A ν scalar in the early universe and $(g-2)_\mu$

Supplemental Material

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In this supplemental material, we show the diagonalization procedure for the active and sterile neutrino mass matrix.

In the UV model, we have given the Yukawa coupling for the three active neutrinos and sterile neutrino $N,\ N'$. To obtain the mass eigenstates and mixing matrices, one needs to diagonalize the 5×5 mass matrix. Since the mass matrix is rank four, one massless active neutrino is guaranteed. The other eigenvalues have been given in the main text with the choice that the Dirac mass term $\lambda_N v_\phi \gg m_N,\ m_{N'}$. Therefore, the active neutrino masses $\tilde{m}_{\nu_i}$ are approximately independent of m_N and $m_{N'}$, because the large mass in the seesaw mechanism is actually $\lambda_N v_\phi$. One can block diagonalize the neutrino mass matrix, separating the active neutrinos from the sterile neutrinos and simultaneously diagonalize the sterile neutrino 2×2 block matrix, by performing the following rotation

$$\begin{pmatrix} \vec{\nu} \\ N \\ N' \end{pmatrix} \approx \begin{pmatrix} \mathbb{I}_{3\times 3} & \frac{\overrightarrow{y_N} v + \overrightarrow{y_{N'}} v'}{\sqrt{2}\lambda_N v_\phi} & \frac{\overrightarrow{y_N} v - \overrightarrow{y_{N'}} v'}{\sqrt{2}\lambda_N v_\phi} \\ -\frac{\overrightarrow{y_{N'}}^{\mathrm{T}} v'}{\lambda_N v_\phi} & \frac{1}{\sqrt{2}} + z & -\frac{1}{\sqrt{2}} + z \\ -\frac{\overrightarrow{y_N}^{\mathrm{T}} v}{\lambda_N v_\phi} & \frac{1}{\sqrt{2}} + z & \frac{1}{\sqrt{2}} + z \end{pmatrix} \begin{pmatrix} \vec{\nu}' \\ \tilde{N} \\ \tilde{N}' \end{pmatrix}. \tag{S1}$$

Here $\vec{\nu}'$ are intermediate active neutrino states defined by the above rotation, while $\tilde{N}$ and $\tilde{N}'$ are already in their mass eigenstates. We have defined $z \equiv (m_N - m_{N'})/(4\lambda_N v_\phi)$ and assumed $m_N > m_{N'}$ to derive the above rotation matrix. The active neutrino mass matrix in the $\vec{\nu}'$ basis is given by

$$\mathcal{M}_{ij} = (y_{N,i} y_{N',j} + y_{N,j} y_{N',i}) \frac{\sqrt{2} v v'}{2\lambda_N v_\phi}, \tag{S2}$$

which can be obtained directly by applying the rotation in Eq. (S1) or by integrating out the heavy sterile neutrinos. After diagonalizing this mass matrix, we can obtain the eigenvalue masses which shows the usual seesaw expression for the neutrino masses, associated with Majorana masses of order $\lambda_N v_\phi$. The massless eigenstate is actually $\vec{v}' \cdot (\overrightarrow{y_N} \times \overrightarrow{y_{N'}})$. From here one can obtain the proper mass eigenstates ν_a, and the mixing angles for both active and sterile neutrinos.