

324 Supporting information

325 Breakdown of Quasigeostrophic(QG) Assumptions

326 The presented FALWA formalism is based on the QG approximation, and hence it is intended for the
327 extratropical circulation. The QG framework simplifies interpretation, and the overall inaccuracy due to the
328 QG approximation is a small price to pay for the huge gain in the ability to analyze finite-amplitude eddies.
329 That being said, there are a couple of regions in which the analysis is known to produce spurious results as
330 a result of the QG assumptions. This is all related to the violation of the assumption that the isentropic
331 surface does not deviate strongly from the horizontal surface in the QG approximation.

332 Tropical upper troposphere

333 In the “stretching term” of QGPV, $\frac{\partial}{\partial z} \left[\frac{\theta - \bar{\theta}}{\partial \theta / \partial z} \right]$ is often negative in the tropical upper troposphere due to a
334 significant vertical variation in $\bar{\theta}(\theta, z, t) - \bar{\theta}(z, t)$, and at times strongly so — that is, it is $O(1)$ as opposed
335 to the small Rossby number assumed in the QG approximation. When multiplied by f , it overwhelms
336 absolute vorticity and creates a negative (positive) QGPV in the Northern (Southern) Hemisphere. Not
337 only is this unphysical (negative QGPV in the Northern Hemisphere would be symmetrically unstable) but
338 it also creates a false impression that a large chunk of air carrying QGPV with the opposite sign has been
339 displaced from the other side of the equator, which would lead to a spuriously large wave activity in the
340 tropical upper troposphere (Nakamura and Solomon, 2010). If $f q_{\text{REF}} < 0$ in a significant fraction of the
341 domain, the inversion algorithm for u_{REF} may also fail to converge.

342 Lowermost stratosphere in the extratropics during the cold season

343 The contribution of the stretching term to the meridional gradient of the zonal-mean QGPV includes
344 $-\frac{f^2}{N^2} \frac{\partial^2 \bar{u}}{\partial z^2}$. This term is positive around the tropopause and reinforces β — in fact, the peak value of this term
345 closely approximates the height of the dynamic tropopause in the QG approximation. (The vertical gradient
346 of $N^2 \propto \frac{\partial \bar{\theta}}{\partial z}$ is horizontally uniform and only defines a broad “tropopause layer.”) However, this term is
347 negative above the extratropical tropopause around 100-150 hPa where \bar{u} is minimal, and it is comparable
348 to β during the cold season. As a result, the zonal-mean QGPV gradient nearly vanishes in this altitude
349 range, causing a spuriously large wave activity (meandering of the QGPV contours). This is due to the QG
350 assumption — similar analysis using Ertel’s PV with the primitive equations in the isentropic coordinate
351 does not show this behavior (Nakamura and Solomon, 2010, 2011). Fortunately, this behavior is limited to
352 one or two analysis layers and does not affect the column averaged wave activity budget adversely.