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Abstract

Cities' Demand-driven Industrial Composition

I develop a two-city model to explain a new stylized fact: high-income cities specialize in income-elastic sectors. The model has heterogeneous income elasticities and mobile agents, and either heterogeneous fundamental productivities or heterogeneous amenities generate the specialization pattern through the home market effect. The model also implies that tradable sector shares affect income inequality across locations by driving the home market effect. I provide empirical evidence of this with U.S. CBSA-level data. Finally, I theoretically and empirically suggest that, through this effect by tradable sector shares, the disproportionate trade cost reduction of business services has been supporting a stable city-size wage premium since 1980.

Persistence of Non-democratic Regimes and Reputation

I develop a model that demonstrates how the reputation of a military can generate different paths of political transitions. When a military has a reputation for not holding on to power for a long time, citizens can tolerate frequent coups. A military with such a reputation voluntarily democratizes to maintain the reputation. This equilibrium replicates the characteristics of the political transition paths of countries like Thailand. When there is no such reputation, the citizens resist a coup to avoid a non-democratic regime. Once the military seizes power, it will never voluntarily democratize, and the non-democratic regime becomes persistent. I show that citizens can choose the combination of frequent coups and voluntary democratizations over buying out their military.

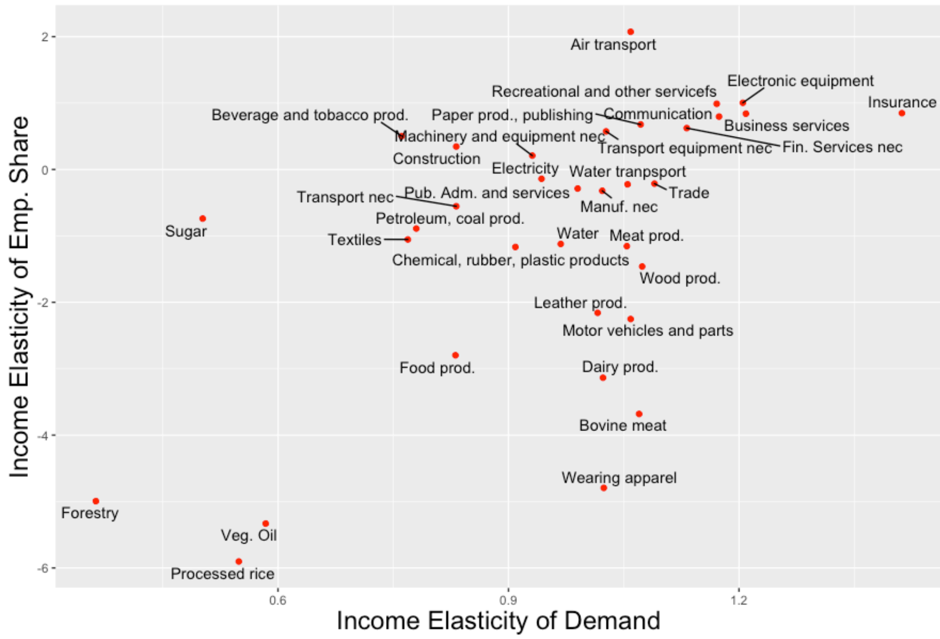
Chapter 1

Cities' Demand-driven Industrial Composition

1.1 Introduction

Industrial composition varies greatly across cities: Detroit, for example, is synonymous with cars and Silicon Valley with computers. Figure 1.1 shows that much of the variation in employment composition across U.S. cities is linked to the income elasticity of demand for that industries' output. Employment shares of industries with high income elasticities of demand, such as recreational services and insurance, are higher in high-income cities like New York City. Industries with low income elasticities of demand, such as sugar and textiles, have larger employment shares in low-income cities.

Figure 1.1: Elasticity of Employment Share with respect to MSA's Income Level and Elasticity of Demand with respect to Income



Income is per capita personal income in 2006 from the Bureau of Economic Analysis. Employment shares are calculated from 2006 County Business Pattern data. When obtaining the income elasticity of employment share, region $\in \{Northeast, Midwest, South, West\}$ is controlled. Income elasticity estimates are from Caron et al. (2020). For details of data description and robustness checks, see Appendix A.1.

This paper investigates this production pattern and the implications both theoretically and empirically. I begin by developing a model that can generate this specialization pattern as an equilibrium outcome. My model is based on Matsuyama (2019); thus, sectors all produce differentiated goods and differ in income elasticities. I extend Matsuyama (2019) by introducing worker mobility and non-tradable sectors. In an equilibrium, a fundamental difference between cities either in productivity or amenities generates the home market effect on the wage rate and on the trade pattern, which is consistent with the specialization pattern in Figure 1.1. In addition, the model implies that the aggregate share of tradable sectors has a significant effect on cross-city wage patterns. Next, I theoretically study comparative statics for two different forms of trade cost reduction that affect the aggregate tradable sector shares. Then, I empirically verify the importance of the aggregate share with U.S. core-based statistical area (CBSA) data. Finally, based on these results, I use the model to explain that a sector-specific trade cost reduction in business services have supported a stable city-size wage premium between 1980 and 2007.

In the production pattern explained as an equilibrium outcome in Figure 1.1, the home market effect plays a key role. In the home market approach first formally theorized by Krugman (1980), the effect is of two types, each of which shares the mechanism from trade costs and an increasing return to scale production. The first affects the wage rate. Other things being equal, the wage rate tends to be higher in larger markets. When firms are exposed to competition with firms in other locations and sharing demands, differences in access to markets due to trade costs drive the difference in the input costs so that the firms' profits are equalized at zero in any location. The second effect affects trade pattern. When the relative market size of sectors varies across regions, regions export goods for which they have relatively large domestic markets. This is because, in the presence of trade costs, local firms are incentivized to operate in a sector that has a relatively larger home market, and this incentive is strong enough to amplify the demand pattern to the production pattern.

In my model, one fundamental difference between cities either in productivity or amenities generates these two home market effects, and this eventually produces the specialization pattern, which is consistent with Figure 1.1. First, a city with better fundamentals (productivity or amenity) attracts workers, which results in a large population, and this generates the home market effect on the wage rate. Second, due to the higher wage, residents in the large city spend relatively more on income elastic sectors and this generates the home market effect on the trade pattern. Hence, in the equilibrium, the fundamentally attractive city becomes larger, offers a higher wage, and specializes in income elastic sectors.

In addition to explaining the production pattern, the model highlights the importance of the aggregate tradable sector share when we study cross-location income inequality. In equilibrium, the wage increases not only with city size but also with the aggregate tradable sector share. This is because the home market effect works only through tradable sectors. The source of the home market effect is competition between firms in different locations. When a sector is non-tradable, that competition does not exist, and the home market effect does not emerge. Thus, the market size that drives the wage is the size of tradable sectors, which is the product of the overall market size and the aggregate tradable sector share. Because in my model the aggregate tradable sector share of a city is endogenously determined, we can use that share to analyze how the economic environment affects cross-location income patterns. To the best of my knowledge, this theme has not been explicitly examined in previous studies of wage patterns, whether across countries or across cities.

The endogenous tradable sector share reveals that the effect on cross-location income inequality of a trade cost reduction can also be the opposite depending on whether it is uniform or sector-specific. In Section 1.4, I provide comparative statics of trade cost reduction in two forms for two models: an international trade model and an urban model. The first form is uniform trade cost reduction; the second is a non-tradable sector that becomes tradable. In the uniform case, the income inequality between countries and between cities always shrinks. In the second case, the income inequality can rise in either setting. A theoretical result for a special case and simulations suggest that income inequality tends to rise when the transitioning sector is income elastic compared to existing tradable sectors. If the addition of the new tradable sector makes the tradable sectors more income elastic as a whole, the location with a higher wage starts to spend relatively more on the tradable sectors, which raises the local wage due to the home market effect on the wage rate.

The comparative statics as well as the following empirical studies suggest that the disproportionate trade cost reduction in business services has been supporting the stable city-size wage premium. According to Head et al. (2009) and Eckert (2019), since 1980, the trade cost of business services has disproportionately declined. Interpreting this as a second type of trade cost reduction in the comparative statics keeps the tractability, and it provides insights into the consequence. The results of the comparative statics suggest that this transition is likely to widen the wage inequality along the dimension of city size because business services have a very high income elasticity. In Section 1.5, I verify this result with U.S. core-based statistical area (CBSA) data between 1980 and 2007 by implementing reduced form wage regressions. The regression model is motivated by an equilibrium condition of the theoretical model, and it entails a simultaneity problem. I solve this problem by employing instrumental variables that have been used in the literature: cities' historical population and measurements of the climate of cities. When the regression model does not control the aggregate tradable sector share, it estimates the city-size wage premium, as measured by the population elasticity of wages. The regression reveals that the premium remained stable or rose slightly over the sample period. In theoretical models that feature either one tradable sector that has a fixed share or all tradable sectors, this could imply a stable trade cost, although this seems counterintuitive. However, given the assumption that business services were non-tradable in 1980 and tradable in 2007, controlling the relevant aggregate tradable sector share changes the interpretation. In the data, the aggregate tradable sector share decreases in city sizes in 1980 and increases in city sizes in 2007, and controlling the relevant aggregate share reveals that the population elasticity of wages has been declining. This finding is consistent with the results of the comparative statics, and it suggests that the transition of business services has been supporting the stable city-size wage premium through the aggregate tradable sector share.

Industrial composition is an important factor for local economies, and not only because an aggregate tradable sector share is a driver of local wages through the home market effect; it also is important because local economic performance is significantly affected by the industries that locate in a city (e.g., Autor et al. (2013)). To fully understand variation in local economic performance it is necessary, first, to understand what determines the mix of industry in cities. The mechanism that drives industrial composition also is important for researchers who want to exploit regional variation in the size of industries. In this paper, I report that industrial composition is related to income level. Given this relationship, regressing dependent variables on sectoral sizes or shares, while not controlling the income level of examined locations, might lead to an omitted variable bias problem because the trend of a dependent variable could be driven by the income level of the locations rather than by sectoral sizes or shares. Understanding the mechanism can help researchers avoid this endogeneity issue.

This paper is the first research on cross-city inter-sectoral specialization patterns to be undertaken from the demand-side perspective. Most previous works on the cross-city specialization pattern has focus on non-demand side factors such as functions in production, (Duranton and Puga (2005) and Henderson and Ono (2008)), the strength of the agglomeration economy (Behrens and Robert-Nicoud (2015)), and the skill supply (Davis and Dingel (2019)). A few works focus on the demand side's effect on cross-city difference (e.g. Handbury (2019)). The closest of these is Dingel (2017), who quantifies the contributions made to the quality specialization of cities by the heterogeneous demand factor and by the skill supply factor, and he creates a model to guide the quantification. There are two major differences between his model and mine. First, we focus on different types of specialization and trade patterns. His work, and, therefore, his model, examines vertical specialization and the intra-sector trade, where different quality goods are gross substitutes. Whereas, my model examines horizontal specialization and inter-sectoral trade, where goods in different sectors are gross complements. Second, agents are immobile in his model, while in mine they are mobile. The mobile-agent assumption fits the urban economy environment, and, moreover, it enables us to study the relationship between the size of a city and its industrial composition. This paper, the first work on cross-city inter-sectoral specialization patterns undertaken from the demand-side perspective, contributes to the literature by reporting the stylized fact and providing the theoretical framework for urban economies.

Furthermore, my work contributes more broadly to studies of the demand-side effect in trade patterns The international trade literature shows theoretically that the demand-side effect plays an important role in determining trade patterns and specializations (e.g. Flam and Helpman (1987), Stokey (1991), Matsuyama (2019), Fajgelbaum et al. (2011)). The closest to mine is Matsuyama (2019) in the sense he analyzes horizontal specialization and inter-sectoral trade. My model extends Matsuyama (2019) by introducing non-tradable sectors, and it reveals a new effect that heterogeneous demand across locations generates. This is the effect on cross-location income through endogenous tradable sector shares, and it is relevant both in international trade and in domestic trade.

The analysis of the business services' transition, implemented from a new perspective, is a unique contribution on its own. Eckert (2019) examines the effect of this transition on the skill premium by focusing on the high skill intensity of business services, but the effect from the high income elasticity has not been studied yet. This paper examines this aspect for the first time.

The rest of this paper is organized as follows. Section 2 develops the model to explain the stylized fact. Section 3 studies comparative statics. Section 4 interprets the evolution of the city-size wage premium through the lens of the model, and Section 5 concludes.

1.2 Model

In this section, I introduce the model of two cities ($c \in \{1,2\}$). The cities fundamentally differ in productivity and amenity. There is a mass of N workers who are freely mobile and homogeneous except for inherent taste over cities. Conditional on location, individual labor supply is inelastic. There are K sectors, and the sectors differ in relative income elasticity (ϵ_k) and tradability (τ_k) as well as productivities (ϕ_k, ψ_k) and preference shifters (β_k). I start with explaining the household problem.

Household

The problem for a worker i is given by

$$\max_{c\in\{1,2\},\{q_k(\nu)\}_{\nu\in\Omega_{ck},k\in K},\{Q_k\}_{k\in K}} \quad U_c \cdot a_c \cdot \delta(i,c) \tag{1.1}$$

$$s.t.\ 1 = \left[\sum_{k\in K} \beta_k^{\frac{1}{\eta}} U_c^{\frac{\epsilon_k}{\eta}} Q_{c,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$Q_{c,k} = \left[\int_{\nu\in\Omega_{c,k}} q_k(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}$$

$$E_c = \sum_{k\in K} \int_{\nu\in\Omega_{c,k}} p_{ck}(\nu) q_k(\nu) d\nu$$

where c is the city to reside in, $q_k(\nu)$ is the consumption of variety ν in sector k, Ω_{ck} is the set of available varieties of sector k in city c, K is the set of sectors, $Q_{c,k}$ is the composite consumption of sector k, β_k is the preference shifter of sector k, E_c is the income in city c, and $a_c > 0$ for $c \in \{1,2\}$, $0 < \epsilon_k < 1$ for all $k \in K$, $0 < \eta < 1$, $\sigma > 1$. By setting $\eta < 1$, I assume sectors are gross complements throughout the paper. The utility consists of three factors: U_c, a_c, and $\delta(i,c)$.

U_c is the utility from goods consumption, and the functional form captures the non-homothetic preference of the consumer. This functional form follows Comin et al. (2021). Matsuyama (2019) uses the same form, properties of which he illustrates in detail in the same paper. When solving this household problem, a convenient property of the functional form of U_c can be seen. The demand function derived from this preference becomes

$$Q_{c,k} = \beta_k P_{c,k}^{-\eta} U_c^{\epsilon_k} E_c^{\eta} \tag{1.2}$$

where $P_{c,k}$ is the price index for sector k in city c and it is defined as $P_{c,k} = \left[\int_{\nu \in \Omega_{c,k}} p_k(\nu)^{1-\sigma} d\nu\right]^{\frac{1}{1-\sigma}}$. This shows that the relative income elasticity of demand (ϵ_k) and price elasticity (η) are separated. While the price elasticity is assumed to be common among sectors, I vary the income elasticity across sectors. Also, the expenditure share of sector k is obtained as follows (see Appendix A.2 for the derivation):

$$m_{c,k} \equiv \frac{P_{c,k} Q_{c,k}}{\sum_{l \in K} P_{c,l} Q_{c,l}} = \frac{\beta_k P_{c,k}^{1-\eta} U_c^{\epsilon_k}}{\sum_{l \in K} \beta_l P_{c,l}^{1-\eta} U_c^{\epsilon_l}} \tag{1.3}$$

This shows that, holding the price indices $\{P_k\}_{k \in K}$ constant, agents with higher utility from goods consumption (U_c) spend relatively more on sectors with high ϵ_k. Another result is the indirect sub-utility, which is implicitly expressed as

$$E_c^{1-\eta} = \sum_{k \in K} \beta_k P_{ck}^{1-\eta} U_c^{\epsilon_k} \tag{1.4}$$

(see Appendix A.2 for the derivation). This illustrates that as U_c rises, agents care particularly about the prices of high ϵ_k goods, on which they spend relatively more.

a_c is the utility from an amenity offered by city c, such as weather, landscape, and historic heritage. While "amenity" generally refers to access to local services and consumer goods (e.g., restaurants), in this model, those local services and goods contribute to U_c when they are consumed.

$\delta(i,c)$ is the idiosyncratic utility shock for the worker i and city c pair. This generates the heterogeneous taste over cities (following Tabuchi and Thisse (2002) and others). I assume $\delta(i,c)$ is distributed i.i.d. across workers and cities according to the Fréchet distribution with shape parameter $1/\gamma$ ($Pr[\delta < x] = e^{-x^{-1/\gamma}}$). Each worker chooses the city that offers the higher utility, taking into account her consumption optimization. Thus, given the indirect utility levels in the two cities, a_1U_1 and a_2U_2, the probability of choosing city 1 for a given agent is derived as $Pr[a_1U_1\delta(i) > a_2U_2\delta(i)] = (a_1U_1)^{1/\gamma} / \left\{(a_1U_1)^{1/\gamma} + (a_2U_2)^{1/\gamma}\right\}$. Since the shock is i.i.d., the cities' population ratio follows.

$$\frac{N_1}{N_2} = \left(\frac{a_1U_1}{a_2U_2}\right)^{1/\gamma} \tag{1.5}$$

Given the same relative utility from goods consumption ($\frac{a_1U_1}{a_2U_2}$), the lower γ is, the greater the population inequality is. This illustrates that the γ measures the dispersion force in this economy. [1]

Production

The production in my model is based on Krugman (1980). For all sectors $k \in K$ there are endogenous sets of varieties, homogeneous firms, and monopolistic competition . Each sector is either tradable with iceberg trade cost $\tau > 1$ or it is non-tradable. Let $\mathbb{T}$ be the set of

[1] If additional variables V_1 and V_2 are introduced s.t. $V_1 = a_1U_1N_1^{-\gamma}, V_2 = a_1U_2N_2^{-\gamma}$, (1.5) becomes $V_1 = V_2$. This shows the isomorphism to agents with homogeneous tastes whose utility function is $V_c = UN_c^{-\gamma}$. N_c^{γ} can be interpreted as a congestion cost from a local population that directly negatively affects the utility and the microfoundation can be provided by a housing sector with an inelastic land supply and a Cobb-Douglas utility function for the fixed expenditure share allocation between housing and goods (Helpman (1998)). However, when the preference is non-homothetic, the price index of aggregate goods consumption depends on the utility level, in which case the fixed expenditure share result does not hold. As a result, the Helpman-type microfoundation does not generate (1.5) .

tradable sectors and $\mathbb{N}$ be the set of non-tradable sectors ($K = \mathbb{T} \cup \mathbb{N}$ and $\mathbb{T} \cap \mathbb{N} = \emptyset$). To let city 1 be fundamentally more productive than city 2, I assume that labor supply by a worker in city 1 is λ (≥ 1) efficiency units of labor for all sectors while that by the same worker in city 2 is one unit for all sectors. Each worker chooses the location of labor they supply. Conditional on location, individual labor supply is inelastic. I let w_c denote the wage rate for an efficiency unit in city c. It follows that the income for households in each city becomes

$$E_1 = \lambda w_1$$

$$E_2 = w_2$$

Each firm in sector k needs to employ ϕ_k units of labor as the fixed cost and ψ_k as the variable cost to produce a unit of variety. The problem for a firm that produces variety ν in sector k in city c is

$$\begin{aligned} \pi_{ck}(\nu) = \max_{p_{cck}(\nu), q_{cck}(\nu), p_{cc'k}(\nu), q_{cc'k}(\nu)} & [p_{cck}(\nu) q_{ck}(\nu) - \psi_k q_{cck}(\nu) w_c] \\ & + \mathbb{1}\{k \in \mathbb{T}\} [p_{cc'k}(\nu) q_{cc'k}(\nu) - \tau \psi_k q_{cc'k}(\nu) w_c] - \phi_k w_c \end{aligned} \tag{1.6}$$

$$\begin{aligned} s.t.\ q_{cck}(\nu) &= p_{cck}(\nu)^{-\sigma} P_{ck}^{\sigma} Q_{ck} \\ q_{cc'k}(\nu) &= p_{cc'k}(\nu)^{-\sigma} P_{c'k}^{\sigma} Q_{c'k} \end{aligned}$$

where $\pi_{ck}(\nu)$ is the profit by optimized production; $(c, c') \in \{(1,2),(2,1)\}$; $p_{cck}(\nu)$ and $p_{cc'k}(\nu)$ are the prices for the markets in city c and c', respectively; and $q_{cck}(\nu)$ and $q_{cc'k}(\nu)$ are the quantities for the markets in city c and c', respectively. In the following part, I omit ν. The terms in the first bracket are the variable profits from selling products in city c, while those in the second are those in city c', which is zero if $k \in \mathbb{N}$. If sector k in city c has non-zero production in an equilibrium, π_{ck} must be zero such that there is no entrant. Similarly, if sector k in city c has zero production in an equilibrium, π_{ck} must be non-positive. This is the zero-profit condition in sector k in city c.

Definition of Competitive Equilibrium

A competitive equilibrium is

$\{N_1, N_2, U_1, U_2, w_1, w_2, E_1, E_2, \{p_{cc'k}, q_{cc'k}\}_{(c,c',k)\in(1,2)\times(1,2)\times K}, \{\Omega_{1k}, \Omega_{2k}\}_{k\in K}\}$ such that

1. households optimize consumption and locational choice as (1.1) where $E_1 = \lambda w_1$ and $E_2 = w_2$ for $c \in \{1, 2\}$,
2. producers optimize production as (1.6) for all $k \in K$ and $c \in \{1, 2\}$,
3. the zero-profit condition holds such that $\pi_{ck} \leq 0$ for all $k \in K$ and $c \in \{1, 2\}$ where equality holds if $q_{cck} + \tau q_{cc'k} \neq \emptyset$,
4. the national labor market clearing condition that $N_1 + N_2 = N$ holds, and
5. the local labor market clearing conditions,

$$\sum_{k\in\mathbb{N}} \int_{\Omega_{1k}} (\psi_k q_{11k} + \phi_k)\, d\nu + \sum_{k\in\mathbb{T}} \int_{\Omega_{1k}} (\psi_k q_{11k} + \tau\psi_k q_{12k} + \phi_k)\, d\nu = \lambda N_1$$

   and

$$\sum_{k\in\mathbb{N}} \int_{\Omega_{2k}} (\psi_k q_{22k} + \phi_k)\, d\nu + \sum_{k\in\mathbb{T}} \int_{\Omega_{2k}} (\psi_k q_{22k} + \tau\psi_k q_{21k} + \phi_k)\, d\nu = N_2$$

   , hold.

Equilibrium Conditions

I now characterize an equilibrium by obtaining simplified conditions. I focus on on equilibria where all sectors have nonzero output in both cities, and, given the optimized production and the demand function, I impose the zero-profit condition on each sector in each city. The first result concerns non-tradable sectors.

Proposition 1. *Given an equilibrium, the expenditure share and the employment share of a non-tradable sector in a city are equalized.*

$$\forall c \in \{1,2\},\ \forall k \in \mathbb{N} \quad m_{ck} = x_{ck} \tag{1.7}$$

where x_{ck} is the employment share of sector k in city c (see Appendix A.3 for the proof). This equalization follows from the zero-profit condition in the sector level. That is, each firm has zero profit and therefore the sector has zero aggregate profit. Since the revenues in the non-tradable sectors are only received from the agents in the same city and the factor payments are made only to the same agents, the zero-profit condition boils down to equation (1.7). Corollary 1 follows.

Corollary 1. *Given an equilibrium, the aggregate expenditure share and the aggregate employment share in a city are equalized both for the non-tradable sectors and for the tradable sectors.*

$$\forall c \in \{1,2\},\ \sum_{k\in\mathbb{N}} m_{ck} = \sum_{k\in\mathbb{N}} x_{ck} \ \textit{and} \ \sum_{k\in\mathbb{T}} m_{ck} = \sum_{k\in\mathbb{T}} x_{ck}$$

This equalization between the aggregate expenditure share and the aggregate employment share in the tradable sectors is a key for the tractability of the model. Another result is Proposition 2.

Proposition 2. *Given an equilibrium, the city sizes (N_1 and N_2), the aggregate tradable sector shares ($\sum_{k\in\mathbb{T}} m_{1k}$, and $\sum_{k\in\mathbb{T}} m_{2k}$), and the income ratio ($\omega = \frac{E_1}{E_2} = \frac{\lambda w_1}{w_2}$) satisfy the following equation.*

$$\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{T}} m_{2k}} \frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^{\sigma} - \rho\omega^{\sigma}}\right] \tag{1.8}$$

This result is obtained by aggregating the zero-profit condition over tradable sectors and making use of $\sum_k m_{ck} = \sum_k x_{ck}$.(see Appendix A.3 for the proof). This summarizes the sector-level zero-profit condition for the tradable sectors into the city level. The LHS is the relative tradable market size of city 1 to city 2. The RHS, which increases in the relative wage of city 1 (ω), shows two things. First, the sector-level force -- that a large local market is accompanied by a higher input cost-- is carried over to the city level. This is the home market effect on the wage rate discussed in Krugman (1980) and Krugman (1991). Second, this home market effect force works only through tradable sectors, and the relative aggregate tradable sector expenditure share matters. For the zero-profit condition to hold in each sector-city pair, the larger city's advantage of lower trade costs for selling to consumers must be exactly offset by higher input costs. This force does not appear in non-tradable sectors because the local wage rate change affects both the demand and the input cost at the same rate. Next, the expenditure shares in an equilibrium are obtained.

Lemma 1. *Given an equilibrium, the expenditure shares of sector $k \in K$ in city 1 and city*

2 can be expressed as follows:

$$m_{1k} = \left[\frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{(1-\rho_k^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{1.9}$$

$$m_{2k} = \left[\frac{1 - \rho_k \lambda^{-\sigma} \omega^{\sigma}}{(1-\rho_k^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{1.10}$$

where

$$\rho_k = \begin{cases} 0 & k \in \mathbb{N} \\ \rho = \tau^{1-\sigma} & k \in \mathbb{T} \end{cases}, \quad \tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}} \phi_k^{\frac{1}{\sigma-1}} \psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}} (\frac{\sigma-1}{\sigma})}\right)^{1-\eta}$$

Making use of $\sum_{k\in K} m_{ck} = 1$, Proposition 3 is obtained.

Proposition 3. *Given an equilibrium, the utility from goods consumption in city 1 and city 2 (U_1 and U_2) can be implicitly expressed as follows:*

$$\left[\frac{\lambda^{-\sigma}}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} = 1 \tag{1.11}$$

$$\left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1 - \rho\lambda^{-\sigma}\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} = 1 \tag{1.12}$$

Holding the relative wage constant, the elasticity of utility from goods consumption with respect to the local population is given by

$$\frac{\partial U_c / U_c}{\partial N_c / N_c} = \frac{1}{\sigma - 1} \frac{1-\eta}{\tilde{\epsilon}_c}$$

where $\tilde{\epsilon}_c = \sum_{k\in K} m_{ck}\epsilon_k$. $1/(\sigma-1)$ is the elasticity of the agglomeration economy or the positive externality in Krugman-type models with homothetic preference. The number of varieties in a location increases with market size and consumers have love-of-variety in their preferences. In this non-homothetic model, the positive externality is adjusted by $(1-\eta)/\tilde{\epsilon}_c$. To understand this, suppose all sectors have homogeneous income elasticity, $\bar{\epsilon}$. The utility becomes explicit, and it is

$$\begin{aligned} U &= \left[\sum_k \tilde{\beta}_k^{\frac{1}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{-\frac{\eta}{\bar{\epsilon}}} \\ &= Y^{\frac{1-\eta}{\bar{\epsilon}}} \end{aligned}$$

where $Y = \left[\sum_k \tilde{\beta}_k^{\frac{1}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$.This shows that, holding Y constant, the greater $\bar{\epsilon}$ is, the smaller the marginal utility is. This relationship between the marginal utility and ϵ_k is carried over to the heterogeneous income elasticity model. The effect on the marginal utility from aggregate consumption is summarized by the average of ϵ_k weighted with the expenditure shares.

Next, I allow ω to move and see whether it amplifies or attenuates the agglomeration economy. Holding the relative tradable sector share constant, it follows from the home market effect on the wage rate (1.8) that

$$\frac{\partial \omega}{\partial N_1} > 0$$

When city 1 becomes larger, the home market effects require a higher relative wage rate in city 1. Consequently, holding the relative tradable share constant,

$$\frac{\partial U_c/U_c}{\partial N_c/N_c} + \frac{\partial U_c/U_c}{\partial \omega/\omega}\frac{\partial \omega/\omega}{\partial N_c/N_c} < \frac{\partial U_c/U_c}{\partial N_c/N_c}$$

as $\frac{\partial U_1}{\partial \omega} < 0$ and $\frac{\partial U_2}{\partial \omega} > 0$ from the equilibrium conditions 1.11 and 1.12. This shows that the movement of ω attenuates the agglomeration economy. This attenuation effect reflects the fact that the increase in the local population is accompanied by a decrease in the other city's population, and the mass of available varieties shipped from the other city decreases. The direction of the net effect is not obvious. To obtain a clear result about the direction in which U_c moves while taking into account all of the effects, I analyze the case in which the sectors are all tradable ($\mathbb{N} = \emptyset$, $K = \mathbb{T}$).[2]After tedious calculations, it can be shown that, if $\mathbb{N} = \emptyset$,

$$\forall \tau > 1, \forall N_1 \in (0, N), \quad \frac{dU_1}{dN_1} > 0$$

$$\begin{cases} \forall N_2 \in (0, N), \quad \frac{dU_2}{dN_2} > 0 & if\ \lambda^{\frac{\sigma}{\sigma-1}} \leq \tau \\ \exists \bar{N}_2 > 0\ s.t. \begin{cases} \frac{dU_2}{dN_2} > 0 & N_2 \in (0, \bar{N}_2) \\ \frac{dU_2}{dN_2} < 0 & N_2 \in (\bar{N}_2, N) \end{cases} & if\ 1 < \tau < \lambda^{\frac{\sigma}{\sigma-1}} \end{cases}$$

The result for city 1 reflects the fact that the increase in city 1's varieties dominates the decreases in city 2's varieties. Reallocation of labor from the unproductive location to the productive one increases the total mass of available varieties. In the case of city 2's expansion, the net effect is the opposite. However, there are still two positive effects: saving the trade cost on average as the share of varieties without the trade cost rises; and the rising purchasing power. The aggregate effect depends on the size of the relative productivity and the trade cost. When the trade cost is large enough relative to the productivity gap, the negative effect from the unproductive labor reallocation is dominated by the combination of the benefit from saving the trade cost and the purchasing power effect. Therefore, population expansion always increases U_2, regardless of the level of N_2. In the other case, when the trade

[2]When the relative tradable sector share is held constant, there are four effects. First, the increase in local population leads to an increase in locally produced varieties. Second, the mass of available varieties shipped from the other city decreases. Third, the rising share of locally produced varieties makes the average price of varieties, other things being equal, cheaper because the average trade cost declines. Fourth, the home market effect raises the purchasing power of the local residents ($\omega \uparrow$ for city 1 residents). The first, third, and fourth effects are positive effects for U_c while the second is negative.

cost is relatively low, the negative productivity effect catches up with the positive effects at some point ($\bar{N}_2$) and U_2 starts to decline.

Finally, an equilibrium is characterized by the seven conditions obtained above: (1.5), (1.8), (1.11), (1.12), the national labor clearing condition ($N_1 + N_2 = N$), and the aggregate tradable sector shares as follows:

$$\sum_{k \in \mathbb{T}} m_{1k} = \left[\frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{(1-\rho_k^2) N_1} \right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$
$$\sum_{k \in \mathbb{T}} m_{2k} = \left[\frac{1 - \rho_k \lambda^{-\sigma} \omega^{\sigma}}{(1-\rho_k^2) N_2} \right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$

These aggregate shares follow from individual tradable sector's share ((1.9) and (1.10)). There are seven unknown variables $\{U_1, U_2, \omega, N_1, N_2, \sum_{k \in \mathbb{T}} m_{1k}, \sum_{k \in \mathbb{T}} m_{2k}\}$ for these seven equations. By making use of the zero-profit conditions for all city-sector pairs, the number of endogenous variables here is fewer than that of the equilibrium definition. Based on these conditions, I now provide some properties of the equilibrium.

The Existence, the Uniqueness, and the Stability of an Equilibrium

First, it can be shown that, given Assumption 1, there exists a unique equilibrium.

Assumption 1. $\gamma > \frac{1-\eta}{(\sigma-1)\min\{\epsilon_k\}_{k \in \mathbb{T}}}$

Proposition 4. *If Assumption 1 holds, there exists a unique equilibrium.*

This corresponds to the sufficient and necessary condition, which Helpman (1998) obtains by numerical simulations, for his model to have a unique stable equilibrium. Compared to his condition ($\gamma > \frac{1}{\sigma-1}$ in this paper's notation), there is an adjusting term, $\frac{1-\eta}{\min\{\epsilon_k\}_{k \in \mathbb{T}}}$. This is sufficient to ensure that whatever the expenditure composition is, the agglomeration force is weaker than the dispersion force.

Next, I analyze whether the unique equilibrium is stable. When U_1 and U_2 can be expressed as functions of only N_1 from (1.8), (1.11), and (1.12), a stable equilibrium is defined as follows.

Definition. A competitive equilibrium is a stable equilibrium if and only if

$$\frac{d\frac{a_1U_1(N_1)}{a_2U_2(N_2)}}{dN_1} < \frac{d\left(\frac{N_1}{N_2}\right)^{\gamma}}{dN_1}$$

[3]This guarantees that, with the migration of agents whose agent-specific utility in city 1 is marginally greater than that of city 2, the expanding city would not experience enough of a relative gain in utility from goods consumption to support the post-migration population allocation. With this condition, the marginal agents in the expanding city find the shrinking one preferable and return to their original location. Consequently, the economy converges back to the original state. For the stability, Proposition 5 is obtained.

Proposition 5. *If Assumption 1 holds, the unique equilibrium is stable.*

When the dispersion force is strong enough, the unique equilibrium is stable.[4]In the rest of the paper, I assume that Assumption 1 holds, and so I focus on the unique and stable equilibrium.

---

[3]With V_1 and V_2 s.t. $V_1 = a_1U_1N_1^{-\gamma}$ and $V_2 = a_2U_2N_2^{-\gamma}$, the condition of the stable equilibrium becomes $\frac{dV_1}{dN_1} < \frac{dV_2}{dN_1}$In the homogeneous agent's interpretation of the model (as discussed in a previous footnote), this guarantees that the expanding city does not offer a higher utility than the shrinking city.

[4]If Assumption 1 does not hold and $\gamma < \frac{1-\eta}{(\sigma-1)\max\{\epsilon_k\}_{k\in\mathbb{T}}}$ and $\mathbb{N} \neq \emptyset$, then all workers living in one city 1 ($N_1 = N$ and $N_2 = N$) become a stable equilibrium. This case corresponds to $\beta\epsilon < 1$ in Helpman (1998)'s prohibitive trade cost. In my model, the existence of a non-tradable sector, rather than a prohibitive trade cost, suffices to generate this result. In the non-tradable sectors, a new marginal city after a deviation from a one-city equilibrium has only a marginal mass of varieties. Therefore, the relative mass of the locally-offered varieties for the preexisting city is unbounded. When $\gamma < \frac{1-\eta}{(\sigma-1)\max\{\epsilon_k\}_{k\in\mathbb{T}}}$, this unbounded gap dominates the dispersion force.

1.3 Cross-City Analysis

In this section, I illustrate how the two cities differ in the equilibrium depending on their fundamental productivities and amenities. I start with a partial equilibrium analysis to show the directions of the forces that the fundamental differences generate. Then, I lay out general equilibrium results in the case where the cities have the same amenity level and differ only in fundamental productivity ($a_1 = a_2$). The results of different amenities with the same productivity are briefly explained (A detailed explanation is provided in Appendix A.4).

Partial Equilibrium Analysis

For the partial equilibrium analysis, we focus on city 1. Making use of the population allocation with Fréchet taste shock (1.5), the utility from goods consumption in city 1 in equilibrium (1.11) and the home market effect (HME) on the wage rate (1.8) can be written as follows:

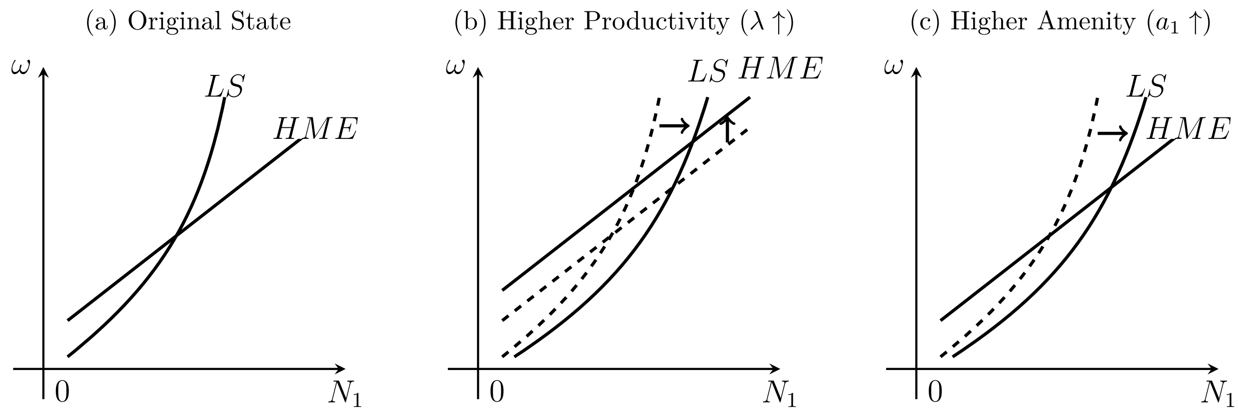
$$1 = \left[\frac{1}{\lambda^{\sigma}}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} N_1^{\gamma\epsilon_k \frac{\sigma-1}{\sigma-\eta} - \frac{1-\eta}{\sigma-\eta}} \left(\frac{V}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$

$$+ \left[\frac{1}{\left(\lambda^{\sigma} + \frac{\omega^{\sigma-1}}{\tau^{\sigma-1}}\left(\frac{N_2}{N_1}\right)\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}}\right)}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in K} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} N_1^{\gamma\epsilon_k \frac{\sigma-1}{\sigma-\eta} - \frac{1-\eta}{\sigma-\eta}} \left(\frac{V}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{1.13}$$

$$N_1 = \frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}} \omega^{2\sigma-1} \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^{\sigma} - \rho\omega^{\sigma}}\right] N_2 \tag{1.14}$$

where $V = a_2 U_2 N_2^{\gamma}$ and w_2 is normalized to 1. In this partial equilibrium analysis, I fix $V(= a_2 U_2 N_2^{\gamma})$, and the tradable sector share ratio $(\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}})$. Then, equation (1.13) can be interpreted as the labor supply curve in city 1, and, given Assumption 1, N_1 increases in ω and therefore in w_1 . As the wage increases, the city attracts more workers. Similarly, given

Figure 1.2: Partial Equilibrium Analysis



Assumption 1, N_1 increases in ω and therefore in w_1 from the HME on the wage rate (1.14). Having a large local market requires a higher input cost to keep the profit at zero. These two curves are depicted in Figure 1.2a. [5]

When city 1 has a higher productivity ($\lambda \uparrow$), the HME curve shifts up and the labor supply curve shifts to the right, as in Figure (1.2b). The HME curve shifts for two reasons. First, the local income linearly increases in local productivity as $\omega = \lambda w_1$. Consequently, the local market size expands and the input cost, w_1, needs to rise to keep the zero profit. The shift of the labor supply curve is driven by additional local varieties and reflects the fact that the number of people increases given the income (ω). Higher productivity increases the local labor supply in terms of the efficiency unit and, therefore, the mass of local varieties. Given the prices of goods, which are linear in the wage rate per efficient unit w_1, this reduces the price indices and raises the local utility from goods consumption, and, therefore, it attracts more workers. Because of the shifts of the two curves, the new intersection is located where

[5]How the curves intersect is not easy to tell from the equations in this part. The depiction here is based on the theoretical results. For cities with asymmetric productivity, the one with higher productivity offers higher local wage and larger population. For cities with asymmetric amenity, the one with higher amenity offers higher local wage and larger population. These are consistent with the labor supply curve intersecting the home market effect curve from below.

both the population and the wage are higher than before.

When city 1 has a higher amenity, this shifts the labor supply curve to the right, although it does not affect the HME curve, as in Figure 1.2c. This reflects the fact that the higher amenity attracts more people, but it does not affect production. As a result, the intersection moves along the HME curve, and both the population and the wage are higher than before. [6]

This analysis, which ignores what is taking place in the other city (N_2, U_2) and the effect through the tradable sector share ratio $(\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}})$, shows the main forces generated by heterogeneous fundamental productivity and amenity. In the next section, I provide the results in general equilibrium for the case in which cities differ only in terms of productivity. Appendix A.4 provides the results for the case in which there is a different amenity and a common productivity. The city that has better fundamental characteristics (productivity or amenity) becomes larger and offers a higher wage.

General Equilibrium with Productivity Difference

In this section, I consider cities that have the same amenity level and differ only in fundamental productivity, and I provide a cross-city analysis for the stable unique equilibrium. The first comparison concerns population allocation and the nominal income ratio. The partial equilibrium analysis (Figure 1.2b) suggests that city 1 has the larger population and the residents receive the higher income. When we consider a general equilibrium and allow additional variables, including N_2, to move, the movement of N_2 attenuates the agglomeration economy in city 1 because of the decrease in the mass of varieties shipped from city 2, as discussed in Section 1.2. As a result, this shortens the shift of the labor supply curve in

[6] While this result is in contrast to what a Rosen-Roback model (Rosen (1979), Roback (1982)) implies, this is not new in the literature. For example, Glaeser and Gottlieb (2009) point out that rising amenities can increase wages because of agglomeration economy.

Figure 1.2b. On the other hand, the shrinking population in city 2 amplifies the home market effect and it shifts further up the HME curve in Figure 1.2b. After all, it can be proven that the qualitative result does not change and Proposition 6 is obtained (see Appendix A.3 for the proof).

Proposition 6. *In the stable unique equilibrium with Assumption 1 and $a_1 = a_2$, the fundamentally productive city is larger and the nominal income ratio is greater than the fundamental productivity ratio.*

$$N_1 > N_2 \text{ and } \omega > \lambda > 1$$

This result is consistent with a stylized fact that the nominal income level is higher in larger cities, even when observable or unobservable workers' characteristics are taken into account (e.g., Behrens and Robert-Nicoud (2015),Glaeser and Mare (2001)).

The next result concerns expenditure shares.

Proposition 7. *In the stable unique equilibrium, given Assumption 1 and $a_1 = a_2$, the sectoral expenditure share ratio of city 1 to city 2 increases in ϵ_k within the non-tradable sectors and within the tradable sectors, respectively.*

Proof. (1.9) and (1.10) provide the expenditure share ratio of a non-tradable sector and a tradable sector, respectively,

$$\forall k \in \mathbb{N}, \quad \frac{m_{1k}}{m_{2k}} = \left[\lambda^{-\sigma}\frac{N_2}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$
$$\forall k \in \mathbb{T}, \quad \frac{m_{1k}}{m_{2k}} = \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{1-\rho\lambda^{-\sigma}\omega^{\sigma}}\frac{N_2}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$

Since $\frac{U_1}{U_2} > 1$, the expenditure share ratio increases in ϵ_k within the non-tradable sectors and within the tradable sectors. □

In the equilibrium, the high-income city spends more expenditure in the income elastic sectors. This difference in the expenditure shares generates the following results in the employment shares.

Proposition 8. *In an equilibrium, for a tradable sector, there is a relationship with the within-tradable employment share ratio and the within-tradable expenditure share ratio such that*

$$\forall k \in \mathbb{T}, \ \frac{\tilde{x}_{1k}}{\tilde{x}_{2k}} = \frac{\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} - \rho\lambda^{\sigma}\omega^{-\sigma}}{1 - \rho\lambda^{\sigma}\omega^{-\sigma}} \frac{1 - \rho\omega^{\sigma}\lambda^{-\sigma}}{1 - \rho\omega^{\sigma}\lambda^{-\sigma}\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}}}$$

where $\tilde{m}_{ck} = \frac{m_{ck}}{\sum_{k\in\mathbb{T}} m_{ck}}$ *and* $\tilde{x}_{ck} = \frac{x_{ck}}{\sum_{k\in\mathbb{T}} x_{ck}}$. *This shows* $\frac{\tilde{x}_{1k}}{\tilde{x}_{2k}}$ *increases in* $\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}}$. *That is, given an equilibrium, the greater the within-tradable expenditure share difference of a sector, the greater that sector's within-tradable employment share difference. Also, this implies that a city becomes the net exporter in sectors for which the city has a greater relative within-tradable expenditure share as follows:*

$$\forall k \in \mathbb{T}, \ \frac{\tilde{x}_{1k}}{\tilde{x}_{2k}} > \frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} \iff \frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} > 1$$

This is the home market effect in the sectoral specialization. The difference in the expenditure pattern is amplified to that of the employment pattern. Unlike Krugman (1980), who assumes an exogenous taste difference to generate the heterogeneous relative demand, here that difference arises endogenously from the non-homothetic preference. The importance and the endogenous formation of the relative demand are the same as in Matsuyama (2019), but my definition of the relative demand is different. The result shown above demonstrates that when non-tradable sectors exist, the relative size of demand should be measured within the tradable sectors. In Matsuyama (2019), all sectors are tradable and the relative demand size is the same whether it is within the overall economy or within the tradable sectors.

Next, I analyze the aggregate tradable and non-tradable sector expenditure shares, which are equal to their employment shares from Corollary 1. In the equilibrium, it follows from equation (1.9) and (1.10) that the ratios of these shares are

$$\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{N}} m_{1k}} = \left[\frac{1-\rho\lambda^{\sigma}\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}$$
$$\frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{N}} m_{2k}} = \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}$$

This shows that the ratio is determined by two forces. The first is the relative price of goods in the tradable sectors, which shows up in the form of the first factor in each RHS. When two cities start to share the varieties in the tradable sectors through inter-city trade, the relative increase in the mass of varieties is greater in city 2 due to the asymmetric city size. This causes a relatively greater reduction in the the price index in the tradable sectors in city 2. Given the assumption that the sectors are gross complements ($\eta < 1$), this implies that the relative expenditure share in the tradable sectors is lower in city 2, other things being equal. The second force is the income elasticities of the two groups. Because $U_1 > U_2$ in the equilibrium, the higher the income elasticities of tradable sectors as a whole are compared to those of the non-tradable sectors, the higher the aggregate tradable sector share is, other things being equal. This observation becomes important when we think about a composition change in each group, which is the focus of the comparative statics in the next section.

Finally, the price index of a sector differs between locations. The sector price index ratio is given by

$$\frac{P_{1k}}{P_{2k}} = \begin{cases} \frac{w_1}{w_2}\left(\frac{\lambda N_1}{N_2}\frac{x_{1k}}{x_{2k}}\right)^{-\frac{1}{\sigma-1}} & k \in \mathbb{N} \\ \frac{1}{\tau}\left(1 + \frac{\tau^{2(\sigma-1)}-1}{\left(\frac{\tau w_2}{w_1}\right)^{\sigma-1}\lambda\frac{N_1}{N_2}\frac{x_{1k}}{x_{2k}}+1}\right)^{\frac{1}{\sigma-1}} & k \in \mathbb{T} \end{cases}$$

In both groups, $\frac{P_{1k}}{P_{2k}}$ decreases in $\frac{x_{1k}}{x_{2k}}$. As the employment share ratio increases in income elasticity within a group, the sector price index decreases in income elasticity within a group. In city 1, the higher expenditure shares on income elastic goods attract firms in those sectors, and the price indices in those sectors become relatively cheap, reflecting relatively more varieties.

Connection to Stylized Fact and Alternative Explanation

In summary, the fundamentally productive location becomes the large and high-income city, and within each group (tradable and non-tradable), the residents allocate their expenditure relatively more towards income elastic sectors, which offer relatively richer varieties. On the supply side, the high-income city specializes in income elastic sectors in the sense that workers are employed relatively more in income elastic sectors within each group, which replicates the sectoral specialization pattern seen in Figure 1.1. While what we observe in the real world are the income level and the sectoral employment of a city, what generates the relationship between them in this model is the fundamental productivity. Moreover, the model predicts that the demand pattern is amplified to the specialization pattern for tradable sectors; that is, the city with the greater relative within-expenditure share becomes the net exporter of that sector. In Appendix A.1, I provide an empirical analysis like that of Figure 1 (except that it has both tradable and non-tradable sectors) that shows that the amplification mechanism is consistent with the data. While the income elasticity estimates in Figure 1 are borrowed from Caron et al. (2020), who estimate them using international trade flow data, the ones in Appendix A.1 are from Comin et al. (2021), who provide estimates for 9 sectors using sectoral employment data from 39 countries. Although the plot has fewer sectors, the sectoral specialization pattern that appears is the same as Figure (1.1). The fact that the pattern becomes clearer in the tradable sectors is consistent with the theory's amplification effect.

In Appendix A.1, I address an alternative explanation of the pattern in Figure 1.1. One may think that the reported positive relationship is driven by an omitted variable--that is, skilled-labor supply in cities. This is a reasonable concern given that skilled workers, who are at the same time high income earners, tend to reside in large cities, and those cities tend to host skill-intensive sectors (Davis and Dingel (2020)). Also, and as is well known, there is a positive correlation between the skill intensity and the income elasticity of a sector (Caron et al. (2014), Caron et al. (2020))). In Appendix A.1f, I address this concern by controlling the college graduate share in the local labor force when the elasticities of employment share are obtained. The positive relationship is robust to this control, and this shows that the alternative explanation cannot deny the mechanism that is proposed by my model.

1.4 Comparative Statics

In this section, I study comparative statics for two types of trade cost reduction: (i) uniform trade cost reduction for the tradable sectors; and (ii) a non-tradable sector that becomes tradable. This illustrates what the model can additionally speak to compared to models with only tradable sectors or ones with one tradable sector with a fixed sectoral share. In addition, this serves as a basis for the case study in the next section. To focus on the major forces I start with an international trade model, which is a model of immobile workers, and then move to the urban model, which features mobile workers

International Trade Model

In the international setting, the model has two locations inhabited by immobile workers. This is a generalized version of Matsuyama (2019) in the sense that it can additionally have non-tradable sectors. Matsuyama (2019) provides comparative statics for uniform trade cost reduction in his analysis of globalization, but globalization can also take other forms. As documented in an appendix by Eckert (2019), communication costs have been drastically declining for decades. As argued in Eckert (2019), this should have disproportionately reduced the trade costs of sectors that heavily rely on communication, such as business services. Yet Head et al. (2009) report that the international trade-diminishing effect of distance in commercial services fell substantially between 1992 ad 2006. Another example of sector-specific trade cost reductions is sectoral trade liberalization. Countries often liberalize international trade for a subset of sectors, such as manufacturing, while they protect agriculture or financial services. The international trade model presented in this section has non-tradable sectors; its sectoral trade cost reduction can be illustrated by a non-tradable sector that becomes tradable. Thus, this model is capable of providing insight into the effects of globalization that takes the form of non-uniform trade cost reductions.

I start with the definition of the competitive equilibrium in this international trade model ("international competitive equilibrium"), which is given by

Definition. International Competitive Equilibrium

Given N_1 and N_2, the international competitive equilibrium is

$$\{U_1, U_2, w_1, w_2, E_1, E_2, \{p_{cc'k}, q_{cc'k}\}_{(c,c',k)\in(1,2)\times(1,2)\times K}, \{\Omega_{1k}, \Omega_{2k}\}_{k\in K}\}$$

such that

1. households in country c for all $c \in \{1,2\}$ optimize consumption choice as follows:

$$\max_{\{q_k(\nu)\}_{\nu\in\Omega_{ck},k\in K},\{Q_k\}_{k\in K}} \quad U_c$$
$$s.t.\ 1 = \left[\sum_{k\in K} \beta_k^{\frac{1}{\eta}} U_c^{\frac{\epsilon_k}{\eta}} Q_{c,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$Q_{c,k} = \left[\int_{\nu\in\Omega_{c,k}} q_k(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}$$
$$E_c = \sum_{k\in K}\int_{\nu\in\Omega_{c,k}} p_{ck}(\nu) q_k(\nu) d\nu$$

   where $E_1 = \lambda w_1$ and $E_2 = w_2$.

2. producers optimize production as (1.6) for all k in K and $c \in \{1,2\}$,

3. the zero-profit condition holds such that $\pi_{ck} \leq 0$ for all k in K and $c \in \{1,2\}$ where equality holds if $\Omega_{ck} \neq \emptyset$,

4. the local labor market clearing conditions,

$$\sum_{k\in\mathbb{N}}\int_{\Omega_{1k}} (\psi_k q_{11k} + \phi_k)\, d\nu + \sum_{k\in\mathbb{T}}\int_{\Omega_{1k}} (\psi_k q_{11k} + \tau\psi_k q_{12k} + \phi_k)\, d\nu = \lambda N_1$$

   and

$$\sum_{k\in\mathbb{N}}\int_{\Omega_{2k}} (\psi_k q_{22k} + \phi_k)\, d\nu + \sum_{k\in\mathbb{T}}\int_{\Omega_{2k}} (\psi_k q_{22k} + \tau\psi_k q_{21k} + \phi_k)\, d\nu = N_2$$

   hold.

The equilibrium is characterized by a subset of the equilibrium conditions of the urban model: (1.8), (1.11), and (1.12). I focus on (1.8), which is the equation of the home market effect on the wage rate, and it is laid out again below.

$$\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{T}} m_{2k}}\frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{\lambda^{\sigma}-\rho\omega^{\sigma}}\right]$$

Without non-tradable sectors ($\frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}} = 1$), this condition implies two things. First, λN_1 and N_2 dictate the income ratio. Specifically, the country with the greater efficiency unit of labor necessarily has the higher wage rate. Here, I assume country 1 has the greater efficiency unit ($\lambda N_1 > N_2$) without loss of generality. Second, with globalization, the relative income of country 2 necessarily rises and income inequality shrinks. When all sectors are tradable, the only way to express globalization is through a uniform trade cost reduction ($\rho\uparrow$). Because ω is the only endogenous variable and it decreases in ρ, the relative income gap narrows. Yet when non-tradable sectors are present, globalization can cause different outcomes. This is so because there is another margin where globalization can take place and because of the existence of the additional endogenous variable, the relative expenditure share of the tradable sectors, $\frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}}$. Suppose sector s transitions from non-tradable to tradable ($\tau_s = \infty \to \tau$ or $\mathbb{T}^{new} = \mathbb{T}^{old}\cup\{s\}$ and $\mathbb{N}^{new} = \mathbb{N}^{old}\setminus\{s\}$). The following theoretical result is obtained for a special case.

Proposition 9. *Suppose $\beta_k \to 0$ for all $k \in \mathbb{T}^{new}$ and the income elasticity of sector s (ϵ_s) is sufficiently large compared to that of the existing tradable sectors ($\mathbb{T}^{old}$) such that the expenditure share ratio of sector s is higher than the ratio of the aggregate tradable sector share ($\frac{m_{1,s}}{m_{2,s}} > \frac{\sum_{k\in\mathbb{T}^{old}} m_{1k}}{\sum_{k\in\mathbb{T}^{old}} m_{2k}}$). Then, the income inequality ($\omega = \frac{E_1}{E_2} = \frac{\lambda w_1}{w_2}$) is higher when sector s is tradable ($K^{new} = \mathbb{N}^{new}\cup\mathbb{T}^{new}$) than when sector s is non-tradable ($K^{old} = \mathbb{N}^{old}\cup\mathbb{T}^{old}$).*

The home market effect on the wage rate (1.8) implies that the greater relative expenditure share of the tradable sectors raises relative income. When a non-tradable sector with a relatively large local demand becomes tradable, the relative tradable expenditure share can increase and so, consequently, can the relative income. Since the high-income country has a relatively greater demand for income-elastic goods, when one of those in non-tradable becomes tradable, income inequality can widen.[7] Proposition (9) has the condition on β_k because the tradable sectors needs to be negligible to the overall economy to obtain the analytical result. When sector s is not negligible in terms of expenditure share within all sectors, the utility in both countries rises, reflecting the cheaper sector price index caused by newly-available foreign varieties. This utility increase additionally changes the relative tradable sector shares due to the non-homothetic preference.

Although the theoretical result is obtained only for the special case, simulations in Appendix A.5 show that this widening income inequality can occur more generally. Moreover, the simulations suggest that the relative income elasticity of the transitioning sector is generally important. When the new tradable sector is sufficiently income elastic relative to the existing sector, the income inequality tends to rise.

Thus, the model illustrates that globalizations' impact on cross-country income inequality can differ depending on the form of the globalization. When globalization takes a form of sectoral trade liberalization, income elasticity is a key factor that determines the direction of the income inequality.

Urban Model

The main forces that we saw in the international trade model are still present, but now there is an additional margin of response: migration. The result in Proposition (9) can also be proven in this case. When tradable sectors are negligible relative to the whole economy, utilities remain unchanged and migration does not take place. Nevertheless, the relative

[7] This argument about the relative tradable expenditure share concerns the relativity between cities and is still valid when the tradable sectors are negligible within a city.

nominal income can change because it is driven by the aggregate tradable sector share.[8] In simulations for more general cases, the share of widening income inequality cases is smaller than in the international trade model because agents move from the high-income large city to the low-income small city in response to the shrinking gap of the utility from goods consumption. Losing population leads to a lower relative wage in the large city, other things being equal, and this attenuates the original effect. Nevertheless, when the new tradable sector is sufficiently income elastic, income inequality can rise in the same way as in the international trade case.

1.5 City-size wage premium and Tradable Business Services

Having described in the previous section the model's predictions about the effect of trade cost reduction, I now demonstrate with core-based US statistical area (CBSA) data that the model can be used to understand the evolution of the income level across cities after 1980 by focusing on business services. This is motivated by reports that business services, which is a very income-elastic sector, has undergone a disproportionate decline in its trade cost. As mentioned above, Head et al. (2009) report the declining distance effect on international trade for a subset of services that includes financial, computer, and communication services. With regards to inter-city trade, Eckert (2019) reports that the distance elasticity of trade cost for business services declined from -2.1 in 1980 to -1.6 in 2000, while that of good sectors stayed stable at -1.6. While my model does not include sector-specific trade costs, it

[8] This is easy to understand if one begins with an autarky case. In the autarky case, the nominal income is undetermined and one can set arbitrary nominal incomes. When tradable sectors exist but are negligible, the nominal income has almost no effect on the economy as a whole. However, the nominal income is determined because there is trade.

features a dichotomy of tradable and non-tradable sectors, and this disproportionate trade cost decline can be interpreted as business services having transitioned from the non-tradable to to the tradable sector. Judging from the comparative statics examined in the previous section, this transition is likely to have pushed up the aggregate tradable sector share in large cities and, consequently, raised the city-size wage premium. A caveat is that my model and, therefore, the following analyses ignore an aspect of business services: they are consumed not by households but by other businesses.[9] If an input-output linkage was introduced, the model would loose the tractability. To provide theoretical results, it is assumed that all varieties are directly consumed by households. Next, I test the model's prediction that the local wage level is positively associated not only with local population size but also with the aggregate tradable sector share, which is the key channel in the comparative statics. Then, assuming that business services were non-tradable in 1980 and became tradable by 2007, I interpret the evolution of the city-size wage premium.

Main regression

The main regression model of this section is based on the equilibrium condition that characterizes the city-level home market effect on the wage rate, (1.8). Since there are more than two cities in the data, I implement reduced form regressions motivated by equation (1.8). The reduced form model for city c motivated by equation (1.8) is given by

$$\log(wage_c) = \alpha + \gamma f(A_c) + \beta_1 \log(pop_c) + \beta_2 \log\left(\sum_{k \in \mathbb{T}} x_{ck}\right) + \varepsilon_c$$

[9]In the empirical part, I assign finance and insurance to business services, although some of their services are consumed directly by households.

where α is the intercept, A_c is the city-specific fundamental productivity, $f(\cdot)$ is an increasing function, and Corollary 1 replaces $\sum_{k\in\mathbb{T}} m_{ck}$ with $\sum_{k\in\mathbb{T}} x_{ck}$. Here, the city-specific fundamental productivity (A_c) is not observable, nor is the proxy easy to obtain. Therefore, I use the following model:

$$\log(wage_c) = \alpha + \beta_1 \log(pop_c) + \beta_2 \log\left(\sum_{k\in\mathbb{T}} x_{ck}\right) + \varepsilon_c \tag{1.15}$$

where the error term includes the productivity term, $\varepsilon_c = \gamma f(A_c) + \varepsilon_c^1$. The OLS estimation of equation (1.15) faces two potential endogeneity problems. First, the population, the aggregate tradable sector shares, and the wage are simultaneously determined because these are all endogenous variables. To address this issue, I use an instrumental variable (IV) that affects $\log(pop_c)$ and $\log\left(\sum_{k\in\mathbb{T}} x_{ck}\right)$ but does not appear in equation (1.15). The theoretical model indicates that a variable that is correlated not with city-specific productivity (A_c) but with city-specific amenity (a_c) satisfies this condition. The amenity affects the equilibrium levels of the variables because a_1 and a_2 appear in the system of equations that characterizes equilibrium, although the amenity is not included in equation (1.15). This becomes clearer when we examine Figure (1.2c) in the cross-city analysis. Although this shows the results of a partial equilibrium analysis, when amenity rises, the equilibrium moves along the curve of (1.8), which shows that amenity helps us identify equation (1.15). Amenity can also work as an IV for the aggregate tradable sector share ($\sum_{k\in\mathbb{T}} x_{ck}$). This is because the aggregate tradable sector share is endogenous, and heterogeneous amenity generates heterogeneous aggregate tradable sector shares in the equilibrium.[10] Also, we can expect variation in the aggregate tradable sector share even when it is conditional on the population size when cities

[10] In the special case where the tradable sectors and the non-tradable sectors are symmetric, the larger city has a higher aggregate tradable sector share ($\sum_{k\in\mathbb{T}} x_{ck}$) irrespective of whether the cities differ in fundamental productivity or amenity.

are heterogeneous in two dimensions -- productivities and amenities. This is because, even when the city size is the same, the equilibrium utility from goods consumption, which affects the expenditure pattern, is different depending on what drives the city size. Intuitively, when a large city's amenity is mediocre, the utility from goods consumption must be high enough to support the city size. On the other hand, when a large city offers an excellent amenity, the utility from goods consumption need not be very high.

Now, I introduce the first IV and assume that it satisfies the conditions of an IV. The IV, widely used in the agglomeration economy literature since Ciccone and Hall (1996), is a city's historical population. Specifically, I use the city's historical population in 1850 and in some cases in 1880. The historical population is expected to be correlated with the current amenity of cities because amenities such as weather, landscape, and historic heritage persist over time. Additionally, and perhaps more importantly, a city that had a large population in 1850 should have had a large number of births locally, which would have created inertia in the next generation's cross-city migration due to moving costs (e.g., social network, housing). This inertia, which can persist over generations, can be interpreted as a city' persistent amenity. In fact, the literature reports that a city's historical population is a strong IV for its modern population pattern. On other hand, it is not clear whether this variable is correlated with a city's productivity. The population of a city in 1850 should be correlated with its fundamental productivity at that time. However, because the characteristics of economic activity in 1850 and today differ greatly, they are unlikely to be strongly correlated. That said, some characteristics of a city are persistent and perhaps are correlated with productivity; these include geographical features or the accumulation of non-traded physical capital or intangible assets. To address this, I control whether a city is located within 50km of the sea or of the Great Lakes and which region a city belongs to. In a robustness check, I further control the share of flat land, of woody wetland, and of emergent/herbaceous wetlands. For the capital and asset accumulation, I do not have a good control, and I assume the effect decays sufficiently over 100 years.

In addition to the historical population, I use two climate variables -- yearly mean of daily temperature above and below 65 degrees Fahrenheit -- provided by Burchfield et al. (2006) and assume they are correlated with a city's amenity. These variables were originally created to measure the demand for air conditioning and heating; Burchfield et al. (2006) employed them as instrumental variables for urban sprawl. I assume these climate variables are not correlated with productivities that are conditional on the region and whether the city is coastal. However, it is impossible to deny that this assumption is violated. For example, the climate can affect the cost of a businesses' air conditioning. Also, it is reasonable to expect that some sectors favor specific types of climates and it might work as an advantage or a disadvantage for the entire city. This is a particular concern in the agriculture, foresty, and fishing sectors, although these sectors' cumulative shares generally are very small (e.g., 1.4% of the total employment share in 1980). To address this issue, I provide a robustness check in which I use the historical population in 1880 as the second IV instead of the climate IVs.

The second endogeneity issue is the sorting of workers across cities. It is well documented in the literature that skilled workers are more prevalent in large cities (e.g., Davis and Dingel (2020) and Combes et al. (2008)). To address this issue I control workers' characteristics (education, age, sex, and race) when I obtain the city-specific wage rate level (for details see Appendix A.6).

Using CBSA-level data with a consistent industrial classification that is based on the database of Eckert et al. (2021) (see Appendix A.6 for the details), I implement the cross-sectional IV regressions for four different years between 1980 and 2007. Because this is thought to be the time period when business services transitioned to being tradable, I use two different definitions of the tradable sectors. $\mathbb{T}_1$ is the set of agriculture, foresty, fishing, mining, and manufacturing, while $\mathbb{T}_2$ is the set that contains the same sectors and, additionally, business services.[11][12]

The OLS regression results are reported in Table 1.1. The coefficients of the log of population and the aggregate tradable sector share are both significantly positive for all specifications and in all years except that of the aggregate tradable sector share with $\mathbb{T}_1$ in 2007. The coefficient of the aggregate tradable sector share is greater with $\mathbb{T}_2$ than with $\mathbb{T}_1$, which possibly reflects the fact that a large city tends to be populated by skilled workers, whom business services intensively employ, and they are, at the same time, high-income earners. Because of the simultaneity problem, I proceed to the IV regression results summarized in Table 1.2.

The IV regression results are consistent with the model's prediction in two respects. First, $\log(pop_c)$ has a significantly positive coefficient in all years except (3) in 2000. Second, and more importantly, the coefficient of the aggregate tradable sector share is significantly positive at the 5 percent level in all years using either definition. The sizes of these coefficients are greater in the IV regressions than in the OLS regressions in all years using either definition. When we take the position that the true tradable sectors did not include business services in 1980 but did in 2007, the significant positive coefficient with $\mathbb{T}_2$ in 1980 and with $\mathbb{T}_1$ in 2007 may look odd at first glance. However, as shown in Figure 1.3 and Figure 1.4, these two variables are actually strongly correlated. In 1980, when the business services share was relatively small, the correlation was very strong, and it remained strong in 2007, although not as strong as in 1980. Thus, it is natural that they share the sign of the coefficient and

[11] 11, 21, and 31-33 in 2012 NAICS.

[12] As in Eckert (2019), in 2012 NAICS, business services consist of Information (51); Finance, insurance, real estate, rental, and leasing (FIRE); and Professional and business services (PROF); excluding Real Estate and Waste management and remediation services .

Table 1.1: OLS regressions of local wage

	log wage					
	1980	1980	1980	1990	1990	1990
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.044***	0.046***	0.041***	0.049***	0.055***	0.044***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.045***			0.059***	
		(0.014)			(0.013)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.108***			0.182***
			(0.031)			(0.028)
Observations	243	243	243	253	253	253

	log wage					
	2000	2000	2000	2007	2007	2007
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.054***	0.060***	0.045***	0.048***	0.050***	0.043***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.005)	(0.005)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.046***			0.010	
		(0.012)			(0.012)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.193***			0.099***
			(0.028)			(0.033)
Observations	262	262	262	262	262	262

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels, respectively.
Region $\in \{Northeast, Midwest, South, West\}$, and whether a city is on an ocean or lake coastline are controlled.

Table 1.2: IV regressions of local wage

	log wage					
	1980	1980	1980	1990	1990	1990
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.034**	0.040***	0.035**	0.049***	0.081***	0.051***
	(0.014)	(0.015)	(0.015)	(0.013)	(0.018)	(0.017)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.126***			0.215***	
		(0.044)			(0.050)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.250***			0.601***
			(0.088)			(0.144)
Cond. F stat. for $\log(pop_c)$	12.9	18.4	16.3	13.1	22.4	18.7
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		18.0	22.6		15.9	16.0
Observations	199	199	199	205	205	205

	log wage					
	2000	2000	2000	2007	2007	2007
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.053***	0.098***	0.034	0.050***	0.084***	0.049***
	(0.012)	(0.020)	(0.022)	(0.014)	(0.020)	(0.015)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.208***			0.151***	
		(0.053)			(0.053)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.808***			0.330**
			(0.277)			(0.143)
Cond. F stat. for $\log(pop_c)$	12.1	18.8	12.6	11.9	16.2	20.9
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		12.8	9.7		10.5	9.8
Observations	210	210	210	210	210	210

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels, respectively.
Region $\in \{Northeast, Midwest, South, West\}$ and whether a city is on an ocean or lake coastline are controlled. $\log(pop_c)$ and $\log(\sum_{k\in\mathbb{T}} m_{ck})$ are instumented by $log(pop_{c,1850}+1)$, mean heating degree-days, and mean cooling degree-days. Samples are weighted with aggregate weights of Census samples in each CBSA, and these are used to derive the city-specific wage. The conditional F statistics are those described by Sanderson and Windmeijer (2014)

this does not imply that the data are inconsistent with the model result. The significantly positive coefficient confirms that the aggregate tradable sector share is important in the cross-city wage determination. What elevates the local wage is not simply the size of the local population; also important is the aggregate share of the tradable sectors. Because the aggregate share of tradable sectors is characterized by substantial variation across cities, this result suggests the importance of understanding a city's industrial composition.

Figure 1.3: Correlation of Two Aggregate Tradable Sector Shares in 1980

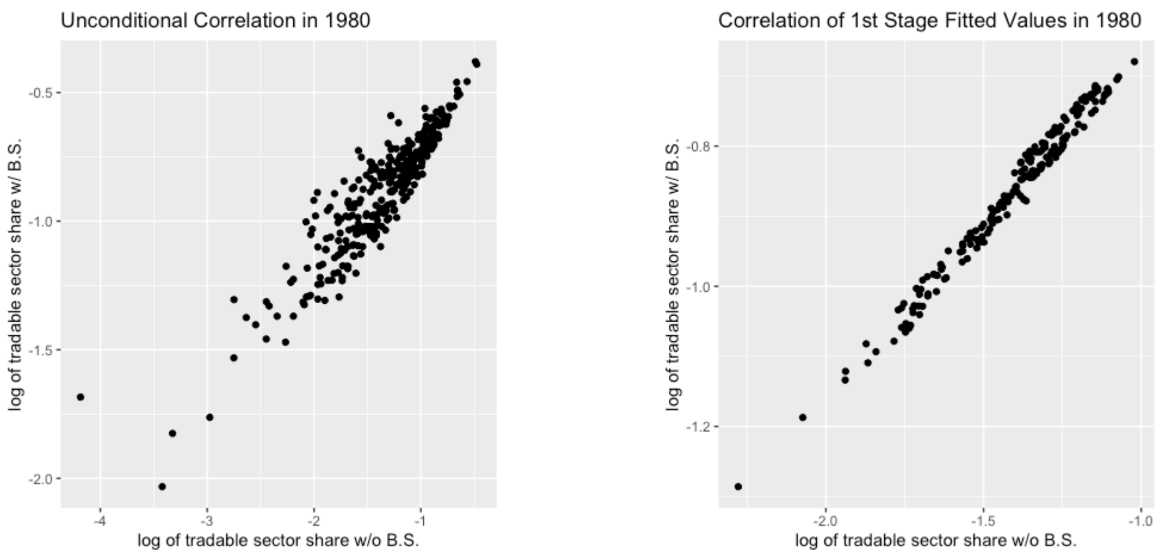
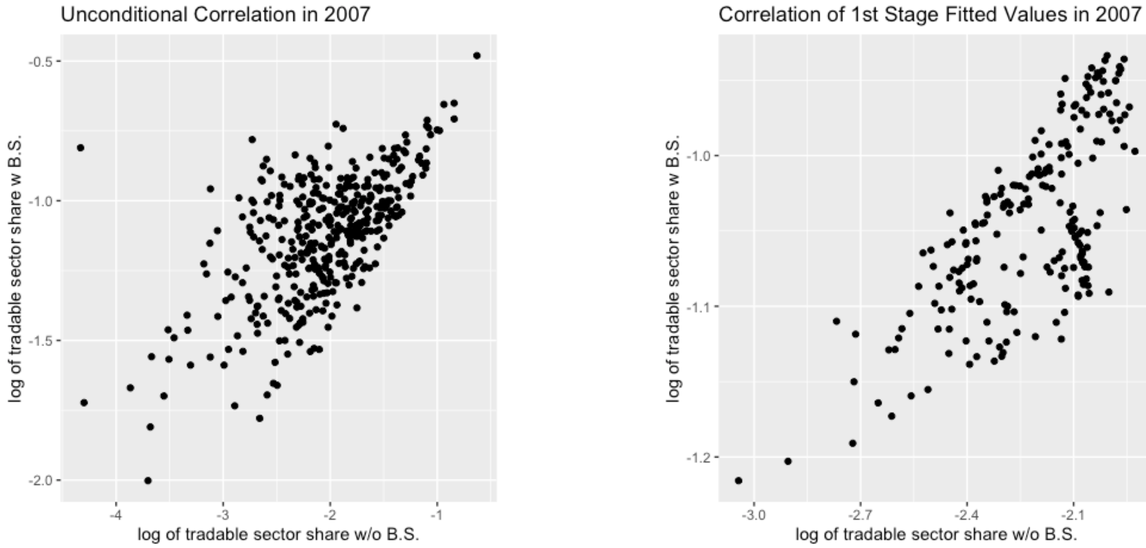


Figure 1.4: Correlation of Two Aggregate Tradable Sector Shares in 2007



Having verified that the aggregate tradable sector margin affects the wage levels, I now provide an interpretation of the wage pattern across cities through the lens of the model. In this part, I take as a given that business services were non-tradable until 1990 and have been tradable since 2000.

There are two key observations. First, the aggregate tradable sector decreases in a city's population size in 1980. The aggregate tradable sector share in 1980 is that of $\mathbb{T}_1$, and the pattern is displayed in the left panel of Figure 1.5. The regression line is slightly downward sloping. Given the cross-city analysis, this suggests that the tradable sectors are sufficiently income inelastic, which is consistent with the estimates prepared by Caron et al. (2020).[13] Second, the aggregate tradable sector increases in a city's population size in 2007. The aggregate tradable sector share in 2007 is that of $\mathbb{T}_2$, and the pattern is displayed in the right panel of Figure 1.5. The regression line is upward sloping, which suggests that the inclusion of the very income elastic sector has raised the aggregate tradable sector share in

[13]The estimates of ϵ_k are Agriculture (0.32), Mining (0.41), and Manufacturing (1.00). The other six sectors are above 1.

large cities, as predicted in the comparative statics.

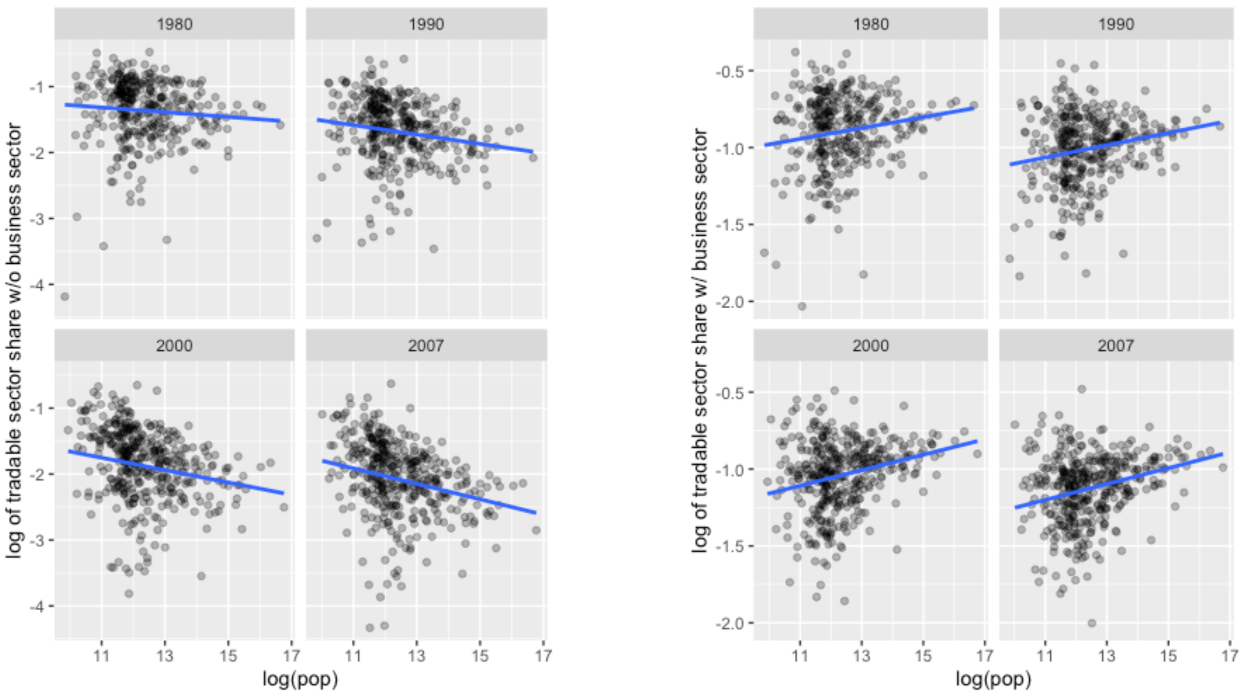
These observations point to a conclusion: the transition of business services has supported the stable city-size wage premium. The city-size wage premium--the unconditional elasticity of wage with respect to the local population--stayed stable or rose slightly during the sample period (mean is 0.042 during the first two periods and is 0.052 during the last two periods in Table 1.2). This could imply a stable trade cost in an all-tradable-sector model. However, controlling the relevant aggregate tradable sector share ($\mathbb{T}_1$ in 1980 and 1990 and $\mathbb{T}_2$ in 2000 and 2007) reveals that the population elasticity of wages has been declining (mean is 0.061 in the first two periods and 0.039 in the last two periods), which could reflect a uniform trade cost reduction. What had been supporting the stable urban wage is the aggregate tradable sector share, which is positively correlated with city sizes after the transition of business services. This is exactly the outcome that the comparative statics predict when the transitioning sector is very income elastic.

Robustness Checks

In this subsection, I provide robustness checks to address two possible concerns. First is the endogeneity of the IVs, as discussed in the previous sub-section. To alleviate concern about the historical population, I add controls that are possibly correlated with the productivity levels of cities in 1850 and today. Specifically, I control three variables separately: (a) share of flat land, (b) share of woody wetlands, and (c) share of emergent/herbaceous wetlands.[14] The results, shown in Table 1.3, closely resemble those in Table 1.2. Although in some years the coefficient of $\log(pop_c)$ lost significance, the aggregate tradable sector share has significantly positive coefficients in all years. A caveat is that the conditional F-statistics for

[14]This is in line with Combes et al. (2010), who use historical population density to instrument local population density in wage regressions while controlling coast, mountain, lakes and waterways.

Figure 1.5: Aggregate Tradable Sector Shares across Cities



log of the aggregate tradable sector share are not high enough in the 2000s. According to the critical values in Stock et al. (2005), 7.4 for $\mathbb{T}_2$ in 2000 locates between 20% and 25% in TSLS Wald test size distortions.[15]

Next, to deal with the possible endogeneity of the climate IVs I replace the climate IVs with the historical population in 1880. The results, laid out in Table 1.4, show that the aggregate tradable sector share has significantly positive coefficients in the majority of the regressions. The fact that the aggregate tradable sector share with $\mathbb{T}_2$ loses its significance in 2000 and 2007 is mainly due to the large standard error caused by its stronger correlation with population size. The sign remains positive.

[15] Since this is conditional F-statistics, the relevant critical value is that of $n = 1$ and $K_2 = 2$ (Sanderson and Windmeijer (2016)).

Table 1.3: IV regressions of local wage with additional controls

	log wage					
	1980	1980	1980	1990	1990	1990
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.034**	0.032**	0.029*	0.045***	0.075***	0.033
	(0.014)	(0.016)	(0.015)	(0.013)	(0.023)	(0.022)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.164**			0.331***	
		(0.064)			(0.084)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.290***			0.875***
			(0.107)			(0.227)
Cond. F stat. for $\log(pop_c)$	6.5	12.9	13.6	6.5	25.4	13.8
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		10.5	16.3		10.6	13.9
Observations	195	195	195	201	201	201

	log wage					
	2000	2000	2000	2007	2007	2007
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.048***	0.104***	0.009	0.038***	0.089***	0.025
	(0.013)	(0.031)	(0.028)	(0.014)	(0.029)	(0.018)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.379***			0.313***	
		(0.115)			(0.100)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.927**			0.573***
			(0.372)			(0.202)
Cond. F stat. for $\log(pop_c)$	5.9	21.5	7.3	5.6	17.5	14.0
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		9.5	7.4		8.6	8.7
Observations	205	205	205	205	205	205

Notes: ***,**, and * denote significant at 1 %, 5%, and 10% levels, respectively.
Region $\in \{Northeast, Midwest, South, West\}$, whether a city is on an ocean or lake coastline, flat land share, woody wetland share, and emergent/herbaceous wetland share are controlled. $\log(pop_c)$ and $\log(\sum_{k\in\mathbb{T}} m_{ck})$ are instrumented by $log(pop_{c,1850}+1)$, mean heating degree-days, and mean cooling degree-days. Samples are weighted with aggregate weights of Census samples in each CBSA, and these are used to derive the city-specific wage. The conditional F statistics are those described in Sanderson and Windmeijer (2014)

Table 1.4: IV regressions of local wage only with historical population

	log wage					
	1980	1980	1980	1990	1990	1990
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.056***	0.046***	0.034**	0.075***	0.067***	−0.010
	(0.011)	(0.012)	(0.016)	(0.011)	(0.014)	(0.051)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.115**			0.232***	
		(0.055)			(0.073)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			0.298**			1.094*
			(0.147)			(0.583)
Cond. F stat. for $\log(pop_c)$	31.3	18.3	10.4	30.4	36.3	14.6
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		12.3	14.5		10.3	16.8
Observations	242	242	242	252	252	252

	log wage					
	2000	2000	2000	2007	2007	2007
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(pop_c)$	0.089***	0.091***	−0.025	0.073***	0.073***	−0.074
	(0.012)	(0.012)	(0.090)	(0.012)	(0.013)	(0.158)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$		0.157***			0.136**	
		(0.055)			(0.063)	
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$			1.318			1.393
			(0.988)			(1.466)
Cond. F stat. for $\log(pop_c)$	28.4	77.6	16.1	26.6	56.5	11.9
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$		7.6	18.8		6.4	18.5
Observations	261	261	261	261	261	261

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels respectively.
Region $\in \{Northeast, Midwest, South, West\}$ and whether a city is on an ocean or lake coastline are controlled. $\log(pop_c)$ and $\log(\sum_{k\in\mathbb{T}} m_{ck})$ are instrumented by $log(pop_{c,1850}+1)$ and $log(pop_{c,1880}+1)$. Samples are weighted with aggregate weights of Census samples in each CBSA, which are used to derive the city-specific wage. The conditional F statistics are those described in Sanderson and Windmeijer (2014).

The second issue I address is an alternative explanation: the comparative advantage of cities in terms of skill-intensity. Suppose there is a city where the residents are mostly skilled workers, whether this is exogenously given or an endogenous result. The city specializes in skill-intensive sectors due to the comparative advantage from the factor supply, and, business services is the major industry. At the same time, there is an agglomeration economy that is strengthened by the high share of skilled workers, and it cannot be predicted by the population size alone.[16] In this case, even if the home market effect through the tradable sector share margin is not present, this city has high aggregate tradable sector shares due to a high business services employment share, and at the same time, it offers a high wage rate even when workers' characteristics are controlled. This story is particularly a concern with $\mathbb{T}_2$ but not with $\mathbb{T}_1$ because the tradable sectors in $\mathbb{T}_1$ are not skill-intensive.[17]I provide a robustness check for this by additionally controlling the local skill supply. Specifically, I include in (1.15) the log of the college graduate share in local employment [18] The results are shown in Table 1.5. The aggregate sector share still has a significantly positive coefficient in all years except 2007, when its significance is 10%. The college graduate share has a positive coefficient that is significant at 10% for three years when business services are not included in the tradable sectors. However, when business services are included, a significant coefficient is not obtained in any year. In my opinion, this shows that the demand channel through the aggregate tradable sector share is robust to the alternative.

Finally, I provide another set of reduced form regressions that additionally take into account access to other cities' markets. I start by generalizing the theoretical model so that

[16]The assumption that local productivity increases not only in the quantity but also in the quality of the local workers is widely used. For example, Davis and Dingel (2019) assume the efficiency of learning in a city increases both in the probability of matching and in the ability of other agents.

[17]According to Caron et al. (2020), the skill intensity is 0.135 for "fishing", 0.141 for "foresty", below 0.2 for all sectors within agriculture, 0.35 for "petroleum, coal products", below 0.38 for all sectors within manufacturing, 0.53 for "financial services" and 0.49 for "business services nec."

[18]The results are very similar when the level of college graduate share is used instead.

Table 1.5: IV regressions of local wage additionaly with log(college graduate share)

	log wage							
	1980	1990	2000	2007	1980	1990	2000	2007
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(college graduate share)	0.07*	0.09*	0.08*	0.05	0.04	−0.01	−0.05	−0.06
	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.09)	(0.08)
$\log(pop_c)$	0.04***	0.08***	0.09***	0.08***	0.03**	0.05***	0.04	0.05***
	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)
$\log(\sum_{k\in\mathbb{T}_1} m_{ck})$	0.13***	0.20***	0.19***	0.13**				
	(0.04)	(0.04)	(0.05)	(0.05)				
$\log(\sum_{k\in\mathbb{T}_2} m_{ck})$					0.26***	0.61***	0.89**	0.43*
					(0.09)	(0.15)	(0.36)	(0.23)
Cond. F stat. for $\log(pop_c)$	23.3	23.1	19.0	16.5	21.7	23.9	24.5	20.7
Cond. F stat. for $\log(\sum_{k\in\mathbb{T}} m_{ck})$	26.7	22.8	18.0	14.0	28.3	17.5	10.4	7.6
Observations	199	205	210	210	199	205	210	210

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels respectively.
Region $\in \{Northeast, Midwest, South, West\}$, whether a city is on an ocean or lake coastline are controlled. $\log(pop_c)$ and $\log(\sum_{k\in\mathbb{T}} m_{ck})$ are instrumented by $log(pop_{c,1850}+1)$, mean heating degree-days, and mean cooling degree-days. Samples are weighted with aggregate weights of Census samples in each CBSA, and these are used to derive the city-specific wage. The conditional F statistics are those described by Sanderson and Windmeijer (2014).

it has more than two cities. The following zero-profit condition for a firm in sector k in city c can be straightforwardly obtained

$$E_c^\sigma = A_c^\sigma \sum_{c' \in \mathbb{C}} \left(\tau_{cc'}^{1-\sigma} P_{c'k}^{\sigma-1} m_{c'k} Y_{c'} \right) \tag{1.16}$$

where $\mathbb{C}$ is the set of cities, E_c is the income per capita (which is equal to $A_c w_c$), A_c is the fundamental productivity, and w_c is the wage rate per efficiency unit in city c, and $Y_c (= N_c E_c)$ is the aggregate income in city c. This form is not specific to the non-homothetic model and the non-homotheticity matters through the determination of $m_{c'k}$. In contrast to the two-city case, it is not feasible to derive a simple system of equations that characterize the equilibrium from this. Nevertheless, I can obtain some relationship between endogenous variables in the equilibrium that are useful in motivating the regression models. First, as in the two-city case, the aggregate tradable sector share is a key variable. Specifically, the aggregate tradable sector shares ($\{\sum_{k \in \mathbb{T}} x_{ck}\}_{c \in \mathbb{C}}$), the aggregate income ($\{Y_c\}_{c \in \mathbb{C}}$), and the inter-city trade costs ($\{\tau_{ci}\}_{c \in \mathbb{C}}$), which are common in tradable sectors, are sufficient statistics for the inter-city wage pattern in an equilibrium. Second, other thing being equal, the wage is higher in the city that has the higher local demand in the tradable sectors, which is $Y_c \sum_{k \in \mathbb{T}} x_{ck}$. Third, other thing being equal, the city that has the lower trade cost to another city has the higher wage. Having these results, I use the following regression model.

$$\log(wage_c) = \alpha_c + \beta \log \left(\sum_{c' \in \mathbb{C}} \tau_{cc'}^{1-\sigma} Y_{c'} \sum_{k \in \mathbb{T}} x_{c'k} \right) + \epsilon_c \tag{1.17}$$

When $\alpha_c = \epsilon_c = 0$, this model satisfies the three relationships. A different way of reaching this model proceeds more directly from (1.16). Aggregating (1.16) over all tradable sectors generates

$$E_c^\sigma = TA_c^\sigma \sum_{k\in\mathbb{T}} \sum_{c'\in\mathbb{C}} \left(\tau_{cc'}^{1-\sigma} P_{c'k}^{\sigma-1} m_{c'k} Y_{c'}\right) \tag{1.18}$$

where T is the number of sectors in $\mathbb{T}$. The summation term, $\sum_{k\in\mathbb{T}} \sum_{c'\in\mathbb{C}} \left(\tau_k^{1-\sigma} P_{c'k}^{\sigma-1} m_{c'k} Y_{c'}\right)$, is what Head and Mayer (2004) call Real Market Potential (RMP). When we assume one tradable sector and ignore the effect from the price index, the summation term becomes $\sum_{c'\in\mathbb{C}} \left(\tau_k^{1-\sigma} Y_{c'}\right)$, which Head and Mayer (2004) call Nominal Market Potential (NMP), and it becomes proportional to the market potential by Harris (1954) when setting $\tau^{1-\sigma} = \frac{1}{distance}$. By dropping the price index, (1.18) becomes (1.17). When the sectors are gross complements, as the main model assumes, ignoring the price effect should be less harmful.[19] Given the utility from goods consumption and the total expenditure, the price index is an increasing function of the expenditure share when sectors are gross complements, which implies that $P_{c'k}^{\sigma-1} m_{c'k}$ and $m_{c'k}$ do not move in the opposite direction.

I set $\tau_{ij}^{1-\sigma} = \frac{1}{distance_{ij}}$, which should be a reasonable approximation given the most of the estimates of elasticity of $\tau_{ij}^{1-\sigma}$ with respect to distance are not far from -1.[20] Finally, to determine whether the tradable sector margin has a non-negligible impact on the wage, I decompose the NMP into two parts. The regression model is given by

$$\log(wage_c) = \alpha_c + \beta_1 \log\left(\sum_{c'\in\mathbb{C}} \frac{Y_{c'}}{distance_{cc'}}\right) + \beta_2 \log\left(\frac{\sum_{c'\in\mathbb{C}} \frac{\sum_{k\in\mathbb{T}} x_{c'k} Y_{c'}}{distance_{cc'}}}{\sum_{c'\in\mathbb{C}} \frac{Y_{c'}}{distance_{cc'}}}\right) + \epsilon_c$$

The coefficient of interest is β_2. A significantly positive result would confirm that the tradable sector margin also affects the nominal wage in this setting. In order to implement this regression, I need IVs again. To account for the city's own portion in $\log\left(\sum_{c'\in\mathbb{C}} \frac{Y_{c'}}{distance_{cc'}}\right)$

[19]Comin et al. (2019) report that in both their regressions with three sectors and their regressions with ten sectors, their estimates suggest that the sectors are gross complements.

[20]In his gravity regressions between US cities Dingel (2017) estimates that the distance elasticity of trade is near negative one.

or NMP, I use the climate IVs. For the rest of the NMP variation, I assume the other cities' market sizes are approximately exogenous to city c and set $\log \sum_{c' \in \mathbb{C} \setminus c} \frac{Y_{c'}}{distance_{cc'}}$ as the second IV. For the ratio term, I construct $\log \frac{\sum_{c' \in \mathbb{C} \setminus c} \frac{\sum_{k \in \mathbb{T}} x_{c'k} Y_{c'}}{distance_{cc'}}}{\sum_{c' \in \mathbb{C} \setminus c} \frac{Y_{c'}}{distance_{cc'}}}$, which is the same as the original variable, except that it does not include the city's own value, and I use it as the third IV. This assumes that not both the market sizes of the other cities and the aggregate tradable sector shares are approximately exogenous to city c. The regression results are summarized in Table 1.6.

In both definitions of tradable sectors, the adjustment term has a significantly positive coefficient in all sample years. The significance of the aggregate tradable sector share is robust to the consideration of access to other cities, and this suggests that the aggregate tradable sector share both in one's own city and in neighboring cities affects the local wage rate.

Table 1.6: IV regressions of local wage with NMP

	log wage					
	1980	1980	1980	1990	1990	1990
	(1)	(2)	(3)	(4)	(5)	(6)
log(NMP)	0.04	0.05*	0.03	0.12***	0.11***	0.11***
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.03)
log(Adjusted NMP1/NMP)		0.15**			0.25***	
		(0.06)			(0.05)	
log(Adjusted NMP2/NMP)			0.31**			0.93***
			(0.13)			(0.19)
Cond. F stat. for log(NMP)	15.5	19.2	22.1	18.9	20.6	21.3
Cond. F stat. for log(Adjusted NMPn/NMP)		24.4	31.0		24.8	21.9
Observations	199	199	199	205	205	205

	log wage					
	2000	2000	2000	2007	2007	2007
	(1)	(2)	(3)	(4)	(5)	(6)
log(NMP)	0.08***	0.08***	0.08*	0.08***	0.09***	0.15***
	(0.03)	(0.03)	(0.04)	(0.02)	(0.03)	(0.05)
log(Adjusted NMP1/NMP)		0.21***			0.13**	
		(0.05)			(0.06)	
log(Adjusted NMP2/NMP)			1.62***			1.10**
			(0.61)			(0.47)
Cond. F stat. for log(NMP)	16.4	17.9	16.7	21.5	27.2	9.5
Cond. F stat. for log(Adjusted NMPn/NMP)		23.0	12.5		26.5	10.2
Observations	210	210	210	210	210	210

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels respectively.
Region $\in \{Northeast, Midwest, South, West\}$ and whether the city is on the coastline of an ocean or lake are controlled. log(NMP) and log(Adjusted NMPn/NMP) are instrumented by their own value after deduction of the local city's values (as detailed in the main text), mean heating degree-days, and mean cooling degree-days. Samples are weighted with aggregate weights of Census samples in each CBSA that are used to derive the city-specific wage. The conditional F statistics are those described in Sanderson and Windmeijer (2014)

1.6 Conclusion

In this paper, I first report a new stylized fact: high-income cities specialize in income elastic sectors. To explain this, I present a two-city model and demonstrate how a fundamental difference in productivity and in amenity generates the patterns of a city's size, income level, and industrial composition, which is consistent with the stylized fact. Next, after studying the comparative statics for two types of trade cost reduction, I test the model's implication that the aggregate tradable sector share is positively correlated with the local wage. In almost all regressions, including robustness checks, the aggregate tradable sector share has a significantly positive coefficient, which suggests the importance of this margin in empirical studies that involve cross-city wage data. Furthermore, it suggests the importance of understanding the mechanism that drives the industrial composition because it affects the cross-city wage pattern through the aggregate tradable sector share. Finally, I analyze the evolution of the city-size wage premium during the period when business services became substantially tradable. The analysis suggests that the transition of the business services –rather than the increase in the uniform trade cost– supported the stable or increasing premium. While I believe this is intuitive and that this new perspective is valuable, I recognize that interpreting the transition in the world of two levels of tradability should have limitations. Thus, in the future, a model that features more than two trade costs might provide more demand-side insights into the transitions' effects.

Chapter 2

Persistence of Non-democratic Regimes and Reputation

2.1 Introduction

Why do some non-democratic regimes persist while others are temporary and repetitive? In Myanmar, the military held power for 23 years until 2011. In contrast, since its transition to a constitutional monarchy in 1932, Thailand has experienced 12 successful coups, and in most cases, the military remained in power only for a relatively short period. Similarly, in Turkey, the military has carried out successful coups four times since 1960, including a "soft coup" in 1997. Although the military ousted political leaders, it did not abandon democracy. In this paper, I theoretically demonstrate that these different paths of regime transitions can be explained by a history-dependent equilibrium.

This work's main thesis is that the characteristics of regime change depend on a military's reputation. After a coup, when a military has built up its reputation to the point that it no longer needs to cling to the power, its optimal strategy is to protect that reputation by voluntarily and quickly democratizing a new regime. It is beneficial for a military to maintain a positive reputation because over the long run its citizens become more tolerant of regime transitions to military rule and stop resisting coups. This allows the military to carry out coups more frequently. On the other hand, when a military does not have a positive reputation, citizens do not expect their military to voluntarily democratize, and they resist coups to avoid lengthy military regimes. Once the military seizes power, the military does not return the power back to the citizens until an unavoidable democratization event (e.g., a revolution or a foreign intervention) occurs. This is so because there is no positive reputation to maintain.

To formally demonstrate how a military's reputation can generate particular political dynamics, such as those found in Thailand and Turkey, I develop a model of a stochastic game between a country's military and its citizens. In each period, a country is either in a democracy or a non-democratic regime. In a democracy, the military can attempt a coup and the citizens can resist it. In a non-democratic regime, the military can democratize. In a non-history dependent equilibria, coups are met with resistance and the military never democratizes. In a history-dependent equilibria where the military holds the positive reputation (" equilibrium with reputation"), the military and the citizens play trigger strategies. As long as the citizens do not respond to coups with resistance, the military will democratize after a coup at the first opportunity. At the same time, where the military allows coups to be followed by democratization, citizens will not resist coups. These trigger strategies form a sub-game perfect Nash equilibrium. Comparing the equilibria, reputation expands the set of states in which the military carries out coups. Citizen concerns are alleviated when a military establishes a reputation for not holding on to power for long periods after it initiates a regime change—that is, when a non-democratic regime phase ends quickly, thus, establishing an equilibrium with reputation. This pattern is characteristic of regime changes in Thailand.

My model shares many features with the model developed by Acemoglu and Robinson (Acemoglu and Robinson (2000a,b, 2001, 2006, 2008)) and Boix (2003). In their models and mine, regime changes are consequences of strategic actions by players. Also, in non-democratic regimes, democratization takes place because an individual or the group in power allows it to happen. Where my model differs from theirs is the motivation to democratize. In their model, democratization is motivated by the threat of revolution and the related problem in redistribution. In my model, democratization is motivated by a military's desire to maintain its positive reputation—a motivation that is historically dependent and contingent.

My work provides a new explanation of the heterogeneous persistence of a regime across countries. Previous research on this issue has been carried out by Acemoglu and Robinson (Acemoglu and Robinson (2001, 2006, 2008)), who provide comparative statics about whether democracy is consolidated and an absorbing state. According to their models, whether consolidation occurs depends on fundamental parameters such as inequality between elites and citizens. In my work, the persistence of a regime depends both on its fundamental parameters and its history.

A military that lacks a positive reputation will cling to power and, thus, the non-democratic regime that it establishes will last a long time. Then, once a military regime collapses and democracy is established, citizen distrust in the military regime persists. Citizen resistance discourages the military from mounting other coups, and the democracy lasts longer than it otherwise might. In contrast, under a military that has a positive reputation, a non-democratic regime established by the military tends to end quickly, and in such a democracy the probability of a coup is high. In this setting, democracy experiences frequent short interruptions, and neither democratic nor non-democratic regimes last very long.

My research is also novel because it models regime changes focusing on its history dependence. The vast majority of preceding works that theoretically explain regime changes in a framework of game theory focus on Markov perfect equilibria (MPE). Although the possibility of history-dependent equilibria has not been ignored, it is generally analyzed as an extension of the MPEs. For example, Acemoglu and Robinson (2006) point out that allowing non-Markovian strategies expands the set of parameter values under which non-democratic regime can survive. This is so because history-dependent strategies help resolve the commitment problem. Acemoglu and Robinson (2000b) show that the main characteristics of their MPE hold in the non-Markovian equilibria. Their focus is the commitment problem and how non-Markovian strategies can alleviate the commitment problem in redistribution for the elites in power. In contrast, my work demonstrates that non-Markovian strategies generate a pattern of regime transitions that is completely different from the pattern characteristic of the MPE.

Although history-dependent equilibria are not limited to those with reputation, in some cases an equilibria with reputation can exist even when others do not. After analyzing the equilibrium with reputation, I provide an analysis of the other history-dependent equilibrium, in which citizens provide a transfer (a sidepayment) to the military in return for the military not attempting a coup. Both the equilibrium with reputation and the equilibrium with transfer entail commitment problems. The military can deviate by not democratizing in the equilibrium with reputation and by attempting a coup in the one with transfer. When the incentive to deviate is too strong, the military cannot commit to either democratization or not attempting a coup, and the corresponding equilibrium does not exist. Because how the fundamental parameters of the model affect the incentive of these deviations differs in the two equilibrium, there are cases in which the two equilibria cannot exist simultaneously. For example, simulations suggest that when a coup has a high chance of success, the deviation becomes disproportionately attractive in the sidepayment equilibrium, and this makes it difficult for the sidepayment equilibrium to exist even when the equilibrium with reputation can exist. One possible interpretation is that the equilibrium with reputation tends to appear when the military is well organized or when the citizens are not sufficiently unified to form a powerful resistance.

The rest of this paper is organized as follows. Section 2 lays out the model. Section 3 analyzes the Markov perfect equilibrium and imposes necessary assumptions. Section 4 shows how and if the equilibrium with reputation exists. Section 5 explains the sidepayment equilibrium and discusses the difference between the two history-dependent equilibria. Section 6 concludes.

2.2 Model

In this section, I develop the main model of regime transitions in which reputation can play the main role. We consider an infinite horizon stochastic game between citizens and the military, wherein the state during period $t+1$ depends on the state during period t and on players' actions during that state. There are two states: democracy (D) and non-democratic regime (N). In the following model sections, I use the terms "state" and "regime" interchangeably. Each group consists of a continuum of homogeneous agents, and the mass are λ and $1-\lambda$ for the citizens and the military, respectively. Because we assume that agents are homogeneous, there is no collective action problem and group $g \in \{c, m\}$ maximizes the expected discounted payoffs

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t r^g(t)$$

where r_t^c and r_t^m are the payoff to the citizens and to the military, respectively, in period t, and $\beta \in (0,1)$ is the common discount factor. I begin by describing each state.

Table 2.1: Sub-states in Democracy

Sub-state k	1	2	3
Citizen's payoff, $c(k)$	c	c	c
Military's payoff, $m(k)$	0	m	$m*$
Probability, $Pr(k)$	s	$1-s-s^*$	s^*

2.2.1 Democracy

In a democratic regime, there are three possible sub-states. The sub-state is i.i.d and the period payoff depends on the sub-state, which is summarized in Table 2.1. While the period payoff to the citizen is constant at $c > 0$, that of the military depends on the sub-state. For the military, sub-state 1 is the worst and sub-state 3 is the best because $0 < m < m^*$. The fluctuating payoff to the military captures the notion that the economic and political power of the military fluctuates. For example, faced with tensions in international relations, a democratic government raises military spending, which can generate new rents for the military. In another example, a newly elected politician pursues transparency in politics, which jeopardizes the military's rent from corruption with politicians.

In each period, the military can attempt a coup; the decision to do so is denoted by μ_t. Without a coup ($\mu_t = 0$), the payoff is finalized, as in Table 2.1. When a coup is attempted ($\mu_t = 1$), it costs the military their entire period payoff, $m(k)$, and the probability of success depends on whether the citizens resist, which is denoted by ϕ_t . Citizen resistance also costs the military a part of its period payoff, $\alpha c(k)$ where $0 < \alpha \leq 1$. When citizens choose to resist ($\phi_t = 1$), they can prevent the regime change with probability $1-p$ where $0 < p < 1$. In this case, an attempted coup succeeds with p. I assume that when there is no resistance ($\phi_t = 0$) the coup is always successful. Only when a coup is successful does the state transition to the non- democratic regime during the next period. Otherwise, democracy lasts and the next period sees a democratic regime. The timing of events is summarized below.

1. The current sub-state $k \in \{1, 2, 3\}$ is revealed and the period payoffs are given to the players.

2. The military decides whether to attempt a coup, which costs their entire period payoff $m(k)$.

3. If a coup is attempted, citizens decide whether to resist, which costs $\alpha c(k)$ where $\alpha \in (0, 1]$.

4. If there is resistance, the coup succeeds with probability $p \in (0, 1)$.

5. If there is no resistance, or if the coup succeeds, the next period starts under a dictatorship.

2.2.2 Non-democratic Regime

In the non-democratic regime, there is only one sub-state. The military's period payoff is $n > 0$ while that of the citizens is $d \geq 0$. I impose an assumption on the relative payoff in the two regimes within each party.

Assumption 2. *For the military, the expected period payoff is higher in a non-democratic regime. For the citizens, it is higher in a democracy.*

$$n > \sum_{k \in \{1,2,3\}} Pr(k)m(k)$$

$$d < c$$

Additionally, to reasonably discipline the payoffs, the expected total economic pie is assumed to be the same in the two regimes.

Assumption 3. *The expected aggregate period payoff of the two parties is the same in the non-democratic regime and in a democracy.*

$$\lambda n + (1-\lambda)d = \lambda \sum_{k\in\{1,2,3\}} Pr(k)m(k) + (1-\lambda)c$$

In the remainder of this paper, I assume Assumptions 2 and 3 hold. The only possible choice in the non-democratic regime is whether the military voluntarily democratizes ($\delta_t \in \{0,1\}$). Democratization ($\delta_t = 1$) is costless and it makes the regime a democracy in the next period. In addition to democratization, a second event can bring about regime transition. If the military does not democratize ($\delta_t = 0$), an exogenous democratization shock realizes with probability $q \in (0,1)$. This captures any kind of events that result in regime collapse. This could include a revolution that originates outside the interactions of this model, an internal conflict within the military, or intervention by a foreign country. The timing of such events during a non-democratic period is summarized below.

1. The military chooses whether to voluntarily democratize (δ_t).
2. If voluntary democratization does not occur ($\delta_t = 0$), an exogenous democratization shock realizes with probability $q \in (0,1)$.
3. If there is neither voluntary democratization ($\delta_t = 0$) nor an exogenous shock, the regime will be a dictatorship during the next period.

2.2.3 Definition of Strategies

Because throughout this paper I analyze how history-dependent strategies can generate outcomes that are distinct from those predicted by Markovian strategies, I here formally define the strategies of the two players.

Definition 1. Given every possible history up to period $t-1$ (h^{t-1}), and sub-state s in period t, the military's strategy σ_t^m is a mapping from (h^{t-1}, s) to the decision of a coup ($\mu_t \in \{0,1\}$), and the decision of democratization ($\delta \in \{0,1\}$).

$$\sigma_t^m : (h^{t-1}, s) \mapsto (\mu_t, \delta)$$

Definition 2. Given every possible history up to period $t-1$ (h^{t-1}), sub-state s in period t, and military's decision to undertake a coup ($\mu \in \{0,1\}$), the citizens' strategy σ_t^c is a mapping from (h^{t-1}, s) to the decision of the resistance ($\phi \in \{0,1\}$).

$$\sigma_t^c : (h^{t-1}, s, \mu_t) \mapsto \phi$$

Here, the regime of period t does not explicitly appear in the definition. This is so because the regime during period t is completely determined by the regime, the actions, and the outcome of the shock during the previous period, all of which are included in h^{t-1}.

2.3 Markov Perfect Equilibrium

I first study a Markov perfect equilibrium (MPE). This part is necessary not only to illustrates the difference between the history dependent and non-dependent equilibria but also to use the MPE as the punishment in the history-dependent equilibrium. In a MPE, the strategies depend only on the current state and are the best responses to each other. That is, $\forall h^{t-1}$, $\sigma_t^m(h^{t-1}, s_t) = \sigma^m(s_t)$ and $\sigma_t^c(h^{t-1}, s_t, \mu_t) = \sigma^c(s_t, \mu_t)$ where $R \in \{D, N\}$. Depending on the parameters, there are many possible MPEs. In this paper, I consider a pure MPE consisting of ($\sigma^{m,MPE}$,$\sigma^{c,MPE}$) s.t.

$$\forall t\{1,\ldots,\infty\},\ \mu_t = \begin{cases} 1 & if\ s_t = 1 \\ 0 & if\ s_t = 2 \\ 0 & if\ s_t = 3 \end{cases},\quad \delta_t = 0$$

$$\forall (t,\mu_t) \in \{1,\ldots,\infty\} \times \{0,1\},\ \phi_t = 1$$

A coup is attempted only in the military's worst sub-state, and it always faced with resistance. In order for ($\sigma^{m,MPE}$,$\sigma^{c,MPE}$) to be a MPE, I introduce assumptions on the parameters in the following sub-sections.

2.3.1 Values for Military

To derive the conditions for (σ^m,σ^c) to be a MPE, I begin by deriving the values for the two players with ($\sigma^{m,MPE}$,$\sigma^{c,MPE}$). In this equilibrium, let the value for the military at the beginning of sub-state k of democracy be $V^a(D,k)$. In sub-state 1, a coup is attempted and it encounters resistance. With probability p, the regime changes and it will receive the value at the beginning of a non-democratic regime, which is defined as $V^a(M)$.

$$V^a(D,1) = \beta\left[pV^a(M) + (1-p)\mathbb{E}V^a(D)\right]$$

where $\mathbb{E}V^a(D) = sV^a(D,1) + (1-s-s^*)V^a(D,2) + s^*V^a(D,3)$.

In the other sub-states, the military does not attempt a coup and it keep the payoffs (m

or m^*). The values are given by

$$V^a(D,2) = m + \beta \mathbb{E} V^a(D)$$
$$V^a(D,3) = m^* + \beta \mathbb{E} V^a(D)$$

In the non-democratic regime, the military receives n and the regime does not change unless exogenous democratization is realized.

$$V^a(M) = n + \beta \left[(1-q)V^a(M) + q\mathbb{E}V^a(D)\right]$$

By combining the four equations, I obtain the solutions to the value functions.

$$\begin{aligned}
\mathbb{E}V^a(D) &= s\beta p \frac{n}{(1-\beta)\left[1-\beta(1-q-sp)\right]} \\
&\quad + \left[\frac{s\beta^2 pq}{(1-\beta)\left[1-\beta(1-q-sp)\right]} + 1\right] \frac{(1-s-s^*)m + s^* m^*}{1-\beta(1-sp)} && (2.1) \\
V^a(M) &= \frac{n\left(1-\beta(1-sp)\right) + \beta q\left[(1-s-s^*)m + s^* m^*\right]}{(1-\beta)\left[1-\beta(1-q-sp)\right]} && (2.2)
\end{aligned}$$

2.3.2 Assumptions for MPE on Military Side

The first condition is that the military attempts a coup in sub-state 1. Since the period payoff in state 1 is 0, the comparison of the expected payoffs becomes

$$\begin{aligned}
&\beta \mathbb{E}V^a(D) \le \beta \left[pV^a(M) + (1-p)\mathbb{E}V^a(D)\right] \\
\Longrightarrow\ & \mathbb{E}V^a(D) \le V^a(M) && (2.3)
\end{aligned}$$

Substituting the value functions produces the following inequality.

$$\mathbb{E}m(k) = (1 - s - s^*)m + s^*m' \leq n$$

This shows that for a military, the non-democratic regime needs to be more attractive than the democracy in terms of the period payoff. This is already assumed in Assumption 2.

Next, the conditions under which the military does not choose to attempt a coup in sub-state 2 or 3 are given by

$$\begin{aligned} \beta\left[pV^a(M) + (1-p)\mathbb{E}V^a(D)\right] &\leq m + \beta\mathbb{E}V^a(D) \qquad (2.4) \\ \beta\left[pV^a(M) + (1-p)\mathbb{E}V^a(D)\right] &\leq m^* + \beta\mathbb{E}V^a(D) \end{aligned}$$

Note that the period payoffs are not included in the LHS because they are the direct costs of a coup and the second inequality is redundant because $m < m^*$. Substituting the values leads to Lemma 2

Assumption 4.

$$\frac{n - \mathbb{E}m(k)}{m} \leq \frac{1 - \beta(1 - q - sp)}{\beta p}$$

Lemma 2. *If Assumption 4 holds, not attempting a coup ($\mu_t = 0$) is the best response for the military to the citizens' Markovian strategy ($\sigma^{c,MPE}$) in sub-state 2.*

For the military not to mount a coup in the middle sub-state, the non-democratic regime cannot be relatively too attractive for the military in two ways. First, the expected relative period payoff ($\frac{n-\mathbb{E}m(k)}{m}$) cannot be too high. Second, the exogenous regime collapse (q) cannot be too unlikely. In addition, the attractive regime cannot be too attainable–that is, the probability of a successful coup without resistance (p) cannot be too high. I assume

Assumption (4) holds in the rest of the paper.

Finally, not voluntarily democratizing the regime needs to be rational, which is guaranteed by

$$\begin{aligned} & \mathbb{E}V^a(D) \leq (1-q)V^a(M) + q\mathbb{E}V^a(D) \\ \implies & \mathbb{E}V^a(D) \leq V^a(M) \end{aligned}$$

This is the same as inequality (2.3) and, therefore, it requires no additional parameter condition.

2.3.3 Values for Citizens

Let the value at the beginning of sub-state k in democracy for citizens be $U^a(D,k)$ and that of the non-democratic regime be $U^a(M)$. Then, the equations for the values in democracy are given by

$$\begin{aligned} U^a(D,1) &= (1-\alpha)c + \beta\left[pU^a(M) + (1-p)\mathbb{E}U^a(D)\right] \\ U^a(D,2) &= c + \beta\mathbb{E}U^a(D) \\ U^a(D,3) &= c + \beta\mathbb{E}U^a(D) \end{aligned}$$

where $\mathbb{E}U^a(D) = sU^a(D,1) + (1-s-s^*)U^a(D,2) + s^*U^a(D,3)$. Only in sub-state 1 the citizens face a coup and they resist it, incurring the cost, αc. In the non-democratic regime, citizens undertake no action and awaits the exogenous regime's collapse while receiving d in each period.

$$U^a(M) = d + \beta\left(q\mathbb{E}U^a(D) + (1-q)U^a(M)\right)$$

Combining the four equations gives the solutions.

$$\mathbb{E}U^a(D) = \frac{(1-\beta+\beta q)(1-s\alpha)c + \beta spd}{(1-\beta)\left(1-\beta(1-sp-q)\right))} \tag{2.5}$$

$$U^a(M) = \frac{\beta q(1-s\alpha)c + (1-\beta+\beta sp)\, d}{(1-\beta)\left(1-\beta(1-sp-q)\right)} \tag{2.6}$$

2.3.4 Assumptions for MPE on Citizens' Side

The only citizens' action that requires a condition on the parameters is the resistance in sub-state 1.

$$c + \beta U^a(M) \leq (1-\alpha)c + \beta\left[pU^a(M) + (1-p)\mathbb{E}U^a(D)\right]$$

By resisting, the citizens lower the probability of a successful coup from 1 to p and they incur the cost, αc. Lemma 3 is obtained by substituting the values.

Assumption 5.

$$\frac{1-\beta(1-s-q)}{\beta(1-p)}\alpha \leq \frac{c-d}{c}$$

Lemma 3. *If Assumption 5 holds, resisting a coup ($\phi_t = 1$) is the best response for citizens to the military's Markovian strategy ($\sigma^{m,MPE}$) in sub-state 1.*

Assumption 5 is more likely to be satisfied by a higher $\frac{c-d}{c}$ and a lower α. That is, a democracy needs to be sufficiently attractive relative to a non-democratic regime for citizens to protect it at the expense of a fraction of their payoff. Also, when the non-democratic regime is stable (low q) or resistance is effective (low p), the citizens are more motivated to resist, and this condition is more likely to be satisfied. I assume Assumption (5) holds in the rest of this paper.

2.4 Equilibrium with Reputation

In this section, I consider an equilibrium in which the military has the reputation of democratizing shortly after a successful coup. More specifically, the two parties play trigger strategies. In a democratic regime, the military attempts a coup in sub-state 1 and 2 and the citizens do not resist. In the non-democratic regime, the military voluntarily democratizes. If either party deviates, both parties start to play ($\sigma^{m,MPE}$,$\sigma^{c,MPE}$) and they transition to the MPE. Formally, the trigger strategy of the citizens is defined as follows.

Definition 3. For $t \in \{1, 2, \ldots \infty\}$, the citizens' trigger strategy $\{\sigma_t^{c,T}\}_{t=1}^{\infty}$ is a strategy such that

$$
\begin{aligned}
&If\ (i)\mu_j = 1\ for\ all\ j \leq t,\ s.t.\ R_j = D, and\ s_j \in \{1, 2\},\\
&\quad (ii)\mu_j = 0\ for\ all\ j \leq t,\ s.t.\ R_j = D, and\ s_j = 3,\ and\\
&\quad (iii)\delta_j = 1\ for\ all\ j \leq t-1\ s.t.\ R_j = N,\\
&then\ \phi_t = 0\\
&Otherwise,\ \phi_t = 1
\end{aligned}
$$

The military's trigger strategy is defined as follows.

Definition 4. For $t \in \{1, 2, \ldots \infty\}$, the military's trigger strategy $\{\sigma_t^{m,T}\}_{t=1}^{\infty}$ is a strategy such that

$$
\begin{aligned}
&If\ t = 1\ or \\
&\quad \phi_j = 0\ for\ all\ j \leq t-1\ s.t.\ s_j \in \{1,2\}, \\
&then\ \mu_t = \begin{cases} 1 & if\ s_t = 1 \\ 1 & if\ s_t = 2 \\ 0 & if\ s_t = 3 \end{cases}, \quad \delta_t = 1 \\
&Otherwise,\ \mu_t = \begin{cases} 1 & if\ s_t = 1 \\ 0 & if\ s_t = 2 \\ 0 & if\ s_t = 3 \end{cases}, \quad \delta_t = 0
\end{aligned}
$$

I now show that, depending on the parameters, $(\sigma_t^{m,T}, \sigma_t^{c,T})$ can form a sub-game perfect Nash equilibrium. As in the MPE, I start by assuming the equilibrium exists and solve for the value functions.

2.4.1 Values for Military

In this equilibrium, let the value for the military at the beginning of sub-state k of democracy be $V^b(D,k)$. In sub-state 1 and 2, the military attempts a coup. It does not face resistance and it succeeds. Accordingly, the values are given by

$$
V^b(D,1) = \beta V^b(M)
$$

$$
V^b(D,2) = \beta V^b(M)
$$

In sub-state 3, no coup is attempted and democracy continues into the next period.

$$V^b(D,3) = m^* + \beta \mathbb{E} V^b(D)$$

where $\mathbb{E}V^b(D) = sV^b(D,1) + (1-s-s^*)V^b(D,2) + s^*V^b(D,3)$.

When the regime during this period is non-democratic, the next period is democratic because of voluntary democratization. It follows that

$$V^b(M) = n + \beta \mathbb{E} V^b(D)$$

From the four equations above, the solutions to the value functions are obtained.

$$\mathbb{E}V^b(D) = \frac{\beta(1-s^*)n + s^*m^*}{(1-\beta)(1+\beta-\beta s^*)} \tag{2.7}$$

$$V^b(M) = \frac{(1-\beta s^*)n + \beta s^* m^*}{(1-\beta)(1+\beta-\beta s^*)} \tag{2.8}$$

2.4.2 Conditions for Military Actions to be Rational

First, I examine the condition of the voluntary democratization. If the military does not give up power, it is subject to the exogenous democratization shock and it moves to the MPE. In both cases, the military receives n. Therefore, the condition is

$$(1-q)\beta V^a(M) + q\beta \mathbb{E} V^a(D) \leq \beta \mathbb{E} V^b(D) \tag{2.9}$$

Substituting the solutions to the value functions ((2.1), (2.2), and (2.7)), this becomes

$$0 \leq \left(\frac{1}{1+\beta-s^*\beta} - \frac{q}{1-\beta(1-q-sp)} \right) (\mathbb{E}m(k) - n)$$

Given the regime, in the two equilibria the payoff the military keeps is the same. In the MPE, they keep m and m^* by not attempting a coup; in the equilibrium with reputation they keep m and m^* because they do not face resistance. Therefore, which equilibrium they prefer depends on which regime they prefer ($\mathbb{E}m(k) - n$) and in which the favorable regime appears more frequently ($\frac{1}{1+\beta-s^*\beta} - \frac{q}{1-\beta(1-q-sp)}$). Under Assumption 2, the military prefers the non-democratic regime and, therefore, the only condition is Condition 1.

Condition 1. (Voluntary democratization) Given the citizens' trigger strategy ($\sigma_t^{c,T}$), and under a non-democratic regime, democratizing ($\delta_t = 1$) is the best response for the military if the following condition is satisfied.

$$\frac{1-\beta\,(1-sp)}{1-s^*\beta} \leq q$$

When the voluntary democratization condition (Condition 1) is satisfied, the expected payoff to the military is greater by democratizing and staying in the equilibrium with reputation than by holding on to the non-democratic regime and transitioning to the MPE. The difference in the expected payoff boils down to the difference in frequency of non-democratic regimes taking into account discounting. Four parameters govern the frequency and for four reasons the condition is likely to be satisfied: 1) the probability of the exogenous regime collapse shock is higher ($q \uparrow$); 2) the probability of sub-state 1 is lower ($s \downarrow$); 3) the probability of a successful coup when faced with resistance is lower ($p \downarrow$); and 4) the probability of sub-state 3 is lower ($s^* \downarrow$).

Next, the military now attempts a coup in sub-state 1 and 2. Otherwise, it would transition to the MPE. Therefore, the comparison of values in each sub-state is the following.

$$\begin{cases} \beta \mathbb{E} V^a(D) \leq \beta V^b(M) \\ m + \beta \mathbb{E} V^a(D) \leq m + \beta V^b(M) \end{cases}$$
$$\Longrightarrow \mathbb{E} V^a(D) \leq V^b(M) \tag{2.10}$$

A coup takes place when the inequality holds. Using the solutions to the value functions ((2.7) and (2.8)), Lemma 4 is obtained

Lemma 4. *(Coup in sub-state 1 and 2) Given the citizens' trigger strategy ($\sigma_t^{c,T}$), Condition 1 is sufficient for attempting a coup ($\mu_t = 1$) to be the best response for the military in sub-state 1 and 2 in democracy.*

Proof. By substituting the value functions, inequality (2.10) becomes

$$\frac{\beta}{1 + \beta - s^* \beta} \leq \frac{1 - \beta(1 - q)}{1 - \beta(1 - q - sp)}$$

This holds if $\frac{1}{1+\beta-s^*\beta} \leq \frac{q}{1-\beta(1-q-sp)}$, which is equivalent to $\frac{1-\beta(1-sp)}{1-s^*\beta} \leq q$. □

When the voluntary democratization condition (Condition 1) is satisfied, the military in a non-democratic regime judges that the frequency of non-democratic regimes is higher in the equilibrium with reputation despite the fact that the next period becomes democratic. When it is in a democratic regime, the next period will be non-democratic. This raises the frequency taking into account the discounting, which is why the voluntary democratization condition is sufficient for the coup in sub-state 1 and 2 to be rational.

Finally, the condition for no coup in sub-state 3 is given by

$$\beta \left[pV^a(M) + (1-p)\mathbb{E}V^a(D) \right] \leq m^* + \beta \mathbb{E}V^b(D) \tag{2.11}$$

According to Lemma 5, we do not need additional conditions for this inequality.

Lemma 5. *(No coup in sub-state 3) Given the citizens' trigger strategy ($\sigma_t^{c,T}$), Assumption 2 and 4 and Condition 1 are sufficient for attempting a coup ($\mu_t = 1$) to be the best response for the military in sub-state 1 and 2 in democracy.*

Proof. We know

$$\begin{aligned} \mathbb{E}V^b(D) &\geq (1-q)V^a(M) + q\mathbb{E}V^a(D) \\ &\geq \mathbb{E}V^a(D) \end{aligned}$$

where the first inequality follows from inequality 2.9 for the voluntary democratization, which is satisfied by Condition 1, and the second inequality follows from inequality 2.3, which holds with Assumption 2. Then, it follows

$$\begin{aligned} m^* + \beta \mathbb{E}V^b(D) &\geq m + \beta \mathbb{E}V^a(D)) \\ &\geq \beta \left[pV^a(M) + (1-p)\mathbb{E}V^a(D) \right] \end{aligned}$$

where the second inequality follows from inequality 2.4 for no coup in sub-state 2 in the MPE, which holds with Assumption 4. Thus, the inequality (2.11) is satisfied. □

2.4.3 Values for Citizens

In this equilibrium, let the value for the citizens at the beginning of sub-state k of democracy be $U^b(D,k)$ and the value for citizens at the beginning of a non-democratic regime be $U^b(M)$. Because the citizens do not resist and the regime changes except in in sub-state 3, the values are given by

$$\begin{aligned} U^b(D,1) &= c + \beta U^b(M) \\ U^b(D,2) &= c + \beta U^b(M) \\ U^b(D,3) &= c + \beta \mathbb{E}U^b(D) \\ U^b(M) &= d + \beta \mathbb{E}U^b(D) \end{aligned}$$

where $\mathbb{E}U^b(D) = sU^b(D,1) + (1-s-s^*)U^b(D,2) + s^*U^b(D,3)$. The solutions are

$$\begin{aligned} \mathbb{E}U^b(D) &= \frac{c + \beta\left(1-s^*\right)d}{\left(1-\beta\right)\left(1-\beta s^* + \beta\right)} \\ U^b(M) &= \frac{\beta c + (1-\beta s^*)d}{\left(1-\beta\right)\left(1-\beta s^* + \beta\right)} \end{aligned} \tag{2.12}$$

2.4.4 Conditions for Citizens' Actions to be Rational

In this equilibrium, the only choice the citizens make is whether to resist in sub-state 1 and 2. They do not prefer resistance when the following inequality holds.

$$(1-\alpha)c + \beta\left[pU^a(M) + (1-p)\mathbb{E}U^a(D)\right] \leq c + \beta U^b(M)$$

By substituting the solutions to the value functions ((2.5), (2.6), and (2.12)), the following is obtained.

$$0 \leq \left\{ \alpha \left[\frac{1}{\beta} - \frac{(1+\beta s-\beta)sp}{1-\beta+\beta q+\beta sp} \right] + \left[\frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp} - \frac{1-\beta s^*}{1-\beta s^*+\beta} \right] \right\} c$$
$$- \left[\frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp} - \frac{1-\beta s^*}{1-\beta s^*+\beta} \right] d$$

The democratization condition (Condition 1) implies $\frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp} - \frac{1-\beta s^*}{1-\beta s^*+\beta} < 0$. Lemma 6 follows

Lemma 6. *Given the military's trigger strategy ($\sigma_t^{m,T}$), not resisting a coup ($\phi_t = 0$) is the best response for the citizens in sub-states 1 and 2 of democracy if*

$$\frac{c-d}{c} \leq \frac{\frac{1}{\beta} - \frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp} s}{\frac{1-\beta s^*}{1-\beta s^*+\beta} - \frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp}} \alpha \tag{2.13}$$

This is likely to be satisfied for three reasons: 1) the probability of sub-state 1 is higher ($s \uparrow$); 2) the probability of a successful coup when faced with resistance is higher ($p \uparrow$); and 3) the probability of sub-state 3 is higher ($s^* \uparrow$). A high s and s^* implies that the probability of sub-state 2 is low. Allowing the military to attempt a coup during an additional sub-state (sub-state 2) worsens the circumstances of citizens, other things being equal, and, therefore, a high probability of such a sub-state incentivizes the citizens to resist. This is the reason why the probability of the additional sub-state cannot be too high. Also, a higher p makes it easier for the condition to be satisfied because it worsens the welfare of the citizens in the MPE and strengthens the incentive to have the equilibrium with the reputation.

2.4.5 Existence of Equilibrium with Reputation

In the previous sub-sections, I derived the additional conditions for the trigger strategies $(\sigma_t^{m,T}, \sigma_t^{c,T})$ to form a sub-game perfect Nash equilibrium. Only if these conditions are satisfied and the assumptions for the MPE hold do the trigger strategies $(\sigma_t^{m,T}, \sigma_t^{c,T})$ form a sub-game perfect Nash equilibrium and does the equilibrium with reputation exist. In this sub-section, I analyze the parameters that satisfy these conditions. We know from Assumption 2, Condition 1, and Lemma 2, 4 and 5, that on the military side, the only conditions that the existence of the equilibrium with reputation requires are the following:

$$0 < \frac{n - \mathbb{E}m(k)}{m} \leq \frac{1 - \beta(1 - q - sp)}{\beta p}$$
$$\frac{1 - \beta\left(1 - sp\right)}{1 - \beta s^*} \leq q$$

The lower bound on the ratio of the payoffs is zero because as long as the non-democratic regime becomes more frequent in the equilibrium with reputation, there is no incentive for the military to deviate even if a non-democratic regime is barely more attractive than a democracy. On the citizens' side, combining the conditions about the resistance in the two equilibria (Lemma (3) and Condition (2.13)), the following interval is obtained.

$$\frac{1 - \beta(1 - s - q)}{\beta(1 - p)}\alpha \leq \frac{c - d}{c} \leq \frac{\frac{1}{\beta} - \frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp} s}{\frac{1-\beta s^*}{1-\beta s^*+\beta} - \frac{(1+\beta s-\beta)p}{1-\beta+\beta q+\beta sp}}\alpha \tag{2.14}$$

On the citizens' side, there is a non-zero lower bound on the ratio of the payoffs. This is so because the citizens need to be incentivized to resist in the MPE but not in the equilibrium with reputation. While the left inequality of (2.14) is more likely to be satisfied with lower q and lower p, the right inequality is more likely to be satisfied with higher q and higher p. This reflects the fact that the left inequality is the condition of resistance while the right inequality is the condition of non-resistance. The less stable a non-democratic regime is ($q \downarrow$) and the higher chance of success resistance has ($p \downarrow$), the more incentivized to resist citizens are. Here, Lemma 7 is obtained.

Lemma 7. $\exists \frac{d}{c} \in [0,1)$ *such that (2.14) if and only if*

$$q \leq \frac{1-\beta(1-s)p}{1-\beta s^*} + (1-s)(1-p)$$
$$\alpha \leq \frac{\beta(1-p)}{1-\beta(1-s-q)}$$

The RHS of the first inequality in Lemma 7 is greater than the LHS of inequality of Condition 1. Therefore, these conditions do not contradict each other. In fact, simulations have confirmed that there exists $\{c, d, m, m^*, n, p, q, s, s^*, \beta, \lambda\} \in R^5_+ \times (0,1)^6$ such that Assumptions 2, 3, and 4, Condition 1, and inequality (2.14) hold.

2.5 Sidepayment Equilibrium

In this section, I study another history-dependent equilibrium in which the citizens offer a transfer to the military in return for no coup. To analyze this case in the the framework, I add one step in each period of a democracy. At the beginning of a democratic period, the citizens can provide a financial transfer per military person $\theta \in [0, \frac{\lambda}{1-\lambda}c]$ to the military. After this transfer, the sub-state of the period is revealed and the players follow exactly the same steps as in the original game. I assume the transfer will not be lost even when a coup is attempted but meets faced with resistance. The transfer cannot be conditioned on any action by the military. Nevertheless, it is possible for the citizens to use another trigger strategy that exploits this option to induce military to not undertake a coup with another trigger strategy. Strategies of this game are defined in general as follows.

Definition 5. Citizens' Strategy with Transfer

The citizens' strategy consists of two sub-strategies $\{\sigma^c_{\theta,t}, \sigma^c_{\phi,t}\}$ s.t.

(i) given every possible history up to period $t-1$ (h^{t-1}), the citizens' transfer strategy $\sigma^c_{\theta,t}$ is a mapping from h^{t-1} to the value of the transfer $\theta_t \in [0,1]$.

$$\sigma^c_{\theta,t} : h^{t-1} \mapsto \theta_t$$

(ii) given every possible history up to period $t-1$ (h^{t-1}), transfer ($\theta_t \in [0, \frac{\lambda}{1-\lambda}c]$), sub-state $s \in \{1,2,3\}$ in period t, and the military's coup decision ($\mu_t \in \{0,1\}$), the citizens' resistance strategy $\sigma^c_{\phi,t}$ is a mapping from $(h^{t-1}, \theta_t, s_t, \mu_t)$ to the decision of the resistance ($\phi_t \in \{0,1\}$).

$$\sigma^c_{\phi,t} : (h^{t-1}, \theta_t, s_t, \mu_t) \mapsto \phi_t$$

Definition 6. Military's Strategy with Transfer

Given every possible history up to period $t-1$ (h^{t-1}), transfer ($\theta_t \in [0, \frac{\lambda}{1-\lambda}c]$), and sub-state in period t (s_t), the military's strategy σ^m_t is a mapping from (h^{t-1}, θ_t, s_t) to the decision to undertake a coup ($\mu_t \in \{0,1\}$) and the decision to democratize ($\delta_t \in \{0,1\}$).

$$\sigma^m_t : (h^{t-1}, \theta_t, s_t) \mapsto (\mu_t, \delta_t)$$

When $\theta = 0$ for any state, this game is identical to the game considered for and the equilibrium with reputation. It is straightforward that all of the previous results, including the existence and the parameter conditions of the MPE and the equilibrium with reputation, are valid without any modification, except that the citizens' strategy now additionally assign $\theta_t = 0$ in all periods in both equilibria.

Now, I study a new history-dependent equilibrium in which citizens provide positive transfer as long as the military does not attempt a coup. [1] This equilibrium can be expressed as the two players play their trigger strategies.[2]

Definition 7. For $t \in \{1, 2, \ldots \infty\}$, the citizens' trigger strategy $\sigma_t^{c,S}(\theta)$ is a strategy such that

$$\begin{aligned} &If\ t = 1\ or\ \mu_j = 0\ for\ all\ j \leq t-1,\ s.t.\ R_j = D \\ &then\ \theta_t = \theta \\ &Otherwise,\ \theta_t = 0\ and\ \phi_t = 1 \end{aligned}$$

Definition 8. For $t \in \{1, 2, \ldots \infty\}$, the military's trigger strategy $\sigma_t^{m,S}(\theta)$ is a strategy such that

$$\begin{aligned} &If\ (i)\ t = 1\ or \\ &\quad (ii)\ \theta_k = \theta\ for\ all\ k \leq t\ s.t.\ R_k = D\ and \\ &\quad \phi_j = 0\ for\ all\ j \leq t-1\ s.t.\ s_j \in \{1, 2\}, \\ &then\ \forall s_t \in \{1, 2, 3\},\ \mu_t = 0 \\ &Otherwise,\ \mu_t = \begin{cases} 1 & if\ s_t = 1 \\ 0 & if\ s_t = 2 \\ 0 & if\ s_t = 3 \end{cases},\quad \delta_t = 0 \end{aligned}$$

As in the previous section, I derive the value functions and find the conditions on the

[1]In the following part, I assume the first period is in democracy ($R_1 = D$).

[2]This definition of this trigger strategy is not complete in the sense that it does not specify ϕ_t for the first case. Since $\mu_j = 0$, the choice of ϕ_t does not affect the analysis, and it can take either 0 or 1.

parameters for these trigger strategies to form a sub-game perfect Nash equilibrium.

2.5.1 Values for Military and Condition

In this equilibrium, let the value for the military at the beginning of sub-state $k \in \{1, 2, 3\}$ of democracy be $V^c(D, k)$. Since no regime change occurs, the values specific to this equilibrium are only determined for democracy. The values in the non-democratic regime are those of the MPE. In a democracy, the military, in addition to its own payoff, receives $\theta > 0$ per person from the citizens before the state is revealed. The values can be written as

$$V^c(D,1) = \theta + \beta \mathbb{E} V^c(D)$$

$$V^c(D,2) = m + \theta + \beta \mathbb{E} V^c(D)$$

$$V^c(D,3) = m^* + \theta + \beta \mathbb{E} V^c(D)$$

where $\mathbb{E}V^c(D) = sV^c(D,1) + (1-s-s^*)V^c(D,2) + s^*V^c(D,3)$. The solution is

$$\mathbb{E}V^c(D) = \frac{\mathbb{E}m(k) + \theta}{1-\beta} \tag{2.15}$$

Now, the condition that the military does not attempt a coup after receiving θ is given by

$$\beta \mathbb{E} V^c(D) \geq \beta \left[pV^a(M) + (1-p)\mathbb{E}V^a(D) \right]$$

The LHS is the value that the military can keep in state 1 by not attempting a coup. Note that the LHS does not contain θ. Regardless of the coup decision, the military can keep the transfer they have already received, and the transfer in the current period cannot work as a disincentive of a coup. The RHS is the value in the case of a coup, which can be calculated with the value functions of the MPE. If this condition is satisfied, no coup is attempted not only in sub-state 1 but also in sub-states 2 and 3. This is because, in sub-states 2 and 3, the military would incur an additional opportunity cost, m or m^*, for a coup. By substituting (2.1), (2.2), and (2.15), the condition is obtained.

Lemma 8. *Given the citizens' trigger strategy ($\sigma_t^{c,S}(\theta)$), not attempting a coup ($\mu_t = 0$) is the best response for the military in all sub-states of democracy if the following condition is satisfied.*

$$\frac{(1-\beta+\beta s)p}{1-\beta(1-q-sp)}\left[n - \mathbb{E}m(k)\right] \leq \theta$$

The transfer needs to be large enough to make the military give up their favored regime.

2.5.2 Values for Citizens and Condition

In this equilibrium let the value for the citizens at the beginning of sub-state $k \in \{1,2,3\}$ of democracy be $U^c(D,k)$. To provide θ per each individual in the military, each citizen needs to pay $\frac{\lambda}{1-\lambda}\theta$. The value functions for all $k \in \{1,2,3\}$ are given by

$$U^c(D,k) = c - \frac{\lambda}{1-\lambda}\theta + \beta\mathbb{E}U^c(D)$$

where $\mathbb{E}U^c(D) = sU^c(D,1) + (1-s-s^*)U^c(D,2) + s^*U^c(D,3)$. Since there is no regime change and the payoff to the citizen is independent of the sub-state, the value is also independent of the sub-state. Let $U^c(D) \equiv U^c(D,1) = U^c(D,2) = U^c(D,3)$. Then, the solution is

$$U^c(D) = \frac{c - \frac{1-\lambda}{\lambda}\theta}{1-\beta} \tag{2.16}$$

If the citizens do not offer θ, the military immediately recognizes the deviation and they transition to the MPE. Therefore, the value the citizen receives by a deviation is that of the MPE and the condition for the citizens to offer θ is given by

$$\forall k \in \{1,2,3\}, \quad U^a(D,k) \leq U^c(D)$$

By substituting (2.5) and (2.16), the condition is obtained.

Lemma 9. *Given the military's trigger strategy ($\sigma_t^{c,S}(\theta)$), providing a transfer θ is the best response for the citizens in all sub-states of democracy if the following condition is satisfied.*

$$\theta \leq \frac{\lambda}{1-\lambda}\beta s \frac{\beta p(c-d) + \alpha(1-\beta+\beta q)c}{1-\beta+\beta q+\beta sp}$$

The transfer cannot be too large for the citizens.

2.5.3 Condition for Existence of a Sidepayment Equilibrium

From Lemmas 8 and 9, θ has the upper and lower bounds. The interval is non-empty if

$$\frac{1-\beta+\beta s}{\beta s}\left[\frac{\frac{n-\mathbb{E}m(k)}{m}}{\beta(\frac{c-d}{c}) + \alpha\frac{1-\beta+\beta q}{p}}\right] \leq \frac{\lambda}{1-\lambda}\frac{c}{m}$$

Using Assumption 3, it follows that

Proposition 10. *There exists $\theta \in (0,c]$ such that $\{\sigma_t^{m,T}(\theta), \sigma_t^{c,T}(\theta)\}_{t=1}^{\infty}$ is a sub-game perfect Nash equilibrium if and only if*

$$\frac{1-\lambda}{\lambda}\frac{n-\mathbb{E}m(k)}{c} \leq \frac{\alpha}{p}\left(1+\frac{\beta}{1-\beta}q\right)\frac{\beta s}{1+\beta s}$$

The citizens use the transfer to avoid the cost of resistance and the reduced period payoff in the non-democratic regime. The more costly the resistance is ($\alpha \uparrow$) and the higher the probability of facing a coup is ($s \uparrow$), the more incentive citizens have to avoid coups and the more likely it is that the condition will be satisfied. In addition, the transfer needs to be high enough for the military to give up their favored regime. The lower the successful rate of a coup is ($p \downarrow$), the smaller the gain from the transition is ($n - \mathbb{E}m(k) \downarrow$); similarly, the less stable the non-democratic regime is ($q \downarrow$), the easier it is to incentive the military not to undertake a coup is and the more likely it is that the condition will be satisfied. Finally, providing a sufficient amount of transfer per capita is easier when the mass of the citizens is greater ($\lambda \uparrow$).

2.5.4 Reputation or Sidepayment?

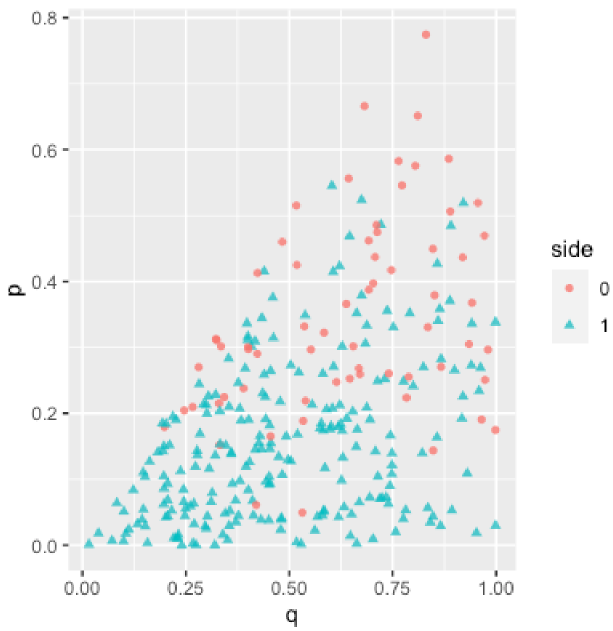
Although the two history-dependent equilibria can both exist, in some cases only one of the two exists. For two reasons the two equilibria cannot exist simultaneously. First, the the division of the additional gain between the two players can be continuous in the transfer but it is discrete in the case of reputation. That is, in some cases the sidepayment equilibrium exists while the equilibrium with reputation does not.

Second and more importantly, the two equilibria have two different commitment problems. First, in the sidepayment case, the military makes the coup decision after receiving the transfer, and a coup does not affect the transfer that the military has already received. Thus, the transfer in the current period does not count towards the return for not attempting a coup, although the transfers during the next period and thereafter count. Even when some amount of transfer during every period makes the military better off than in the MPE, attempting a coup after receiving the initial transfer, a deviation, can make them more better off. The more likely the coup is to be successful (higher p) and the more stable the non-democratic regime is (lower q), the more tempted the military is to deviate and the more difficult it becomes for the military to commit to no coup. This can be seen in the condition for the sidepayment equilibrium in Proposition 10, which is more difficult to satisfy with higher p and lower q. Second, in the reputation case, the deviation by the military takes the form of not democratizing. The military makes the democratization decision after the citizens provide cooperation in the form of no resistance. Thus, like the transfer and the coup decision in the sidepayment equilibrium, the absence of resistance in the previous period does not directly affect the democratization decision. Also, the more stable the non-democratic regime is (lower q) and the more likely a coup is to be successful (higher p), the more attractive the MPE becomes and the more tempted the military is to deviate. Therefore, it becomes more difficult for the military to be committed to democratization. This can be seen in the voluntary democratization condition.

Although in general all parameters affect the severity of these two problems differently, the effects of p are very different. This can be understood by considering the military's incentive to deviate. In the equilibrium with reputation, the military compares the frequency of the non-democratic regime in the MPE and that of the equilibrium with reputation, as in the voluntary democratization condition (Condition 1). High p increases the frequency in the MPE, and this is the only way in which p affects the incentive to deviate. On the other hand, p affects the incentive in the sidepayment equilibrium in two ways. First, high p makes the MPE more attractive in the same way that it does in the equilibrium with reputation. Second, high p makes it more likely that the deviation will be successful. A deviation is successful only if it provides a gain before the opponent's punishment kicks in. In the equilibrium with reputation, the deviation necessarily provides a short-term gain because it is done by not democratizing and there is no uncertainty. On the other hand, the deviation in the sidepayment equilibrium results in a short-term gain only if the coup succeeds. Higher p increases the chance of success. Therefore, we can guess that higher p increases the incentive for the military to deviate more in the equilibrium with sidepayment, and this makes it more likely that the equilibrium with reputation will exist rather than the sidepayment equilibrium. Simulation results are consistent with this guess. In the simulations, I randomized the set of parameters, $\{\beta, s, s^*, q, p, c, d, m, m^*, n, \alpha\}$, such that Assumptions 2 and 3 are satisfied. For those parameter sets that satisfy all of the assumptions and the conditions for the existence of the equilibrium with reputation (Assumption 2, Condition 1, and inequality (2.14)), Figure 2.1 plots p and q. For each set, I additionally examine whether it satisfies the conditions that the existence of the sidepayment equilibrium requires. If it does, the shape of the point is a triangle. Otherwise, it is round. The most obvious pattern in this plot is that the upper bound of p is a linear function of q, which reflects the voluntary democratization condition (Condition 1). The second and more important point is that points with high p tend to be round. That is, the sidepayment equilibrium is unlikely to exist with high p given a parameter set that supports the equilibrium with reputation. This finding is consistent with the guess.

These results suggest that the equilibrium with reputation tends to appear when the military is well organized or when citizens are not sufficiently unified to powerfully resist the military. In such an environment, a coup is likely to be successful, which implies that the military demands too expensive sidepayments in return for no coup so that the sidepayment equilibrium does not emerge. Nevertheless, in some cases, the equilibrium with reputation can exist.

Figure 2.1: Simulation Results of Existence of Reputation and Sidepayment Equilibria



2.6 Conclusion

In this paper, I develop a simple model that demonstrates how the reputation of a military can generate different paths of political transitions. When a military has a reputation for voluntarily democratizing, citizens do not resist coups, and they can even tolerate frequent coups. A military with such a reputation has an incentive to maintain it. In the model, this interaction is illustrated by an equilibrium with trigger strategies. In the equilibrium, coups take place often and the non-democratic regime is not persistent because it is always democratized by the military, which replicates the characteristics of the political transition paths of countries like Thailand and Turkey. On the other hand, when there is no such reputation, the citizens resist a coup to avoid a non-democratic regime. Once the military overcomes the resistance and seizes power, it will never voluntarily democratize, and the non-democratic regime becomes persistent. This is illustrated by a Markov perfect equilibrium in the model that corresponds to the paths in countries like Myanmar.

The model also shows why citizens do not buy out the military to prevent transitions to non-democratic regimes. In the model's second history-dependent equilibrium, citizens provide a transfer to the military in every period only if the military has never attempted a coup. This history-dependent strategy can incentivize the military to stop attempting coups. However, when a coup is very likely to be successful, the military has a strong incentive to mount a coup and the citizens cannot afford to provide enough of a transfer to dissuade the military from doing so. Meanwhile, the first history-dependent equilibrium, or the equilibrium with reputation, is relatively robust to a coup that would have a high successful rate because the deviation by the military does not involve a coup. This result suggests that when a coup has a high success rate buying out the military is not effective, and so the equilibrium with reputation tends to appear.

Analyzing political transitions from the perspective of history-dependent equilibria has a great potential, and this work presents a first step towards realizing it. This paper's model is simplified in order to highlight its most important features. For example, the period payoffs are exogenously given and there is no production or redistribution. Introducing additional features, such as endogenous production or redistributions, in future research might increase the sophistication of the model's results. Also, this would generate additional means of cooperation between the two parties and, therefore, additional history-dependent equilibria. While it is not reasonable to investigate every possible equilibrium, it would be worthwhile in future research to map some of those additional history- dependent equilibria onto political transition paths in the real world.

References

Acemoglu, D. and Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier.

Acemoglu, D. and Robinson, J. A. (2000a). Democratization or repression? *European Economic Review*, 44(4-6):683–693.

Acemoglu, D. and Robinson, J. A. (2000b). Why did the west extend the franchise? democracy, inequality, and growth in historical perspective. *The quarterly journal of economics*, 115(4):1167–1199.

Acemoglu, D. and Robinson, J. A. (2001). A theory of political transitions. *American Economic Review*, 91(4):938–963.

Acemoglu, D. and Robinson, J. A. (2006). *Economic origins of dictatorship and democracy.* Cambridge University Press.

Acemoglu, D. and Robinson, J. A. (2008). Persistence of power, elites, and institutions. *American Economic Review*, 98(1):267–93.

Autor, D. H., Dorn, D., and Hanson, G. H. (2013). The china syndrome: Local labor market effects of import competition in the united states. *American Economic Review*, 103(6):2121–68.

Behrens, K. and Robert-Nicoud, F. (2015). Agglomeration theory with heterogeneous agents. *Handbook of regional and urban economics*, 5:171–245.

Boix, C. (2003). *Democracy and redistribution.* Cambridge University Press.

Burchfield, M., Overman, H. G., Puga, D., and Turner, M. A. (2006). Causes of sprawl: A portrait from space. *The Quarterly Journal of Economics*, 121(2):587–633.

Caron, J., Fally, T., and Markusen, J. (2020). Per capita income and the demand for skills. *Journal of International Economics*, 123:103306.

Caron, J., Fally, T., and Markusen, J. R. (2014). International trade puzzles: A solution linking production and preferences. *The Quarterly Journal of Economics*, 129(3):1501–1552.

Carrico, C., Jones, L., and Tsigas, M. E. (2012). Disaggregate us labor statistics for the usage 2.0 and gtap applied general equilibrium models. *Available at SSRN 2169415*.

Ciccone, A. and Hall, R. E. (1996). Productivity and the density of economic activity. *The American Economic Review*, 86(1):54–70.

Combes, P.-P., Duranton, G., and Gobillon, L. (2008). Spatial wage disparities: Sorting matters! *Journal of urban economics*, 63(2):723–742.

Combes, P.-P., Duranton, G., Gobillon, L., and Roux, S. (2010). Estimating agglomeration economies with history, geology, and worker effects. In *Agglomeration economics*, pages 15–66. University of Chicago Press.

Comin, D., Lashkari, D., and Mestieri, M. (2021). Structural change with long-run income and price effects. *Econometrica*, 89(1):311–374.

Davis, D. R. and Dingel, J. I. (2019). A spatial knowledge economy. *American Economic Review*, 109(1):153–70.

Davis, D. R. and Dingel, J. I. (2020). The comparative advantage of cities. *Journal of International Economics*, 123:103291.

Dingel, J. I. (2017). The determinants of quality specialization. *The Review of Economic Studies*, 84(4):1551–1582.

Duranton, G. and Puga, D. (2005). From sectoral to functional urban specialisation. *Journal of urban Economics*, 57(2):343–370.

Eckert, F. (2019). Growing apart: Tradable services and the fragmentation of the us economy. *mimeograph, Yale University*.

Eckert, F., Fort, T. C., Schott, P. K., and Yang, N. J. (2021). Imputing missing values in the us census bureau's county business patterns. Technical report, National Bureau of Economic Research.

Fajgelbaum, P., Grossman, G. M., and Helpman, E. (2011). Income distribution, product quality, and international trade. *Journal of political Economy*, 119(4):721–765.

Flam, H. and Helpman, E. (1987). Vertical product differentiation and north-south trade. *The American Economic Review*, pages 810–822.

Glaeser, E. L. and Gottlieb, J. D. (2009). The wealth of cities: Agglomeration economies and spatial equilibrium in the united states. *Journal of Economic Literature*, 47(4):983–1028.

Glaeser, E. L. and Mare, D. C. (2001). Cities and skills. *Journal of labor economics*, 19(2):316–342.

Handbury, J. (2019). Are poor cities cheap for everyone? non-homotheticity and the cost of living across us cities. Technical report, National Bureau of Economic Research.

Harris, C. D. (1954). The, market as a factor in the localization of industry in the united states. *Annals of the Association of American Geographers*, 44(4):315–348.

Head, K. and Mayer, T. (2004). The empirics of agglomeration and trade. In *Handbook of regional and urban economics*, volume 4, pages 2609–2669. Elsevier.

Head, K., Mayer, T., and Ries, J. (2009). How remote is the offshoring threat? *European Economic Review*, 53(4):429–444.

Helpman, E. (1998). The size of regions. In D. Pines, E. Sadka, I. Z., editor, *Topics in Public Economics. Theoretical and Empirical Analysis*, pages 33–54. Cambridge University Press.

Henderson, J. V. and Ono, Y. (2008). Where do manufacturing firms locate their headquarters? *Journal of Urban Economics*, 63(2):431–450.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review*, 70(5):950–959.

Krugman, P. (1991). Increasing returns and economic geography. *Journal of political economy*, 99(3):483–499.

Matsuyama, K. (2019). Engel's law in the global economy: Demand-induced patterns of structural change, innovation, and trade. *Econometrica*, 87(2):497–528.

Nunn, N. (2007). Relationship-specificity, incomplete contracts, and the pattern of trade. *The Quarterly Journal of Economics*, 122(2):569–600.

Roback, J. (1982). Wages, rents, and the quality of life. *Journal of Political Economy*, 90(6):1257–1278.

Rosen, S. (1979). *Wage-Based Indexes of Urban Quality of Life*. Baltimore: Johns Hopkins University Press.

Saiz, A. (2010). The geographic determinants of housing supply. *The Quarterly Journal of Economics*, 125(3):1253–1296.

Sanderson, E. and Windmeijer, F. (2016). A weak instrument f-test in linear iv models with multiple endogenous variables. *Journal of econometrics*, 190(2):212–221.

Schroeder, J. P. (2016). Historical population estimates for 2010 u.s. states, counties and metro/micro areas, 1790-2010. *Retrieved from the Data Repository for the University of Minnesota*.

Silva, J. S. and Tenreyro, S. (2006). The log of gravity. *The Review of Economics and statistics*, 88(4):641–658.

Stock, J. H., Yogo, M., et al. (2005). Testing for weak instruments in linear iv regression. *Identification and inference for econometric models: Essays in honor of Thomas Rothenberg*, 80(4.2):1.

Stokey, N. L. (1991). The volume and composition of trade between rich and poor countries. *The Review of Economic Studies*, 58(1):63–80.

Tabuchi, T. and Thisse, J.-F. (2002). Taste heterogeneity, labor mobility and economic geography. *Journal of Development Economics*, 69(1):155–177.

Appendix A

Appendix to Cities' Demand-driven Industrial Composition

A.1 Data and Robustness Check of Stylized Fact

In this appendix, I first describe the data that are the basis of Figure 1.1. Second, I address a possible omitted variable bias in the positive relationship and provide another set of empirical results to show that the the same production pattern can be found with an alternative identification. Finally, I show that the demand side mechanism has considerable explanatory power.

Data

To create Figure 1.1, I borrow estimates of income elasticities from Caron et al. (2020). Using 2007 international trade data for 109 countries, these scholars estimate the elasticities for 49 sectors. The elasticity varies from 0.137 for "Processed rice" to 1.311 for "Financial services nec". For employment data, I use datasets from Country Business Patterns (CBP). CBP

Table A.1: Distribution of Income and Population across MSAs in 2006

	Min	Q1	Q2	Q3	Max
Income	$18,720	$30,190	$33,383	$37,878	$83,059
Population	98,695	166,923	352,204	782,757	19,200,372

provides employment data for sectors classified annually according to the North American Industry Classification System (NAICS) in metropolitan areas. The classification in Caron et al. (2020), which is different from NAICS, is called GTAP. In most cases, one GTAP code corresponds to multiple 3-digit or 4-digit NAICS codes. Following Carrico et al. (2012), and mapping NAICS data to GTAP, I create employment data by GTAP. For income level of MSAs, I use per capita personal income available in the Bureau of Economic Analysis. Table A.1 displays the distribution of income levels and population in 2006. The largest MSA in 2006 in this sample is New York-Newark-Jersey City, (NY-NJ-PA), which had a population of 19,200,372, while the smallest is Ocean City, NJ, which had a population of 98,695. Income level varies substantially across MSAs: $18,720 in McAllen-Edinburg-Mission, TX, is the lowest and $83,059 in Bridgeport-Stamford-Norwalk, CT, is the highest.

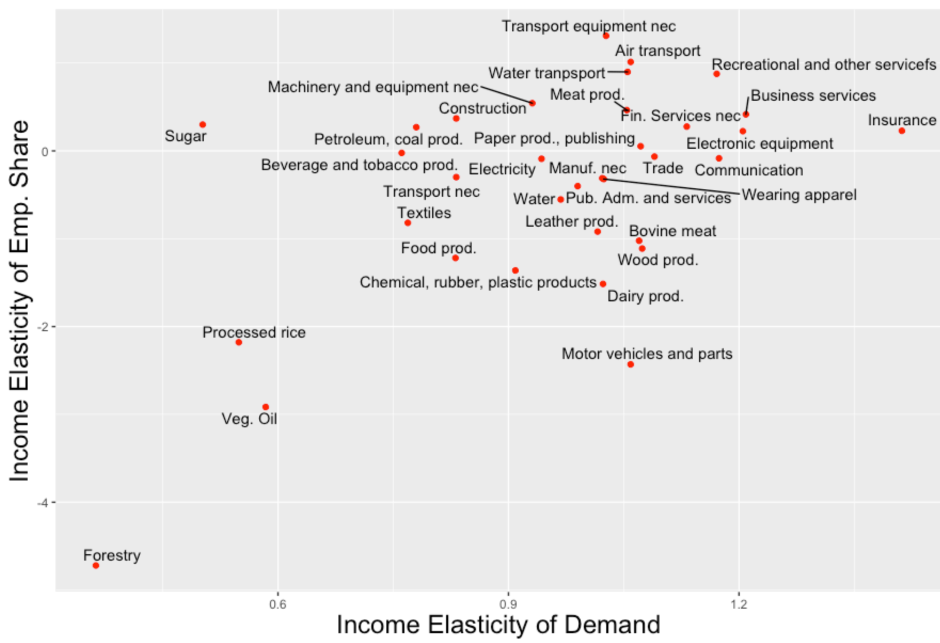
Controlling Skill Supply

I address the omitted variable concern for the positive relationship in Figure 1.1. Figure A.1 shows that the positive relationship is robust to this concern. The college graduate share in the labor force in the MSA[1] is controlled when the elasticities of employment share are obtained. With these revised elasticities in the y-axis, it can be seen that the positive relationship becomes, if anything, more clear. The slope of the regression line in Figure A.1 implies, for example, that the employment ratio of a sector with an income elasticity 1.2 over that with of 0.8 becomes 1.72 times higher in a MSA in which per capita personal income is

[1]Data are from Census via IPUMS.

one standard deviation above the average compared to a sector in which per capita personal income is one standard deviation below the average. In other words, this relationship has a significant impact on cities' industrial composition.

Figure A.1: Elasticity of Employment Share with respect to MSA's Income Level Conditioned on Skill Supply and Elasticity of Demand with respect to. Income



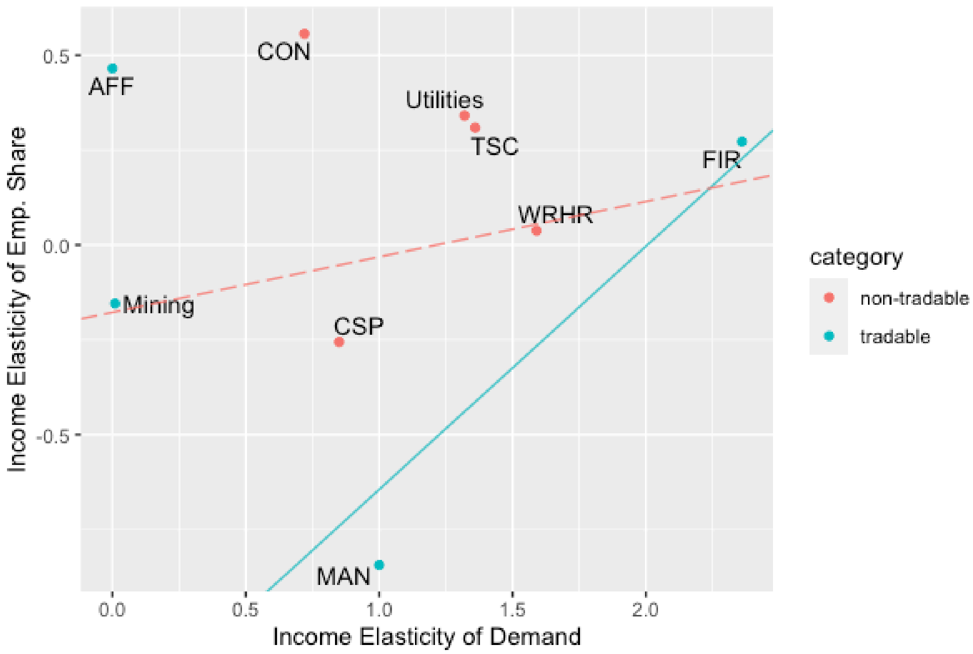
The income of a MSA is per capita personal income in 2006; from the Bureau of Economic Analysis. Employment shares are calculated from 2006 CBP data. When obtaining the income elasticity of employment share, the college graduate share in the labor force and the region $\in \{Northeast, Midwest, South, West\}$ are controlled. Income elasticity estimates are fromCaron et al. (2020).

Including Non-tradable Sectors

The previous exercises use income elasticity estimates obtained from international trade data. Therefore, the scope of these estimates is limited to tradable sectors. To determine the degree to which the model's prediction with non-tradable sectors is consistent with the actual production pattern, I use the income elasticity estimates (which include non-tradables) calculated by Comin et al. (2021) for 9 sectors: (1) agriculture, forestry and fishing; (2) mining and quarrying; (3) manufacturing; (4) public utilities; (5) construction; (6) wholesale and retail trade, hotels and restaurants; (7) transport, storage and communication; (8) finance, insurance, and real estate; and (9) community, social and personal services. The procedure is largely the same as before except for two differences. First, because the Comin et al. (2021) sector classification is based on Groningen's 10-Sector Database , I follow the US Census concordance from 2002 NAICS to ISIC Rev3.1., and so I map the MSA's sectoral employment data in 4 digit NAICS to the 9 sectors [2]. Second, the employment shares are defined within the non-tradable sectors and the tradable sectors following the model's implication: (1) agriculture, (2) mining, (3) manufacturing, and (8) finance are classified as tradable and the rest are classified as as non-tradable. The result is shown in Figure A.2. The dashed (red) line is the weighted regression line of the elasticity of the within-employment share of the non-tradable sectors on the income elasticities, with the U.S. aggregate sectoral employment as the weights. The solid (blue) line is that of the tradable sectors. Although the small sample size should not be ignored, it is noteworthy that this graph is consistent with the model predictions in two respects. First, whether or not the sectors are tradable, Figure A.2 shows the same positive relationship as in Figure 1.1, which the model successfully generates. Second, the positive relationship is stronger for tradable sectors, which is consistent with the model's prediction that the home market effect amplifies the expenditure share difference.

[2] When one NAICS code corresponds to multiple codes in ISIC, I allocate the value evenly.

Figure A.2: Elasticity of the Within-Employment Share with respect to MSA's Income Level Conditioned on Skill Supply and Elasticity of Demand with respect to Income with Comin et al. (2021) estimates.



Income is per capita personal income in 2006 from the Bureau of Economic Analysis. The within-employment share is derived as the share of employment of a sector in the corresponding group (tradable or non-tradable) in each 2006 MSA in CBP. When obtaining the income elasticity of the within employment share, the college graduate share in the labor force and the region $\in \{Northeast, Midwest, South, West\}$ are controlled. Income elasticity estimates are from Comin et al. (2021) and are for OECD countries.

Alternative Specification

I test the association of the income level and the income elasticities of the local industries with alternative identification. This test follows Nunn (2007), and the regression model is given by

$$
\begin{aligned}
y_{mk} = &\alpha \cdot \exp(\beta \cdot \epsilon_k \cdot \log(Income_m) + \delta \cdot \theta_k \cdot \log(College_m) \\
&+ \gamma_m D_m + \gamma_k D_{k,region})) \cdot e_{mk}
\end{aligned} \tag{A.1}
$$

where

$$
\begin{aligned}
y_{mk} &: \text{employment level of industry } k \text{ in MSA } m \\
\epsilon_k &: \text{income elasticity of demand in industry } k \\
Income_m &: \text{per capita personal income in MSA } m \\
\theta_k &: \text{skill intensity of goods in industry} k \\
College_m &: \text{college graduate share in labor force in MSA } m \\
D_m &: \text{MSA dummy variable} \\
D_{k,region} &: (\text{industry} \times \text{region} \in \{Northeast, Midwest, South, West\}) \text{ dummy variable} \\
e_{mk} &: \text{error term for MSA } m \times \text{ industry } k
\end{aligned}
$$

The coefficient of interest is β. When β is positive, the employment level rises more for high ϵ_k as the income level rises, which is consistent with the stylized fact and the model prediction. Two points are noteworthy. First, as the model shows, I implement level regressions by the Poisson Pseudo Maximum Likelihood (PPML) estimation. As discussed in Silva and Tenreyro (2006), log-linear regressions require a very specific condition on error terms to obtain consistent estimators. Moreover, in log-linear estimations, it is problematic when zeros are contained in the data. On the other hand, PPML provides consistent estimators that do not require this condition, and it is efficient with various error term patterns. For this reason, PPML is very common in gravity equation estimations in international trade where zeros are prevalent and error terms show heteroskedasiticy. In my dataset, 17% of the sample is zero. To address these zeros, I use PPML, and I set employment levels (instead of employment shares) as the dependent variable so that the error terms show a heteroskedasticity pattern that is suitable for PPML in terms of efficiency. Second, I control the supply side effect using the skill-intensity of sectors, as in the main empirical work. This time, the interaction term, $\theta_k \cdot \log(College_m)$, is used to implement the control, and the estimates of skill intensities are again borrowed from Caron et al. (2020), who obtained them through a structural estimation that had both heterogeneous skill-intensities and income elasticities.

Because this regression is cross-sectional, I implement it for three different years separately to make use of the annual data. The results are shown in Figure A.3. The results with PPML are shown in the left three columns, while those in the right three columns show log-linear regressions after 1 is added to every observation to take care of zeros. In all of the results, β is significantly positive, which is consistent with the stylized fact and the model prediction. The size of β is between 1.9 and 2.3 in PPML. For example, to see the impact of this β using 2006 data, suppose, first, that City A has a per capita income that is higher than the sample mean ($34,765) by one standard deviation ($7,381) and, second, that City B has a per capita income that is lower than the sample mean by one standard deviation. Then, the employment ratio of a sector with $\epsilon_k = 1.2$ over that with $\epsilon_k = 0.8$ ($\frac{N_{\epsilon_k=1.2}}{N_{\epsilon_k=0.8}}$) is 1.48 times greater in City A than in City B.

It is not straightforward to measure the explanatory power of the PPML estimators because technically they are obtained by a maximum likelihood estimation. To construct a measurement for the PPML estimators, I calculate the estimated employment shares implied by the fitted values for the employment levels and obtain the residual sum of squares (RSS) from the difference between the estimated shares and the actual data. Using this RSS, a

Figure A.3: Regression Result of Alternative Specification

	employment			log(employment+1)		
	2006	2011	2016	2006	2011	2016
	(1)	(2)	(3)	(4)	(5)	(6)
β	2.257***	2.263***	1.928***	2.391***	2.420***	1.724***
	(0.221)	(0.215)	(0.211)	(0.549)	(0.486)	(0.430)
δ	2.505***	3.258***	3.367***	2.889***	3.262***	3.750***
	(0.234)	(0.227)	(0.250)	(0.667)	(0.633)	(0.663)
Observations	8,636	8,670	8,228	8,636	8,670	8,228
R^2				0.841	0.842	0.843
Adjusted R^2				0.833	0.835	0.835

Notes: ***Significant at the 1 percent level.
In the log regressions, the standard errors are clustered by MSA.

measurement which analogous to R^2 in linear regressions is constructed and summarized in Table A.2. In the first row, RSS_{FE} is the RSS when the regression has only the FE effects of MSA and industry × region and TSS is the residual sum of squares by the unconditional mean of the employment shares. Table A.2 shows that the FE effects explain 93% of TSS in all of the three years. In the second row, $RSS_{FE+skill}$ is obtained by the regression that has the skill interaction term as an additional control. Given an industry and a region, the skill supply effect explains 9%-12% of the variation. Similarly, in the third row, $RSS_{FE+income}$ is obtained by the regression, here with the income interaction term instead of the skill interaction, and the income effect explains 12%-13% of the variation. Finally, in the last row, $RSS_{FE+skill+income}$ is by the regression that has the same controls as (A.1), and the skill and income effects jointly capture 18%-22% of the variation. These results suggest that, given an industry and a region, the income effect can explain a significant portion of the

Table A.2: Explanatory Power

	2006	2011	2016
$1-\frac{RSS_{FE}}{TSS}$	0.93	0.93	0.93
$1-\frac{RSS_{FE+skill}}{RSS_{FE}}$	0.10	0.12	0.09
$1-\frac{RSS_{FE+income}}{RSS_{FE}}$	0.12	0.13	0.13
$1-\frac{RSS_{FE+skill+income}}{RSS_{FE}}$	0.18	0.22	0.19

variation in the employment share, even after controlling the skill supply. Moreover, the explanatory power is not smaller than that of the skill effect.

A.2 Derivation of Demand Function and Indirect Utility

Derivation of Demand Function

The FOCs are

$$\begin{aligned}
U :& 1+\mu\frac{\eta}{\eta-1}\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \left[\sum_k \frac{\epsilon_k}{\eta}\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}-1} Q_k^{\frac{\eta-1}{\eta}}\right] = 0 \\
Q_k :& \mu\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \left[\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{-1}{\eta}}\right] = \xi \\
q(\nu) :& \xi\left[\int_{\Omega_k} q(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{1}{\sigma-1}} q^{-\frac{1}{\sigma}} = \zeta p(\nu)
\end{aligned}$$

where μ, ξ, and ζ are lagrange multipliers. Note $\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}$ can be removed because it is 1.

First, derive the usual result with CES.

$$P_k Q_k^{\frac{1}{\sigma}} = p(\nu) q(\nu)^{\frac{1}{\sigma}}$$

With this result, the FOC w.r.t Q_k, and that w.r.t $q(\nu)$,

$$\mu \left[\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}} \right] = \zeta P_k Q_k$$
$$\implies \frac{\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} = \frac{P_k Q_k}{P_l Q_l} \tag{A.2}$$

Next, using (A.2),

$$\begin{aligned} E &= \sum_k P_l Q_l \frac{\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \\ &= \frac{P_l Q_l}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}} \\ &= \frac{P_l Q_l}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \end{aligned}$$
$$\implies Q_l = E^\eta \beta_l U^{\epsilon_l} P_l^{-\eta} \tag{A.3}$$

$$E^\eta \beta_l U^{\epsilon_l} P_l^{1-\eta}$$

Derivation of Indirect Utility

I can implicitly express the indirect utility by plugging (A.3) into the definition of C,

$$\begin{aligned} 1 = & \left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} \left[E^\eta \beta_k U^{\epsilon_k} P_k^{-\eta}\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ = & E^\eta \left[\sum_k \beta_k U^{\epsilon_k} P_k^{1-\eta}\right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

A.3 Proof of Propositions and Lemma

Proof of Proposition 1

I begin by substituting the optimized production into the zero-profit condition. As for the optimized production, the optimized prices are as follows.

$$\forall k \in K,\ p_{cck} = p_{ck} = \frac{\sigma}{\sigma - 1}\psi_k w_c$$
$$\forall k \in \mathbb{T},\ p_{cc'k} = \tau p_{ck} = \frac{\sigma}{\sigma - 1}\tau\psi_k w_c$$

Then, the zero-profit condition implies $\pi_{ck} = 0$ for all k in K and $c \in \{1, 2\}$. It follows for all (c, c') in $\{(1, 2), (2, 1)\}$,

$$\forall k \in \mathbb{N},\ q_{ck}\psi_k w_c \left[\frac{1}{\sigma - 1}\right] - \phi_k w_c = 0$$
$$\forall k \in \mathbb{T},\ \left(q_{ck} + q_{cc'k}\tau\right)\psi_k w_c \left[\frac{1}{\sigma - 1}\right] - \phi_k w_c = 0$$

The total labor demand by a firm in sector k in city c, L_{ck}, is pinned down as

$$L_{ck} = \begin{cases} q_{ck}\psi_k + \phi_k = \sigma\phi_k & k \in \mathbb{N} \\ \\ \left(q_{ck} + q_{cc'k}\tau\right)\psi_k + \phi_k = \sigma\phi_k & k \in \mathbb{T} \end{cases} \tag{A.4}$$

The labor demand is determined solely by the fixed cost. Now, I use a normalization. It can be shown that β_k, ϕ_k and ψ_k affect the equilibrium values of $N_1, N_2, w_1, w_2, U_1, U_2$ only through $\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k$. Therefore, given any set of parameters, replacing $\{\beta_k, \phi_k, \psi_k\}_{k\in K}$ by $\left\{\tilde{\beta}_k, \frac{1}{\sigma}, \frac{\sigma-1}{\sigma}\right\}_{k\in K}$ where $\tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}}(\frac{\sigma-1}{\sigma})}\right)^{1-\eta}$ does not affect the equilibrium values of those variables. One caveat is that the price index is affected by this change. Let $\tilde{P}_k$ be the new price index given $\left\{\tilde{\beta}_k, \frac{1}{\sigma}, \frac{\sigma-1}{\sigma}\right\}_{k\in K}$. Then, $P_k = (\frac{1}{\sigma})^{\frac{1}{\sigma-1}}(\frac{\sigma-1}{\sigma})\tilde{P}_k$. Following Matsuyama (2019), I set $\psi_k = \frac{\sigma-1}{\sigma}$ and $\phi_k = \frac{1}{\sigma}$ so that $p_{ck} = w_c$ for all $k \in K$ and $L_{ck} = 1$, which requires that β_k is replaced by $\tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}}(\frac{\sigma-1}{\sigma})}\right)^{1-\eta}$. Then, it follows from equation (A.4) and the normalization that the aggregate supply of goods by a firm is given by

$$\forall k \in \mathbb{N},\ q_{ck} = 1$$

$$\forall k \in \mathbb{T},\ q_{ck} + q_{cc'k}\tau = 1$$

Next, to equate demand to supply, I derive the aggregate demand for a variety in sector k in city c. I let D_{ck} denote the aggregate demand, and it is as follows:

$$D_{ck} = p_{ck}^{-\sigma}A_{ck}$$

where

$$A_{ck} = N_c \tilde{P}_{ck}^{\sigma} Q_{ck} + \rho_k N_{c'} \tilde{P}_{c'k}^{\sigma} Q_{c'k}$$

$$\rho_k = \begin{cases} 0 & k \in \mathbb{N} \\ \rho = \tau^{1-\sigma} & k \in \mathbb{T} \end{cases}$$

Equating demand (D_{ck}) and supply (q_{ck} for $k \in \mathbb{N}$ and $q_{ck} + q_{cc'k}\tau$ for $k \in \mathbb{T}$) with $p_{ck} = w_c$ gives, $\forall c \in \{1, 2\}, \forall k \in K$

$$1 = w_c^{-\sigma} A_{ck} \tag{A.5}$$

This equates supply and demand, and it reflects the zero-profit condition. This illustrates that, given a sector, the city with greater aggregate demand has a higher wage and, given a city, this equates the aggregate demands across sectors because the wage is common. To make use of equation (A.5), I use two different expressions of the demand function to substitute for the Q_{ck} that is contained in A_{ck}.

$$Q_{ck} = \begin{cases} \tilde{P}_{ck}^{-1} E_c m_{ck} \\ \tilde{\beta}_k \tilde{P}_{ck}^{-\eta} E_c^{\eta} U_c^{\epsilon_k} \end{cases} \tag{A.6}$$

The first follows from equation (1.3) and the second from equation (1.2). With the first expression, equation (A.5) becomes

$$w_c^{\sigma} = N_c \tilde{P}_{ck}^{\sigma-1} E_c m_{ck} + \rho_k N_{c'} \tilde{P}_{c'k}^{\sigma-1} E_{c'} m_{c'k} \tag{A.7}$$

Proposition 1 follows from (A.7) for a non-tradable sector $k \in \mathbb{N}$

$$\begin{aligned} w_1^\sigma &= \tilde{P}_{1k}^{\sigma-1} N_1 E_1 m_{1k} \\ &= \frac{N_1 E_1 m_{1k}}{x_{1k} \lambda N_1 w_1^{1-\sigma}} \\ \implies x_{1k} &= m_{1k} \end{aligned} \tag{A.8}$$

where I use the price index $\tilde{P}_{1k}^{1-\sigma} = x_{1k} \lambda N_1 (w_1)^{1-\sigma}$ for $k \in \mathbb{N}$ and x_{ck} is the employment share in sector k in city c

Proof of Corollary (1)

This follows from (A.8) as follows:

$$\begin{aligned} \sum_{k \in \mathbb{N}} x_{1k} &= \sum_{k \in \mathbb{N}} m_{1k} \\ \implies 1 - \sum_{k \in \mathbb{T}} x_{1k} &= 1 - \sum_{k \in \mathbb{T}} m_{1k} \\ \implies \sum_{k \in \mathbb{T}} x_{1k} &= \sum_{k \in \mathbb{T}} m_{1k} \end{aligned} \tag{A.9}$$

Proof of Proposition 2

This follows from zero-profit conditions for a tradable sector $k \in \mathbb{T}$ ((A.7)) for the two cities that

$$\frac{w_1^\sigma - \rho w_2^\sigma}{1 - \rho^2} = \tilde{P}_{1k}^{\sigma-1} N_1 E_1 m_{1k} \tag{A.10}$$

$$= \frac{N_1 \lambda w_1 m_{1k}}{x_{1k} \lambda N_1 w_1^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma}}$$

where I use the price index $\tilde{P}_{1k}^{1-\sigma} = x_{1k}\lambda N_1 (w_1)^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma}$ and x_{ck} is the employment share in sector k in city c. It follows that

$$x_{1k}\lambda N_1 w_1^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma} = (1-\rho^2)\frac{N_1 \lambda w_1 m_{1k}}{w_1^{\sigma} - \rho w_2^{\sigma}} \tag{A.11}$$

Aggregate over tradable sectors and use the income ratio $\omega = \frac{E_1}{E_2} = \frac{\lambda w_1}{w_2}$,

$$\sum_{k\in\mathbb{T}} x_{1k}\lambda^{\sigma} N_1 + \rho \sum_{k\in\mathbb{T}} x_{2k} N_2 \omega^{1-\sigma} = (1-\rho^2)\frac{N_1 \sum_{k\in\mathbb{T}} m_{1k}}{\lambda^{-\sigma} - \rho\omega^{-\sigma}}$$

Transforming this making use of(A.9), the following city size ratio is obtained.

$$\frac{N_1}{N_2} = \frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}} \omega^{2\sigma-1} \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^{\sigma} - \rho\omega^{\sigma}}\right]$$

Proof of Lemma 1

I use the second form of the demand function (A.6). (A.5) becomes

$$w_c^{\sigma} = \tilde{\beta}_k \tilde{P}_{ck}^{\sigma-\eta} N_c E_c^{\eta} U_c^{\epsilon_k} + \rho_k \tilde{\beta}_k \tilde{P}_{c'k}^{\sigma-\eta} N_{c'} E_{c'}^{\eta} U_{c'}^{\epsilon_k} \tag{A.12}$$

It follows from A.12 of sector $k \in \mathbb{T} \cup \mathbb{N}$ for the two cities ((A.12)),

$$\frac{w_1^{\sigma} - \rho_k w_2^{\sigma}}{1-\rho_k^2} = \tilde{\beta}_k \tilde{P}_{1k}^{\sigma-\eta} N_1 E_1^{\eta} U_1 \tag{A.13}$$

I eliminate $\tilde{P}_{nk}$ using (1.3). From (1.3) and (1.4),

$$\begin{aligned}\tilde{P}_{1k} &= \left[\frac{m_{1k}\left(\sum_l \tilde{\beta}_l U_1 \tilde{P}_{1l}^{1-\eta}\right)}{\tilde{\beta}_k U_1}\right]^{\frac{1}{1-\eta}} \\ &= \left[\frac{m_{1k}E_1^{1-\eta}}{\tilde{\beta}_k U_1}\right]^{\frac{1}{1-\eta}}\end{aligned}$$

Plug this into (A.13),

$$\begin{aligned}\frac{w_1^\sigma - \rho_k w_2^\sigma}{1-\rho_k^2} &= \tilde{\beta}_k \left[\frac{m_{1k}E_1^{1-\eta}}{\tilde{\beta}_k U_1}\right]^{\frac{\sigma-\eta}{1-\eta}} N_1 E_1^\eta U_1 \\ m_{1k}^{\frac{\sigma-\eta}{1-\eta}} &= \frac{w_1^\sigma - \rho_k w_2^\sigma}{1-\rho_k^2}\frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} U_1}{N_1 (\lambda w_1)^\sigma} \\ &= \frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{1-\rho_k^2}\frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} U_1}{N_1} \\ m_{1k} &= \left[\frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{(1-\rho_k^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}\end{aligned}$$

Proof of Proposition 8

For a tradable sector in city 1, from (A.11),

$$x_{1k}\lambda^\sigma N_1 + \rho x_{2k} N_2 \omega^{\sigma-1} = (1-\rho^2)\frac{N_1 m_{1k}}{\lambda^{-\sigma} - \rho\omega^{-\sigma}} \quad \text{(A.14)}$$

The counterpart of this with m_{2k} can be obtained as

$$\rho x_{1k}\lambda^\sigma N_1 + x_{2k} N_2 \omega^{\sigma-1} = (1-\rho^2)\frac{N_2 m_{2k}}{\omega^{-\sigma} - \rho\lambda^{-\sigma}}\omega^{-1} \quad \text{(A.15)}$$

Solve (A.14) and (A.15) for x_{1k} and x_{2k} using 1.8.

$$\frac{x_{1k}}{\sum_{k\in\mathbb{T}} x_{1k}} = \frac{\frac{m_{1k}}{\sum_{k\in\mathbb{T}} m_{1k}} - \rho\lambda^{\sigma}\omega^{-\sigma}\frac{m_{2k}}{\sum_{k\in\mathbb{T}} m_{2k}}}{1-\rho\lambda^{\sigma}\omega^{-\sigma}}$$
$$\frac{x_{2k}}{\sum_{k\in\mathbb{T}} x_{2k}} = \frac{\frac{m_{2k}}{\sum_{k\in\mathbb{T}} m_{2k}} - \rho\omega^{\sigma}\lambda^{-\sigma}\frac{m_{1k}}{\sum_{k\in\mathbb{T}} m_{1k}}}{1-\rho\omega^{\sigma}\lambda^{-\sigma}}$$

The employment share ratio immediately follows from this.

Proof of Proposition 4 and Proposition 5

I introduce new variables, V_1 and V_2, defined as

$$V_1 = a_1 U_1 N_1^{-\gamma}$$
$$V_2 = a_2 U_2 N_2^{-\gamma}$$

Using them, the equilibrium conditions can be rewritten as follows:

$$N_1^{\frac{1-\eta}{\sigma-\eta}} = \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} \tag{A.16}$$

$$N_2^{\frac{1-\eta}{\sigma-\eta}} = \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} \tag{A.17}$$

$$\left(\frac{N_2}{N_1}\right)^{\frac{\sigma-1}{\sigma-\eta}} = \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \tag{A.18}$$

$$V_1 = V_2$$
$$N = N_1 + N_2$$

I prove the existence and uniqueness of the equilibrium by showing that V_1 and V_2 can be expressed as monotone continuous functions of N_1 and that they have a unique intersection by the intermediate value theorem.

(i) V_1 and V_2 can be expressed as functions of N_1 First, I show that V_1 decreases in N_1 and that V_2 increases in N_1. It follows from(A.16) and (A.17) that

$$\frac{\partial V_1(N_1,\omega)}{\partial\omega} < 0, \frac{\partial V_2(N_2,\omega)}{\partial\omega} > 0$$

Then, notice that, given N_1, N_2, the RHS of (A.18) decreases in ω, taking into account $\frac{\partial V_1(\omega,N_1)}{\partial\omega}$ and $\frac{\partial V_2(\omega,N_2)}{\partial\omega}$. Also, given N_1, N_2,

$$\lim_{\omega\to\rho^{\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \to +\infty \tag{A.19}$$

$$\lim_{\omega\to\rho^{-\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \to 0 \tag{A.20}$$

Therefore, given N_1, N_2, it uniquely pins down ω_1 and, consequently, V_1 and V_2 from (A.16) and (A.17), respectively. With the workers clearing condition, V_1 and V_2 are functions of N_1.

(ii) ω increases in N_1 Suppose ω weakly decreases in N_1. Then, $V_1N_1^{\gamma}$ must strictly increase and $V_2N_2^{\gamma}$ strictly decreases in N_1 to satisfy (A.16) and (A.17), respectively. Then, ω needs to strictly increase in N_1 to satisfy (A.18). This contradicts that ω weakly decreases in N_1. Therefore, ω strictly increases in N_1. Assuming the differentiability,

$$\frac{d\omega}{dN_1} > 0$$

(iii) V_1 and V_2 are continuous in N_1 Given N_1 (and therefore N_2), (A.16) and (A.17) imply that $V_1 N_1^{\gamma}$ and $V_2 N_2^{\gamma}$ are continuous in ω on $(\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}})$, decrease and increase in ω, respectively, and are nonzero. Then, given N_1, the RHS of (A.18) is a continuous decreasing function of ω. Combined with (A.19) and (A.20), it follows that for all $N_1 \in (0, N)$ there exists $\omega \in (\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}}\lambda)$. Since ω increases in N_1, this implies ω is continuous on $N_1 \in (0, N)$. It immediately follows from (A.16) and (A.17) that V_1 and V_2 are continuous in $N_1 \in (0, N)$.

(iv) V_1 decreases and V_2 increases in N_1 Transform (A.16) and (A.17),

$$\begin{aligned} 1 = {} & \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\ & + \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \end{aligned} \tag{A.21}$$

$$\begin{aligned} 1 = {} & \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\ & + \left[\frac{1 - \rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \end{aligned} \tag{A.22}$$

Given Assumption 1, it follows that

$$\frac{\partial V_1(N_1,\omega)}{\partial N_1} < 0, \ \frac{\partial V_2(N_2,\omega)}{\partial N_2} < 0$$

Combining the results so far, the signs of the total derivatives can be obtained.

$$\frac{dV_1(N_1,\omega)}{dN_1} = \frac{\partial V_1(N_1,\omega)}{\partial N_1} + \frac{\partial V_1(N_1,\omega)}{\partial \omega_1}\frac{d\omega}{dN_1} < 0$$
$$\frac{dV_2(N_2,\omega)}{dN_1} = \frac{\partial V_2(N_2,\omega)}{\partial N_2}\frac{dN_2}{dN_1} + \frac{\partial V_2(N_2,\omega)}{\partial \omega}\frac{d\omega}{dN_1} > 0$$

(iv) $V_1 > V_2$ when $N_1 \to 0$ and $V_1 < V_2$ when $N_1 \to N$ and unique intersection When $N_1 \to 0$, $\omega \to \rho^{\frac{1}{\sigma}}\lambda$. Suppose $V_1 \not\to \infty$. This contradicts (A.21). Therefore, $V_1 \to \infty$ $(N_1 \to 0)$. Meanwhile, V_2 is bounded following from (A.22). The same can be done when $N_1 \to 0$ or $N_2 \to N$. Thus,

$$\lim_{N_1 \to 0} V_1 > \lim_{N_1 \to 0} V_2$$
$$\lim_{N_1 \to N} V_1 < \lim_{N_1 \to N} V_2$$

Since V_1 and V_2 are a continuous decreasing function and a continuous increasing function of N_1, respectively, there is a unique intersection at $N_1 \in (0, N)$ by the intermediate value theorem. In addition, this shows that it is unstable for all workers to live in one location because after marginal migration, the utility of the small new city is greater than that of the large original city, and additional migration follows.

Proof of Proposition of 6

First, I prove that the productive city becomes larger by showing that the intersection of V_1 and V_2 in the proof of 4 is located in $N_1 \in (\frac{N}{2}, N)$. When $N_1 = N_2 = \frac{N}{2}$, suppose $V_1 \leq V_2$. Then, (A.16) and (A.17) implies that it is necessary that

$$\omega_{|\frac{N}{2}} > \lambda$$

Substitute $\frac{1-\rho\lambda^{-\sigma}\omega^{-\sigma}}{\lambda^{-\sigma}-\rho\omega^{\sigma}}$ in (A.18) with (A.16) and (A.17).

$$\begin{aligned}
\left(\frac{N_2}{N_1}\right)^{\frac{\sigma-1}{\sigma-\eta}} &= \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \\
&= \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\lambda\omega^{-1}\right)^{2\sigma-1} \lambda^{1-\sigma} \\
&\cdot \left[\frac{1-\left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\frac{N_2}{N_1}\right)^{\frac{1-\eta}{\sigma-\eta}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&\cdot \left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{1-\left[\frac{\lambda^{-\sigma}}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\frac{N_2}{N_1}\right)^{\frac{1-\eta}{\sigma-\eta}}\right]^{\frac{\sigma-1}{1-\eta}}
\end{aligned}$$

Evaluate this equation at $N_1 = N_2 = \frac{N}{2}$.

$$
\begin{aligned}
1 =& \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\lambda \omega_{|\frac{N}{2}}^{-1}\right)^{2\sigma-1} \\
&\cdot \lambda^{1-\sigma} \left[\frac{1 - \left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&\cdot \left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{1 - \left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&< \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \\
&\cdot \left[\frac{1 - \left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{1 - \left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&= \left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}+1} \left[\frac{1 - \left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{1 - \left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 \left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}}
\end{aligned}
$$

where the inequality follows from $\lambda < \omega_{|\frac{N}{2}}$. When $V_1 \leq V_2$, the value of the RHS is always smaller than 1. This contradicts the inequality. Therefore, $V_1 > V_2$ at $N_1 = \frac{N}{2}$, which implies that the unique intersection is located in $N_1 \in (\frac{N}{2}, N)$ because V_1 decreases and V_2 increases in N_1.

As for the wage level, think about the equilibrium conditions when $N_1 = N_2$. Suppose

$\omega \leq \lambda$.

$$1 \leq \sum_{k\in\mathbb{N}} \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \left[\frac{1}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1 + \sum_{k\in\mathbb{T}} \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \left[\frac{1}{(1+\rho)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1$$

$$1 \geq \sum_{k\in\mathbb{N}} \left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} + \sum_{k\in\mathbb{T}} \left[\frac{1}{(1+\rho)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}$$

$$\begin{aligned} 1 &\geq \frac{\lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \left[\frac{1-\rho}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\left[\frac{1-\rho}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda \\ &= \frac{\lambda^{\frac{\sigma-1}{\sigma-\eta}\eta} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \end{aligned}$$

Since the first and the second imply $U_1 > U_2$, the RHS of the third is greater than 1. This contradicts the inequality in the third. Therefore, $\omega(N_1 = N_2 = \frac{N}{2}) > \lambda$. Because $N_1 > \frac{N}{2}$, and ω increases in N_1, $\omega > \lambda$.

A.4 Equilibrium with Asymmetric Amenity

In this appendix, I impose symmetric fundamental productivity ($\lambda = 1$) and show how the asymmetric amenity ($a_1 \neq a_2$) generates a cross-city difference. I let city 1 have a higher amenity ($a_1 > a_2$) without loss of generality. The system of equations characterizing an equilibrium is given by

$$
\begin{aligned}
1 &= \left[\frac{1}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
1 &= \left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{T}} m_{2k}} \frac{N_1}{N_2} &= \omega^{2\sigma-1}\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right] \\
\sum_{k\in\mathbb{T}} m_{1k} &= \left[\frac{1-\rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\sum_{k\in\mathbb{T}} m_{2k} &= \left[\frac{1-\rho\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\frac{a_1 U_1}{a_2 U_2} &= \left(\frac{N_1}{N_2}\right)^{\gamma}
\end{aligned}
$$

Let V_1 and V_2 be

$$V_1 = a_1 U_1 N_1^{-\gamma}$$

$$V_2 = a_2 U_2 N_2^{-\gamma}$$

The system becomes

$$\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1$$

$$\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1$$

$$\frac{\left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}}{\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}} \frac{N_1}{N_2} = \omega^{2\sigma-1} \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right]$$

$$V_1 = V_2$$

Consider the relative level of V_1 and V_2 at $N_1 = N_2 = \frac{N}{2}$ from the first three equations.

$$\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1$$

$$\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1$$

$$\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}} = \omega^{2\sigma-1} \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}}$$

Suppose $\omega > 1$. The first and the second equations imply $\frac{V_1}{a_1} > \frac{V_2}{a_1}$. However, the third implies $\frac{V_1}{a_1} < \frac{V_2}{a_1}$. These contradict each other. So, when $N_1 = N_2 = \frac{N}{2}$, the first three equations imply $\omega \leq 1$. Then, it follows from the third

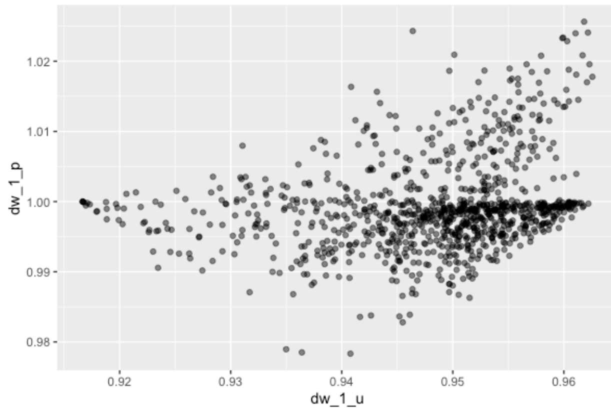
$$\frac{V_1(N_1 = \frac{N}{2})}{a_1} \geq \frac{V_2(N_1 = \frac{N}{2})}{a_1} \implies V_1(N_1 = \frac{N}{2}) \geq \frac{a_1}{a_2} V_2(N_1 = \frac{N}{2}) > V_2(N_1 = \frac{N}{2})$$

Therefore, it follows from the intermediate value theorem that the the equilibrium is located in $N_1 \in (\frac{N}{2}, N)$ when Assumption 1 holds. In addition, since ω increases in N_1, the equilibrium wage level is higher in city 1 than in city 2 ($\omega = \frac{w_1}{w_2} > 1$).

A.5 Simulation of Trade Cost Decline

The simulations are implemented for both the international trade model and the urban model. In both models, I generated 1,000 samples of equilibrium using 1,000 sets of parameters, and then I introduced the shocks. In the international trade case, the population in the two countries are set equal. The model used for simulations has three sectors. Sector 1 and Sector 2 are non-tradable while Sector 3 is tradable. Fixing the non-sector characteristics ($\lambda = 2, \sigma = 4, \eta = 0.3, \tau = 1.2, N = 100,000$), I randomized the sector characteristics, $\{\beta_k\}$ and $\{\epsilon_k\}$, to generate 1,000 samples. After solving the initial equilibrium, two changes were added separately: (i) a uniform trade cost reduction from $\tau = 1.2$ to $\tau = 1.1$; and (ii) sector 1 became tradable ("sectoral trade cost reduction"). Figure A.4 shows the two simulation results. First, the uniform trade cost reduction shrinks income inequality in all of the cases. Second, under the sectoral trade cost reduction, income inequality widens in a substantial share of samples (281 out of 1,000). To see what drives the heterogeneous effect under the sectoral trade cost reduction, I plotted the relative income changes against the difference in

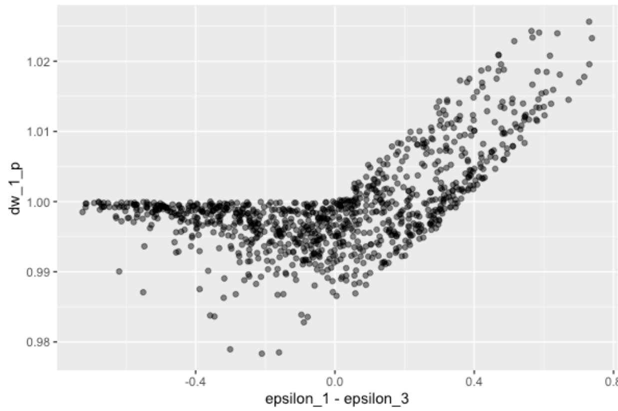
Figure A.4: Relative Nominal Income Change in Two Types of Globalization in International Model



The values of the X axis are 1+the proportional change in the value of the relative income of country 1 to country 2 after the uniform trade cost reduction. The values of the Y axis are the value that occurs after Sector 1 becomes tradable.

the income elasticities between the existing tradable sector (sector 3) and the new one (sector 1) in Figure A.5; this is motivated by the theoretical result in the special case. The graph clearly shows, first, that the income elasticity difference is a key variable that determines the direction in which income inequality changes and, second, that it affects the direction in the same way that it does in the special case. Income inequality can widen when the new tradable sector is sufficiently income elastic relative to the existing one, which is the case in $\epsilon_1 - \epsilon_3 \gg 0$.

Figure A.5: Relative Nominal Income Change and the Income elasticity Difference in the International Model



The values of X axis are $\epsilon_1 - \epsilon_3$ and those of Y axis are (1+the proportional change) of the relative income of country 1 to country 2 after Sector 1 becomes tradable.

A.6 Data of Comparative Statics

CBSA sectoral employment data

For sectoral employment data I use the County Business Patterns database prepared by Eckert et al. (2021). By imputing missing data and mapping between different industry classifications, these data provide a county-level panel that classifies industries according to a consistent set of 2012 NAICS codes. To create a CBSA-level panel that is based on this, I use the CBSA to Federal Information Processing Series (FIPS) County Crosswalk prepared by the National Bureau of Economics Research (NBER).

City-Specific Wage

To obtain city-level hourly wage data, I primarily follow Davis and Dingel (2020) and Acemoglu and Autor (2011). The datasource is Census data via IPUMS, and I limit samples to white males 25-to-55 years of age who worked at least 40 weeks during the previous year and who usually work at least 35 hours per week. The individual hourly wage is "INCWAGE" divided by both "UHRSWORK" and "WKSWORK1". I drop samples whose hourly wage was below $1.675 in 1982 dollars or that each year were in the top 0.5 percentile. For the 1980 data, I map samples from the county group (PWCNTYGP) to CBSA. For the 1990 data, I map samples from PWPUMA to CBSA; for the 2000 and 2007 data, I map samples from PUMA00 to CBSA. In each case, the crosswalk is made from that between the corresponding geographic unit to counties and that between counties to CBSA. In a geographic unit, I map the unit to the CBSA only if the CBSA's population share in the geographic unit exceeds 80%. After the mapping, I regress the log of the individual hourly wage on potential experience (age minus 18), the square of potential experience, the education level, and dummies of CBSAs with Census weights (PERWT). In the case of the education level, there are two groups each year. For 1980, I only use samples of either "Grade 12" (EDUCD 60) or "4 years of college" (EDUCD 100). In the other years, I use only samples of either "High school graduate or GED" (EDUCD 62) or "Bachelor's degree " (EDUCD 101). The fixed effects of CBSAs in this regression are the CBSA-specific hourly wages.

Historical Population

The data source is Schroeder (2016). The core of his data set is a series of historical population estimates for each U.S. decennial census year, 1790-2010, for all U.S. counties and county equivalents, using spatially fixed 2010 county definitions. I use his historical population estimates for CBSAs in 2009 definition.

Coast, Flat Land Share, and Wetland Shares

The primary data source is the dataset of Saiz (2010). The data are provided for the old definition of MSA. For each CBSA, I map a county to the old MSA using SSA to FIPS CBSA and Metropolitan and MSA County Crosswalk by the NBER. Then, to calculate the population share of the old MSAs in each CBSA, I use county population from the BEA. Using these shares, I calculate the weighted average of "S_LAND_50", "FLAT_SHARE_50_15", "lu91", "lu92." The CBSA whose weighted average of "S_LAND_50" is less than 100 is assigned 1 in the coastal dummy variable. In addition, if CBSA is located in a state that does not have a coastline on any ocean or the Great Lakes, the CBSA is assigned 0. For the flat land share and the two wetland shares, the weighted averages are used.

College Graduate Share

The data source is the Census via IPUMS. It is calculated as the share of samples that have a degree not lower than a Bachelor's degree (or Grade 12 in 1980) among those 25-55 who work for wage. The mappings from Census geographic units to CBSAs and those of city-specific wages are done in the same way.

NMP

I calculate county-level NMP first using the County Distance Database from the NBER and income data from the Bureau of Economic Analysis. The CBSA-level NMP is the average NMP within CBSA weighted with the county population. In the case of the NMP that takes into account the aggregate tradable sector share, I follow the same steps.