^q \gamma^\mu \sigma^a Q_L^q)$	$-\frac{1}{2} g_2^4 C_{(DW)}$
$\left[\begin{array}{l} (\bar{L} \Gamma_e e)(\bar{e} \Gamma_e^\dagger L) + (\bar{Q} \Gamma_d d)(\bar{d} \Gamma_d^\dagger Q) \\ + (\bar{Q} \Gamma_u u)(\bar{u} \Gamma_u^\dagger Q) \\ + ((\bar{L} \Gamma_e e)(\bar{d} \Gamma_d^\dagger Q) + \text{h.c.}) \\ - ((\bar{L}^j \Gamma_e e) \epsilon_{jk} (\bar{Q}^k \Gamma_u u) + \text{h.c.}) \\ - ((Q^j \Gamma_d d) \epsilon_{jk} (\bar{Q}^k \Gamma_u u) + \text{h.c.}) \end{array} \right]$	$2C_{(2,4)}$

Table 4.6: A dictionary to convert our Wilson coefficient results from Table 4.4 to the Wilson coefficients defined in Ref. [435].

Wilson Coefficient in Ref. [435]	Value per our Table 4.4
c_{WW}	$-\frac{1}{8} (2C_{(WW)} + C_{(W)} - 4C_{(2,4)})$
c_{BB}	$-\frac{1}{8} (2C_{(BB)} + C_{(B)} - 4C_{(2,4)})$
c_{WB}	$-\frac{1}{8} (2C_{(BW)} + C_{(W)} + C_{(B)} - 8C_{(2,4)})$
c_H	$-C_{(4,2),C} + \frac{1}{2}C_{(4,2),B} + C_{(4,2),A}$ $+ \frac{3}{2}g_2^4 C_{(DW)} - \frac{3}{4}g_2^2 (C_{(W)} - 2C_{(2,4)})$
c_T	$-\frac{1}{2}C_{(4,2),B} + \frac{1}{2}g_1^4 C_{(DB)} - \frac{1}{4}g_1^2 (C_{(B)} - 2C_{(2,4)})$
$c_L^{(3)l}$	$-\frac{1}{2}g_2^4 C_{(DW)} + \frac{1}{8}g_2^2 (C_{(W)} - 2C_{(2,4)})$
$c_{LL}^{(3)l}$	$-\frac{1}{4}g_2^4 C_{(DW)}$
c_L^l	$\frac{1}{2}g_1^4 C_{(DB)} - \frac{1}{8}g_1^2 (C_{(B)} - 2C_{(2,4)})$
c_R^e	$g_1^4 C_{(DB)} - \frac{1}{4}g_1^2 (C_{(B)} - 2C_{(2,4)})$

addition of a new term in the amplitude itself.¹²

The results of Ref. [435] can then be used to read off the value of σ_{Zh} ; in Table 4.6, we supply an explicit dictionary to transform our Wilson coefficients to the notational conventions of Ref. [435]. In all our results, we utilize the (G_F, M_Z, α_e) input parameter set.

Apart from the independent re-computation of σ_{Zh} beginning from the basis of operators and Wilson coefficients in Table 4.5 to cross-check against the results of Ref. [435], we also verified the correctness of the fairly complicated manipulations which were used to remove the operator $\mathcal{O}_{(DW)}$ (see Eq. (4.20) and the text following) in arriving at Eq. (4.24). To do so, we augmented the SM Lagrangian with $\mathcal{O}_{(DW)}$ alone and considered its effects on σ_{Zh} directly, rather than eliminating this operator by EOM; we obtained the same results via either method.

Furthermore, since the ILC measurement projections assume a polarized electron beam

12. Note that $C_{(WW)} = C_{(BB)} = \frac{1}{2}C_{(BW)}$ and $C_{(W)} = C_{(B)}$ in our model, so the Wilson coefficients of the operators $H^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} H$ and $H^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} H$ remain equal, and equal to one-half of the Wilson coefficient of the operator $H^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} H$. This ultimately follows from the fact that there are no new Higgs-charged-charged vertices in the full theory picture, and guarantees that there is no leading-order (one-loop) correction to the $h \rightarrow \gamma\gamma$ decay rate. This would not be true if we had introduced a charged partner to the χ to maintain custodial symmetry, as we discussed earlier.

with $(P_{e-}, P_{e+}) = (-0.8, +0.3)$ [556], in the course of our independent computation, we also computed the polarized-beam cross section. In the notation of Ref. [435], the polarized cross section is obtained from the unpolarized cross section by making the following simple replacements in the quantities defined either in their eq. (2.3) or in their table 2: multiply every appearance of g_L^2 in their F_{SM} and $F_{1,2}$, as well as every appearance of g_L in their $F_{3,4}$, by $(1 - P_{e-})(1 + P_{e+})$. Similarly, multiply every appearance of g_R^2 in their F_{SM} and $F_{1,2}$, as well as every appearance of g_R in their $F_{3,5}$, by $(1 + P_{e-})(1 - P_{e+})$. No changes are needed to the factors appearing explicitly in their eq. (3.9).

In Figure 4.3 we show the regions in the mass parameter space (m_χ, M_F) which, for the given fixed value of κ , would yield a value of σ_{Zh} in conflict with the projected 95%-confidence limits as given in Table 4.1. We can see that the best sensitivity comes from the CEPC and FCC-ee. At the CEPC, the lower bounds on M_F range from 590 GeV to 7 TeV for $\kappa \sim 1.0 - 4.0$ ¹³ in the limit where $m_\chi \ll M_F$. These lower limits rise to approximately 660 GeV to 10 TeV for $m_\chi \sim M_F/4$, which is roughly where we begin questioning the validity of the EFT. (The limits on M_F in the region where the EFT results are valid are more sensitive to the value of m_χ than the electroweak precision limits.) The FCC-ee constraints are again a little stronger than CEPC constraints. Lower limits on M_F in the regime $m_\chi \ll M_F$ range from approximately 630 GeV to 7.8 TeV for $\kappa \sim 1.0$ to 4.0.

The constraints obtained at the ILC are somewhat weaker in all three of the considered scenarios than either the CEPC or FCC-ee constraints. Note that care was taken here in combining limits where runs are at different energies as $(\Delta\sigma_{Zh})/\sigma_{Zh}^{\text{SM}}$ is energy-dependent (see comment in caption of Table 4.1). The most optimistic ILC scenario with several ab^{-1} of data yields lower limits on M_F for $m_\chi \ll M_F$ that are approximately 490 GeV and 5.9 TeV, respectively, for $\kappa \sim 1.0$ and 4.0.

In none of these cases are the limits from the precision Higgsstrahlung measurement

13. $\kappa = 4.0$ is a fairly large coupling; we display these results only with the explicit point of indicating that a very large coupling is required to probe this region of parameter space.

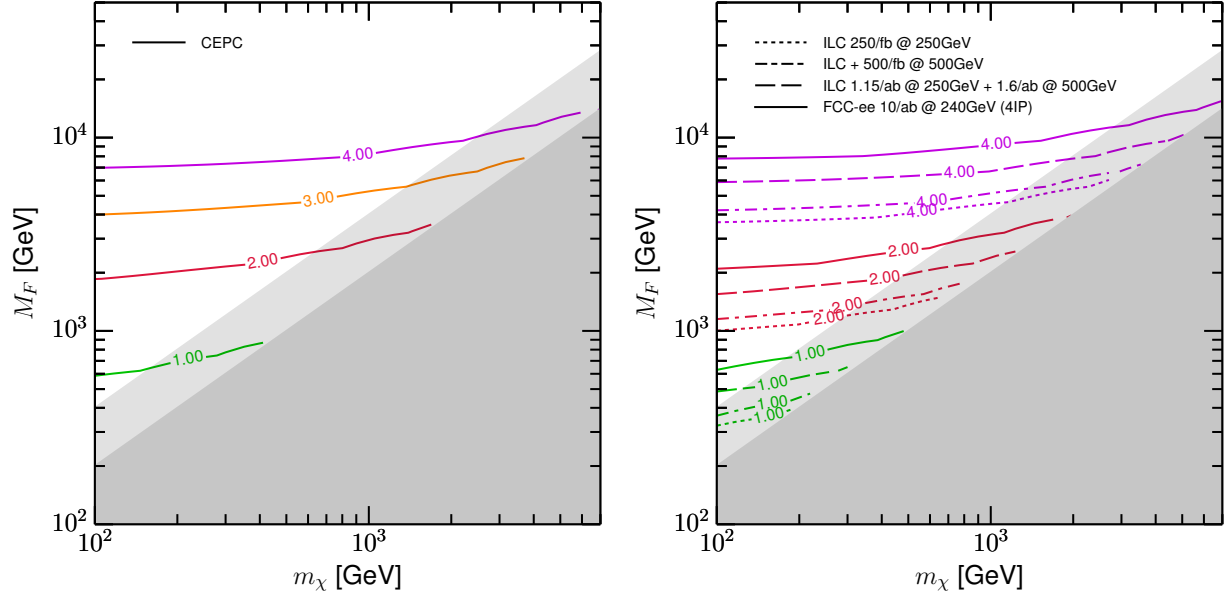


Figure 4.3: One-parameter 95%-confidence exclusion regions ($-2\log[\mathcal{L}/\mathcal{L}_0] \gtrsim 3.84$) from precision measurements of σ_{Zh} , using results computed in the EFT. We present these as boundaries in the allowed mass parameter space for fixed representative values of κ , as annotated on each line; the unshaded region to the lower-left of these lines is excluded for the given value of κ . Absent the full loop computation, it is not possible to quote an error on this EFT-based result, but based on the comparison of the EFT and full-theory computations for the EWPO results, it is probable that the EFT results here are questionable in the light shaded region, $M_F/4 \lesssim m_\chi \lesssim M_F/2$, and are almost certainly invalid in the dark-shaded region, $m_\chi \gtrsim M_F/2$, where the results have consequently been masked. See Table 4.1 for the assumed sensitivities of the experiments.

competitive with the electroweak precision programmes at these future colliders in imposing constraints on this specific model; nevertheless, these results do demonstrate that the σ_{Zh} measurement would provide a strong complimentary constraint on closely allied models where the T parameter is dialled away, as we discussed previously.

Although a large number of operators contribute to σ_{Zh} , a partial and heuristic understanding of the generic difference in the strength of these limits can be obtained by examining a subset of the operators relevant for the generation of T and $\Delta\sigma_{Zh}$. Suppose $\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{a}{2\Lambda^2}(H^\dagger D_\mu H - \text{h.c.})^2 + \frac{b}{2\Lambda^2}(\partial_\mu |H|^2)^2$. It can then easily be shown that $T = (av^2)/(\alpha_e \Lambda^2) \sim 10^2 a(v^2/\Lambda^2)$, and a little more work shows that $\Delta\sigma_{Zh}/\sigma_{Zh} \approx$

$-(v^2/\Lambda^2)(b + 0.83a)$;¹⁴ note that the two operators contribute almost equally to the Higgsstrahlung cross section. The anticipated one-parameter 95%-confidence measurement uncertainties on T (restricted to $S = U = 0$) are 2.0×10^{-2} , 1.5×10^{-2} , and 8.9×10^{-3} for the CEPC baseline, ‘Improved $\Gamma_Z, \sin^2 \theta$ ’, and ‘Improved $\Gamma_Z, \sin^2 \theta, m_t$ ’ scenarios, respectively.¹⁵ The resulting 95%-confidence lower bounds on $\Lambda/\sqrt{|a|}$ are approximately 20 TeV, 23 TeV and 29 TeV, respectively. On the other hand, the 95%-confidence measurement uncertainty on the Higgsstrahlung cross section at CEPC is projected to be $\sim 1\%$ (corresponding to the 68%-confidence projection $\Delta\sigma_{Zh}/\sigma_{Zh} = 0.5\%$ in Table 4.1), which yields the limit $\Lambda/\sqrt{|b + 0.83a|} \gtrsim 2.5$ TeV. It is clear that the latter bounds are significantly weaker than the EWPO constraints for roughly equally sized Wilson coefficients, which is the scenario one would expect when these operators are generated at the same loop order (provided of course that there is no symmetry—e.g., custodial—that prevents this). Parametrically, it is clear that the relative strength of the limits can be traced directly to the enhancement of T by a factor of $1/\alpha_e$.

4.4.4 Loop Computation of Electroweak Precision Observables

By design, the EFT we have constructed is valid only in the region of parameter space where $M_F \gg m_\chi$, and it is in this region of parameter space where this model supplies a UV completion of the fermionic Higgs Portal operator $H^\dagger H \bar{\chi} \chi$. One could of course also construct other EFTs valid in the region $m_\chi, M_F \gg v \sim m_W$, but for similar particle masses, $m_\chi \sim M_F$; or for the opposite mass hierarchy, $m_\chi \gg M_F$. Given the amount of work required to obtain the results already presented, it appears to be more efficient to simply perform the full one-loop computations of S, T ; this computation is detailed in Appendix C.1 and was performed in the mass-eigenstate basis of neutral fermions, accounting

14. The ‘0.83’ here is the numerical value of a fairly complicated function of the gauge couplings, assuming $g_1 = 0.358$ and $g_2 = 0.648$.

15. These are obtained from the values in Table 4.1 as $\sigma_{T,1 \text{ parameter}}^{95\% \text{ CL}} = 1.96 \sigma_T \sqrt{1 - \rho_{ST}^2}$.

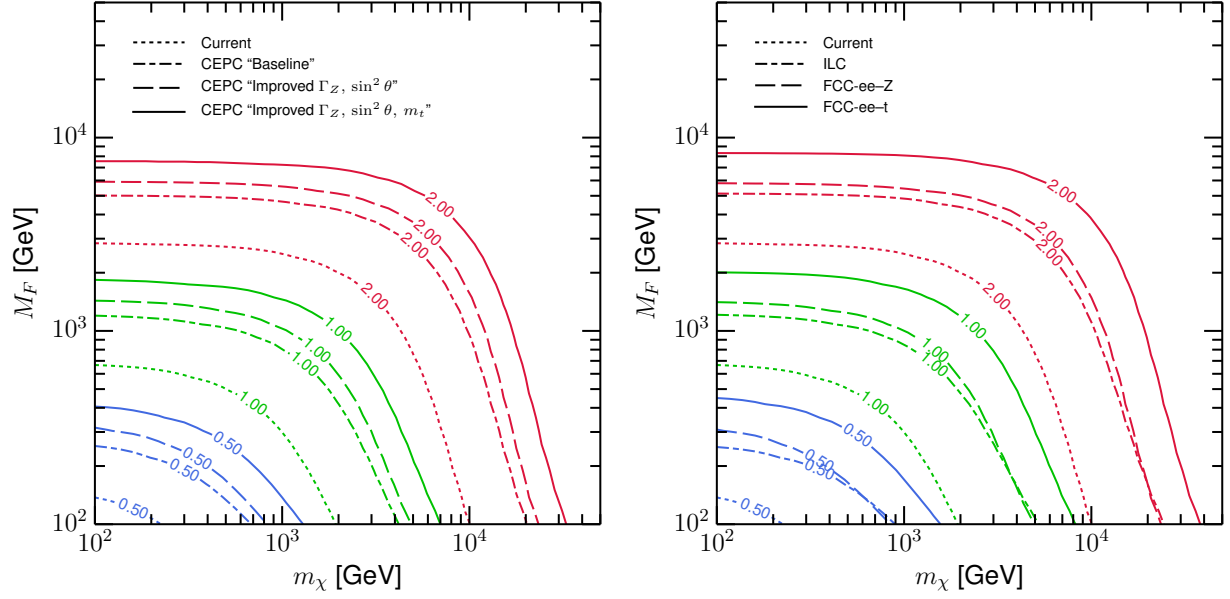


Figure 4.4: As for Figure 4.2, two-parameter 95%-confidence exclusion regions ($-2\log[\mathcal{L}/\mathcal{L}_0] \gtrsim 5.99$) from measurement of the S and T parameters, except these results are computed using the full one-loop results described in Section 4.4.4 and hence are valid for the full range of masses shown. Note that m_χ and M_F are the input mass-parameters, and are not the physical masses of the neutral fermions; see Appendix C.1.

for mixing angle effects. The full one-loop computation of σ_{Zh} is beyond the scope of the analysis presented in this chapter, as we have already established from the EFT results that the Higgsstrahlung bounds are expected to be somewhat weaker than those from EWPO.

The exclusion regions due to precision electroweak constraints arising from the full loop computation of S, T are shown in Figure 4.4. In the limit where $m_\chi \ll M_F$ it is clear that the results agree well with the EFT computation (cf. Figure 4.2), the small differences being ascribable to our neglect in the EFT of dimension-8-and-higher operators, as well as the running between scales. Note that if the full result for S or T is expanded out to $\mathcal{O}(v^2/M_F^2)$ to find the contribution to the result ascribable to dimension-6 operators, we find all the same analytic (in the masses) pieces of the result as in the EFT computation, but naturally also find non-analytic factors $\sim \log[m_\chi^2/M_F^2]$ which are absent in our EFT result as we neglected the running of the Wilson coefficients between scales $\mu = M_F$ and $\mu = m_\chi$; however, for the contributions to S and T , these are suppressed by at least the third power of m_χ/M_F and

are thus negligibly small whenever the EFT which we constructed is valid.

As we have already discussed the results in the region $m_\chi \ll M_F$ in connection with the EFT results in Section 4.4.2, we focus here on the parameter space not covered by the EFT. Of course, in this region, the theory does not provide a UV completion of the fermionic Higgs Portal operator $H^\dagger H \bar{\chi} \chi$, although in the limit $m_\chi \gg M_F$ it would provide a completion of the fermionic-doublet Higgs Portal operator $H^\dagger H \bar{F} F$.

The most obvious point is that a significantly larger region of the m_χ parameter space at small M_F is ruled out than vice versa. For example, the fully improved CEPC results indicate that if $M_F = 100$ GeV, then $m_\chi < 33$ TeV can be constrained for $\kappa \sim 2.0$, compared to $M_F < 7.7$ TeV being constrained if $m_\chi = 100$ GeV. This pattern is generic for all of the experiments; it traces its origin to the fact that for $m_\chi \ll M_F$, S and T are both positive and, roughly speaking, $|S| \sim 0.1|T|$ near the exclusion limit, whereas for $M_F \ll m_\chi$, T is positive and S is negative, with $|S| \sim |T|$ near the exclusion limit (indeed, $|S| > |T|$ is possible here). The larger deviation from $(S, T) = (0, 0)$ in this region leads to the stronger limits. There is also a stronger dependence of the boundary of the exclusion region on M_F at large m_χ than vice versa. Both of these behaviours are due to the fact that, parametrically, $S \sim \Delta M/M_F$ where ΔM is the mass-splitting in the doublet: $\Delta M \equiv M_F^{\text{neutral}} - M_F^{\text{charged}} = v(\kappa^2/2)[v/(M_F - m_\chi)]$. This parametric dependence arises because S is zero in the weak-isospin-symmetric limit of the theory, and because it is always F fermions which couple directly to W and Z . This implies that the leading contributions are $S \sim \kappa^2 v^2/M_F^2$ for $m_\chi \ll M_F$, but $S \sim -\kappa^2 v^2/(M_F m_\chi)$ for $M_F \ll m_\chi$; clearly then, at light M_F and heavy m_χ , $|S|$ will be larger than for the opposite mass hierarchy.

The slightly different shapes of the ILC and FCC-ee exclusion regions when $m_\chi \gtrsim M_F$, as seen in the right plot of Figure 4.4, arise from the different alignments of the 68%-coverage likelihood contours in the (S, T) plane [555] for the ILC compared to those for FCC-ee, which in turn is due to the different projected sensitivities to m_W , $\sin^2 \theta_{\text{eff}}^l$ and Γ_Z at each collider (see also Ref. [581]). With reference to figure 1 in Ref. [555], it is clear that if $S, T > 0$

and $|S| \ll |T|$ (as is the case for $m_\chi \ll M_F$), the projected limits for FCC-ee-Z¹⁶ are more constraining than those for the ILC; whereas if $T > 0$ and $S \sim -T$ (as is the case for $M_F \ll m_\chi$), the projected ILC and FCC-ee-Z limits are about equally constraining, with the ILC actually marginally better, as is visible in the right panel of our Figure 4.4 for $M_F \lesssim 150$ GeV.

4.5 Conclusions

In this chapter we examined two possible models which UV-complete the CP-even fermionic Higgs Portal operator $H^\dagger H \bar{\chi} \chi$, and investigated how future precision electroweak and precision Higgsstrahlung measurements at the proposed ILC, FCC-ee and CEPC high-energy e^+e^- colliders could constrain these models. In the first model, the ‘scalar completion’, a scalar singlet acts as a mediator between the vector-like Dirac fermion χ and the SM Higgs field H , with a Yukawa coupling $\kappa_S S \bar{\chi} \chi$ to the χ and a coupling $a m_S S |H|^2$ to the Higgs doublet. In the second model, the ‘fermionic completion’, a vector-like $SU(2)_W$ doublet of Dirac fermions couples to the χ and H field with a Yukawa-like interaction $\kappa \bar{F} H \chi + \text{h.c.}$

For the scalar completion, we estimated that the effect of the new χ particle will be subdominant (i.e., loop-suppressed) for the computation of the electroweak precision variables S and T , and the Higgs couplings to the SM particles, as compared to the dominant effect of the singlet mediator itself. The limits which can be placed on the SM augmented with such a scalar-singlet mediator have already been extensively studied in the literature (e.g., Ref. [560]): a precision Higgsstrahlung cross section measurement $(\Delta\sigma_{Zh})/\sigma_{Zh} \sim 0.5\%$ can place a 95%-confidence upper limit on m_S/a around 2.5 TeV, while the precision electroweak limits are only marginally weaker (despite S and T being only loop-induced by running).

Our main focus in this chapter was on the fermionic completion. We constructed an EFT valid in the limit $v \lesssim m_\chi \ll M_F$ and examined the 95%-confidence exclusion reach on the mass parameter space, for a variety of projected sensitivities for the precision electroweak

16. Denoted as TLEP-Z in Ref. [555].

and Higgsstrahlung measurements at future colliders. Provided the coupling constant κ is $\mathcal{O}(1)$, we found that the precision electroweak limits are very powerful and primarily driven by $T \sim (\alpha_\kappa/\alpha_e)S \gg S$, owing to the violation of custodial symmetry in this model. Already with current limits, the 95%-confidence exclusion reach on M_F for $m_\chi \ll M_F$ is up to $M_F \sim 675 \text{ GeV}$ [2.9 TeV] for $\kappa = 1.0$ [2.0]. For the most optimistic projections we consider for the various possible configurations of the ILC, CEPC and FCC-ee colliders, these lower limits rise to 1.2 TeV [5.3 TeV], 1.8 TeV [7.7 TeV], and 2.0 TeV [8.5 TeV], respectively (see Figure 4.2).

For this model, the precision electroweak limits are generically more powerful than the precision Higgsstrahlung cross section measurement due to the parametric enhancement of T over $(\Delta\sigma_{Zh})/\sigma_{Zh}$ by $\sim 1/\alpha_e$. Nevertheless, precise measurement of the latter also yields good complementary exclusion reach, which is useful because closely allied UV completions in which the T parameter can be made small will still be fairly strongly constrained by this measurement (other probes, such as $h \rightarrow \gamma\gamma$, may also become important). The most optimistic 95%-confidence exclusion lower limits from Higgsstrahlung on M_F , in the limit $m_\chi \ll M_F$, for the various possible configurations of the ILC, CEPC and FCC-ee colliders, are 490 GeV [1.5 TeV], 590 GeV [1.9 TeV], and 620 GeV [2.1 TeV], for $\kappa = 1.0$ [2.0], respectively (see Figure 4.3).

We also considered the full one-loop computation of the electroweak precision observables in the more general mass parameter space, where this model does not necessarily provide a UV completion for the CP-even fermionic Higgs Portal as we have defined it. The one-loop computation was found to agree well with the EFT computation for $m_\chi \ll M_F$. The model is significantly more constrained for the ‘opposite’ mass hierarchy $m_\chi \gg M_F$ owing to the fact that S and T are comparably large in this region; for example, we find 95%-confidence lower limits on m_χ for $M_F = 100 \text{ GeV}$ are typically a factor of 3–5 times higher than the corresponding lower limits on M_F for $m_\chi = 100 \text{ GeV}$ (see Figure 4.4).

Additionally for the fermionic completion, the appearance in the EFT analysis of opera-

tors modifying the coupling of the Higgs to SM fermions (see Table 4.5) raises the prospect of a modification to the rate for $h \rightarrow b\bar{b}$, which will be measured to fairly high accuracy at lepton colliders; we defer investigation of this point to future work.

Overall, we see that the sensitivities possible at future e^+e^- machines through measurements of precision electroweak observables and the Higgsstrahlung cross section will allow significant improvements in the exclusion reach for the CP-even fermionic Higgs Portal over current limits, pushing into the (multi-)TeV range of particles masses, well beyond the direct reach of the LHC.

CHAPTER 5

RELAXATION OF THE COMPOSITE HIGGS LITTLE HIERARCHY

This chapter contains material from Ref. [7]: B. Batell, M. A. Fedderke, and L.-T. Wang, *Relaxation of the Composite Higgs Little Hierarchy*, [arXiv:1705.09666](#).¹

We describe a composite Higgs scenario in which a cosmological relaxation mechanism naturally gives rise to a hierarchy between the weak scale and the scale of spontaneous global symmetry breaking. This is achieved through the scanning of sources of explicit global symmetry breaking by a relaxion field during an exponentially long period of inflation in the early universe. We explore this mechanism in detail in a specific composite Higgs scenario with QCD-like dynamics, based on an ultraviolet $SU(N)_{\text{TC}}$ ‘technicolour’ confining gauge theory with three Dirac technifermion flavours. We find that we can successfully generate a hierarchy of scales $\xi \equiv \langle h \rangle^2 / F_\pi^2 \gtrsim 1.2 \times 10^{-4}$ (i.e., compositeness scales $F_\pi \sim 20$ TeV) without tuning. This evades all current precision bounds on our (custodial violating) model. While directly observing the heavy composite states in this model will be challenging, a future electroweak precision measurement programme can probe most of the natural parameter space for the model. We also highlight signatures of more general composite Higgs models in the cosmological relaxation framework, including some implications for flavour and dark matter.

5.1 Introduction

The cosmological relaxation scenario of Graham, Kaplan, and Rajendran [413] provides a novel approach to the Hierarchy Problem of the Standard Model (SM). In this scenario, the vacuum expectation value (vev) of an axion field [97, 98], dubbed the relaxion, slowly rolls

1. Ref. [7] is copyright © 2017 by the Authors.

through a trans-Planckian excursion down a very flat shift-symmetry-breaking potential during an exponentially long period of low-scale inflation, in the process dynamically ‘scanning’ the value of the Higgs squared-mass parameter. Although the bare Higgs squared-mass parameter can be assumed natural (i.e., positive and of the order of the cutoff), it is eventually scanned through zero, triggering spontaneous electroweak symmetry breaking (EWSB). The breaking of electroweak symmetry gives rise to a back-reaction² on the flat relaxion scanning potential which, combined with energy dissipation from Hubble friction, stalls the relaxion rolling, dynamically locking in a small, technically natural value of the Higgs vev and Higgs mass. While some of the required ingredients may appear rather exotic from an effective field theory perspective, the scenario offers a fresh perspective on the Hierarchy Problem, in the spirit of the self-organized criticality proposal of Ref. [343], and is worthy of further exploration (see Refs. [582–606] for some recent studies). See also Refs. [410–412, 414, 415] for other cosmological approaches to Naturalness.

The relaxion models presented in Ref. [413] are based on the QCD axion, or an extended strong dynamics with a non-QCD axion. These simple models are able to extend the cutoff of the SM to scales that are parametrically larger than the weak scale, but still well below the GUT or Planck scales. In other words, these simple models are not able to fully address the ‘Big’ Hierarchy Problem, but instead can offer a solution to the ‘Little’ Hierarchy Problem—i.e., a way to understand the absence of new particles at the LHC, as well as deviations from the SM predictions in precision flavour, electroweak, and Higgs measurements. While it is possible that more complex models (perhaps with additional scanning fields [582]) can extend the cutoff further into the ultraviolet (UV) and perhaps even all the way to the Planck scale, one can also imagine that the new physics that emerges at the cutoff is of a more conventional type, such as supersymmetry or compositeness, which shields the Higgs against arbitrary short-distance physics. Supersymmetric completions of the relaxion were

2. In the simplest realization [413], the back-reaction is supplied by the emergence of the periodic QCD vacuum potential [40–48] following EWSB, with $V_{\text{QCD}} \propto m_\pi^2 f_\pi^2 \propto m_{u,d} \propto v$.

investigated in Refs. [588, 593].

In this chapter, we consider the relaxion scenario in the context of composite Higgs (CH) models [401, 607–611] (see Ref. [399] for a recent review). In such models, the Big Hierarchy Problem is ameliorated by the assumption that the Higgs is a composite object of heavy ‘technifermions’ bound together by the agency of strong ‘technicolour’ (TC) gauge dynamics [612–614]. Above the confinement scale, the theory is one of free fermion constituents whose masses are technically natural. As in QCD, dimensional transmutation accounts for the hierarchy between the ultimate cutoff scale (e.g., the GUT or Planck scale) and the confinement scale of the composite theory. Below the confinement scale, the theory is that of the pseudo-Nambu–Goldstone bosons (pNGBs) [14–19]—four of which comprise the Higgs doublet—of a spontaneous global symmetry breaking triggered by the strong dynamics when it confines. Due to explicit global symmetry breaking, the pNGB Higgs develops a potential, and vacuum misalignment arguments dictate that the Higgs vev in such models is expected to be of the same order as the compositeness scale, whereas phenomenological viability of CH models demands that the Higgs vev, $\langle h \rangle = 246$ GeV, should lie somewhat below the compositeness scale, F_π . This is summarized by the well-known requirement

$$\xi \equiv \langle h \rangle^2 / F_\pi^2 \ll 1, \quad (5.1)$$

which encapsulates the Little Hierarchy Problem in CH models.

Our aim in this chapter is to demonstrate, within the context of an explicit CH model, that a large hierarchy between the weak scale and the global symmetry breaking scale, Eq. (5.1), can be achieved in a technically natural fashion by invoking the cosmological relaxation mechanism. The essential feature is that as the relaxion evolves in the early universe, it scans the techniquark masses, which provide a source of explicit global symmetry breaking. Since the Higgs potential is controlled by such explicit symmetry breaking, this manifests in the low energy effective theory as a scanning of the Higgs potential, allowing

the relaxation mechanism to be implemented in a manner similar to Ref. [413].

UV completions of CH models based on strong technicolour dynamics generally give rise to the cosets $SU(N_F)/SO(N_F)$, $SU(N_F)/Sp(N_F)$, and $[SU(N_F) \times SU(N_F)]/SU(N_F)$, when N_F technifermions are in a real, pseudoreal, or complex representation, respectively, of the technicolour gauge group [615, 616]. While the relaxation mechanism can be implemented with any of these cosets, we will construct and investigate a concrete model with QCD-like dynamics, based on an $SU(N)_{\text{TC}}$ gauge group with $N_F = 3$ Dirac flavours (an ‘ $L + N$ ’ model). This leads to the global symmetry breaking pattern $SU(3) \times SU(3) \times U(1) \rightarrow SU(3) \times U(1) \supset SU(2)_W \times U(1)_Y$. Indeed, this theory can in many ways be viewed as a scaled up copy of QCD. Interestingly, this is the smallest in the class of QCD-like cosets, $[SU(N_F) \times SU(N_F)]/SU(N_F)$, which furnishes a Higgs doublet. However, this coset is not usually considered for CH models since it does not contain the custodial symmetry group, $SU(2) \times SU(2)$ [571], which protects against large tree-level corrections to the electroweak precision T parameter [552–554]. In our scenario, however, this coset can indeed be viable since the relaxation mechanism will naturally generate the large hierarchy in Eq. (5.1), allowing the T parameter to be adequately suppressed and compatible with precision electroweak measurements.

In our construction, the relaxion is taken to have axion-like couplings to both the technicolour $SU(N)_{\text{TC}}$ gauge group and the QCD $SU(3)_C$ gauge group. An appropriate chiral rotation of the technifermion fields leads to a coupling of the relaxion to the techniquark masses. As the techniquark mass terms explicitly break the global symmetry and contribute to the composite Higgs potential, this coupling provides the basis for the scanning mechanism. We construct the low-energy Chiral Lagrangian, taking into account the large radiative corrections to the Higgs potential due to the top quark, and show that the potential contains the requisite relaxion–Higgs couplings to effect electroweak symmetry breaking and halt the relaxion evolution once it dynamically rolls through some critical value. The strong-CP problem is addressed as in Ref. [413] with a slope-drop mechanism: we assume the scanning

potential for the relaxion arises from a coupling to the inflaton, such that it dominates the rolling during inflation but disappears post-inflation, allowing the effective QCD θ -angle to relax to small values.

By design, the relaxation mechanism pushes the dynamics stabilizing the weak scale to higher scales, making experimental confirmation of the scenario more challenging. While there is no guarantee that the framework can be fully tested with near-term experiments, there are certainly some experimental opportunities worth pursuing. In the specific model studied here, the spectrum of the pNGB sector in our model consists of a light composite Higgs state, and four additional ultra-heavy composite technimesons, which are either neutral or only charged under the electroweak (EW) gauge group. It will be challenging to directly probe such a heavy spectrum, even at proposed future hadron colliders such as the SPPC [382] or FCC-hh [389–391]. More promisingly, future improvements in the measurements of electroweak precision observables (EWPO) such as the T parameter at the ILC [383–387], CEPC [382], or FCC-ee [388] have the potential [555] to probe this model over most of the natural parameter space. More generally, there is potentially a diverse set of experimental probes for this and other composite Higgs theories within the cosmological relaxion framework, including tests of flavour and CP violation, electroweak precision measurements, dark-matter and axion searches, and collider searches for new states. We will highlight some of these opportunities.

Our work is not the first to consider the cosmological relaxation mechanism of Ref. [413] in the context of composite Higgs models; however, important details of our model differ significantly from previous work [584, 605]. In particular, we consider in our work only a single composite Higgs doublet, whereas Refs. [584, 605] both analyze Type-I Two Higgs Doublet Models with one elementary and one composite Higgs doublet. Another major difference is that we require the additional axion-like coupling of the relaxion to QCD to stall its rolling, following Ref. [413], whereas in Refs. [584, 605], the relaxion rolling is stalled by virtue of the Higgs-vev-dependent barriers for ϕ that appear as a result of the non-QCD

strong gauge dynamics.

The rest of this chapter is structured as follows: we begin in Section 5.2 with a discussion of a simplified ‘cartoon’ picture of the mechanism we wish to explore in this chapter, in order to orient the reader before we delve into the detailed construction and analysis of our explicit model; in this section we also review both the post-inflation slope-drop mechanism of Ref. [413] as it applies to our model to solve the strong-CP problem, and the clockwork mechanism [436, 437] that may potentially generate the requisite super-Planckian axion decay constants for our model. In Section 5.3 we begin our explicit model construction, by presenting the constituent UV model that defines the underlying theory for the CH sector; we also specify the effective four-fermion interactions that give rise to the CH Yukawa couplings, specify the relaxion sector, and make some initial manipulations to the model to allow construction of the low-energy Chiral Lagrangian. In Section 5.4 we explicitly construct the Chiral Lagrangian describing the composite states of the theory defined in Section 5.3, and we extract those terms from the Chiral Lagrangian which are required to obtain the spectrum of the theory and understand its vacuum structure. Section 5.5 contains our detailed analysis of the effective potential for the model, along with our analysis of the properties of the broken and symmetric electroweak phases of the theory. The relaxion potential is discussed in Section 5.6. We present a summary of our analytical results and a numerical investigation of the model parameter space in Section 5.7. A general discussion of some additional phenomenologically interesting considerations applicable to both our model, and more general composite Higgs models with large F_π , is given in Section 5.8. We conclude in Section 5.9. Appendix D.1 gives the closed-form expression for a general exponentiated $SU(3)$ matrix, which is of some utility in our analysis. Appendix D.2 contains a more general analysis of the EWSB dynamics of our full model, in which we relax one of the simplifying assumptions made in Section 5.5.

5.2 A Simplified ‘Cartoon’ Model

In order to facilitate a better understanding of the ideas we will explore in this chapter, we present in this section a cartoon picture of the mechanism that we develop in greater detail in the sections to follow.

In order to successfully implement the relaxion mechanism of Ref. [413] in a composite Higgs model, we need to engineer three essential components: (a) a CH–relaxion coupling, (b) a potential for the relaxion which is sufficiently flat and which causes the field to slow-roll in the correct direction in field space to trigger dynamical EWSB, and (c) a mechanism to create barriers in the relaxion potential that stall its slow-roll once EWSB is triggered.

We will achieve (a) and (c) by assuming that the relaxion ϕ is an axion of both the strongly coupled TC gauge group that confines to yield the composite Higgs state, and of QCD (see Section 5.2.5):

$$\mathcal{L} \supset \frac{g_s^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}} \right] \text{Tr} [G_{\mu\nu} \tilde{G}^{\mu\nu}] + \frac{g_{\text{TC}}^2}{16\pi^2} \frac{\phi}{F} \text{Tr} [G_{\text{TC}\mu\nu} \tilde{G}_{\text{TC}}^{\mu\nu}], \quad (5.2)$$

where f and F are dimensionful parameters with $F \gg f$.

Appropriate to the level of our cartoon picture in this section, we will discuss only an approximate low-energy pseudo-Nambu–Goldstone boson (pNGB) description of the composite sector of the theory, via the Chiral Lagrangian. If $\mathcal{U}$ is the matrix-valued field of pNGBs which include among them the composite Higgs state h , then once the axion-type couplings are rotated into the technifermion mass matrices in the underlying constituent theory, and we include the large, dominant radiative effects of the top quark,³ the following terms will

3. There are also subdominant effects from gauge loops. These will not change the qualitative picture we explore in this chapter, and we ignore them.

appear in the effective potential for the model:

$$V \sim -c_m \Lambda_{\text{TC}} F_\pi^2 \text{Tr} [M \mathcal{U} e^{i\phi/F} + \text{h.c.}] - \frac{c_t y_t^2 N_c \Lambda_{\text{TC}}^2 F_\pi^2}{16\pi^2} |\text{Tr} [\mathcal{U} \cdot \Delta]|^2 + V_\phi(\phi) + V_{\text{QCD}} \quad (5.3)$$

$$\sim -c_m \Lambda_{\text{TC}} F_\pi^2 m \cos\left(\frac{h}{F_\pi}\right) \cos\left(\frac{\phi}{F}\right) - \frac{c_t y_t^2 N_c \Lambda_{\text{TC}}^2 F_\pi^2}{16\pi^2} \sin^2\left(\frac{h}{F_\pi}\right) + V_\phi(\phi) + V_{\text{QCD}}, \quad (5.4)$$

where c_t and c_m are perturbatively incalculable $\mathcal{O}(1)$ numbers, F_π is the compositeness scale associated with spontaneous global flavour symmetry breaking, $\Lambda_{\text{TC}} \approx (4\pi/\sqrt{N})F_\pi$ is the cutoff scale of the composite theory, and we have taken m to be a representative mass of the technifermions. Furthermore, in Eq. (5.4), $V_\phi(\phi)$ is an additional relaxion potential which will be discussed in Section 5.2.3, and Δ is the appropriate projection operator that extracts the part of $\mathcal{U}$ to which the top quark couples (i.e., the Higgs doublet). We also emphasize that Eq. (5.4) is highly schematic—much of the development in the following sections is precisely to deal with the more complicated structures that actually appear when evaluating Eq. (5.3) in a realistic theory. Nevertheless, this simplified picture captures the essential features of the model. It also suffices for the present discussion to merely assume that V_{QCD} is a cosine periodic potential:

$$V_{\text{QCD}} \sim -\Lambda^4 \cos\left(\frac{\phi}{f} - \theta_{\text{QCD}}\right), \quad (5.5)$$

where Λ^4 depends linearly on the Higgs vev $\langle h \rangle$ through its dependence on the quark masses m_q : $\Lambda^4 \sim m_\pi^2 f_\pi^2 \propto m_q \propto \langle h \rangle$.

5.2.1 Electroweak Symmetry Breaking

The dynamical picture to bear in mind is that while the relaxion is slow-rolling down its potential during an exponentially long period of low-scale inflation, the other fields respond by assuming their instantaneous equilibrium vacuum expectation values, such that the effective potential is minimized with ϕ held fixed. Therefore, before we return to a discussion of the

relaxion rolling, consider first the dynamics of the h field per Eq. (5.4); we will ignore the dynamics of any other states in the theory—this topic will consume much of our attention in the concrete model we analyse in the following sections.

If we define $\cos(\phi_{\text{crit.}}/F) \equiv (c_t y_t^2 N_c \Lambda_{\text{TC}})/(8\pi^2 c_m m) > 0$ [we assume $c_t, c_m > 0$], then the minimization condition for the potential Eq. (5.4) in the h -direction is

$$\partial_h V \propto \sin\left(\frac{\langle h \rangle}{F_\pi}\right) \left[\frac{\cos(\phi/F)}{\cos(\phi_{\text{crit.}}/F)} - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \right] = 0, \quad (5.6)$$

where we have ignored correction terms $\sim \Lambda^4/(\Lambda_{\text{TC}} F_\pi^2 \langle h \rangle) \ll 1$. We see that $\langle h \rangle = 0$ is always a solution to Eq. (5.6); whether or not there are other solutions in the region near $\langle h \rangle/F_\pi \approx 0$ depends on the relative sizes of $\cos(\phi/F)$ and $\cos(\phi_{\text{crit.}}/F)$. For $\cos(\phi/F) > \cos(\phi_{\text{crit.}}/F)$, the $[\dots]$ -bracket in Eq. (5.6) cannot be set to zero for any value of h , so no additional solution(s) exists; however, for $\cos(\phi/F) < \cos(\phi_{\text{crit.}}/F)$, two additional solutions to Eq. (5.6) appear symmetrically around $h = 0$. A graphical sketch of the situation is shown in the left panel of Figure 5.1, both for $\cos(\phi/F) > \cos(\phi_{\text{crit.}}/F)$ and vice versa. Clearly, if ϕ rolls to larger values from some initial value near $\phi \sim 0$, as it crosses $\phi_{\text{crit.}}$ it triggers a dynamical destabilization of the $h = 0$ solution leading to a dynamically generated spontaneous EWSB. Per the mechanism developed in Ref. [413], once the h field gets a non-zero vev $\langle h \rangle$, the QCD barriers grow in size and rapidly stall the slow-roll of the relaxion field ϕ in the vicinity of $\phi_{\text{crit.}}$, locking in a small, technically natural $\langle h \rangle$.

Note that for the discussion in the previous paragraph to work, we must demand that $0 < \cos(\phi_{\text{crit.}}/F) \leq 1$, which implies a lower bound on the masses of the fermions charged under the strong dynamics (assuming that $c_t, c_m > 0$):

$$m \geq N_c \frac{c_t}{c_m} \frac{y_t^2}{8\pi^2} \Lambda_{\text{TC}}. \quad (5.7)$$

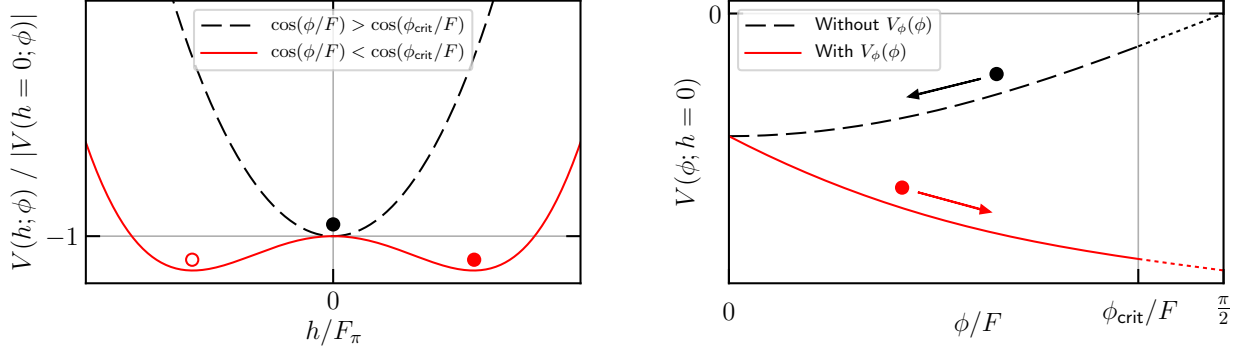


Figure 5.1: **Left panel:** a sketch plot of the potential for the Higgs at fixed ϕ -field value, $V(h; \phi)$. For $\phi < \phi_{\text{crit.}}$, the potential $V(h; \phi)$ has a stable minimum at the origin $h = 0$, yielding an EW-symmetric vacuum (dashed black line). For $\phi > \phi_{\text{crit.}}$, dynamical EWSB is triggered, and the Higgs field h rolls to a minimum displaced from the origin (solid red line). The dots indicate the stable minima; the EWSB minima are degenerate, and we select $h > 0$ by convention. **Right panel:** a sketch plot of the relaxion potential in the EW-symmetric phase, $V(\phi; h = 0)$. The solid (red) line shows the potential with the $V_\phi(\phi)$ term (plotted here for $\gamma \sim 1$; see Section 5.2.3), and the dashed (black) line shows the potential without the $V_\phi(\phi)$ term [in both cases, the lines are dotted in the region $\phi \in [\phi_{\text{crit.}}, \pi/2]$, as the EW-symmetric ($h = 0$) potential is not relevant for the ϕ rolling after EWSB had been triggered]. As indicated by the respective dots and arrows, without the $V_\phi(\phi)$ term, the relaxion field naturally wants to roll from its initial value back toward the origin at $\phi = 0$; with the $V_\phi(\phi)$ term, the rolling direction is reversed: ϕ will roll toward $\phi_{\text{crit.}}$, triggering dynamical EWSB.

5.2.2 The Higgs Mass

Some mild residual tuning is required to obtain the correct Higgs mass. From Eqs. (5.4) and (5.6), it is straightforward to derive that⁴

$$m_h^2 = 4c_t \left(\frac{N_c}{N} \right) \left[\frac{y_t}{\sqrt{2}} \langle h \rangle \text{sinc} \left(\frac{\langle h \rangle}{F_\pi} \right) \right]^2 = 4c_t \left(\frac{N_c}{N} \right) m_t^2. \quad (5.8)$$

Therefore, assuming that the relaxion mechanism has already selected the correct value of the Higgs vev $\langle h \rangle$ (and by implication the correct top mass m_t), the residual tuning can be estimated by comparing the expected $c_t \sim \mathcal{O}(1)$ with the required

$$c_t \sim \left(\frac{N}{N_c} \right) \left(\frac{1}{2} \frac{m_h}{m_t} \right)^2 \sim 0.1 \left(\frac{N}{3} \right). \quad (5.9)$$

4. We remind the reader that $\text{sinc}(x) \equiv \sin(x)/x \simeq 1 - x^2/6 + \mathcal{O}(x^4)$ for small x .

The residual tuning for the Higgs mass is thus at worst on the order of an additional one-in-ten tuning, and may even be much milder if $N \sim 10$.

5.2.3 The Relaxion Potential

In order to achieve condition (b) and obtain a sufficiently flat relaxion potential which also drives the relaxion field toward the critical value, we will need to add in an extra potential term for the relaxion, $V_\phi(\phi)$. This is illustrated in the right panel of Figure 5.1: without an additional term $V_\phi(\phi)$ in the potential, during the EW-symmetric phase the relaxion would roll in the incorrect direction in field space to give rise to the dynamical EWSB we have just discussed. Following Ref. [413], we thus add a linear potential for the relaxion, to obtain the correct rolling. For convenience, we will choose to parametrize this term as $V_\phi(\phi) = -\gamma(c_m\Lambda_{\text{TC}}F_\pi^2m/F)\phi$, where γ is an entirely free parameter of as-yet-unknown size controlling the slope of the linear potential. In the EW-symmetric phase, we would then have

$$V(\phi; h = 0) \sim -c_m\Lambda_{\text{TC}}F_\pi^2m \left[\cos\left(\frac{\phi}{F}\right) + \gamma\frac{\phi}{F} \right]; \quad (5.10)$$

assuming slow-roll in the EW-symmetric phase this yields

$$\partial_t\phi \propto -\partial_\phi V(\phi; h = 0) = c_m\frac{\Lambda_{\text{TC}}F_\pi^2m}{F} \left[\gamma - \sin\left(\frac{\phi}{F}\right) \right], \quad (5.11)$$

which provides a lower bound $\gamma \gtrsim 1$ such that $\partial_t\phi > 0$ for $\phi \in [0, \phi_{\text{crit.}}]$. With an appropriate V_ϕ added in, the rolling direction is now correct, as is illustrated again in the right panel of Figure 5.1 (which is shown schematically for $\gamma \sim 1$).

In order for the QCD barriers to be effective in stopping the rolling of the relaxion field at $\phi^* = \phi_{\text{crit.}} + \delta\phi$ where $0 < \delta\phi \ll \phi_{\text{crit.}}$, the slope of the QCD barriers and the ‘relaxion rolling

slope' must match approximately at the stopping point during inflation. If we naïvely⁵ were to consider that $\gamma \sim 1$ then, up to $\mathcal{O}(1)$ numbers,

$$c_m \frac{\Lambda_{\text{TC}} F_\pi^2 m}{F} \sim \frac{\Lambda^4}{f} \quad \Rightarrow \quad F \sim c_m \frac{\Lambda_{\text{TC}} F_\pi^2 m}{\Lambda^4} f \quad (\gamma \sim 1). \quad (5.12)$$

Taking $\Lambda_{\text{TC}} \sim 80 \text{ TeV}$, $F_\pi \sim \Lambda_{\text{TC}}(\sqrt{N}/4\pi) \sim 20 \text{ TeV}$ for $N \sim 10$, $m \sim 3 \text{ TeV}$, $\Lambda \sim \sqrt{m_\pi f_\pi} \approx 110 \text{ MeV}$,⁶ and $c_m = 1$, this implies that $F \sim (7 \times 10^{20}) f$. This means that with the usual QCD Peccei–Quinn [99–104] symmetry breaking scale $f \sim 10^{11} \text{ GeV}$, this model will require $F \sim 7 \times 10^{31} \text{ GeV}$ if we want the compositeness scale on the order of 20 TeV; we will comment on the viability of such a large dimensionful scale ($F \gg m_{\text{Pl.}}$) in Section 5.2.5.

However, we know from Ref. [413] that using the QCD barriers to stop the relaxion rolling results in a severe strong-CP problem if a significant non-QCD relaxion slope persists to the present day, because the stopping point for the relaxion is displaced from the minimum of the QCD potential. In order to alleviate this constraint, we will utilize the mechanism of post-inflation slope-drop proposed in Ref. [413]: the basic idea of this mechanism is that the slope of the relaxion scanning potential $V_\phi(\phi)$ should originate via a coupling to a field σ during inflation, $\gamma = \gamma(\sigma)$, such that $\gamma = \gamma_i$ during inflation, but when the σ field rolls to end inflation, it causes the slope of the scanning potential to disappear: $\gamma \rightarrow 0$. Thus, if we were to naïvely assume that $\gamma_i \sim 1$ during inflation, and that slope-drop mechanism sends $\gamma \rightarrow 0$ at the end of inflation as σ rolls, the strong-CP problem would not be alleviated (see Figure 5.2).

Suppose then that instead of considering the parameter regime $\gamma_i \sim 1$, we consider $\gamma_i \gg 1$, such that the relaxion rolling slope is entirely dominated by the linear contribution from the coupling to the σ -field. Then the estimate Eq. (5.12) is modified by an additional

5. Since the dynamical origin of the additional slope will in general be different from the strong TC dynamics, $\gamma \sim 1$ would appear to require some accidental coincidence.

6. We used the QCD neutral pion mass $m_\pi \approx 135 \text{ MeV}$ [38], and the QCD pion decay constant $f_\pi \approx 93 \text{ MeV}$.

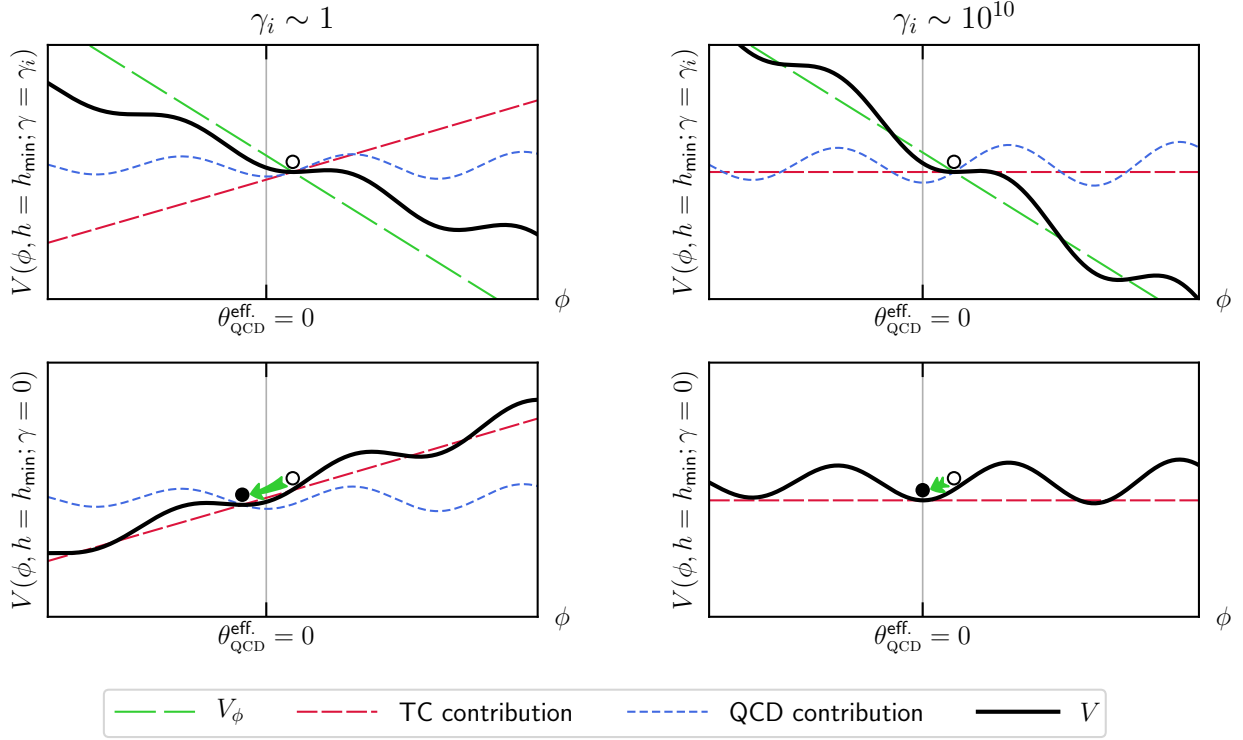


Figure 5.2: Sketch plot of the effects of post-inflation slope-drop for the cases of $\gamma_i \sim 1$ (left column) and $\gamma_i \sim 10^{10}$ (right column); the top row shows the potentials during inflation ($\gamma = \gamma_i$), while the bottom row shows the situation after slope drop ($\gamma \rightarrow 0$). In all panels, the solid, thick (black) curve shows the total ϕ potential with h pinned to its minimum, with the open (black) circle showing where the relaxion stalls rolling before slope drop (i.e., with $\gamma = \gamma_i$), and the solid (black) circle (bottom row only) showing where the relaxion would settle after slope-drop ($\gamma = 0$). The (green) arrows (bottom row only) indicate how the relaxion rolls from its initial stalling position to the position it assumes once the scanning slope drops. The long-dashed (green) line (top row only) shows the contribution to the total potential from the ‘scanning potential’ before slope drop (i.e., with $\gamma = \gamma_i$), the medium-dashed (red) line shows the contribution from the strong dynamics, and the short-dashed (blue) line shows the QCD contribution. In order to display all the individual potential contributions on the same axes, the zero-point of each individual contribution to the potential has been independently shifted so that all the contributions cross at the point where the relaxion stalls [i.e., the three dashed (coloured) lines sum up to the solid (black) line plus a constant offset], such that the important slope information from each contribution at that point is clearly visible. The vertical line marked $\theta_{\text{QCD}}^{\text{eff}} = 0$ is the point where the effective QCD θ -angle would vanish in the dip in the periodic QCD potential just prior to the initial relaxion stalling point.

factor of γ_i :

$$\gamma_i c_m \frac{\Lambda_{\text{TC}} F_\pi^2 m}{F} \sim \frac{\Lambda^4}{f} \quad \Rightarrow \quad F \sim \gamma_i c_m \frac{\Lambda_{\text{TC}} F_\pi^2 m}{\Lambda^4} f \quad (\gamma_i \gg 1). \quad (5.13)$$

Now, taking the same parameter estimates as just below Eq. (5.12), this implies that $F \sim \gamma_i \times (7 \times 10^{31}) \text{ GeV}$. For large γ_i , F is proportionally larger than the estimate at Eq. (5.12), so that the overall slope of the additional potential term, $\partial_\phi V_\phi = \gamma_i (c_m \Lambda_{\text{TC}} F_\pi^2 m / F) \propto \gamma_i / F$, remains of the correct size to cancel against the QCD barriers and stop the rolling, while the contribution to the relaxion potential from the strong TC dynamics is highly suppressed. It is this suppression of the contribution from the strong dynamics in this region of parameter space that allows the slope-drop mechanism to work (see Figure 5.2).

The estimate for the post-inflation settling point for the relaxion is obtained from Eqs. (5.4), (5.5) and (5.10) with $\gamma \rightarrow 0$:

$$\partial_\phi V = 0 \quad \Rightarrow \quad \left| \frac{c_m \Lambda_{\text{TC}} F_\pi^2 m}{F} \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \sin\left(\frac{\phi}{F}\right) \right| \sim \left| \frac{\Lambda^4}{f} \sin\left(\frac{\phi}{f} - \theta_{\text{QCD}}\right) \right|. \quad (5.14)$$

Post slope-drop, the relaxion will only roll a distance $|\Delta\phi| \sim f$ from its initial stopping point (see Figure 5.2), so the changes in $\sin(\phi/F)$ and in $\langle h \rangle$ will be negligible. Taking $\sin(\phi/F) \sim \mathcal{O}(1)$, using that the scanning mechanism selects $\cos(\langle h \rangle / F_\pi) \sim 1$ (i.e., $\xi \ll 1$) for the purposes of this estimate, and noting that $\theta_{\text{QCD}}^{\text{eff.}} = (\theta_{\text{QCD}} - \phi/f) \bmod 2\pi$, we estimate that

$$\frac{c_m \Lambda_{\text{TC}} F_\pi^2 m}{F} \sim \frac{\Lambda^4}{f} \left| \sin\left(\theta_{\text{QCD}}^{\text{eff.}}\right) \right|; \quad (5.15)$$

but using Eq. (5.13), we then have

$$\left| \sin\left(\theta_{\text{QCD}}^{\text{eff.}}\right) \right| \sim \frac{1}{\gamma_i} \quad \Rightarrow \quad \left| \theta_{\text{QCD}}^{\text{eff.}} \right| \sim \frac{1}{\gamma_i}. \quad (5.16)$$

Therefore, we must have $\gamma_i \sim 10^{10}$ to obtain an appropriately small QCD θ -angle, $|\theta_{\text{QCD}}^{\text{eff.}}| \sim 10^{-10}$. We emphasize that the large dimensionless parameter γ_i is merely an artefact of our parametrization of $V_\phi(\phi)$; the physical content of the statement that $\gamma_i \gg 1$ is that the slope of the relaxion potential contributed by the strong dynamics should be highly suppressed compared to the slope of $V_\phi(\phi)$; i.e., F must be taken larger than it would be were $\gamma_i \sim 1$ (see Section 5.2.5).

5.2.4 Self-Consistency

In addition to the ‘slope matching’ condition Eq. (5.13), Ref. [413] presented a number of restrictions on the relaxion mechanism which must be satisfied to achieve self-consistency. As applied to this cartoon model, these restrictions are as follows:

Vacuum energy domination. The total change in the relaxion energy density as ϕ rolls must be a sub-leading correction to the energy density driving inflation. The relaxion ϕ must roll from an initial position in field space near $\phi = 0$ to a value near $\phi_{\text{crit.}}$ in order to trigger EWSB. While the exact value for $\phi_{\text{crit.}}$ depends on parameter choices, we can generically assume that because ϕ enters the Higgs potential as $\cos(\phi/F)$, $\phi_{\text{crit.}}$ will not be orders of magnitude different from F . Therefore,

$$V_I = 3H_I^2 m_{\text{Pl.}}^2 \gg \Delta V(\phi, h) \sim \gamma_i c_m \Lambda_{\text{TC}} F_\pi^2 m \quad \Rightarrow \quad H_I \gtrsim \left(\gamma_i \frac{c_m \Lambda_{\text{TC}} F_\pi^2 m}{3m_{\text{Pl.}}^2} \right)^{\frac{1}{2}}, \quad (5.17)$$

where $m_{\text{Pl.}} = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck mass, V_I is the energy density driving inflation, and H_I is the Hubble constant during inflation.

Barrier formation. The Hubble scale must be low enough that the QCD barriers form:

$$H_I \lesssim \Lambda. \quad (5.18)$$

Classical beats quantum. It is necessary to impose a constraint that classical rolling of the relaxion dominates over quantum fluctuations, so that following inflation each patch of the universe obtains a vev of order the weak scale. In a Hubble time,

$$\Delta\phi_{\text{cl.}} \sim \dot{\phi}\Delta t_H \approx \frac{1}{3} \frac{|\partial_\phi V|}{H_I^2} \sim \gamma_i \frac{c_m \Lambda_{\text{TC}} F_\pi^2 m}{3F H_I^2}, \quad \text{while} \quad \Delta\phi_{\text{quantum}} \sim H_I \quad (5.19)$$

so

$$\Delta\phi_{\text{cl.}} \gtrsim \Delta\phi_{\text{quantum}} \quad \Rightarrow \quad H_I \lesssim \left(\gamma_i \frac{c_m \Lambda_{\text{TC}} F_\pi^2 m}{3F} \right)^{\frac{1}{3}}. \quad (5.20)$$

Sufficiently many e -folds. For ϕ to roll a distance on the order of F in field space given N_e e -folds of inflation requires that

$$F \sim \Delta\phi \approx \dot{\phi}\Delta t \approx \dot{\phi} \frac{N_e}{H_I} \approx \frac{1}{3} \frac{|\partial_\phi V|}{H_I^2} N_e \quad \Rightarrow \quad N_e \sim \frac{3H_I^2 F^2}{\gamma_i c_m \Lambda_{\text{TC}} F_\pi^2 m}. \quad (5.21)$$

Combining Eqs. (5.13), (5.17), and (5.20) gives an upper limit on F :⁷

$$F \leq \left(\frac{3m_{\text{Pl.}}^6 f}{\Lambda^4} \right)^{\frac{1}{3}} = (8 \times 10^{41} \text{ GeV}) \left(\frac{f}{10^{11} \text{ GeV}} \right)^{\frac{1}{3}}. \quad (5.22)$$

Using Eq. (5.13), $\Lambda_{\text{TC}} \approx (4\pi/\sqrt{N})F_\pi \approx 4F_\pi \times (10/N)^{1/2}$, and writing⁸

$$m \approx N_c \frac{c_t}{c_m} \frac{y_t^2}{8\pi^2} \Lambda_{\text{TC}} \approx \frac{c_t}{25c_m} \Lambda_{\text{TC}} \quad (5.23)$$

7. Using Eq. (5.18) in place of Eq. (5.20) gives a much weaker upper limit for all reasonable values of f .

8. These estimates are consistent with the parameter choices appearing just below Eq. (5.13).

—which saturates Eq. (5.7)—transforms Eq. (5.22) into an upper limit on Λ_{TC} , the UV cutoff of the composite theory:

$$\Lambda_{\text{TC}} \lesssim \left(\frac{128\pi^4 \sqrt[3]{3}}{c_t y_t^2 N N_c} \right)^{\frac{1}{4}} \gamma_i^{-\frac{1}{4}} \left(\frac{\Lambda^4 m_{\text{Pl}}^3}{f} \right)^{\frac{1}{6}} \quad (5.24)$$

$$= (3 \times 10^7 \text{ GeV}) \left(\frac{N}{10} \right)^{-\frac{1}{4}} \left(\frac{1}{\gamma_i} \right)^{\frac{1}{4}} \left(\frac{f}{10^{11} \text{ GeV}} \right)^{-\frac{1}{6}} \quad (5.25)$$

$$= (8 \times 10^4 \text{ GeV}) \left(\frac{N}{10} \right)^{-\frac{1}{4}} \left(\frac{10^{10}}{\gamma_i} \right)^{\frac{1}{4}} \left(\frac{f}{10^{11} \text{ GeV}} \right)^{-\frac{1}{6}} \quad (5.26)$$

$$= (8 \times 10^4 \text{ GeV}) \left(\frac{N}{10} \right)^{-\frac{1}{4}} \left(\frac{\theta_{\text{QCD}}^{\text{eff.}}}{10^{-10}} \right)^{\frac{1}{4}} \left(\frac{f}{10^{11} \text{ GeV}} \right)^{-\frac{1}{6}}. \quad (5.27)$$

We therefore find that the relaxation mechanism can indeed explain in a technically natural fashion a large hierarchy between the UV cutoff of the composite theory and the weak scale; that is, we have provided a realization of the picture advocated from the outset in which compositeness addresses the Big Hierarchy Problem while cosmological relaxation explains the Little Hierarchy. We also remark here that the parameter point discussed earlier, $\Lambda_{\text{TC}} \sim 80 \text{ TeV}$, does not obviously run afoul of any of the self-consistency conditions when the effective QCD θ -angle is appropriately small. Additionally, Eqs. (5.13) and (5.20) together provide the most stringent available upper bound on the scale of inflation:

$$V_I^{\frac{1}{4}} \lesssim \left(\frac{\sqrt{3}\Lambda^4}{f} \right)^{\frac{1}{6}} m_{\text{Pl}}^{\frac{1}{2}} = (6 \times 10^6 \text{ GeV}) \left(\frac{f}{10^{11} \text{ GeV}} \right)^{-\frac{1}{6}}, \quad (5.28)$$

while Eqs. (5.13) and (5.17) together provide a lower bound on the scale of inflation:

$$V_I^{\frac{1}{4}} \gtrsim \Lambda \left(\frac{F}{f} \right)^{\frac{1}{4}} = (6 \times 10^6 \text{ GeV}) \left(\frac{f}{10^{11} \text{ GeV}} \right)^{-\frac{1}{4}} \left(\frac{F}{8 \times 10^{41} \text{ GeV}} \right)^{\frac{1}{4}}. \quad (5.29)$$

The number of e -folds is bounded by Eqs. (5.17) and (5.21):

$$N_e \gtrsim \left(\frac{F}{m_{\text{Pl}}} \right)^2 = (1 \times 10^{47}) \left(\frac{F}{8 \times 10^{41} \text{ GeV}} \right)^2. \quad (5.30)$$

As in the original relaxion model [413], these results indicate that an extremely long period of low-scale inflation, and an ultra-trans-Planckian relaxion field excursion are required to make the model viable.

Beginning in Section 5.3, we devote significant effort to the presentation and detailed analysis of a concrete model that realizes the mechanism which we have just described schematically.

5.2.5 Clockwork Mechanism

Achieving the hierarchy $F \gg f$ required for the viability of our model requires further model building. One possibility is the ‘clockwork’ mechanism of Refs. [436, 437], which we will briefly review here as it applies to our model; see Refs. [617–619] for earlier work in the context of inflation and Refs. [620–625] for further recent theoretical and phenomenological investigations of the clockwork. In particular, see Ref. [622] for a model similar to the one we advocate here.

The clockwork mechanism postulates the existence of $(M + 1)$ complex scalar fields φ_j ($j = 0, 1, \dots, M$) interacting via the Lagrangian [436]

$$\mathcal{L} \supset \sum_{j=0}^M \left(|\partial_\mu \varphi_j|^2 + \mu^2 |\varphi_j|^2 - \lambda |\varphi_j|^4 \right) + \epsilon \sum_{j=0}^{M-1} \left(\varphi_j^\dagger \varphi_{j+1}^3 + \text{h.c.} \right), \quad (5.31)$$

where $\epsilon \ll \lambda$. If $\epsilon = 0$, this Lagrangian would exhibit a global $[U(1)]^{M+1}$ symmetry [436]; all $(M + 1)$ of these global symmetries are spontaneously broken such that $\varphi_j \equiv \frac{1}{\sqrt{2}}(f_0 + \rho_j) \exp[-i\pi_j/f_0]$ with $f_0 \equiv \mu^2/\lambda$, giving rise to $(M + 1)$ NGBs, π_j [436]. When $\epsilon \neq 0$, M of the $U(1)$ symmetries are additionally explicitly broken, which masses-up M of the NGBs.

The residual unbroken global $U(1)$ has an interesting pattern of charges, with φ_j having charge $q_j = 3^{-j}$ [436]; the corresponding massless NGB is taken to be the relaxion. The radial modes have masses $m_{\rho_j} \approx \sqrt{2\lambda}f_0$ [622].

In order to obtain couplings of the relaxion to both QCD and the strong TC dynamics, we assume a KSVZ-type [101, 103] axion model, with vector-like fermions charged under QCD coupled to the field φ_K (where K is an integer obeying $0 \leq K \ll M$), and fermions charged under the strong TC dynamics coupled to the field φ_M (cf. Ref. [622]). These fermions will obtain masses $m_{K,M} = y_{K,M}f_0/\sqrt{2}$, where $y_{K,M}$ are the Yukawa couplings of the $\varphi_{K,M}$ fields to the fermions. Integrating out these fermions in the usual way generates the usual axion couplings of the π_K and π_M fields to, respectively, the QCD and TC field strength tensors.

Ref. [436] supplied the general procedure for the diagonalization of the tri-diagonal mass matrix for the π_j that arises from the explicit breaking terms in Eq. (5.31); this procedure was elaborated on in detail in Refs. [620, 622]. Letting the massless NGB be the relaxion ϕ , and calling the other mass-eigenstate pNGB fields a_n (these fields are sometimes called the ‘gears’ of the clockwork mechanism [620]), it is straightforward to show that the Lagrangian for the (p)NGBs after the vector-like fermions are integrated out can be expressed in terms of the mass-eigenstate fields as [620, 622]

$$\begin{aligned} \mathcal{L} \supset & \frac{1}{2}(\partial_\mu \phi)^2 + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}}^0 \right] \text{Tr} [G_{\mu\nu} \tilde{G}^{\mu\nu}] + \frac{(g_{\text{TC}})^2}{16\pi^2} \frac{\phi}{F} \text{Tr} [G_{\text{TC} \mu\nu} \tilde{G}_{\text{TC}}^{\mu\nu}] \\ & + \sum_{n=1}^M \left[\frac{1}{2}(\partial_\mu a_n)^2 - \frac{1}{2}m_n^2 a_n^2 + \frac{(g_s)^2}{16\pi^2} \frac{a_n}{f_K^{(n)}} \text{Tr} [G_{\mu\nu} \tilde{G}^{\mu\nu}] + \frac{(g_{\text{TC}})^2}{16\pi^2} \frac{a_n}{f_M^{(n)}} \text{Tr} [G_{\text{TC} \mu\nu} \tilde{G}_{\text{TC}}^{\mu\nu}] \right], \end{aligned} \quad (5.32)$$

where m_n are pNGB masses that obey $2\sqrt{\epsilon}f_0 \leq m_n \leq 4\sqrt{\epsilon}f_0$ [620, 622]; $f_j^{(n)}$ are decay constants which are generically roughly of the order $\sqrt{(M+1)/2}f_0$ [620, 622]; and,

crucially [436, 620, 622],

$$f \equiv \frac{3}{2\sqrt{2}} 3^K f_0 \approx 3^K f_0 \quad \text{and} \quad F \equiv \frac{3}{2\sqrt{2}} 3^M f_0 \approx 3^M f_0. \quad (5.33)$$

With moderate K and M , this exponential enhancement of the decay constants makes it straightforward to engineer large hierarchies: $F \gg f \gg$ the weak scale.

Two scenarios suggest themselves: (a) $f_0 \sim f$ [i.e., $K = 0$ and $M \gg 1$], and (b) $f_0 \sim$ a few (tens of) TeV $\ll f$ [i.e., $1 \ll K \ll M$].

Scenario (a): $f_0 \sim f$. To obtain $F \sim \gamma_i \times (6.6 \times 10^{31}) \text{ GeV} \sim 6.6 \times 10^{41} \text{ GeV}$ for $\gamma_i \sim 10^{10}$ with $f \sim 10^{11} \text{ GeV}$, we need $M \sim 65$. The only light field is the relaxion, while the radial modes and gears have masses of order $\sqrt{2\lambda}f$ and $3\sqrt{\epsilon}f$, respectively. The radial modes are thus heavy and, provided ϵ is not exponentially small, so too are the gears. Additionally, the vector-like fermions charged under QCD and the strong TC dynamics—which were integrated out to give rise to the dimension-5 axion couplings—have masses on the order of $yf/\sqrt{2}$; assuming that $y \sim \mathcal{O}(1)$ (or at least not exponentially small), these too are unobservably heavy. While the mechanism thus allows for the exponential scale separation $F \gg f \gg$ the weak scale, there are no additional experimental signatures which are accessible at any current or proposed collider, as all the additional new states are extremely massive.

Scenario (b): $f_0 \sim$ a few (tens of) TeV $\ll f$. Taking $F \sim 6.6 \times 10^{41} \text{ GeV}$, $f \sim 10^{11} \text{ GeV}$, and $f_0 \sim 10 \text{ TeV}$, we find that we need $K \sim 15$ and $M \sim 79$. The radial modes and gears have masses of order $\sqrt{2\lambda}f_0$ and $3\sqrt{\epsilon}f_0$, respectively. While the radial modes masses are thus around a few (tens of) TeV since $\lambda \sim 1$, the gears could easily have masses below a TeV for reasonably small ϵ . Moreover, the couplings of the gears and radial modes to the QCD and TC field strength tensors have ‘decay constants’ of order $\sim 3f_0$ and $\sim 6f_0$, respectively. Additionally, the coloured and TC-charged vector-like fermions that were integrated out to give rise to the dimension-5 axion couplings would also have masses

on the order of $y f_0/\sqrt{2}$. Therefore, in this scenario we expect that additional experimental signatures would be accessible at current or future colliders: the gears and radial modes, and the coloured fermions from the KSVZ mechanism, could all presumably be produced strongly if they are light enough.

5.3 Constituent Model

We begin our concrete model construction by specifying in detail the underlying constituent UV model for the technicolour dynamics. This underlying constituent model can be broken into three distinct components: (a) the CH sector, (b) the terms which give rise to the CH–SM Yukawa couplings, and (c) the relaxion sector. We discuss each of these components in turn in Sections 5.3.1–5.3.3.

5.3.1 Composite Higgs Sector

Our CH model is constructed to exhibit the global (TC-flavour) symmetry breaking pattern $SU(3)_L \times SU(3)_R \times U(1)_V \rightarrow SU(3)_V \times U(1)_V \rightarrow SU(2) \times U(1) \times U(1)$, where the first step arises from spontaneous chiral symmetry breaking, and the second arises due to the addition of explicit breaking terms (technifermion masses, Yukawas, and gauge couplings). A remaining global $SU(2) \times U(1)$ is gauged and identified as the SM electroweak gauge group, $SU(2)_W \times U(1)_Y$.

The gauge group which will confine to give rise to the composite Higgs state is assumed to be an $SU(N)$ group under which all SM field content transforms as singlets.⁹ The SM is augmented by three (Dirac) technifermions which transform in the fundamental of this gauged TC group, hereinafter referred to as $SU(N)_{TC}$. In order to allow for the existence of a bound state of the technifermions with the correct SM quantum numbers to be interpreted as the Higgs, we demand that the three (Dirac) technifermion species consist of an $SU(2)_W$

9. See, e.g., Refs. [626–634] for some recent studies of UV-complete composite Higgs models with strong TC dynamics.

doublet $\mathbb{L}$, and an $SU(2)_W$ -singlet $\mathbb{N}$; additionally, we demand that $\mathbb{L}$ carry SM $U(1)_Y$ hypercharge of $+1/2$, while $\mathbb{N}$ is taken to be neutral under $U(1)_Y$. For the remainder of this chapter, we will write all fermion fields as left-handed¹⁰ two-component Weyl spinors;¹¹ our naming conventions for the two-component Weyl fermions, and the matter-field gauge charges, are given in Table 5.1. Note that given those charges, $LN^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{2})_{+1/2}$, where contraction of the TC fundamental and anti-fundamental indices on L and N^c , respectively, is understood. As these are the quantum numbers of the SM Higgs, the requisite composite Higgs state will be formed after confinement.

Introducing the N additional hypercharged $SU(2)_W$ -doublets L and L^c will modify the running of the couplings g_1 and g_2 . If N is taken too large, there is a possibility that Landau poles in these couplings may occur below the Planck scale. A simple check indicates that with $N \lesssim 14$, no such poles should appear in either coupling.

In order to impose the global $SU(3)_L \times SU(3)_R$ TC-flavour symmetry in our Lagrangian and make it manifest, we arrange L and N into a $(\mathbf{3}, \mathbf{1})$ TC-flavour multiplet χ , and we arrange L^c and N^c into a $(\mathbf{1}, \bar{\mathbf{3}})$ TC-flavour multiplet χ^c :

$$\chi \equiv \begin{pmatrix} L \\ N \end{pmatrix} \quad \text{and} \quad \chi^c \equiv \begin{pmatrix} L^c \\ N^c \end{pmatrix}. \quad (5.34)$$

If $U_L \equiv \exp[i\alpha_L^a T^a]$ and $U_R \equiv \exp[i\alpha_R^a T^a]$ are, respectively, forward $SU(3)_L$ and $SU(3)_R$ transformations, we then have $\chi \rightarrow U_L \chi$ and $\chi^c \rightarrow \chi^c U_R^\dagger$. Note also that the χ and χ^c transform in the fundamental and anti-fundamental, respectively, of the gauged $SU(N)_{\text{TC}}$ group, given the representations assigned in Table 5.1.

10. That is, transforming under the $(\frac{1}{2}, 0)$ representation of the Lorentz group [35, 36].

11. We generally follow the notational conventions of Ref. [35]. Explicitly, in the Weyl basis, a Dirac fermion $\mathbb{F}$ can be expressed in terms of the two-component Weyl fermions F and F^c as

$$\mathbb{F} = \begin{pmatrix} F_\alpha \\ [(F^c)^\dagger]^{\dot{\alpha}} \end{pmatrix},$$

where α is a $(\frac{1}{2}, 0)$ Lorentz spinor index, and $\dot{\alpha}$ is a $(0, \frac{1}{2})$ Lorentz spinor index.

Description	Field Name	$(SU(N)_{\text{TC}}, SU(3)_C, SU(2)_W)_{U(1)_Y}$
SM quark doublet	$Q \equiv \begin{pmatrix} U \\ D \end{pmatrix}$	$(\mathbf{1}, \mathbf{3}, \mathbf{2})_{+1/6}$
SM down-type quark singlet	D^c	$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{+1/3}$
SM up-type quark singlet	U^c	$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-2/3}$
Technifermion ‘L’ doublet	L	$(\mathbf{N}, \mathbf{1}, \mathbf{2})_{+1/2}$
Technifermion ‘R’ doublet	L^c	$(\bar{\mathbf{N}}, \mathbf{1}, \bar{\mathbf{2}})_{-1/2}$
Technifermion ‘L’ singlet	N	$(\mathbf{N}, \mathbf{1}, \mathbf{1})_0$
Technifermion ‘R’ singlet	N^c	$(\bar{\mathbf{N}}, \mathbf{1}, \mathbf{1})_0$

Table 5.1: Gauge charges and representations for the fermion matter content. For the non-Abelian factors we give the representation, and for the hypercharge group we give the charge. All fields are assumed to be two-component left-handed [i.e., $(\frac{1}{2}, 0)$] Weyl fermions. ‘Singlet’ and ‘doublet’ in the descriptions refer to the $SU(2)_W$ representations of the matter fields; ‘L’ and ‘R’ in the technifermion descriptions refer to which of the $SU(3)_{\text{L,R}}$ TC-flavour groups the fields transform under. SM generation indices are suppressed. A superscript c is considered to be an integral part of the field name and merely denotes that the field transforms in the anti-fundamental of the relevant gauge groups; i.e., L and L^c each represent two independent, unrelated degrees of freedom (L^c is not a conjugate of L). We omit the SM lepton fields here as they are not relevant to our discussion.

Thus far, the TC-gauge-coupling and kinetic terms for the χ and χ^c fields can be written in manifestly TC-flavour and $SU(N)_{\text{TC}}$ invariant fashion as

$$\mathcal{L} \supset i\chi^\dagger (\bar{\sigma} \cdot D) \chi + i\chi^c (\sigma \cdot D) (\chi^c)^\dagger, \quad (5.35)$$

where

$$D_\mu \{ \chi, (\chi^c)^\dagger \} \supset \left(\partial_\mu - ig_{\text{TC}} A_\mu^{\text{TC}} \right) \{ \chi, (\chi^c)^\dagger \}, \quad (5.36)$$

where we have suppressed all indices, and have written the matrix-valued TC gauge field $A_\mu^{\text{TC}} \equiv (A_\mu^{\text{TC}})^n (T_{\text{TC}})^n$, where $n = 1, \dots, (N^2 - 1)$, with $(T_{\text{TC}})^n$ the generators of the fundamental representation of $SU(N)_{\text{TC}}$.

Gauging the $SU(2)_W$ subgroup of the TC-flavour group is straightforward. We assume that the matrix representatives of the generators T^a of the $SU(3)_{\text{L,R}}$ transformations are given by $T^a = \frac{1}{2}\lambda^a$, where λ^a are the usual Gell-Mann matrices [39] (the Dynkin index is

$\frac{1}{2}$).¹² In this convention, $\lambda^{\tilde{a}} = \begin{pmatrix} \sigma^{\tilde{a}} & 0 \\ 0 & 0 \end{pmatrix}$ where (here, and throughout) $\tilde{a} = 1, 2, 3$, and $\sigma^{\tilde{a}}$ are the usual Pauli matrices. Therefore, $SU(2)_W$ is gauged by simply adding to the covariant derivative D_μ the term $D_\mu \{\chi, (\chi^c)^\dagger\} \supset -ig_2 W_\mu^{\tilde{a}} T^{\tilde{a}} \{\chi, (\chi^c)^\dagger\}$, and demanding that under a forward $SU(2)_W$ gauge transform parametrized by $\alpha^{\tilde{a}}$ we have

$$\{\chi, (\chi^c)^\dagger\} \rightarrow U_V \{\chi, (\chi^c)^\dagger\} \quad \text{and} \quad W_\mu \rightarrow U_V W_\mu U_V^\dagger + \frac{i}{g_2} U_V \partial_\mu U_V^\dagger, \quad (5.37)$$

where

$$U_V \equiv \exp \left[i \alpha^{\tilde{a}} T^{\tilde{a}} \right] \quad \text{and} \quad W_\mu \equiv W_\mu^{\tilde{a}} T^{\tilde{a}}. \quad (5.38)$$

Gauging $U(1)_Y$ is marginally more complicated. Recall that $[T^8, T^{\tilde{a}}] = 0$ for the $SU(3)$ generators as defined above; this may lead one to conclude that T^8 is the $U(1)_Y$ generator because it commutes with the $SU(2)_W$ generators. However, the matrix representative of T^8 is

$$T^8 = \frac{1}{2} \lambda^8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}; \quad (5.39)$$

since the 3-3 component of this representative is non-vanishing, T^8 will act non-trivially on the N component of χ . But this cannot then be the SM $U(1)_Y$ hypercharge generator since N is uncharged under $U(1)_Y$. This problem is easily remedied by noting that the true global symmetry of Eq. (5.35) is $U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$ at the classical level; while it is well known that the $U(1)_A$ is anomalous [40–48], the $U(1)_V$

12. We have suppressed TC-flavour indices here and have just written T^a as the generators for either simple $SU(3)_{L,R}$ factor in the TC-flavour group. At the risk of being pedantic, we should really write separate generators T_L^a and T_R^a , and be consistent in the usage of each throughout. The matrix representatives of these generators are numerically equal as matrices, but as generators they are distinct objects as they carry different types of indices.

symmetry remains good at the quantum level. This additional $U(1)_V$ has a generator equal to the identity on TC-flavour space: $T^X \equiv \mathbb{1}_3$, which obviously commutes with all the T^a . We can thus form the hypercharge generator

$$Y \equiv \frac{1}{\sqrt{3}}T^8 + Q_X T^X = \begin{pmatrix} (Q_X + \frac{1}{6})\mathbb{1}_{2 \times 2} & 0 \\ 0 & Q_X - \frac{1}{3} \end{pmatrix}, \quad (5.40)$$

where Q_X is chosen appropriately to the field on which Y acts. For the χ and $(\chi^c)^\dagger$ fields, we need $Q_X[\chi] = Q_X[(\chi^c)^\dagger] = \frac{1}{3}$ to obtain the hypercharge assignments for $L^{(c)}$ and $N^{(c)}$ in Table 5.1. We then gauge $U(1)_Y$ by adding to the covariant derivative D the term $D_\mu \{\chi, (\chi^c)^\dagger\} \supset -ig_1 B_\mu Y \{\chi, (\chi^c)^\dagger\}$, and demanding that under a forward $U(1)_Y$ transformation parametrized by α , we have

$$\{\chi, (\chi^c)^\dagger\} \rightarrow \exp[i\alpha Y] \{\chi, (\chi^c)^\dagger\} \quad \text{and} \quad B_\mu \rightarrow B_\mu + \frac{1}{g_1} \partial_\mu \alpha. \quad (5.41)$$

We can also include mass terms for the χ and χ^c fields by including the explicit $SU(3)_L \times SU(3)_R$ -breaking terms

$$\mathcal{L} \supset -\chi^c M \chi + \text{h.c.} = -\text{Tr}[M \chi \chi^c] + \text{h.c.}, \quad (5.42)$$

where $\text{Tr}[\dots]$ is over the TC-flavour indices, and where M is a mass matrix in TC-flavour space which in the mass-eigenbasis must respect the residual global $SU(2) \times U(1)$ symmetry in order not to explicitly break the electroweak gauging:

$$M = \begin{pmatrix} m_L \mathbb{1}_{2 \times 2} & 0 \\ 0 & m_N \end{pmatrix}. \quad (5.43)$$

Note that while M explicitly breaks $SU(3)_L \times SU(3)_R \rightarrow SU(2)_V \times U(1)_V$,¹³ if M is given

13. At least if $m_L \neq m_N$; if $m_L = m_N$, it only breaks $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$.

the usual spurionic transformation $M \rightarrow U_R M U_L^\dagger$, then Eq. (5.42) is a spurionic TC-flavour invariant.

To summarize, the underlying constituent theory for the CH sector is specified by the Lagrangian terms

$$\mathcal{L} \supset i\bar{\chi}^\dagger (\bar{\sigma} \cdot D) \chi + i\chi^c (\sigma \cdot D) (\chi^c)^\dagger - \text{Tr} [M \chi \chi^c] + \text{h.c.}, \quad (5.44)$$

where

$$D_\mu \left\{ \chi, (\chi^c)^\dagger \right\} = \left(\partial_\mu - ig_{\text{TC}} A_\mu^{\text{TC}} - ig_2 W_\mu^{\tilde{a}} T^{\tilde{a}} - ig_1 B_\mu Y \right) \left\{ \chi, (\chi^c)^\dagger \right\}. \quad (5.45)$$

We return to the low-energy description of this sector by means of the Chiral Lagrangian in Section 5.4.

5.3.2 Contact Interactions that lead to Yukawas

In order for the low-energy Chiral Lagrangian describing the CH model to exhibit the correct Higgs Yukawa couplings to the SM quarks, we must write couplings of the technifermions to the SM quarks in the constituent theory. In this work we simply write down the required four-fermion operators coupling the SM fermions to the techniquark bilinear condensate, remaining agnostic about their underlying UV origin. Various mechanisms exist to generate such couplings, such as extended technicolour [635, 636], partial compositeness [637], or bosonic technicolour [638–640]; exploring their detailed consequences in this context goes beyond the scope of this work.

In order to write these couplings in a fashion which will allow spurionic TC-flavour symmetries to be made manifest, we arrange the SM field content into incomplete TC-flavour multiplets. There are of course multiple ways to do this. We assign U^c to an incomplete $(\mathbf{1}, \mathbf{3})$ of TC-flavour which we denote $U_{\mathbf{3}_R}^c$, and we assign D^c to an incomplete $(\bar{\mathbf{3}}, \mathbf{1})$ of

TC-flavour which we denote $D_{\bar{\mathbf{3}}_L}^c$:

$$U_{\mathbf{3}_R}^c \equiv \begin{pmatrix} 0 \\ 0 \\ U^c \end{pmatrix} \quad \text{and} \quad D_{\bar{\mathbf{3}}_L}^c \equiv \begin{pmatrix} 0 \\ 0 \\ D^c \end{pmatrix}. \quad (5.46)$$

The spurionic $SU(3)_L \times SU(3)_R$ transformations of these incomplete multiplets are $U_{\mathbf{3}_R}^c \rightarrow U_R U_{\mathbf{3}_R}^c$ and $(D_{\bar{\mathbf{3}}_L}^c)^\dagger \rightarrow U_L (D_{\bar{\mathbf{3}}_L}^c)^\dagger$. In order to obtain the correct hypercharge assignments for U^c and D^c under the gauging prescription developed above, we set $Q_X[U_{\mathbf{3}_R}^c] = -\frac{1}{3}$ and $Q_X[(D_{\bar{\mathbf{3}}_L}^c)^\dagger] = 0$.

In close analogy to the requirement in the SM to use the two opposite-hypercharge $SU(2)_W$ -fundamental fields H and $\widetilde{H}$ (following the notation of Ref. [32]) to write the SM Yukawas, we will need to embed the quark doublets in incomplete multiplets in two different ways. If Q is in the fundamental $\mathbf{2}$ of $SU(2)_W$, then the field $\widehat{Q} \equiv i\sigma^2 Q$ (in components, $\widehat{Q}^i \equiv \epsilon^{ij} Q_j$ where ϵ is the anti-symmetric invariant symbol of $SU(2)$ with $\epsilon^{12} = +1$) is in the conjugate¹⁴ anti-fundamental $\bar{\mathbf{2}}$ of $SU(2)_W$. We will embed Q in an incomplete $(\mathbf{1}, \mathbf{3})$ of TC-flavour which we denote $Q_{\mathbf{3}_R}$, and embed $\widehat{Q}$ in an incomplete $(\bar{\mathbf{3}}, \mathbf{1})$ of TC-flavour which we denote $Q_{\bar{\mathbf{3}}_L}$:

$$Q_{\mathbf{3}_R} \equiv \begin{pmatrix} Q \\ 0 \end{pmatrix} \quad \text{and} \quad Q_{\bar{\mathbf{3}}_L} \equiv \begin{pmatrix} \widehat{Q} \\ 0 \end{pmatrix}. \quad (5.47)$$

The spurionic $SU(3)_L \times SU(3)_R$ transformations of these incomplete multiplets are thus $Q_{\mathbf{3}_R} \rightarrow U_R Q_{\mathbf{3}_R}$ and $(Q_{\bar{\mathbf{3}}_L})^\dagger \rightarrow U_L (Q_{\bar{\mathbf{3}}_L})^\dagger$; note also that the action of the gauged $SU(2)_W$ subgroup of TC-flavour automatically gives the correct $SU(2)_W$ action on the quark doublets given these embeddings since we have placed the fundamental $\mathbf{2}$ of $SU(2)_W$ in the

14. Since the spinorial $\mathbf{2}$ of $SU(2)$ is a pseudo-real representation [39], it is unitarily equivalent to the conjugate $\bar{\mathbf{2}}$ representation. There is thus no distinction between $\mathbf{2}$ and $\bar{\mathbf{2}}$; nevertheless, we maintain the bar to match up with the notation for the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)$ into which the fermions are embedded.

fundamental $\mathbf{3}$ of $SU(3)_R$, and the anti-fundamental $\bar{\mathbf{2}}$ of $SU(2)_W$ in the anti-fundamental $\bar{\mathbf{3}}$ of $SU(3)_L$. In order to obtain the correct hypercharge assignment for Q under the gauging prescription developed above, we set $Q_X[Q_{\mathbf{3}_R}] = 0$ and $Q_X[(Q_{\bar{\mathbf{3}}_L})^\dagger] = -\frac{1}{3}$.

Armed with these embeddings of the SM quarks, and noting that $\chi\chi^c \sim (\mathbf{3}, \bar{\mathbf{3}})$ under $SU(3)_L \times SU(3)_R$, there are only two independent four-fermion spurionic TC-flavour invariants with zero net Q_X charge which can be formed which couple $\chi\chi^c$ to SM quarks:

$$D_{\bar{\mathbf{3}}_L}^c (\chi\chi^c) Q_{\mathbf{3}_R} \quad \text{and} \quad Q_{\bar{\mathbf{3}}_L} (\chi\chi^c) U_{\mathbf{3}_R}^c, \quad (5.48)$$

along with their Hermitian conjugates. By construction, the spurionic TC-flavour invariance (and net zero Q_X charge) of these terms implies actual invariance under the gauged electroweak subgroup of the TC-flavour group.

We thus add the following contact terms to the Lagrangian¹⁵

$$\mathcal{L} \supset + \frac{(Y_D)_p^q}{\Lambda_y^2} \left[D_{\bar{\mathbf{3}}_L}^c \right]^p (\chi\chi^c) [Q_{\mathbf{3}_R}]_q + \text{h.c.} + \frac{(Y_U)_p^q}{\Lambda_y^2} [Q_{\bar{\mathbf{3}}_L}]_q (\chi\chi^c) [U_{\mathbf{3}_R}^c]^p + \text{h.c.}, \quad (5.49)$$

where Λ_y is a dimensionful scale that will be fixed later to obtain canonically normalized Yukawas in the Chiral Lagrangian, and $Y_{U,D}$ are Yukawa matrices in SM-generation space, which we have indexed explicitly: $p, q = 1, 2, 3$.

While Eq. (5.49) is useful in that it displays manifest spurionic TC-flavour invariance, an alternative form not expressed in terms of incomplete TC-flavour multiplets is more useful

15. To be explicit, the full index structure here is

$$\mathcal{L} \supset + \frac{(Y_D)_p^q}{\Lambda_y^2} \left[D_{\bar{\mathbf{3}}_L}^c \right]^{\alpha, \bar{j}, a, p} [\chi]_{\bar{j}, m}^\beta [\chi^c]_{\beta}^{\hat{i}, m} [Q_{\mathbf{3}_R}]_{\alpha, \hat{i}, a, q} + \text{h.c.} + \frac{(Y_U)_p^q}{\Lambda_y^2} [Q_{\bar{\mathbf{3}}_L}]_{a, q}^{\alpha, \bar{j}} [\chi]_{\bar{j}, m}^\beta [\chi^c]_{\beta}^{\hat{i}, m} [U_{\mathbf{3}_R}^c]_{\alpha, \hat{i}}^{a, p} + \text{h.c.},$$

where $\alpha, \beta = 1, 2$ are $(\frac{1}{2}, 0)$ spinorial Lorentz indices, $a = 1, 2, 3$ is an $SU(3)_C$ colour index, $p, q = 1, 2, 3$ are SM generation indices, $\bar{j} = 1, 2, 3$ is an $SU(3)_L$ index, $\hat{j} = 1, 2, 3$ is an $SU(3)_R$ index, and $m = 1, \dots, N$ is an $SU(N)_{TC}$ index. Per the conventions of Ref. [35], for all indices other than the spinorial Lorentz indices, a lowered position denotes a fundamental index and a raised position denotes an anti-fundamental index. Repeated indices are obviously summed.

for computational use. To write this alternative form, we define projection operators

$$\left[P_k\right]_{\hat{i}}^{\bar{j}} \equiv \delta_{\hat{i}}^3 \delta^{\bar{j}}_k \quad \text{and} \quad \left[\tilde{P}^k\right]_{\hat{i}}^{\bar{j}} \equiv \delta_{\hat{i}}^k \delta^{\bar{j}}_3, \quad (5.50)$$

where δ is the Kronecker- δ symbol and where the raised $\bar{j} = 1, 2, 3$ is an $SU(3)_R$ anti-fundamental index, the lowered $\hat{i} = 1, 2, 3$ is an $SU(3)_L$ fundamental index, and the lowered (raised) $k = 1, 2$ is an (anti-)fundamental $SU(2)_W$ index. Then Eq. (5.49) can be alternatively written as

$$\mathcal{L} \supset +\frac{1}{\Lambda_y^2} (D^c Y_D Q_j) \text{Tr} [\tilde{P}^j (\chi \chi^c)] + \text{h.c.} - \frac{1}{\Lambda_y^2} (U^c Y_U Q_i) \epsilon^{ij} \text{Tr} [P_j (\chi \chi^c)] + \text{h.c.}, \quad (5.51)$$

where $\text{Tr}[\dots]$ is a trace over $SU(3)_L \times SU(3)_R$ indices, ϵ is as before the antisymmetric invariant symbol of $SU(2)$, and we have suppressed the implied SM-generation indices in this form. Although the SM quark fields are no longer in (incomplete) multiplets of TC-flavour, we can maintain a spurionic invariance of this expression under $SU(3)_L \times SU(3)_R$ by assigning the spurionic transformation rule $P \rightarrow U_R P U_L^\dagger$, where $P \in \{P_j, \tilde{P}^j\}$.

In the low-energy Chiral Lagrangian, the terms in Eq. (5.51) will give rise to, *inter alia*, the requisite quark Yukawa terms.

5.3.3 Relaxion Sector

The relaxion sector of the theory consists of a real pseudoscalar field ϕ , the relaxion [413], which we take to have dimension-5 effective axion couplings both to QCD and to the $SU(N)_{\text{TC}}$ gauge group (see Section 5.2.5); additionally, we allow for the existence of an additional potential $V_\phi(\phi)$ for the relaxion field:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \phi)^2 + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}}^0 \right] \text{Tr} [G_{\mu\nu} \tilde{G}^{\mu\nu}] + \frac{(g_{\text{TC}})^2}{16\pi^2} \frac{\phi}{F} \text{Tr} [G_{\text{TC}\mu\nu} \tilde{G}_{\text{TC}}^{\mu\nu}] - V_\phi(\phi), \quad (5.52)$$

where G and G_{TC} are, respectively, the matrix-valued gauge field strength tensors for the $SU(3)_C$ and $SU(N)_{\text{TC}}$ gauge groups, $\tilde{G}_{(\text{TC})\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G_{(\text{TC})}^{\alpha\beta}$ are the respective dual field strength tensors, and g_s and g_{TC} are the respective gauge couplings. In Eq. (5.52), θ_{QCD}^0 is a bare QCD θ -term; we do not write an analogous independent θ_{TC}^0 angle, as it could be absorbed into an unobservable shift of ϕ and θ_{QCD}^0 . Additionally, f is the usual QCD Peccei–Quinn symmetry breaking scale, and F is a PQ-symmetry breaking scale which, as we mentioned in Section 5.2, needs to be taken exponentially larger than f : $F \gg f$. Indeed, as we discussed in Section 5.2 and as we will find in more detail in Section 5.7, F will be required to be many orders of magnitude larger than the Planck scale; see Section 5.2.5 for further discussion.

The additional potential $V_\phi(\phi)$ is required to obtain the correct dynamical rolling of the relaxion field and will be discussed in more detail in Section 5.6.

5.3.4 Chiral Rotations

Before constructing the Chiral Lagrangian, we perform a $U(1)_A$ chiral rotation of the χ, χ^c fields to rotate the $SU(N)_{\text{TC}}$ -relaxion coupling into the technifermion mass matrix:

$$\chi \rightarrow e^{i\phi/6F} \chi \tag{5.53}$$

$$\chi^c \rightarrow e^{i\phi/6F} \chi^c. \tag{5.54}$$

Following the method of Fujikawa [69, 70] to include the anomalous [67, 68] transformation of the measure of the functional integral under this transformation, we find that this rotation

results in the following form for the Lagrangian:

$$\begin{aligned}
\mathcal{L} = & \mathcal{L}_{\text{SM}, H=0} + \frac{1}{2}(\partial_\mu \phi)^2 - V_\phi(\phi) + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}}^0 \right] \text{Tr} [G^{\mu\nu} \tilde{G}_{\mu\nu}] \\
& + i\chi^\dagger \bar{\sigma}^\mu D_\mu \chi + i\chi^c \sigma^\mu D_\mu (\chi^c)^\dagger - \text{Tr} [M(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& + \frac{1}{\Lambda_y^2} (D^c Y_D Q_j) \text{Tr} [P^j(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& - \frac{1}{\Lambda_y^2} (U^c Y_U Q_i) \epsilon^{ij} \text{Tr} [P_j(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& - \frac{\partial_\mu \phi}{6F} (J_A^0)^\mu - \frac{\phi}{F} \left[\frac{(g_2)^2}{16\pi^2} \frac{N}{3} \text{Tr} [W_{\mu\nu} \tilde{W}^{\mu\nu}] + \frac{(g_1)^2}{16\pi^2} \frac{N}{3} \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} \right], \tag{5.55}
\end{aligned}$$

where $W_{\mu\nu}$ are the matrix-valued $SU(2)_W$ field strength tensors, $B_{\mu\nu}$ is the $U(1)_Y$ hypercharge field strength tensor, and we have also now included the non-Higgs part of the SM Lagrangian, which we denote $\mathcal{L}_{\text{SM}, H=0}$ (by which notation we mean the usual SM Lagrangian with the elementary Higgs doublet H set equal to zero); $(J_A^0)^\mu \equiv [\chi^\dagger \bar{\sigma}^\mu \chi - \chi^c \sigma^\mu (\chi^c)^\dagger]$ is the $U(1)_A$ axial current; and the covariant derivative is still given by Eq. (5.45).

As a final step before we pass to the Chiral Lagrangian, we rotate to the mass-eigenstate basis for the SM quarks via the usual CKM manipulations [62, 63]. Following the exposition of Ref. [32], and letting $X \in \{U, D\}$ for the remainder in this paragraph (with X always to be read consistently as either U or D in every formula), it is always possible to write $Y_X \equiv P_X Q_X y_X Q_X^\dagger$ where P_X and Q_X are unitary matrices in SM-generation space and y_X are diagonal matrices whose entries are the real positive square roots of the eigenvalues of the Hermitian matrix $Y_X Y_X^\dagger$. We define $V_{\text{CKM}} \equiv Q_U^\dagger Q_D$. The necessary chiral quark rotation to bring the fields to the mass-eigenstate basis is then given by $X^c \rightarrow X^c Q_X^\dagger P_X^\dagger$ and $X \rightarrow Q_X X$,

which shifts $\theta_{\text{QCD}}^0 \rightarrow \theta_{\text{QCD}}^0 - \arg \det Y_U Y_D \equiv \theta_{\text{QCD}}$. The final result on the Lagrangian is

$$\begin{aligned}
\mathcal{L} = & \mathcal{L}_{\text{SM}, H=0, \text{NO QUARKS}} \\
& + iU^\dagger \bar{\sigma}^\mu \hat{D}_\mu U + iD^\dagger \bar{\sigma}^\mu \hat{D}_\mu D + i(D^c)^\dagger \sigma^\mu \hat{D}_\mu (D^c)^\dagger + i(U^c)^\dagger \sigma^\mu \hat{D}_\mu (U^c)^\dagger \\
& + \frac{g_2^2}{2} W_\mu^3 (U^\dagger \bar{\sigma}^\mu U - D^\dagger \bar{\sigma}^\mu D) + \frac{g_2^2}{\sqrt{2}} \left(W_\mu^+ U^\dagger \bar{\sigma}^\mu V_{\text{CKM}} D + W_\mu^- D^\dagger \bar{\sigma}^\mu V_{\text{CKM}}^\dagger U \right) \\
& + \frac{1}{2} (\partial_\mu \phi)^2 - V_\phi(\phi) + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}} \right] \text{Tr} [G^{\mu\nu} \tilde{G}_{\mu\nu}] \\
& + i\chi^\dagger \bar{\sigma}^\mu D_\mu \chi + i\chi^c \sigma^\mu D_\mu (\chi^c)^\dagger - \text{Tr} [M(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& + \frac{1}{\Lambda_y^2} (D^c y_D D) \text{Tr} [P^2(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& - \frac{1}{\Lambda_y^2} (U^c y_U U) \text{Tr} [P_2(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& + \frac{1}{\Lambda_y^2} (D^c y_D V_{\text{CKM}}^\dagger U) \text{Tr} [P^1(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& + \frac{1}{\Lambda_y^2} (U^c y_U V_{\text{CKM}} D) \text{Tr} [P_1(\chi\chi^c)] e^{i\phi/3F} + \text{h.c.} \\
& - \frac{\partial_\mu \phi}{6F} (J_A^0)^\mu - \frac{\phi}{F} \left[\frac{(g_2)^2}{16\pi^2} \frac{N}{3} \text{Tr} [W_{\mu\nu} \tilde{W}^{\mu\nu}] + \frac{(g_1)^2}{16\pi^2} \frac{N}{3} \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} \right], \tag{5.56}
\end{aligned}$$

where $\hat{D}_\mu \equiv \partial_\mu - ig_s G_\mu - ig_1 B_\mu Y$ (we have explicitly extracted the $SU(2)_W$ gauge couplings to the quarks) with G_μ the matrix-valued $SU(3)_C$ gauge fields, and B_μ and Y the $U(1)_Y$ hypercharge field and hypercharge operator respectively; $W_\mu^\pm \equiv (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$; and the notation ‘NO QUARKS’ on the SM part of the Lagrangian indicates that we have explicitly extracted and displayed all the quark-dependent terms.

5.4 Chiral Lagrangian

To proceed with the analysis of our model, Eq. (5.56), we must now pass to the theory of the bound states of χ and χ^c fermions after the $SU(N)_{\text{TC}}$ group confines. We therefore construct the Chiral Lagrangian (see, e.g., Ref. [34]) based on the global $SU(3)_L \times SU(3)_R \times U(1)_V$ (spurionic) TC-flavour symmetry exhibited by Eq. (5.56). On confinement,

$SU(3)_L \times SU(3)_R \times U(1)_V \rightarrow SU(3)_V \times U(1)_V$ owing to the spontaneous emergence of a chiral condensate $\langle \chi \chi^c \rangle \sim -\Lambda_{\text{TC}} F_\pi^2 \mathbb{1}_3$ (naïve dimensional analysis [NDA] estimate [641, 642]). The Chiral Lagrangian is therefore the theory of the eight ‘technipions’ π_{TC}^a —the pseudo-Nambu–Goldstone bosons of the spontaneously broken global $SU(3)_A$ symmetry. We assume that the excitation associated with the anomalous $U(1)_A$ symmetry, η'_{TC} , is massive enough to have been integrated out of the theory.

5.4.1 Matrix-Valued Technipion Field $\mathcal{U}$

The fundamental object in the construction of the Chiral Lagrangian is the matrix-valued field of the technipions, $\mathcal{U}$, which is assumed to transform in a $(\mathbf{3}, \bar{\mathbf{3}})$ of the $SU(3)_L \times SU(3)_R$ TC-flavour group:¹⁶

$$\mathcal{U} \equiv \exp \left[\frac{2i}{F_\pi} \Pi \right] \quad \text{where} \quad \Pi \equiv \pi_{\text{TC}}^a T^a, \quad (5.57)$$

with T^a the $SU(3)$ generators, and where F_π is the dimensionful compositeness scale. We define $\eta_{\text{TC}} \equiv \pi_{\text{TC}}^8$, $iH^+ \equiv (\pi_{\text{TC}}^4 - i\pi_{\text{TC}}^5)/\sqrt{2}$ and $iH^0 \equiv (\pi_{\text{TC}}^6 - i\pi_{\text{TC}}^7)/\sqrt{2}$; then Π is explicitly given by

$$\Pi = \frac{1}{2} \left(\begin{array}{c|c} \frac{1}{\sqrt{3}} \eta_{\text{TC}} \mathbb{1}_2 + \tilde{\Pi} & i\sqrt{2} H \\ \hline -i\sqrt{2} H^\dagger & -\frac{2}{\sqrt{3}} \eta_{\text{TC}} \end{array} \right), \quad (5.58)$$

where

$$\tilde{\Pi} = \left(\begin{array}{c|c} \pi_{\text{TC}}^3 & \pi_{\text{TC}}^1 - i\pi_{\text{TC}}^2 \\ \hline \pi_{\text{TC}}^1 + i\pi_{\text{TC}}^2 & -\pi_{\text{TC}}^3 \end{array} \right) \quad \text{and} \quad H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. \quad (5.59)$$

16. See Appendix D.1 for the general, closed-form expression for the matrix $\mathcal{U}$ in terms of the π_{TC}^a fields.

For reasons to become clear shortly, we will rewrite Eq. (5.59) as

$$\tilde{\Pi} = V_{\xi}^{\dagger} \left(\frac{\pi_{\text{TC}}^0}{\sqrt{2}\pi_{\text{TC}}^-} \middle| \frac{\sqrt{2}\pi_{\text{TC}}^+}{-\pi_{\text{TC}}^0} \right) V_{\xi} \quad \text{and} \quad H \equiv \frac{1}{\sqrt{2}} V_{\xi}^{\dagger} \begin{pmatrix} 0 \\ h \end{pmatrix}, \quad (5.60)$$

where $V_{\xi} \equiv \exp [i\xi^{\tilde{a}}(x)\tau^{\tilde{a}}]$. Under $SU(3)_{\text{V}}$ transformations, $\mathcal{U}$ transforms as $\mathcal{U} \rightarrow V_3 \mathcal{U} V_3^{\dagger}$, which implies that $\Pi \rightarrow V_3 \Pi V_3^{\dagger}$; in particular, since an $SU(2)$ subgroup of $SU(3)_{\text{V}}$ is gauged as $SU(2)_{\text{W}}$, an $SU(2)_{\text{W}}$ gauge transformation V acts with

$$V_3 \equiv \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \quad \text{where} \quad V \equiv \exp [i\alpha^{\tilde{a}}(x)\tau^{\tilde{a}}]; \quad (5.61)$$

alternatively,

$$\tilde{\Pi} \rightarrow V \tilde{\Pi} V^{\dagger}, \quad H \rightarrow V H, \quad \text{and} \quad \eta_{\text{TC}} \rightarrow \eta_{\text{TC}}. \quad (5.62)$$

We will work in a gauge where the $\xi^{\tilde{a}}$ in Eq. (5.60) are gauged away [i.e., a local gauge transformation with $\alpha^{\tilde{a}} = \xi^{\tilde{a}}$ is made to Eq. (5.60)]. That is, Eqs. (5.57), (5.58) and (5.60) define the matrix-valued field $\mathcal{U}$ in terms of its five physical degrees of freedom $\pi_{\text{TC}}^0, \pi_{\text{TC}}^{\pm}, \eta_{\text{TC}}$, and h provided we simply replace $V_{\xi}^{(\dagger)} \rightarrow \mathbb{1}_2$ in Eq. (5.60). After this gauge choice is made, it will turn out to be most convenient to also make the following $SO(2)$ rotation in field space

$$\begin{pmatrix} \kappa_{\text{TC}} \\ \omega_{\text{TC}} \end{pmatrix} \equiv \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \begin{pmatrix} \pi_{\text{TC}}^0 \\ \eta_{\text{TC}} \end{pmatrix} \quad \text{with} \quad \vartheta = \frac{\pi}{6}, \quad (5.63)$$

and work instead with the five degrees of freedom $\omega_{\text{TC}}, \kappa_{\text{TC}}, \pi_{\text{TC}}^{\pm}$, and h .

5.4.2 Chiral Lagrangian

A careful analysis of the spurionic symmetries of Eq. (5.56), and the anomalies in the various axial currents, yields the Chiral Lagrangian corresponding to Eq. (5.56), at leading order order in spurions and momenta:

$$\begin{aligned}
\mathcal{L} = \mathcal{L}_{\text{SM}, H=0} &+ \frac{1}{2}(\partial_\mu \phi)^2 - V_\phi(\phi) + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} - \theta_{\text{QCD}} \right] \text{Tr} [G^{\mu\nu} \tilde{G}_{\mu\nu}] \\
&+ \frac{F_\pi^2}{4} \text{Tr} [(D_\mu \mathcal{U})^\dagger (D^\mu \mathcal{U})] + c_m \Lambda_{\text{TC}} F_\pi^2 \text{Tr} [M \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \\
&- \frac{F_\pi}{\sqrt{2}} (D^c y_D D) \text{Tr} [P^2 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} + \frac{F_\pi}{\sqrt{2}} (U^c y_U U) \text{Tr} [P_2 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \\
&- \frac{F_\pi}{\sqrt{2}} \left(D^c y_D V_{\text{CKM}}^\dagger U \right) \text{Tr} [P^1 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \\
&- \frac{F_\pi}{\sqrt{2}} (U^c y_U V_{\text{CKM}} D) \text{Tr} [P_1 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \\
&- \frac{N}{8\pi^2} \left[\frac{(g_2)^2}{2} \text{Tr} \left[\Pi_\phi \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix} \right] \text{Tr} [W_{\mu\nu} \tilde{W}^{\mu\nu}] + (g_1)^2 \text{Tr} [\Pi_\phi Y^2] B_{\mu\nu} \tilde{B}^{\mu\nu} \right. \\
&\quad \left. + 2g_1 g_2 \text{Tr} [\Pi_\phi Y T^{\tilde{b}}] B_{\mu\nu} \tilde{W}^{\tilde{b}\mu\nu} \right],
\end{aligned} \tag{5.64}$$

where $\Pi_\phi \equiv -\frac{i}{2} \ln \mathcal{U} + (\phi/6F) \mathbb{1}_3$; $D_\mu \mathcal{U} \equiv \partial_\mu \mathcal{U} - i[v_\mu, \mathcal{U}]$ with $v_\mu \equiv g_1 B_\mu Y + g_2 W_\mu^{\tilde{a}} T^{\tilde{a}}$; $\Lambda_{\text{TC}} \approx 4\pi F_\pi / \sqrt{N}$ is the cutoff scale for this effective description; c_m is a perturbatively incalculable $\mathcal{O}(1)$ constant; we have absorbed the quark kinetic and gauge-coupling terms which were explicitly displayed in Eq. (5.56) back into $\mathcal{L}_{\text{SM}, H=0}$; and we have demanded canonically normalized Yukawa terms, which, up to a perturbatively incalculable constant, suggests $\Lambda_y^2 \sim \sqrt{2} F_\pi \Lambda_{\text{TC}} \approx (\sqrt{2N}/4\pi) \cdot \Lambda_{\text{TC}}^2$. While this last relation would seem to indicate all flavour structures are generated at the same scale, we emphasize that this is not necessarily the case—different structures could be generated with a hierarchy of scales and/or Wilson coefficients, depending on the mechanism underlying flavour.

5.4.3 Yukawas

It is straightforward to show that

$$\mathcal{L} \supset \frac{F_\pi}{\sqrt{2}} (U^c y_U U) \text{Tr} [P_2 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \quad (5.65)$$

$$\supset -\frac{1}{\sqrt{2}} (U^c y_U U) h \exp \left[i \frac{\phi}{3F} - i \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right] \text{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) + \text{h.c.}, \quad (5.66)$$

and

$$\mathcal{L} \supset -\frac{F_\pi}{\sqrt{2}} (D^c y_D D) \text{Tr} [P^2 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \quad (5.67)$$

$$\supset -\frac{1}{\sqrt{2}} (D^c y_D D) h \exp \left[i \frac{\phi}{3F} - i \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right] \text{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) + \text{h.c.}, \quad (5.68)$$

where

$$\bar{\pi}_{\text{TC}} \equiv \sqrt{h^2 + \omega_{\text{TC}}^2} \quad \text{and} \quad \text{sinc } x \equiv \frac{\sin x}{x}, \quad (5.69)$$

and where these results are correct to all orders in F_π for the uncharged fields, but we have ignored any (interaction) terms involving the electromagnetically charged (EM-charged) states $\sim \pi_{\text{TC}}^+ \pi_{\text{TC}}^-$. The other types of couplings of the $\mathcal{U}$ field to the SM quarks are (minimally) three-point interactions (after the h gets a vev) involving the EM-charged technipions $\pi_{\text{TC}}^\pm$:

$$\mathcal{L} \supset -\frac{F_\pi}{\sqrt{2}} \left(D^c y_D V_{\text{CKM}}^\dagger U \right) \text{Tr} [P^1 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \quad (5.70)$$

$$\supset -\frac{i}{2} \left(D^c y_D V_{\text{CKM}}^\dagger U \right) \frac{h\pi_{\text{TC}}^-}{F_\pi} \exp \left[i \frac{\phi}{3F} - i \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right] \left[1 + \mathcal{O}(F_\pi^{-1}) \right] + \text{h.c.}, \quad (5.71)$$

$$\mathcal{L} \supset -\frac{F_\pi}{\sqrt{2}} (U^c y_U V_{\text{CKM}} D) \text{Tr} [P_1 \mathcal{U}] e^{i\phi/3F} + \text{h.c.} \quad (5.72)$$

$$\supset +\frac{i}{2} (U^c y_U V_{\text{CKM}} D) \frac{h\pi_{\text{TC}}^+}{F_\pi} \exp \left[i \frac{\phi}{3F} - i \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right] \left[1 + \mathcal{O}(F_\pi^{-1}) \right] + \text{h.c.}, \quad (5.73)$$

where we have kept only those terms with no additional charged fields.

Eqs. (5.66) and (5.68) include, *inter alia*, the Yukawas that give rise to the SM quark masses; however, they also contain unwanted phases unless $\langle \phi/(3F) - \kappa_{\text{TC}}/(\sqrt{3}F_\pi) \rangle = 0$. We therefore perform a further chiral field redefinition on (every generation of) the SM quarks:

$$\{D, D^c, U, U^c\} \rightarrow \{D, D^c, U, U^c\} \cdot \exp \left[-\frac{i}{2} \left(\frac{\phi}{3F} - \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right) \right]. \quad (5.74)$$

This has a number of effects: (a) it removes the exponential factors in Eqs. (5.66) and (5.68), so that the Lagrangian contains the straightforward Yukawa terms

$$\mathcal{L} \supset -\frac{1}{\sqrt{2}} (U^c y_U U + D^c y_D D) h \text{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) + \text{h.c.}; \quad (5.75)$$

(b) it removes the explicit exponential factors in Eqs. (5.71) and (5.73); (c) the QCD θ -angle shifts: $\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}} + (6\kappa_{\text{TC}})/(\sqrt{3}F_\pi) - i(2\phi)/F$; and (d) an additional term is added to the Lagrangian:

$$\mathcal{L} \supset -\frac{1}{2} \left[U^c \sigma^\mu (U^c)^\dagger - U^\dagger \bar{\sigma}^\mu U + (U^{(c)} \leftrightarrow D^{(c)}) \right] \partial_\mu \left[\frac{\phi}{3F} - \frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} \right]. \quad (5.76)$$

5.4.4 Expanding the Chiral Lagrangian

In order to analyse Eq. (5.64) [as modified per the discussion in Section 5.4.3], it is necessary to write it out in terms of the physical degrees of freedom of $\mathcal{U}$: ω_{TC} , κ_{TC} , $\pi_{\text{TC}}^\pm$, and h . By making use of Eq. (D.3) [see Appendix D.1], it is in principle possible to do this exactly in closed form. However, our primary interest here will be those terms which have a bearing on the spectrum of the theory, and the effective potential for the neutral scalar fields. As such, we will not need all the higher-order interaction terms in their full generality.

In addition to the Yukawa terms we already discussed in Section 5.4.3, the terms of interest to us are: (a) the kinetic and potential terms for the relaxion, and its coupling to QCD; (b) the kinetic terms for the states ω_{TC} , κ_{TC} , and h ; (c) the kinetic terms for the states $\pi_{\text{TC}}^\pm$, their kinetic mixing terms with $W_\mu^\pm$ owing to our gauge choice discussed in

Section 5.4.1, and the mass terms for the $W^\pm$ and Z bosons; (d) the mass terms for the EM-charged technipions; and (e) the terms which give rise to the full tree-level potential for the EM-neutral scalars ω_{TC} , κ_{TC} , and h . We discuss each of these terms in turn.

(a) After the chiral rotation in Section 5.4.3, the relaxion-dependent terms in the first line of Eq. (5.64) are

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \phi)^2 - V_\phi(\phi) + \frac{(g_s)^2}{16\pi^2} \left[\frac{\phi}{f} \left(1 + \frac{2f}{F} \right) - \frac{2\sqrt{3}}{F_\pi} \kappa_{\text{TC}} - \theta_{\text{QCD}} \right] \text{Tr} [G^{\mu\nu} \tilde{G}_{\mu\nu}]. \quad (5.77)$$

The term proportional to $f/F \ll 1$ can be dropped.

(b) The kinetic terms for ω_{TC} , κ_{TC} , and h are contained in the term $\frac{F_\pi^2}{4} \text{Tr} [(D_\mu \mathcal{U})^\dagger (D^\mu \mathcal{U})]$ in Eq. (5.64). We are only interested in the two-derivative terms which are potentially quadratic in the fields after the EM-neutral scalars possibly obtain vevs; we can thus always neglect any terms which contain un-differentiated $\pi_{\text{TC}}^\pm$ fields, but we need to keep terms to all orders in the un-differentiated ω_{TC} , κ_{TC} , and h fields. It is straightforward to obtain these terms by sending $\pi_{\text{TC}}^\pm \rightarrow 0$ in the definition for $\mathcal{U}$ before directly exponentiating and inserting the result into the relevant terms in Eq. (5.64). The terms which arise are

$$\begin{aligned} \mathcal{L} \supset & \frac{1}{2}(\partial_\mu \kappa_{\text{TC}})^2 + \frac{1}{2}(\partial_\mu h)^2 \left[\frac{h^2}{\bar{\pi}_{\text{TC}}^2} + \frac{\omega_{\text{TC}}^2}{\bar{\pi}_{\text{TC}}^2} \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \right] \\ & + \frac{1}{2}(\partial_\mu \omega_{\text{TC}})^2 \left[\frac{\omega_{\text{TC}}^2}{\bar{\pi}_{\text{TC}}^2} + \frac{h^2}{\bar{\pi}_{\text{TC}}^2} \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \right] \\ & + (\partial_\mu h)(\partial^\mu \omega_{\text{TC}}) \frac{h\omega_{\text{TC}}}{\bar{\pi}_{\text{TC}}^2} \left[1 - \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \right]. \end{aligned} \quad (5.78)$$

(c) Given our gauge choice, the $\pi_{\text{TC}}^\pm$ fields can possibly kinetically mix with the $W_\mu^\pm$. The relevant kinetic terms for $\pi_{\text{TC}}^\pm$, the kinetic mixing terms, and the gauge boson mass terms are also contained in the term $\frac{F_\pi^2}{4} \text{Tr} [(D_\mu \mathcal{U})^\dagger (D^\mu \mathcal{U})]$ in Eq. (5.64). Similar arguments to those made at (b) apply about which terms need to be kept to find all possible contributions to the two-derivative, one-derivative-one-gauge-boson, or two-gauge-boson terms which are

possibly quadratic in the fields after the EM-neutral scalars possibly obtain vevs. To extract these terms, we make use of the exact closed-form expression Eq. (D.3), expanded out to quadratic order in the charged technipion fields. The relevant terms are

$$\begin{aligned}
\mathcal{L} \supset & \partial_\mu \pi_{\text{TC}}^+ \partial^\mu \pi_{\text{TC}}^- \left(\begin{aligned} & 2F_\pi^2 \frac{h^2 + (\omega_{\text{TC}} + \sqrt{3}\kappa_{\text{TC}})^2}{(\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)^2} \left[1 - \cos\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \right] \\ & - 2F_\pi \frac{\bar{\pi}_{\text{TC}}^2(\omega_{\text{TC}} + 2\sqrt{3}\kappa_{\text{TC}}) + 3\kappa_{\text{TC}}^2\omega_{\text{TC}}}{(\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)^2} \sin\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \end{aligned} \right) \\
& - ig_2 \left(W_\mu^+ \partial^\mu \pi_{\text{TC}}^- - W_\mu^- \partial^\mu \pi_{\text{TC}}^+ \right) \left(\begin{aligned} & \frac{F_\pi^2(\sqrt{3}\kappa_{\text{TC}} + \omega_{\text{TC}})}{\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2} \left[1 - \cos\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \right] \\ & - \frac{F_\pi(\bar{\pi}_{\text{TC}}^2 + \sqrt{3}\kappa_{\text{TC}}\omega_{\text{TC}})}{\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2} \sin\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \end{aligned} \right) \\
& + \frac{g_2^2 F_\pi^2}{2} \left[1 - \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) - \frac{\omega_{\text{TC}}}{F_\pi} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\sqrt{3}\kappa_{\text{TC}}}{F_\pi}\right) \right] W_\mu^- W^\mu + \\
& + \frac{g_1^2 + g_2^2}{8} h^2 \text{sinc}^2\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) Z^2. \tag{5.79}
\end{aligned}$$

(d) The mass terms for the EM-charged technipions are obtained from the terms $c_m \Lambda_{\text{TC}} F_\pi^2 \text{Tr}[M\mathcal{U}] e^{i\phi/3F} + \text{h.c.}$ in Eq. (5.64). The relevant terms in the expansion are those proportional to $\pi_{\text{TC}}^+ \pi_{\text{TC}}^-$, and are again obtained using the exact closed-form expression

Eq. (D.3), expanded out to quadratic order in the charged technipion fields:

$$\begin{aligned}
\mathcal{L} \supset & c_m \Lambda_{\text{TC}} \pi_{\text{TC}}^+ \pi_{\text{TC}}^- \\
& \times \left[\begin{aligned}
& -4F_\pi^2(m_L - m_N) \frac{h^2}{(\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)^2} \left[\cos\left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) - \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \right] \\
& + 6\sqrt{3}F_\pi(m_L - m_N) \frac{\kappa_{\text{TC}} h^2 (\bar{\pi}_{\text{TC}}^2 - \kappa_{\text{TC}}^2)}{\bar{\pi}_{\text{TC}}^2 (\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)^2} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \\
& + 4F_\pi m_L \frac{\sqrt{3}\kappa_{\text{TC}} + \omega_{\text{TC}}}{\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2} \sin\left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) \\
& + 2F_\pi \frac{\sqrt{3}(m_L + m_N)\kappa_{\text{TC}} h^2 + 2m_L \omega_{\text{TC}} (\bar{\pi}_{\text{TC}}^2 + \sqrt{3}\kappa_{\text{TC}} \omega_{\text{TC}})}{\bar{\pi}_{\text{TC}}^2 (\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)} \\
& \quad \times \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \\
& - 2 \frac{(m_L + m_N) h^2 + 2m_L \omega_{\text{TC}} (\sqrt{3}\kappa_{\text{TC}} + \omega_{\text{TC}})}{(\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2)} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right)
\end{aligned} \right],
\end{aligned} \tag{5.80}$$

which in the equal-mass limit $m_L = m_N \equiv m$, which we will needed later, is given by

$$\begin{aligned}
\mathcal{L} \stackrel{\text{equal masses}}{\supset} & 4c_m m \Lambda_{\text{TC}} \pi_{\text{TC}}^+ \pi_{\text{TC}}^- \\
& \times \left[\begin{aligned}
& F_\pi \frac{\sqrt{3}\kappa_{\text{TC}} + \omega_{\text{TC}}}{\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2} \left[\cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) + \sin\left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) \right] \\
& - \frac{\bar{\pi}_{\text{TC}}^2 + \sqrt{3}\kappa_{\text{TC}} \omega_{\text{TC}}}{\bar{\pi}_{\text{TC}}^2 - 3\kappa_{\text{TC}}^2} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right)
\end{aligned} \right].
\end{aligned} \tag{5.81}$$

(e) Finally, the terms that give the (tree-level) contribution to the scalar potential for the EM-neutral technipions are also contained in the terms $c_m \Lambda_{\text{TC}} F_\pi^2 \text{Tr}[M\mathcal{U}] e^{i\phi/3F} + \text{h.c.}$ in Eq. (5.64). The relevant terms are those with no EM-charged technipions [i.e., we again send $\pi_{\text{TC}}^\pm \rightarrow 0$ in the expression for $\mathcal{U}$, before directly exponentiating and inserting the result into

the relevant terms in Eq. (5.64)]:

$$\begin{aligned}
\mathcal{L} \supset & + 2F_\pi^2 \Lambda_{\text{TC}} c_m (m_L + m_N) \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \\
& + 2F_\pi^2 \Lambda_{\text{TC}} c_m m_L \cos\left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) \\
& + 2F_\pi^2 \Lambda_{\text{TC}} c_m (m_L - m_N) \frac{\omega_{\text{TC}}}{F_\pi} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right). \tag{5.82}
\end{aligned}$$

5.5 Effective Potential and Electroweak Symmetry Breaking

Eqs. (5.75)–(5.82) contain all the terms from the Chiral Lagrangian Eq. (5.64) which will be relevant for our further analysis. The immediate next step is to understand the effective potential for the EM-neutral scalars in more detail, in order to understand which of the EM-neutral scalars κ_{TC} , ω_{TC} , and h obtain vevs as the relaxion field ϕ slow-rolls.¹⁷

5.5.1 Effective Potential

After QCD quark confinement, and reading off the relevant terms from Eqs. (5.77) and (5.82), we have the following tree-level contributions to the effective potential (recalling that $\mathcal{L} \supset -V$):

$$\begin{aligned}
V_{\text{eff., tree}} = & V_\phi(\phi) + V_{\text{QCD}}(h, \kappa_{\text{TC}}, \omega_{\text{TC}}, \phi) \\
& - 2F_\pi^2 \Lambda_{\text{TC}} c_m (m_L + m_N) \cos\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \cos\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \\
& - 2F_\pi^2 \Lambda_{\text{TC}} c_m m_L \cos\left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) \\
& - 2F_\pi^2 \Lambda_{\text{TC}} c_m (m_L - m_N) \frac{\omega_{\text{TC}}}{F_\pi} \text{sinc}\left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi}\right) \sin\left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right). \tag{5.83}
\end{aligned}$$

17. Since the relaxion field ϕ is assumed to be slow-rolling down its potential until it stalls, we will always assume that the fields κ_{TC} , ω_{TC} , and h take vevs such that the instantaneous minimum—with ϕ held fixed—of the effective potential is obtained.

QCD Contribution

Although the detailed form of the QCD contribution to Eq. (5.83) depends on the exact details of QCD confinement and the SM quark masses, the only properties that will be relevant are that the potential is (a) periodic, and (b) proportional to the light SM quark masses, which owing to the Yukawa couplings Eq. (5.75) implies proportionality to $|h \operatorname{sinc}(\bar{\pi}_{\text{TC}}/F_\pi)|$. We will thus take the approximate form, based on Eqs. (5.75) and (5.77), for the QCD contribution:

$$V_{\text{QCD}}(h, \kappa_{\text{TC}}, \omega_{\text{TC}}, \phi) \approx -\Lambda_{\text{low}}^3 \left| h \operatorname{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \right| \cos \left[\frac{\phi}{f} - \frac{2\sqrt{3}\kappa_{\text{TC}}}{F_\pi} - \theta_{\text{QCD}} \right]. \quad (5.84)$$

In this normalization, $\Lambda_{\text{low}}^3 h \operatorname{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \sim m_\pi^2 f_\pi^2$. For $h \operatorname{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) = v_{\text{SM}} \approx 246 \text{ GeV}$, we then have $\Lambda_{\text{low}} \approx 8.5 \text{ MeV}$. Note also that the effective QCD θ -angle is given by

$$\theta_{\text{QCD}}^{\text{eff.}} = \theta_{\text{QCD}} - \frac{\phi}{f} + \frac{2\sqrt{3}\kappa_{\text{TC}}}{F_\pi}. \quad (5.85)$$

Radiative Corrections

Important one-loop radiative corrections to the potential for the composite pNGB states arise from top quark loops owing to the large top Yukawa, $y_t \approx 1$.¹⁸ The impact of the top loops is in principle finite and calculable (e.g., on the lattice), but cannot be computed in the Chiral Lagrangian framework because the corrections are quadratically divergent, and are thus sensitive to physics at the cutoff scale Λ_{TC} of the low-energy effective description (which we cannot perturbatively match to the [known] UV completion as the latter is strongly coupled at the matching scale). Nevertheless, we can estimate their size using NDA [642],

18. As mentioned in footnote 3, we ignore the subdominant gauge loops, as they do not qualitatively alter the dynamics of our model.

and add to the effective potential a contribution

$$V_{\text{eff.}} \supset -c_t N_c \frac{y_t^2}{16\pi^2} \Lambda_{\text{TC}}^2 h^2 \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right), \quad (5.86)$$

where c_t is an $\mathcal{O}(1)$ constant which is incalculable in perturbation theory, and $N_c = 3$ is the number of QCD quark colours; note the dependence on $y_t^2 h^2 \text{sinc}^2(\bar{\pi}_{\text{TC}}/F_\pi)$, which is proportional to m_t^2 per Eq. (5.75). The sign here is crucially important, but is also not calculable within the Chiral Lagrangian framework for reasons similar to those advanced above about the size of the top loop correction; we nevertheless assume that the sign is negative, as is obtained from a naïve perturbative loop computation (see, e.g., Ref. [627] for discussion of this point).

Equal-Mass Limit

For the remainder of this chapter we will work in the equal-mass limit $m_L = m_N$; this case is most amenable to straightforward analysis, and yields all the desired properties. In Appendix D.2, we revisit the more complicated case of unequal masses, $m_L \neq m_N$. The important conclusion from the analysis in Appendix D.2 is that the equal-mass limit is not in any way special from the point of view of its physical properties: much the same qualitative picture of the EWSB dynamics is obtained for $m_L \neq m_N$ as for $m_L = m_N$, and we thus do not lose any qualitative features by making the simplifying equal-mass assumption.

Combining Eqs. (5.83)–(5.86), and setting $m_L = m_N \equiv m$, we obtain the following contributions to the one-loop effective potential:

$$\begin{aligned} V_{\text{eff.}} \supset & V_\phi(\phi) - \Lambda_{\text{low}}^3 \left| h \text{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \right| \cos \left[\frac{\phi}{f} - \frac{2\sqrt{3}\kappa_{\text{TC}}}{F_\pi} - \theta_{\text{QCD}} \right] \\ & - 2F_\pi^2 \Lambda_{\text{TC}} c_m m \left[2 \cos \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \cos \left(\frac{\kappa_{\text{TC}}}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) + \cos \left(\frac{2\kappa_{\text{TC}}}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) \right] \\ & - 2F_\pi^2 \Lambda_{\text{TC}} c_m m \epsilon_t \frac{h^2}{F_\pi^2} \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right), \end{aligned} \quad (5.87)$$

where we have defined

$$\epsilon_t \equiv \frac{c_t}{c_m} N_c \frac{y_t^2}{32\pi^2} \frac{\Lambda_{\text{TC}}}{m} > 0. \quad (5.88)$$

5.5.2 Electroweak-Symmetric Phase

In the electroweak-symmetric (EW-symmetric) phase of the theory, it is straightforward to show that

$$\langle h \rangle = 0, \quad \langle \kappa_{\text{TC}} \rangle = 0, \quad \langle \omega_{\text{TC}} \rangle = 0, \quad (5.89)$$

and, assuming slow-roll of the relaxion field ϕ ,

$$\partial_t \phi \propto -\partial_\phi V \Big|_{\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0} = -\partial_\phi V_\phi - 2 \frac{\Lambda_{\text{TC}} F_\pi^2 c_m m}{F} \sin\left(\frac{\phi}{3F}\right). \quad (5.90)$$

We will return to a discussion of the rolling of the relaxion in Section 5.6; for now let us focus on the other properties of this phase, for a fixed value of the relaxion field ϕ . It is straightforward to see from Eqs. (5.78) and (5.79) that κ_{TC} , ω_{TC} , $\pi_{\text{TC}}^\pm$, and h have canonical kinetic terms, that there is no $\pi_{\text{TC}}^\pm - W_\mu^\pm$ kinetic mixing, and that the $W^\pm$ and Z bosons are massless (as is of course required for this phase). Moreover, ignoring in the squared-mass matrix¹⁹ off-diagonal entries proportional to F_π/F (which ratio is exponentially small) that mix ϕ with ω_{TC} and κ_{TC} , that matrix is diagonal and the squared-masses of the scalars are

$$m_{\kappa_{\text{TC}}}^2 = m_{\omega_{\text{TC}}}^2 = m_{\pi_{\text{TC}}^\pm}^2 = 4c_m m \Lambda_{\text{TC}} \cos\left(\frac{\phi}{3F}\right), \quad (5.91)$$

$$\text{and} \quad m_h^2 = 4c_m m \Lambda_{\text{TC}} \left[\cos\left(\frac{\phi}{3F}\right) - \epsilon_t \right]; \quad (5.92)$$

19. Defined for the EM-neutral scalars as $M_{XY}^2 \equiv \partial_X \partial_Y V_{\text{eff.}}|_{\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0}$ with $X, Y \in \{h, \omega_{\text{TC}}, \kappa_{\text{TC}}, \phi\}$; for the EM-charged scalars, one can simply read off the mass from Eq. (5.81).

since ϕ is still rolling in this phase, $m_\phi^2 \equiv \partial_\phi^2 V|_{\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0}$ has no physical interpretation.

In exact parallel with the ‘cartoon’ model of Section 5.2, we see from Eq. (5.92) that the EW-symmetric phase is thus stable so long as $\cos(\phi/3F) > \cos(\phi_{\text{crit.}}/3F) > 0$, where

$$\cos\left(\frac{\phi_{\text{crit.}}}{3F}\right) \equiv \epsilon_t. \quad (5.93)$$

We emphasize the perhaps obvious point that the solution Eq. (5.89) exists independent of the values of any of the other parameters in the theory; this will be important to bear in mind when we discuss the evolution of the potential with changing ϕ in Section 5.5.3.

5.5.3 Broken Phase

In the broken phase of the theory, we find that both h and κ_{TC} obtain vevs, which are determined by the following relations:

$$\cos\left(\frac{\langle h \rangle}{F_\pi}\right) = \frac{1}{\epsilon_t} \cos\left[\frac{\phi}{3F} - \frac{1}{2} \arctan\left(\frac{2 \sin\left(\frac{\phi}{3F}\right) [\cos\left(\frac{\phi}{3F}\right) - \epsilon_t]}{\cos\left(\frac{2\phi}{3F}\right) + 2\epsilon_t \cos\left(\frac{\phi}{3F}\right)}\right)\right], \quad (5.94)$$

$$\tan\left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi}\right) = \frac{2 \sin\left(\frac{\phi}{3F}\right) [\cos\left(\frac{\phi}{3F}\right) - \epsilon_t]}{\cos\left(\frac{2\phi}{3F}\right) + 2\epsilon_t \cos\left(\frac{\phi}{3F}\right)}, \quad (5.95)$$

$$\langle \omega_{\text{TC}} \rangle = 0. \quad (5.96)$$

Per Eq. (5.78), since $\langle \omega_{\text{TC}} \rangle = 0$, h and κ_{TC} have canonical kinetic terms, and there is no h - ω_{TC} kinetic mixing. The field ω_{TC} does not however have a canonical kinetic term: $\mathcal{L} \supset \frac{1}{2}(\partial_\mu \omega_{\text{TC}})^2 \text{sinc}^2(\langle h \rangle/F_\pi)$; we will return to this point below.

A Deeper Investigation of the EWSB Minimum

Prior to any further examination of the properties of this phase (masses, etc.), it is worthwhile to examine the results Eqs. (5.94) and (5.95) in more detail in the vicinity of $\phi = \phi_{\text{crit.}}$, as

this clarifies the physical situation tremendously. Suppose that $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$, where $|\delta| \ll 1$. Expanding Eqs. (5.94) and (5.95) in powers of δ , we find

$$\cos\left(\frac{\langle h \rangle}{F_\pi}\right) = 1 - \frac{3\epsilon_t \delta \sqrt{1 - \epsilon_t^2}}{4\epsilon_t^2 - 1} + \mathcal{O}(\delta^2) \quad (5.97)$$

$$\tan\left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi}\right) = -2\delta \frac{1 - \epsilon_t^2}{4\epsilon_t^2 - 1} + \mathcal{O}(\delta^2). \quad (5.98)$$

Clearly, Eq. (5.97) has a real solution for $\langle h \rangle$ only if

$$\frac{\delta}{4\epsilon_t^2 - 1} \geq 0 \quad \text{and} \quad 0 < \epsilon_t \leq 1, \quad (5.99)$$

where we used that $\epsilon_t > 0$ [Eq. (5.88)]. There are two regimes that satisfy these constraints:

(a) $\delta > 0$ and $1/2 < \epsilon_t \leq 1$, and (b) $\delta < 0$ and $0 < \epsilon_t < 1/2$. In either case (a) or (b), Eqs. (5.97) and (5.98) have two approximate solutions in the vicinity of $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ (cf. Section 5.2):

$$\frac{\langle h \rangle}{F_\pi} = \pm \left[6\delta \frac{\epsilon_t \sqrt{1 - \epsilon_t^2}}{4\epsilon_t^2 - 1} + \mathcal{O}(\delta^2) \right]^{\frac{1}{2}} \quad \text{and} \quad \frac{\langle \kappa_{\text{TC}} \rangle}{F_\pi} = -\sqrt{3}\delta \frac{1 - \epsilon_t^2}{4\epsilon_t^2 - 1} + \mathcal{O}(\delta^2). \quad (5.100)$$

Note also that $\partial_h^2 V_{\text{eff.}}|_{\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0} = -4c_m m \delta \Lambda_{\text{TC}} \sqrt{1 - \epsilon_t^2} + \mathcal{O}(\delta^2)$.

Suppose then that $0 < \epsilon_t < 1/2$. As we noted above, the solution at $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ also exists for any value of the parameters. If additionally $\delta < 0$, then both solutions Eq. (5.100) exist [case (b)], for a total of three solutions for $\langle h \rangle$ and $\langle \kappa_{\text{TC}} \rangle$ in the vicinity of $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ (and $\langle \omega_{\text{TC}} \rangle = 0$). These solutions merge as $\delta \rightarrow 0$ from below. Once $\delta > 0$, the solutions Eq. (5.100) no longer exist, leaving only the solution $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ in the vicinity of $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$. Moreover, since $\partial_h^2 V_{\text{eff.}}|_{\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0}$ is positive for $\delta < 0$ and negative for $\delta > 0$, the solution $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ is stable for $\delta < 0$ and unstable for $\delta > 0$. Further analysis shows that the solutions Eq. (5.100) are unstable if $0 < \epsilon_t < 1/2$. Therefore, we find that for $0 < \epsilon_t < 1/2$, the model exhibits a so-called ‘subcritical pitchfork

bifurcation' at $\delta = 0$ (see, e.g., Ref. [643]). As δ approaches zero from below, a stable solution exists at the origin in field space; two additional solutions—both unstable—exist nearby in field space. As δ gets nearer zero, the two unstable solutions approach the stable one, and they merge at $\delta = 0$ (i.e., $\phi = \phi_{\text{crit.}}$). Once $\delta > 0$, there are no stable solutions left in the vicinity of the original stable solution. Indeed, in this case, once $\delta > 0$, the nearest minimum of the potential occurs for $|\langle h \rangle / F_\pi| \approx \pi$ and $|\langle \kappa_{\text{TC}} \rangle / F_\pi| \sim \mathcal{O}(1)$. This is clearly the incorrect behaviour for a reasonable EWSB transition.

Consider then the other case, $1/2 < \epsilon_t \leq 1$. For $\delta < 0$, the solutions Eq. (5.100) do not exist, and the only solution in the vicinity of $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ is $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ itself. If $\delta > 0$, then both solutions Eq. (5.100) exist [case (a)], for a total of three solutions for $\langle h \rangle$ and $\langle \kappa_{\text{TC}} \rangle$ in the vicinity of $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ (and $\langle \omega_{\text{TC}} \rangle = 0$). These solutions separate from each other as δ grows more positive. Moreover, the solution $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ is stable for $\delta < 0$ and unstable for $\delta > 0$. Further analysis shows that the solutions Eq. (5.100) are stable if $1/2 < \epsilon_t \leq 1$. Therefore, we find that for $1/2 < \epsilon_t \leq 1$, the model exhibits a so-called 'supercritical pitchfork bifurcation' at $\delta = 0$. For $\delta < 0$, a stable solution exists at the origin in field space, and no other solutions exist nearby. Once $\delta > 0$, two new stable solutions appear in the vicinity of the origin in field space, and the solution at the origin becomes unstable. The system will relax to one or the other of these new stable solutions, which slowly separate from $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = 0$ as δ becomes increasingly positive. This is the behaviour we need, and closely mirrors the behaviour of the 'cartoon' model of Section 5.2; see Figure 5.1

An alternative analysis is also instructive. Consider $V_{\text{eff.}}$ evaluated at $\langle \omega_{\text{TC}} \rangle = 0$ and with $\langle \kappa_{\text{TC}} \rangle$ fixed at the solution Eq. (5.95). Expanding $V_{\text{eff.}}$ in powers of h for fixed δ , we find

$$\frac{V_{\text{eff.}}|_{\langle \kappa_{\text{TC}} \rangle \text{ from Eq. (5.95)} \atop \langle \omega_{\text{TC}} \rangle = 0}}{c_m \Lambda_{\text{TC}} F_\pi^2 m} \approx \left(-6\epsilon_t + 6\delta\sqrt{1 - \epsilon_t^2} \right) - 2\delta\sqrt{1 - \epsilon_t^2} \left(\frac{h^2}{F_\pi^2} \right) + \frac{4\epsilon_t^2 - 1}{6\epsilon_t} \left(\frac{h^4}{F_\pi^4} \right) + \dots \quad (5.101)$$

Therefore, for $1/2 < \epsilon_t \leq 1$, the quartic coupling is positive, with the h squared-mass parameter positive for $\delta < 0$ and negative for $\delta > 0$, exactly as required to obtain a slow separation of the EWSB minimum from the EW-symmetric minimum as δ increases through zero. For $0 < \epsilon_t < 1/2$, the h squared-mass parameter is still positive for $\delta < 0$ and negative for $\delta > 0$; however, the quartic coupling is negative in both cases. Thus, the moment the h squared-mass parameter runs negative as δ increases through zero, the h field rolls off to a large field value.²⁰

A parameter space restriction is thus required [cf. Eq. (5.7)]:

$$1/2 < \epsilon_t \leq 1 \quad \Leftrightarrow \quad \frac{c_t N_c y_t^2}{32\pi^2 c_m} \leq \frac{m}{\Lambda_{\text{TC}}} < \frac{c_t N_c y_t^2}{16\pi^2 c_m}. \quad (5.102)$$

The technifermion masses may thus be no larger than a loop factor smaller than Λ_{TC} ; such a choice is, however, technically natural. Note also that this implies a very mild restriction on the value of $\phi_{\text{crit.}}$: $0 \leq (\phi_{\text{crit.}}/F) < \pi$.

Properties of the Broken Phase

Again, we will return to a discussion of the rolling of the relaxion in Section 5.6; for now let us focus again on the other properties of this phase for a fixed value of the relaxion field ϕ .

Canonically normalizing the ω_{TC} field by sending $\omega_{\text{TC}} \rightarrow \omega_{\text{TC}}/\text{sinc}(\langle h \rangle/F_\pi)$, and evaluating the broken-phase scalar squared-mass matrix

$$M_{XY}^2 \equiv \left. \frac{\partial^2 V_{\text{eff.}}}{\partial X \partial Y} \right|_{\substack{\langle h \rangle \text{ from Eq. (5.94)} \\ \langle \kappa_{\text{TC}} \rangle \text{ from Eq. (5.95)} \\ \langle \omega_{\text{TC}} \rangle = 0 \\ \phi = \phi_{\text{crit.}} + 3F\delta}} \quad \text{with} \quad X, Y \in \{h, \kappa_{\text{TC}}, \omega_{\text{TC}}, \phi\}, \quad (5.103)$$

ignoring all terms suppressed by one or more powers of the (exponentially small) ratio F_π/F ,

20. Strictly speaking, the expansion for fourth-order in h does not allow one to make this latter conclusion because, e.g., the sixth-order term could stabilize a nearby minimum. Our conclusion here is nevertheless correct, and is based on evaluation of the full (unexpanded) potential.

and ignoring (small) QCD corrections everywhere except in the squared-mass of the relaxation field, we find that the squared-masses of the scalars²¹ are

$$m_{\text{phys. Higgs}}^2 = \frac{2}{3} c_m m \Lambda_{\text{TC}} \times \left[2 \cos \left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) + 4 \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) - 3\epsilon_t \cos \left(\frac{2\langle h \rangle}{F_\pi} \right) \right] \\ \times \left[- \left(\left[2 \cos \left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) - 2 \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) \right]^2 + 3\epsilon_t \cos \left(\frac{2\langle h \rangle}{F_\pi} \right) + 12 \sin^2 \left(\frac{\langle h \rangle}{F_\pi} \right) \sin^2 \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) \right]^{\frac{1}{2}} \right] \quad (5.104)$$

$$= \left(8c_m m \Lambda_{\text{TC}} \sqrt{1 - \epsilon_t^2} \right) \delta + \mathcal{O}(\delta^2), \quad (5.105)$$

$$m_{\text{phys. } \kappa_{\text{TC}}}^2 = \frac{2}{3} c_m m \Lambda_{\text{TC}} \times \left[2 \cos \left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) + 4 \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) - 3\epsilon_t \cos \left(\frac{2\langle h \rangle}{F_\pi} \right) \right] \\ \times \left[+ \left(\left[2 \cos \left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) - 2 \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) \right]^2 + 3\epsilon_t \cos \left(\frac{2\langle h \rangle}{F_\pi} \right) + 12 \sin^2 \left(\frac{\langle h \rangle}{F_\pi} \right) \sin^2 \left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) \right]^{\frac{1}{2}} \right] \quad (5.106)$$

$$= 4c_m m \Lambda_{\text{TC}} \epsilon_t \left[1 - 4\delta \frac{(2\epsilon_t^2 - 1)\sqrt{1 - \epsilon_t^2}}{\epsilon_t(4\epsilon_t^2 - 1)} + \mathcal{O}(\delta^2) \right], \quad (5.107)$$

21. The h and κ_{TC} fields mix; the physical Higgs is mostly h ; the ‘physical κ_{TC} ’ is the mostly κ_{TC} state. Note that if we did not ignore the terms $\sim F_\pi/F$ in the mass matrix, the ϕ would also mix with the h and κ_{TC} , with a mixing angle $\sim F_\pi/F \ll 1$. This mixing of the CP-even and CP-odd scalars is allowed since the ϕ vev is non-zero, which breaks CP.

$$m_{\omega_{\text{TC}}}^2 = 4c_m m \Lambda_{\text{TC}} \frac{\cos\left(\frac{\langle\kappa_{\text{TC}}\rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) + \epsilon_t \left[\text{sinc}\left(\frac{\langle h \rangle}{F_\pi}\right) - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \right]}{\text{sinc}\left(\frac{\langle h \rangle}{F_\pi}\right)} \quad (5.108)$$

$$= 4c_m m \Lambda_{\text{TC}} \epsilon_t \quad (\text{exactly}), \quad (5.109)$$

and

$$m_\phi^2 \simeq \left[\frac{\partial^2 V_\phi(\phi)}{\partial \phi^2} + \frac{\Lambda_{\text{low}}^3 F_\pi^2}{f^2} \sin\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\phi}{f} - \frac{2\sqrt{3}\langle\kappa_{\text{TC}}\rangle}{F_\pi} - \theta_{\text{QCD}}\right) \right]_{\substack{\langle h \rangle \text{ from Eq. (5.94)} \\ \langle \kappa_{\text{TC}} \rangle \text{ from Eq. (5.95)} \\ \phi = \phi_{\text{crit.}} + 3F\delta}} \quad (5.110)$$

where in the first two results we have used Eqs. (5.94) and (5.95), and $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$, have expanded in powers of δ , and have kept only leading terms, as we expect that the QCD barriers will stall the relaxion in the vicinity of $\phi_{\text{crit.}}$ (i.e., at $0 < \delta \ll 1$); see Section 5.6. The expression for $m_{\omega_{\text{TC}}}^2$ at Eq. (5.109) is exact owing to the relation $\cos\left(\frac{\langle\kappa_{\text{TC}}\rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) = \epsilon_t \cos\left(\frac{\langle h \rangle}{F_\pi}\right)$ in the broken phase, which can easily be verified using Eqs. (5.94) and (5.95). Note also that m_ϕ^2 is only interpretable as the present-day squared-mass of the relaxion field once it has stopped rolling after the post-inflation slope-drop discussed in Section 5.2.3 has occurred; see Section 5.6.

It is also straightforward to read off the W and Z -boson squared-masses from Eq. (5.79) (note that we had to keep the $\mathcal{O}(\delta^2)$ terms in the expansions Eq. (5.100) in order to obtain the $\mathcal{O}(\delta^2)$ terms here correctly):

$$m_W^2 = \frac{g_2^2}{2} F_\pi^2 \left[1 - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\sqrt{3}\langle\kappa_{\text{TC}}\rangle}{F_\pi}\right) \right] \quad (5.111)$$

$$= \frac{3}{2} g_2^2 F_\pi^2 \delta \left[\frac{\epsilon_t \sqrt{1 - \epsilon_t^2}}{4\epsilon_t^2 - 1} + \frac{3}{2} \delta \frac{2 - 3\epsilon_t^2 + 2\epsilon_t^4}{(4\epsilon_t^4 - 1)^2} + \mathcal{O}(\delta^2) \right], \quad (5.112)$$

and

$$m_Z^2 = \frac{g_1^2 + g_2^2}{4} F_\pi^2 \sin^2 \left(\frac{\langle h \rangle}{F_\pi} \right) \quad (5.113)$$

$$= \frac{3}{2} (g_1^2 + g_2^2) F_\pi^2 \delta \left[\frac{\epsilon_t \sqrt{1 - \epsilon_t^2}}{4\epsilon_t^2 - 1} + \frac{3}{2} \delta \frac{1 - 2\epsilon_t^2 + 2\epsilon_t^4}{(4\epsilon_t^4 - 1)^2} + \mathcal{O}(\delta^2) \right], \quad (5.114)$$

which imply a tree-level contribution to the T parameter of

$$\alpha_e T \equiv \frac{1}{m_W^2} [\Pi_{W^+W^-}(0) - c_W^2 \Pi_{ZZ}(0)] \quad (5.115)$$

$$= 1 - \frac{g_2^2}{g_1^2 + g_2^2} \frac{m_Z^2}{m_W^2} \quad (5.116)$$

$$= 1 - \frac{1}{2} \frac{\sin^2 \left(\frac{\langle h \rangle}{F_\pi} \right)}{1 - \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\sqrt{3} \langle \kappa_{TC} \rangle}{F_\pi} \right)} \quad (5.117)$$

$$= \frac{3}{2} \delta \frac{\sqrt{1 - \epsilon_t^2}}{\epsilon_t (4\epsilon_t^2 - 1)} + \mathcal{O}(\delta^2), \quad (5.118)$$

where α_e is measured at the Z -pole, and $c_W \equiv g_2 / \sqrt{g_1^2 + g_2^2}$ is the cosine of the weak mixing angle. Note that $\alpha_e T \propto \xi$ per Eqs. (5.1) and (5.100) (cf. eq. (16) of Ref. [611]).

Owing to (a) the fact that $\pi_{TC}^\pm$ do not have canonical kinetic terms, and (b) the kinetic mixing of the $\pi_{TC}^\pm$ with the $W_\mu^\pm$, two manipulations are required before the $\pi_{TC}^\pm$ masses can be read off from Eqs. (5.79) and (5.81): (1) we send $W_\mu^\pm \rightarrow W_\mu^\pm \pm i\alpha \partial_\mu \pi_{TC}^\pm$, with α chosen to eliminate the kinetic mixing term in Eq. (5.79):

$$\alpha = \frac{2}{g_2} \frac{\langle h \rangle \sin \left(\frac{\langle h \rangle}{F_\pi} \right) \sin \left(\frac{\sqrt{3} \langle \kappa_{TC} \rangle}{F_\pi} \right) - \sqrt{3} \langle \kappa_{TC} \rangle \left[1 - \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\sqrt{3} \langle \kappa_{TC} \rangle}{F_\pi} \right) \right]}{\left(\langle h \rangle^2 - 3 \langle \kappa_{TC} \rangle^2 \right) \left[1 - \cos \left(\frac{\langle h \rangle}{F_\pi} \right) \cos \left(\frac{\sqrt{3} \langle \kappa_{TC} \rangle}{F_\pi} \right) \right]}, \quad (5.119)$$

which does not impact the W mass but does modify the $\pi_{TC}^\pm$ kinetic term (and of course induces couplings to the $\pi_{TC}^\pm$ for all fields that couple to the $W_\mu^\pm$); and (2) we rescale the

$\pi_{\text{TC}}^\pm$ fields to achieve a canonical kinetic term:

$$\pi_{\text{TC}}^\pm \rightarrow \pi_{\text{TC}}^\pm \times \frac{\langle h \rangle^2 - 3\langle \kappa_{\text{TC}} \rangle^2}{\sqrt{2}F_\pi \langle h \rangle} \frac{\sqrt{1 - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right)}}{\cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right) - \cos\left(\frac{\langle h \rangle}{F_\pi}\right)}. \quad (5.120)$$

The squared-mass of the $\pi_{\text{TC}}^\pm$ can then be read off from the rescaled Eq. (5.81) as

$$\begin{aligned} m_{\pi_{\text{TC}}^\pm}^2 &= 2c_m m \Lambda_{\text{TC}} \frac{\langle h \rangle^2 - 3\langle \kappa_{\text{TC}} \rangle^2}{\langle h \rangle^2} \frac{1 - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right)}{\left[\cos\left(\frac{\langle h \rangle}{F_\pi}\right) - \cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right)\right]^2} \\ &\times \left[\frac{\langle h \rangle}{F_\pi} \sin\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) \right. \\ &\quad \left. - \frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi} \left[\cos\left(\frac{\langle h \rangle}{F_\pi}\right) \sin\left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) + \sin\left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) \right] \right] \end{aligned} \quad (5.121)$$

$$= 2c_m m \Lambda_{\text{TC}} \epsilon_t \frac{\langle h \rangle^2 - 3\langle \kappa_{\text{TC}} \rangle^2}{F_\pi^2} \frac{1 - \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right)}{\left[\cos\left(\frac{\langle h \rangle}{F_\pi}\right) - \cos\left(\frac{\sqrt{3}\langle \kappa_{\text{TC}} \rangle}{F_\pi}\right)\right]^2} \text{sinc}\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left(\frac{\langle h \rangle}{F_\pi}\right) \quad (5.122)$$

$$= 4c_m m \Lambda_{\text{TC}} \epsilon_t \left[1 - \frac{1}{2} \delta \frac{(13\epsilon_t^2 - 6)\sqrt{1 - \epsilon_t^2}}{\epsilon_t(4\epsilon_t^2 - 1)} + \mathcal{O}(\delta^2) \right], \quad (5.123)$$

where at Eq. (5.122) we have used the exact relations $\cos\left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) = \epsilon_t \cos\left(\frac{\langle h \rangle}{F_\pi}\right)$ and $\cos\left(\frac{\langle h \rangle}{F_\pi}\right) \sin\left(\frac{\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F}\right) + \sin\left(\frac{2\langle \kappa_{\text{TC}} \rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F}\right) = 0$, which follow from Eqs. (5.94) and (5.95). At Eq. (5.123), we have used Eqs. (5.94) and (5.95), and have expanded in powers of δ .

5.5.4 Summary

In summary, we have found that for the parameter range $1/2 < \epsilon_t \leq 1$, the theory has a stable EW-symmetric vacuum solution $\langle h \rangle = \langle \kappa_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle = 0$ while $\cos(\phi/3F) >$

$\cos(\phi_{\text{crit.}}/3F) \equiv \epsilon_t$. This vacuum solution destabilizes if ϕ is larger than $\phi_{\text{crit.}}$, and if $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$ (with $0 < \delta \ll 1$), we find a Higgs-vev $\langle h \rangle \propto \sqrt{\delta} F_\pi$, a κ_{TC} -vev $\langle \kappa_{\text{TC}} \rangle \propto \delta F_\pi$, a light physical Higgs mass ($m_h^2 \propto m \Lambda_{\text{TC}} \delta$), light W and Z masses ($m_{W,Z}^2 \propto F_\pi^2 \delta$), and four heavy states ($M^2 \propto m \Lambda_{\text{TC}}$)—two of these states are EM-neutral and two are charged. A tree-level T parameter $\alpha_e T \propto \delta$ is generated.

5.6 Relaxion Potential

In order to exploit the observations summarized in Section 5.5.4, we desire to have the relaxion field initially in the range $\phi \in [0, \phi_{\text{crit.}})$, and have the field slow-roll out to larger values of ϕ over time: $\partial_t \phi > 0$. In exactly the same fashion as discussed in Section 5.2, this will trigger dynamical EWSB as ϕ crosses $\phi_{\text{crit.}}$, giving rise to increasingly large QCD barriers to the rolling, which will stall the relaxion shortly after it crosses $\phi_{\text{crit.}}$, while $0 < \delta \ll 1$, per the mechanism of Ref. [413].

To this end, examine again Eq. (5.90), which gives the gradient of the potential with respect to ϕ in the EW-symmetric phase. As the second term in Eq. (5.90) is negative on $\phi \in [0, \phi_{\text{crit.}})$, the first term must be made positive to obtain the correct rolling direction.

Following Ref. [413] and our discussion in Section 5.2.3, we add a linear term for the ϕ , which explicitly breaks the residual discrete ϕ shift symmetry. As in Section 5.2, we write this term as follows (the additional factor of 2 here compared to $V_\phi(\phi)$ in Section 5.2.3 is merely a convenient rescaling of γ):

$$V_\phi(\phi) = -\gamma \frac{2\Lambda_{\text{TC}} F_\pi^2 c_m m}{F} \phi, \quad (5.124)$$

where the free numerical prefactor $\gamma = \gamma(\sigma)$ is again assumed to take a value $\gamma_i \sim 10^{10}$ during inflation (see Section 5.2.3). This implies that, during inflation, [cf. Eq. (5.11)]

$$\partial_t \phi \propto -\partial_\phi V|_{\text{EW-symmetric}} = 2 \frac{\Lambda_{\text{TC}} F_\pi^2 c_m m}{F} \left[\gamma_i - \sin\left(\frac{\phi}{3F}\right) \right]. \quad (5.125)$$

Taking $\gamma_i \gg 1$ certainly guarantees that $\partial_t \phi > 0$ in the EW-symmetric phase.

In the broken phase, we require the relaxion to stop rolling once the QCD barriers become sufficiently large. The stopping condition $\partial_\phi V_{\text{eff.}} = 0$ is

$$\begin{aligned} & \frac{\Lambda_{\text{low}}^3}{f} h \operatorname{sinc} \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \sin \left[\frac{\phi}{f} - \frac{2\sqrt{3}\langle\kappa_{\text{TC}}\rangle}{F_\pi} - \theta_{\text{QCD}} \right] \\ & \approx \frac{2\Lambda_{\text{TC}}F_\pi^2 c_m m}{F} \left[\gamma + \frac{2}{3} \cos \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right) \sin \left(\frac{\langle\kappa_{\text{TC}}\rangle}{\sqrt{3}F_\pi} - \frac{\phi}{3F} \right) - \frac{1}{3} \sin \left(\frac{2\langle\kappa_{\text{TC}}\rangle}{\sqrt{3}F_\pi} + \frac{\phi}{3F} \right) \right]. \end{aligned} \quad (5.126)$$

Here, during inflation $\gamma = \gamma_i$ for the initial stalling of the relaxion, but $\gamma \rightarrow 0$ when the post-inflation slope-drop occurs and ϕ settles to its new minimum. Evaluating this at $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$ in the broken phase as defined by Eqs. (5.94)–(5.96), and expanding in powers of δ everywhere except for the ϕ/f term in the argument of the sine term on the LHS of Eq. (5.126), we find during inflation that the relaxion stalls when

$$\begin{aligned} & \frac{2\Lambda_{\text{TC}}F_\pi^2 c_m m}{F} \left[\gamma_i - \sqrt{1 - \epsilon_t^2} \right] \\ & \approx \sqrt{\delta} \frac{\sqrt{6}F_\pi \Lambda_{\text{low}}^3}{f} \frac{(1 - \epsilon_t^2)^{\frac{1}{4}} \sqrt{\epsilon_t}}{\sqrt{4\epsilon_t^2 - 1}} \sin \left[\frac{3F}{f} \left(\arccos(\epsilon_t) + \delta \right) - \theta_{\text{QCD}} \right] + \mathcal{O}(\delta). \end{aligned} \quad (5.127)$$

This cannot be solved for δ exactly in closed form, and no small- δ expansion of the argument of the sine term is possible owing to the large δ -prefactor proportional to $F/f \gg 1$. However, we can make progress by assuming that the sine factor on the RHS of Eq. (5.127) is equal to 1 to obtain the approximate solution for the relaxion stalling during inflation:

$$\delta \approx \frac{2}{3} \frac{f^2 F_\pi^2 c_m^2 m^2 \Lambda_{\text{TC}}^2}{F^2 \Lambda_{\text{low}}^6} \frac{4\epsilon_t^2 - 1}{\epsilon_t \sqrt{1 - \epsilon_t^2}} \left[\gamma_i - \sqrt{1 - \epsilon_t^2} \right]^2 \approx \frac{2}{3} \frac{f^2 F_\pi^2 c_m^2 m^2 \Lambda_{\text{TC}}^2}{F^2 \Lambda_{\text{low}}^6} \frac{4\epsilon_t^2 - 1}{\epsilon_t \sqrt{1 - \epsilon_t^2}} \gamma_i^2, \quad (5.128)$$

where in the latter approximate equality we have used $\gamma_i \gg \sqrt{1 - \epsilon_t^2} \sim \mathcal{O}(1)$. Self-consistency of the expansion effectively demands that, up to $\mathcal{O}(1)$ factors, $F \gg \gamma_i f F_\pi c_m m \Lambda_{\text{TC}} / \Lambda_{\text{low}}^3$. Since we have that $\Lambda_{\text{low}}^3 \langle h \rangle \sim \Lambda^4$ [cf. Eqs. (5.5) and (5.84)], so

that $\Lambda_{\text{low}}^{-3} \sim \sqrt{\xi} F_\pi \Lambda^{-4}$, we require here that $F \gg \sqrt{\xi} \gamma_i f F_\pi^2 c_m m \Lambda_{\text{TC}} / \Lambda^4$; this should be compared to Eq. (5.13), which indicates that in the cartoon model we had $F \sim \gamma_i f F_\pi^2 c_m m \Lambda_{\text{TC}} / \Lambda^4$: the parametrics examined for the cartoon model thus correctly imply $\xi \ll 1$.

By the intermediate value theorem, the actual solution of Eq. (5.127) must occur for a value of δ no more than $\Delta\delta = \frac{2\pi}{3} \frac{f}{F} \ll 1$ greater than the approximate solution Eq. (5.128). Although it is not obvious parametrically, the shift $\Delta\delta$ turns out to be numerically small compared to δ . Eq. (5.128) is thus a sufficient approximate solution to be used everywhere except when an expression depends on a (co)sine factor with an argument containing a contribution proportional to ϕ/f . The only other place that this occurs is in the relaxion squared-mass [note that terms proportional to $(F_\pi/F)^2 \ll 1$ have been ignored here]:

$$m_\phi^2 \approx \frac{\Lambda_{\text{low}}^3 F_\pi}{f^2} \sin\left(\frac{\langle h \rangle}{F_\pi}\right) \cos\left[\theta_{\text{QCD}} - \frac{\phi}{f} + \frac{2\sqrt{3}\langle\kappa_{\text{TC}}\rangle}{F_\pi}\right] \begin{matrix} \langle h \rangle \text{ from Eq. (5.94)} \\ \langle \kappa_{\text{TC}} \rangle \text{ from Eq. (5.95)} \end{matrix} . \quad (5.129)$$

However, for precisely the reason that this expression is sensitive to shifts of size $\Delta\delta \sim f/F$ (i.e., $\Delta\phi \sim f$), the relaxion mass will change after the post-inflation slope-drop as the relaxion rolls a distance of order $|\Delta\phi| \sim f$ to its new settling point (see Section 5.2.3); on the other hand, all the other estimates we have made will not be significantly impacted by this small change in ϕ due to the slope drop. Since the argument of the cosine factor in Eq. (5.129) is just $\theta_{\text{QCD}}^{\text{eff}}$ [which from comparing Eq. (5.126) with $\gamma = \gamma_i$ and with $\gamma \rightarrow 0$, is easily seen to be a factor of $\gamma_i \sim 10^{10}$ smaller than its $\mathcal{O}(1)$ value at initial stalling; cf. Eq. (5.16)], it follows that the post-slope-drop relaxion mass can be estimated by setting the cosine factor to 1, and using Eqs. (5.100) and (5.128) in the $\sin(\langle h \rangle / F_\pi)$ term:

$$m_\phi^2 \approx \frac{2\Lambda_{\text{TC}} F_\pi^2 c_m m}{fF} \gamma_i \approx \frac{\Lambda_{\text{low}}^3 \langle h \rangle}{f^2} \approx \frac{m_\pi^2 f_\pi^2}{f^2} \quad (\text{post slope-drop}), \quad (5.130)$$

where we have used the stopping relation estimated from Eq. (5.126) in the second step above, and the estimates $\Lambda_{\text{low}}^3 \langle h \rangle \sim \Lambda^4 \sim m_\pi^2 f_\pi^2$ in the third step [see the discussions just

below Eqs. (5.12) and (5.128)]. Note that the expression appearing on the RHS of the first approximate equality in Eq. (5.130) is $F/f \gg 1$ larger than the $\mathcal{O}(F_\pi^2/F^2)$ terms we neglected in Eq. (5.129). We thus see that the relaxion mass is expected to obey the standard scaling relation of a generic QCD axion.

5.7 Summary and Numerical Results

In this section, we present a summary of our analytical results, as well as selected numerical results.

In order to present the analytical results in the cleanest fashion possible, we first exchange all appearances of m for ϵ_t using Eq. (5.88), and we replace $\Lambda_{\text{TC}} \rightarrow 4\pi F_\pi/\sqrt{N}$. Next, keeping the terms at $\mathcal{O}(\delta^2)$ in the expansion for $\langle h \rangle^2/F_\pi^2$ which were not shown explicitly at Eq. (5.100), we invert that expansion to obtain δ as a power series in $\langle h \rangle^2/F_\pi^2$ correct to $\mathcal{O}(\langle h \rangle^4/F_\pi^4)$, and insert this inverted expansion into the various results from Sections 5.5 and 5.6 that had previously been expanded in powers of δ . In this fashion, all results other than $\langle h \rangle^2/F_\pi^2$ can be expressed as a power series in $\langle h \rangle^2/F_\pi^2$, while $\langle h \rangle^2/F_\pi^2$ is expressed as a power series in δ , which is estimated by Eq. (5.128). In the broken phase, this procedure leaves us with the following results:

$$\delta \approx \left(c_t y_t^2 \frac{N_c}{N} \right)^2 \frac{f^2 F_\pi^6}{F^2 \Lambda_{\text{low}}^6} \frac{\gamma_i^2}{\epsilon_t^2} \left[\frac{4\epsilon_t^2 - 1}{6\epsilon_t \sqrt{1 - \epsilon_t^2}} \right], \quad (5.131)$$

$$\frac{\langle h \rangle^2}{F_\pi^2} = \frac{6\epsilon_t \sqrt{1 - \epsilon_t^2}}{4\epsilon_t^2 - 1} \delta + \frac{3(2\epsilon_t^4 - 2\epsilon_t^2 + 3)}{(4\epsilon_t^2 - 1)^2} \delta^2 + \dots, \quad (5.132)$$

$$\frac{\langle \kappa_{\text{TC}} \rangle}{F_\pi} = -\frac{\sqrt{1 - \epsilon_t^2}}{2\sqrt{3}\epsilon_t} \frac{\langle h \rangle^2}{F_\pi^2} + \dots, \quad (5.133)$$

$$\frac{m}{F_\pi} \equiv \frac{1}{8\pi} \left(\frac{c_t}{c_m} y_t^2 \frac{N_c}{\sqrt{N}} \right) \frac{1}{\epsilon_t}, \quad (5.134)$$

$$\frac{m_t}{F_\pi} = \frac{y_t}{\sqrt{2}} \frac{\langle h \rangle}{F_\pi} \left[1 - \frac{1}{6} \frac{\langle h \rangle^2}{F_\pi^2} + \dots \right], \quad (5.135)$$

and

$$\frac{m_{\text{phys. Higgs}}^2}{F_\pi^2} = \frac{2}{3} \left(c_t y_t^2 \frac{N_c}{N} \right) \frac{4\epsilon_t^2 - 1}{\epsilon_t^2} \frac{\langle h \rangle^2}{F_\pi^2} \left[1 - \frac{2}{3\epsilon_t^2} \frac{\langle h \rangle^2}{F_\pi^2} + \dots \right], \quad (5.136)$$

$$\frac{m_{\text{phys. } \kappa_{\text{TC}}}^2}{F_\pi^2} \approx \frac{m_{\pi_{\text{TC}}^\pm}^2}{F_\pi^2} \approx \frac{m_{\omega_{\text{TC}}}^2}{F_\pi^2} = 2 \left(c_t y_t^2 \frac{N_c}{N} \right), \quad (5.137)$$

$$\frac{m_W^2}{F_\pi^2} = \frac{g_2^2}{4} \frac{\langle h \rangle^2}{F_\pi^2} \left[1 - \frac{1}{3} \left(1 - \frac{3}{4\epsilon_t^2} \right) \frac{\langle h \rangle^2}{F_\pi^2} + \dots \right], \quad (5.138)$$

$$\frac{m_Z^2}{F_\pi^2} = \frac{g_1^2 + g_2^2}{4} \frac{\langle h \rangle^2}{F_\pi^2} \left[1 - \frac{1}{3} \frac{\langle h \rangle^2}{F_\pi^2} + \dots \right], \quad (5.139)$$

$$\hat{T} \equiv \alpha_e T = \frac{1}{4\epsilon_t^2} \frac{\langle h \rangle^2}{F_\pi^2} + \dots, \quad (5.140)$$

$$G_F = \frac{g_2^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}\langle h \rangle^2} \left[1 + \frac{1}{3} \left(1 - \frac{3}{4\epsilon_t^2} \right) \frac{\langle h \rangle^2}{F_\pi^2} + \dots \right], \quad (5.141)$$

$$m_\phi^2 \approx \frac{\langle h \rangle \Lambda_{\text{low}}^3}{f^2} \approx \frac{m_\pi^2 f_\pi^2}{f^2} \quad (\text{post slope-drop}). \quad (5.142)$$

To investigate these results numerically, we fix $c_m = c_t = 1$, $N_c = 3$, $\Lambda_{\text{low}} = 8.5 \text{ MeV}$, $f = 10^{11} \text{ GeV}$, and $\gamma_i = 10^{10}$. Using as input the values $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ [38], $m_t = 173.21(51)(71) \text{ GeV}$ [38], and $m_{\text{phys. Higgs}} = 125.09(24) \text{ GeV}$ [38], we scan over N and F_π , solving for y_t , $\langle h \rangle$, and ϵ_t using Eqs. (5.135), (5.136), and (5.141). We also fix $\alpha_e(m_Z) \approx 1/127.950(17)$ [38] to be able to compute T using the values of ϵ_t and $\langle h \rangle$ thus obtained.²² The results of this numerical analysis are shown in Figures 5.3 and 5.4; the former shows the phenomenologically interesting results for T and $m_{\text{phys. } \kappa_{\text{TC}}} \approx m_{\pi_{\text{TC}}^\pm} \approx m_{\omega_{\text{TC}}}$, while the latter shows the values of F , m , Λ_{TC} , and ϵ_t that are required at each point in parameter space. Also shown in Figure 5.3 are the current and projected 95%-confidence one-parameter upper limits on the T parameter (taken at fixed $S = U = 0$) for a variety of proposed collider

22. We do not require the values of g_1 and g_2 to discuss the relevant phenomenology. They could be obtained by, e.g., additionally fixing $m_Z = 91.1876(21) \text{ GeV}$ [38], or by performing a global EW fit with the additional tree-level shifts indicated above accounted for; however, as is well-known even in the SM, it would be necessary to include the one-loop SM corrections in order to obtain accurate values here.

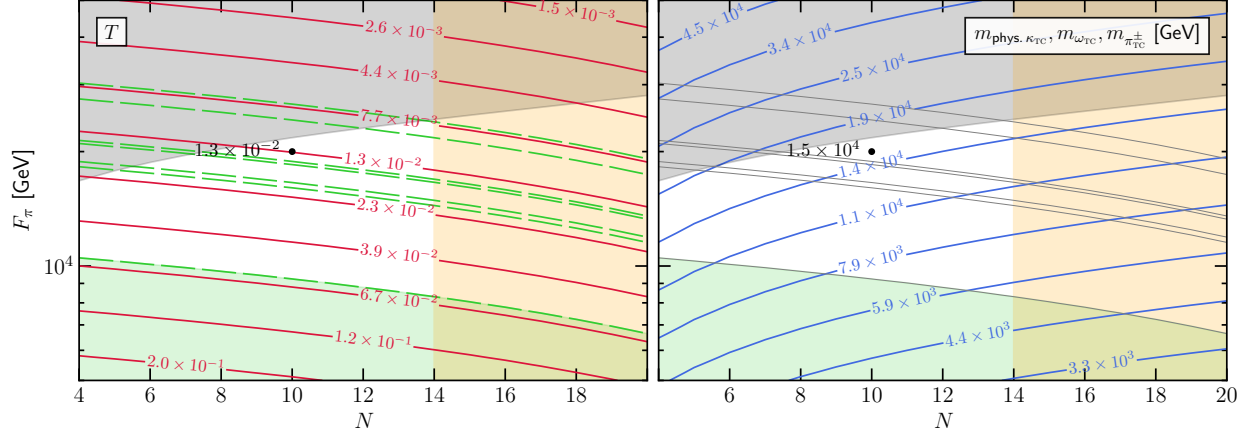


Figure 5.3: Numerical results for T and $m_{\text{phys. } \kappa_{\text{TC}}} \approx m_{\omega} \approx m_{\pi_{\text{TC}}^{\pm}}$, as a function of N and F_{π} , with $c_m = c_t = 1$, $N_c = 3$, $\Lambda_{\text{low}} = 8.5 \text{ MeV}$, $f = 10^{11} \text{ GeV}$, $\gamma_i = 10^{10}$, and $\alpha_e(m_Z) \approx 1/127.950(17)$ [38] fixed, and with y_t , $\langle h \rangle$, and ϵ_t adjusted to obtain $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ [38], $m_t = 173.21(51)(71) \text{ GeV}$ [38], and $m_{\text{phys. Higgs}} = 125.09(24) \text{ GeV}$ [38]. The dashed (green) lines in the right panel are the one-parameter 95%-confidence exclusion regions on the T parameter (taken at fixed $S = U = 0$ and computed using the results in Table 1 of Ref. [6]^a), for a variety of current and future colliders: from bottom to top, these lines represent the ‘Current’, ‘CEPC “Baseline”’, ‘ILC’, ‘FCC-ee-Z’, ‘CEPC “Improved $\Gamma_Z, \sin^2 \theta$ ”’, ‘CEPC “Improved $\Gamma_Z, \sin^2 \theta, m_t$ ”’, and ‘FCC-ee-t’ projections of Ref. [555]. These same contours are indicated by light (grey) lines on the left panel. The green shaded region on each panel is excluded by the current T -parameter constraints. The grey shaded region in each panel is where the consistency relation Eq. (5.143) is not satisfied. The orange shaded region indicates $N > 14$, where Landau poles in g_1 or g_2 may appear below the Planck scale (see Section 5.3.1). Also shown on each panel (black dots) is the parameter point $N = 10$ and $F_{\pi} = 20 \text{ TeV}$, along with the value of the respective quantity at that point; this parameter point is singled out for discussion in the text.

^a To be explicit, the one-parameter 95%-confidence upper bound on the T parameter (taken at $S = U = 0$), and assuming that the best fit point is $(S, T) = (0, 0)$, is given by $1.96 \sigma_T \sqrt{1 - \rho_{\text{ST}}^2}$, where σ_T and ρ_{ST} are given in Table 1 of Ref. [6], which supplies a convenient parametrization of the results of Ref. [555].

configurations for the ILC, CEPC, and FCC-ee (these limits are taken from the presentation in Ref. [6] of the limits examined in Ref. [555], and assume that the best-fit point for the global electroweak fit is at $(S, T) = (0, 0)$; cf. Ref. [76]).

A benchmark parameter point of interest is $N = 10$ and $F_{\pi} = 20 \text{ TeV}$, indicated by the black dots on Figures 5.3 and 5.4. At this point, and with c_m , c_t , N_c , Λ_{low} , f , and γ_i fixed as above, we have $\Lambda_{\text{TC}} = 80 \text{ TeV}$, $F = 5.2 \times 10^{41} \text{ GeV}$, $T = 1.3 \times 10^{-2}$, $m_{\text{phys. } \kappa_{\text{TC}}} \approx m_{\omega_{\text{TC}}} \approx m_{\pi_{\text{TC}}^{\pm}} = 15 \text{ TeV}$, $m = 1.2 \text{ TeV}$, $\epsilon_t = 0.61$, $y_t = 0.99$, and $\langle h \rangle = 246 \text{ GeV}$. Additionally,

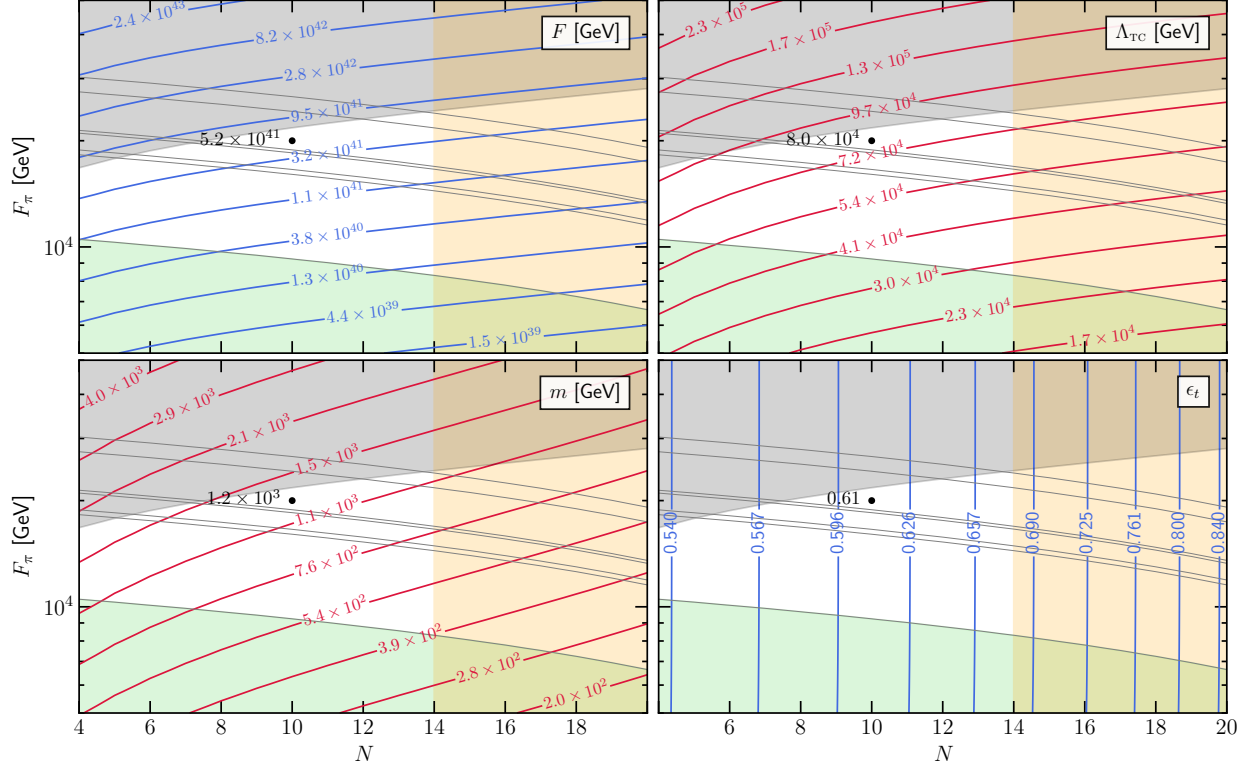


Figure 5.4: As for Figure 5.3, but showing the phenomenologically less interesting results for F , Λ_{TC} , m , and ϵ_t , as a function of N and F_π .

we find $\theta_{\text{QCD}}^{\text{eff.}} \approx \frac{\pi}{2}$ at the initial stalling point. Post slope-drop, we have $m_\phi \approx 120 \mu\text{eV}$ and $|\theta_{\text{QCD}}^{\text{eff.}}| = 7.9 \times 10^{-11}$, which is small enough to evade the constraint from the neutron electric dipole moment (EDM) limits, $|d_n| \sim 3 \times 10^{-16} |\theta_{\text{QCD}}^{\text{eff.}}| < 3 \times 10^{-26} \text{e cm}$ [38, 644].

In Figures 5.3 and 5.4, we also show the limits of the parameter region in which the self-consistency conditions discussed in Section 5.2.4 are satisfied. The essential content of the conditions for the cartoon model are captured by Eq. (5.24) which, taking $\Lambda_{\text{TC}} = 4\pi F_\pi / \sqrt{N}$, expresses a constraint on the upper limit of F_π of about 20 TeV. For the full model, essentially the same parametric estimate is obtained, with the exception of a weak ϵ_t -dependence:

$$F_\pi \lesssim \left(\frac{\sqrt[3]{3}N}{2\gamma_i c_t y_t^2 N_c} \right)^{\frac{1}{4}} (2\epsilon_t)^{\frac{1}{4}} \left(\frac{\Lambda^4 m_{\text{Pl.}}^3}{f} \right)^{\frac{1}{6}} \approx (20 \text{ TeV}) \times \left(\frac{N}{10} \right)^{\frac{1}{4}} \times \left(\frac{\epsilon_t}{0.61} \right)^{\frac{1}{4}}. \quad (5.143)$$

For $N = 10$, this translates to a lower bound $\xi \gtrsim 1.2 \times 10^{-4}$.

The massive bound states here are on the order of 10 TeV, and are charged only under the electroweak gauge group; as such they would be extremely hard to see at the proposed SPPC and FCC-hh high-energy hadron colliders. The T parameter is however a highly relevant probe: although current experiments are not sensitive to values of T as small as those obtained at the benchmark point $N = 10$ and $F_\pi = 20$ TeV, with improved Z -pole measurements and a top-threshold scan, both CEPC and FCC-ee would be able to exclude at 95% confidence not only this benchmark point, but almost the entirety of the model parameter space in which the relaxion consistency conditions are satisfied, and in which Landau poles in the couplings $g_{1,2}$ are not expected below the Planck scale.

5.8 Other Considerations

The cosmological relaxion mechanism generates a technically natural weak scale when the cutoff of the SM effective field theory is at much higher scales. In our context, we have seen that we are able to push the cutoff Λ_{TC} to scales of order 100 TeV. Despite the high scale of new physics, it is possible to imagine low energy probes of scenario, including precision electroweak, flavour, and CP tests, as well as signatures of heavy technibaryon composite dark matter. Similar considerations would follow from a tuned composite Higgs model (see, e.g., Refs. [645, 646]); searches for the relaxion are therefore crucial to test the scenario, although connecting the low- and high-energy dynamics may be challenging. In this section we will make a few remarks concerning these issues.

5.8.1 *Flavour*

In this chapter we have taken a bottom-up approach to flavour, writing only the minimal effective operators that generate the Yukawa couplings. The relaxion mechanism allows us to push Λ_{TC} to higher scales than in typical quasi-natural composite Higgs models, and this implies that the scale of flavour dynamics may also be at a higher scale. This generally

eases the constraints from anomalous flavour changing neutral currents (FCNCs) and potentially allows for simpler UV models of flavour (i.e., without necessarily requiring large anomalous dimensions of the composite operators, walking dynamics, etc. [647–654]). Nevertheless, it is not possible in our scenario to push the flavour scale arbitrarily high due to the self-consistency conditions in the inflation sector. Thus it may still be possible to have experimental signatures of flavour and CP violation within the reach of current and future experiments.

5.8.2 *Relaxion Phenomenology*

A chief prediction of this scenario is the existence of a QCD (rel-)axion, which can be tested through a variety of techniques, depending on its underlying couplings to the SM; see, e.g., Ref. [586] for a review of axion phenomenology. The classic probes of the relaxion–photon coupling include helioscopes, light-shining-through-walls experiments, and observations a variety of astrophysical systems in which relaxions may be produced. Furthermore, since the effective QCD θ -angle is expected to be small but non-zero, it may be possible to probe the relaxion–gluon coupling with improved measurements of the static neutron EDM. The relaxion may also form some or all of the dark matter, and there are numerous proposals expected to make significant inroads in the axion dark matter parameter space; see, e.g., Refs. [298, 300, 301, 655–661].

5.8.3 *Custodial Models*

The coset studied in this work does not admit a custodial symmetry, leading generically to a large T parameter if the compositeness scale is below about 10 TeV (see Figure 5.3). As we have shown, the relaxion mechanism can allow a high compositeness scale, but this means it will be challenging to directly search for the additional composite pNGBs and resonances at colliders. On the other hand, one can certainly consider cosets that are custodially symmetric. For such models, it is conceivable that the compositeness scale is as low as a few

TeV, and that the relaxion addresses only a mild Little Hierarchy. This would potentially open the window for observation of new heavy composite particles at the LHC and future hadron colliders.

5.8.4 Composite Dark Matter

Another potentially interesting consequence of the general scenario is heavy composite dark matter in the form of a technibaryon (see, e.g., Refs. [662–668]). The technibaryon in our scenario is expected to have a mass $m_B \sim N\Lambda_{\text{TC}} \sim 100\text{--}1000\text{ TeV}$, which is, interestingly, in the correct range for a thermal relic cosmology. In general composite Higgs models, one must take care to ensure that the lightest technibaryon is a neutral state if a dark-matter interpretation is desired. Furthermore, if the lightest state carries hypercharge the scenario may face strong constraints from direct detection experiments due to the tree-level Z -boson exchange. Possible phenomenological implications range from scattering in direct detection experiments, to indirect signals from decaying dark matter.

5.9 Conclusion

In this chapter, we have examined how the cosmological relaxation mechanism of Ref. [413] can be utilized to dynamically generate the Little Hierarchy in a composite Higgs model based on underlying strong $SU(N)_{\text{TC}}$ technicolour dynamics with three Dirac fermion flavours, leading to the global chiral symmetry breaking $SU(3)_L \times SU(3)_R \times U(1)_V \rightarrow SU(3)_V \times U(1)_V$. The relaxion was given anomaly-like couplings to both the technicolour and QCD gauge groups, with the former giving rise to the requisite coupling of the relaxion to the pNGB Higgs in the low-energy Chiral Lagrangian description, and the latter giving rise to the back-reaction on the relaxion slope required to stall the relaxion rolling after QCD chiral symmetry breaking and confinement [413]. The hierarchy of axion decay constants $f \ll F$ required by our model was engineered using a clockwork mechanism [436, 437, 618]. We

found that an additional potential term for the relaxion is required in this model to obtain the correct slow-roll direction for the relaxion field during the requisite exponentially long period of low-scale inflation. With the additional term in the potential, the technifermion masses—a source of explicit global chiral symmetry breaking—are scanned as the relaxion field rolls, which in turn results in the scanning of the term in the Higgs potential which opposes EWSB. Eventually, this scanning results in the importance of the dominant top-loop radiative corrections to the Higgs potential increasing relative to the contributions arising from the technifermion masses, leading to dynamical EWSB provided the technifermion masses are chosen to be roughly a loop factor below the cutoff scale of the composite theory. We utilized the post-inflation slope-drop mechanism of Ref. [413] to obtain an acceptably small QCD θ -angle in this framework.

We conclude that Little Hierarchies on the order of $\xi \equiv \langle h \rangle^2 / F_\pi^2 \sim \mathcal{O}(10^{-4})$ can be generated by our model (i.e., $F_\pi \sim 20$ TeV, with $\Lambda_{\text{TC}} \sim 80$ TeV for $N = 10$) while remaining within the region of parameter space in which the relaxion model is self-consistent, and without running afoul of the QCD θ -angle constraint. Phenomenological signatures of this (custodial violating) model include an electroweak T parameter large enough that high-precision measurements at proposed e^+e^- Higgs factories could explore essentially the entire viable parameter space for the model at the 95%-confidence exclusion level; a set of electroweak-charged states with masses on the order of 10 TeV, which would be challenging to observe at next-generation hadron colliders owing to large backgrounds and low rates; observables related to the existence of a QCD-like axion; and—depending on how this part of the model is implemented in detail—possibly also additional strongly charged states associated with the clockwork mechanism. Additionally, within the general scenario of composite Higgs models with cutoffs on the order of 100 TeV (whether tuned, or arising from relaxation as we have considered here), there may be interesting signatures associated with flavour physics, or technibaryon dark matter.

CHAPTER 6

IRRUPTION OF MASSIVE PARTICLE SPECIES DURING INFLATION

This chapter contains material from Ref. [8]: M. A. Fedderke, E. W. Kolb, and M. Wyman, *Irruption of massive particle species during inflation*, *Phys. Rev.* **D91** (2015) 063505 [arXiv:1409.1584].¹

In this chapter, we examine the creation of a massive particle species during inflation in a variety of settings. While this is a fascinating phenomenon in its own right (and has a long history [304]), it also has relevance for the dark-matter problem: for cold thermal relics, it is well known that partial-wave unitarity arguments [203] combined with the restriction that the relic not overclose the universe, limit the largest mass of that relic to be on the order of a few hundred TeV (the exact bound depends on the detailed assumptions made). However, there is no *a priori* reason that a dark-matter particle cannot be a supermassive particle (such as a WIMPzilla [185, 187–189]) with a mass around 10^{13} GeV or higher, so long as the production mechanism for the species is non-thermal.

All species of (non-conformally coupled) particles are produced during inflation so long as their mass M is not too much larger than H , the expansion rate during inflation. It has been shown that if a particle species that is normally massive ($M \gg H$) couples to the inflaton field in such a way that its mass vanishes, or at least becomes small ($M < H$), for a particular value of the inflaton field, then not only are such particles produced, but an irruption² of that particle species can occur during inflation. We pay particular attention to models which realize such an irruptive production mechanism in this chapter, and perform a detailed numerical and analytical study of their properties.

1. Ref. [8] is copyright © 2015 by the American Physical Society. Reuse and adaptation by permission, per APS dissertation copyright policy [3].

2. ‘Irruption, *n.* The action of bursting or breaking in; a violent entry, inroad, incursion, or invasion, esp. of a hostile force or tribe. *spec.* An abrupt local increase in the numbers of a species of animals.’ [669]

6.1 Introduction

The epoch of inflation [419–421] is the highest energy physical process to which we are ever likely to have observational / experimental access. Recent (c. 2014) observational evidence for degree-scale B -mode polarization of the Cosmic Microwave Background radiation [670] suggests³ that inflation occurred at or near the Grand Unified Theory scale with an expansion rate during inflation of $H \sim 10^{14}$ GeV, and that the inflaton field traversed a field space distance much larger than the Planck scale during the inflationary epoch (e.g., Refs. [674, 675]).

For physicists, this situation is fortuitous, because very-high-energy-scale inflation provides us with a significant opportunity to uncover new physical laws. This is because as the inflaton traverses a great (super-Planckian) distance in field space, it is possible to uncover new ultraviolet physical effects and probe the couplings of the inflaton to other particles. The inflaton must be coupled to other particle species since after inflation ends the inflaton energy density must be converted to entropy by reheating or preheating. Presumably this is related to the coupling of the inflaton to ‘light’ particle species.⁴ However the inflaton might also be coupled to additional ‘heavy’ particle species. If the inflaton couples to a particle species of mass that is always much larger than the inflaton mass or the expansion rate of the universe, the heavy field can be integrated out to form an effective field theory describing the inflaton field and its coupling to light degrees of freedom. In this case, the heavy field will not be dynamically important during inflation.

In this chapter we investigate the possibility that the inflaton couples to another particle

3. See also Refs. [671, 672] for an early critical discussion of galactic dust foregrounds in connection with these results. More recent joint BICEP2/*Keck*/*Planck* [673] and BICEP2/*Keck* [430] analyses incorporating data at multiple frequencies, obtain degree-scale CMB B -mode polarization results consistent with a polarized dust foreground contribution [430] in the region of the sky initially investigated by BICEP2 in Ref. [670]. At present, the Collaborations report only an upper limit on the primordial tensor-to-scalar ratio $r_{0.05} \leq 0.07$ at 95% confidence [430].

4. Unless otherwise specified, ‘light’ and ‘heavy’ particle species refers to the mass of a species compared to the mass of the inflaton, which is approximately the expansion rate of the universe during inflation in the inflation model we study.

species in such a way that the mass of the additional species depends on the value of the inflaton field. Thus, as the inflaton field evolves, the mass of the other particle species will change with the value of the inflaton field. We will consider several toy models in which the mass of the additional field vanishes (or at least becomes ‘light’) for a particular value of the inflaton field. For that particular value of the inflaton field, and only for that value, the additional field becomes dynamically important. One consequence of the fact that for a particular value of the inflaton field the new particle is dynamically important, is that while the field is light it can be produced via the usual mechanism of particle creation during inflation. This may result in a sudden growth, or irruption, in the population of the new particle species, which we dub the ‘irrupton’.

String theory provides a calculable framework for articulating the general statement that the inflaton might be coupled to heavy fields, so let us describe some characteristic examples from the literature of string cosmology. One of the most successful early models for inflation within string theory was brane inflation [676], wherein the separation of two higher dimensional (mem)branes played the role of the inflaton. After extensive study of this framework it was realized that the string-theoretic context of brane inflation excluded the possibility of super-Planckian field excursions and hence observable gravitational waves [677]. Intuitively, this is because the higher dimensional Calabi-Yau geometries in which brane inflation was thought to operate cannot be enlarged well beyond the string (and hence, 4-dimensional Planck) scales without their constituting elements generating an overwhelming back-reaction. Since this phenomenon was general (if not universal), the ‘model space’ for inflation in string theory was generally thought to involve small volumes and field excursions; see, e.g., Ref. [678] for a review.

Nonetheless, the breadth of the string landscape (and theorists’ capacity for imagination) is also large (super-Planckian, in fact). A significant breakthrough in this direction was the model of axion monodromy [679], a proof-of-principle calculation that geometries could exist in string theory that permitted super-Planckian field excursions. Although the well-

studied models of monodromy are in some conflict with recent (c. 2014) putative primordial gravitational wave discoveries [670] (although see footnote 3)—because their gravitational wave production is too small to explain the putative detection!—they offer hope that string-theoretic constructions can agree with the cosmological data we have. Moreover, it suggests that the higher dimensional geometry necessary to embed high-energy-scale inflation within string theory will have to be something very different from the simple geometries most often studied in the past.

Summing up, we are in a remarkable situation: if string theory is the correct description of quantum gravity, then the putative observation of primordial gravitational waves tells us that the inflationary epoch can in principle affect, and hence potentially generate, many ultra-UV phenomena previously thought of chiefly as technical arcana of higher dimensional geometries. Put more simply, if inflation is near the GUT scale and the inflaton field traverses super-Planckian distances, it can shake and rattle fields (e.g., moduli that describe and shape that geometry) as it rolls.

In this context, then, it becomes of intense interest to understand the full range of observable phenomena that may result from such an inflationary epoch. A relatively less well-studied possibility in this research space is the creation of new particles during the inflationary period. Within the picture we have outlined, following the seminal work of Ref. [680], this can be thought of as the physical manifestation of enhanced symmetry points within the geometrical space of fields that characterize the extra dimensions (see also Refs. [681] and [682]). In such a set-up, the vanishing of effective masses corresponds to a momentary enhancement of symmetry, which we expect on general grounds to be a dynamical attractor. By the same token, it is worth going beyond the relatively simple effective theory described in Ref. [680], where a dynamically varying mass was captured by a simple $g\phi^2\chi^2$ potential (where ϕ is the inflaton and χ the representative extra modular field), to richer dynamical systems.

To that end, we extend the study of particle creation beyond the canonical $g\phi^2\chi^2$ model,

which has been a familiar friend since its introduction in the context of preheating after inflation [423]. In particular, although we continue to work in the context of a simple two scalar-field model, we will study in detail two models that encapsulate the phenomenon of a field that is heavy (and thus dynamically unimportant) before, during, and after inflation, except at a particular value of the inflaton field during inflation.

The two models we investigate in detail have different starting points for coupling the irrupton to the inflaton. In our first model, the irrupton and inflaton fields are coupled in the scalar potential through a inflaton-field-dependent mass term for the irrupton. We will refer to this as the ‘potentially coupled’ case. For the second model we study, the coupling of the irrupton to the inflaton traces to the kinetic interactions of the inflaton field and the irrupton. We will refer to this as the ‘kinetically coupled’ case. We do not suggest that the two models we investigate span the space of all possibilities for massive particle species irruption during inflation; indeed there are many other possibilities one might consider. We concentrate on the two models we have chosen in order to understand the issues that we anticipate will be generic to any model of massive species irruption.

While previous studies have considered potentially coupled models [189, 683–686], this work will also focus on kinetic interactions between the two fields. Such interactions are characteristic of the supergravity limit of string theory (see, e.g., Ref. [687] and references therein). Complete formal perturbative analysis of such systems has been done [688–690], but the resulting system is complex and can describe a surprising range of nontrivial phenomena, including reduced speed-of-sound effective dynamics [691], step-like features in the effective potential for the inflaton [692], and temporarily non-adiabatic evolution of the inflaton itself [693–695].

In the original model for superheavy dark-matter production [189], the simple fact of the highly energetic (yet still adiabatic) inflationary background was exploited to generate a tiny number density of ultra-heavy particles that could play the role of dark matter. The mechanism we will describe is a generalization of that approach, where the existence of non-trivial

multi-field dynamics will allow the prospective dark-matter particle to become effectively light (or even temporarily tachyonic) during inflation, thus allowing it to be produced. Once created, the mass of the particle then varies strongly with the value of the inflaton, turning the former into an end-of-inflation very massive particle, with a mass greater than 10^{13} GeV.

In Section 6.2 we discuss the criteria for species irruption. We then review a model for creation of massive particles during creation under the condition that the particle mass is unaffected by the value of the inflaton field. While there is no species irruption in this model, it serves as a useful baseline in understanding irruption in models where the species mass does depend on the inflaton field. We then discuss the adiabatic conditions that must be violated for particle creation to occur. Also in Section 6.2 we describe the potentially coupled and kinetically coupled models. Finally, in that section we discuss the expressions for the number density of the particles in terms of the Bogoliubov coefficient and the issue of initial conditions.

In Section 6.3 we review the relationship between the Bogoliubov coefficient and the present number density assuming the produced particle is stable. Section 6.4 discusses the numerical methods we employ as well as the limitations of our numerical study. In Section 6.5 we present the numerical results in the three models we consider. We comment on possible implications of irruption of massive particle species and conclude in Section 6.6. Appendices E.1 and E.2 contain longer technical derivations of some results that we will have occasion to refer to multiple times in this chapter.

6.2 Irruption of Particle Species During Inflation

In this section we discuss the creation of particles during inflation. When we refer to a ‘massive’ particle, we mean a particle species with a mass at the end of inflation larger than the expansion rate of the universe during inflation.

The idea of creation of particles during any phase of the expansion of the universe traces back to the (largely forgotten) 1939 paper of Schrödinger, *The proper vibrations of the ex-*

panding universe [304]. Here, we briefly summarize what we have learned about cosmological particle creation in 75 years since that paper:

1. For a particle species to be created during the expansion of the universe it must participate in the breaking of conformal invariance. This is usually accomplished by a mass for the field and/or a non-conformal coupling of a scalar field to the Ricci scalar.
2. Particle (in this case, the inflaton) creation during inflation is the origin of the temperature and density perturbations seen as temperature anisotropies in the background radiation.
3. Particle (in this case, the graviton) creation during inflation is the origin of the primordial gravitational waves (tensor modes) that can be deduced from the background radiation polarization pattern.
4. In the inflationary phase the expansion rate of the universe H is nearly constant, and with the assumption of adiabatic initial conditions for each quantum mode, creation of particles of mass m larger than H is suppressed by a factor of $\exp(-m/H)$.
5. Gravitationally created particles of mass comparable to (or slightly larger than) H , if stable, would be a candidate for dark matter [185, 186, 188].
6. Particles of mass larger than H can only be created during inflation if one is willing to accept some sort of trans-Planckian particle creation [696].

The above considerations assume that the mass, the couplings, and the kinetic term of the field are constant. The situation changes if the particle couples to the inflaton, which evolves during inflation.

In the first proposal studying creation of massive particles during inflation, Refs. [185, 189] assumed the existence of a fermion field ψ that has a Yukawa coupling to the inflaton ϕ of the form $\mathcal{L}_Y = \lambda\phi\bar{\psi}\psi$ and a Lagrangian mass term of the form $\mathcal{L}_M = -M_0\bar{\psi}\psi$. For non-zero values of ϕ , the mass of the ψ would be $M(\phi) = M_0 - \lambda\phi$, where M_0 is the mass at $\phi = 0$.

This leads to a critical value of the inflaton field, $\phi_* = M_0/\lambda$, where the mass vanishes. Even if $M_0 \gg H$, there will be a resonant production of ψ during inflation when the inflaton field is around ϕ_* . Note that in large-field models of inflation, ϕ is of order M_{Pl} , so one can have resonant production even if $M_0 \gg H$. Several papers extended this work to the analogous situation for a scalar field χ coupled to the inflaton via a Lagrangian effective mass term of the form $\mathcal{L}_M^{\text{eff}} = -\frac{1}{2}g^2(\phi - \phi_*)^2\chi^2$ [683–685]. Again, there is a critical value ϕ_* of the inflaton field where the mass vanishes and particle creation can occur.

6.2.1 A Simple Non-irruptive Model

Before turning to the complicated cases of ϕ -dependent mass terms, let us consider the simple case of production of particles of fixed mass M in the expanding universe; such a model has been used in, e.g., the context of inflationary production of superheavy dark matter (e.g., Refs. [185, 188]). We start with a particle of mass M with action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} M^2 \chi^2 - \frac{1}{2} \xi \mathcal{R} \chi^2 \right], \quad (6.1)$$

where ξ is a constant in the coupling term of the scalar field to the Ricci scalar $\mathcal{R}$. In this chapter we will assume a flat Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime with mostly minus signature: $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$, where η is conformal time. In the flat FLRW background, $\mathcal{R} = 6a''/a^3$, where $'$ denotes $d/d\eta$ (dot will denote d/dt). The equation of motion for χ is

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\eta^{ij}\partial_i\partial_j\chi}{a^2(t)} + M^2\chi + \xi\mathcal{R}\chi = 0. \quad (6.2)$$

Now we mode expand $\chi = \sum_k \hat{a}_k u_k + \hat{a}_k^\dagger u_k^*$ and make the plane wave ansatz

$$u_k(\mathbf{x}, t) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}a(t)} \chi_k(t), \quad (6.3)$$

where $\mathbf{k}$ is the comoving momentum.⁵ This gives the mode equation for χ_k (here and below, $k \equiv |\mathbf{k}|$, not to be confused with the four-momentum)

$$\ddot{\chi}_k + H\dot{\chi}_k + \left[\frac{k^2}{a^2} - \frac{a''}{a^3} (1 - 6\xi) + M^2 \right] \chi_k = 0, \quad (6.4)$$

where we used $a^{-1}a'' = a\ddot{a} + \dot{a}^2$. Passing to conformal time using $a^{-2}\chi_k'' = \ddot{\chi}_k + H\dot{\chi}_k$, the mode equation takes the form

$$\chi_k'' + \omega_k^2(\eta)\chi_k = 0, \quad (6.5)$$

as is indeed expected for a sensible mode expansion. In Eq. (6.5), $\omega_k^2(\eta)$ is given by

$$\begin{aligned} \omega_k^2(\eta) &= k^2 - \frac{a''}{a} (1 - 6\xi) + a^2 M^2 \\ &= a^2 H^2 \left[\left(\frac{k}{aH} \right)^2 + H^{-2} \left[M^2 - (1 - 6\xi) a^{-3} a'' \right] \right]. \end{aligned} \quad (6.6)$$

Note that if $a^{-3}a''(1 - 6\xi) = M^2$, the total coupling of the field is conformal, although (provided $M \neq 0$ and $\xi \neq 1/6$, respectively) the mass term and the Ricci scalar term individually break conformal symmetry. Since we will eventually encounter sufficient complexity, we will assume henceforth that $\xi = 0$; that is, we make the choice that χ is a ‘minimally’ coupled scalar field.

In this simple model we can see the underlying cause of particle creation. For a static case, a is constant (so a'' vanishes) and M is constant as well, so ω_k^2 is constant in conformal time. If we choose at some initial time a pure outgoing wave (positive frequency mode),

$$\chi_k(\eta) = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k\eta}, \quad (6.7)$$

5. The choice of normalization is such that $\chi_k \cdot (\chi_k^*)' - \chi_k'^* \cdot \chi_k = i$ gives modes normalized with respect to the usual inner product $(u_k, u_{k'}) = -i \int_{\Sigma} d\Sigma^\alpha [-g_\Sigma]^{1/2} u_k \overset{\leftrightarrow}{\partial}_\alpha u_{k'}^* = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, etc. We take the usual creation/annihilation algebra $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, etc. (see, e.g., pp. 44–45 of Ref. [697]).

then it will remain a solution without admixture onto incoming waves (negative frequency modes). If $\omega_k(\eta)$ is not constant (in our simple case due to the a''/a term, and the mass term), the previous statement need not be true. One might try an adiabatic approximate solution of the form

$$\chi_k(\eta) = \frac{1}{\sqrt{2\omega_k(\eta)}} e^{-i \int \omega_k(\eta) d\eta}. \quad (6.8)$$

This (zeroth-order) adiabatic solution (see, e.g., Ref. [697] for a discussion of the adiabatic approximation in this context) is constructed to satisfy the equation of motion (EOM) Eq. (6.5) up to terms of $\mathcal{O}(|\omega'_k/\omega_k^2|^2, |\omega''_k/\omega_k^3|)$, and so the conditions for the adiabatic solution to be a good approximation are that

$$\mathcal{A} \equiv \left| \frac{\omega'_k}{\omega_k^2} \right|^2 \ll 1 \quad \text{and} \quad \mathcal{B} \equiv \left| \frac{\omega''_k}{\omega_k^3} \right| \ll 1. \quad (6.9)$$

If either condition fails, the adiabatic solution is not a good approximation. When the solution is non-adiabatic the incoming and outgoing modes (Schrödinger's ‘proper vibrations’) are mixed: particles are created.⁶ Notice that, in the inflationary universe, a'' is positive. This means that it is possible to have a ‘tachyonic’ mass for χ : i.e., $\omega_k^2 < 0$.

Using the simple scalar field model above, let us see where the evolution is non-adiabatic. The problem is very simple if we consider the evolution of the scalar field in a de Sitter background. In de Sitter space, H is constant, and conformal time and the scale factor are related by $a = -1/\eta H$, where η is in the range $-\infty \leq \eta \leq 0$. With the definitions $x \equiv k\eta$ ($0 < x^2 < \infty$) and $\gamma = M^2/H^2 - 2$, where $-2 < \gamma < \infty$,

$$\left| \frac{\omega'_k}{\omega_k^2} \right|^2 = \left| \frac{\gamma^2}{(x^2 + \gamma)^3} \right|, \quad \text{and} \quad \left| \frac{\omega''_k}{\omega_k^3} \right| = \left| \frac{\gamma^2}{(x^2 + \gamma)^3} + \frac{3\gamma}{(x^2 + \gamma)^2} \right|. \quad (6.10)$$

6. Here we are glossing over the fact that one can only really speak of particles if the evolution is adiabatic [697].

Note that $\gamma = 0$ is a special point where there would not be particle creation. This happens if $M^2/H^2 = 2$. As mentioned above, this is where $(a''/a^3)(1 - 6\xi) = M^2$ (for $\xi = 0$, as we consider here). The condition for non-adiabatic particle creation, violation of Eq. (6.9), is satisfied when $x^2 \simeq -\gamma$, or $k^2\eta^2 = k^2/a^2H^2 \simeq -\gamma = 2 - M^2/H^2$. For $M^2/H^2 \ll 2$ the evolution is non-adiabatic at $k/aH \sim 1$, i.e., when a momentum mode crosses the comoving Hubble radius. On the other hand, for $M^2/H^2 \gg 1$ the evolution is always very nearly adiabatic and particle creation is suppressed. The two lessons we have learned from this model are

1. for $M \gg H$, one should not expect significant production of particles
2. for $M \ll H$ particle creation is continuous, since for any value of conformal time (or, equivalently the scale factor) there is a comoving momentum mode k that satisfies $k^2\eta^2 = 2$.

This simple model assumes that the mass of the χ field remains constant. However, if the mass varied during inflation, one might imagine that today the mass of the species is much larger than H during inflation, but for some period during inflation, the mass vanished (or at least became much less than the value of H) due to the species coupling to the inflaton. If this occurs there can be an irruption of the particle species, but only while the mass is less than $\mathcal{O}(H)$. This is exactly what occurs in the models discussed in the next two subsections.

For the remainder of this chapter, we employ a simple ‘chaotic’ inflation model rather than this fixed de Sitter background. Production of massive states during inflation has also been considered for hybrid inflation models, as well as natural inflation models [190], and the results seem to imply that the phenomenon of massive particle production is generic.

6.2.2 Massive Particle Species Irruption from a Potential Term

For the first irruption model, consider the following simple two-field model

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi, \phi) \right]. \quad (6.11)$$

The ϕ field will serve as the inflaton. As the purpose of this chapter is to study the sudden increase in the population of the particle species (i.e., the irruption of the particle species) χ , we refer to χ as the ‘irrupton’.

We will assume the simplest inflaton potential and a simple inflaton–irrupton interaction term that will serve our purpose (we refer to this as the ‘potentially coupled’ irrupton):

$$V(\phi) = \frac{1}{2} m^2 \phi^2, \quad (6.12)$$

$$U(\chi, \phi) = \frac{1}{2} g^2 (\phi - \phi_*)^2 \chi^2. \quad (6.13)$$

During inflation we will assume that $\dot{\phi} > 0$, $\dot{\phi} < 0$, and $\phi_* > 0$. The effective mass of the irrupton is $M(\phi) = g |\phi - \phi_*|$; today, we have $\phi = 0$ and $M(0) = g\phi_*$. We wish to choose ϕ_* such that $U(\chi, \phi)$ vanishes during inflation: since the inflaton field excursion in this model is super-Planckian, ϕ_* may be chosen to be in excess of M_{Pl} , and the mass of the irrupton today may be of the scale of the Planck mass.⁷ We will discuss possible implications of this observation in Section 6.6.

7. Note that the recent BICEP2 detection of degree-scale CMB B -mode polarization [670] initially hinted at primordial tensor modes in the background and thus suggested that the inflaton mass and the expansion rate during inflation were of order 10^{14} GeV, and that inflaton field excursions during inflation were indeed super-Planckian (although see Refs. [671, 672] for early critical analysis of galactic dust foregrounds in connection with this result). Subsequent joint BICEP2/*Keck*/*Planck* [673] and BICEP2/*Keck* [430] analyses have concluded that the degree-scale CMB B -mode detection of Ref. [670] is consistent with a polarized dust foreground contribution, and provide only upper limits on the primordial tensor-to-scalar of $r_{0.05} \leq 0.07$ at 95% confidence [430]. These official collaboration analyses indicating a significant dust foreground appeared after Ref. [8], the paper on which this chapter is based, was published. Although the disappearance of these observational hints that ϕ may indeed have executed a super-Planckian excursion do somewhat lessen the motivation for considering ϕ_* to be quite so large, this does not impact the fact that the models under study in this chapter remain intrinsically interesting owing to their rich dynamics, because the latter does not depend strongly on ϕ_* being super-Planckian.

The equation of motion for the (spatially homogeneous) inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = -g^2(\phi - \phi_*)\chi^2 \approx 0, \quad (6.14)$$

where the approximate equality reflects that fact that we will ignore the back-reaction on the classical inflaton field (and hence on the metric) induced by the irrupton.⁸ For the chosen potential, inflation ends at $\phi \simeq 0.2M_{\text{Pl}}$, and 50 e -folds before the end of inflation corresponds to $\phi \simeq 2.8M_{\text{Pl}}$.

The equation of motion for the irrupton is

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\eta^{ij}\partial_i\partial_j\chi}{a^2(t)} + g^2(\phi - \phi_*)^2\chi = 0. \quad (6.15)$$

Employing the same mode expansion as before and again making the plane wave ansatz as in Eq. (6.3), the mode equation for χ_k is

$$\ddot{\chi}_k + H\dot{\chi}_k + \left[\frac{k^2}{a^2} - \frac{a''}{a^3} + g^2(\phi - \phi_*)^2 \right] \chi_k = 0. \quad (6.16)$$

Passing to conformal time, the mode equation again takes the form

$$\chi_k'' + \omega_k^2(\eta)\chi_k = 0, \quad (6.17)$$

but now $\omega_k^2(\eta)$ is given by

$$\omega_k^2(\eta) = k^2 - \frac{a''}{a} + a^2 M_{\text{eff}}^2 = a^2 H^2 \left[\left(\frac{k}{aH} \right)^2 + H^{-2} (M_{\text{eff}}^2 - a^{-3}a'') \right], \quad (6.18)$$

$$M_{\text{eff}}^2 = g^2(\phi - \phi_*)^2 \equiv M_g^2(\nu - \nu_*)^2, \quad (6.19)$$

8. One can, and we will, justify this *a posteriori* for the present purposes by showing that the energy density extracted by irrupton production is a negligible fraction of the inflaton energy density. See however Refs. [189, 686] for a discussion of observational effects in the cosmic microwave background which can arise in this model, or its fermionic cognate, when the back-reaction is not ignored.

where $M_g \equiv gM_{\text{Pl}}$, and we have defined the variable $\nu \equiv \phi/M_{\text{Pl}}$, where M_{Pl} is the Planck mass, for future convenience.

Just as before, species irruption will occur when one of the conditions in Eq. (6.9) is violated. For sure, ω'_k and ω''_k are more complicated in this model than in the model of Section 6.2.1, but if we make use of what we learned in Section 6.2.1, we expect that irruption will occur when ω_k^2 passes through zero. Since $(k/aH)^2$ is positive, for ω_k^2 to pass through zero we must have $H^{-2} (M_{\text{eff}}^2 - a^{-3}a'') < 0$. A graph of $H^{-2} (M_{\text{eff}}^2 - a^{-3}a'')$ as a function of cosmic time (not conformal time) is given in Figure 6.1. Regions in the evolution where the foregoing quantity is negative are shown by the dashed part of the curve. Irruption will occur in and around these dashed regions.

A basic understanding of the results to be expected for this case may be obtained if we make a couple of simple approximations. First, recall that

$$\frac{a''}{a^3} = \frac{4\pi G}{3} (\rho - 3p) = \frac{8\pi G}{3} \left(2\rho - \frac{3}{2}\dot{\phi}^2 \right), \quad (6.20)$$

where the first equality holds for any FLRW model, and the second equality holds if the energy density is dominated by the inflaton (regardless of inflaton potential). One could numerically solve the inflaton field equation for $\dot{\phi}$, or use the slow-roll approximation for $\dot{\phi}$; however, to get a rough idea of what is expected we can make a cruder (but still reasonable) approximation and ignore the $\dot{\phi}$ term in Eq. (6.20) altogether, with the result $a^{-3}a'' \sim 2H^2$.

For this inflaton model, $\phi \sim 10^5 H$ during inflation, so unless g is quite small, M_{eff} will be much larger than H except in a very narrow range near $\phi = \phi_*$. So for irruption we may write $M_{\text{eff}}^2 = g^2 \delta\phi^2$ where $\delta\phi = \phi - \phi_*$.

Using $M_{\text{eff}}^2 = g^2 \delta\phi^2$ and $a^{-3}a'' = 2H^2$, the square bracket in Eq. (6.18) becomes $(k/aH)^2 - 2 + g^2 \delta\phi^2/H^2$. We expect irruption when $(k/aH)^2 \sim 2 - g^2 \delta\phi^2/H^2 \sim 2 - g^2 \delta\phi^2/m^2$, where we have made the further approximation that $H \sim m$, the inflaton mass.

We can draw a couple of expected results from these crude approximations:

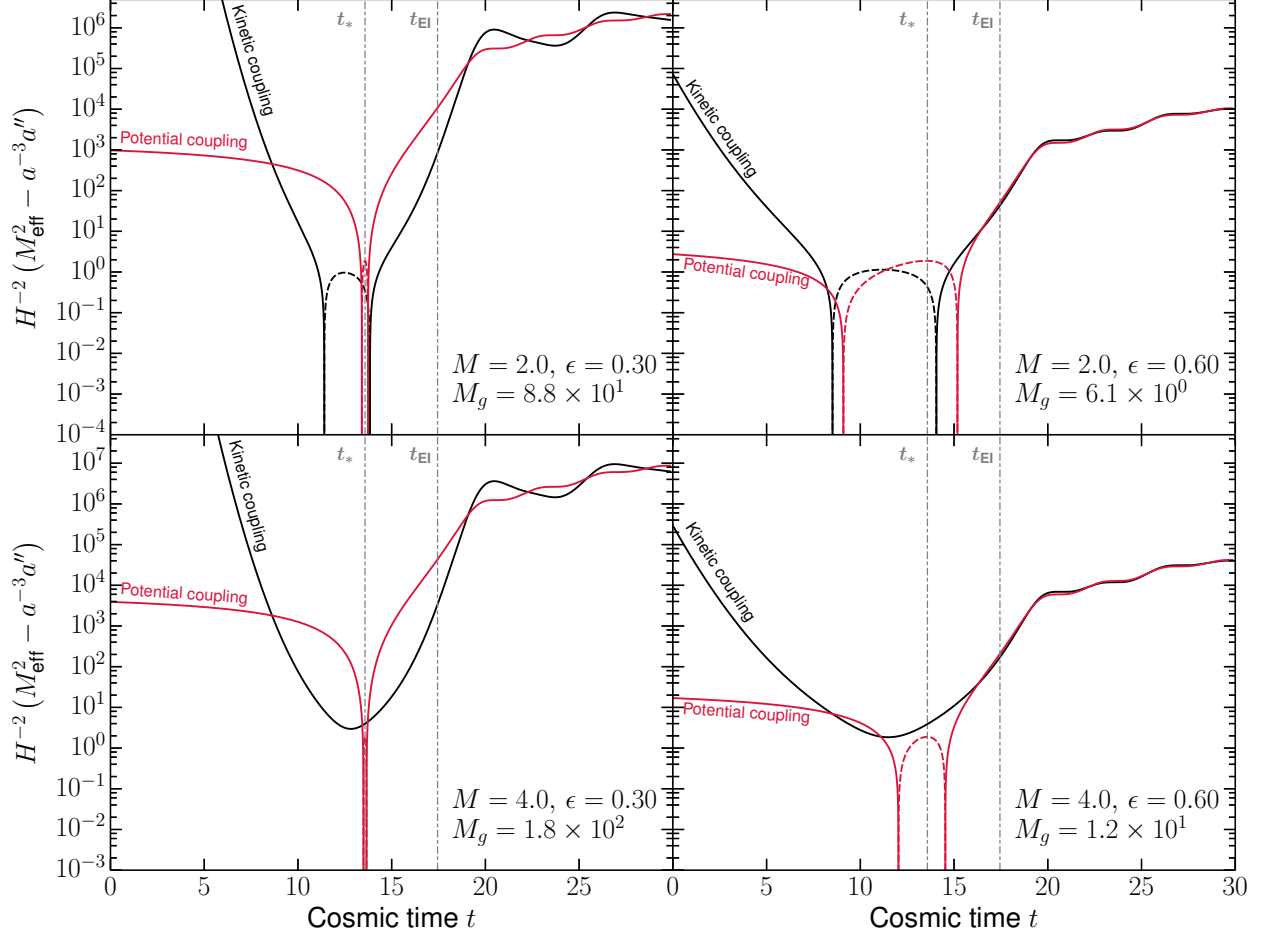


Figure 6.1: A graph of $H^{-2}(M_{\text{eff}}^2 - a^{-3}a'')$ as a function of cosmic time t in two models, for four choices of parameters. The models are for inflaton–irrupton coupling through either the potential term (discussed in Section 6.2.2) or through the kinetic term (discussed in Section 6.2.3). Where the lines are dashed, $H^{-2}(M_{\text{eff}}^2 - a^{-3}a'') < 0$. The dashed vertical lines denoted ‘ t_{EI} ’ mark the end of the inflationary phase. Time is in units of m^{-1} , where m is the inflaton mass, and $t = 0$ corresponds to the time when $\phi = 3M_{\text{Pl}}$. The inflaton field has the value $\phi = \phi_* = 0.8M_{\text{Pl}}$ at $t = 13.6$, indicated by the dashed vertical lines marked ‘ t_* ’. Model parameters are chosen such that in each individual plot, the irrupton masses are the same at $\phi \equiv 0$ and $a'' = 0$ (i.e., at late time) for the two models.

1. The condition for irruption, $\omega_k^2 \sim 0$, obtains for only for a rather narrow range of $\delta\phi$: $|\delta\phi| \lesssim m/g$. Since $m \sim 10^{13} \text{ GeV}$, $M_{\text{Pl.}} \sim 10^{19} \text{ GeV}$, and $\phi \sim M_{\text{Pl.}}$, irruption occurs for $\delta\phi \ll \phi$ unless $g \lesssim 10^{-6}$.
2. The duration of irruption will increase as g decreases, roughly as $|\delta\phi| \sim m/g$.
3. Since $g^2\delta\phi^2/H^2$ is positive definite, the largest that k/aH can be during irruption is of order unity.
4. The spectrum of produced particles is peaked; for large g , it is possibly strongly peaked. This is different from the model in Section 6.2.1.
5. Particles with present mass much larger than H can be created during inflation if the particle species couples to the inflaton in such a way that its effective mass vanishes during inflation.
6. In this model, if g is not much smaller than about 0.1, the present mass of the irrupton can be larger than the Planck mass. We will speculate on the implications and complications of this fact in Section 6.6.

In Section 6.2.3 we will consider an even more complicated irruption model. It will prove very useful to understand the results of that model on the basis of the results of the two simpler models we have already discussed in Sections 6.2.1 and 6.2.2.

6.2.3 *Massive Particle Species Irruption from a Non-canonical Kinetic Term*

Now consider the following two-field model in which the inflaton is coupled to the irrupton through the irrupton kinetic term (we refer to this as the ‘kinetically coupled’ irrupton):

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \frac{1}{2} e^{2f(\phi)} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi, \phi) \right], \quad (6.21)$$

where ϕ is the inflaton and χ is the irrupton. Here, U and V are potentials defined such that the U contains only potential terms depending on χ and χ - ϕ interaction terms, while V contains all terms depending on ϕ only.

We again take ϕ to be a spatially homogeneous classical field, governed by the EOM

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = f_{,\phi}e^{2f(\phi)}(\partial\chi)^2 - U(\phi, \chi)_{,\phi} \approx 0, \quad (6.22)$$

where the approximate equality above reflects that fact that we will again ignore the back-reaction on the classical inflaton field (and hence the metric) induced by the χ field. In terms of the dimensionless field $\nu \equiv \phi/M_{\text{Pl}}$, we have

$$\ddot{\nu} + 3H\dot{\nu} + m^2\nu = 0, \quad (6.23)$$

where we have specialized to $V(\phi) = \frac{1}{2}m^2\phi^2$. Ignoring the back-reaction on the metric is equivalent to ignoring the contribution of the χ field to the total energy density, so from the Friedmann equation it follows that

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{4\pi}{3}} \left(\dot{\nu}^2 + m^2\nu^2 \right)^{1/2}. \quad (6.24)$$

For our numerical work in this chapter we will solve the classical field equation for ν assuming that $\nu(t_0 = 0) = 3$ and that the inflaton field undergoes initial slow-roll $\dot{\nu}(t_0 = 0) = -1/\sqrt{12\pi}$, yielding roughly 57 e -folds of inflation. We also fix the normalization of the scale parameter to be $a(t_{\text{EI}}) = 1$ where $t_{\text{EI}} \approx 17.5$ is the end of inflation (defined to be the moment when $\ddot{a}(t_{\text{EI}}) = 0$, or $w = -1/3$) which gives $H(t_{\text{EI}}) \approx 0.50$.⁹

The irrupton field χ is governed by the field equation

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\eta^{ij}\partial_i\partial_j\chi}{a^2(t)} + 2f_{,\phi}\dot{\phi}\dot{\chi} + e^{-2f(\phi)}U_{,\chi} = 0. \quad (6.25)$$

9. We denote all parameters at the end of inflation by the subscript ‘EI’.

Specializing to $U(\chi, \phi) \equiv \frac{1}{2} \mathcal{U}(\phi) \chi^2$, passing to the ν variable and making a field redefinition $\mu = e^{f(\nu)} \chi$ to eliminate the mixed derivative term,¹⁰ this becomes

$$\ddot{\mu} + 3H\dot{\mu} + \left[\frac{\eta^{ij} \partial_i \partial_j}{a^2} - f_{,\nu} (\ddot{\nu} + 3H\dot{\nu}) - \dot{\nu}^2 (f_{,\nu\nu} + f_{,\nu}^2) + e^{-2f(\nu)} \mathcal{U}(\nu) \right] \mu = 0. \quad (6.26)$$

We again mode-expand $\mu = \sum_k \hat{a}_k u_k + \hat{a}_k^\dagger u_k^*$ and make the plane wave ansatz

$$u_k(\mathbf{x}, t) = \frac{e^{i\mathbf{k} \cdot \mathbf{x}}}{(2\pi)^{3/2} a(t)} \mu_k(t), \quad (6.27)$$

where $\mathbf{k}$ is the comoving momentum. This gives the mode equation for μ_k

$$\ddot{\mu}_k + H\dot{\mu}_k + \left[\frac{k^2}{a^2} - \frac{a''}{a^3} + m^2 \nu f_{,\nu} - \dot{\nu}^2 (f_{,\nu\nu} + f_{,\nu}^2) + e^{-2f(\nu)} \mathcal{U}(\nu) \right] \mu_k = 0, \quad (6.28)$$

where we used Eq. (6.23) to simplify.

In this chapter we will specialize to the potential $\mathcal{U}(\nu) = M^2$ and we assume that the function $f(\phi)$ takes the form $f(\phi) = -(\phi - \phi_*)^2 / 2\epsilon^2 M_{\text{Pl}}^2$, which implies $f(\nu) = -(\nu - \nu_*)^2 / 2\epsilon^2$. We then have $f_{,\nu} = -(\nu - \nu_*) / \epsilon^2$ and $f_{,\nu\nu} = -1/\epsilon^2$, so that the mode equation can be written in the form:

$$\ddot{\mu}_k + H\dot{\mu}_k + \frac{\omega_k^2(\eta)}{a^2} \mu_k = 0 \quad \Leftrightarrow \quad \mu_k'' + \omega_k^2(\eta) \mu_k = 0 \quad (6.29)$$

$$\text{where} \quad \omega_k^2(\eta) = k^2 - \frac{a''}{a} + a^2 M_{\text{eff}}^2 = a^2 H^2 \left[\left(\frac{k}{aH} \right)^2 + H^{-2} (M_{\text{eff}}^2 - a^{-3} a'') \right] \quad (6.30)$$

$$\text{and} \quad M_{\text{eff}}^2 = M^2 \exp \left[\frac{(\nu - \nu_*)^2}{\epsilon^2} \right] - \frac{m^2}{\epsilon^2} \nu (\nu - \nu_*) + \frac{\dot{\nu}^2}{\epsilon^2} \left[1 - \frac{1}{\epsilon^2} (\nu - \nu_*)^2 \right], \quad (6.31)$$

which implies that the late-time ($\nu, \dot{\nu} \approx 0$) effective mass of the μ field is given by $M_{\text{eff}}^\infty = M e^{\nu_*^2 / 2\epsilon^2}$, which may be much larger than M if $\epsilon \ll \nu_*$. In all our numerical work, we will take $\nu_* = 0.8$; the solution of the mode equation becomes increasingly numerically

10. At the level of the action, this redefinition canonically normalizes the μ kinetic term: $S \supset \int d^4x \sqrt{-g} \frac{1}{2} \dot{\mu}^2$.

intractable as ν_* is increased, or as ϵ is decreased.

Note that as $\epsilon \rightarrow \infty$, $M_{\text{eff}} \rightarrow M = \text{constant}$. Thus in the limit of large ϵ , the kinetically coupled irrupton model approaches the simple model of Section 6.2.1, albeit in a different inflationary background, as we already noted.

Before we discuss the numerical calculation of species irruption in this model we can observe similarities and differences between it and the potentially coupled irruption model. In Figure 6.1 we show the function $H^{-2}(M_{\text{eff}}^2 - a^{-3}a'')$ for a variety of parameter choices; there are some general observations we can draw from this figure:

1. One very general result is that the kinetically coupled irrupton mass is very large at early time.
2. For some model parameters, the function can appear similar to that for the potentially coupled irrupton model (see, e.g., the upper-right plot).
3. For some model parameters, the function never becomes negative (see, e.g., the lower plots).
4. The region where the function is negative is not centred on the time when $\phi = \phi_*$.

6.2.4 *Irrupton Number Density and Irrupton Initial Conditions*

Now let us turn to some general considerations relevant to the numerical calculations of irruption, which we will detail further in Section 6.4. As noted, the form of the mode equation for μ_k on the right in Eq. (6.29) is an ‘harmonic oscillator’ equation with η -dependent frequency in conformal time; however, we find it more convenient¹¹ for our numerical work to use the form on the left of Eq. (6.29) in terms of cosmic time t .¹² We also emphasize that

11. The relation $dt = a d\eta$ implies that for exponentially small a , as occurs in the early inflationary epoch, a very large range of η must be covered to cover even a small range of t . Since the 57 e -folds of inflation in our computation last $\Delta t \approx 18$ in the units in which we perform the computation, whereas $a(0) \sim 10^{-25}$, this would be a significant problem.

12. Here, and throughout the remainder of the chapter, we use μ_k generically to mean ‘the mode function’. Whenever the constant- M or potentially coupled models are under discussion, μ_k should of course be read

if ω_k^2 anywhere runs negative (as it may do owing to the presence of the $-a''/a$ term, which would be absent if the irrupton were coupled conformally to the metric [$\xi = 1/6$], rather than minimally), the mode becomes ‘tachyonic’ and the mode function diverges exponentially.¹³

During the non-adiabatic phase, the notion of particle number is ambiguous [697], but in both the early- and late-time regimes, where the expansion is adiabatic with respect to any given mode, the notion of a particle number regains physical validity. In order to extract the number of particles produced in mode k , we make use of the method of Bogoliubov coefficients (see, e.g., Refs. [697, 698]). We specify some initial conditions (equivalent to the in-vacuum choice; see below), then numerically integrate the mode equation for μ_k to some late time after inflation has ceased ($\ddot{a} < 0$), and use this solution to extract the Bogoliubov coefficient β_k giving the overlap of the exact solution to the mode equation subject to the early-time-vacuum initial conditions, with the exact solution which is pure negative-frequency at late time. This gives the differential comoving number-density of particles present in mode $\mathbf{k}$ in the asymptotic late-time regime as

$$\begin{aligned} (2\pi)^3 \frac{dn}{d^3k} &\equiv |\beta_k|^2 = \frac{\omega_k}{2} \left[|\mu_k|^2 + \omega_k^{-2} |\mu'_k|^2 \right] - \frac{1}{2} \\ &= \frac{\omega_k}{2} \left[|\mu_k|^2 + \frac{a^2}{\omega_k^2} |\dot{\mu}_k|^2 \right] - \frac{1}{2}. \end{aligned} \quad (6.32)$$

Clearly, the identification of the RHS of Eq. (6.32) as the modulus squared of a complex quantity is only sensible if the RHS is real and positive. If $\omega_k^2 < 0$ (the tachyonic regime), the RHS is imaginary and cannot be identified as $|\beta_k|^2$. While the identification of the RHS as $|\beta_k|^2$ is sensible if $\omega_k^2 > 0$, it can only be interpreted as a particle density in the adiabatic regime, $\mathcal{A} = |\omega'_k/\omega_k^2|^2 \ll 1$ and $\mathcal{B} = |\omega''_k/\omega_k^3| \ll 1$. We will frequently have recourse to refer to the differential comoving number density n_k^c of particles per logarithmic interval of

instead as χ_k : cf. Eq. (6.29) and Eqs. (6.5) and/or (6.17).

13. Despite this exponential growth, the evolution does always exactly preserve the norms defined in footnote 5.

$k \equiv |\mathbf{k}|$, which is given by

$$n_k^c \equiv \frac{dn}{d \ln k} \equiv \frac{k^3}{2\pi^2} |\beta_k|^2, \quad (6.33)$$

as well as the total physical number density n^p of particles, which is given by

$$n^p = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} |\beta_k|^2 = \frac{1}{a^3} \int_{-\infty}^{\infty} n_k^c d \ln k. \quad (6.34)$$

If ω_k^2 does run negative anywhere, the resulting tachyonic evolution of the mode function will show up as an exponential enhancement in the particle production since μ_k and $\dot{\mu}_k$ will be exponentially larger at late time.¹⁴ On the other hand, if $\omega_k^2 \gg 0$ is everywhere too large (e.g., if we choose k much larger than other parameters), then there is never appreciable particle creation as the mode will suffer neither tachyonic evolution nor a strongly non-adiabatic background spacetime expansion with respect to that mode.

The question of which initial conditions to specify for the mode function μ_k is subtle since their choice defines which basis of solutions are used for the mode expansion, making the question equivalent to the deep issue of the choice of vacuum in a non-static spacetime [697]. For practical reasons of numerical stability, we choose initial conditions such that we start in an instantaneous Minkowski vacuum:¹⁵

$$\mu_k(t_0^k) = \frac{e^{i\pi/4}}{\sqrt{2\omega_k(t_0^k)}}, \quad \text{and} \quad \dot{\mu}_k(t_0^k) = -i \frac{\omega_k(t_0^k)}{a(t_0^k)} \mu_k(t_0^k), \quad (6.35)$$

for mode k at a time $t = t_0^k$ chosen such that, for the mode in question, $\omega_k(t_0^k)/a(t_0^k)$ is equal to a very large threshold value (at least $5 \times 10^3 m$). That is, we pick the vacuum state $|0\rangle$ such that $\hat{a}_k(t_0^k)|0\rangle = 0$, which implies that $n_k^c(t_0^k) = 0$. We immediately note that this

14. While this statement is true, the issue is a bit more subtle than it appears. It will be discussed in more detail in Section 6.5.

15. These initial conditions would obtain by requiring that the exact solution matches onto the zeroth-order adiabatic approximate solution at $t = t_0^k$, correct up to terms of zeroth adiabatic order. That is, these initial conditions specify a ‘zeroth-order adiabatic vacuum’. See, e.g., Section 3.5 of Ref. [697].

choice of initial conditions does indeed specify a different (zeroth-order adiabatic) vacuum for every mode, as t_0^k is mode-dependent; however, since up to and including this time, the adiabaticity parameters $|\omega'_k/\omega_k^2|^2 \ll 1$ and $|\omega''_k/\omega_k^3| \ll 1$ are very small, and the bases of mode functions specified by the imposition of the initial conditions at t_0^k or $t_0^{k'}$ differ from each other only by terms of adiabatic order greater than zero, the family of vacua are all approximately equivalent. Indeed, we have explicitly verified that by increasing the threshold value of $\omega_k(t_0^k)/a(t_0^k)$, the amount of particle production we find does not change. There is one subtlety involved here, mainly relevant for the kinetically coupled case: if M_{eff} is very large at early time (and/or k is very small), ω_k^2/a^2 may cross the large threshold value while dominated by the M_{eff}^2 term, which could possibly lead to a situation where t_0^k comes after the mode in question crosses outside the Hubble radius at t_{HC} (see Figures 6.7 and 6.8). This would be potentially problematic on conceptual grounds, and we thus choose to present results only for modes which satisfy $t_0^k < t_{\text{HC}}$.

6.3 Present-Day Abundance for Stable Irruptions

Before we turn to a more detailed discussion of our numerical work, we specify how the comoving number density of irruptions produced in the final e -folds of inflation is translated into a physical present-day relic abundance assuming the irruption is stable. We follow the argument of Ref. [185], which we summarize here.

The irruptions are produced mainly during the final stages of inflation, which we assume to be followed by a brief matter-dominated phase characterized by coherent inflaton oscillations about the potential minimum $\phi = 0$. At the onset of oscillations, the universe is in a low-entropy frozen state owing to the inflationary expansion; however, after some number of inflaton oscillations, during which time the overwhelming majority of the energy density of the universe is contained in the inflaton field, the inflaton energy density is converted to radiation, heating the universe to some high temperature T_{RH} following which, in the standard thermal history of the universe, there is no significant further entropy production

and the universe in the large simply undergoes adiabatic expansion to the present time.

Following Ref. [185], we consider first the epoch after the inflaton energy density has been converted to radiation. The irrupton is non-relativistic (NR), so its energy density is $\rho_I = M_{\text{eff}}^\infty n^{\text{P}}$, with n^{P} given by Eq. (6.34) and where M_{eff}^∞ is the late-time effective irrupton mass (see Eq. (6.41) below). Under adiabatic expansion the comoving entropy density is constant, so it follows that the ratio

$$\frac{\rho_I}{\rho_R} \frac{g_{\text{eff}}}{h_{\text{eff}}} T \quad (6.36)$$

is constant, where ρ_R is the radiation energy density, and g_{eff} (h_{eff}) is the effective number of relativistic degrees of freedom relevant for the computation of ρ_R (entropy density s); see also Appendix B.1.1. The present-day relic abundance of the heavy irrupton species is thus

$$\Omega_I h^2|_0 = \Omega_R h^2|_0 \left(\frac{T_{\text{RH}}}{T_0} \right) \left(\frac{h_{\text{eff}}}{g_{\text{eff}}} \right)_0 \left(\frac{\rho_I}{\rho_R} \right)_{\text{RH}}, \quad (6.37)$$

where ‘RH’ denotes quantities evaluated at the moment of matter-radiation equality at the completion of (p)reheating, $T_0 = 2.7255 \text{ K}$ [167], $\Omega_R h^2|_0 = 4.15 \times 10^{-5}$, $g_{\text{eff},0} = 3.38$, $h_{\text{eff},0} = 3.91$ [120], and we have taken $g_{\text{eff}} = h_{\text{eff}}$ at the completion of (p)reheating.

It remains to determine $(\rho_I/\rho_R)_{\text{RH}}$. To do this, we note that during the matter-dominated inflaton-oscillation epoch, the overwhelming majority of the energy density is in the NR inflaton coherent oscillations, so the Friedmann equation yields $\rho_\phi \approx \rho_{\text{tot}} = 3M_{\text{Pl}}^2 H^2/8\pi$. The irruptons carry a subdominant component of the energy density (see Section 6.6) and are already NR at this epoch, so we still have $\rho_I = M_{\text{eff}}^\infty n^{\text{P}}$ with n^{P} given by Eq. (6.34) (the mass of the irrupton is fairly well approximated by its asymptotic large-time value M_{eff}^∞ after a few inflaton oscillations). The ratio $(\rho_I/\rho_\phi)_{\text{osc.}} \sim H^{-2} a^{-3} \sim t^{-2w}$ is thus constant for a pure-matter era ($w = 0$). This means that we may extract ρ_I at any point during the matter-dominated era once ρ_I has stabilized. In order to relate $(\rho_I/\rho_\phi)_{\text{osc.}}$ to $(\rho_I/\rho_R)_{\text{RH}}$ we must now make two assumptions:

1. the entire energy density of the inflaton coherent oscillations ends up in radiation after the inflaton oscillations decay, and
2. the transition between the matter-dominated oscillation epoch and the radiation-dominated epoch happens fairly quickly so that (a) the universe does not expand significantly during the transition which would cause the ratio $\rho_I/\rho_{\text{total}} = \rho_I/(\rho_R + \rho_\phi)$ to change non-trivially, and (b) there is no significant further entropy production once in the radiation dominated era.

Under these assumptions, it is a good approximation to set $(\rho_I/\rho_\phi)_{\text{osc.}} \approx (\rho_I/\rho_R)_{\text{RH}}$ and to apply Eq. (6.37). Putting this all together, we have

$$\Omega_I h^2|_0 \approx \Omega_R h^2|_0 \left(\frac{T_{\text{RH}}}{T_0} \right) \left(\frac{h_{\text{eff}}}{g_{\text{eff}}} \right)_0 \times \left(\frac{3M_{\text{Pl}}^2}{8\pi} H^2(\tilde{t}) \right)^{-1} \left[\frac{M_{\text{eff}}^\infty}{a^3(\tilde{t})} \int_{-\infty}^{\infty} n_k^c(\tilde{t}) d \ln k \right], \quad (6.38)$$

where $\tilde{t}$ is some reference time during the matter-dominated inflaton oscillation era at which we choose to extract the particle spectrum from our numerical solutions (see Section 6.4). In order to make contact with our numerical simulations in which we work in units for k , H , and t which are based on $m = 1$, we can re-write the above result as

$$\begin{aligned} \Omega_I h^2|_0 \approx \frac{8\pi}{3} (\Omega_R h^2)_0 \left(\frac{T_{\text{RH}}}{10^9 \text{ GeV}} \right) \left(\frac{10^9 \text{ GeV}}{T_0} \right) \left(\frac{h_{\text{eff}}}{g_{\text{eff}}} \right)_0 \left(\frac{m}{10^{13} \text{ GeV}} \right)^2 \left(\frac{10^{13} \text{ GeV}}{M_{\text{Pl}}} \right)^2 \\ \times \frac{M_{\text{eff}}^{\infty'}}{H'^2(\tilde{t}) a^3(\tilde{t})} \int_{-\infty}^{\infty} n_{k'}^c(\tilde{t}) d \ln k', \end{aligned} \quad (6.39)$$

or

$$\Omega_I h^2|_0 \times \left(\frac{T_{\text{RH}}}{10^9 \text{ GeV}} \right)^{-1} \times \left(\frac{m}{10^{13} \text{ GeV}} \right)^{-2} \approx 1.1 \times 10^6 \times \left[\frac{M_{\text{eff}}^{\infty'}}{H'^2(\tilde{t}) a^3(\tilde{t})} \times \int_{-\infty}^{\infty} n_{k'}^c(\tilde{t}) d \ln k' \right], \quad (6.40)$$

where the values of H' , $M_{\text{eff}}^{\infty'}$, and $n_{k'}$ are all extracted from the numerical solutions of the ν EOM and the mode equation for μ_k , in the units where $m = 1$. For the three models we

have discussed in this chapter, M_{eff}^∞ is given by

$$M_{\text{eff}}^{\infty'} = \begin{cases} M/m & \text{constant } M \\ (M_g/m)\nu_* = 0.8M_g & \text{potentially coupled} \\ (M/m)\exp(\nu_*^2/2\epsilon^2) & \text{kinetically coupled} \end{cases} . \quad (6.41)$$

Also, in the matter-dominated era after inflation, $H'^2(\tilde{t}) a^3(\tilde{t}) \simeq 0.18$.

We shall henceforth drop the primes on all of these quantities with the understanding that all are measured in units of m .

6.4 Numerical Methods and Issues

In order to calculate the final value of the Bogoliubov coefficient, β_k , and hence the number density of the irrupton, in principle we require the late-time values of $|\mu_k|^2$ and $|\dot{\mu}_k|^2$ [see Eq. (6.32)]. The straightforward procedure to obtain these values is to integrate the seven-dimensional system of first-order coupled ordinary differential equations (odes) for the scale factor a , the inflaton field value ν and its derivative $\dot{\nu}$, and the real and imaginary components of the irrupton mode function and its first derivatives; that is, the system of equations governing the vector

$$\mathbf{F}[t] \equiv \left\{ a, \nu, \dot{\nu}, \text{Re } \mu_k, \text{Im } \mu_k, \text{Re } \dot{\mu}_k, \text{Im } \dot{\mu}_k \right\}. \quad (6.42)$$

In some cases we will consider, this straightforward procedure is impractical. One issue is that, for large irrupton mass, the irrupton oscillation frequency may be much, much larger than the inflaton oscillation frequency. We will also see that for some parameters, we require integration of the system very deep into the matter-dominated era after inflation. Integrating the seven-dimensional system is thus occasionally unwieldy and computationally limited. Therefore, we will have occasion to employ a different computational strategy that

can be used in the regimes where there is no tachyonic phase. We details these methods in this section.

6.4.1 *Integration of the Full Seven-Dimensional System*

For the numerical integration of the seven-dimensional system of odes, we utilize the Runge–Kutta Dormand–Prince 8(53) algorithm `dop853` natively implemented in the `scipy v0.13` module for `python v2.7`, and cross-check our results against at least one other solver (`scipy`’s native `lsoda` algorithm) for a large subset of parameter values, finding agreement for most input parameter choices, and noting that it is fairly obviously the `lsoda` algorithm which fails when the two numerical solutions disagree.

In implementing these algorithms we find it necessary to impose very strict error tolerances and small maximum step sizes δt , as a variety of egregious numerical issues arise in solving the system of equations owing to the magnitude of some terms which enter. For example, an incomplete cancellation at early times between the positive and negative terms in Eq. (6.32)—arising from tolerances which are too loose—can cause $|\beta_k|^2$ to jump unphysically in the first few time steps to a value many orders of magnitude larger than it should be; this erroneously large value places a floor on how small a value of $|\beta_k|^2$ can be probed in our simulations at late times, which causes complications in tracking certain features in the particle spectra.

Furthermore, the cosmic-time oscillation frequency ω_k/a can become very large in both the early and late evolution of the inflationary epoch, making it computationally infeasible (given the number of solutions required to scan over parameter space) to perform sufficiently small δt time steps when the oscillation frequency ω_k/a of the mode function is larger than about $\mathcal{O}(10^4)$.¹⁶ This necessitates a subdivision of the computation. As we ignore the back-reaction due to the irrupton on the spacetime metric, as a first step we solve

16. As was alluded to above (see footnote 11), the opposite side of the same coin is that by going to conformal time, the domain of the η integration becomes infeasibly long to step forward any reasonable amount of cosmic time.

the restricted three-dimensional problem for $\tilde{\mathbf{F}}[t] = \{a, \nu, \dot{\nu}\}$, specifying initial conditions $\tilde{\mathbf{F}}[0] = \{1.60928 \times 10^{-25}, 3, -1/\sqrt{12\pi}\}$, where the value of $a(0)$ was set by the requirement that $a(t_{\text{EI}}) = 1$. We use this solution to find the time t_0^k (or more precisely, the nearest sampled time-step earlier than this) where $\omega_k/a \sim 5 \times 10^3$ and is decreasing. As discussed above, this is the point at which we specify the mode function μ_k to match the zeroth-order approximate adiabatic solution (i.e., specify the in-vacuum). Having identified this point, we then switch to integrating the full seven-dimensional system¹⁷ $\mathbf{F}[t]$ starting with initial conditions

$$\mathbf{F}[t_0^k] = \left\{ \tilde{a}(t_0^k), \tilde{\nu}(t_0^k), \dot{\nu}(t_0^k), \frac{1}{2\sqrt{\omega_k(t_0^k)}}, \frac{1}{2\sqrt{\omega_k(t_0^k)}}, \frac{\sqrt{\omega_k(t_0^k)}}{2\tilde{a}(t_0^k)}, -\frac{\sqrt{\omega_k(t_0^k)}}{2\tilde{a}(t_0^k)} \right\}, \quad (6.43)$$

where the quantities with a tilde are obtained from the integration of the restricted three-dimensional problem, and the μ_k initial conditions are precisely those defined in Eq. (6.35). We obtain $|\beta_k|^2$ from this full solution via Eq. (6.32); an example of the time-evolution of $|\beta_k|^2$ thus extracted is shown in Figure 6.2.

We extract the final particle spectrum using the ‘asymptotic’ value of $|\beta_k|^2$ deep in the matter-dominated inflaton oscillation era. When $|\beta_k|^2$ has stabilized to a damped oscillation about a constant central value at large time, we extract the asymptotic value by averaging over the last few oscillations at some late time. However, for certain parameter choices, $|\beta_k|^2$ does not stabilize to oscillations about a constant central value even by $t = 500$, but is rather still executing oscillation about a downward-drifting central value. Although for such modes a stable asymptotic value for $|\beta_k|^2$ is achieved if the mode equation is integrated for sufficiently long (see Appendix E.1), this is not computationally feasible via straightforward solution of the seven-dimensional system, and below we describe the method we use to obtain the asymptotic value of $|\beta_k|^2$ at very late time in such cases.

17. For reasons of numerical accuracy, we prefer this approach to interpolating the already-known solution for the restricted three-dimensional problem and solving just the four-dimensional system for μ_k .

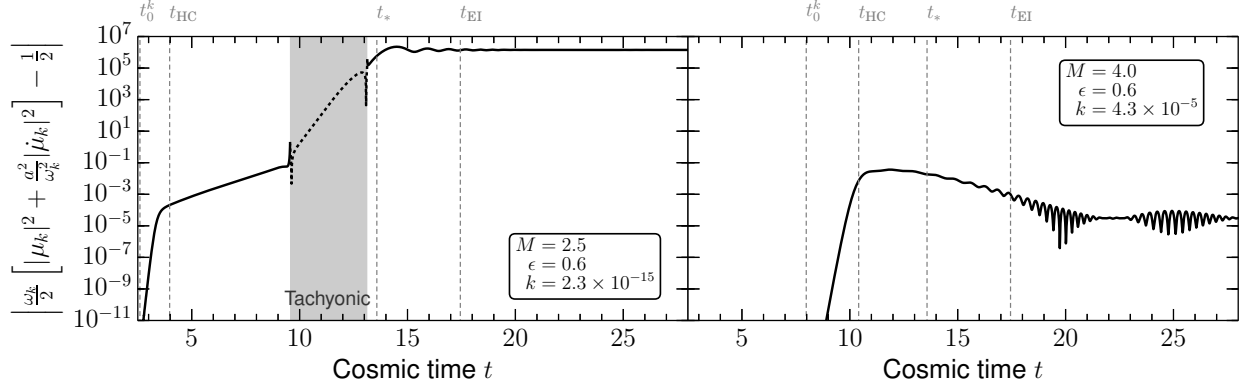


Figure 6.2: The time evolution of $\left| \frac{\omega_k}{2} \left(|\mu_k|^2 + (a^2/\omega_k^2) |\dot{\mu}_k|^2 \right) - \frac{1}{2} \right|$ for two representative cases for the kinetically coupled irruption, as extracted from the numerical solution for the choices of parameters $M = 2.5$, $\epsilon = 0.6$, and $k = 2.3 \times 10^{-15}$ (left plot); and $M = 4.0$, $\epsilon = 0.6$, and $k = 4.3 \times 10^{-5}$ (right plot). If $\omega_k^2 > 0$, $\frac{\omega_k}{2} \left[|\mu_k|^2 + (a^2/\omega_k^2) |\dot{\mu}_k|^2 \right] - \frac{1}{2}$ is real and positive, and equal to $|\beta_k|^2$. The evolution during the tachyonic phase (grey band, left plot) is presented for demonstrative purposes only, in order to indicate the rapid increase during this period; we caution however that there is no fashion in which this quantity can be interpreted as a particle number during the tachyonic phase. At both early and late times, the background spacetime evolution is sufficiently adiabatic with respect to this mode that $|\beta_k|^2$ can indeed be interpreted as the occupation number for mode $\mathbf{k} = k\hat{\mathbf{k}}$; note that the late-time behaviour (i.e., after the end of inflation) shows that $|\beta_k|^2$ undergoes damped oscillation about a constant non-zero value, indicating particle production has occurred. The times $t_0^k, t_{\text{HC}}, t_*$, and t_{EI} indicate, respectively, the times when the initial conditions Eq. (6.43) were imposed, when the mode crosses the Hubble radius ($k = aH$), when $\nu = \nu_*$, and when inflation ends ($\ddot{a} = 0$).

6.4.2 The Iterative Solution Approach

In this section we describe another approach to the calculation of $|\beta_k|^2$ that can be used only in the absence of a tachyonic phase. This approach is amenable to a very useful and computationally much less demanding iterative solution approach when $|\beta_k|$ remains small. It also is the formalism we use to extract the late-time asymptotic value of $|\beta_k|^2$ for model parameters where this quantity has not stabilized to its asymptotic value by $t \sim$ a few hundred, which generically only occurs for modes without a tachyonic phase.

If ω_k is real (no tachyonic behaviour), the solution to the mode equation may be written

as (see, e.g., Ref. [699])

$$\mu_k = \frac{\alpha_k(t)}{\sqrt{2\omega_k}} e^{-i\Phi(t)} + \frac{\beta_k(t)}{\sqrt{2\omega_k}} e^{+i\Phi(t)} \quad (6.44)$$

where the accumulated phase $\Phi(t)$ is given by [obviously $\Phi_0 \equiv \Phi(t_0^k)$]

$$\Phi(t) \equiv \int_{t_0^k}^t \frac{\omega_k(t')}{a(t')} dt' + \Phi_0. \quad (6.45)$$

Eq. (6.44) is a solution to the mode equation Eq. (6.29) if $\alpha(t)$ and $\beta(t)$ satisfy the coupled equations of motion

$$\dot{\alpha}_k = \frac{\dot{\omega}_k}{2\omega_k} \beta_k e^{2i\Phi} \quad \text{and} \quad \dot{\beta}_k = \frac{\dot{\omega}_k}{2\omega_k} \alpha_k e^{-2i\Phi}, \quad (6.46)$$

which also implies that

$$\dot{\mu}_k = -i \frac{\omega_k}{a} \left[\frac{\alpha_k(t)}{\sqrt{2\omega_k}} e^{-i\Phi(t)} - \frac{\beta_k(t)}{\sqrt{2\omega_k}} e^{+i\Phi(t)} \right]. \quad (6.47)$$

In line with our previous discussion, we take initial conditions

$$\mu_k(t_0^k) = \frac{1}{\sqrt{2\omega_k(t_0^k)}} e^{i\pi/4} \quad \text{and} \quad \dot{\mu}_k(t_0^k) = -i \frac{\omega_k(t_0^k)}{a(t_0^k)} \mu_k(t_0^k), \quad (6.48)$$

where the threshold value of ω_k/a which dictates the value of t_0^k can be taken to be much larger than $\omega_k(t_0^k)/a(t_0^k) = 5 \times 10^3$ in this method provided we have a sufficiently accurate background solution for $\{a, \nu, \dot{\nu}\}$; we utilize a threshold value of $\omega_k(t_0^k)/a(t_0^k) = 10^7$, and have checked explicitly (for a subset of parameter values) that the results are insensitive to this parameter provided it remains large. These initial conditions imply that (Φ_0 is of course

arbitrary; we simply take the value consistent with our previous discussion)

$$\alpha_k(t_0^k) = 1, \quad \beta_k(t_0^k) = 0, \quad \text{and} \quad \Phi_0 = -\frac{\pi}{4}. \quad (6.49)$$

The occupancy number for the mode $\mathbf{k} = k\hat{\mathbf{k}}$ is given by

$$n_{\mathbf{k}} = \frac{1}{2} [|\alpha_k|^2 + |\beta_k|^2 - 1] = |\beta_k|^2 = |\alpha_k|^2 - 1, \quad (6.50)$$

where we have used the Wronskian condition $|\alpha_k|^2 - |\beta_k|^2 = 1$ at the last two steps; this condition follows from demanding correctly normalized modes per footnote 5.

This approach is amenable to an iterative solution when $|\beta_k|$ is small and the evolution is nearly adiabatic (we will quantify this statement shortly). To develop the iterative solution we introduce a formal small parameter ϵ by rescaling $\dot{\omega}_k/\omega_k$:

$$\frac{\dot{\omega}_k}{\omega_k} \rightarrow \epsilon \frac{\dot{\omega}_k}{\omega_k}, \quad (6.51)$$

and expand the solutions as a formal power series in ϵ :

$$\alpha_k(t) \equiv \sum_{n=0}^{\infty} \epsilon^{2n} \alpha_k^{(2n)}(t) \quad \text{and} \quad \beta_k(t) \equiv \sum_{n=0}^{\infty} \epsilon^{2n+1} \beta_k^{(2n+1)}(t), \quad (6.52)$$

in terms of which we impose the initial conditions on the $\alpha_k^{(n)}(t)$ and $\beta_k^{(n)}(t)$ as

$$\alpha_k^{(0)}(t) \equiv 1, \quad \Phi_0 = -\frac{\pi}{4} \quad (6.53)$$

$$\alpha_k^{(2n)}(t_0^k) = \beta_k^{(2n-1)}(t_0^k) = 0 \quad \text{for} \quad n \geq 1. \quad (6.54)$$

Substitution into the equations of motion for α_k and β_k , followed by equating coefficients of ϵ^n to zero for all n , then sending $\epsilon \rightarrow 1$ at the end of the process, yields the governing

equations for $\alpha_k^{(n)}$ and $\beta_k^{(n)}$:

$$\begin{aligned}\dot{\alpha}_k^{(0)} &= 0 \\ \dot{\beta}_k^{(2n+1)} &= \frac{\dot{\omega}_k}{2\omega_k} \alpha_k^{(2n)} e^{-2i\Phi} \quad \text{for } n \geq 0 \\ \dot{\alpha}_k^{(2n+2)} &= \frac{\dot{\omega}_k}{2\omega_k} \beta_k^{(2n+1)} e^{+2i\Phi} \quad \text{for } n \geq 0,\end{aligned}\tag{6.55}$$

which has the following solutions consistent with the initial conditions:

$$\begin{aligned}\alpha_k^{(0)}(t) &\equiv 1, \quad \Phi_0 = -\frac{\pi}{4} \\ \beta_k^{(2n+1)}(t) &= \int_{t_0^k}^t \frac{\dot{\omega}_k(t')}{2\omega_k(t')} \alpha_k^{(2n)}(t') e^{-2i\Phi(t')} dt' \quad \text{for } n \geq 0 \\ \alpha_k^{(2n+2)}(t) &= \int_{t_0^k}^t \frac{\dot{\omega}_k(t')}{2\omega_k(t')} \beta_k^{(2n+1)}(t') e^{+2i\Phi(t')} dt' \quad \text{for } n \geq 0.\end{aligned}\tag{6.56}$$

From these expressions, the iterative solution method is obvious. Convergence is generally obtained after only a few iterations; we always truncate the series for $|\beta_k|^2$ at the tenth iterate and use this value in Eq. (6.50) to obtain n_k .

Note however that to utilize this method, we still need a very accurate solution for the background $\tilde{\mathbf{F}}[t] = \{a, \nu, \dot{\nu}\}$ out to whatever time we choose to run the solution; this must be obtained by solving the (restricted three-dimensional set of) odes for these fields per the methods discussed in Section 6.4.1. Although dramatically less computationally intensive than solving the full seven-dimensional set of odes, this is still computationally time-consuming if we wish to have the solutions out to very late time in order to track $|\beta_k|^2$ all the way to its asymptotic value. In Section 6.4.3 we describe how, in our actual numerical implementation of this iterative method, we have made a modification to the procedure just outlined which allows this problem too to be avoided.

In order to understand the conditions for the iterative solution to be a good approximation, consider the first-order solution $\beta_k^{(1)}$, which yields the lowest-order approximation for

the occupancy number $n_{\mathbf{k}}$:

$$n_{\mathbf{k}}^{(1)} = \left| \beta_{\mathbf{k}}^{(1)} \right|^2 = \left| \int_{t_0^k}^t \frac{\dot{\omega}_{\mathbf{k}}(t')}{2\omega_{\mathbf{k}}(t')} e^{-2i\Phi(t')} dt' \right|^2. \quad (6.57)$$

Since in the absence of a tachyonic phase, $\omega_{\mathbf{k}}(t)$ is everywhere positive and real, $\dot{\Phi}(t) = \omega(t)/a(t) > 0$: the accumulated phase is a strictly increasing function of time. Therefore, we can recast the first-order approximation using the accumulated phase as the integration variable:

$$n_{\mathbf{k}}^{(1)} = \left| \beta_{\mathbf{k}}^{(1)} \right|^2 = \frac{1}{4} \left| \int_{\Phi_0}^{\Phi(t)} \frac{\omega'_{\mathbf{k}}(t')}{\omega_{\mathbf{k}}^2(t')} \Big|_{t'=t'(\Phi)} e^{-2i\Phi} d\Phi \right|^2. \quad (6.58)$$

In the asymptotic late-time regime $t \rightarrow \infty$, we have $\Phi(t) \rightarrow \infty$, so that

$$n_{\mathbf{k}}^{(1)}(t \rightarrow \infty) = \frac{1}{4} \left| \int_{\Phi_0}^{\infty} \frac{\omega'_{\mathbf{k}}}{\omega_{\mathbf{k}}^2} e^{-2i\Phi} d\Phi \right|^2 \leq \int_{\Phi_0}^{\infty} \left| \frac{\omega'_{\mathbf{k}}}{\omega_{\mathbf{k}}^2} \right|^2 d\Phi = \int_{\Phi_0}^{\infty} \mathcal{A} d\Phi, \quad (6.59)$$

where we used the Cauchy-Schwarz inequality, and $\mathcal{A}$ is the adiabaticity parameter defined in Eq. (6.9). The appearance in the integral Eq. (6.58) of the ‘square-root’ (with phase) of the adiabaticity parameter $\mathcal{A}$ refines our previous argument that the size of the adiabaticity parameters limits the amount of particle production. It is in fact the total time-integrated magnitude of the adiabaticity parameter which provides a hard upper limit to the amount of particle production. Note of course that modifications to the adiabaticity parameter near its maximum clearly impact the upper bound more strongly.

It is important to note that the simple expression in Eq. (6.59) is only an extremely crude upper bound to $n_{\mathbf{k}}^{(1)}(t \rightarrow \infty)$; we do not expect this bound to be saturated as it represents the integral of the envelope of the highly oscillatory integrand, rather than the integrand itself. Phase cancellations are important.

6.4.3 Late-Time Solution in the Matter-Dominated Era

Well after inflation ends, the dynamics of the expansion of the universe are those of a matter-dominated (MD) model with the energy density from the oscillating inflaton field (of course, the energy in the inflaton field must eventually be converted to radiation, so this MD phase is not of infinite duration; however, absent postulating a specific model for this process, we cannot assess its impact on our results). Where necessary, we exploit this fact to extend our solution for $|\beta_k|^2$ very deep into the MD era, in order to extract its asymptotic late-time value without having to solve a set of odes.

For an MD phase, the scale factor evolves as $a(t) \propto t^{2/3}$. Suppose that we choose a reference time t_{ref} deep into the MD era following inflation (in our computations, we use $t_{\text{ref}} \sim 10^3$); the scale factor and expansion rate may then be written as

$$a(t) = a_{\text{ref}} \left(\frac{t - \tau}{t_{\text{ref}} - \tau} \right)^{2/3} \quad \text{and} \quad H = \frac{2}{3} \frac{1}{t - \tau}, \quad (6.60)$$

where τ is the effective bang time assuming an MD universe all the way back to the singularity (τ has no actual physical significance whatsoever; this is merely the effective interpretation of the parameter). In this late-time MD era, the inflaton field and its time derivative are given by

$$\begin{aligned} \nu(t) &= A \frac{\sin(t - \tau)}{t - \tau} + B \frac{\cos(t - \tau)}{t - \tau}, \\ \dot{\nu}(t) &= A \frac{\cos(t - \tau)}{t - \tau} - A \frac{\sin(t - \tau)}{(t - \tau)^2} - B \frac{\sin(t - \tau)}{t - \tau} - B \frac{\cos(t - \tau)}{(t - \tau)^2}, \end{aligned} \quad (6.61)$$

where A and B are given by

$$\begin{aligned} A &\equiv \left(\dot{\nu}_{\text{ref}}(t_{\text{ref}} - \tau) + \nu_{\text{ref}} \right) \cos(t_{\text{ref}} - \tau) + \nu_{\text{ref}}(t_{\text{ref}} - \tau) \sin(t_{\text{ref}} - \tau), \\ B &\equiv - \left(\dot{\nu}_{\text{ref}}(t_{\text{ref}} - \tau) + \nu_{\text{ref}} \right) \sin(t_{\text{ref}} - \tau) + \nu_{\text{ref}}(t_{\text{ref}} - \tau) \cos(t_{\text{ref}} - \tau). \end{aligned} \quad (6.62)$$

For the late-time MD evolution we require $\dot{\omega}_k/\omega_k$. There are three terms in the expression for $\omega_k^2(t)$: k^2 , a''/a , and $a^2 M_{\text{eff}}^2$. At late-time in the MD era, the three terms scale as t^0 , $t^{-2/3}$, and $t^{4/3}$, respectively, so at late time we will use $\omega_k^2(t) \simeq a^2(t) M_{\text{eff}}^2$. This leads to $\dot{\omega}_k/\omega_k = H + \dot{M}_{\text{eff}}/M_{\text{eff}}$.

At late time,

$$\begin{aligned}
M_{\text{eff}} &= M && \text{(constant } M) \\
M_{\text{eff}} &= M_g(\nu_* - \nu) && \text{(potentially coupled)} \\
M_{\text{eff}} &\simeq M \exp \left[(\nu - \nu_*)^2 / 2\epsilon^2 \right] && \text{(kinetically coupled).}
\end{aligned} \tag{6.63}$$

For all cases, we can write for the late-time MD era

$$\frac{\dot{\omega}_k}{2\omega_k} = \frac{1}{2}H - \delta \cdot \dot{\nu}, \tag{6.64}$$

where $H(t)$ is given by Eq. (6.60); $\dot{\nu}$ is given by Eq. (6.61); and for $\nu \ll \nu_*$, $\delta = 0$, $\delta = 1/2\nu_*$, and $\delta \simeq \nu_*/2\epsilon^2$ for the constant- M , potentially coupled, and kinetically coupled models, respectively.

We are now positioned to find the expressions for $\alpha(t)$ and $\beta(t)$ in the MD era. First consider the accumulated phase $\Phi(t)$:

$$\begin{aligned}
\Phi(t) &\equiv \Phi_0 + \int_{t_0^k}^t \frac{\omega_k(t')}{a(t')} dt' \approx \Phi_0 + \int_{t_0^k}^{t_{\text{ref}}} \frac{\omega_k(t')}{a(t')} dt' + M_{\infty}(t - t_{\text{ref}}) \\
&\equiv \Phi_{\text{ref}} + M_{\infty}(t - t_{\text{ref}}),
\end{aligned} \tag{6.65}$$

where Φ_0 is the accumulated phase at t_0^k ; Φ_{ref} is evaluated in the full numerical evolution; and $M_{\infty} = M_g\nu_*$ in the potentially coupled model, while $M_{\infty} = Me^{\nu_*^2/2\epsilon^2}$ in the kinetically coupled model. We have assumed here that $|\nu| \ll \nu_*$.

Supposing that we have obtained $\alpha_{k,\text{ref}}^{(n)} = \alpha_k^{(n)}(t_{\text{ref}})$ and $\beta_{k,\text{ref}}^{(n)} = \beta_k^{(n)}(t_{\text{ref}})$ (e.g., by the methods of Section 6.4.2), we may now use our MD-era expressions to extend these to later

times $t > t_{\text{ref}}$ by iterating (for $n \geq 0$)

$$\beta_k^{(2n+1)}(t) = \beta_{k,\text{ref}}^{(2n+1)} + \frac{1}{2}e^{-2i\Phi_{\text{ref}}} \int_{t_{\text{ref}}}^t \left[H(t_1) - 2\delta \cdot \dot{\nu}(t_1) \right] \alpha_k^{(2n)}(t_1) e^{-2iM_\infty(t_1-t_{\text{ref}})} dt_1, \quad (6.66)$$

$$\alpha_k^{(2n+2)}(t) = \alpha_{k,\text{ref}}^{(2n+2)} + \frac{1}{2}e^{+2i\Phi_{\text{ref}}} \int_{t_{\text{ref}}}^t \left[H(t_1) - 2\delta \cdot \dot{\nu}(t_1) \right] \beta_k^{(2n+1)}(t_1) e^{+2iM_\infty(t_1-t_{\text{ref}})} dt_1. \quad (6.67)$$

In our actual numerical implementation for the alternative solution method based on α and β , we utilize the iterated method implicit in Eq. (6.56) without change for $t \leq t_{\text{ref}}$, but for $t > t_{\text{ref}}$ we obtain $\alpha_k^{(n)}$ and $\beta_k^{(n)}$ with the iterated method implicit in Eqs. (6.66) and (6.67).

One could of course also use a hybrid method in which the full seven-dimensional system of odes is integrated through any tachyonic regions, and the iterative solution method is used to evolve the solution forward to very late time starting from some time after the tachyonic phase ends. We never find this to be necessary.

Using the techniques we have expounded upon in this section, we develop an analytical understanding of the late-time asymptotic behaviour of $|\beta_k|^2$ in Appendix E.1.

6.4.4 *Parameters Scanned and Breakdown of Methods Employed*

For the kinetically coupled case, we have completed a scan over the parameters (M, ϵ) at fixed $\nu_* = 0.8$ in the ranges $M \in [0.2, 7]$, $\epsilon > 0.2$ – 0.25 (the smallest computationally feasible lower cutoff here is somewhat dependent on the choice of M). The largest ϵ investigated was effectively infinite (specifically, $\epsilon = 10^{30}$), to allow us to compare our numerical investigations to the simple model of Section 6.2.1 in the context of the chaotic-inflation background, which has been previously investigated in Ref. [188]. Additionally, we investigated the M -dependence of the spectra up to $M = 9$ at fixed $\epsilon = 0.6$.

For the potentially coupled case, we have completed a scan over M_g in the range $M_g \in [1.2, 3.7 \times 10^4]$. In Table 6.1, we summarize explicitly which numerical method was used in

Table 6.1: A summary of which numerical method has been used to obtain the late-time asymptotic value of $|\beta_k|^2$, organized by figure number. The end-time t_f used in these solutions varies and is taken to be ‘late enough’ in the sense that we can reliably extract the asymptotic value of $|\beta_k|^2$. This can be as short as $t_f = 100$ for cases which quickly go to their late-time asymptotic value (e.g., $M = 2$ at small ϵ), or as long as $t_f \sim 4.5 \times 10^5$ for particularly stubborn cases which take a very long time to ‘ring down’ (e.g., $M > 4$ at very large or very small k); a full listing of the values of t_f used would not be enlightening. In this table ‘7D’ refers to the straightforward solution of the full seven-dimensional system of equations for the background fields and the mode equation as described in Section 6.4.1, and ‘Iter.’ refers to the iterated solution method for α and β described in Section 6.4.2, taken where necessary with the late-time MD-era modification described in Section 6.4.3.

Figure Number and Description	Parameter Range or Identifier	Method Employed
Figure 6.3 ($\epsilon \rightarrow \infty$)	$k \gtrsim 0.5$ and $M \geq 1.5$; or $M \geq 5.0$	Iter.
	$k \lesssim 0.5$ and/or $M < 1.5$	7D
Figure 6.4 (Potentially coupled)	—	7D
Figure 6.5 (upper plot, $\epsilon = 0.6$)	$k \gtrsim 0.1$ and $M \geq 0.6$	Iter.
	All others	7D
Figure 6.5 (lower plot, $\epsilon = 0.6$, large M)	All, solid lines	Iter.
	Selected, open circles	7D
Figure 6.6 (upper plot, $M = 2.0$)	$k \gtrsim 0.6$ and $\epsilon \geq 0.4$	Iter.
	$k \lesssim 0.6$ and $\epsilon \geq 0.4$	7D
	$k \gtrsim 0.4$ and $\epsilon = 0.3$	Iter.
	$k \lesssim 0.4$ and $\epsilon = 0.3$	7D
	$k \gtrsim 6 \times 10^{-2}$ and $\epsilon = 0.2$	Iter.
	$k \lesssim 6 \times 10^{-2}$ and $\epsilon = 0.2$	7D
Figure 6.6 (both lower plots, $M = 4.0$)	All, solid lines	Iter.
	Selected, open circles	7D

obtaining the various spectra we present in Section 6.5; the general rule of thumb is that we use the iterative solution method wherever possible, but directly solve the full seven-dimensional set of coupled odes whenever a tachyonic phase is present.

Where we choose to present results for Ωh^2 , the particle spectrum (extracted as detailed above) is integrated over k per Eq. (6.40) in the maximal numerically sampled range¹⁸ $k \in [10^{-20}, 20]$ to obtain a relic abundance Ωh^2 , provided that the numerical results we have extracted indicate that this integral converges in the infrared ($M < 1$ results at large ϵ in the kinetically coupled case are problematic in this regard), and subject to modification in the upper limit of the integral to avoid obvious numerical artefacts which enter at large k (read: very small $|\beta_k|^2$).

6.5 Results

In this section we present the numerical results for particle species irruption either in the model of inflaton–irrupton coupling through the potential term (the model discussed in Section 6.2.2), or in the model of inflaton–irrupton coupling through the kinetic term (the model discussed in Section 6.2.3). The most general models of these types have either two or three free functions: an inflaton potential $V(\phi)$, an inflaton–irrupton potential $U(\chi, \phi)$, and for the kinetically coupled case only, a function $f(\phi)$ that describes the coupling of the inflaton to the kinetic term of the irrupton. For $V(\phi)$ we choose the simple inflaton potential $V(\phi) = \frac{1}{2}m^2\phi^2$. In the potentially coupled case, for $U(\chi, \phi)$ we choose the same potential as in Section 6.2.2: $U(\chi, \phi) = \frac{1}{2}g^2(\phi - \phi_*)^2\chi^2 \equiv \frac{1}{2}M_g^2(\nu - \nu_*)^2\chi^2$; for the kinetically coupled case, for $U(\chi, \phi)$ we choose the same potential as in Section 6.2.3: $U(\chi, \phi) = \frac{1}{2}M^2\chi^2$. Finally, for $f(\phi)$ in the kinetically coupled case, we choose $f(\phi) = -(\phi - \phi_*)^2/2\epsilon^2 M_{\text{Pl}}^2 \equiv -(\nu - \nu_*)^2/2\epsilon^2$. Thus, the two or three free functions are described, respectively, in terms

18. If necessary, we occasionally work outside this range to capture a relevant feature: for example, at $M = 0.2$ and $\epsilon = 0.8$ for the kinetically coupled case, n_k^c peaks around $k \sim 10^{-19}$, and we wish to capture this behaviour fully to get an accurate particle number, so we extend the range of integration down to $k = 10^{-22}$.

of either three parameters: $\{m, M_g, \nu_*\}$, or four parameters: $\{m, M, \nu_*, \epsilon\}$. From background radiation measurements, $m \sim 10^{13}$ GeV. We will express M and M_g in units of m , and t in units of m^{-1} . For the chaotic-inflation model we consider, ranges of ϕ in the observable region of the background radiation are $0.2 \lesssim (\phi/M_{\text{Pl}} \equiv \nu) \lesssim 3$. We will make the choice $\phi_*/M_{\text{Pl}} \equiv \nu_* = 0.8$ for all numerical results presented. With these choices for m and ν_* , the one parameter we will vary for the potentially coupled irrupton is $\{M_g\}$ (in units of m), and the two parameters we will vary for the kinetically coupled case are $\{M, \epsilon\}$ (again, M in units of m , and ϵ dimensionless).

6.5.1 Behaviour of Spectra: Description

Minimally Coupled Scalar Field

We note that the $\epsilon \rightarrow \infty$ limit of the kinetically coupled irrupton is a minimally coupled scalar field of mass M in a background chaotic-inflation model. This model was described in Section 6.2.1 (but for de Sitter space with constant H , not in chaotic inflation with a slowly evolving H , as for the numerical results of this section). Our result for the comoving number density of produced particles as a function of M in this limit is shown in Figure 6.3. This case has been previously considered in the literature (e.g., Ref. [188]) for the same inflationary regime we utilize, and where available we reproduce known results well (the blue circles in the figure are sample points taken from Ref. [188]).

The qualitative behaviour of the curves in Figure 6.3 is different for different M . For $M < 1$, there is a slow growth of n_k^c in the infrared, corresponding to $|\beta_k|^2 \sim k^{-3-x}$ for some $x > 0$, leading to an infrared (IR) divergence in the number of particles produced.¹⁹ For $M > 1$ the spectra decrease in the far infrared, corresponding to $|\beta_k|^2 \sim k^{-3+x}$ for some $x > 0$. For $M = 1$, the spectrum of n_k^c is consistent with being exactly flat in the IR. There is a sharp drop in the spectra for $k \gtrsim 1$ for $M > 1$; the drop is more gradual for small

19. Presumably this IR divergence is cut off if inflation has a finite duration.

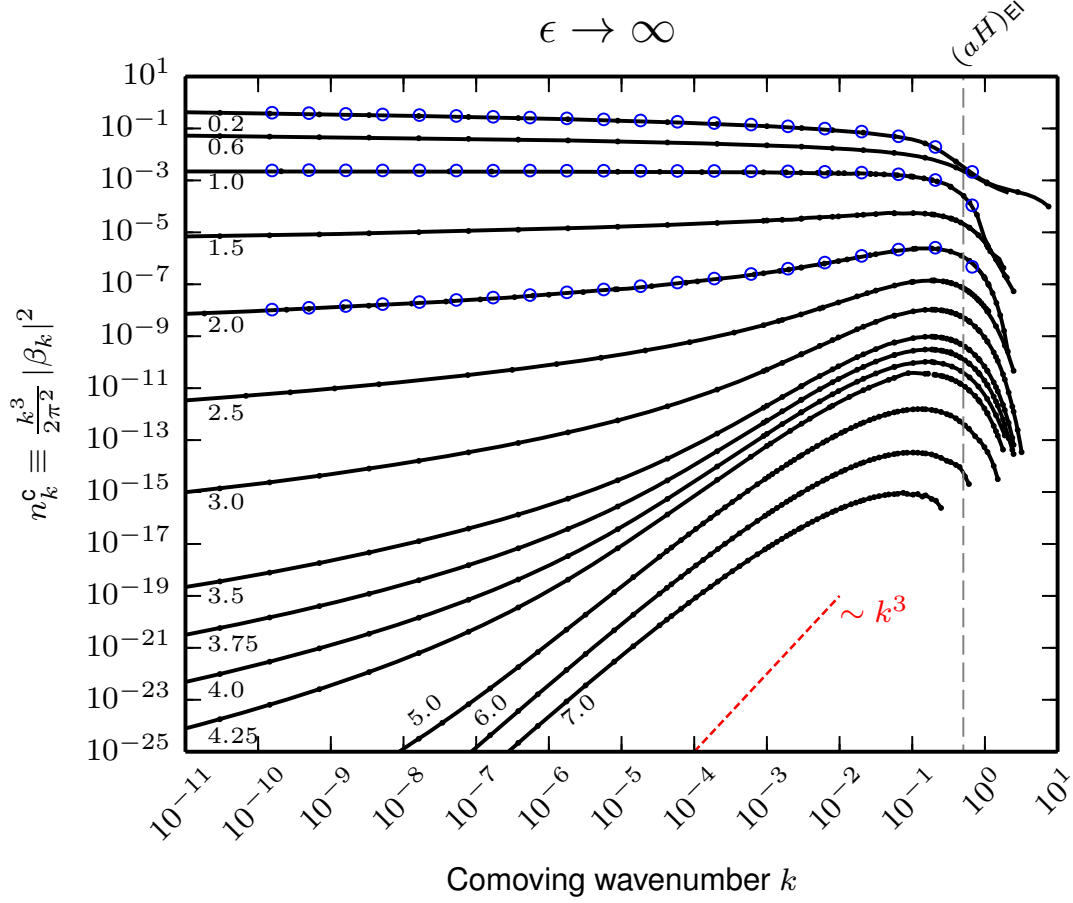


Figure 6.3: Comoving particle-density spectra n_k^c as a function of comoving k (in units of m) for a minimally coupled scalar field with constant mass M (as annotated; also in units of m) as discussed in Section 6.2.1, but using the chaotic-inflation model to describe the evolution of the background (i.e., the $\epsilon \rightarrow \infty$ limit of the kinetically coupled irruption of Section 6.2.3). Different curves are for various heavy-particle masses M . For comparison, the thin (red) dashed line indicates the scaling of n_k^c with k if the scaling were k^3 (constant $|\beta_k|^2$). The (blue) circles on the lines for $M = 0.2$, 1 , and 2 are sampled from spectra in Ref. [188]. The vertical dashed line denotes $k = a_{\text{EI}} H_{\text{EI}}$. See Table 6.1 for a summary of the numerical methods that were applied to obtain these results.

M , and a small bump is even evident in the spectrum for $M = 0.2$ around $k \sim 0.5$. These features will be explained in Section 6.5.2.

Potentially or Kinetically Coupled Scalar Field

Representative particle spectra for the potentially coupled irrupton as a function of M_g are given in Figure 6.4, while those for the kinetically coupled irrupton as a function of M are given for fixed $\epsilon = 0.6$ in Figure 6.5, and as a function of ϵ for two fixed values of M ($M = 2$ and $M = 4$) in Figure 6.6. These figures capture all the important features we have observed in our numerical work. Indeed, for the specific potential and kinetic couplings we have considered for the irrupton, many interesting features are present. Before explaining the causes of these features in the spectra in Section 6.5.2, we qualitatively describe the scalings of the spectra with k and M_g for the potentially coupled irrupton; or with k , M , and ϵ for the kinetically coupled irrupton.

Consider first the spectra for the potentially coupled irrupton shown in Figure 6.4. In all cases with $M_g \gtrsim 5$, the spectra scale as $n_k^c \sim k^3$ in the infrared region, which implies $|\beta_k|^2 \sim k^0$. Provided also that $M_g \lesssim 10$, this behaviour is valid until a threshold value $k = k_*$, beyond which the spectra increase more slowly than k^3 , but have no simple power-law scaling. The spectra then peak, more sharply for larger M_g , before showing a steep drop-off in the UV region. For much larger M_g , the spectra instead roll off exponentially fast directly from the $n_k^c \sim k^3$ regime. For the smallest values of M_g , the IR behaviour does not enter an approximate k^3 scaling regime (at least for the numerically sampled range of $k \gtrsim 10^{-16}$) and, based on arguments we present below, we do not necessarily expect that such a regime would exist for small k when $M_g \lesssim 3$. We note that for $M_g \lesssim 60$ in the results we have presented there is a strict ordering in the size of n_k^c : at fixed k , n_k^c is smaller the larger M_g becomes. This behaviour is modified at very large $M_g \gtrsim 200$, where the IR spectrum stops decreasing with increasing M_g , and instead approaches from above the limit $n_k^c = k^3/2\pi^2$ (which implies $|\beta_k|^2 = 1$); on the other hand, the spectrum near the peak

begins to increase again with increasing M_g , approaching the same limit from below, while the peak itself shifts further to the UV.

For the kinetically coupled irrupton, consider first the case of fixed ϵ and fairly small M in the upper plot of Figure 6.5. In the infrared region, the spectra scale as $n_k^c \sim k^3$, which implies $|\beta_k|^2 \sim k^0$. This scaling behaviour is valid up until an ‘elbow’ in the spectra that occurs for a value of k that depends on M (and ϵ). After the elbow, for increasing k there is a slow decrease in n_k^c with k for $M \lesssim 1$, or a slow increase in n_k^c for $M \gtrsim 1$. For $M \lesssim 2.5$, the spectra scale as a power law in k in this intermediate- k regime, with an M -dependent power which is less than 3. This behaviour continues until $k \sim 10^{-1}$, and thereafter there is a steep decrease in n_k^c . There is however a pronounced bump in the n_k^c spectra at small M ($M = 0.2$) and $k \sim 1$; this does not occur for $M \gtrsim 0.2$. Just as in the model of a minimally coupled scalar field with constant M , there is a general trend of decreasing particle production with increasing M .

Consider now the impact of varying ϵ when M is fairly small, starting with the case of $M = 2$ shown in the upper plot in Figure 6.6. First of all, we note the good agreement of the $\epsilon \rightarrow \infty$ limit of our results with those from Ref. [188], which are shown by the blue circles. The next feature to note is that, just as in Figure 6.5, $n_k^c \propto k^3$ in the infrared for finite ϵ . Again, at some value of k there is an elbow after which the spectrum grows (for $\epsilon \gtrsim 0.4$, as a power law) more slowly than k^3 , and there is again a steep drop in n_k^c for large k . As ϵ decreases, the spectra become more peaked, and decrease in magnitude.

Finally consider the large- M cases: $M = 4$ in the lower plots of Figure 6.6, and large M at fixed $\epsilon = 0.6$ in the lower plot of Figure 6.5. For these values of M , the n_k^c spectra scale as k^3 (i.e., constant $|\beta_k|^2$) in the deep infrared, show oscillations (provided that ϵ is not too small) when transitioning from this behaviour to a scaling steeper than k^3 in an intermediate region of k (corresponding to a bump in $|\beta_k|^2$), then peak at $k \sim 10^{-1}$, and finally decrease rapidly at larger k . Particularly in the vicinity of the peak in the n_k^c spectra in the lower-left plot in Figure 6.6 (see the inset plot), there is only a weak dependence on ϵ .

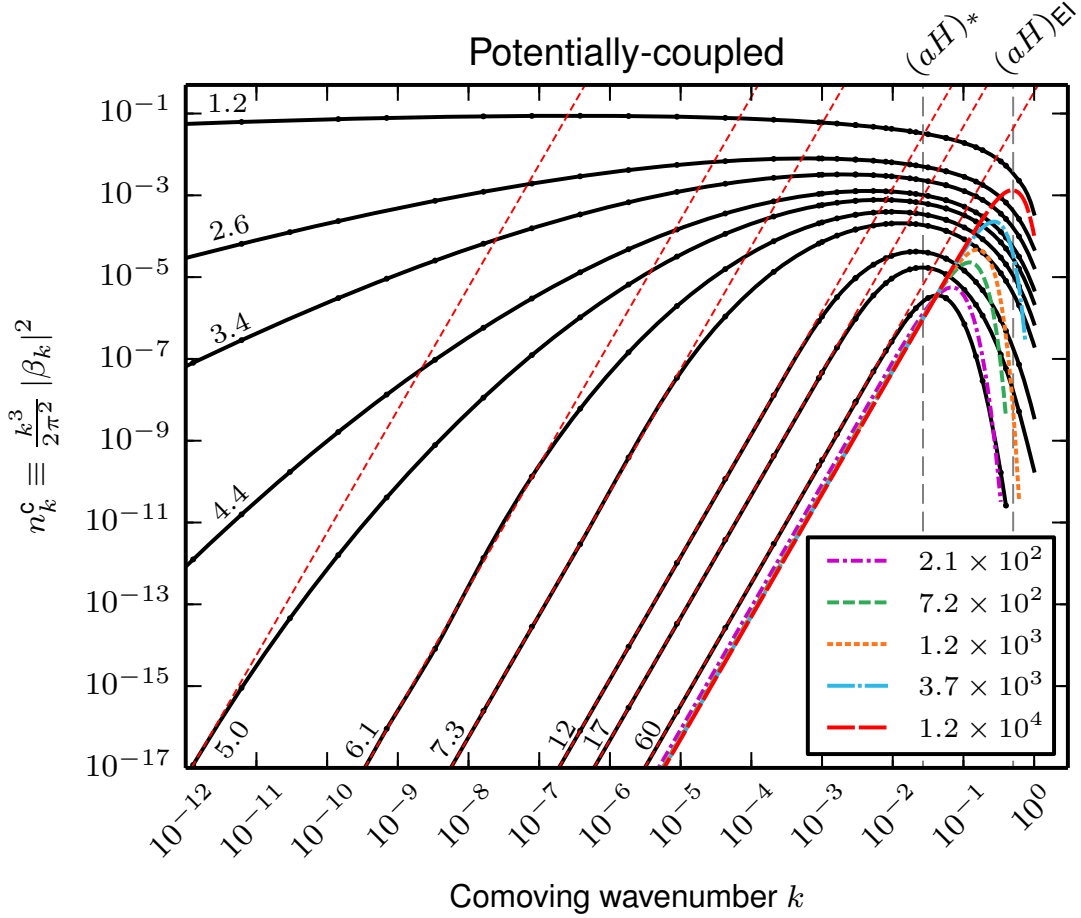


Figure 6.4: Comoving particle-density spectra n_k^c for the potentially coupled irrupton as a function of comoving k (in units of m) for various parameter choices $M_g \equiv gM_{\text{Pl}} = 1.22 \times 10^6 g$ (as annotated; in units of m). The choice of M_g is indicated either by the black numbers annotating the lines, or by the numbers in the legend labelling the differently styled (and coloured) lines. These latter cases are plotted differently to aid the reader visually, and also because these large- M_g spectra show qualitatively different behaviour, which is discussed in the text. The thin (red) dashed lines illustrate an n_k^c scaling proportional to k^3 (constant $|\beta_k|^2$). The vertical dashed lines denote $k = a_* H_*$ and $k = a_{\text{EI}} H_{\text{EI}}$, where $*$ denotes the values at the instant where $\phi = \phi_*$. See Table 6.1 for a summary of the numerical methods that were applied to obtain these results.

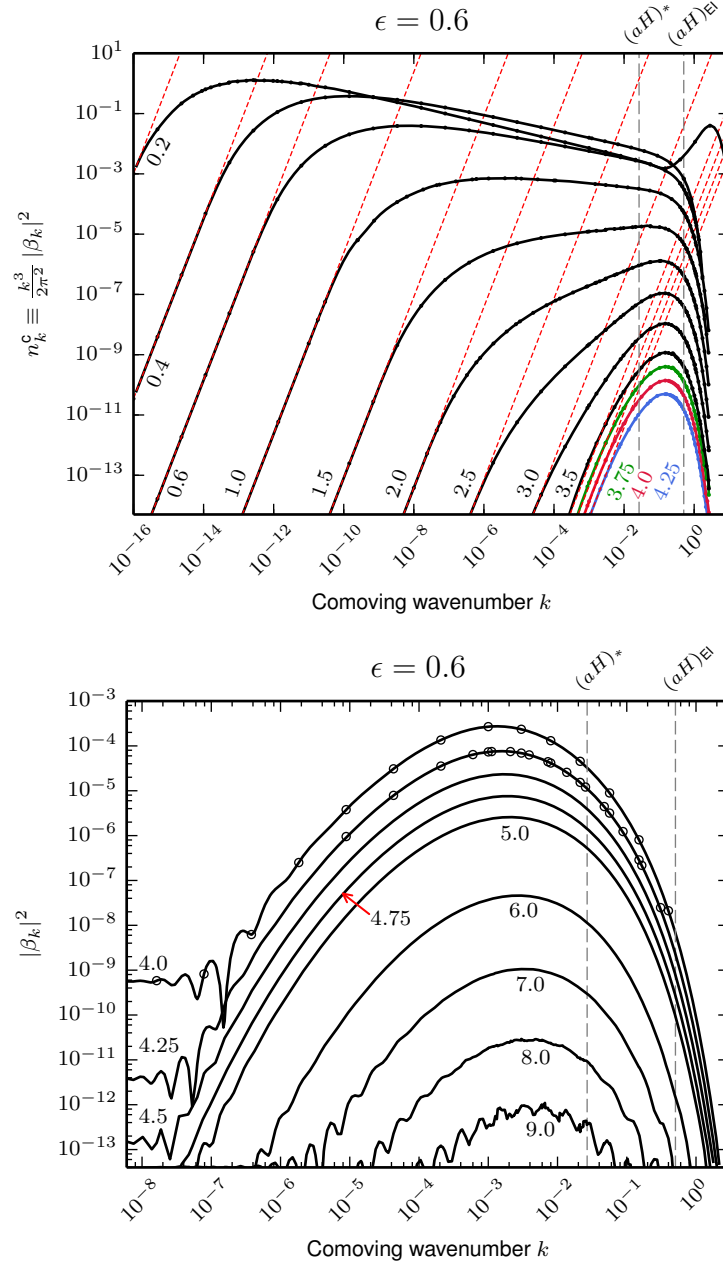


Figure 6.5: Comoving particle-density spectra n_k^c (upper plot) and the late-time Bogoliubov coefficient (i.e., mode- $\mathbf{k}$ occupancy number) $|\beta_k|^2$ (lower plot) for the kinetically coupled irrupton as a function of comoving k (in units of m) for various heavy-particle masses M (as annotated; also in units of m), with fixed $\epsilon = 0.6$. We choose to present $|\beta_k|^2$ rather than n_k^c in the lower plot for greater clarity in these very-large- M cases. In the upper plot, the thin (red) dashed lines illustrate a scaling proportional to k^3 (constant $|\beta_k|^2$). The vertical dashed lines denote $k = a_* H_*$ and $k = a_{\text{EI}} H_{\text{EI}}$, where $*$ denotes the values at the instant where $\phi = \phi_*$. See Table 6.1 for a summary of the numerical methods that were applied to obtain these results, and the meaning of the open circles and solid lines in the lower plot.

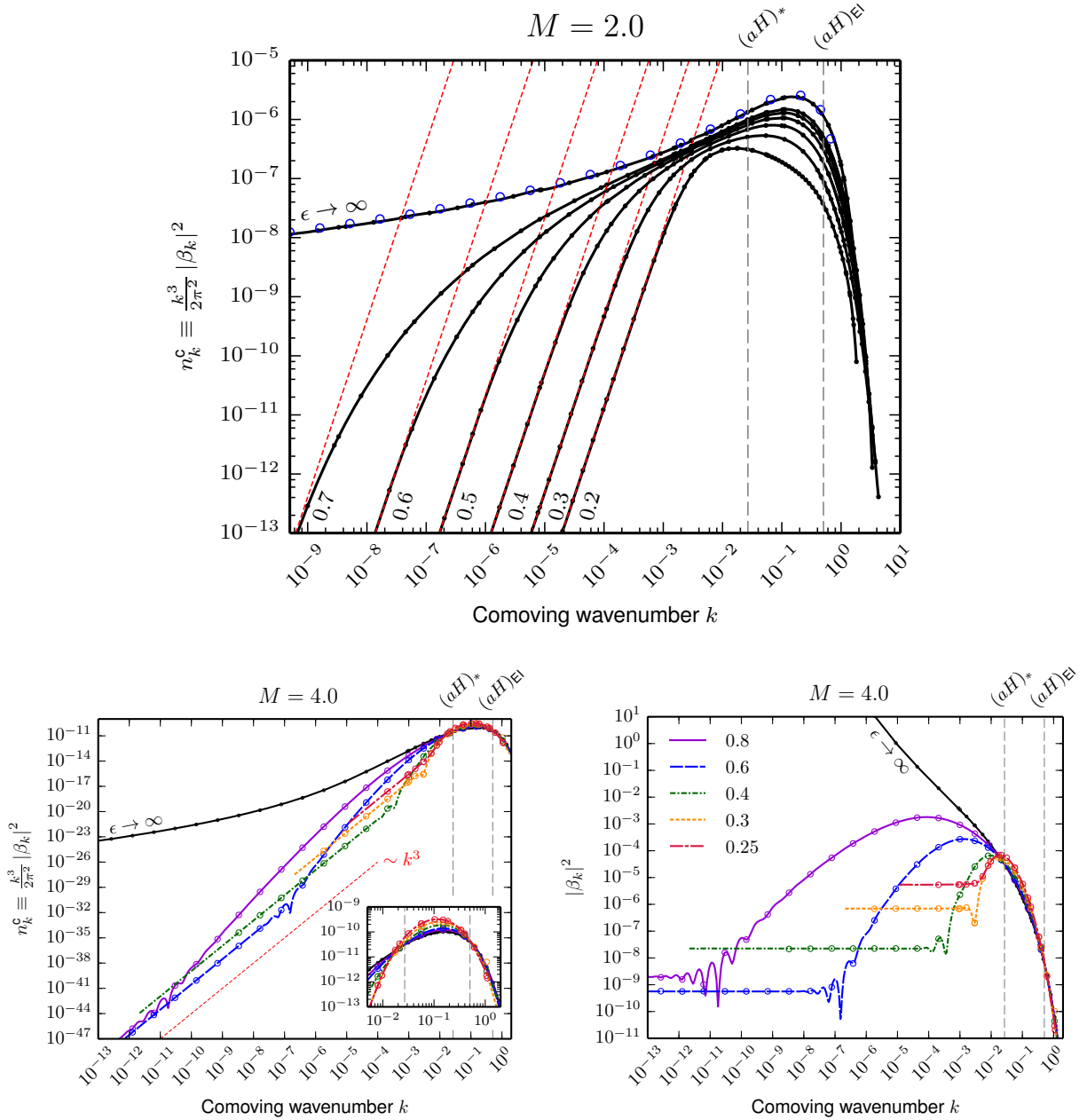


Figure 6.6: Comoving particle density spectra n_k^C as a function of comoving k (in units of m) for the kinetically coupled irrupton, for various choices of ϵ with fixed $M = 2$ (upper plot) and $M = 4$ (lower-left plot). The thin (red) dashed lines illustrate a scaling proportional to k^3 (constant $|\beta_k|^2$). The (blue) circles on the line for $\epsilon \rightarrow \infty$ in the upper plot are sampled from a spectrum in Ref. [188]. The inset in the lower-left plot shows detail near the peak in the spectra. Also shown for the case of $M = 4$ are the values of $|\beta_k|^2$ (lower-right plot). The vertical dashed lines denote $k = a_* H_*$ and $k = a_{\text{EI}} H_{\text{EI}}$, where $*$ denotes the values at the instant where $\phi = \phi_*$. See Table 6.1 for a summary of which numerical methods were applied to obtain these results, and the meaning of the open circles and solid lines in the lower plots.

6.5.2 Behaviour of Spectra: Discussion and Explanations

Now that we have described the spectra, we turn to explanations for their behaviour. The presence or absence of the tachyonic phase, along with its duration, is crucial for understanding the spectra. For modes which can run tachyonic, the behaviour of ω_k^2 clearly dictates the duration of the tachyonic phase and hence the amount of particle production that can occur, as the exponential increase in the mode functions during this phase is the dominant effect. For modes which never run tachyonic, we will show that a good understanding of the behaviour of the adiabaticity parameters leads to a good understanding of the characteristics of the spectrum.

We begin with a discussion of the IR behaviour of the spectra. In the case where M is constant (i.e., the $\epsilon \rightarrow \infty$ limit for the kinetically coupled irrupton) and not much larger than H (Figure 6.3), a mode of comoving momentum k enters the period of tachyonicity almost immediately after crossing outside the comoving Hubble radius, $R_H^c(t) = [a(t)H(t)]^{-1}$, since at this time, the sum $M^2 + k^2/a^2$ becomes comparable to the $-a''/a^3$ term in the expression for ω_k^2/a^2 at Eq. (6.6). (Note that k^2/a^2 is a rapidly falling function of t since a is growing exponentially; at the equality point, all three terms are of roughly the same size in our parameter region of interest. After equality, k/a rapidly becomes completely negligible for all subsequent evolution. See Figures 6.7 and 6.8.) This means that the Hubble-crossing time is a reliable indicator of the onset of the tachyonic phase, and that the exit time from the tachyonic phase is independent of the value of k ; modes of smaller k thus spend much longer in such a tachyonic phase than modes of larger k , which implies that $|\beta_k|^2$ grows without bound at small k , explaining the absence of a $n_k^c \sim k^3$ scaling regime in the infrared region for the constant- M case.

However, in the case of the potentially or kinetically coupled irruptons with a running M_{eff} , the situation is different. We discuss first the scaling with k for the kinetically coupled case. In Figure 6.7 we plot ω_k^2/a^2 as given in Eq. (6.30), along with the magnitude of the three terms contributing to it: k^2/a^2 , $|a''/a^3|$, and M_{eff}^2 . The heavy dashed curve is where

the given momentum mode is tachyonic, which requires $k^2/a^2 + M_{\text{eff}}^2$ to be less than the magnitude of a''/a^3 (recall that during inflation, $a'' > 0$). In the far IR (illustrated by $k = 10^{-14}$) k is sufficiently small that k^2/a^2 drops below M_{eff}^2 early in the evolution, and so the onset of tachyonicity is determined by when M_{eff}^2 drops below $|a''/a^3|$, implying that the onset of tachyonicity no longer closely tracks Hubble-crossing (see also the left plot of Figure 6.2). As k increases, eventually it—and not M_{eff} —will determine the onset of the tachyonic phase. Let us call the crossover point k_* ; the value of k_* will be the value of k for which $\omega_k^2/a^2 \approx k_*^2/a^2 \approx M_{\text{eff}}^2 \approx |a''/a^3|$.²⁰ From Figure 6.7 we see that occurs at $k = 2.5 \times 10^{-7}$ for $M = 2, \epsilon = 0.6$, which agrees well with the cross-over point (i.e., the ‘elbow’) in the spectrum shown in the Figure 6.5. For larger $k > k_*$ (illustrated by $k = 10^{-2}$ in Figure 6.7), the duration of the tachyonic phase is shorter.

To understand the resultant scaling with k , we make the crude approximation (reasonable for $\epsilon \gtrsim 0.4$) that ω_k^2/a^2 is approximately constant during the tachyonic phase: $\omega_k^2/a^2 \approx -\Omega^2$ where $\Omega^2 > 0$ is a k -independent constant (see Figure 6.7). As ω_k^2 is negative (i.e., $i\omega_k \in \mathbb{R}$), the expression for $|\beta_k|^2$ in Eq. (6.32) is inapplicable during the tachyonic phase itself; but the expression can be used immediately before and after the tachyonic phase when $\omega_k^2 > 0$. Since the value of ω_k^2/a^2 passes through zero, it will be equal to the same small positive value at times both immediately before, and immediately after, the tachyonic phase (see Figure 6.7). Consider then the ratio of the values of $|\beta_k|^2$ at those times, which we denote ‘before’ and ‘after’:

$$\frac{|\beta_k|_{\text{after}}^2}{|\beta_k|_{\text{before}}^2} \sim \frac{(\omega_k)_{\text{after}}}{(\omega_k)_{\text{before}}} \frac{|\mu_k|_{\text{after}}^2}{|\mu_k|_{\text{before}}^2} = \frac{a_{\text{after}}}{a_{\text{before}}} \frac{|\mu_k|_{\text{after}}^2}{|\mu_k|_{\text{before}}^2}, \quad (6.68)$$

where we have used that $|\dot{\mu}_k|^2 = |a^{-1}\mu'_k|^2 \propto |\mu_k|^2$ (which will be obvious from the form of the solution shown below) and have neglected the constant term in Eq. (6.32). We must now

20. The value of k_* is exponentially sensitive to the value of ϵ since the flatness of M_{eff}^2 , which is clearly (see Figure 6.7) the most important factor for deciding where M_{eff}^2 and k^2/a^2 become of roughly the same size, is directly set by this parameter.

estimate $|\mu_k|_{\text{after}}/|\mu_k|_{\text{before}}$. The mode function satisfies $\mu_k'' + \omega_k^2 \mu_k = 0$, which under our assumption of constant negative $\omega_k^2/a^2 = -\Omega^2$ becomes $\mu_k'' - a^2 \Omega^2 \mu_k = 0$. Since H does not change very much over the short duration of the tachyonic phase, we will use the de Sitter result $\eta = -1/aH$, so the mode equation becomes $\eta^2 \mu_k'' - (\Omega^2/H^2) \mu_k = 0$, whose growing mode solution is $\mu_k = (-\eta)^{\frac{1}{2}(1-\sqrt{1+4\Omega^2/H^2})} = (aH)^{\frac{1}{2}(\sqrt{1+4\Omega^2/H^2}-1)}$. We thus estimate that

$$\frac{|\beta_k|_{\text{after}}^2}{|\beta_k|_{\text{before}}^2} \sim \left(\frac{a_{\text{after}}}{a_{\text{before}}} \right)^\zeta \quad \text{where} \quad \zeta = \sqrt{1 + 4\Omega^2/H^2} > 0. \quad (6.69)$$

Now, in the deep IR ($k < k_*$), we have already noted that the values of a when entering and leaving the tachyonic phase are independent of k , so we expect $|\beta_k|_{\text{after}}^2/|\beta_k|_{\text{before}}^2$ to be independent of k . Since $n_k^c \sim k^3 |\beta_k|^2$, n_k^c will be proportional to k^3 .

But for $k > k_*$, the onset of the tachyonic phase is determined by $k^2/a^2 = |a''/a^3| \sim 2H^2$. So now $a_{\text{before}} \sim k$ whereas a_{after} is still k -independent, so that $|\beta_k|_{\text{after}}^2/|\beta_k|_{\text{before}}^2 \sim k^{-\zeta}$ which implies $n_k^c \sim k^{3-\zeta}$. This correctly captures the observed behaviour of power-law scaling of n_k^c with a power less than 3 above the elbow for the cases $M \lesssim 2.5$ (i.e., those with a tachyonic phase) in Figure 6.5.²¹ Note however that this conclusion is predicated on a period of approximate constancy of ω_k^2/a^2 , which is not a very good approximation when ϵ is small; therefore, we would not expect a power-law intermediate regime for n_k^c at small ϵ , but rather a spectrum with a more constantly evolving slope, as is becoming evident in the $\epsilon = 0.2, 0.3$ spectra at $M = 2$ in the upper plot of Figure 6.6.

Eventually, as k increases further, the tachyonic phase disappears altogether and the

21. A quantitative estimate of the scaling power $(3 - \zeta)$ requires an estimate for Ω^2/H^2 . A well-motivated approximation is to take it equal to the largest value of $|(\omega_k^2/a^2)/H^2|$ attained during the tachyonic phase. There is no simple closed-form expression for this value, but it can be easily (and accurately) estimated assuming that a, ν and $\dot{\nu}$ take their slow-roll values. With such an estimate, we find scaling powers offset systematically high by about 0.15 compared to the values extracted from linear fits to the power-law section of the spectra above the elbow in Figure 6.5 for all values of M from 0.2 to 2.5 ($\epsilon = 0.6$). The offset notwithstanding, we capture the M -dependence very well. There is also some weak ϵ -dependence in the power law above the elbow (see Figure 6.6, upper plot) which arises from the curvature of ω_k^2/a^2 during the tachyonic phase; our crude approximate approach here is manifestly inadequate to capture this.

evolution becomes more adiabatic, suppressing particle production even further.

The deep-IR scaling, $n_k^c \sim k^3$, for the potentially coupled irruption arises for exactly the same reason as for the kinetically coupled irruption, provided M_g is not very large: the onset of tachyonicity is governed by the k -independent condition $M_{\text{eff}}^2 \approx a''/a^3$ for small enough k , provided that M_g is sufficiently large. Since $k = 10^{-12}$ is the smallest comoving momentum value shown in Figure 6.4, we would only expect such a regime to be manifest in the results presented if $M_g \gtrsim 4.6$ (obtained from solving $M_{\text{eff}}^2 \approx k^2/a^2 \approx a''/a^3$ for M_g at $k = 10^{-12}$). For smaller M_g , one would need to probe smaller k to enter the $n_k^c \sim k^3$ regime; however, once $M_g \lesssim 3$, it is always the case that $M_{\text{eff}}^2 < a''/a^3$ for all $\nu > 0.8$ (even if ν is allowed to run much larger than 3; i.e., more e -folds of inflation are allowed) and so M_{eff}^2 cannot come to dominate in setting the onset of tachyonicity for any value of k , and we would thus not necessarily expect an $n_k^c \sim k^3$ scaling regime to exist in such cases (i.e., we do not expect an irruption of limited duration; the production duration and characteristics are more similar to the constant- M case). The intermediate scaling where n_k^c increases more slowly than k^3 , if present, once again occurs once the onset of tachyonicity becomes governed by when $k^2/a^2 \approx a''/a^3$. Although in the kinetically coupled case this regime gave rise to a simple intermediate power-law scaling, such a regime does not manifest itself for the potentially coupled case.

On the other hand, at very large M_g , the duration of any possible tachyonic phase becomes too small to result in an exponentially large increase in the size of the mode function.²² Instead, it begins to look more like an impulsive ‘kick’ to the mode function localized very sharply around $t = t_*$, as ω_k^2/a^2 very rapidly falls from a very large value to some small

22. In the IR, we expect this regime to be entered roughly when the duration of the tachyonic phase is short enough that the mode function cannot increase in size by much more than an e -fold during this phase. We assume for the sake of this argument that $\omega_k^2/a^2 \approx -a''/a^3 \approx -\frac{8\pi}{3}\nu_*^2 \approx \text{const.}$ during the tachyonic phase, and take the duration of the tachyonic phase, Δt , to be limited both before and after $t = t_*$ by where $M_{\text{eff}}^2 = a''/a^3$, giving $M_g^2 \dot{\nu}_*^2 (\Delta t/2)^2 \approx \frac{8\pi}{3}\nu_*^2$ which implies $\Delta t = 4\sqrt{2\pi/3} |\nu_*/\dot{\nu}_*| M_g^{-1}$. Requiring no more than n e -folds of increase for μ_k during the tachyonic phase demands that we set $|\omega_k/a|\Delta t \leq n$, leading to $M_g \geq (16\pi/3) \nu_*^2/|\dot{\nu}_*|n \approx 67n$. Taking $n \approx 1$ –2 gives good qualitative agreement with the value of M_g for which there is a quantitative behaviour change in the IR in Figure 6.4.

negative value before rapidly increasing again. This results in $|\beta_k|^2$ jumping from essentially zero to its late-time value almost instantaneously; we develop this argument further into a quantitative analytical prediction for the shape of the spectrum in Appendix E.2. However, since this process is still k -independent when k is sufficiently small so as to not significantly modify ω_k^2/a^2 at $t = t_*$, it also results in a $n_k^c \sim k^3$ scaling in the IR. This is quantitatively confirmed by Eq. (E.28) which shows that, in the large- M_g limit, $n_k^c \rightarrow k^3/2\pi^2$ ($|\beta_k|^2 \rightarrow 1$) for small k .

Even if no tachyonic phase or impulsive kick is present owing to the relative sizes of the contributions to ω_k^2/a^2 , the $n_k^c \sim k^3$ infrared scaling can still obtain for the kinetically coupled irruption; for instance, the cases with $M > 3$ in Figure 6.5, or the $M = 4$ cases in Figure 6.6. Although the explanation of this particular behaviour does not require the full machinery we are about to develop, we will nevertheless have recourse to the same ideas to explain other features in the non-tachyonic cases, so we pause to carefully develop the arguments here. Firstly, we note that can obtain a good qualitative understanding of the behaviour of the spectra for non-tachyonic cases by examining the lowest-order solution $\beta_k^{(1)}$ in the iterated-solution method of Section 6.4.2, particularly in the form as given in Eq. (6.58): we see that the size of the (square root of) the adiabaticity parameter $|\omega'_k/\omega_k^2|^2$ defines the envelope bounding the rapidly oscillating phase factor $e^{2i\Phi}$. Provided that this envelope varies only slowly as the phase advances by π , neighbouring excursions in the positive and negative directions, of both the real and imaginary parts of the integrand, cancel nearly completely when integrated over. On the other hand, if the envelope varies rapidly as the phase advances by π , neighbouring excursions cancel incompletely. Although in either case it is possible to obtain transient values of $|\beta_k^{(1)}|^2$ which are large, a non-zero late-time asymptotic value of $|\beta_k^{(1)}|^2$ occurs as a result of the accumulated incomplete cancellations between neighbouring excursions over the full evolution out to $\Phi \rightarrow \infty$ (see Figure 6.9). To illustrate the point explicitly, consider a toy model in which we assume that ω'_k/ω_k^2 is an exact Gaussian with standard deviation $\sigma = \pi n$; we then find that the late-time

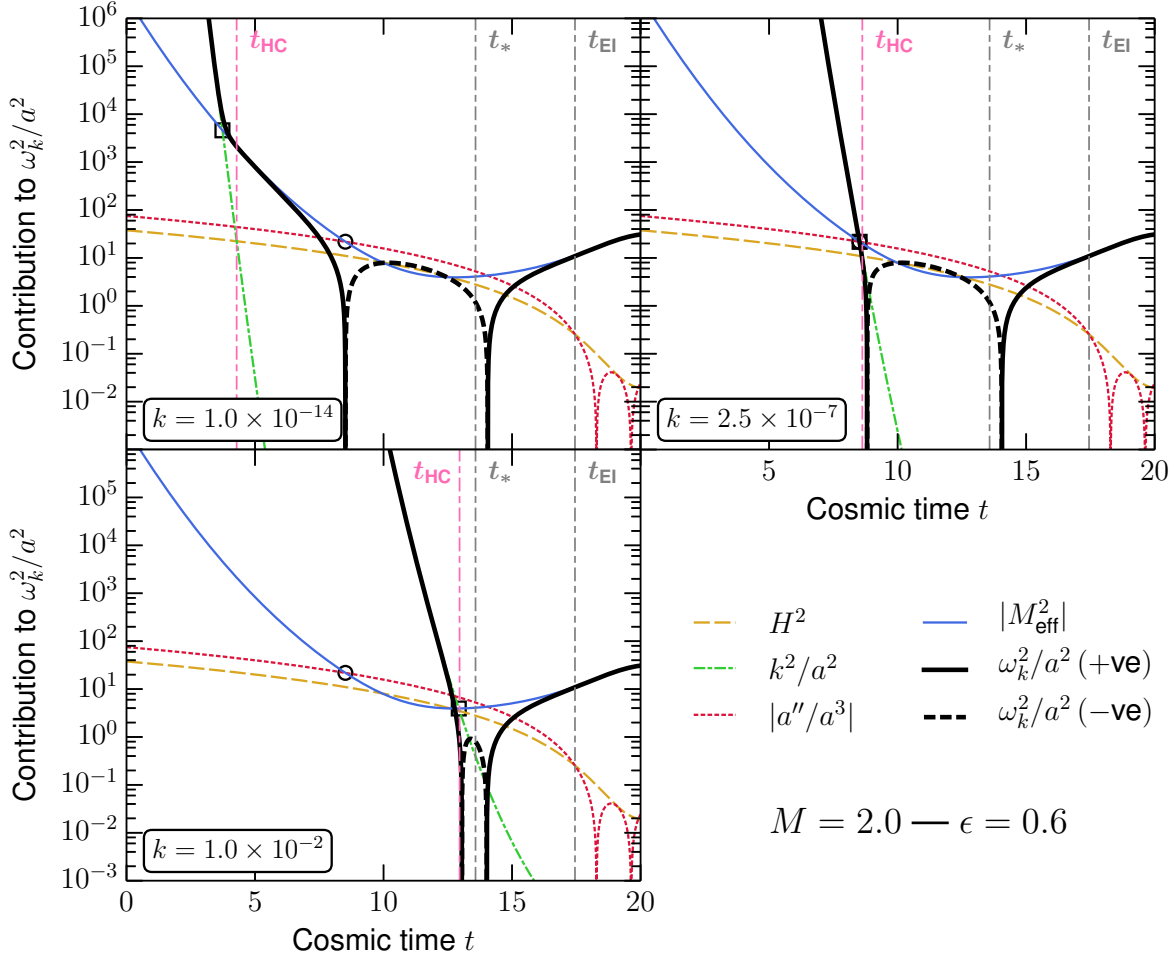


Figure 6.7: The oscillation frequency ω_k^2/a^2 and its three contributions for the kinetically coupled irruption [see Eq. (6.30)], plotted as a function of cosmic time t . The open circle in each plot indicates the point where two of the contributions to ω_k^2/a^2 , namely M_{eff}^2 and $|a''/a^3|$, are approximately equal. The point where k^2/a^2 and M_{eff}^2 first intersect is denoted by the open square. The value of k at the ‘elbow’ break-point, k_* , is the value of k for which $k_*^2/a^2 \approx |a''/a^3| \approx M_{\text{eff}}^2$; i.e., the open circle and the square coincide. From the upper right plot, we see that this occurs for $k = k_* = 2.5 \times 10^{-7}$. For $k \ll k_*$ (illustrated by the case $k = 10^{-14}$), the onset of the tachyonic phase is determined by M_{eff}^2 and is independent of k . Since the duration of the tachyonic phase is independent of k , $|\beta_k|^2$ should be independent of k , and $n_k^c \propto k^3$. For $k \gg k_*$ (illustrated by the case $k = 10^{-2}$), the onset of the tachyonic phase is determined by k^2/a^2 , and the duration of the tachyonic phase is reduced, which implies that $|\beta_k|^2$ will decrease with k and that n_k^c will grow more slowly than k^3 . It is clear that for even larger k , there may be no tachyonic phase at all, which exponentially suppresses the particle number produced. Note that larger values of ϵ flatten M_{eff}^2 , giving a longer maximum duration of the tachyonic phase, while larger values of the M move the minimum of M_{eff}^2 upward, which suppresses particle production by reducing the duration of the tachyonic phase (which argument holds until such a phase ceases to exist; see Figure 6.8 for further consideration of this case).

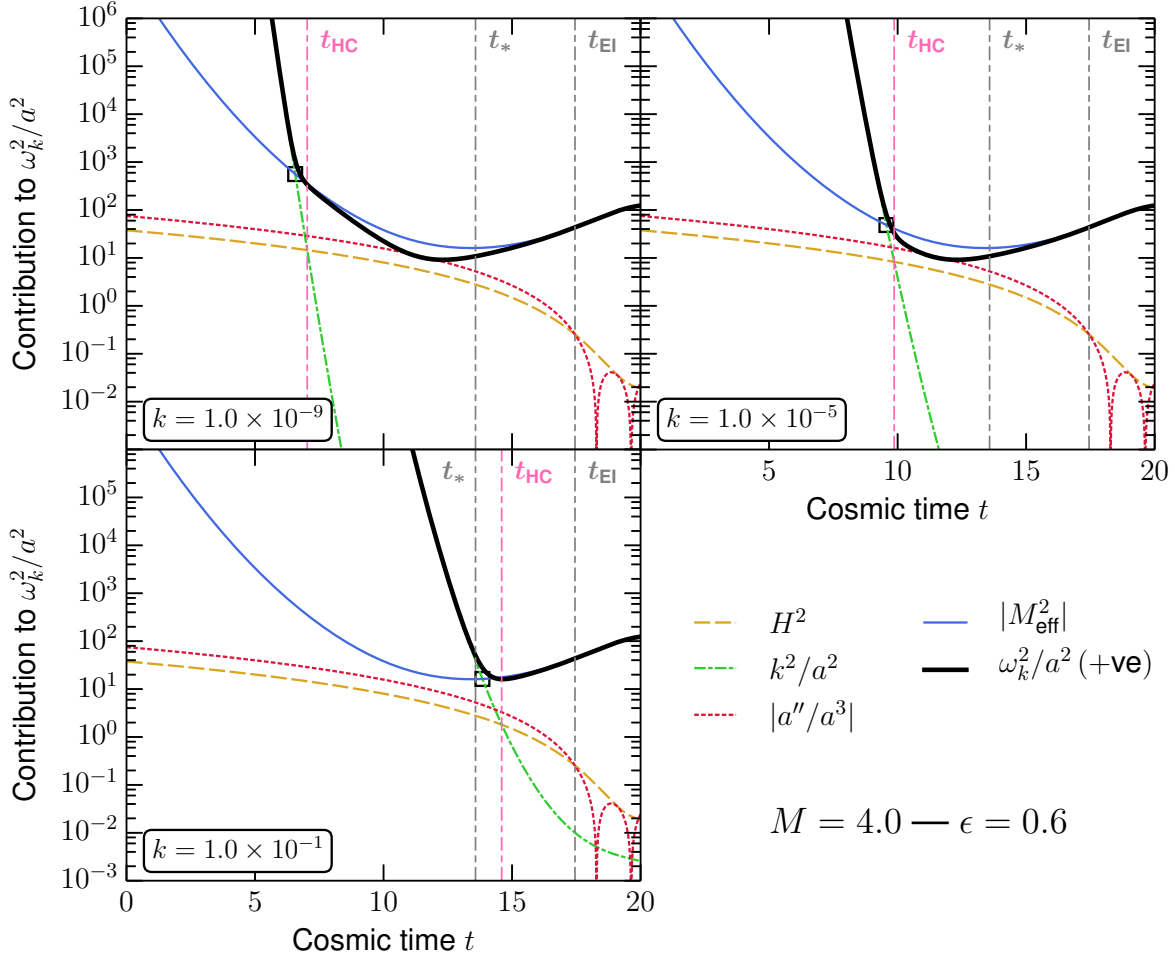


Figure 6.8: As for Figure 6.7, except this series of plots of the various contributions to ω_k^2/a^2 for varying k is shown for a case where no tachyonic phase is present (M is larger here than in Figure 6.7). Nevertheless, the minimum value of ω_k^2/a^2 still becomes k -independent at small k , which leads to an $n_k^c \sim k^3$ infrared behaviour in the particle spectrum. As k becomes very large (lower plot), the minimum value of ω_k^2/a^2 is clearly shifted upward, which leads to the large- k suppression in the spectra.

asymptotic value of $|\beta_k^{(1)}|^2 \propto e^{-4\pi^2 n^2}$. Clearly, the wider the Gaussian (i.e., the slower the envelope varies), the smaller the late-time asymptotic value of $|\beta_k^{(1)}|^2$. Although these arguments can be formalized, our goal here is simply to build qualitative intuition for the behaviour of the spectrum.

In order to apply this intuition to understand the $n_k^c \sim k^3$ IR scaling of the spectra, we note firstly that the adiabaticity parameters are larger and evolving rapidly with increasing Φ when ω_k^2/a^2 is near its minimum; for sufficiently small k , they are generally smaller and (provided that $t \gtrsim t_{\text{HC}}$) evolving more slowly with increasing Φ when ω_k^2/a^2 is large; for $t \lesssim t_{\text{HC}}$ the adiabaticity parameters evolve fairly rapidly but are damped to very small values exponentially quickly as the phase Φ is decreased, because ω_k^2/a^2 generally increases much more rapidly with decreasing phase Φ when dominated by k^2/a^2 than by M_{eff}^2 (see Figure 6.8). Therefore, for small values of k , and viewed as a function of increasing phase Φ (see Figure 6.9), the envelope modulating the rapid phase oscillation starts exponentially small, fairly rapidly rises to some small value around the time of Hubble-radius crossing, evolves fairly slowly for some duration of increasing phase Φ , grows in size and evolves more rapidly as ω_k^2/a^2 goes through its minimum, then decreases in size again as ω_k^2/a^2 increases in size, and finally executes small amplitude oscillations with the same period at which ω_k^2/a^2 oscillates around a constant in the MD era. Crucially however, most of that evolution is completely k -independent, and if k is further decreased, the only modification to the envelope is to add in an additional duration of fairly slow evolution of the envelope at early time, by shifting to earlier times the point around t_{HC} where the envelope rises from its initial, very small value (compare the upper plots of Figure 6.9). As such, for sufficiently small k , we do not expect that significant additional incomplete cancellation between neighbouring excursions in the additional oscillations can occur as k is further decreased. As a result we expect qualitatively that $|\beta_k|^2$ should become k -independent at sufficient small k , leading to $n_k^c \sim k^3$.

Furthermore, by virtue of the fact that neighbouring oscillations near the maximum of

the envelope become more rapidly incommensurate in size if the envelope decreases more rapidly from its maximum with changing phase, such as occurs for cases of smaller ϵ , we expect that the plateau value of $|\beta_k|^2$ should be larger, as observed (at least for $\epsilon \leq 0.6$) in Figure 6.6. We can mock up the plateau behaviour in the same toy model discussed above by supposing that the Gaussian envelope is simply cut off sharply to zero at a point ζ standard deviations before its maximum, which leads to a late-time asymptotic value of $|\beta_k^{(1)}|^2 \propto e^{-4\pi^2 n^2} \left| 1 - \text{Erf}\left(-\sqrt{2}(\zeta - i\pi n)\right) \right|^2$, which for $\zeta > \pi n = \sigma$ is essentially independent of ζ , and is larger if n is smaller. Translating back to the language of our actual model we see that this captures all the salient features: for k sufficiently small that t_{HC} occurs a number of phase oscillations before ω_k^2/a^2 goes through its minimum, the spectrum would be t_{HC} - and thus k -independent, and this plateau would be at a higher value if ϵ were smaller.

In almost all cases of either constant or running effective masses, as the effective mass term increases in size,²³ fewer particles are produced both at fixed k and overall. Qualitatively, the reason is clear: heavier modes experience changes in the background spacetime ‘more adiabatically’. We mean one of two things here: either (a) the duration of any possible tachyonic phase is shortened as the effective mass grows (see, e.g., Figure 6.7), or (b) in the more extreme case where the effective mass is so large that no tachyonic phase at all is present (i.e., $M_{\text{eff}}^2 > |a''/a^3|$ at all times during inflation; see, e.g., Figure 6.8), the minimum value of ω_k^2/a^2 reached during inflation increases in size as the effective mass increases, which generally correlates with a decrease in the maximum size of the adiabaticity parameters. This not only collapses the envelope modulating the phase factor $e^{2i\Phi}$ in Eq. (6.58) (see the lower-right plot of Figure 6.9) but also results in the envelope evolving more slowly with phase, since larger ω_k/a results in a smaller Δt to get the same $\Delta\Phi \approx (\omega_k/a)\Delta t$. Therefore, by arguments similar to those just advanced, production is suppressed in the latter case.

The obvious exception in our results to this general rule of decreasing n_k^c with increasing

23. To be concrete: for the minimally coupled (constant mass) case of Section 6.2.1 (in the chaotic-inflation background), we mean increasing M ; for the potentially coupled of Section 6.2.2, we mean increasing M_g ; and for the kinetically coupled case of Section 6.2.3 we mean increasing M and/or decreasing ϵ .

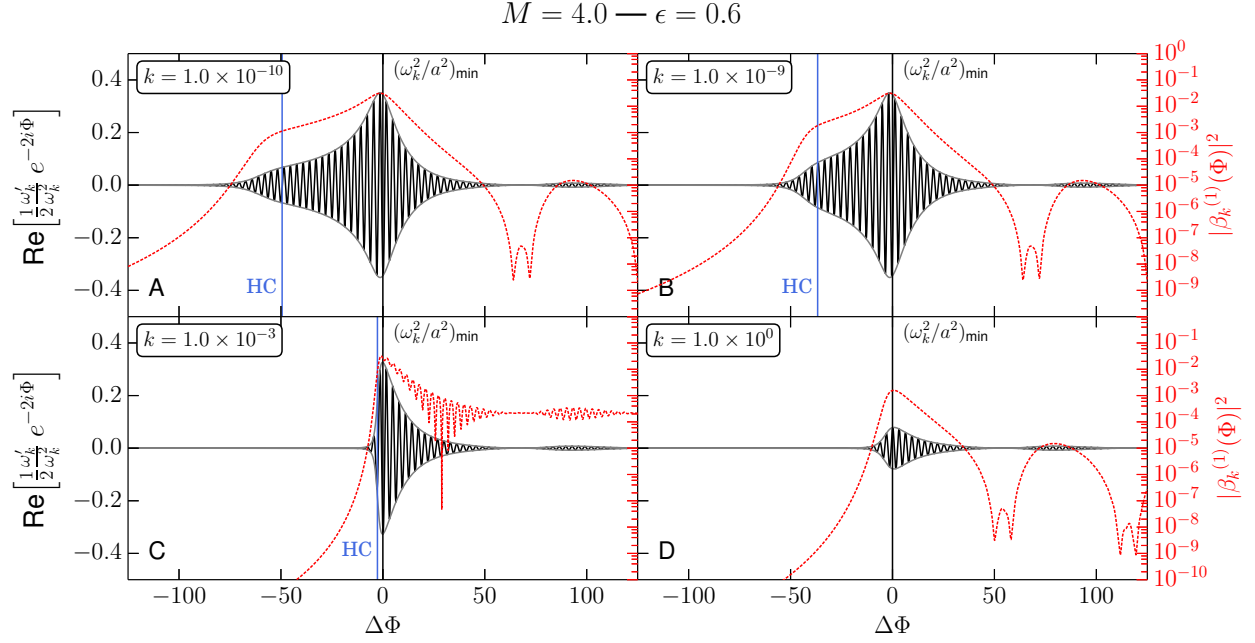


Figure 6.9: The envelope $\pm(1/2) \left| \omega'_k / \omega_k^2 \right|$ (grey) bounding the real part of the rapidly oscillating integrand $(1/2)(\omega'_k / \omega_k^2) e^{-2i\Phi}$ in Eq. (6.58) (black) for the first iterate $\beta_k^{(1)}$, for a variety of values of k for the case $M = 4$ and $\epsilon = 0.6$ for the kinetically coupled irruption, plotted as a function of the phase deviation $\Delta\Phi$ from when ω_k^2/a^2 is minimized. The imaginary part of the integrand shows similar behaviour; we omit it for clarity. We also show $|\beta_k^{(1)}(\Phi)|^2$ per Eq. (6.58) (right-scale on each axes; red dotted line) for illustrative purposes to indicate the impact of the shape of the envelope on the evolution of $|\beta_k|^2$. ‘HC’ denotes Hubble-radius crossing; the mode with $k = 1$ is always sub-Hubble-radius sized. The relevant comparisons we intend to reader to make from this series of plots are between plots A and B, and between plot C and plots A or D; in particular, we caution the reader that the approximate order-of-magnitude equality of the value of $|\beta_k|^2$ at large positive $\Delta\Phi$ in plot D (large k), and the values of $|\beta_k|^2$ in plots A and B (small k), are coincidental consequences of the values of k we have chosen to display (see lower plot of Figure 6.5), so no deep significance should be attached to that approximate equality.

mass parameters occurs in the large- M_g results for the potentially coupled case in Figure 6.4. For $k < k_{\text{th}} \approx 4 \times 10^{-2}$, as M_g increases, the spectrum goes to a limiting value of $|\beta_k|^2 = 1$ from above, while for larger (fixed) $k > k_{\text{th}}$, the spectrum goes to the same limit from below. The comoving momentum k_{peak} where the n_k^c spectrum peaks, shifts over to the UV roughly as $k_{\text{peak}} \propto (M_g)^{0.500(5)}$ (from fits to the largest- M_g numerical results we have). We already argued in footnote 22 that once $M_g \gtrsim 70$, the duration-of-tachyonicity argument for understanding the results breaks down and we enter the regime in which the analytical expressions we develop in Appendix E.2 apply. Indeed, examining Eq. (E.28) immediately explains many of the observed ‘anomalous’ features at large- M_g : at fixed k , as M_g gets larger the exponent in Eq. (E.28) goes to zero, and $|\beta_k|^2 \rightarrow 1$. Since the exponent is, for the parameter values we give in Appendix E.2, positive for $k \lesssim 4 \times 10^{-2}$, and negative for larger k , it is also clear that the limit should indeed be approached from above for $k < k_{\text{th}}$ and from below for larger k . Finally, once $k \gg H_*^2, \dot{H}_*$, the exponent scales proportional to $(-k^2/M_g)$, and so for equal exponential fall-offs from $|\beta_k|^2 = 1$, k has to increase as $\sqrt{M_g}$, consistent with the peak shift to the UV seen in the numerical results.

The remaining point we wish to clarify in connection with this discussion is how this specific analytical understanding of the scaling of the spectra at large M_g is consistent, in the regime where no tachyonic phase exists, with the more general picture outlined just above; i.e., that the amount of production in this regime is limited by the maximum size of the adiabaticity parameters. If we were discussing the kinetically coupled case, it is obvious that increasing M necessarily monotonically increases the minimum value of M_{eff} , and hence could only increase the minimum value of ω_k^2/a^2 , suppressing production. However, for the potentially coupled case, M_{eff} always goes to zero at $\nu = \nu_*$, and the resulting M_g dependence of the minimum value of ω_k^2/a^2 (and hence the maximum value of the adiabaticity parameters) is not immediately clear. In particular, if our general arguments are to hold, the behaviour of the adiabaticity parameters with increasing M_g must necessarily be non-monotonic, because we observe the spectrum to first decrease and then increase again at

large k as M_g is increased.

We begin by making the important observation that the threshold value k_{th} is additionally the boundary, in the limit of large M_g , between the cases where a (brief, impulsive) tachyonic phase exists ($k < k_{\text{th}}$), and when one does not exist ($k > k_{\text{th}}$): to see this, note that in the $M_g \rightarrow \infty$ limit, the minimization of ω_k^2/a^2 occurs at $\nu = \nu_*$, so in this limit k_{th} is simply estimated by requiring that $\omega_{k_{\text{th}}}^2/a^2|_{\nu=\nu_*} = 0$, which implies that $k_{\text{th}} \approx \sqrt{4\pi a_*^2/3 \times (2\nu_*^2 - 1/12\pi)} \approx 3.7 \times 10^{-2}$ where we have used $a''/a^3 = 4\pi/3 \times (2\nu^2 - \dot{\nu}^2)$, the slow-roll value $\dot{\nu} \approx -1/\sqrt{12\pi}$, and have taken $a_* \equiv a(\nu = \nu_*) = 1.6 \times 10^{-2}$ from our full numerical solutions.

Therefore, for $k > k_{\text{th}}$, no tachyonic phase exists for the large- M_g cases, and the amount of particle production should indeed be governed by the size of the adiabaticity parameters $|\omega'_k/\omega_k^2|^2$ and $|\omega''_k/\omega_k^3|$ (as well as phase cancellations, as we have previously argued); this reinforces that it is necessary to demonstrate non-monotonic behaviour of those parameters with increasing M_g for all $k > k_{\text{th}}$, if our arguments are to be consistent. In Figure 6.10 we plot the maximum value of the adiabaticity parameter $|\omega''_k/\omega_k^3|$ ($\max\{|\omega'_k/\omega_k^2|^2\}$ behaves similarly) as a function M_g at fixed k , as evaluated in our numerical work. It is clear that the requisite non-monotonic behaviour is indeed present.

It is instructive to understand the origin of this non-monotonic M_g -dependence of $\max\{|\omega''_k/\omega_k^3|\}$. We observe that it is usually true that the maximum value of $|\omega''_k/\omega_k^3|$ is reached very near to the point where ω_k^2/a^2 reaches a minimum. As we have already noted, for large M_g the latter function is minimized at $\nu \approx \nu_*$; imposing this condition kills all M_g -dependence in $|\omega''_k/\omega_k^3|$ except for a term which scales as M_g^2 in the numerator [i.e., the surviving term arising from twice differentiating M_{eff}^2 as given in Eq. (6.19)], which gives rise to the M_g^2 scaling.

On the other hand, once M_g gets small enough, the minimum of ω_k^2/a^2 (and hence the maximum of $|\omega''_k/\omega_k^3|$) is no longer reached at $\nu \approx \nu_*$. Assuming for the sake of this argument that k is sufficiently large that, even though M_g is small, the $-a''/a^3$ term in ω_k^2/a^2 always

remains negligible, the minimum is reached at a later time, when $\nu \approx \tilde{\nu}$ (note: $\tilde{\nu} < \nu_*$), where $\tilde{\nu}$ is defined to be the point where the decreasing function k^2/a^2 is equal to the increasing (on $\nu < \nu_*$) function M_{eff}^2 , yielding $\omega_k^2(\nu = \tilde{\nu}) \approx 2k^2$. This requires $\nu_* - \tilde{\nu} = k/(\tilde{a}M_g) > 0$; taking the slow-roll expression $\tilde{a} \equiv a(\nu = \tilde{\nu}) \approx a_* \exp[-2\pi(\tilde{\nu}^2 - \nu_*^2)]$ as a good approximation results in a transcendental equation for $\tilde{\nu}$, which we solve numerically. If in evaluating the adiabaticity parameters $|\omega_k''/\omega_k^3|$ and $|\omega_k'/\omega_k^2|^2$ we additionally take $\dot{\nu} \equiv -1/\sqrt{12\pi}$ and drop all $\ddot{\nu}$ terms (consistent with the slow-roll approximation for a), we find the adiabaticity parameters scale approximately as

$$\left\{ |\omega_k''/\omega_k^3|, |\omega_k'/\omega_k^2|^2 \right\} \propto \frac{1}{M_g^2} \frac{\tilde{\nu}^2}{(\nu_* - \tilde{\nu})^2} \left[1 + \frac{1}{2\pi\nu(\nu_* - \nu)} \right]. \quad (6.70)$$

Using the numerically obtained values of $\tilde{\nu}$, we find that the factor multiplying M_g^{-2} scales roughly proportional to M_g for large enough k , leading to an approximate scaling of $\left\{ |\omega_k''/\omega_k^3|, |\omega_k'/\omega_k^2|^2 \right\} \propto M_g^{-1}$ in the regime of large k and small M_g . This compares favourably with the large- k results in Figure 6.10. The behaviour for smaller k at small M_g would be obtained from a similar argument, in which one did not neglect the $-a''/a^3$ term, but still assumed that k and M_g were sufficiently large to prevent a tachyonic phase.²⁴

The question then naturally arises as to where the cross-over point between the two regimes occurs, as this should give a good estimate of when $\max\{|\omega_k''/\omega_k^3|\}$ goes through its turning point, which in turn indicates at what value of M_g one would expect the spectrum (at a fixed value of k) to stop decreasing, and instead enter the regime where it increases again back to the limiting value of unity as we discussed above. Since we expect the g^2 scaling to obtain whenever ω_k^2/a^2 is minimized around $\nu \approx \nu_*$, we can estimate that once

24. Indeed, the divergences evident in Figure 6.10 for small k and M_g are precisely due to these cases allowing for a tachyonic phase. For small M_g , since ω_k^2/a^2 is not minimized at $\nu = \nu_*$, our earlier argument that $k_{\text{th}} = 4 \times 10^{-2}$ is the threshold value beyond which no tachyonic phase is present, is not applicable. Instead, the threshold value k_{th} should be obtained from the full condition $\min_{\nu} \{\omega_{k_{\text{th}}}^2\} = 0$, and this typically requires a larger threshold value than $k_{\text{th}} = 4 \times 10^{-2}$ if ω_k^2/a^2 is minimized at $\nu < \nu_*$. The appearance of divergences in Figure 6.10 related to the existence of a tachyonic phase is thus not in contradiction with our earlier statements.

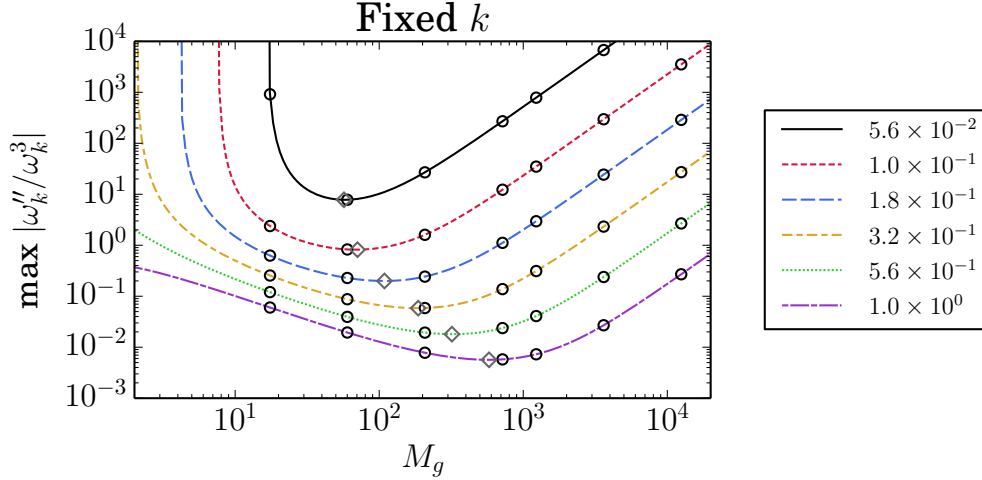


Figure 6.10: The maximum values attained during inflation of the adiabaticity parameter $|\omega''_k/\omega_k^3|$ (the behaviour of the other adiabaticity parameter $|\omega'_k/\omega_k^2|^2$ is similar) in the potentially coupled irrupton model at representative fixed k (see legend) as a function of M_g (in units of m). The black circles indicate the relevant values $M_g = \{17, 60, 2.1 \times 10^2, 7.2 \times 10^2, 1.2 \times 10^3, 3.7 \times 10^3, 1.2 \times 10^4\}$ (see Figure 6.4). The divergences evident at small M_g for some of the smaller- k curves indicate that these modes allow for a tachyonic phase. The minimum of each curve is indicated by a grey diamond.

$\nu_* - \tilde{\nu} \gtrsim 0.1\nu_*$, this scaling may begin to break down. Again throwing away the $-a''/a$ term in ω_k^2/a^2 , and solving for the value of $M_g^{\min}$ required to achieve such a deviation assuming $a \approx a_*$, we find that $M_g^{\min} \approx \sqrt{4\pi k^2/0.1a_*^2} \approx 7 \times 10^2 k$; this compares fairly well with the values of M_g for which $\max\{|\omega''_k/\omega_k^3|\}$ reaches its minimum in Figure 6.10. Of course, the fairly good numerical agreement here is sensitive to the exact value of a and the assumed small deviation from ν_* which is used in this argument; however, the fact that $M_g^{\min} \propto k$ is a fairly robust prediction, and is consistent with the scaling with k of the minima of the curves shown in Figure 6.10 for sufficiently large k .

We now turn attention to the far-ultraviolet (UV) behaviour of the spectra. To understand the rapid drop-off in the particle spectra once $k \gtrsim 10^{-1}$ (see Figures 6.3 to 6.6), one must consider three qualitatively distinct cases: (a) for small enough k there exists a broad region of tachyonic behaviour of the mode function (e.g., for $M \lesssim 3$ and $\epsilon = 0.6$ for the kinetically coupled irrupton [Figure 6.5], and the potentially coupled irrupton results for $M_g \lesssim 60$ [Figure 6.4]), (b) for small enough k there is a very narrow tachyonic region which

looks more like the impulsive kick we discussed above (e.g., for the $M_g \gtrsim 200$ results in Figure 6.4), and (c) there is never any tachyonic behaviour of the mode functions for any value of k (e.g., for $M \gtrsim 3$ for the kinetically coupled irrupton).

We begin with case (a) by noting again that a temporary but fairly broad tachyonic instability in the mode function, such as occurs in this case at smaller k , leads naturally to exponentially more particle production than in a case where no such instability exists. The important observation is that once $k^2 > \max\{a''/a\}$, it is impossible for the mode function to become tachyonic at any point during its evolution, regardless of the value of M_{eff}^2 . Therefore, for large enough k , we naturally expect the spectrum (or more exactly, $|\beta_k|^2$) to show a rapid fall-off compared to the values it obtains at smaller k . A simple-minded estimate for when this criterion is satisfied, obtained by assuming that the slow-roll regime is always valid, yields $k \gtrsim 0.4$, which is of the same order of magnitude as the point beyond which the n_k^c spectra are observed to drop off exponentially fast in Figure 6.5 and at small M_g in Figure 6.4.²⁵

Case (b) is handled by our analytical treatment in Appendix E.2: once the value of k is large enough so that the k^2/a^2 term in ω_k^2/a^2 is significant, the spectrum rolls off exponential quickly as $\exp\left[-\left(\pi/M_g|\dot{\nu}_*|a_*^2\right)k^2\right]$ from $|\beta_k|^2 \approx 1$. We have already noted above that the quantitative prediction arising from Eq. (E.28) for the location of the peak in the n_k^c spectra, which coincides with the drop-off of $|\beta_k|^2$, does indeed match well with the numerical results shown in Figure 6.4.

Case (c) requires a little more care, since the evolution can never be tachyonic for any value of k ; however, by increasing k beyond the point where Hubble-crossing is occurring around $\nu = \nu_*$, the minimum value of ω_k^2/a^2 achieved during inflation becomes a monotonically increasing function of k (see, e.g., Figure 6.8). We have already argued that the minimum value of ω_k^2/a^2 anti-correlates with the maximum value of the adiabaticity pa-

25. This estimate is about 30% too large compared to the same criterion evaluated in our full numerical solution. Also, adding a (positive) effective mass term only helps to make ω_k^2/a^2 more positive, so the estimate shown is really an upper bound which we do not expect to be saturated.

rameters, so as this minimum value increases the particle production is suppressed as the envelope bounding the phase factor $e^{2i\Phi}$ in Eq. (6.58) collapses. The simple-minded estimate here for when the spectrum starts to drop is thus obtained from setting $k = aH$ at $\nu = \nu_*$; assuming that slow-roll is valid, this gives $k \gtrsim 0.1$, which is again in good agreement with the observed behaviour in Figures 6.5 and 6.6.

The foregoing comments about the UV spectra do have a clear exception in the slower drop-off / small bump around $k \sim 5$ in the constant- M spectra for $M \lesssim 1$ (Figure 6.3) and the pronounced bump around $k \sim 5$ in the kinetically coupled irrupton spectrum for small M ($M = 0.2$) and $\epsilon = 0.6$ (Figure 6.5). This bump clearly has a different physical origin to the rest of the spectrum; an examination of the time-evolution of $|\beta_k|^2$ and the behaviour of ω_k^2/a^2 indicates that it arises from modes which become light during the first few coherent inflaton oscillations immediately after the end of inflation, leading to their excitation either in the usual matter-dominated Friedmann expansion phase, or in the transition out of the inflationary phase. Again, once k is sufficiently large, even this effect is suppressed.

We now discuss the some features the large- M kinetically coupled irrupton spectra (Figure 6.6) to which we drew attention above: the bump in $|\beta_k|^2$ at intermediate k (corresponding to $n_k^c \sim k^{3+x}$ for some $x > 0$) and the nearly universal behaviour of the spectra near the peak.

Consider first the bump in $|\beta_k|^2$ at intermediate k , which occurs provided that ϵ is not too small; see Figure 6.6. The reason for this feature can be traced back to the behaviour of (the square root of) the adiabatic parameter $|\omega'_k/\omega_k^2|^2$ per our earlier argument about the shape of the envelope of the rapid oscillations in the integrand in Eq. (6.58). For such intermediate- k cases, ω_k^2/a^2 is dominated by k^2/a^2 until the former is very near its minimum, and from our general observation of an anti-correlation in the sizes of ω_k^2/a^2 and the adiabaticity parameters it follows that this envelope is very small until very close to its maximum, making it highly asymmetric near that maximum (see the lower-left plot in Figure 6.9); this leads to a significantly larger accumulated incomplete cancellation between

neighbouring oscillations when viewed in the late-time regime. The appearance of oscillations in the spectra in the transition from the IR k^3 scaling to this ‘bump’ regime can also be qualitatively understood as the effect of first the positive, and then the negative, excursions just before the maximum being alternately larger as the envelope opens up on the low-phase / low- Φ side as k is decreased from, e.g., the situation pictured in the lower-left plot of Figure 6.9. Furthermore, the fact (which we noted above) that neighbouring excursions become more rapidly incommensurate in size as the envelope decreases more rapidly in size from its maximum with changing phase, such as happens in the cases of smaller ϵ , makes this argument more marginal in such cases, explaining why the oscillations in smaller- ϵ cases are less pronounced (see Figure 6.6). Again, our toy model captures the essential elements of this behaviour: $|\beta_k^{(1)}|^2 \propto e^{-4\pi^2 n^2} \left| 1 - \text{Erf} \left(-\sqrt{2} (\zeta - i\pi n) \right) \right|^2$ increases rapidly from its plateau value as soon as $\zeta < n\pi$ (read: as t_{HC} approaches the time at which ω_k^2/a^2 is minimized), and also demonstrates some dips and wiggles, more pronounced for larger n (read: for larger ϵ) provided it is not too large, in the transition from the plateau to this regime of growth.

The second feature of interest is the fairly universal behaviour (i.e., approximately independent of changing ϵ) for $k \gtrsim 10^{-2}$ for $M = 4$ in Figure 6.6. Since the n_k^c spectra are strongly peaked near $k \sim 10^{-1}$, it follows that to a fairly good approximation the total number of particles produced, n^p , thus also becomes approximately independent of ϵ . This is clearly an important qualitative feature of this result, and can be explained by appealing to arguments similar to those already advanced about the maximum size of, and behaviour near the maximum of, the adiabaticity parameters in this regime, via their role of defining the envelope of oscillations in Eq. (6.58). Briefly, since at large k , ω_k^2/a^2 is dominated by k^2/a^2 until quite late during inflation, ω_k^2/a^2 has no ϵ -dependence until very late during inflation. This implies that the evolution of the adiabaticity parameters is nearly identical for different values of ϵ until the k^2/a^2 term red-shifts away sufficiently to expose the ϵ -dependence in M_{eff} , and hence in the adiabaticity parameters. However, the resulting ϵ -dependence of the maximum value of the adiabaticity parameters, and their behaviour near that maximum, is

very mild in the $k \gtrsim 10^{-2}$ regime for all the values of ϵ we have examined, which implies that as a gross approximation, the amount of particle production should be approximately the same for each ϵ value, roughly as observed in Figure 6.6. Going beyond this gross approximation, we note that the small excess in the production at $\epsilon = 0.25$ versus that at $\epsilon = 0.8$ in the regime between $k \sim 10^{-2}$ and $k \sim 5 \times 10^{-1}$ (see the inset in the lower-left plot in Figure 6.6) is borne out in the slightly larger maximum value of the adiabaticity parameter attained for the former case compared to the latter.

6.6 Discussion and Applications

While inflation is a phenomenological success, the particle-physics foundations upon which a complete theory of inflation can be built are yet to be set. Even assuming that the dynamics of inflation may be described in terms of a scalar field (the inflaton), we do not know whether the inflaton is a ‘fundamental’ scalar field describable in terms of a ultraviolet-complete theory, or whether it should be considered within the framework of an effective field theory. We do not yet understand how inflation began (e.g., whether inflation is eternal), or how inflation ended (preheating, reheating, etc.). We also do not know how the inflaton couples to other fields.

If the universe did undergo an early phase of inflation, then one probe of the dynamics results from particle production during inflation. One relic of particle production during inflation is the ‘scalar’ curvature fluctuations resulting from creation of inflaton quanta during inflation. A second relic is the ‘tensor’ perturbations (gravitational waves) resulting from creation of the transverse, traceless component of the gravitational field (viz., gravitons). Measurements of the scalar and (especially) the tensor perturbation spectra and possible non-Gaussian signatures will go a long way toward untangling the foundations of inflation.

In addition to the inflaton and the graviton, other fields will be produced during inflation if conformal symmetry is broken through either a mass term or a non-conformal coupling to gravity. We considered such a model in Section 6.2.1. This model has been considered

before in, e.g., Refs. [185, 188]; here, we extended the range of numerical results to larger mass and larger comoving wavenumber and developed a clearer physical understanding of the expected results in ranges of parameters not accessible to numerical techniques.

Another avenue of exploration is the possibility that the inflaton may couple to massive particles, where again ‘massive’ is with respect to the expansion rate during inflation. Of particular interest is the possibility that the additional field may become massless (or at least light compared to H) during inflation as a result of its coupling to the inflaton. In this case there may be a ‘resonant’ production of the particle species at a particular value of the inflaton field. This is what we call an ‘irruption’ of the massive particle species, and why we refer to the additional scalar field as the ‘irrupton’.

While it is impossible to imagine all possible inflaton–irrupton couplings, the two models we considered exhibit a range of final spectra that should encompass a wide range of possibilities.

The first model was a potentially coupled irrupton, which couples to the inflaton via a term in the potential. This possibility was first proposed in a similar model by Ref. [189], and explored in Refs. [683–686]. Here, we developed numerical and analytical techniques that allowed us to extend the numerical range of study and to understand the behaviour of the resulting spectra with parameters where a numerical calculation is problematic.

We also consider a new model for irrupton–inflaton coupling; specifically a non-canonical kinetic-term coupling between a heavy scalar field with a canonical mass parameter and the scalar (inflaton) field which drives inflation. By canonically normalizing the heavy scalar kinetic term, for our choice of coupling, we found a time-dependent exponential enhancement of the effective mass of the canonically normalized heavy scalar which allows it to briefly become as light (or lighter than) H , but otherwise to be much heavier at both early and late times. By using the method of Bogoliubov coefficients, and numerically solving the equations of motion for the inflaton field, the scale factor, and the mode equations for the heavy scalar field, we determined the number of these heavy scalar particles produced

gravitationally by the non-adiabatic expansion of the background spacetime during inflation acting on quantum fluctuations of the heavy scalar field. From this we extracted the final irrupton particle spectra.

The particle spectra in the two models were found to exhibit a variety of complex behaviours attributable to the time dependence of the effective mass, with the most generic feature being an infrared cutoff in the spectra compared to the minimally coupled WIMPzilla which has been previously extensively studied in the literature, in addition to the usual well-known UV fall-off. These spectra become increasingly peaked toward scales which cross the comoving Hubble radius near the end of inflation as the mass parameter M or M_g increases. For the second model, they additionally become more peaked as the non-canonical kinetic term increasingly singles out a specific inflaton field value as important (i.e., the parameter ϵ decreases in size).

Using the particle spectra which we computed in Section 6.5, we arrive at an important result of our work: the relic density of these heavy particles, assuming they are stable, as a function of the heavy-particle late-time effective mass; these results are shown in Figures 6.11 and 6.12.²⁶ From these results, we conclude that the effect of singling out in the non-canonical kinetic coupling, a single inflaton field value as more important (i.e., decreasing ϵ) at fixed sufficiently large M , is to increase both the late-time effective mass of the heavy particle and the relic abundance. (For smaller M , the effect of increasing ϵ is first to decrease the relic abundance while increasing the late-time effective mass of the heavy particle, but this behaviour is short-lived and as ϵ gets smaller, the mass and relic abundance increase together again.) As a result, we find explicitly that we can produce heavy particles with late-time effective masses more than three orders of magnitude larger than the inflation mass

26. Note that in integrating n_k^c over k to extract the total particle number n^p via Eq. (6.34), a problem arose in some cases (i.e., the kinetically coupled irrupton at $M \leq 1$ as $\epsilon \rightarrow \infty$), indicated in Figure 6.11 by (green) squares, due to the non-convergence of the integral owing to IR-divergent behaviour in n_k^c . We simply cut off the integration in the IR at $k = 10^{-20}$, as this was the smallest value we sampled; this corresponds to modes that crossed the Hubble radius roughly 48 e -folds before the end of inflation. The marked results may very well underestimate the total particle number produced, and they should thus be interpreted with caution. These results are also larger than those from Ref. [188], as we have probed smaller values of k .

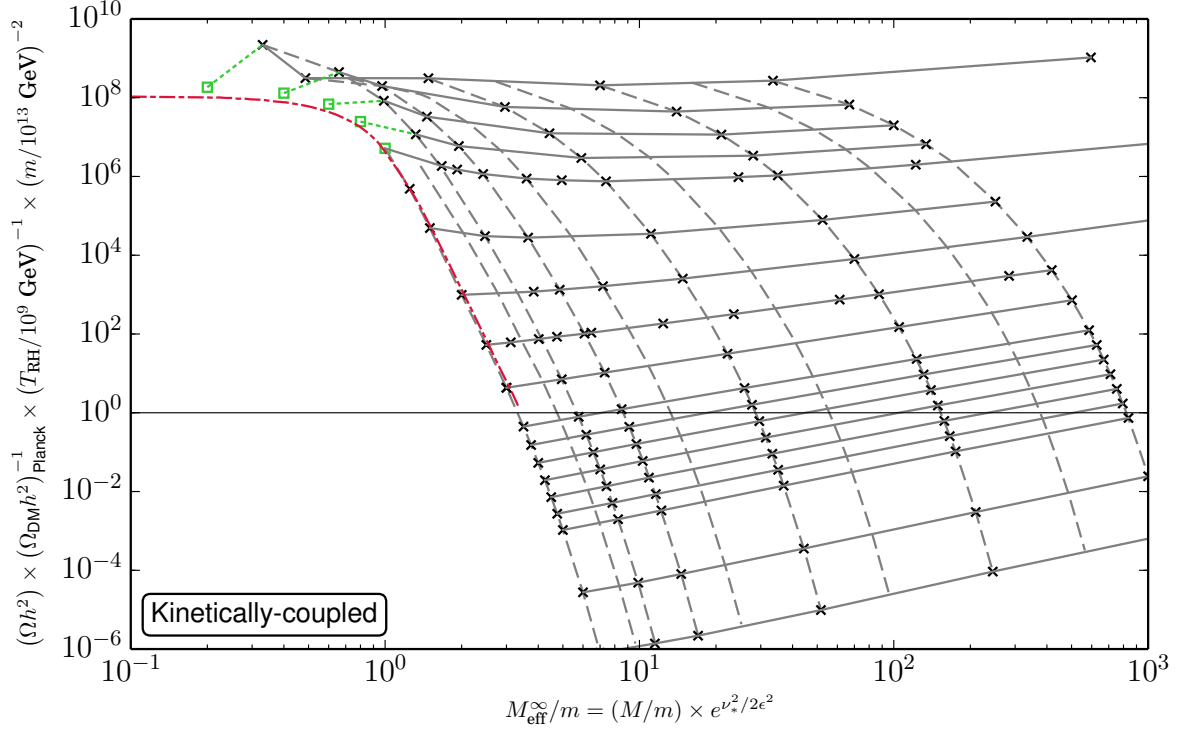


Figure 6.11: The present-day relic mass-density of stable irruptons for the kinetically coupled model in units of $(\Omega_{\text{DM}} h^2)_{\text{Planck}}^{-1} \times (T_{\text{RH}}/10^9 \text{ GeV})^{-1} \times (m/10^{13} \text{ GeV})^{-2}$, where $(\Omega_{\text{DM}} h^2)_{\text{Planck}} = 0.1186$ [85], as a function of the late-time effective mass M_{eff}^∞ . Crosses (black) represent points we have explicitly sampled in our numerical computations. Solid (grey) lines join points at constant M/m (from top to bottom: $M = 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 3.75, 4.0, 4.25, 4.5, 4.75, 5.0, 6.0, 7.0$), and dashed (grey) lines join points at constant ϵ (from left to right: $\epsilon = \infty, 1.0, 0.8, 0.6, 0.5, 0.4, 0.35, 0.3, 0.27, 0.25$), where interpolation between sampled points has been performed. The (green) square points are parameter values where it was necessary to cut the spectrum off in the infrared to obtain a finite value. The dash-dot (red) line is the ‘constant- M ’ result taken from Ref. [188], which our $\epsilon \rightarrow \infty$ results recover well except at small M ; see footnote 26.

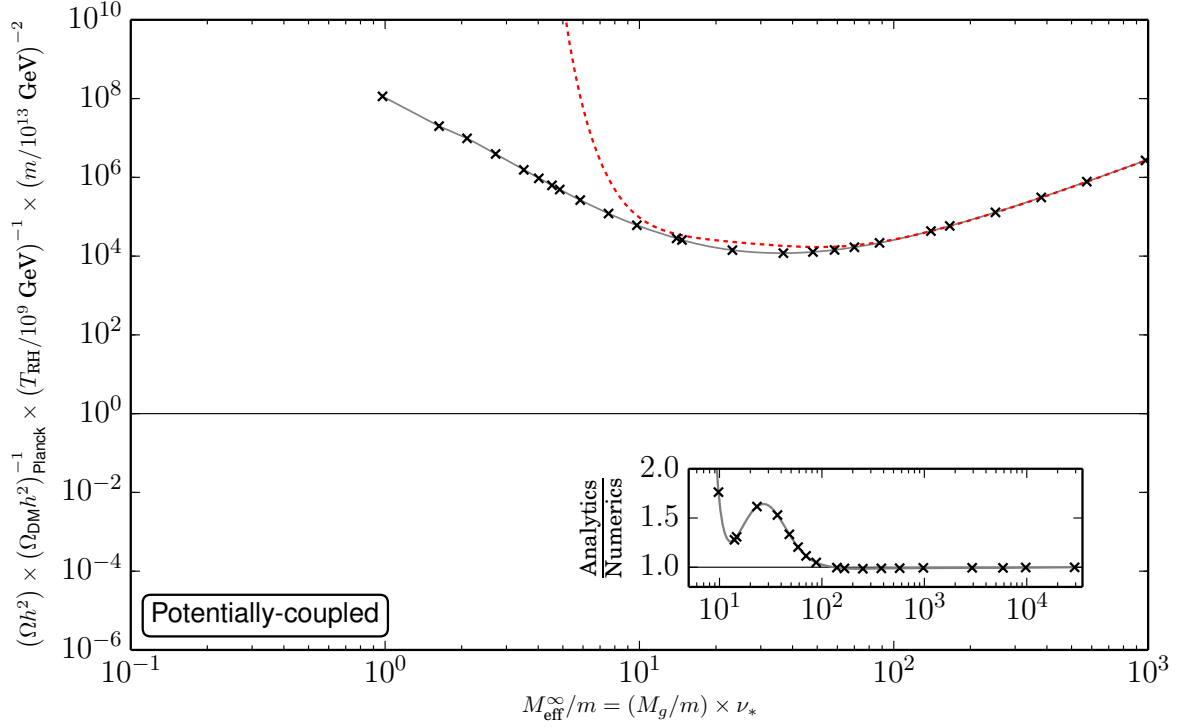


Figure 6.12: The present-day relic mass-density of stable irruptons for the potentially coupled model in units of $(\Omega_{\text{DM}} h^2)_{\text{Planck}}^{-1} \times (T_{\text{RH}}/10^9 \text{ GeV})^{-1} \times (m/10^{13} \text{ GeV})^{-2}$, where $(\Omega_{\text{DM}} h^2)_{\text{Planck}} = 0.1186$ [85], as a function of the late-time effective mass M_{eff}^∞ . Crosses (black) represent points we have explicitly sampled in our numerical computations. The solid (grey) line is a (log-log) cubic spline interpolant between the sampled points, while the dashed (red) line is the result of our analytical expression Eq. (E.28). We show the ratio of the analytical to numerical result in the inset, including also numerically sampled points up to the mass $M_{\text{eff}}^\infty/m = 2.9 \times 10^4$ which are not shown on the main axes as they lie exactly on the $\Omega h^2 \propto M_g^{5/2}$ extrapolation line.

$[m \sim 2H(t_{EI})]$, yet with sufficient relic abundance to saturate the Planck result for $\Omega_{\text{DM}}h^2$, which is in marked contrast to the usual minimally coupled WIMPzilla whose mass must be around $3.3m$ to achieve the same result (see our ‘ $\epsilon \rightarrow \infty$ ’ results, which agree well with those of Ref. [188]). We expect that the mechanism should remain operative for even higher effective masses, possibly even up to the Planck mass scale, for suitable parameter choices.

We note that to obtain the ratio of the irrupton mass-density to the inflaton mass-density during the inflaton oscillation phase, one should multiply the quantity plotted in Figures 6.11 and 6.12, $(\Omega h^2) \times (\Omega_{\text{DM}} h^2)^{-1}_{\text{Planck}} \times (T_{\text{RH}}/10^9 \text{ GeV})^{-1} \times (m/10^{13} \text{ GeV})^{-2}$, by $5.8 \times 10^{-19} \times (m/10^{13} \text{ GeV})^2$. This implies that the irrupton mass-density is a very small fraction of the total since the maximum value of the plotted quantity is around 10^9 for all cases we have considered, and finally supplies the *a posteriori* justification (promised in footnote 8) for ignoring it in the dynamics of expansion.

We will now recapitulate some of the important results we have obtained for the various models investigated in this chapter, and make some additional comments. Consider first the constant- M model:

1. The only parameter in the model is M (always expressed in units of the inflaton mass, $m \simeq 10^{13} \text{ GeV}$).
2. The spectra are slightly red for $M < 1$, slightly blue for $1 < M \lesssim 2$, and increasingly peaked around $10^{-2} \lesssim k \lesssim 1$ for $2 \lesssim M$. There is a sharp drop in the spectra for $k > 1$.
3. The spectra of produced particles decreases very rapidly with M for $M \gtrsim 1$; the decrease in the spectra with M for wavenumbers in the range $10^{-2} \lesssim k \lesssim 10^0$ is still substantial, but less pronounced than in the infrared.
4. If the produced particle is stable, $\Omega h^2 = 0.12$ is obtained for $M \simeq 3.3$ provided $T_{\text{RH}} = 10^9 \text{ GeV}$. (This can be inferred from the $\epsilon \rightarrow \infty$ results in Figure 6.11.)

For the potentially coupled irruption model, some of our important results are:

1. There are two parameters in this model, $M_g = gM_{\text{Pl}}$, and ν_* . The late-time value of the irruption mass is $M_{\text{eff}}^\infty = M_g\nu_*$. Our numerical investigation fixed $\nu_* = 0.8$ which corresponds to four e -folds before the end of inflation.
2. For $M_g < 3$ the spectrum is similar in shape to the constant- M model, owing to the existence in both cases of a broad tachyonic phase whose duration depends on k . That is, at small M_g this model has continuous particle production over extended durations rather than a localized irruption.
3. The spectrum for the potentially coupled case is much larger than the corresponding spectrum for the constant-mass model with $M_g = M$.
4. For still larger M_g , say $M_g \gtrsim 5$ or so (see Figure 6.4), the spectrum becomes increasingly peaked around $10^{-2} \lesssim k \lesssim 1$.
5. The irruption production mechanism is too efficient at producing particles to allow for their interpretation as a possible superheavy DM candidate unless the reheat temperature is fairly low: the total number of particles produced over-saturates the Planck result for Ωh^2 [85] by a factor of at least $10^4 \times (T_{\text{RH}}/10^9 \text{ GeV}) \times (m/10^{13} \text{ GeV})^2$ for all choices of M_g which we have studied (see Figure 6.12). This conclusion is, however, very sensitive to the value of ν_* : since $\Omega h^2 \sim \nu_* a_*^3$ [see Eq. (E.33)], had we taken $\nu_* \gtrsim 1.1$ (greater than eight e -folds before the end of inflation), the minimum value of $(\Omega h^2) \times (\Omega_{\text{DM}} h^2)_{\text{Planck}}^{-1} \times (T_{\text{RH}}/10^9 \text{ GeV})^{-1} \times (m/10^{13} \text{ GeV})^{-2}$ would drop below 1, and it would be possible to attain the right relic abundance even for $T_{\text{RH}} \sim 10^9 \text{ GeV}$.
6. For fixed M_g , our understanding of how the behaviour of M_{eff} impacts the spectra leads us to conclude on general grounds that as ν_* is increased, the n_k^c spectrum will have the same general shape but will (a) shift to the infrared because smaller values of k are needed to allow for strongly non-adiabatic / tachyonic behaviour if M_{eff} has

its zero earlier during inflation when a_* is smaller, (b) broaden on the low- k side due to an extension of the tachyonic phase to earlier times, and (c) decrease in amplitude owing to the greater dilution of an NR species if it is created earlier during inflation. The dilution effect is always dominant, leading to an exponential suppression of the abundance of particles as measured by Ωh^2 after the end of inflation, as ν_* is increased linearly. These points are all explicitly borne out by our large- M_g analytical expressions Eqs. (E.31) and (E.33), as discussed further in Appendix E.2.

7. Our analytical expressions in Appendix E.2 also indicate that at large M_g , this model produces more particles with increasing M_g : $n^{\text{P}} \propto M_g^{3/2}$ and $\Omega h^2 \propto M_g^{5/2}$; our numerical results agree with these scalings.

Finally, we turn to some additional final comments on the kinetically coupled irrupton model:

1. There are three parameters in this model, M , ϵ , and ν_* . The late-time value of the irrupton mass is $M_{\text{eff}}^\infty = M e^{\nu_*^2/2\epsilon^2}$. For $\epsilon \rightarrow \infty$ the constant- M model is recovered. M_{eff}^∞ is exponentially sensitive to ν_*/ϵ .
2. The simple large- M scaling behaviour with changing ϵ evident in Figure 6.11 traces its origin to the insensitivity of the spectra, in the regions where they contribute dominantly to the particle number integral in Eq. (6.34), to changing values of ϵ (at fixed M) which we also noted with regard our discussion of Figure 6.6 in Section 6.5: for example, n^{P} increases by a factor of only about 2 between $\epsilon = 0.80$ and $\epsilon = 0.25$ for $M = 4$, yet $\Omega_{\text{DM}} h^2$ increases by a factor 2 orders of magnitude larger than that. That is, the clean scaling behaviour of Ωh^2 with varying ϵ at fixed M seen in Figure 6.11 at large M arises predominantly through the parametric dependence on ϵ of the horizontally and vertically plotted quantities [both proportional to $e^{\nu_*^2/2\epsilon^2}$; see Eqs. (6.40) and (6.41)] rather than through the impact on the particle spectra of varying ϵ *per se*.

3. The simple large- M scaling with increasing M arises jointly from parametric dependence causing M_{eff}^{∞} to increase linearly, while the spectra themselves undergo an exponentially fast decrease, fairly uniform over a fairly wide range of M , in normalization, thus causing the relic abundance (proportional to n^{P}) to drop, its parametric scaling proportional to M notwithstanding. The integrated spectrum itself can be very well fit over the range from $M = 4$ to 8 by a function of the form $n^{\text{P}} \sim e^{-aM+bM^2}$ where a and b are some positive real numbers and $b \ll a$.
4. The more complex behaviour at smaller M is due the more non-trivial dependence of the particle spectra in the vicinity of their maxima (e.g., Figure 6.6) on M and ϵ , in addition to the parametric dependence just discussed. As an example, consider the case of varying ϵ at small M . Two effects compete against one another: with decreasing ϵ , the normalization of the particle spectrum drops at fixed small M (e.g., the upper plot of Figure 6.6), but the late-time effective mass rises proportional to $e^{\nu_*^2/2\epsilon^2}$. Since Ωh^2 is proportional to $M_{\text{eff}}^{\infty} n^{\text{P}}$, this gives rise to the turning-point behaviour around $\epsilon = 0.4$ seen in, e.g., the $M = 1$ results because at small ϵ , the latter effect wins whereas at larger ϵ , the former effect does.
5. Our general understanding of the behaviour of M_{eff} leads us to conclude that if a tachyonic phase is already present, further increasing ν_* will increase the duration of that phase, and will shift it to slightly earlier times. While the former effect would argue for an increase in the n_k^{C} spectra, the latter effect leads to a greater dilution of the NR species as it is produced earlier. Simulations indicate that the latter effect wins, causing the amplitude of the n_k^{C} spectra to decrease with increasing ν_* . The extension of the tachyonic phase to earlier times causes the spectrum to broaden on the low- k side, while its truncation at later times causes the UV drop-off to happen for slightly smaller k . In other words, with increasing ν_* , the n_k^{C} spectrum gets smaller at fixed k , and broadens as a function of k , and moves to the IR.

6. In contrast, if no tachyonic phase exists (e.g., at large M), increasing ν_* can cause more complex changes to the spectrum. This is perhaps most easily discussed by way of an example; for instance, consider the case of $M = 4$. As ν_* is increased from 0.4 to 0.8, the n_k^c spectrum increases in amplitude, broadens, and the peak moves to the IR due to the fact that $\min\{\omega_k^2/a^2\}$ decreases and is attained at slightly earlier times (the increase in the production from the former effect swamps the dilution implied by the latter effect). As ν_* is increased further from 0.8 to 1.2, the spectrum continues to move to the IR, but its amplitude drops as the dilution effect becomes the dominant. Eventually, for large enough ν_* , a tachyonic phase develops, and point (5) begins to apply.

All the models we consider are capable of producing massive particles during inflation. For the constant-mass model, the efficacy of particle creation drops precipitously for $M \gtrsim H$. Also, for constant-mass models with masses light enough for appreciable particle production, the spectrum of produced particles tends to be flat. We considered two models with varying mass: a ‘potentially coupled’ model and a ‘kinetically coupled’ model. Both of these models are capable of producing an irruption of a particle species as the effective mass of the species vanishes or becomes small compared to H . Both of these models are able to produce particles with mass (after inflation) much larger than H . They are also capable of producing highly peaked spectra.

While the models we considered do not exhaust the space of models with varying particle mass due to the coupling of the particle to the inflaton, they do encompass a remarkable range of phenomena and results.

While a complete study of the applications of massive particle production is outside of the scope of this work, we conclude by remarking on some possible implications and applications.

1. *Back-reaction on the Inflaton Field:* The original motivation for studying irruption of massive particles during inflation was the back-reaction of particle production on the inflaton field, which can lead to features in the scalar density spectrum [189]. It was

later demonstrated that there are additional effects on the density spectrum due to the scattering of the produced particles with the inflaton field [685, 686]. All of these studies assumed a potentially coupled irrupton.

Both of the aforementioned effects are sensitive to the magnitude and the duration of particle irruption. As we have shown, a wide range of possibilities for magnitude and duration are possible just in the two irrupton models we have considered. We have shown that for a same-mass (M_{eff}^∞) particle species, the spectrum and amplitude of produced particles may differ greatly between the potentially coupled and the kinetically coupled models. This will have a large effect on the calculation of the effect of the back-reaction on the inflaton field as well as irrupton–inflaton scattering after production.

2. *Superheavy Dark-Matter (‘WIMPzilla’) Production:* The concept of dark matter produced by particle creation during inflation was proposed within the framework of the constant-mass model [185, 188]. The idea is that the particle coupling to the inflaton is stable and is produced gravitationally during inflation in the correct abundance to be the relic dark matter. As we can see from Figure 6.11, the correct relic mass-density in this model is obtained if $M \simeq 3.3m$. This is in broad agreement with previous analyses.

We can now extend this idea to models with species irruption. The results for a potentially coupled irrupton model are shown in Figure 6.12. The interesting result is that for fairly small values of ν_* , there is no value of the model parameters that do not overproduce the particle, provided the reheat temperature is not quite low. For this model, Ωh^2 actually has a minimum around $M_{\text{eff}}^\infty \approx 30$ as a result of the fact that n^{P} does not continue to monotonically decrease with increasing M_g when the latter is large (Figure 6.4): once $\partial n^{\text{P}}/\partial M_g \sim M_g^{-1}$, the relic density (proportional to $M_g n^{\text{P}}$) will go through a minimum. Once $\nu_* \gtrsim 1.1$ however, the correct relic abundance can

be obtained (even for $T_{\text{RH}} \approx 10^9 \text{ GeV}$) due to the additional dilution of the NR species owing to it being produced earlier during inflation.

For a kinetically coupled irrupton, M_{eff}^∞ can be (many) orders of magnitude larger than $3.3m$ and still give rise to the right abundance of particles (see Figure 6.11); in fact, it could naïvely be as large as the Planck mass (i.e., it can be a Planckon). We will remark briefly on the possibility of super-Planckian particle production below.

3. *Isocurvature Modes:* Isocurvature modes are produced in the WIMPzilla scenario [302, 303] because the dark matter is produced by the dynamics of the coupled irrupton-inflaton system and the curvature perturbations are produced by the inflaton dynamics alone.

Again, the calculation of amplitude and spectrum of the isocurvature component will differ in the constant-mass case and either of the kinetically or potentially coupled irrupton models. This is particularly important as the limits to the isocurvature model become more stringent [700].

4. *Non-Gaussian Features:* Non-Gaussian features in the scalar perturbation spectrum will occur in all of the models we study. Isocurvature perturbations were considered in the WIMPzilla scenario in Ref. [303] assuming a constant- M model. If there is an irruption of massive particles during inflation there is another source of non-Gaussianity called infrared cascading in Ref. [685] (see Ref. [686] for a review). The model used in that study is the potentially coupled model of this chapter. The calculations for both of these effects (WIMPzilla and infrared cascading) will be modified in the kinetically coupled scenario.

5. *Planck-Mass Particle Production:* As we noted above, the efficacy of the either the potentially or kinetically coupled irrupton production mechanism with regard to large-mass particles is expected to extend beyond the regime we have explicitly investigated numerically. This raises the prospect that by dialling the parameters M, ϵ and ν_*

(kinetically coupled case) or M_g and ν_* (potentially coupled case), we could naïvely raise the late-time (as well as early-time) effective mass of the irruptons to $M_{\text{eff}}^\infty \gtrsim M_{\text{Pl}}$ while still maintaining a non-negligible abundance.

This raises the possibility that inflation could be sensitive to particles²⁷ with mass larger than the Planck mass. In this scenario, except for an extremely narrow region near $\phi = \phi_*$, the Planckian state can be integrated out of the effective field theory describing inflation. But it is exactly near $\phi = \phi_*$ that the particles will be produced, and when they are produced they are light. Only after the inflaton continues to evolve past $\phi = \phi_*$ will the particle regain its proper Planckian mass. Presumably then the ‘particle’ becomes a classical black hole, and description of its dynamics in terms of single-particle excitations of a fundamental scalar field is then inappropriate. Furthermore, with Planck-mass particles present, the local spacetime will be subject to significant back-reaction, and if a black-hole forms, it would look locally like Schwarzschild, rather than homogeneous quasi-de Sitter, so it is clear that our results in this regime are on shaky footing. At some point our simple model must break down and it is therefore unclear whether it does actually allow for the production of trans/super-Planckian particles. This is an interesting open question.

27. Ref. [701] argues that these states are not properly quantum particles, but black holes.

CHAPTER 7

CONCLUSION

The studies presented in this dissertation run a gamut of particle physics phenomenology topics lying at the intersection of the fields of Higgs physics, particle dark matter, and early universe cosmology. In Chapter 2, we studied how astrophysical gamma-ray flux measurements can be used to constrain effective field theory descriptions of classes of models in which cold thermal-relic WIMPs annihilate into electroweak gauge bosons and/or the recently discovered Higgs boson, finding that current indirect detection experiments already constrain interesting regions of parameter space. We extended the theme of effective operator analyses of thermal-relic WIMP dark matter to the consideration of the coupling of such a dark-matter candidate to the Higgs boson via the well-motivated Higgs Portal scenario in Chapter 3, and studied how both collider and direct detection experiments place severe constraints of large parts of the available parameter space for the effective description and, by implication, many classes of models which could give rise to such a coupling. Higgs Portals were also our topic of interest in Chapter 4, in which we studied in detail a singlet-doublet UV completion of one of the dimension-5 fermionic Higgs Portal operators which we had considered in Chapter 3, finding that proposed ‘Higgs factory’ e^+e^- colliders operating in the few-hundred GeV range could probe for such a completion via their precise measurements of the electroweak S and T parameters, and the Higgsstrahlung cross section σ_{Zh} . The Higgs sector was again our topic in Chapter 5, in which we considered in detail how the Little Hierarchy in a specific composite Higgs model could be generated without resorting to unnatural tuning, by making use of the cosmological relaxation mechanism which has recently been proposed in the literature, and explored possible signatures of this scenario. Finally, in Chapter 6 we returned to the question of potential particle dark-matter candidates, examining the production in the inflationary epoch of the early universe of late-time supermassive particles whose masses become momentarily light during inflation, leading to an irruptive burst of production of such particles which is easily large enough to explain the

observed cosmological relic abundance of dark matter.

Independent of the specific results of each of these studies, perhaps the broadest conclusions that can be drawn from this body of work are that there is rich variety of theoretical ideas which extend across these subfields of particle physics; that the interplay of ideas from these diverse fields leads to novel and interesting research directions; and that the vibrant and fast-evolving experimental programmes in collider physics, direct detection, indirect detection, and cosmological measurements together provide—and in future will continue to provide—complementary avenues both to constrain, and provide guidance for the development of, theories of new physics that aim to address big open questions in particle physics.

APPENDIX A

APPENDICES TO CHAPTER 2

This appendix contains material from Ref. [4]: M. A. Fedderke, E. W. Kolb, T. Lin, and L.-T. Wang, *Gamma-ray constraints on dark-matter annihilation to electroweak gauge and Higgs bosons*, *JCAP* **1401** (2014) 001 [arXiv:1310.6047].¹

A.1 Numerical Factors in Eqs. (2.1) and (2.7)

Despite the correct answer having been known for decades [514, 524], there is occasionally some confusion about various factors of 2 in Eqs. (2.1) and (2.7). We thus wish to give a succinct summary of the origin of the relative factors of 2 in these expressions that depend on whether self-conjugate or non-self-conjugate DM is assumed.

Consider first Eq. (2.1): per Ref. [120], the Boltzmann equations that must be solved to obtain the relic abundances for non-self-conjugate DM are

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle [n\bar{n} - n_{\text{EQ}}\bar{n}_{\text{EQ}}] \quad (\text{A.1})$$

$$\frac{d\bar{n}}{dt} + 3H\bar{n} = -\langle\sigma v\rangle [n\bar{n} - n_{\text{EQ}}\bar{n}_{\text{EQ}}], \quad (\text{A.2})$$

where n and $\bar{n}$ the particle and anti-particle abundances, respectively. For symmetric DM, $n = \bar{n}$ and $n_{\text{EQ}} = \bar{n}_{\text{EQ}}$, so that

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle [n^2 - n_{\text{EQ}}^2] \quad (\text{A.3})$$

is the Boltzmann equation governing the particle abundance n . The total abundance is $n_{\text{total}} = n + \bar{n} = 2n$.

For self-conjugate DM, each annihilation eliminates two DM particles so the RHS of

1. Ref. [4] is copyright © 2014 by the Authors, and is under a Creative Commons Attribution (CC BY) 3.0 Licence [1].

Eq. (A.3) gets multiplied by 2; however, for N total self-conjugate particles, one can obtain a maximum $N(N-1)/2 \rightarrow N^2/2$ (in the thermodynamic limit) annihilations, so the RHS must also be multiplied by a compensating factor of $1/2$ to account for this; equivalently, this additional factor of $1/2$ can be understood [514, 524] to avoid a double-counting of the initial-state phase space for identical particles since, in deriving Eq. (A.3), the initial state momenta were integrated over.

Since n in both cases is governed by the same differential equation, Eq. (A.3), and since $n_{\text{total}} = 2n$ for non-self-conjugate DM but $n_{\text{total}} = n$ for self-conjugate DM, it follows that, for equal DM mass and cross section, the relic abundance for non-self-conjugate DM is twice that for self-conjugate DM, as shown in Eq. (2.1).

Now consider Eq. (2.7): all other things being equal, the annihilation photon flux for (symmetric) non-self-conjugate DM is proportional to $n\bar{n} = n^2 = (n_{\text{total}}/2)^2 = n_{\text{total}}^2/4 = M^2\rho_{\text{DM}}^2/4$; for self-conjugate DM, the flux is proportional to $n^2/2 = n_{\text{total}}^2/2 = M^2\rho_{\text{DM}}^2/2$ where the factor of $1/2$ here is the same initial-state phase-space factor described above. As a result, the RHS of Eq. (2.7) for self-conjugate DM is a factor of 2 larger than that for non-self-conjugate DM (see also Section 5 of Ref. [471], and pp. 54–55 in Ref. [35]).

A.2 Inverse Compton Scattering

In addition to prompt photons, there may be gamma rays resulting from Inverse Compton Scattering (ICS) of any high-energy electrons/positrons injected into the GC radiation field by DM annihilation or decay (see, e.g., Ref. [471]). ICS is most relevant for annihilation modes which inject energy primarily in the form of $e^\pm$, in particular the direct e^+e^- or $\mu^+\mu^-$ annihilation modes.²

For the heavy quark or diboson primary final states relevant for most of the operators, the effect of ICS at high energies is subdominant compared to the prompt gamma rays produced

2. For $\tau^+\tau^-$, ICS is not as important since over 60% of τ decays go to hadronic final states [38].

by pion decay. Although the ICS contribution can dominate at lower photon energies, the most constraining limits for continuum emission typically come from the 10–100 GeV energy range, so we do not expect significant changes to the results for these operators.

There are some operators which have direct annihilation to charged fermions. However, even for these operators, the charged lepton annihilation modes are relevant only for masses below the thresholds for diboson final states, and here the inclusive limits are already constraining. For example, in the case of operator IX-2 (Figure 2.16) the fermion contributions are proportional to m_f^2 , so only the $\tau^+\tau^-$ annihilation mode is relevant, and only for $M \lesssim 100$ GeV. For all other operators where fermion final states are present, the charged leptons typically comprise no more than 40% of the total charged fermion branching fraction, and are only relevant below a few hundred GeV (with the exception of Operator XX-1, Figure 2.37). We thus do not expect a very large correction to the limits we have set when the ICS photons are included.

Here we demonstrate that including ICS will not change limits significantly by giving a quantitative estimate for operators with charged fermion final states. We will make use of the results of Ref. [471], which gives pre-computed Green functions $I_{\text{IC}}(E_s, E_\gamma, l, b)$ that allow one to convert from an electron injection spectrum to an ICS photon spectrum, taking into account the various $e^\pm$ energy-loss mechanisms in the GC.

It is important to note that the morphologies of the prompt and ICS photon signals will not in general be exactly the same, since the ICS signal depends on the galactic radiation fields as well as the energy loss and diffusion of the injected electrons. As detailed in Section 2.3.3, our primary source of inclusive spectrum limits is Ref. [451]; the limits in that reference are extracted for $E_\gamma \in [0.1, 100]$ GeV from the upper limit of the normalization of a prompt-photon signal template $\propto \int_{\text{LOS}} \rho^2(r[s, l, b]) ds$ smoothed over 0.5° regions. The limits set in that analysis are therefore not applicable for photon signal components which do not follow approximately the same morphology as prompt photons. That said, we find morphological differences give rise to only a factor of a few variation in the ratio of the ICS

to prompt signal when considered over $\mathcal{O}(10^\circ)$ regions. We therefore take the approximation that the limits in Ref. [451] are applicable to the combined ICS and prompt photon signals.

Since we are not however reproducing the template subtraction analysis, we must make a choice for the ROI over which to extract the effective ICS spectrum in order to make a direct comparison with the cross section limits in Ref. [451]. Motivated by the cuspy nature of the expected signal morphologies, and the fact that the photon signal used in setting the limits in that reference is mostly contained within a region of about 3° around the GC, we chose to utilize a 5° radius circular region centred on the GC in extracting the effective ICS spectra. Furthermore, at very high injection energies, the high-energy ICS signal is more constrained to the Galactic Centre, since electrons must be at high energy to create high-energy photons via single Klein-Nishina-regime Compton scatters, and this only happens near the point of production because the electrons lose energy as they propagate out of the GC region [471].

We integrate the ICS signal over this ROI and normalize by the line-of-sight integral for that region:

$$\begin{aligned} \left. \frac{dN_\gamma^{\text{ICS}}}{dE_\gamma} \right|_{\text{ROI}} &= \frac{1}{E_\gamma^2} \times \left[\int_{m_e}^M dE_s \left(\sum_f \text{Br}_f \frac{dN_{e^\pm}^{f, \text{inj.}}}{dE_s} \right) \int_{\text{ROI}} d\Omega I_{\text{IC}}(E_s, E_\gamma, l, b) \right] \\ &\times \left[\int_{\text{LOS-ROI}} d\Omega \frac{ds}{r_\odot} \left(\frac{\rho(r)}{\rho_\odot} \right)^2 \right]^{-1}. \end{aligned} \quad (\text{A.4})$$

This gives an effective ROI-averaged per-annihilation spectrum of up-scattered ICS photons, in terms of the constituent electron injection spectra $dN_{e^\pm}^{f, \text{inj.}}/dE_s$, for final state f , where $E_s \in [m_e, M]$ is the energy of the injected electron, E_γ is the energy on the up-scattered ICS photon, and l, b, s, r are coordinates and distances as defined in the main text of Chapter 2. A fairly conservative estimate for the systematic uncertainty here is $\pm 50\%$ due to the choice of ROI. Note finally that the diffusion setups MIN, MED, MAX (discussed in Ref. [471]) give largely similar results for an ROI of this size near the GC.

Explicitly checking the shift in the limits set using the results of Ref. [451], we find that the present-day DM annihilation cross section σv limits are strengthened by no more than

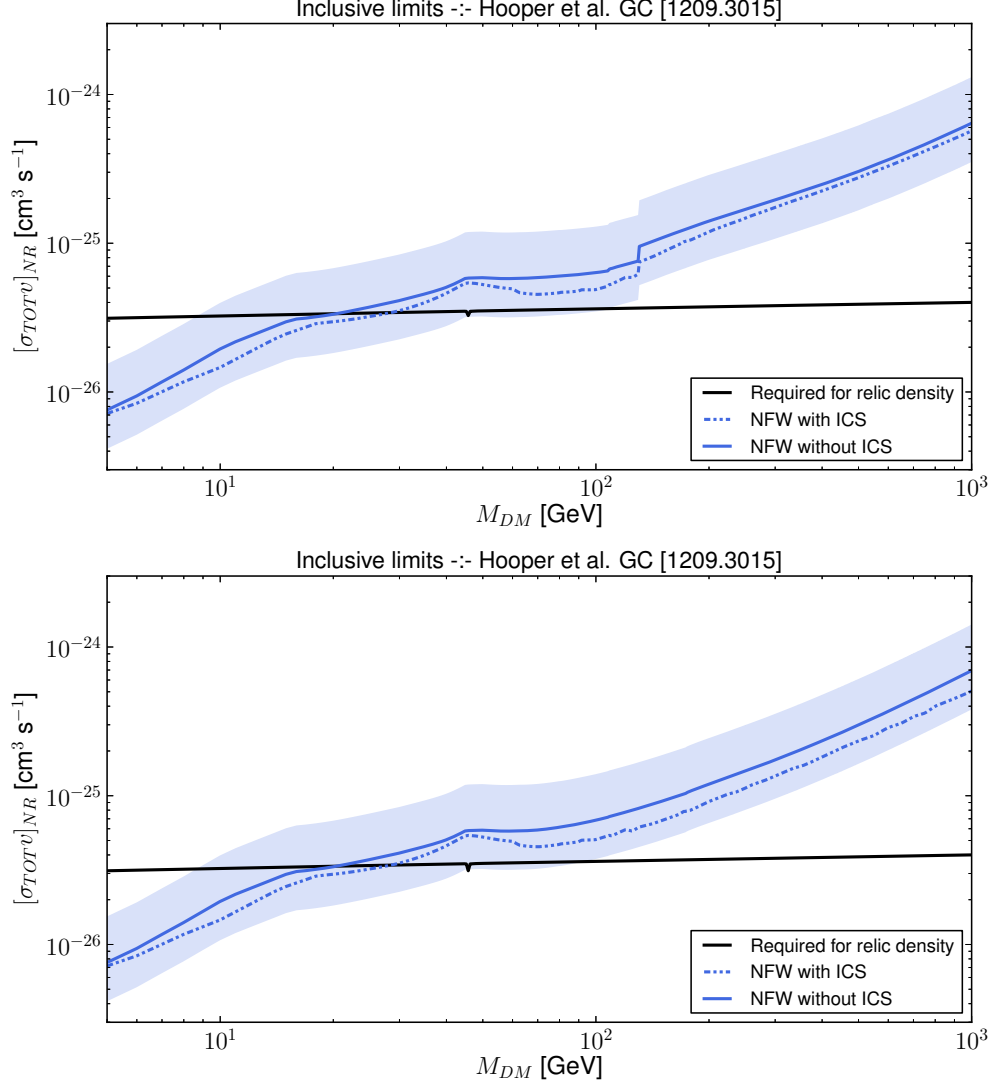


Figure A.1: An example of the impact on the inclusive σv limits from Ref. [451] when Inverse Compton Scattering (ICS) is included (subject to the approximations discussed in the text). These results are for operators XVIII-1 (top panel) and XX-1 (bottom panel) and are representative of the most dramatic effect seen for any operator: even so, the effect on the limits is well within the astrophysical uncertainties on the halo profile normalization (blue shaded band). The small downward blips on the black lines near $M \sim 45$ GeV are unphysical plotting artefacts.

40% at any M for any operator, which is well within the astrophysical uncertainty due to the normalization of the halo profile. The resulting effect on the limits is shown for two representative cases in Figure A.1.

A.3 Estimate of Systematic Uncertainties

It is useful to have an approximate knowledge of the size of various systematic uncertainties which impact our results and which we have not explicitly included in the main body of Chapter 2. We summarize the systematics in Table A.1. It is relatively clear that the astrophysical (halo profile normalization) uncertainties dominate the systematics, but depending on the result and mass range in question, it may be prudent to take the total systematic uncertainty as something up to twice as large as the astrophysical uncertainty.

A.4 Exposure for Line Fit in Annular Region

In Section 2.3.7 we describe the application of the line-fit results of Ref. [453] to operators with both $\gamma\gamma$ and γZ lines. However, Ref. [453] supplied only fitted values of $N_{\gamma\gamma} + N_{\gamma Z}$. In order to apply their results, a necessary ingredient is the exposure: the mission-time-integrated effective observing area multiplied by the observing time, averaged over the ROI. In this Appendix we summarize our calculation of the exposure for the reported data set used in Ref. [453].

We have reproduced the data extraction reported in Ref. [453] using the Fermi ScienceTools `v9r31p1` software [476] with the publicly available Fermi-LAT weekly data files [702]. To be explicit, we have considered data on the time interval 239557447–356400002 (in Mission Elapsed Time), with photon energy range 5–200 GeV in an annular region centred on the GC with inner and outer radii of 1° and 3° respectively. We use the `P7_V6_ULTRACLEAN` data set and filter the data using ‘option 2’ as recommended for a diffuse analysis [703].

Table A.1: Estimated maximum systematic uncertainties from various effects, quoted as the percentage variation in the σv limit: $f \equiv 100 \times \left(\frac{[\sigma v]_{\text{effect accounted for}}^{95\% \text{ CL UL}}}{[\sigma v]_{\text{limit as presently set}}^{95\% \text{ CL UL}}} \right)$ (so $f < 0$ means a more stringent limit with the effect properly accounted for, while $f > 0$ means the limit as currently set is too stringent). Values estimated by us are marked with a *; values taken from the relevant cited literature source are unmarked. For comparison, the typical variation between the most conservative normalization for the most conservative profile choice and the most aggressive normalization for the cuspiest profile is about an order of magnitude.

Result(s) affected	Description of effect	Variation in result f
All	Halo rescaling or J -factor computation	$\pm 5\%^*$
Ref. [449] line limits	Variations in the fitted signal strength due to the finite spacing between tested values of the scanned line peak energy and estimate of the energy resolution.	$+12\% / -7\%$
Ref. [449] line limits	Variations in the effective exposure used in converting the fitted number of photons in a line to a flux.	$\pm 16\%$
Ref. [449] line limits	Fake or masked signals: for low WIMP masses (a few tens of GeV), up to roughly the size of the statistical uncertainties, which are about a factor of 2.	$+100\% / -50\%$ for low masses, dropping rapidly as mass increases.
Ref. [449] line limits	Correctly accounting for the γZ photon linewidth.	$+7\%$
Ref. [449] line limits	Mismodelling of the photon line shape for the cases with both $\gamma\gamma$ and γZ peaks in the vicinity of $M \sim 80\text{GeV}$ and $M \sim 160\text{GeV}$.	$+10\%^*$
Ref. [450] line limits	Overall systematics estimate.	$+50\% / -50\%$

Table continued on next page ...

Table A.1: ... continued from previous page.

Result(s) affected	Description of effect	Variation in result f
Ref. [451] inclusive limits	Subtraction of additional background associated with Galactic Ridge or central gamma-ray point source.	-50%
Ref. [451] inclusive limits	Uncertainty in the exact value of the correction factor of 2.8 necessary to resolve [474] the discrepancy noted in the main text; see Section 2.3.3.	$\pm 10\%^*$
Ref. [451] inclusive limits	Including ICS photons in setting the limit (estimated using the 5° ROI normalization; the systematic uncertainty on the normalization of ICS spectrum itself is about $\pm 50\%$ owing to our approximate treatment).	$-40\%^*$
Ref. [452] continuum limits	Excluding from the limit computation the two most uncertain spherical dwarf measurements.	$+50\%$
Ref. [433] line fit	Overall systematics estimate.	$+15\% / -20\%$

Table A.2: The J -factors for the $1-3^\circ$ annular ROI defined in Ref. [453]. Unless otherwise indicated, these results assume the central normalization and parameter values from Table 2.1.

Halo Profile	J -factor [$10^{21} \text{ GeV}^2 \text{ cm}^{-5}$]
NFWc ($\gamma = 1.3$)	32.50
Einasto	15.05
NFW	9.02
Isothermal	0.361
Einasto with $R_\odot = 8.5 \text{ kpc}$ and $\rho_\odot = 0.4 \text{ GeV/cm}^3$	21.41

Histogramming the photon counts in the annular ROI we reproduce almost exactly³ the counts presented in Tables II and III of Ref. [453].

We compute the full-sky exposure map as a function of photon energies near 125–150 GeV, and subselect the annular ROI from this full-sky exposure map (exposure values in angular bins). The ROI-averaged exposure is given by

$$\mathcal{E}_{\text{ROI}} = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} \mathcal{E}(\Omega) d\Omega \approx \frac{1}{\Delta\Omega} \sum_i \mathcal{E}_i \cos b_i \Delta l \Delta b, \quad (\text{A.5})$$

where $\Delta\Omega = 7.65 \times 10^{-3}$ sr, $\mathcal{E}_i$ is the exposure value in angular bin i , and the sum runs over all angular bins (which have angular size $\Delta l \times \Delta b = 0.1^\circ \times 0.1^\circ$). Strictly speaking, one should weight the exposure by the signal morphology before performing the integration in Eq. (A.5); however, the very small size of the ROI together with the almost uniform exposure over the ROI make neglecting this subtlety a good approximation. We find that the exposure does not vary significantly over the energy range 125–150 GeV, and takes a value of $\mathcal{E}_{\text{ROI}} \approx 1.05 \times 10^{11} \text{ cm}^2 \text{ s}$.

The photon flux corresponding to the line is given by $(N_{\gamma\gamma} + N_{\gamma Z})/\mathcal{E}_{\text{ROI}}$. We find the total normalization $N_{\gamma\gamma} + N_{\gamma Z}$ for each operator using a 2D polynomial interpolation of the contour map given in Figure 2 of Ref. [453], extracting the value of $N_{\gamma\gamma} + N_{\gamma Z}$ on the operator-specific (black) curves indicated in Figure 2.3. Finally, we compute the J -factors necessary to extract cross sections from these fluxes given the central normalization values in Table 2.1; the results are in Table A.2. Applying Eq. (2.22), we thus interpret the fit results in terms of the annihilation cross section to lines, $(2\sigma_{\gamma\gamma} + \sigma_{\gamma Z})v$; we plot this quantity in our result plots on the regions of M parameter space where the delta-log-likelihood for the double-line fit is $\leq 3\sigma$, $\leq 2\sigma$ and $\leq 1\sigma$ relative to the best fit point of Ref. [453] (see Figure 2.3).

To check our work, we compare with the analysis of a single line presented in Ref. [433].

3. The only discrepancies seem to be associated with the exact boundaries of the energy bins, or with a slightly different choice of time interval.

We compute the value⁴ of $\sigma_{\gamma\gamma}v = (8\pi M^2/2J)N_{\gamma\gamma}/\mathcal{E}_{\text{ROI}}$ assuming $\theta_{\gamma z/\gamma\gamma} = 0$, $N_{\gamma\gamma} = 33$ and $M = 130 \text{ GeV}$, along with the Einasto profile J -factor at the bottom of Table A.2, which has been computed using the indicated parameter values taken from Ref. [433]. We obtain $\sigma_{\gamma\gamma}v = 3.2 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}$, which is a factor of 3 larger than the central value ($\sigma_{\gamma\gamma}v = 1.1 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}$) reported in Ref. [433]. There is thus some tension between these two analyses, and one might reasonably estimate a similar degree of uncertainty in the results of this analysis which are shown in the various result plots in Section 2.4.

A.5 Updates on the Current Status of the 130 GeV Photon Line and the Various Limits Considered in Chapter 2

A.5.1 Update on the Status of the 130 GeV Photon Line

Chapter 2 is based on Ref. [4], which was completed in mid- to late-2013; at that time, there was some evidence for a c. 130 GeV photon line in Galactic Centre photon data from the Fermi-LAT experiment [432, 433, 449]. The 3.7-year Fermi-LAT Collaboration analysis [449] showed 3.3σ local significance (1.6σ global significance) for such a spectral feature at 133 GeV using their **Pass 7** (**P7REP_CLEAN**) data reconstruction, although they also showed that, using the same data reconstruction, a similar feature with a local significance of 2.0σ existed in their Earth-limb data [449], which was expected to be featureless [449]; this was suggestive of a possible (partially) systematic origin for the 133 GeV feature in the GC data [449], although the Collaboration cautioned that ‘more data and study [were] needed to clarify the origin of [the] feature’ [449].

The 5.8-year Fermi-LAT Collaboration analysis [231] has subsequently been released (incorporating also systematic uncertainties omitted in Ref. [449]) and showed that: (a) using the **Pass 8** (**P8_CLEAN**) reconstruction on the original 3.7-year data examined in

4. The coefficient of 8 rather than 16 is correct here; we are comparing to a limit set under the assumption of self-conjugate DM; see Eq. (2.7)

Ref. [449] reduces the local significance of the feature at 133 GeV to 2.0σ [231]; (b) using the `P7REP_CLEAN` reconstruction of the full 5.8-year data set, the local significance for the feature at 133 GeV is reduced to 2.1σ [231]; (c) using the `P8_CLEAN` reconstruction of the full 5.8-year data set yields a local significance for the feature at 133 GeV of 0.7σ [231]; and (d) the largest local significance for any line feature in the new analysis is 2.8σ for a feature at 115 GeV (0.4σ global significance) [231]. It is also worthwhile noting that the Collaboration still sees a feature at 133 GeV in the Earth-limb data [231] in the `Pass 8` reconstruction of the 5.8-year data at 2.5σ local significance, which is a slight increase in the local significance of this anomalous feature compared to the `P7_REP_CLEAN` reconstruction of the 3.7-year data in the analysis of Ref. [449]; additionally, the `P7REP_CLEAN` reconstruction of the 5.8-year data places this feature at a smaller local significance (1.4σ) than the `P8_CLEAN` reconstruction [231]. This signal is however not present in their Galactic Plane (GP) masked control region on the sky [231]. Ref. [231] observes that these facts indicate that there may be some remaining ‘small systematic effect’ [231] in the `P8_CLEAN` reconstruction for energies around 130 GeV, but concludes that the decreased significance (for this reconstruction) of the 133 GeV feature in the larger data set is ‘consistent with most of the original feature originating from a statistical fluctuation’ [231].

The H.E.S.S. Collaboration has also recently released an analysis [232] (see also Appendix A.5.3) which would be sensitive to the putative 130 GeV signal, if one assumes an Einasto DM halo profile centred at the location of the original 130 GeV excess: $(l, b) = (-1.5^\circ, 0^\circ)$ [704]. Ref. [232] finds no evidence for an excess at 130 GeV with the flux reported in Ref. [433] under this off-centre Einasto DM profile assumption; the updated search would not however be sensitive to the signal if a GC-aligned Einasto DM profile were assumed [232].

Taken together, these new results strongly suggest that the line signal near 130 GeV originally reported in Ref. [432] was not a real feature ‘on the sky’. Despite the disappearance of this potentially interesting line signal with increased data and improved analysis, in this dis-

sertation we will continue to refer to the existence of a c. 130 GeV photon line as a potentially interesting signal, as many of the considerations associated with our analysis would apply more broadly to any putative line signal that could be found in future. We have inserted references to this Appendix throughout Chapter 2 whenever we discuss the 130 GeV line signal, as a reminder to consider the discussion there in light of these updated results.

A.5.2 Update on the Fermi-LAT Line Search Limits

The Fermi-LAT Collaboration has recently updated their line search analysis [231] by (a) using their **Pass 8 (P8_CLEAN)** data reconstruction instead of the **Pass 7 (P7REP_CLEAN)** reconstruction used in the analysis of Ref. [449]; (b) accounting for systematic uncertainties in Ref. [231] which were not accounted for in Ref. [449]; (c) extending the analysis to the full 5.8-year data set in Ref. [231] from the 3.7-year data set used in Ref. [449]; and (d) enlarging the gamma-ray energy range considered in the analysis to 0.2–500 GeV in Ref. [231], from the range of 5–300 GeV analysed in Ref. [449] (Ref. [705] had previously extended the analysis into the 0.1–100 GeV gamma-ray energy range for the 5.2-year data set). The new analysis found no globally significant photon lines in any of their GC ROIs [231] (see also Appendix A.5.1) and set updated limits. As shown in Figure 8 of Ref. [231], the limits set by the two analyses in Refs. [449] and [231] are not dramatically different in the region where both limits are available, 5–300 GeV: (a) for the **R3** ROI for $E_\gamma \sim 100\text{--}300$ GeV, the 5.8-year limits are on average a factor of a few stronger than the 3.7-year limits [231]; (b) for the **R41** ROI for $E_\gamma \sim 5\text{--}15$ GeV, the 5.8-year limits are on average a factor of a few weaker than the 3.7-year limits [231]; and (c) for the **R90** ROI for $E_\gamma \sim 5\text{--}40$ GeV, the 5.8-year limits are on average a factor of a few weaker than the 3.7-year limits [231]. For all other ROIs, at almost all other energies, the two limits are in agreement within the 2σ statistical variation in the expected location of the updated limits [231]. Therefore, apart from the enlargement of the range of WIMP masses over which the updated Fermi-LAT line limits [231] would be constraining (owing to the larger range of gamma-ray energies examined in the latter analysis), we do not

expect that the limits on the operators which we show in Chapter 2 would be significantly altered (in the mass ranges for which we shown our results) were we to re-run our analysis with the updated limits from Ref. [231]. As a consequence, we have not updated our results account for these new developments.

A.5.3 Update on the H.E.S.S. Line Search Limits

The H.E.S.S. Collaboration has recently supplemented the line search results of Ref. [450] with an analysis in Ref. [232] reporting flux limits on the energy range 100 GeV–2 TeV from a 15.2hr data set, for a 0.4° -radius ROI centred at $(l, b) = (-1.5^\circ, 0^\circ)$. This differs from the original 1° -radius GC-centred ROI (with $|b| \leq 0.3^\circ$ masked) used in Ref. [450] (see also Section 2.3.2). This offset (from the GC) was motivated by the original Fermi-LAT 130 GeV excess [432, 433] having a best fit location of $(l, b) = (-1.5^\circ, 0^\circ)$ [704]. Although the new analysis [232] further covers the 300–500 GeV gap between the previous Fermi-LAT [449] and H.E.S.S. analyses [450], Ref. [232] explicitly warns that direct comparison of interpreted σv cross section limits obtained from these the flux limits in Refs. [450] and [232] is not apt, as the different assumptions made in the two analyses as to where the DM halo profile was centred dramatically modify the J -factors given the small, non-overlapping ROIs [232]. In particular, per Figure 3 of Ref. [232], assuming an Einasto DM halo profile aligned to the GC, the Monte Carlo (MC) expected limits on σv from the new analysis [232] are about an order of magnitude less constraining than those from the previous analysis in Ref. [450] (or, indeed, those from the updated Fermi-LAT analysis, Ref. [231]), and would be about an order of magnitude too weak to be sensitive to the putative 130 GeV excess of Refs. [432, 433]. However, under the assumption of an (Einasto) DM halo profile centred at the location $(l, b) = (-1.5^\circ, 0^\circ)$ [704] of the c. 2013 putative 130 GeV excess [432, 433], the σv limits from the new H.E.S.S. analysis [232] become comparable to those of the updated Fermi-LAT analysis [231] in the region where they overlap (Ref. [232] notes that this comparison is allowed as the off-centre halo profile only modifies the J -factor for the Fermi-LAT analysis

[231] by about 1%, in contrast with the comparison to Ref. [450]). Additionally, under these assumptions, Ref. [232] would be sensitive (at around the 95%-confidence exclusion level) to the putative 130 GeV excess, but finds no statistically significant evidence for such an excess [232]. While these additional results increase confidence that the 130 GeV Fermi-LAT excess was indeed not ‘on the sky’, we implicitly assume throughout Chapter 2 that the DM halo is GC-centred, so we would not expect the updated analysis in Ref. [232] to modify our limit results, other than allowing their extension to lower DM mass. However, given that the MC sensitivity estimate for the limits under the GC-centred halo assumption for 15.2h of data is on the order of $\sigma_{\gamma\gamma\nu} \sim 10^{-25}\text{--}10^{-26}\text{cm}^3\text{s}^{-1}$ in the DM mass range 100 GeV–2 TeV per Figure 3 of Ref. [232] [under a GC-centred Einasto DM halo profile assumption, and for self-conjugate DM—which gives a limit a factor of 2 stronger than for non-self-conjugate DM; see Eq. (2.7)], we would not expect these extended limits to be particularly constraining for our operators either. Therefore, apart from this summary of the supplemental analysis, we have not updated our analysis of the H.E.S.S. line limits on our operators to account for the new results in Ref. [232].

A.5.4 Update on the Fermi-LAT (and DES) MW-Dwarf Limits

The Fermi-LAT dwarf limits [452] we used in Ref. [4], as presented in Chapter 2, were based on a 2-year data set and are by now quite old, having been initially published in 2011. As we note in Section 2.3.4, as we were finishing Ref. [4] (the work on which Chapter 2 and this Appendix is based) an updated analysis based on a 4-year data set [475] was released by the Fermi-LAT Collaboration; see Section 2.3.4 for discussion. The Fermi-LAT Collaboration has subsequently released an analysis based on a 6-year data set [706] and, more recently, a joint analysis has been released by the Fermi-LAT and DES Collaborations [707], including in the analysis a number of the newly discovered, or candidate, MW-dwarf companions from the Dark Energy Survey [708], bringing the total number of MW-dwarfs and MW-dwarf-candidates in the analysis to 45 [707]. As there have been a number of updates in the

data processing and analysis approaches of this analysis since Ref. [452], we do not attempt to summarize these technical improvements here, and will merely satisfy ourselves with an overview of how the most recent limits [707] have improved since those released in Ref. [452]. Ref. [707] presents limits on σv assuming 100% branching fraction to either $b\bar{b}$ or $\tau^+\tau^-$ final states as a function of the DM mass, from the larger of the final-state kinematic threshold or 2 GeV, up to 10 TeV; Ref. [452] presents results on σv assuming 100% branching fraction to $b\bar{b}$, $\tau^+\tau^-$, $\mu^+\mu^-$, or W^+W^- final states as a function of the DM mass, from the larger of the final-state kinematic threshold or 5 GeV, up to 1 TeV. Both analyses include their estimated uncertainties on the J -factors for the dwarfs in their results [452, 707]. Comparing these results where possible (same final state, same DM mass range) we find that (a) in the DM mass range 6 GeV–1 TeV and for the $b\bar{b}$ final state, the improvement is quite flat with changing DM mass: the latest σv limits from Ref. [707] are on average a factor c. 2.1 better than those from Ref. [452], ranging from an improvement by a factor of c. 1.6 around 6 GeV to an improvement by a factor of c. 3.2 around 1 TeV; (b) in the DM mass range 5 GeV to c. 70 GeV for the $\tau^+\tau^-$ final state, the improvement is quite flat with changing DM mass: the latest σv limits from Ref. [707] are on average a factor c. 2.2 better than those from Ref. [452], ranging from an improvement by a factor of c. 1.6 around 20 GeV to an improvement by a factor of c. 3.6 around 5 GeV, and (c) in the DM mass range c. 70 GeV–1 TeV for the $\tau^+\tau^-$ final state, the improvement depends strongly on the DM mass: the latest σv limits from Ref. [707] are a factor of c. 2.7 better than those from Ref. [452] around 70 GeV, strengthening to an improvement by a factor of c. 25 around 1 TeV. The two regimes (b) and (c) for the $\tau^+\tau^-$ final state can be traced [707] to the ‘kink’ at a DM mass of about 100 GeV in the limit lines shown in Figure 9 of Ref. [707], below which the limits are weaker than the median expected limit (but still within the 68%-containment region for the expected limit location [707]) and above which the limits are stronger than the median expected limit (lying at about the lower edge of the 95%-containment region for the expected limit location [707]). A similar feature (albeit less pronounced) is also seen in the $b\bar{b}$ limit line in the same Figure

in Ref. [707], but is not impactful for the specific comparison at hand as it occurs for a DM mass around 1 TeV, which is at the upper range of masses for which limits were reported in Ref. [452]; these ‘kink’ features are explicitly noted, highlighted, and discussed in Ref. [707].

We thus expect that an update to our MW-dwarf limits discussed in Section 2.3.4 to account for the results of Ref. [707] would (a) allow us to extend the analysis to larger DM mass, and (b) yield some potentially significant improvements compared to the results we present there: these improvements should be on average about a factor of 2–3 for lower DM mass per the above discussion, and stronger for larger DM masses. (Note that although the more dramatic improvements for the $\tau^+\tau^-$ final state at large DM mass are not likely to be directly relevant for us since very few of our operators have significant $\tau^+\tau^-$ branching fractions for larger DM mass, we would still expect that the underlying flux limits driving these larger improvements for larger DM mass would manifest as larger improvements in the limits set with other final states too.) However, the limitations on our interpretation and implementation of these dwarf limits as applied to our operators, which we discussed in Section 2.3.4, remain even with the latest results of Ref. [707]. This summary of the updates that would be expected notwithstanding, we have not updated our results to explicitly account for the newer Fermi-LAT/DES MW-dwarf limits from Ref. [707].

APPENDIX B

APPENDICES TO CHAPTER 3

This appendix contains material from Ref. [5]: M. A. Fedderke, J.-Y. Chen, E. W. Kolb, and L.-T. Wang, *The Fermionic Dark Matter Higgs Portal: an effective field theory approach*, *JHEP* **1408** (2014) 122 [[arXiv:1404.2283](#)].¹

B.1 Additional Considerations for Solving the Boltzmann Equation

For completeness, we present in this section summaries of some well-known results that are relevant to the solution of the Boltzmann equation.

B.1.1 Selected Thermodynamics Results

We start with a collection of some thermodynamics results which are relevant for the relic density computation outlined in Section 3.5. All of these results are in standard textbooks; see, e.g., Refs. [120, 709].

Consider a single relativistic free particle species i characterized by Fermi–Dirac (+; FD) or Bose–Einstein (−; BE) statistics, with g_i degrees of freedom (we use a counting where particles and anti-particles are counted as different species, so $g_i = 3$ for a massive vector, $g_i = 2$ for any kind of fermion or for a massless vector, and $g_i = 1$ for any kind of scalar). In local thermodynamic equilibrium, the number density, energy density and entropy density for a single species are given, respectively, by

$$n_i(T_i) = \frac{g_i}{2\pi^2} T_i^3 I_1^\pm(x_i = m_i/T_i) \quad (\text{single species}), \quad (\text{B.1})$$

$$\rho_i(T_i) = \frac{g_i}{2\pi^2} T_i^4 I_2^\pm(x_i = m_i/T_i) \quad (\text{single species}), \quad (\text{B.2})$$

1. Ref. [5] is copyright © 2014 by the Authors, and is under a Creative Commons Attribution (CC BY) 4.0 Licence [2].

and

$$s_i(T_i) = \frac{g_i}{2\pi^2} T_i^3 \left[\frac{4}{3} I_2^\pm(x_i = m_i/T_i) - \frac{x_i^2}{3} I_0^\pm(x_i = m_i/T_i) \right] \quad (\text{single species}), \quad (\text{B.3})$$

while the total energy and entropy densities are, respectively, conventionally expressed as

$$\rho(T) = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4; \quad g_{\text{eff}}(T) \equiv \frac{15}{\pi^4} \sum_i g_i \left(\frac{T_i}{T} \right)^4 I_2^\pm(x_i = m_i/T_i), \quad (\text{B.4})$$

$$s(T) = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3; \quad h_{\text{eff}}(T) \equiv \frac{15}{\pi^4} \sum_i g_i \left(\frac{T_i}{T} \right)^3 \left[I_2^\pm(x_i = m_i/T_i) - \frac{x_i^2}{4} I_0^\pm(x_i = m_i/T_i) \right], \quad (\text{B.5})$$

where the sum is over all species i (counting particles and anti-particles independently)² and the integrals $I_n^\pm(x)$ are defined as

$$I_n^\pm(x_i) \equiv \int_{x_i}^{\infty} \frac{y^n \sqrt{y^2 - x_i^2}}{e^y \pm 1} dy. \quad (\text{B.6})$$

These integrals have the following limits (relativistic; $x \rightarrow 0$) or asymptotic forms (non-relativistic; $x \rightarrow \infty$):

$$I_0^\pm(x \rightarrow 0) = \begin{cases} \frac{\pi^2}{12} & \text{FD} + \\ \frac{\pi^2}{6} & \text{BE} - \end{cases}; \quad (\text{B.7})$$

$$I_1^\pm(x \rightarrow 0) = \begin{cases} \frac{3\zeta(3)}{2} & \text{FD} + \\ 2\zeta(3) & \text{BE} - \end{cases}; \quad (\text{B.8})$$

$$I_2^\pm(x \rightarrow 0) = \begin{cases} \frac{7\pi^4}{120} & \text{FD} + \\ \frac{\pi^4}{15} & \text{BE} - \end{cases}; \quad (\text{B.9})$$

2. Note that even if the SM neutrinos are Dirac, the SM-singlet ν^c states are not to be included in the sum as their only SM-interactions are neutrino-mass suppressed and they are thus not populated, and do not reach thermal equilibrium, in the early universe before neutrino decoupling at $T \sim 1$ MeV [710].

and

$$I_n^\pm(x \rightarrow \infty) = \mp \sqrt{\frac{\pi}{2}} x^{n+1/2} \left[\text{Li}_{3/2}(\mp e^{-x}) + \left(\frac{12n+3}{8x} \right) \text{Li}_{5/2}(\mp e^{-x}) + \mathcal{O}(x^{-2}e^{-x}) \right], \quad (\text{B.10})$$

where $\zeta(3) = 1.20205690 \dots$ and $\text{Li}_n(x)$ is the polylogarithm.

In the above expressions, T_i is the temperature of species i . For all species except for neutrinos, T_i is just the photon temperature because massive species decouple after they go non-relativistic. Neutrinos however remain relativistic after decoupling around $T \sim 1$ MeV, while electrons only decouple at lower temperature. To account for this, consider the following argument taken from Ref. [120]. After the neutrino decoupling, the co-moving entropies in each of the two decoupled baths (neutrinos on one hand; electrons, positrons and photons on the other hand) are separately conserved. Thus, the neutrinos will maintain their distribution at decoupling (Fermi–Dirac), but with a temperature that redshifts as $T_\nu \sim a^{-1}$ where a is the FLRW scale factor (because $s_\nu a^3 \sim \text{const.}$). On the other hand, the photon bath is reheated after neutrino decoupling because the entropy in the electrons and positrons is dumped into photons when the former decouple. Since the comoving entropy in photons, electrons and positrons is conserved, we have that $s_{\gamma,e^\pm} a^3 \propto s_{\gamma,e^\pm} T_\nu^{-3} \sim \text{const.}$ Using that the species initially share a common temperature, it is then straightforward to derive that

$$\frac{T}{T_\nu(x_e)} = \left(\frac{h_{\text{eff}}^{\gamma,e^\pm}(x=0)}{h_{\text{eff}}^{\gamma,e^\pm}(x_e)} \right)^{\frac{1}{3}} = \left(\frac{11}{4} \right)^{\frac{1}{3}} \left[1 + \frac{30}{\pi^4} \left(I_2^+(x_e) - \frac{x_e^2}{4} I_0^+(x_e) \right) \right]^{-\frac{1}{3}}, \quad (\text{B.11})$$

where $x_e = m_e/T$. In the NR limit $x_e \rightarrow \infty$, $T/T_\nu = 1.40$.

To fully follow the thermal history of the universe when solving the Boltzmann equation as outlined in Section 3.5, one must also account for the QCD phase transition causing a loss of the light quark and gluon degrees of freedom around $T = T_{\text{QCD}} \sim 150$ MeV. In our numerical work in Section 3.5, we accounted for this phase transition by simply smoothly

turning off T_q and T_g for $T < T_{\text{QCD}}$ by writing $T_q = T_g = \frac{1}{2}T [1 + \tanh((T - T_{\text{QCD}})/\delta T)]$ where we took $\delta T \sim 150$ MeV. This is of course a somewhat crude approach; for more detailed treatments, see, e.g., Refs. [711].

B.1.2 Boltzmann Equation Numerics

Our solution of the Boltzmann equation Eq. (3.18) in Section 3.5 followed Refs. [120, 524]; we summarize the relevant results here. The Boltzmann equation Eq. (3.18) is first recast to the form

$$\frac{dY}{dx} = +\frac{\langle\sigma v\rangle}{3H}\frac{ds}{dx}\left[Y^2 - Y_{\text{EQ}}^2\right] = -\langle\sigma v\rangle\sqrt{\frac{\pi}{45}}\frac{MM_{\text{Pl}}}{x^2}\sqrt{g_*(x)}\left[Y^2 - Y_{\text{EQ}}^2\right] \quad (\text{B.12})$$

$$\text{where} \quad \sqrt{g_*(x)} \equiv \frac{h_{\text{eff}}(x)}{\sqrt{g_{\text{eff}}(x)}}\left[1 - \frac{x}{3}\frac{1}{h_{\text{eff}}(x)}\frac{dh_{\text{eff}}(x)}{dx}\right], \quad (\text{B.13})$$

$Y \equiv n/s$, and $x \equiv M/T$. To arrive at this form, Eq. (3.18) is combined with the equation for comoving entropy conservation $\dot{s} + 3Hs = 0$, and the Friedmann equation and the expressions for ρ and s given at Eqs. (B.4) and (B.5), respectively, are used [120, 524]. Since $Y_{\text{EQ}} \equiv n_{\text{EQ}}/s$ we have (recall that $g_{\text{DM}} = 3$ for a massive vector, $g_{\text{DM}} = 2$ for any kind of fermion or a massless vector, and $g_{\text{DM}} = 1$ for any kind of scalar)

$$Y_{\text{EQ}}(x) = \frac{45}{4\pi^4}\frac{g_{\text{DM}}}{h_{\text{eff}}(x)}I_1^\pm(x). \quad (\text{B.14})$$

In principle, this equation is solved by specifying the initial condition $Y(x_0) = Y_{\text{EQ}}(x_0)$ at some point $x_0 \lesssim 1.0$ [524], and then evolving it out to large $x \gtrsim 400$ until the equilibrium plateau $Y_\infty \equiv Y(x \rightarrow \infty)$ is reached.

Since this is computationally intensive, we followed Ref. [524] and solved the Boltzmann equation until the condition $Y(T_F) = (1 + \delta)Y_{\text{EQ}}(T_F)$ was met at some $T_F = M/x_F$ to be determined numerically; we took $\delta = 1.5$ [524]. Per Ref. [524], we then dropped the Y_{EQ}

term in Eq. (B.13), and simply integrated (in the ordinary sense) to obtain Y_∞ :

$$\frac{1}{Y_\infty} = \frac{1}{Y(T_F)} - \int_{x_F}^{\infty} \left[-\langle \sigma v \rangle \sqrt{\frac{\pi}{45}} \frac{MM_{\text{Pl}}}{x^2} \sqrt{g_*(x)} \right] dx. \quad (\text{B.15})$$

B.2 Selected Results in Terms of (M_0, θ)

It is interesting to view the limits we have presented in the main text of Chapter 3 also in terms of (M_0, θ) , as these are the parameters which appear in the manifestly gauge-invariant Lagrangian Eq. (3.1). However, as discussed in Section 3.2, the map $(M, \xi) \mapsto (M_0, \theta)$ [see Eqs. (3.8) and (3.10)] is not necessarily one-to-one (injective) once we impose the relic density constraint $\Lambda = \Lambda(M, \xi)$. This makes the general presentation of our results in terms of (M_0, θ) challenging. However, provided the physical mass of the DM particle, M , satisfies the constraint $M \geq m_W$, we can present some restricted results. The form of $\Lambda(M, \xi)$ is sufficiently simple in this region (see Figure 3.5) that the map $(M, \xi) \mapsto (M_0, \theta)$ constrained to the domain $M > m_W$ is indeed one-to-one; or, viewed in the other direction, the map $(M_0, \theta) \mapsto (M, \xi)$ is single-valued if restricted to the range $M > m_W$. This allows us to present the results of our analysis of the LUX direct detection bounds [506], which are the only ones relevant in the regime $M > m_W$, in terms of the variables $(M_0, \cos \theta)$; we do this in the plots in the right column of Figure B.1. We stress that, owing to the considerations already outlined, the plots in the right column of Figure B.1 cannot be interpreted naïvely as showing regions of $(M_0, \cos \theta)$ which are ruled out by direct detection assuming the correct relic abundance; they may only be interpreted in this fashion if additionally one assumes the constraint on the physical DM mass, $M \geq m_W$. Also shown in the left column of Figure B.1 are the direct detection constraints in terms of $(M, \cos \xi)$; these results are a subset of those already shown in Figure 3.7, where they were presented as a function of $(M, \cos^2 \xi)$. The re-presentation here is to facilitate more direct comparison between the nature of the constrained regions of parameter space when viewed in each set of variables.

In the large M (or M_0) regions the constrained parameter space is broadly similar for the

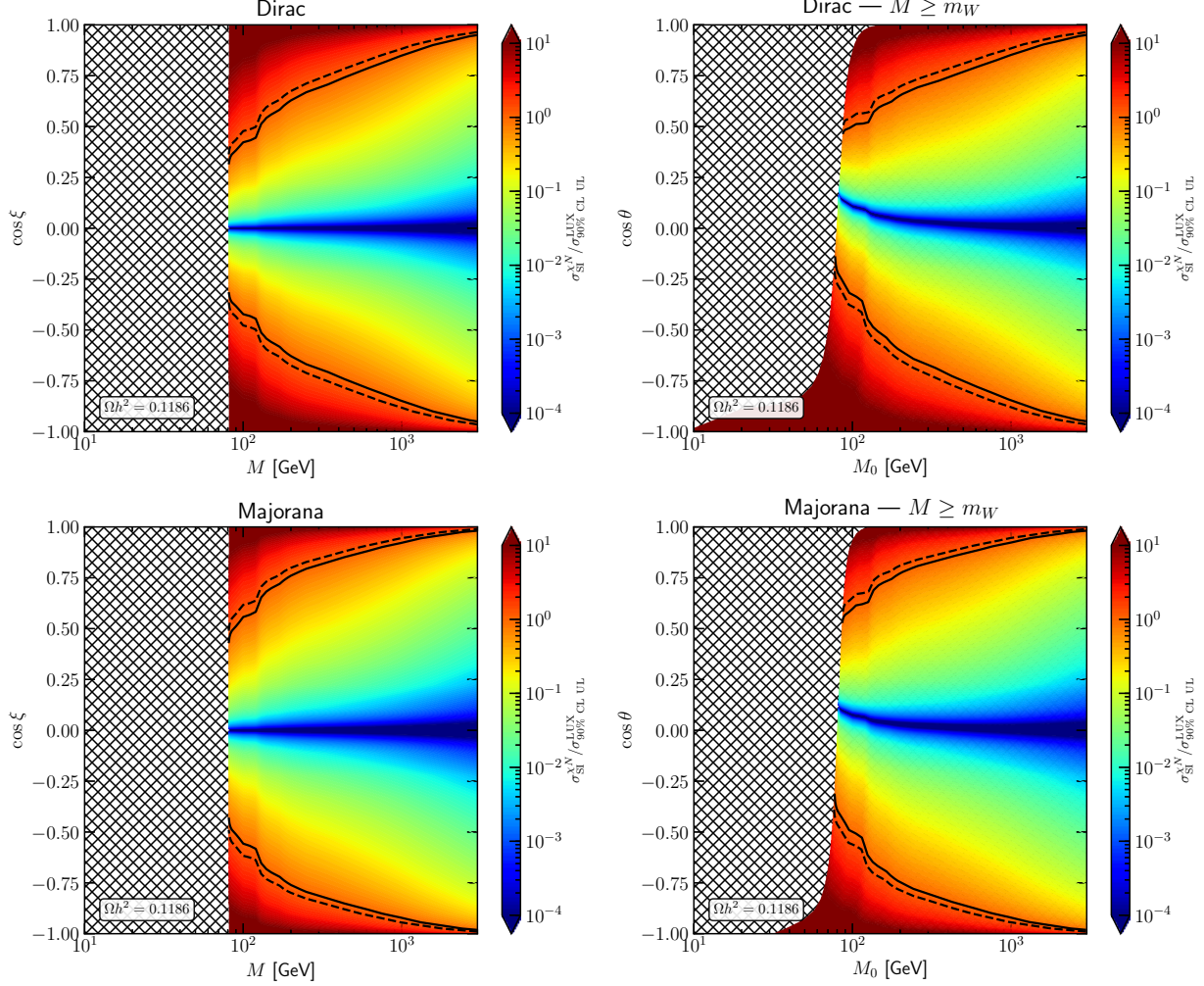


Figure B.1: Direct detection bounds computed in the same way as for Figure 3.7, in the regions where the physical DM mass satisfies the constraint $M \geq m_W$, presented in terms of the variables $(M, \cos \xi)$ in the left column and $(M_0, \cos \theta)$ in the right column, for Dirac fermion DM (top row) and Majorana fermion DM (bottom row). All features are as described in the caption of Figure 3.7, with the exception that the hatching here indicates that $M < M_W$. Important caveats stated in the main text apply to the interpretation of the results in the right column. These results are computed using the 2013 LUX direct detection bounds from Ref. [506].

two sets of variables, but as either of these parameters (or Λ) decreases, the nature of the constrained regions begins to differ due to the increasing importance of the chiral rotation, Eq. (3.5): we note in particular that the constraints in terms of $(M_0, \cos \theta)$ [assuming $M \geq m_W$] are not symmetric about $\cos \theta = 0$, whereas those in terms of $(M, \cos \xi)$ are symmetric (provided we continue to ignore the t - and u - channel diagrams; see Figure 3.6). The origin of this asymmetry is already manifest in Eq. (3.10).

B.3 Discussion of the Direct Detection Cross Section

Our result for the SI direct detection cross section in Section 3.6, Eq. (3.23), requires some clarification.

In a direct detection experiment, the observable is the differential rate dR/dE_R of interaction events per detector mass, as a function of the nuclear recoil energy E_R and is given by (see, e.g., Refs. [212, 485, 712])

$$\frac{dR}{dE_R} = \frac{2\rho_0}{M} \int_{v > v_{\min}(E_R)} d^3v v f_G(\mathbf{v}_D + \mathbf{v}) \frac{d\sigma_{\chi T}}{dQ^2} \quad (v \equiv \sqrt{\mathbf{v} \cdot \mathbf{v}}) \quad (\text{B.16})$$

$$\text{where } v_{\min}(E_R) \equiv \sqrt{\frac{M_T E_R}{2\mu_{\chi T}^2}}. \quad (\text{B.17})$$

Assuming that $\frac{d\sigma_{\chi T}}{dQ^2} \sim v^{-2}$ (which is the usual case for ‘non-velocity suppressed’ cross sections), Eq. (B.16) can be written in the following form:

$$\frac{dR}{dE_R} = \frac{\rho_0 A^2}{2M\mu_{\chi N}^2} \cdot \sigma_{\chi N}^0(E_{R,0}) \cdot \left(\frac{E_R}{E_{R,0}}\right)^n \mathcal{F}_0(E_R) |F(E_R)|^2 \quad (\text{B.18})$$

$$\text{with } \sigma_{\chi N}^0(E_{R,0}) \equiv \frac{4\mu_{\chi N}^2}{A^2} (2M_T E_{R,0})^n \left[v^2 \frac{1}{Q^{2n}} \frac{d\sigma_{\chi T}}{dQ^2} \right]_{Q^2=0}, \quad (\text{B.19})$$

In the above expressions,

- $\frac{d\sigma_{\chi T}}{dQ^2}$ is the differential WIMP–target cross section;

•

$$\mathcal{F}_0(E_R) \equiv \int_{v > v_{\min}(E_R)} d^3v \frac{f_G(\mathbf{v}_D + \mathbf{v})}{v} \quad (v \equiv \sqrt{\mathbf{v} \cdot \mathbf{v}}) \quad (\text{B.20})$$

$$\text{where } v_{\min}(E_R) \equiv \sqrt{\frac{M_T E_R}{2\mu_{\chi T}^2}}, \quad (\text{B.21})$$

with f_G the galactic-rest-frame DM velocity distribution function [712, 713] and $\mathbf{v}_D$ the detector velocity, accounts for the distributions of velocities $\mathbf{v}$ of the incoming DM;

- A is the atomic mass number of the nuclear target;
- ρ_0 is the local DM density;
- $\mu_{\chi N} \equiv \frac{MM_N}{M+M_N}$ with M the DM mass and M_N the nucleon mass, is the WIMP–nucleon reduced mass; similarly, $\mu_{\chi T} \equiv \frac{MM_T}{M+M_T}$ is the WIMP–nuclear-target reduced mass;
- $E_{R,0}$ is an arbitrary reference recoil energy which cancels in the final result;
- n is an integer chosen such that $\sigma_{\chi N}^0(E_{R,0})$ has a finite, non-zero value in the $E_R \rightarrow 0$ ($Q^2 \rightarrow 0$) limit; and
- $F(E_R)$ is a nuclear form factor normalized to $F(0) = 1$, usually taken to be the Helm form factor for SI couplings (see, e.g., Refs. [214, 485, 506, 712, 713]).

In any experimental setting, this rate must be smeared with a detector resolution function and corrected by the detector efficiency function. Note that the expression in the $[\dots]$ -brackets in Eq. (B.18) is independent of the DM speed v per our assumption that $\frac{d\sigma_{\chi T}}{dQ^2} \sim v^{-2}$.

For Lagrangians $\mathcal{L}^{\text{SS}} = c_q^{\text{SS}} \bar{\chi} \chi \bar{q} q$ and $\mathcal{L}^{\text{PS}} = c_q^{\text{PS}} \bar{\chi} i \gamma_5 \chi \bar{q} q$, respectively, it can be shown that the leading behaviour is [714–717]

$$\sigma_{\chi N}^{0, \text{SS}}(E_{R,0}) = \frac{\mu_{\chi N}^2}{\pi} \left[f_n^{\text{SS}} + \frac{Z}{A} (f_p^{\text{SS}} - f_n^{\text{SS}}) \right]^2 \quad (n = 0) \quad (\text{B.22})$$

$$\sigma_{\chi N}^{0, \text{PS}}(E_{R,0}) = \frac{m_T E_R^0}{2M^2} \frac{\mu_{\chi N}^2}{\pi} \left[f_n^{\text{PS}} + \frac{Z}{A} (f_p^{\text{PS}} - f_n^{\text{PS}}) \right]^2 \quad (n = 1), \quad (\text{B.23})$$

where Z is the atomic number;

$$\frac{f_N}{M_N} = \sum_{q=u,d,s} \frac{c_q}{m_q} f_{Tq}^N + \frac{2}{27} \left(\sum_{Q=c,b,t} \frac{c_Q}{m_Q} \right) f_{TG}^N, \quad (\text{B.24})$$

for $N = p, n$ [485] (we drop the superscripts SS and PS here, but they are understood to be common); and where f_{Tq}^N and f_{TG}^N are the usual scalar matrix elements (see, e.g., Ref. [510] for a recent review). Note that we have kept only those terms which correspond to the leading $E_R \propto Q^2$ dependence in each case.

For our couplings in Chapter 3, we have $c_q^{\text{SS}} = -\frac{m_q \cos \xi}{\Lambda m_h^2}$ and $c_q^{\text{PS}} = -\frac{m_q \sin \xi}{\Lambda m_h^2}$. Assuming exact isospin symmetry, writing $f_N = m_N \left[\sum_{q=u,d,s} f_{Tq}^N + \frac{2}{9} f_{TG}^N \right]$ (it is irrelevant whether we use $N = p, n$ here since they yield identical values in the isospin-symmetric limit; technically, we utilize the average nucleon mass), and noting that there is no interference term in the cross section in the NR limit between the Lagrangian terms for SS and PS couplings, we can write the total differential rate for our interactions as

$$\frac{dR}{dE_R} = \frac{\rho_0 A^2}{2M \mu_{\chi N}^2} \left[\frac{\mu_{\chi N}^2 \cos^2 \xi}{\pi} \frac{f_N^2}{m_h^4 \Lambda^2} + \frac{m_T E_R \mu_{\chi N}^2 \sin^2 \xi}{2M^2 \pi} \frac{f_N^2}{m_h^4 \Lambda^2} \right] \mathcal{F}_0(E_R) |F(E_R)|^2 \epsilon(E_R). \quad (\text{B.25})$$

The usual LUX spin-independent cross section limits [214, 506] are actually limits on the value of $\sigma_{\chi N}^0$ in Eq. (B.18) for the $n = 0$ case. For the scalar part of the coupling, the appropriate quantity to compare to the LUX limit is the scalar-coupling contribution to the $[\dots]$ -bracket in Eq. (B.25); that is, we should compare to the LUX limit the quantity

$$\frac{\mu_{\chi N}^2 \cos^2 \xi}{\pi} \frac{f_N^2}{m_h^4 \Lambda^2}, \quad (\text{B.26})$$

this agrees with the scalar contribution to the total WIMP–nucleon cross section which we wrote in Eq. (3.23).

Even a cursory examination of the pseudoscalar contribution to Eq. (B.25) shows that

(as we mentioned in Section 3.6.1) a correct extraction of the limit on the pseudoscalar interaction requires dealing with additional non-trivial E_R -dependence in the differential rate, beyond the usual form factor E_R -dependence. The correct way to do this is to modify the profile likelihood analysis that the LUX Collaboration [214, 506] performs on their underlying event data to account for the modified DM event model; as we mentioned in Section 3.6.1, such an analysis is far beyond our intended scope. We can however make an argument that the pseudoscalar contribution to Eq. (B.25) can be approximately treated as per Eq. (3.23) in the main text.

Suppose for the sake of the argument here that we ignore the Earth's motion in the galaxy and assume that $f_G = \frac{1}{4\pi v_0^2} \delta(v - v_0)$, which is the assumption that all the DM is incident on the detector with the same speed v_0 . This is of course unphysical; a more careful treatment would use, e.g., a truncated Maxwellian DM velocity distribution in the Galactic rest frame [214, 506, 713], and would minimally account for the average velocity of the Earth in the galaxy. The simpler assumption allows us to make more analytical progress, and should not yield results that are too far from correct, provided that v_0 is taken to be on the order of the average speed of DM particles incident on the detector under a truncated Maxwellian velocity profile assumption: $v_0 \sim 200\text{--}300\text{km/s}$.

In any event, in this case, $\mathcal{F}_0(E_R) = v_0^{-1} \Theta\left((2\mu_{\chi T}^2 v_0^2 / m_T) - E_R\right)$ where $\Theta(x)$ is the Heaviside function; that is, E_R can only take values $E_R \in [0, E_R^{\max}]$ where $E_R^{\max} = 2v_0^2 \frac{\mu_{\chi T}^2}{m_T}$. Thus, the pseudoscalar contribution in the $[\dots]$ -bracket in Eq. (B.25) has an average value on the allowed recoil energy range of

$$\frac{1}{2} \left(\frac{\mu_{\chi T}}{M} \right)^2 v_0^2 \frac{\mu_{\chi N}^2}{\pi} \frac{\sin^2 \xi}{m_h^4} \frac{f_N^2}{\Lambda^2}. \quad (\text{B.27})$$

Note that since the quantity being averaged is linear, this is just equivalent to evaluating the quantity at the average recoil energy $\bar{E}_R = v_0^2 \frac{\mu_{\chi T}^2}{m_T}$. Here we can also replace $\sin^2 \xi = 1 - \cos^2 \xi \rightarrow 1$ since the error we make in doing so is proportional to $v_0^2 \cos^2 \xi$, which is

a subdominant correction to the scalar contribution to Eq. (B.25), at the same order as terms we already neglected in computing that dominant contribution. But having made that replacement, the average pseudoscalar contribution to the $[\dots]$ -bracket in Eq. (B.25) can be approximated as

$$\frac{1}{2} \left(\frac{\mu_{\chi T}}{M} \right)^2 v_0^2 \frac{\mu_{\chi N}^2}{\pi} \frac{1}{m_h^4} \frac{f_N^2}{\Lambda^2}; \quad (\text{B.28})$$

this agrees with the contribution to the total SI cross section which we wrote in Eq. (3.23). Therefore, our treatment in Section 3.6 is equivalent to assuming all the DM incident on the detector is at the same speed v_0 ; assuming that we can neglect the additional E_R -dependence in the differential recoil rate, simply replacing the additional E_R -dependent term with its average over the allowed range of recoil energies in this approximation; and keeping only the leading- E_R (i.e., Q^2) behaviour.

We acknowledge that these approximations are somewhat crude; absent performing a representative profile likelihood analysis for comparison, we cannot be quantitative in our statement of exactly how crude they are, but the given that our simplifying approximations are reasonably well motivated, we would estimate that a more careful analysis would be quite unlikely to yield a result which was stricter by more than one to two orders of magnitude. As we argued already in Section 3.6.1, an accuracy of one to two orders of magnitude in our estimate for the pseudoscalar contribution to the direct detection cross section is sufficient to still obtain accurate parameter space limits given current experimental sensitivities; this would of course need to be revisited when future tonne-scale experiments such as XENON-1T [219] or LZ [220] report results from their full exposure.

Note also that the analysis here is predicated on using the current LUX bounds [214, 506] on the SI cross section, obtained using their standard analysis strategy, to extract an approximate bound on the pseudoscalar contribution. It is however likely that a more optimized LUX analysis (as discussed in, e.g., Ref. [718]) would be possible for operators

whose differential rate is skewed to larger E_R ; such an analysis may yield stronger constraints than those we estimate here.

B.4 Update on Spin-Independent Direct Detection Limits

All of the direct detection results shown in the main text of Chapter 3 were based on the 2013 LUX SI direct detection limits [506] which were the state-of-the-art when Ref. [5], the paper on which Chapter 3 is based, was published; however, there have been subsequent significant improvements in the state-of-the-art direct detection bounds [214]. In this appendix, we aim to discuss the impact of these improvements on our results.

Figure B.2 shows the improvement in the LUX 90% confidence spin-independent (SI) cross section upper limits from the 2013 analysis [506] to the 2016 analysis [214]. As we discussed in Chapter 3, the Higgs width limits already robustly rule out the whole of the parameter space below c. 55 GeV (the exact number depends on the exact assumptions made; see Chapter 3); above this mass, the direct detection limits are the most relevant. Therefore, we can confine ourselves to a consideration of the improvement in the direct detection bounds from LUX in the region $M \gtrsim 55$ GeV. As is demonstrated in Figure B.2, on this mass range, the limits have improved by about a factor of 7.6 (independent of M) from the 2013 analysis [506] to the 2016 analysis [214], where the former is available ($M \leq 5$ TeV). The latter analysis also extends to much larger DM mass: $M \leq 100$ TeV. In conjunction with the results shown in Figure 3.7, it is straightforward to read off where the limits would now lie, taking into account the new LUX results [214]: merely find the contour $\sigma_{\text{SI}}^{\chi^N} / \sigma_{90\% \text{ CL UL}}^{\text{LUX 2013}} = 1/7.6 \approx 0.13$ on the colourmap in Figure 3.7. Because the colourmap is a hard to read precisely, and because the indication is that the updated LUX limits from Ref. [214] are significantly more constraining on the $M\text{--}\cos^2\xi$ parameter space than those from Ref. [506], we explicitly present an updated version of Figure 3.7 in Figure B.3, showing exactly where the limit line now lies. As expected, the remaining parameter space which was indicated in Figure 3.7 has now been severely further constrained, and even the resonant Portal region [434] is almost

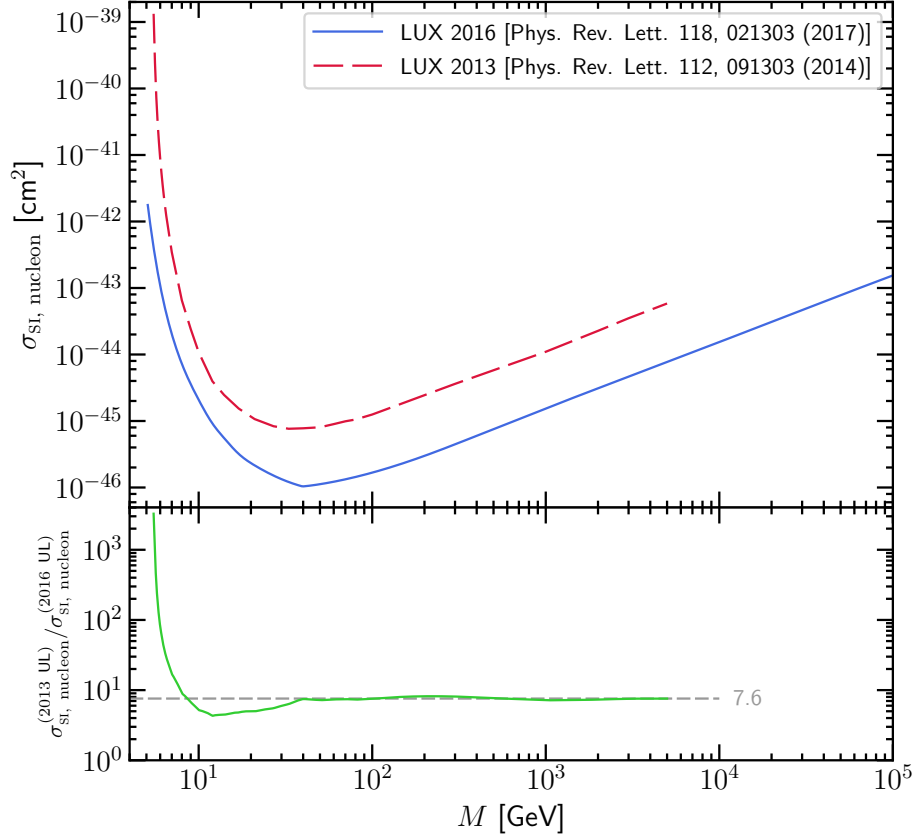


Figure B.2: The improvement in the LUX 90%-confidence spin-independent (SI) cross section upper limits from the 2013 analysis [506] (dashed red line, top panel) to the 2016 analysis [214] (solid blue line, top panel). The lower panel shows the ratio (solid green line) of the 2013 to 2016 limits, indicating that above $M = 40$ GeV, the improvement is approximately a factor of 7.6, roughly independent of the assumed DM mass (indicated by the dashed grey line in the lower panel). Between 8 and 40 GeV, the improvement is more modest, while below 8 GeV, improvements in the analysis techniques that lowered the detector recoil energy threshold in the analysis of Ref. [214] compared to that of Ref. [506] imply dramatically better limits in the former compared to the latter. In the analysis we presented in Chapter 3, we utilized the LUX limits from Ref. [506]. See the text of Appendix B.4 for a discussion of the estimated impact of the improved limits from Ref. [214] on our results presented in Chapter 3.

ruled out for the mostly scalar case for the Dirac fermion choice; Figure B.4 shows a similarly updated version of Figure 3.8. Note that we could also have extended the region of DM mass parameter space presented in Figure B.3 to take advantage of the larger range of DM masses ($M \leq 100 \text{ TeV}$ [214]) that are considered in the analysis in Ref. [214], but as this would require much more significant portions of the analysis presented in Chapter 3 (i.e., Ref. [5]) to be re-run to extract the necessary EFT suppression scales Λ in the higher mass region, such an update is beyond the intended scope of this appendix. Additionally, perturbative unitarity issues (i.e., $M > 2\pi\Lambda$) would most likely invalidate the EFT analysis before M reaches 100 TeV, so the effective increase in the range of masses which can be probed in this EFT analysis with the updated LUX limits [214] will not be quite as dramatic as one might naïvely expect.

As this dissertation was being finalized, the first results from the XENON-1T experiment were released in preprint [719]; they are consistent with a background-only hypothesis [719]. At large DM mass, the 90%-confidence SI cross section limits show a marginal improvement over the 2016 LUX results of Ref. [214], with a more significant improvement of up to a factor of approximately 2 in the 10–40 GeV DM mass range [719].

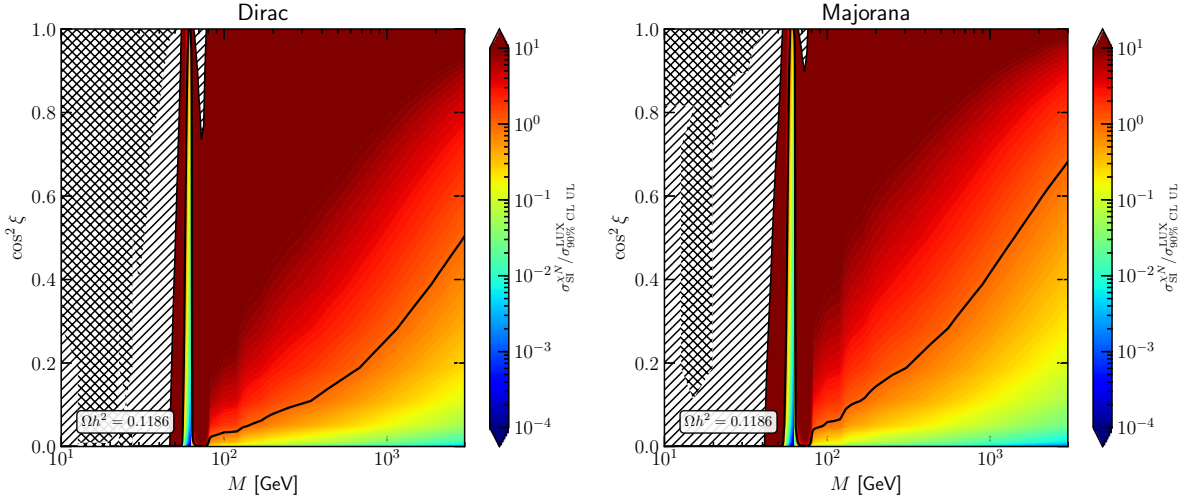


Figure B.3: Colourmaps representing interpolated values of $\sigma_{\text{SI}}^{\chi^N} / \sigma_{90\% \text{ CL UL}}^{\text{LUX}}$, with the solid black line showing the equality of the computed cross section and the updated (2016) LUX limits [214] (but still restricting to a maximum DM mass of 3 TeV). As in Figure 3.7, redder points ‘above’ the black line are excluded, bluer points ‘below’ the black line are allowed. The singly hatched region is where $\Lambda < \langle v \rangle$; the doubly hatched region at low mass is where no Λ value can be found to obtain the correct relic density. Comparison with Figure 3.7, which is the cognate plot using the 2013 limits from LUX [506], indicates that the updated LUX limits have resulted in much more of the parameter space being ruled out for these Portal couplings. Note that there is still a mass region near $M \approx m_h/2$ that is allowed for any value of ξ (this is the resonant Higgs Portal scenario [434]), but the region around $\xi = 0$ is now severely constrained, especially for the Dirac case.

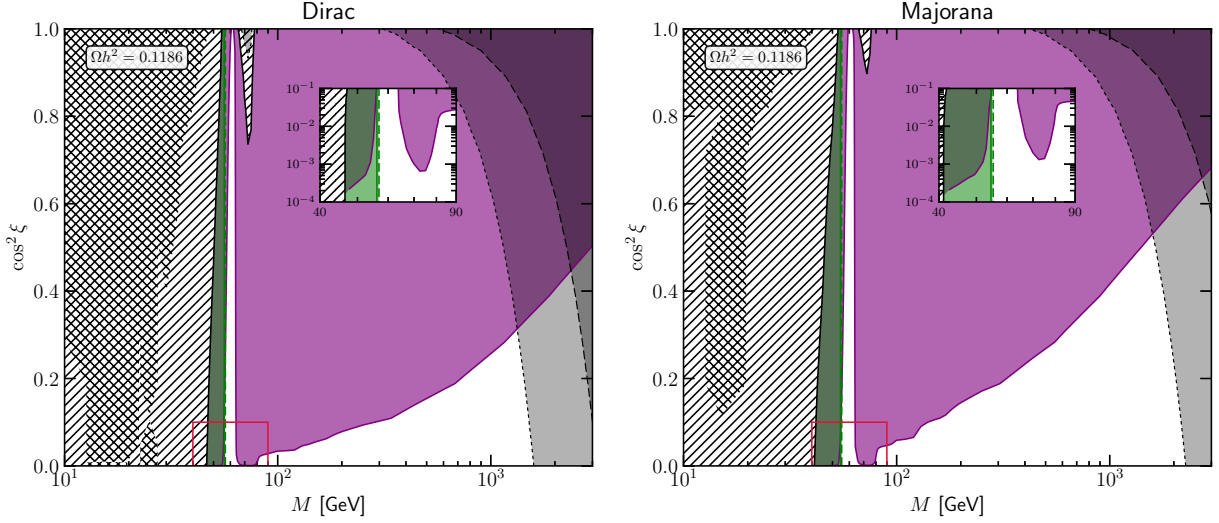


Figure B.4: Combined limits keeping only Λ^{-2} terms in all cross sections, using the updated (2016) LUX limits [214] (but still restricting to a maximum DM mass of 3 TeV); comparison to Figure 3.8 indicates that much more of the parameter space is now excluded in the large-mass region, and that the resonant Higgs Portal scenario [434] is now more constrained, particularly for the mostly scalar coupling for Dirac fermion DM. The grey regions bounded by black lines indicate the regions where $\Lambda \leq 2M$ (dotted line, light grey) and $\Lambda \leq M$ (dashed line, dark grey). The singly hatched region is where $\Lambda < v$. For reference, the simplest perturbative UV completion requires $\Lambda \gtrsim M/2\pi$. The doubly hatched region at low mass is where no Λ value can be found to obtain the correct relic density. The purple shaded regions bounded by purple lines are the regions excluded by LUX [214] direct detection limits at 90%-confidence (solid line); in contradistinction to Figure 3.8, we do not show a 95%-confidence exclusion on this plot. The green regions bounded by green lines indicate regions excluded by invisible width constraints arising from a global fit to data [507] with the SM-Higgs couplings floating (solid line) or fixed to SM values (dashed line); for clarity, we do not show the limits from the CMS direct width constraints [508] as they are slightly weaker. The insets show detail for small $\cos^2 \xi$ for masses $M \in [40, 90]$ GeV (i.e., the regions indicated by a red boxes on the main axes) and indicate that the coupling must be nearly pure pseudoscalar for masses around 70 GeV to not conflict with data.

APPENDIX C

APPENDICES TO CHAPTER 4

This appendix contains material from Ref. [6]: M. A. Fedderke, T. Lin, and L.-T. Wang, *Probing the fermionic Higgs portal at lepton colliders*, *JHEP* **1604** (2016) 160 [arXiv:1506.05465].¹

C.1 One-Loop Computation of S and T

The one-loop computation of S and T discussed in Section 4.4.4 was performed in dimensional regularization in the $\overline{\text{MS}}$ scheme. The IJ self-energy (where I and J are SM vector bosons) arises from diagrams such as those in Figure C.1. For particles A and B with masses M_A and M_B , respectively, running in the loop, we find a contribution

$$\Pi_{IJ;AB}(p^2) = \frac{g_{IAB} \cdot g_{JAB}}{12\pi^2} \times \left[\begin{aligned} & A_0(M_A^2) + A_0(M_B^2) - (M_A^2 + M_B^2) + \frac{1}{3}p^2 - p^2 B_0(p^2, M_A^2, M_B^2) \\ & + \frac{1}{2}B_0(p^2, M_A^2, M_B^2) \left((M_A - M_B)^2 - 4M_A M_B \right) \\ & + \frac{1}{2} \frac{B_0(p^2, M_A^2, M_B^2) - B_0(0, M_A^2, M_B^2)}{p^2} (M_A^2 - M_B^2)^2 \\ & + \text{counter-terms} \end{aligned} \right] \quad (\text{C.1})$$

where A_0 and B_0 are the usual Passarino-Veltman scalar integrals [720]. Expanding $\Pi(p^2)$ in powers of the gauge-boson momentum as $\Pi(p^2) = \Pi(0) + p^2 \Pi'(0) + \dots$, we find, at the

1. Ref. [6] is copyright © 2016 by the Authors, and is under a Creative Commons Attribution (CC BY) 4.0 Licence [2].

$\overline{\text{MS}}$ scale μ ,

$$\begin{aligned} \Pi_{IJ;AB}(0) &= \frac{g_{IAB} \cdot g_{JAB}}{16\pi^2} \\ &\times \left[\begin{aligned} &M_A^2 - 4M_A M_B + M_B^2 \\ &+ \frac{1}{3} (M_A^2 - M_B^2)^{-1} \begin{bmatrix} 2M_A^2 M_B^2 \log(M_B^2/M_A^2) \\ -2M_A^2 (6M_A M_B - 3M_A^2 + M_B^2) \log(\mu^2/M_A^2) \\ +2M_B^2 (6M_A M_B - 3M_B^2 + M_A^2) \log(\mu^2/M_B^2) \end{bmatrix} \end{aligned} \right] \end{aligned} \quad (\text{C.2})$$

$$\longrightarrow 0 \quad \text{as } M_A \rightarrow M_B, \quad (\text{C.3})$$

and

$$\begin{aligned} \Pi'_{IJ;AB}(0) &= \frac{g_{IAB} \cdot g_{JAB}}{16\pi^2} \times \left(-\frac{2}{9} \right) [M_A^2 - M_B^2]^{-3} \\ &\times \left[\begin{aligned} &(M_A^2 - M_B^2) \begin{pmatrix} 6M_A^2 (M_A^2 - M_B^2) \log(\mu^2/M_A^2) \\ +6M_B^2 (M_B^2 - M_A^2) \log(\mu^2/M_B^2) \\ +9M_A^3 M_B - 16M_A^2 M_B^2 + 9M_A M_B^3 + 2M_A^4 + 2M_B^4 \end{pmatrix} \\ &+ 6M_A^2 M_B^2 (-3M_A M_B + M_A^2 + M_B^2) \log(M_A^2/M_B^2) \end{aligned} \right] \end{aligned} \quad (\text{C.4})$$

$$\longrightarrow \frac{g_{IAB} \cdot g_{JAB}}{16\pi^2} \times \left(-\frac{4}{3} \right) \log(\mu^2/M^2) \quad \text{as } M_A \rightarrow M_B \equiv M. \quad (\text{C.5})$$

In the broken phase of the theory, the mass-eigenstate basis of new fermions is $\{n_1, n_2, C\}$, where

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \chi \\ N \end{pmatrix}, \quad (\text{C.6})$$

with the mixing angle given by

$$\tan 2\theta = \frac{\sqrt{2}\kappa v}{M_F - m_\chi}. \quad (\text{C.7})$$

These particles have eigenmasses

$$M_1 = \frac{1}{2} \left[m_\chi + M_F - \left[(M_F - m_\chi)^2 + 2\kappa^2 v^2 \right]^{1/2} \right], \quad (\text{C.8})$$

$$M_2 = \frac{1}{2} \left[m_\chi + M_F + \left[(M_F - m_\chi)^2 + 2\kappa^2 v^2 \right]^{1/2} \right], \quad \text{and} \quad M_C = M_F. \quad (\text{C.9})$$

The W^+W^- , ZZ , $Z\gamma$ and $\gamma\gamma$ self-energy functions and their derivatives can be immediately found by summing the contributions from Eqs. (C.2)–(C.5) over all the allowed new-physics particles which run in the loop (see Table C.1), using the following results for the couplings:

$$g_{ZCC} = \frac{g_2^2 - g_1^2}{2\sqrt{g_1^2 + g_2^2}}, \quad g_{\gamma CC} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad (\text{C.10})$$

$$g_{Z11} = -\frac{\sqrt{g_1^2 + g_2^2}}{2} \sin^2 \theta, \quad g_{W^+C1} = g_{W^-1C} = -\frac{g_2}{\sqrt{2}} \sin \theta, \quad (\text{C.11})$$

$$g_{Z22} = -\frac{\sqrt{g_1^2 + g_2^2}}{2} \cos^2 \theta, \quad g_{W^+C2} = g_{W^-2C} = \frac{g_2}{\sqrt{2}} \cos \theta, \quad (\text{C.12})$$

$$\text{and } g_{Z12} = g_{Z21} = \frac{\sqrt{g_1^2 + g_2^2}}{2} \sin \theta \cos \theta. \quad (\text{C.13})$$

S and T can then be computed from their definitions, Eqs. (4.8) and (4.13), and the results shown here; we have checked explicitly that all terms depending on μ cancel in the results for S and T , as they should. Note that T arises as a small difference between two numerically large terms, and so care is required to obtain a numerically accurate result if these terms are not manually cancelled.

Figure C.1: A one-loop contribution to the IJ self-energy function arising from particles A and B running in the loop.

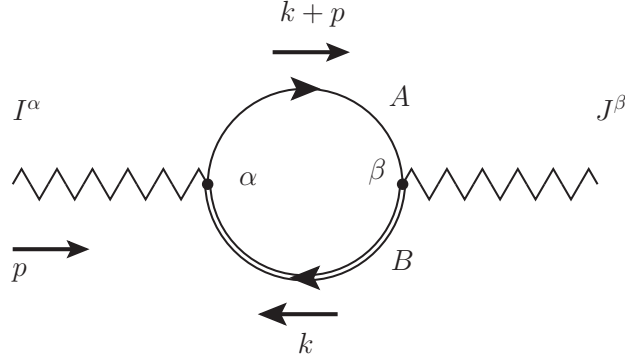


Table C.1: The new-physics particles (A, B) which run in the self-energy loop shown in Figure C.1.

Self-energy IJ	Particles (A, B) in loop
W^+W^-	$(C, 1), (C, 2)$
ZZ	$(1, 1), (2, 2), (1, 2), (2, 1), (C, \overline{C})$
$Z\gamma$	$(C, \overline{C})$
$\gamma\gamma$	$(C, \overline{C})$

C.2 A Clarification on Matching in the Presence of Mixing

This appendix aims to further clarify a point about the matching at the scale $\mu = M_F$ which may not be immediately familiar to the reader (see also footnote 6 of Chapter 4): when $\langle H \rangle \neq 0$, the mass-eigenstate χ field we write in the intermediate-energy EFT² should not be thought to be identified with the (mass-eigenstate admixture) χ field in the UV theory; rather the χ field in the EFT should instead be identified with the light-mass-eigenstate n_1 in the UV theory. This fact notwithstanding, it is common practice, although something of an abuse of notation, to still call the field in the EFT ‘ χ ’.

Consider first a general argument. We have presented both the EFT and UV theory in terms of $SU(2)_W \times U(1)_Y$ -symmetric operators. However, because (a) the limit $\langle H \rangle \rightarrow 0$ is smooth in the sense that the F doublet in the UV theory remains sufficiently massive

2. In this appendix, ‘EFT’ always refers to the intermediate-energy EFT, after the F field is integrated out at $\mu = M_F$, but before the χ field is integrated out at $\mu = m_\chi$. This is the EFT described by the operators and coefficients in Tables 4.2 and 4.3.

($M_F \gg \kappa v, m_\chi$) to be integrated out, and that the EFT power-counting is in $\kappa\langle H\rangle/M_F$, so no EFT operators or Wilson coefficients diverge as $\langle H\rangle \rightarrow 0$; (b) the only source of EWSB in either the UV theory or the EFT is the existence of a non-zero $\langle H\rangle$; and (c) the EFT is explicitly constructed to match the amplitudes of the UV theory at $\mu = M_F$; it follows that when $\langle H\rangle \neq 0$, the EFT must by construction capture all the $\langle H\rangle$ -dependent effects which are present in the UV theory, at each order in the EFT power counting.

Put another way, up to the truncation order of the EFT, one obtains the same physical predictions from the EFT by (a) matching in the broken phase ($\langle H\rangle \neq 0$) of the UV-theory by integrating out the heavy-mass-eigenstates n_2 and C , and writing the EFT in terms of the light-eigenstate n_1 and the physical Higgs field h ; or (b) matching in the unbroken phase ($\langle H\rangle = 0$), integrating out the F doublet whole and writing the EFT in terms of (the EFT field) χ and the $SU(2)_W$ -doublet field H , and then turning on a non-zero $\langle H\rangle$ in the EFT after the matching. Once $\langle H\rangle$ is turned on, the EFT automatically includes the corrections to the χ field which allow its identification with the same physical degree of freedom represented by the light-mass-eigenstate n_1 of the UV theory.

We emphasize that it is by construction that the EFT must reproduce the UV-theory amplitudes at $\mu = M_F$, and so must capture the effect of the mixing of the fermions χ and N in the UV-theory. In the $SU(2)_W$ -symmetric EFT, the effects of this mixing are present in the form of the higher-dimension operators of the EFT. We now show examples of this by explicit computation.

C.2.1 Example 1: Two-Point Functions of the UV-Theory n_1 Field and the EFT Field χ

The clearest demonstration that the EFT field χ represents the same physical degree of freedom as the UV-theory field n_1 is to show that these two fields have the same two-point function; or, equivalently, that when they both have canonical kinetic terms in their respective Lagrangians, they have the same physical mass (up to corrections at the truncated

order in the EFT power-counting).

The UV-theory n_1 field has a canonical kinetic term and mass given in Eq. (C.8). Expanding this result out for $M_F \gg \kappa v, m_\chi$ gives

$$M_1 \approx m_\chi - \frac{\kappa^2 v^2}{2M_F} - \frac{m_\chi \kappa^2 v^2}{2M_F^2} + \frac{\kappa^4 v^4 - 2m_\chi^2 \kappa^2 v^2}{4M_F^3} + \mathcal{O}(M_F^{-4}). \quad (\text{C.14})$$

The EFT χ field does not have canonical kinetic terms. The relevant higher-dimensional operators in the EFT are (see Tables 4.2 and 4.3)

$$\mathcal{L}_{\text{EFT}} \supset i\bar{\chi}\not{\partial}\chi - m_\chi\bar{\chi}\chi + \frac{c_5}{M_F} \mathcal{O}_5 + \frac{c_{6B}}{M_F^2} \mathcal{O}_{6B} + \frac{c_{7A}}{M_F^3} \mathcal{O}_{7A} + \mathcal{O}(M_F^{-4}) \quad (\text{C.15})$$

$$\begin{aligned} &= i\bar{\chi}\not{\partial}\chi - m_\chi\bar{\chi}\chi + \frac{\kappa^2}{M_F} (H^\dagger H)\bar{\chi}\chi + \frac{1}{2} \frac{\kappa^2}{M_F^2} (H^\dagger H)i(\bar{\chi}\not{\partial}\chi - \text{h.c.}) \\ &\quad - \frac{1}{2} \frac{\kappa^2}{M_F^3} (H^\dagger H)(\bar{\chi}\square\chi + \text{h.c.}) + \mathcal{O}(M_F^{-4}). \end{aligned} \quad (\text{C.16})$$

Breaking electroweak symmetry by setting $\langle H \rangle = (0, v/\sqrt{2})^T$, the relevant terms become

$$\mathcal{L}_{\text{EFT}} \supset i\bar{\chi}\not{\partial}\chi \left[1 + \frac{\kappa^2 v^2}{2M_F^2} \right] - \bar{\chi}\chi \left[m_\chi - \frac{\kappa^2 v^2}{2M_F} \right] - \frac{\kappa^2 v^2}{4M_F^3} \left[\bar{\chi} \overleftarrow{\square} \chi + \bar{\chi} \overrightarrow{\square} \chi \right] + \mathcal{O}(M_F^{-4}), \quad (\text{C.17})$$

This can be recast to canonical form by performing a field redefinition, which does not change the physical content of the theory:

$$\chi \rightarrow \left[1 - \frac{\kappa^2 v^2}{4M_F^2} - \frac{m_\chi \kappa^2 v^2}{4M_F^3} \right] \chi - \frac{\kappa^2 v^2}{4M_F^2} \frac{i\not{\partial}\chi}{M_F} + \mathcal{O}(M_F^{-4}). \quad (\text{C.18})$$

In terms of the redefined field, we have

$$\mathcal{L}_{\text{EFT}} \supset i\bar{\chi}\not{\partial}\chi - \bar{\chi}\chi \left[m_\chi - \frac{\kappa^2 v^2}{2M_F} - \frac{m_\chi \kappa^2 v^2}{2M_F^2} + \frac{\kappa^4 v^4 - 2m_\chi^2 \kappa^2 v^2}{4M_F^3} \right] + \mathcal{O}(M_F^{-4}). \quad (\text{C.19})$$

Comparing Eqs. (C.14) and (C.19), we have established the desired result.

C.2.2 Example 2: Z -Boson Coupling to the EFT Field χ

We now look at an example for the EFT χ interaction terms. Consider that the UV-theory χ field is an exact SM-singlet, while $N \subset F$ has SM gauge couplings. When $\langle H \rangle \neq 0$, the mass-basis rotation/mixing, Eq. (C.6), generates SM gauge couplings for both the $n_{1,2}$ mass-eigenstate fermions; see Eqs. (C.10)–(C.13).

If the EFT field χ were to be identified exactly with the χ field in the UV theory, it would necessarily follow that the former should have no SM gauge-couplings; if instead the EFT field χ were identified with the light mass-eigenstate n_1 of the UV theory, we would expect the former to couple to, e.g., the Z -boson. The second alternative is the correct one, as the following short argument shows.

Consider that in the UV theory the coupling constant of the n_1 vector current to the Z -boson is $g_{Z11} = -\frac{1}{2}\sqrt{g_1^2 + g_2^2} \sin^2 \theta$ where $\tan 2\theta = \sqrt{2}\kappa v/(M_F - m_\chi)$; see Eqs. (C.7) and (C.11). In the limit where $M_F \gg \kappa v, m_\chi$, we have

$$g_{Z11} \approx -\frac{1}{4}\sqrt{g_1^2 + g_2^2} \frac{\kappa^2 v^2}{M_F^2} + \dots \quad (\text{C.20})$$

Meanwhile, in the EFT the following operator is present:

$$\mathcal{L}_{\text{EFT}} \supset \frac{c_{6A}}{M_F^2} \mathcal{O}_{6A} \supset -\frac{1}{4}\sqrt{g_1^2 + g_2^2} \frac{\kappa^2 v^2}{M_F^2} \bar{\chi} \gamma_\mu \chi Z^\mu + \dots \quad (\text{C.21})$$

(Note: the field redefinition in Example 1 above does not modify this leading-order coupling.)

One can readily see that the leading-order coupling of the Z -boson with the vector-current of the EFT χ field has the same sign and magnitude, and appears at the same order in the EFT power counting, as the leading-order coupling of the Z -boson with the vector-current of the UV-theory n_1 field. Again, we draw the reader's attention to the fact that it is a higher-dimension operator in the EFT that has captured an effect of UV-theory fermion mixing when $\langle H \rangle \neq 0$.

APPENDIX D

APPENDICES TO CHAPTER 5

This appendix contains material from Ref. [7]: B. Batell, M. A. Fedderke, and L.-T. Wang, *Relaxation of the Composite Higgs Little Hierarchy*, [arXiv:1705.09666](#).¹

D.1 $SU(3)$ Exponentiation

The generic form of the exponentiated $SU(3)$ matrix $\mathcal{U}$ in terms of the technipion fields can be expressed in closed form [721]. Following Ref. [721], if we define

$$H \equiv 2 \frac{\pi_{\text{TC}}^a T^a}{\sqrt{\pi_{\text{TC}}^a \pi_{\text{TC}}^a}}, \quad \theta \equiv \frac{\sqrt{\pi_{\text{TC}}^a \pi_{\text{TC}}^a}}{F_\pi}, \quad \text{and} \quad \varphi \equiv \frac{1}{3} \left[\arccos \left(\frac{3\sqrt{3}}{2} \det H \right) - \frac{\pi}{2} \right], \quad (\text{D.1})$$

then

$$\mathcal{U} \equiv \exp \left[\frac{2i}{F_\pi} \pi_{\text{TC}}^a T^a \right] = \exp[i\theta H] \quad (\text{D.2})$$

$$= \sum_{k=0}^2 \left[H^2 + \frac{2}{\sqrt{3}} H \sin \left(\varphi + \frac{2\pi k}{3} \right) - \frac{1}{3} \mathbb{1}_3 \left(1 + 2 \cos \left[2 \left(\varphi + \frac{2\pi k}{3} \right) \right] \right) \right] \\ \times \frac{\exp \left[\frac{2}{\sqrt{3}} i\theta \sin \left(\varphi + \frac{2\pi k}{3} \right) \right]}{1 - 2 \cos \left[2 \left(\varphi + \frac{2\pi k}{3} \right) \right]}. \quad (\text{D.3})$$

This expression of great utility in providing a closed-form expression for $\mathcal{U}$ that can be expanded out to find certain relevant terms, as outlined in, e.g., Section 5.5.

D.2 Unequal Masses, $m_L \neq m_N$

In Section 5.5.1, we specialized to the case $m_L = m_N \equiv m$, as this simplified the presentation of the analysis of the effective potential in Chapter 5. The aim of this appendix is to

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demonstrate that qualitatively the same EWSB picture is obtained for the case $m_L \neq m_N$.

In particular, we will demonstrate that over a non-negligible region of parameter space, the most important characteristics of the equal-mass case carry over to the unequal-mass case: (a) an initially stable EW-symmetric solution is destabilized as ϕ rolls through a critical value, (b) the destabilization is due to the squared-eigenmass corresponding to the h -field changing sign when evaluated at the EW-symmetric solution, while the other squared-eigenmasses remain positive there, and (c) $\phi = \phi_{\text{crit.}}$ is a supercritical pitchfork bifurcation point for the system.

As in Section 5.5.1, the one-loop effective potential is again obtained by combining Eqs. (5.83)–(5.86), except we now define

$$\epsilon_t \equiv \frac{3c_t N_c |y_t|^2 \Lambda_{\text{TC}}}{32\pi^2 c_m (2m_L + m_N)}, \quad (\text{D.4})$$

which reduces to the definition Eq. (5.88) in the $m_L = m_N \equiv m$ limit; the top-loop contribution to the potential is then written as [cf. Eqs. (5.86) and (5.87)]

$$V \supset -\frac{2}{3} F_\pi^2 \Lambda_{\text{TC}} c_m (2m_L + m_N) \epsilon_t \frac{h^2}{F_\pi^2} \text{sinc}^2 \left(\frac{\bar{\pi}_{\text{TC}}}{F_\pi} \right). \quad (\text{D.5})$$

We also define

$$m_N \equiv z m_L. \quad (\text{D.6})$$

In this appendix, we will ignore the $V_\phi(\phi)$ and V_{QCD} contributions to the potential, and present an analysis of the EWSB dynamics analogous to that in Sections 5.5.2 and 5.5.3, working in the limit $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$ where $|\delta| \ll 1$; the value of $\phi_{\text{crit.}}$ for the unequal mass case will be defined at Eq. (D.14) below.

Evaluating the minimization conditions $\partial_X V = 0$ for $X \in \{h, \kappa_{\text{TC}}, \omega_{\text{TC}}\}$, we find that

the EW-symmetric phase is given by

$$\langle h \rangle = 0, \quad \text{and} \quad \langle \kappa_{\text{TC}} \rangle = \frac{1}{\sqrt{3}} \langle \omega_{\text{TC}} \rangle \quad [\text{i.e., } \langle \pi_{\text{TC}}^0 \rangle = 0, \langle \eta_{\text{TC}} \rangle \neq 0], \quad (\text{D.7})$$

with $\langle \omega_{\text{TC}} \rangle$ implicitly defined as a function of ϕ by (for $z \neq 1$)

$$\sin\left(\frac{\phi}{3F}\right) = \pm \frac{\sin\left(\frac{2\langle \omega_{\text{TC}} \rangle}{3F_\pi}\right) + z \sin\left(\frac{4\langle \omega_{\text{TC}} \rangle}{3F_\pi}\right)}{\sqrt{1 + z^2 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}} \quad \text{for} \quad \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{3F_\pi}\right) \leq z \cos\left(\frac{4\langle \omega_{\text{TC}} \rangle}{3F_\pi}\right). \quad (\text{D.8})$$

There are two regions of parameter space to consider.

D.2.1 Case 1

The first case is $\cos(2\langle \omega_{\text{TC}} \rangle/3F_\pi) < z \cos(4\langle \omega_{\text{TC}} \rangle/3F_\pi)$; this can only be obtained in the vicinity of $\phi_{\text{crit.}}$ if $z > 1$.

We will phrase the physical squared-eigenmasses in terms of $\langle \omega_{\text{TC}} \rangle$ instead of ϕ because inverting Eq. (D.8) in closed form is not straightforward. In the EW-symmetric phase, the three squared-eigenmasses of the canonically normalized² h - κ_{TC} - ω_{TC} system are, as a function of $\langle \omega_{\text{TC}} \rangle$,

$$m_h^2 = \frac{2}{3} c_m \Lambda_{\text{TC}} m_L \left[\frac{3(z^2 - 1)}{\sqrt{z^2 + 1 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}} - 2(z + 2)\epsilon_t \right], \quad (\text{D.9})$$

$$m_2^2 = 4c_m \Lambda_{\text{TC}} m_L \frac{z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right) - 1}{\sqrt{z^2 + 1 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}}, \quad (\text{D.10})$$

2. At $\langle h \rangle = 0$ and $\langle \omega_{\text{TC}} \rangle \neq 0$, the h field has a non-canonical kinetic term $\mathcal{L} \supset \frac{1}{2}(\partial_\mu h)^2 \text{sinc}^2(\langle \omega_{\text{TC}} \rangle/F_\pi)$; cf. Eq. (5.78). We must thus rescale $h \rightarrow h/\text{sinc}(\langle \omega_{\text{TC}} \rangle/F_\pi)$ to canonically normalize. The result for m_h^2 at Eq. (D.9) is shown after this rescaling has been effected.

and

$$m_3^2 = \frac{4}{3} c_m \Lambda_{\text{TC}} m_L \frac{(2z+1)(z-1) + 2z \sin^2 \left(\frac{\langle \omega_{\text{TC}} \rangle}{F_\pi} \right)}{\sqrt{z^2 + 1 - 2z \cos \left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi} \right)}}. \quad (\text{D.11})$$

It is easy to see that m_3^2 is always positive on $z > 1$; m_h^2 will change sign at $\phi = \phi_{\text{crit.}}$ [condition (a)] if

$$\frac{3(z-1)}{2(z+2)} < \epsilon_t < \frac{3(z+1)}{2(z+2)}, \quad (\text{D.12})$$

while demanding that m_2^2 is still positive at this point [condition (b)] implies that

$$\epsilon_t > \frac{3\sqrt{z^2-1}}{2(z+2)}. \quad (\text{D.13})$$

Here, $\phi_{\text{crit.}}$ is given by

$$\cos \left(\frac{\phi_{\text{crit.}}}{3F} \right) = \left\{ 1 - \frac{4\epsilon_t^2(z+2)^2}{9(z^2-1)^2} \left[\sin \left\{ \frac{1}{3} \arccos \left[\frac{1}{2z} \left(1 + z^2 - \frac{9(z^2-1)^2}{4\epsilon_t^2(z+2)^2} \right) \right] \right\} \right. \right. \\ \left. \left. + z \sin \left\{ \frac{2}{3} \arccos \left[\frac{1}{2z} \left(1 + z^2 - \frac{9(z^2-1)^2}{4\epsilon_t^2(z+2)^2} \right) \right] \right\} \right] \right\}^2 \right\}^{\frac{1}{2}} \quad (\text{D.14})$$

$$\stackrel{z \simeq 1}{\equiv} \epsilon_t - \frac{z-1}{6\epsilon_t} + \dots \quad (\text{D.15})$$

It remains to satisfy condition (c): that the destabilization point at $\phi = \phi_{\text{crit.}}$ is a supercritical pitchfork bifurcation. As in the equal-mass case, this can be done by demonstrating that, in addition to $\langle h \rangle = 0$, two other solutions for $\langle h \rangle$ exist for $\delta > 0$, and that these separate from $\langle h \rangle = 0$ as δ increases in size. Because the solution with $\langle h \rangle = 0$ is stable for $\phi < \phi_{\text{crit.}}$ and unstable for $\phi > \phi_{\text{crit.}}$, the existence of two such additional extrema in the potential in the vicinity for $\langle h \rangle = 0$ for $\phi > \phi_{\text{crit.}}$ (i.e., $\delta > 0$) suffices to demonstrate the

supercriticality of the pitchfork bifurcation.

We need only study the vevs in the perturbative limit $0 < \delta \ll 1$. We will expand in formal power series

$$\langle \omega_{\text{TC}} \rangle = \langle \omega_{\text{TC}} \rangle^{\text{EW-sym}}(\delta) + \sum_{j=0} \delta^{2j+1} \langle \omega_{\text{TC}} \rangle^{(2j+1)}, \quad (\text{D.16})$$

$$\langle \kappa_{\text{TC}} \rangle = \langle \kappa_{\text{TC}} \rangle^{\text{EW-sym}}(\delta) + \sum_{j=0} \delta^{2j+1} \langle \kappa_{\text{TC}} \rangle^{(2j+1)}, \quad (\text{D.17})$$

$$\langle h \rangle = \sqrt{\delta} \sum_{j=0} \delta^j \langle h \rangle^{(j)}, \quad (\text{D.18})$$

where $\langle \omega_{\text{TC}} \rangle^{\text{EW-sym}}(\delta)$ and $\langle \kappa_{\text{TC}} \rangle^{\text{EW-sym}}(\delta)$ are the vevs for ω_{TC} and κ_{TC} in the EW-symmetric phase, so that the coefficients $\langle h \rangle^{(j)}$, $\langle \kappa_{\text{TC}} \rangle^{(2j+1)}$, and $\langle \omega_{\text{TC}} \rangle^{(2j+1)}$ parametrize the deviations from the EW-symmetric solution. In the EW-symmetric phase we know that we have $\langle \kappa_{\text{TC}} \rangle^{\text{EW-sym}}(\delta) = (1/\sqrt{3}) \langle \omega_{\text{TC}} \rangle^{\text{EW-sym}}(\delta)$. We expand

$$\langle \omega_{\text{TC}} \rangle^{\text{EW-sym}}(\delta) \equiv \omega_{\text{TC}}^{\text{crit.}} + \delta \cdot \langle \omega_{\text{TC}} \rangle^{\text{EW-sym slope}} + \dots, \quad (\text{D.19})$$

where $\omega_{\text{TC}}^{\text{crit.}}$ is the solution to $m_h^2 = 0$ [see Eq. (D.9)]:

$$\omega_{\text{TC}}^{\text{crit.}} = \frac{F\pi}{2} \arccos \left\{ \frac{1}{2z} \left[1 + z^2 - \left(\frac{3(z^2 - 1)}{2\epsilon_t(z + 2)} \right)^2 \right] \right\}, \quad (\text{D.20})$$

and $\langle \omega_{\text{TC}} \rangle^{\text{EW-sym slope}}$ is obtained by inserting the expansion Eq. (D.19) and the definition $\phi = \phi_{\text{crit.}} + 3F \cdot \delta$ into the EW-symmetric solution Eq. (D.8), expanding in powers of δ and equating coefficients:

$$\langle \omega_{\text{TC}} \rangle^{\text{EW-sym slope}} = \frac{9F\pi(z^2 - 1)}{3(z^2 - 1) + 4(z + 2)^2 \epsilon_t^2}. \quad (\text{D.21})$$

To find the values of the other expansion coefficients in Eqs. (D.16)–(D.18), we substitute all relevant expansions into the extremization conditions $\partial_X V = 0$ for $X \in \{h, \kappa_{\text{TC}}, \omega_{\text{TC}}\}$

and expand in powers of δ to find a formal power series whose individual coefficients we equate to zero to obtain a hierarchical system of equations which we can solve recursively to find $\langle h \rangle^{(j)}$, $\langle \kappa_{\text{TC}} \rangle^{(2j+1)}$, and $\langle \omega_{\text{TC}} \rangle^{(2j+1)}$; the lowest-order solutions $\langle h \rangle^{(0)}$, $\langle \kappa_{\text{TC}} \rangle^{(1)}$, and $\langle \omega_{\text{TC}} \rangle^{(1)}$ suffice to determine the nature of the bifurcation. The resulting expressions correctly reduce to the results Eq. (5.100) in the $z \rightarrow 1$ equal-mass limit, but are too lengthy to display here explicitly. Nevertheless, analysis of these expressions allows us to conclude that the two additional solutions for $\langle h \rangle^{(0)}$ exist for $0 < \delta \ll 1$, and that the pitchfork bifurcation is thus supercritical, if the following conditions are met [we additionally impose $\epsilon_t > 0$ by definition]:

$$\left[0 < \epsilon_t < \frac{3\sqrt{z^2 - 1}}{2(z + 2)} \quad \text{OR} \quad \frac{3z}{2(z + 2)} < \epsilon_t < \frac{3(z + 1)}{2(z + 2)} \right] \quad \text{AND} \quad \frac{3(z - 1)}{2(z + 2)} < \epsilon_t < \frac{3(z + 1)}{2(z + 2)}. \quad (\text{D.22})$$

The most stringent constraint for the region $z > 1$ is thus

$$\frac{3z}{2(z + 2)} < \epsilon_t < \frac{3(z + 1)}{2(z + 2)}. \quad (\text{D.23})$$

D.2.2 Case 2

The second case is $\cos(2\langle \omega_{\text{TC}} \rangle / 3F_\pi) > z \cos(4\langle \omega_{\text{TC}} \rangle / 3F_\pi)$. The relevant part of this region of parameter space occurs for $z < 1$ when $\phi \approx \phi_{\text{crit.}}$; although there is also some part of this region of parameter space at $z > 1$ when $\phi \approx \phi_{\text{crit.}}$, it turns out not to be interesting for our purposes, and we will not discuss it.

We follow a similar analysis as for the previous case. In the EW-symmetric phase, the three squared-eigenmasses of the canonically normalized h - κ_{TC} - ω_{TC} system are, as a function

of $\langle \omega_{\text{TC}} \rangle$,

$$m_h^2 = \frac{2}{3} c_m \Lambda_{\text{TC}} m_L \left[\frac{3(1-z^2)}{\sqrt{z^2 + 1 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}} - 2(z+2)\epsilon_t \right], \quad (\text{D.24})$$

$$m_2^2 = \frac{4}{3} c_m \Lambda_{\text{TC}} m_L \frac{1 - 2z^2 + z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}{\sqrt{z^2 + 1 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}}, \quad (\text{D.25})$$

$$m_3^2 = 4c_m \Lambda_{\text{TC}} m_L \frac{(1-z) + 2z \sin^2\left(\frac{\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}{\sqrt{z^2 + 1 - 2z \cos\left(\frac{2\langle \omega_{\text{TC}} \rangle}{F_\pi}\right)}}. \quad (\text{D.26})$$

It is again easy to see that m_3^2 is always positive on $0 < z < 1$; m_h^2 will change sign at $\phi = \phi_{\text{crit.}}$ [condition (a)] if

$$\frac{3(1-z)}{2(z+2)} < \epsilon_t < \frac{3(z+1)}{2(z+2)}, \quad (\text{D.27})$$

while demanding that m_2^2 is still positive at this point [condition (b)] implies that

$$\epsilon_t > \frac{\sqrt{3}\sqrt{1-z^2}}{2(z+2)}, \quad (\text{D.28})$$

although this latter constraint is only applicable on $\frac{1}{2} < z < 1$, where a solution for $m_2^2 = 0$ exists. The value of $\phi_{\text{crit.}}$ is given by Eq. (D.14) without change.

We perform the same perturbative expansion and solution as for the previous case to find the requirements to satisfy condition (c), supercriticality of the pitchfork bifurcation. The value of $\omega_{\text{TC}}^{\text{crit.}}$ changes sign relative to the solution in Eq. (D.20), while $\langle \omega_{\text{TC}} \rangle^{\text{EW-sym}}$ slope is still given by Eq. (D.21). The solutions for $\langle h \rangle^{(0)}$, $\langle \kappa_{\text{TC}} \rangle^{(1)}$, and $\langle \omega_{\text{TC}} \rangle^{(1)}$ are again too lengthy to show here explicitly.

Once again, the relevant conclusion that can be drawn from this procedure is that the two additional solutions for $\langle h \rangle^{(0)}$ exist for $0 < \delta \ll 1$, and that the bifurcation is thus

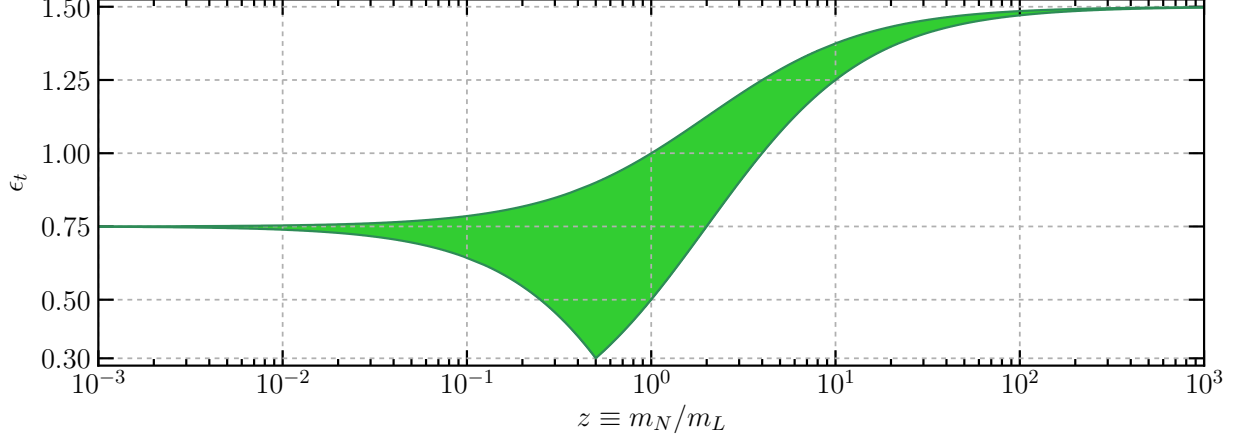


Figure D.1: The green shaded area on this z - ϵ_t plane is the region of parameter space satisfying Eq. (D.31), in which qualitatively similar EWSB dynamics as for the equal-mass case examined in Chapter 5 are obtained.

supercritical, if the following conditions are met [we additionally impose $\epsilon_t > 0$ by definition]:

$$\left[0 < \epsilon_t < \frac{3\sqrt{1-z^2}}{2(z+2)} \quad \text{OR} \quad \frac{3z}{2(z+2)} < \epsilon_t < \frac{3(z+1)}{2(z+2)} \right] \quad \text{AND} \quad \frac{3(1-z)}{2(z+2)} < \epsilon_t < \frac{3(z+1)}{2(z+2)}. \quad (\text{D.29})$$

The most stringent constraint for the region $z < 1$ is thus

$$\max \left\{ \frac{3(1-z)}{2(z+2)}, \frac{3z}{2(z+2)} \right\} < \epsilon_t < \frac{3(z+1)}{2(z+2)}. \quad (\text{D.30})$$

D.2.3 Summary

In summary, the unequal-mass case exhibits qualitatively similar behaviour to the equal-mass case analysed in Chapter 5 when the following conditions are met:

$$\max \left\{ \frac{3|z-1|}{2(z+2)}, \frac{3z}{2(z+2)} \right\} < \epsilon_t < \frac{3(z+1)}{2(z+2)}. \quad (\text{D.31})$$

This region of parameter space is displayed in Figure D.1. Note that for $z = 1$, the constraint correctly reduces to $1/2 < \epsilon_t < 1$; cf. Eq. (5.102).

APPENDIX E

APPENDICES TO CHAPTER 6

This appendix contains material from Ref. [8]: M. A. Fedderke, E. W. Kolb, and M. Wyman, *Irruption of massive particle species during inflation*, *Phys. Rev.* **D91** (2015) 063505 [[arXiv:1409.1584](#)].¹

E.1 Late-Time Asymptotics

It is instructive to develop a further analytical understanding of the late-time behaviour of $|\beta_k|^2$, building from our discussion in Section 6.4.3. We examine here the expression for the first-order iteration, $\beta_k^{(1)}(t)$, for $t \gg t_{\text{ref}}$. Starting with Eq. (6.66) for $n = 1$, and using $\alpha_k^{(0)}(t) = 1$, we have

$$\beta_k^{(1)}(t) = \beta_{k,\text{ref}}^{(1)} + \frac{1}{2} e^{-2i\Phi_{\text{ref}}} \int_{t_{\text{ref}}}^t \left[H(t_1) - 2\delta \cdot \dot{\nu}(t_1) \right] e^{-2iM_{\infty}(t_1 - t_{\text{ref}})} dt_1. \quad (\text{E.1})$$

Applying Eq. (6.61), the integral in Eq. (E.1) can be written in terms of exponential integral functions

$$\text{Ei}(y) \equiv - \int_{-y}^{\infty} \frac{e^{-t}}{t} dt. \quad (\text{E.2})$$

Two standard integrals will enter:

$$\mathcal{I}_1(y; \tilde{y}, \xi) \equiv \int_0^y \frac{e^{-i\xi(y_1 + \tilde{y})}}{y_1 + \tilde{y}} dy_1 = \text{Ei}(-i\xi(y + \tilde{y})) - \text{Ei}(-i\xi\tilde{y}), \quad (\text{E.3})$$

$$\mathcal{I}_2(y; \tilde{y}, \xi) \equiv \int_0^y \frac{e^{-i\xi(y_1 + \tilde{y})}}{(y_1 + \tilde{y})^2} dy_1 = e^{-i\xi\tilde{y}} \left[\frac{1}{\tilde{y}} - \frac{e^{-i\xi y}}{y + \tilde{y}} \right] - i\xi \left[\text{Ei}(-i\xi(y + \tilde{y})) - \text{Ei}(-i\xi\tilde{y}) \right]. \quad (\text{E.4})$$

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Here, we will have either $\xi = 2M_\infty$ or $\xi = 2M_\infty \pm 1$, and $y \equiv t - t_{\text{ref}}$, $\tilde{y} \equiv t_{\text{ref}} - \tau$, and $y + \tilde{y} = t - \tau$ (all are positive, provided $M_\infty > 1/2$). Finally, we can write the desired late-time expression for $\beta_k(t)$:

$$\begin{aligned} \beta_k^{(1)}(t) \approx & \beta_{k, \text{ref}}^{(1)} + \frac{1}{2} e^{-2i(\Phi_{\text{ref}} - M_\infty \tilde{y})} \left\{ \frac{2}{3} \mathcal{I}_1(y; \tilde{y}, 2M_\infty) \right. \\ & - \delta \left[(A + iB) \mathcal{I}_1(y; \tilde{y}, 2M_\infty - 1) + (A - iB) \mathcal{I}_1(y; \tilde{y}, 2M_\infty + 1) \right] \\ & \left. - i\delta \left[(A + iB) \mathcal{I}_2(y; \tilde{y}, 2M_\infty - 1) - (A - iB) \mathcal{I}_2(y; \tilde{y}, 2M_\infty + 1) \right] \right\}, \quad (\text{E.5}) \end{aligned}$$

where A and B are given by Eq. (6.62).

We are interested in the late-time asymptotic limit of Eq. (E.5). Since the exponential integral function has a branch cut one must be careful when making asymptotic expansions:

$$\text{Ei}(\pm ia) \rightarrow \pm i\pi + e^{\pm ia} \left[\mp \frac{i}{a} - \frac{1}{a^2} + \dots \right] \quad \text{as } a \rightarrow +\infty \quad (a \in \mathbb{R}). \quad (\text{E.6})$$

Thus, the late-time ($y \rightarrow \infty$) asymptotic expansions for the two standard integrals are

$$\mathcal{I}_1(y \rightarrow \infty; \tilde{y}, \xi) \sim - \left[i\pi + \text{Ei}(-i\xi\tilde{y}) \right] + e^{-i\xi(y+\tilde{y})} \left[\frac{i}{\xi y} - \frac{1 + i\xi\tilde{y}}{\xi^2 y^2} + \dots \right] \quad (\text{E.7})$$

$$\mathcal{I}_2(y \rightarrow \infty; \tilde{y}, \xi) \sim \frac{e^{-i\xi\tilde{y}}}{\tilde{y}} + i\xi \left[i\pi + \text{Ei}(-i\xi\tilde{y}) \right] + \frac{e^{-i\xi(y+\tilde{y})}}{y} \left[\frac{i}{\xi y} - 2 \frac{1 + i\xi\tilde{y}}{\xi^2 y^2} + \dots \right]. \quad (\text{E.8})$$

Finally we may expand in the large-mass limit ($\xi \rightarrow \infty$), yielding

$$\begin{aligned} \mathcal{I}_1(y \rightarrow \infty; \tilde{y}, \xi \rightarrow \infty) \sim & -\frac{i}{\xi} \left[e^{-i\xi\tilde{y}} \frac{1}{\tilde{y}} - e^{-i\xi(y+\tilde{y})} \left(\frac{1}{y} - \frac{\tilde{y}}{y^2} \right) + \dots \right] \\ & + \frac{1}{\xi^2} \left[e^{-i\xi\tilde{y}} \frac{1}{\tilde{y}^2} - e^{-i\xi(y+\tilde{y})} \frac{1}{y^2} + \dots \right] + \dots, \quad (\text{E.9}) \end{aligned}$$

$$\begin{aligned} \mathcal{I}_2(y \rightarrow \infty; \tilde{y}, \xi \rightarrow \infty) \sim & -\frac{i}{\xi} \left[e^{-i\xi\tilde{y}} \frac{1}{\tilde{y}^2} - e^{-i\xi(y+\tilde{y})} \left(\frac{1}{y^2} - \frac{2\tilde{y}}{y^3} \right) + \dots \right] \\ & + \frac{2}{\xi^2} \left[e^{-i\xi\tilde{y}} \frac{1}{\tilde{y}^3} - e^{-i\xi(y+\tilde{y})} \frac{1}{y^3} + \dots \right] + \dots. \quad (\text{E.10}) \end{aligned}$$

After a bit of manipulation, the late-time-large-mass asymptotic value of $\beta_k^{(1)}$ can thus be written as

$$\begin{aligned} \beta_k^{(1)}(t \rightarrow \infty) \approx & \beta_{k,\text{ref}}^{(1)} - i \frac{e^{-2i\Phi_{\text{ref}}}}{6M_\infty} \frac{1}{t_{\text{ref}} - t_0} \left[1 - 3\delta \cdot (t_{\text{ref}} - t_0) \dot{\nu}_{\text{ref}} \right] \\ & - i \frac{e^{-2i\Phi_{\text{ref}}}}{6M_\infty} \frac{e^{-2iM_\infty(t-t_{\text{ref}})}}{t - t_{\text{ref}}} \left[1 - 3\delta \cdot \begin{pmatrix} (\nu_{\text{ref}} + (t_{\text{ref}} - t_0)\dot{\nu}_{\text{ref}}) \cos(t - t_{\text{ref}}) \\ -\nu_{\text{ref}}(t_{\text{ref}} - t_0) \sin(t - t_{\text{ref}}) \end{pmatrix} \right]. \end{aligned} \quad (\text{E.11})$$

Armed with this expression, we can immediately see the qualitative features of the late-time, large-mass solution:

1. A non-zero (in general) late-time value of

$$|\beta_k^{(1)}(t \rightarrow \infty)|^2 = \left| \beta_{k,\text{ref}}^{(1)} - i \frac{e^{-2i\Phi_{\text{ref}}}}{6M_\infty} \frac{1}{t_{\text{ref}} - t_0} [1 - 3\delta (t_{\text{ref}} - t_0) \dot{\nu}_{\text{ref}}] \right|^2 \quad (\text{E.12})$$

arising from the constant term squared.

2. A fast oscillation at frequency $2M_\infty$, whose amplitude is modulated at frequency m ,² which damps out as $1/t$. This arises from the cross-term between the constant term and the damped term. Schematically, this contribution takes the form $\frac{1}{t} \cos(2M_\infty t + \phi) \left[1 + \zeta \cos(mt + \phi') \right]$. Although we have not displayed it here, if we expanded $\beta_k^{(1)}$ itself up to $1/t^2$, there would also be a similar term damped at $1/t^2$, but this will be subdominant.
3. A slow oscillation at frequency m on top of a constant $1/t^2$ contribution. This arises from the damped term squared. Schematically, this contribution takes the form $\frac{1}{t^2} \left[1 + \zeta' \cos(mt + \phi'') \right]^2$.

². Here and below, we have restored general m ; Eq. (E.11) is expressed such that t is measured in units of m^{-1} .

We see all these behaviours in the numerical solutions; in particular, we have performed Fourier analysis (FFT) on selected late-time numerical solutions and found strong peaks in the Fourier power spectrum at all the expected frequencies (including sidebands generated by the modulations).

E.2 Analytics for the Potentially Coupled Case

This appendix is based on ideas and methods to be found in Section VII B of Ref. [699].

We consider the potentially coupled case at large M_g (larger than any other scale in the problem, except possibly k). In this case, the duration of production is extremely short around $t = t_*$, and we need only consider the solution of the mode equation in this short interval, taking $|\beta_k|^2 = 0$ identically at some short time just before $t = t_*$, and extracting the asymptotic value of $|\beta_k|^2$ shortly after $t = t_*$ ('short' will be made precise below). As a result of this observation, we may Taylor expand $a(t)$, $H(t)$, etc. in the mode equation around the point $t = t_*$, and we will keep only terms up to t^2 .

Rather than solve the mode equation Eq. (6.16) for χ_k , we solve the equation for $f_k(t) \equiv a^{1/2}\chi_k(t)$ [686]:

$$\ddot{f}_k + \left[M_g^2(\nu - \nu_*)^2 + \frac{k^2}{a^2} - \frac{9}{4}H^2 - \frac{3}{2}\dot{H} \right] f_k = 0, \quad (\text{E.13})$$

where we used the fact that $a''/a^3 = 2H^2 + \dot{H}$.

We may now Taylor expand the relevant functions to second-order in powers of t , for the present purposes re-zeroing t at $\nu = \nu_*$:

$$M_g^2(\nu - \nu_*)^2 = M_g^2 \dot{\nu}_*^2 t^2 \equiv k_*^4 t^2, \quad (\text{E.14})$$

$$\frac{k^2}{a^2} = \frac{k^2}{a_*^2} \left[1 - 2H_* t + (2H_*^2 - \dot{H}_*) t^2 \right], \quad (\text{E.15})$$

$$H^2 = H_*^2 + 2H_* \dot{H}_* t + (\dot{H}_*^2 + H_* \ddot{H}_*) t^2, \quad (\text{E.16})$$

and

$$\dot{H} = \dot{H}_* + \ddot{H}_* t + \frac{1}{2} \left. \frac{d^3 H}{dt^3} \right|_* t^2, \quad (\text{E.17})$$

where we defined $k_* \equiv \sqrt{M_g |\dot{\nu}_*|}$. To evaluate the higher derivatives here in terms of ν and $\dot{\nu}$, one can exploit Eq. (6.24), $\dot{H} = -4\pi\dot{\nu}^2$, and $\ddot{\nu} = -3H\dot{\nu} - \nu$, all of which are exact relations when the back-reaction is ignored and the irrupton energy density is assumed to be a very small fraction of the inflaton energy density. We thus have

$$\ddot{f}_k + [pt^2 + qt + r] f_k = 0, \quad (\text{E.18})$$

where we have defined

$$p = k_*^4 + \frac{k^2}{a_*^2} [2H_* - \dot{H}_*] - \frac{9}{4} (\dot{H}_*^2 + H_* \ddot{H}_*) - \frac{3}{2} \left. \frac{d^3 H}{dt^3} \right|_* \quad (\text{E.19})$$

$$q = -2H_* \frac{k^2}{a_*^2} - \frac{9}{2} H_* \dot{H}_* - \frac{3}{2} \ddot{H}_* \quad (\text{E.20})$$

$$r = \frac{k^2}{a_*^2} - \frac{9}{4} H_*^2 - \frac{3}{2} \dot{H}_*. \quad (\text{E.21})$$

Defining

$$z \equiv \sqrt{2} p^{1/4} \left(t + \frac{q}{2p} \right) \quad \text{and} \quad c \equiv \frac{1}{2\sqrt{p}} \left(\frac{q^2}{4p} - r \right), \quad (\text{E.22})$$

the equation for f_k can be written as

$$\frac{d^2 f_k}{dz^2} + \left(\frac{1}{4} z^2 - c \right) f_k = 0. \quad (\text{E.23})$$

The solutions to this equation can be written in terms of confluent hypergeometric func-

tions ${}_1F_1$ (see §9.2 of Ref. [722], or Ref. [723]):

$$f_k = e^{-iz^2/4} \left[a {}_1F_1\left(\frac{1}{4} - \frac{i}{2}c, \frac{1}{2}, \frac{i}{2}z^2\right) + b e^{i\pi/4} z {}_1F_1\left(\frac{3}{4} - \frac{i}{2}c, \frac{3}{2}, \frac{i}{2}z^2\right) \right], \quad (\text{E.24})$$

where a and b are integration constants to be chosen to specify the adiabatic in-vacuum.

Recall that $f_k = a^{1/2} \chi_k$ and the in-vacuum solution for χ_k must reduce to $\chi_k \rightarrow (1/2\omega_k)^{1/2} \exp\left(-i \int^t dt' \omega_k/a\right)$ as $t \rightarrow -\infty$. Therefore, we must have

$$f_k \rightarrow \left(\frac{a}{2\omega_k}\right)^{\frac{1}{2}} \exp\left(-i \int^t dt' \omega_k/a\right) \quad (\text{E.25})$$

as $t \rightarrow -\infty$. We now Taylor expand ω_k^2 and look sufficiently far from $t = 0$: $|t| \gg k_*^{-1}$. Assuming that the mass term is the largest contribution to ω_k^2/a^2 , the latter is dominated by the t^2 term, and up to small corrections like $\dot{H}_*/k_*^4$, etc., we then have that $\omega_k/a \approx \sqrt{p}t \approx p^{1/4}z/\sqrt{2}$. Therefore, the in-vacuum asymptotic early-time form of f_k must be $f_k \rightarrow \exp(+iz^2/4)/\left(2^{1/4}p^{1/8}\sqrt{-z}\right)$ as $z \rightarrow -\infty$, where we have ignored the presence of a possible irrelevant overall phase and have dropped small terms. Note the opposite sign to the naïve expectation appears in the exponent since the $t < 0$ form of ω_k must be used.

By demanding this early-time asymptotic form, we recover the values of a and b for the correctly normalized modes which give an in-vacuum; although these expressions are not particularly enlightening, we present them here for completeness:

$$a = -\frac{(-1)^{7/8}2^{ic/2}}{4\pi p^{1/8}} e^{-3\pi c/4} \left(1 + e^{2\pi c}\right) \Gamma\left(\frac{ic}{2} + \frac{1}{4}\right) \Gamma\left(\frac{1}{2} - ic\right), \quad (\text{E.26})$$

$$b = -\frac{(-1)^{3/8}2^{ic/2}}{2^{3/2}\pi p^{1/8}} e^{-3\pi c/4} \left(1 + e^{2\pi c}\right) \Gamma\left(\frac{ic}{2} + \frac{3}{4}\right) \Gamma\left(\frac{1}{2} - ic\right). \quad (\text{E.27})$$

To extract α_k and β_k we simply look at the late-time behaviour of $f(z)$ as $z \rightarrow \infty$: α_k is the coefficient of the term $\exp(-iz^2/4)/\left(2^{1/4}p^{1/8}\sqrt{z}\right)$ and β_k is the coefficient of the term

$\exp(+iz^2/4)/(2^{1/4}p^{1/8}\sqrt{z})$. The results are

$$|\alpha_k|^2 = 1 + e^{2\pi c} \quad \text{and} \quad |\beta_k|^2 = e^{2\pi c} = \exp\left[\frac{\pi}{\sqrt{p}}\left(\frac{q^2}{4p} - r\right)\right], \quad (\text{E.28})$$

which clearly satisfy the Wronskian condition $|\alpha_k|^2 - |\beta_k|^2 = 1$.

The (red) dashed line in Figure 6.12 is obtained via numerical integration of Eq. (E.28) using Eq. (6.40): $\Omega h^2 \times (\Omega h^2)_{\text{Planck}}^{-1} \times (T_{\text{RH}}/10^9 \text{ GeV})^{-1} \times (m/10^{13} \text{ GeV})^{-2} \approx (2.7043 \times 10^6) \times M_g \nu_* \times \int_0^\infty k^2 |\beta_k|^2 dk$. The relevant parameters in terms of which p , q , and r can be evaluated must be taken from our numerical background solutions: when $\nu = \nu_* = 0.8$, we have $H_* = 1.6697$, $\dot{\nu}_* = -0.1599$, and $a_* = 0.0160$.

At very large M_g further simplifications are possible:

$$p \approx k_*^4 = M_g^2 \dot{\nu}_*^2, \quad (\text{E.29})$$

$$c \approx -\frac{r}{2\sqrt{p}} \approx \frac{1}{2M_g|\dot{\nu}_*|} \left[\frac{9}{4}H_*^2 + \frac{3}{2}\dot{H}_* - \frac{k^2}{a_*^2} \right], \quad (\text{E.30})$$

so that using $\dot{H}_* = -4\pi\dot{\nu}_*^2$,

$$\begin{aligned} |\beta_k|^2 &\approx \exp\left[\frac{\pi}{M_g|\dot{\nu}_*|} \left(\frac{9}{4}H_*^2 + \frac{3}{2}\dot{H}_* - k^2/a_*^2\right)\right] \\ &\approx \exp\left[\frac{\pi}{M_g|\dot{\nu}_*|} \left(\frac{9}{4}H_*^2 - 6\pi\dot{\nu}_*^2 - k^2/a_*^2\right)\right]. \end{aligned} \quad (\text{E.31})$$

Expressing H^2 in terms of $V(\phi)$ and $\dot{\phi}$, and using $\dot{H} = -(4\pi/M_{\text{Pl}}^2)\dot{\phi}^2$, we have

$$\frac{9}{4}H_*^2 + \frac{3}{2}\dot{H}_* = \frac{9}{4}H_*^2 \left[\frac{V(\phi_*) - \frac{1}{2}\dot{\phi}_*^2}{V(\phi_*) + \frac{1}{2}\dot{\phi}_*^2} \right] = -\frac{9}{4}H_*^2 w_*, \quad (\text{E.32})$$

which is always positive during inflation (the equation of state at any point $\nu = \nu_*$ during inflation satisfies $w_* < -1/3$). Therefore, it is clear that $|\beta_k|^2$ is flat for small k and approaches unity from above for large M_g . It also drops rapidly once the k^2 term drives the exponent negative, but at fixed large k approaches unity from below as M_g is further

increased. Granted, this expression is invalid when k dominates ω_k , but since the rapid drop-off sets in by this point for large enough M_g , we can simply neglect this regime.

Furthermore, if we confine ourselves to consideration of values of ν_* such that slow-roll inflation is still a very good approximation around $t = t_*$, it follows that $\dot{\nu}_*$ is very nearly independent of ν_* , so H_*^2 as given by Eq. (6.24) is proportional to ν_*^2 . We also have $a_* \propto e^{-2\pi\nu_*^2}$. Therefore, the deep-IR value $|\beta_{k=0}|^2 \propto \exp\left(3\pi^2\nu_*^2/M_g|\dot{\nu}_*|\right)$ increases exponentially quickly as ν_* is increased linearly. However, the value $k = k_1$ required to cause a one- e -fold drop-off in $|\beta_k|^2$ from this IR value decreases exponentially quickly roughly as $k_1 \propto a_* H_* \propto \nu_* e^{-2\pi\nu_*^2}$. The net result is that as ν_* is increased linearly, the n_k^c spectrum broadens on the low- k side (since $|\beta_{k=0}|^2$ is larger), yet peaks at a much smaller value of k , and as a result has a much smaller maximum value.

Integrating the approximate very-large- M_g spectrum, Eq. (E.31), over all k yields

$$\begin{aligned} \frac{1}{2\pi^2} \int_0^\infty k^2 |\beta_k|^2 dk &= \frac{1}{2\pi^2} \int_{-\infty}^\infty k^3 |\beta_k|^2 d \ln k \\ &= \frac{a_*^3}{8\pi^3} \left(M_g |\dot{\nu}_*|\right)^{3/2} \exp \left[\frac{3\pi}{4} \frac{3H_*^2 - 8\pi\dot{\nu}_*^2}{M_g |\dot{\nu}_*|} \right] \\ &\approx \left(\frac{a_* \sqrt{|\dot{\nu}_*|}}{2\pi} \right)^3 M_g^{3/2}. \end{aligned} \tag{E.33}$$

This shows us that in the large- M_g limit, the total number of produced particles goes like $n^p \propto M_g^{3/2}$, and that $\Omega h^2 \propto M_g^{5/2}$, both of which increase with increasing M_g . Also, as ν_* increases, both n^p and Ωh^2 (measured at the same fixed time t in the MD era following the end of inflation) drop exponentially quickly as a_*^{+3} , as expected for the dilution of an NR species being produced earlier during inflation.

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