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Interpretation of excited Ω_b signals

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Recently LHCb reported the discovery of four extremely narrow excited Ω_b baryons decaying into $\Xi_b^0 K^-$. We interpret these baryons as bound states of a b-quark and a P-wave ss-diquark. For such a system there are exactly five possible combinations of spin and orbital angular momentum. We predict two of spin $1/2$, two of spin $3/2$, and one of spin $5/2$, all with negative parity. We favor identifying the observed states as those with spins $1/2$ and $3/2$, and give a range of predicted masses for the one with spin $5/2$. We update earlier predictions for these states based on the five narrow excited Ω_c states reported by LHCb. An alternative picture of the states in which one of $J = 1/2$ is extremely wide and hence not seen by LHCb is discussed.

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I. INTRODUCTION

Recently LHCb reported the discovery of four extremely narrow excited Ω_b baryons decaying into $\Xi_b^0 K^-$ [1], with masses and widths shown in Table I. We quote also our favored spin-parity assignment for these states. This result follows upon the earlier observation by LHCb of five very narrow excited Ω_c baryons [2], which we interpreted as P-wave excitations of the ss diquark with respect to the c quark [3]. The global significance of the two lowest states is below 3σ, so our assignments are subject to possible change with additional data.

The discovery of the five excited Ω_c states raised some questions, which we addressed:

(a) *Why five states? Are there more in the css system?* There are exactly five $1P$ excitations if the ss diquark remains in its color-triplet spin-1 ground state. In an alternative picture the three lowest states are $1P$ excitations while the two highest are $1/2^+$ and $3/2^+$ $2S$ radial excitations.

(b) *Why are they so narrow?* States with no nonstrange quarks (u or d) do not couple directly to pions, closing important low-threshold channels.

(c) *What are their spin-parity assignments?* We favored $J^P = (1/2^-, 1/2^-, 3/2^-, 3/2^-, 5/2^-)$ for the observed states, in order of increasing mass. An alternative assignment was $(3/2^-, 3/2^-, 5/2^-, 1/2^+, 3/2^+)$.

(d) *Can one understand the mass pattern?* Yes; the favored pattern, based on contributions of spin-orbit, spin-spin, and tensor force interactions, was uniquely selected out of $5! = 120$ possible permutations of the five states.

(e) *Are there other similar states with different quark content, in particular very narrow excited Ω_b baryons?* LHCb has now observed four out of the five predicted $1P$ excitations [1], leaving a fifth to be predicted and observed.

The same questions can be asked for the four observed Ω_b states. Which of the expected five Ω_b states is missing, and what is its mass? Is the spin-weighted average of the $1P$ excitations consistent with expectation?

In Sec. II we comment on P-wave bss baryons. We then analyze spin-dependent forces for the bss system in Sec. III, building upon similar results [3] obtained previously for the negative-parity Ω_c states. We evaluate the energy cost for a P-wave bss excitation in Sec. IV, compare our present results with our earlier predictions for the Ω_b system in Sec. V, discuss alternative interpretations of the spectrum in Sec. VI, and conclude in Sec. VII.

II. *P*-WAVE $b(ss)$ SYSTEM

We retrace steps in [3] leading to five excitations of the ss diquark in a relative P wave with respect to a b quark. Consider the (ss) in $b(ss)$ to be an S-wave color $\bar{\mathbf{3}}_c$ diquark. Then it must have spin $S_{ss} = 1$. This spin can be combined with the spin $1/2$ of the b quark to a total spin $S = 1/2$ or $3/2$. States with relative orbital angular

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TABLE I. Masses, widths, and 90% (95%) confidence level upper limits on natural widths of $\Omega_b = bss$ candidates reported by the LHCb Collaboration [1]. The proposed values of spin-parity J^P are ours.

				Significances (σ)	
State	Mass (MeV)	Width (MeV)	Proposed J^P	Local	Global
$\Omega_b(6316)^0$	$6315.64 \pm 0.31 \pm 0.07 \pm 0.50$	$<2.8(4.2)$	$1/2^-$	3.6	2.1
$\Omega_b(6330)^0$	$6330.30 \pm 0.28 \pm 0.07 \pm 0.50$	$<3.1(4.7)$	$1/2^-$	3.7	2.6
$\Omega_b(6340)^0$	$6339.71 \pm 0.26 \pm 0.05 \pm 0.50$	$<1.5(1.8)$	$3/2^-$	7.2	6.7
$\Omega_b(6350)^0$	$6349.88 \pm 0.35 \pm 0.05 \pm 0.50$	$<2.8(3.2)$	$3/2^-$	7.0	6.2
		$1.4^{+1.0}_{-0.8} \pm 0.1$			

momentum $L = 1$ between the spin-1 diquark and the b quark are

$$(L = 1) \otimes (S = 1/2) = (J = 1/2, 3/2),$$
$$(L = 1) \otimes (S = 3/2) = (J = 1/2, 3/2, 5/2). \quad (1)$$

All five states have negative parity P. Those with $J^P = 1/2^-$ decay to $\Xi_b^0 K^-$ in an S-wave, while those with $J^P = 3/2^-, 5/2^-$ decay to $\Xi_b^0 K^-$ in a D-wave.

The LHCb experiment sees only four of the predicted five P-wave excitations in the $\Omega_b = bss$ system [1]. Only four of the five predicted Ω_c states are seen by Belle in e^+e^- collisions [4]; the omitted state is the heaviest, $\Omega_c(3119)$. This makes sense as kinematic suppression is greatest for the heaviest state. For an initial state with no heavy flavor, the minimum mass recoiling against a css state such as $\Omega_c(3119)$ is $M(\Omega_c) = 2695.2 \pm 1.7$ MeV while typical e^+e^- c.m.s. energy is $M(\Upsilon(4S)) = 10579.4 \pm 1.2$ MeV [5]. In keeping with our identification of the $\Omega_c(3119)$ as the state with $J^P = 5/2$, we shall assume that it is the $J^P = 5/2^-$ Ω_b which is missing, and focus on the mass range above $M(\Omega_b(6350))$ for it.

III. SPIN-DEPENDENCES OF MASSES

The masses of the P-wave excitations of the ss diquark with respect to b are split by spin-orbit forces, a tensor force, and hyperfine interactions, leading to a spin-dependent potential [3]

$$\begin{aligned} V_{SD} = {} & a_1 \boldsymbol{L} \cdot \boldsymbol{S}_{ss} + a_2 \boldsymbol{L} \cdot \boldsymbol{S}_Q \\ & + b[-\boldsymbol{S}_{ss} \cdot \boldsymbol{S}_Q + 3(\boldsymbol{S}_{ss} \cdot \boldsymbol{r})(\boldsymbol{S}_Q \cdot \boldsymbol{r})/r^2] \\ & + c \boldsymbol{S}_{ss} \cdot \boldsymbol{S}_Q. \end{aligned} \quad (2)$$

States with the same J but different S mix with one another, so the mass shift operators $\Delta\mathcal{M}_{1/2,3/2}$ may be written as 2×2 matrices in bases labeled by $S = 1/2, 3/2$:

$$\begin{aligned} \Delta\mathcal{M}_{1/2} = {} & \begin{bmatrix} \frac{1}{3}a_2 - \frac{4}{3}a_1 & \frac{\sqrt{2}}{3}(a_2 - a_1) \\ \frac{\sqrt{2}}{3}(a_2 - a_1) & -\frac{5}{3}a_1 - \frac{5}{6}a_2 \end{bmatrix} + b \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix} \\ & + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \end{aligned} \quad (3)$$

$$\begin{aligned} \Delta\mathcal{M}_{3/2} = {} & \begin{bmatrix} \frac{2}{3}a_1 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) \\ \frac{\sqrt{5}}{3}(a_2 - a_1) & -\frac{2}{3}a_1 - \frac{1}{3}a_2 \end{bmatrix} \\ & + b \begin{bmatrix} 0 & -\sqrt{5}/10 \\ -\sqrt{5}/10 & \frac{4}{5} \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \end{aligned} \quad (4)$$

$$\Delta\mathcal{M}_{5/2} = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c. \quad (5)$$

The spin-weighted sum of these mass shifts is zero:

$$\sum_J (2J + 1)\Delta\mathcal{M}_J = 0. \quad (6)$$

Note that the sums of eigenvalues of $\Delta\mathcal{M}_{1/2}$ and $\Delta\mathcal{M}_{3/2}$ are equal to the traces of the corresponding matrices, making the verification of Eq. (6) simple.

There are four measured masses and four independent parameters leading to four mass *shifts* with respect to a spin-weighted average for which one needs the fifth mass. Thus the determination of the constants a_1, a_2, b, c has one free parameter which we may take as $\mathcal{M}_{5/2}$. We identify the four known masses as shown in Table I.

The spin-weighted average mass $\bar{M} = \sum_J [(2J+1)\mathcal{M}_J]/18$ is linear in the unknown mass $\mathcal{M}_{5/2}$, with slope 1/3. Anticipating the optimal fit $\mathcal{M}_{5/2} = 6358$ MeV [cf. the discussion following Eq. (10)], $\bar{M}$ can be rewritten in terms of the deviation from this fit,

$$\bar{M} = 6344 \text{ MeV} + (1/3)(\mathcal{M}_{5/2} - 6358 \text{ MeV}). \quad (7)$$

The limited range of $\bar{M}$ will be of use when we study the P-wave excitation energy.

We now determine the parameters a_1, a_2, b, c from the masses in Table I. The measured masses permit one to write two identities which are helpful in finding solutions.

We denote the two eigenvalues of $\mathcal{M}_{1/2}$ by M_1 and M_2, and the two eigenvalues of $\mathcal{M}_{3/2}$ by M_3 and M_4. A shorthand for $\mathcal{M}_{5/2}$ is M_5. We find

$$a_2 = -\frac{4}{9}(M_1+M_2) - \frac{2}{9}(M_3+M_4) + \frac{4}{3}M_5 - c - \frac{8}{3}a_1 \quad (8)$$

$$b = -\frac{5}{3}a_1 - \frac{5}{9}(M_1+M_2) + \frac{5}{9}(M_3+M_4). \quad (9)$$

Varying $\mathcal{M}_{5/2}$ above 6350 MeV, we find solutions for the ranges 6355.4 MeV $< \mathcal{M}_{5/2} <$ 6382.5 MeV and 6379.9 MeV $< \mathcal{M}_{5/2} <$ 6406.9 MeV, as shown in the left-hand and right-hand panels of Fig. 1, respectively, with two branches for a_1, b, and c, and a single branch for a_2.

The constants in the figure may be compared with those favored in a fit to excited Ω_c states [3]:

$$a_1 = 26.95\ \text{MeV}, \qquad a_2 = 25.74\ \text{MeV},$$
$$b = 13.52\ \text{MeV}, \qquad c = 4.07\ \text{MeV}. \quad (10)$$

It was argued in Ref. [3] that the hyperfine term c should be no larger for the Ω_b system than for the excited Ω_c states. In that case one selects the lower branch of the dashed (red) ellipse in the left-hand panel, which is correlated with the upper branch of the solid (black) upper ellipse, and favors values of $M_{5/2}$ in the range of 6356 to 6366 MeV, with the most probable (lowest) value of c for $M_{5/2} = 6358$ MeV. Specifically, for that value we find

$$a_1 = 10.34\ \text{MeV}, \qquad a_2 = 6.37\ \text{MeV},$$
$$b = 7.02\ \text{MeV}, \qquad c = 3.07\ \text{MeV}. \quad (11)$$

The smaller value of a_1 in comparison with the value for Ω_c in Eq. (10) shows the limitation of the extrapolation in Ref. [3], which predicted them to be equal, but other parameters are within their predicted ranges.

IV. ENERGY COST OF A *P*-WAVE EXCITATION

We estimated the P-wave excitation energy for a ss diquark bound to a b quark with relative orbital angular momentum $L = 1$ in Sec. V of Ref. [3]. A crude value of 300 MeV was obtained. It was necessary to anticipate the hyperfine splitting in the S-wave bss ground state, as only the $\Omega_b(1/2^+)$ has been seen, with mass $M(\Omega_b) = (6046.1 \pm 1.7)$ MeV. With an estimated hyperfine splitting between $\Omega_b(1/2^+)$ and $\Omega_b^*(3/2^+)$ of 24 MeV, the spin-weighted average of the $1/2^+$ and $3/2^+$ S-wave bss states was estimated to be 6062 MeV. Subsequently we noted [6] that P-wave excitation energies obeyed an approximate linear relation

$$\Delta E_{P-S} = \Sigma B + 417.4\ \text{MeV} - 0.214\mu_R, \quad (12)$$

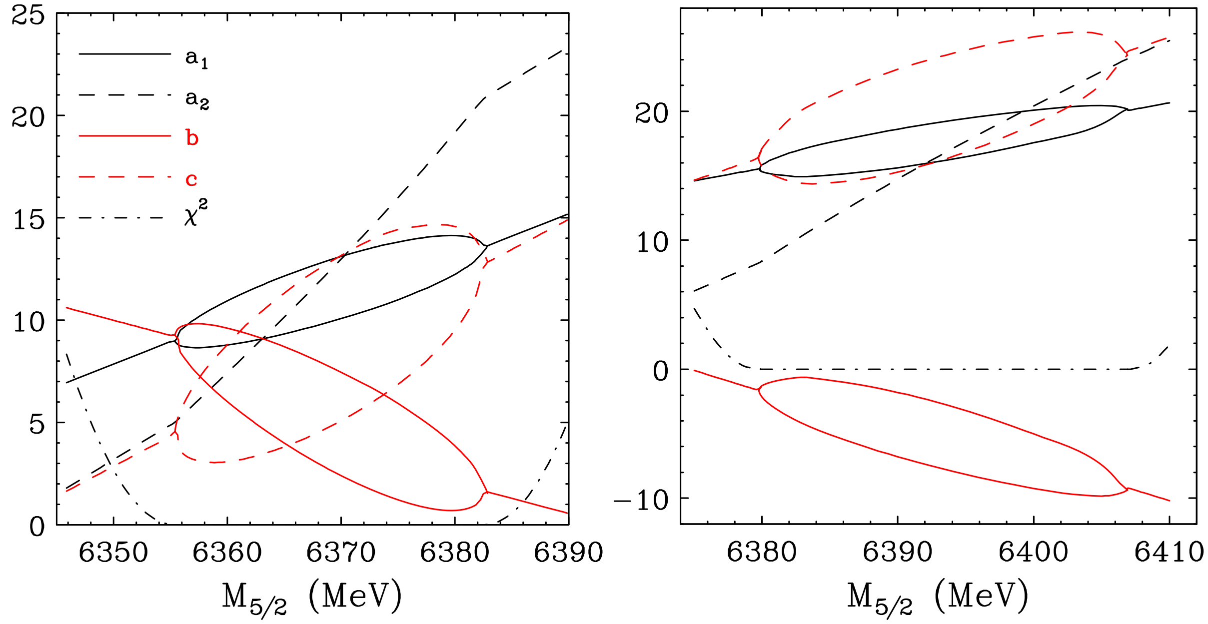

FIG. 1. Dependence of spin-dependent constants (in MeV) on $\mathcal{M}_{5/2}$. The two branches are: 6355.4 MeV $< \mathcal{M}_{5/2} <$ 6382.5 MeV (left panel), and 6379.9 MeV $< \mathcal{M}_{5/2} <$ 6406.9 MeV (right panel). Upper solid (black) ellipse: a_1; dashed single-valued line: a_2; lower solid (red) ellipse: b; dashed (red) ellipse: c. χ^2 (defined in the text): dash-dotted lines, nonzero only outside range of $M_{5/2}$ giving solutions. Ellipses coalesce into a single line for values of $M_{5/2}$ giving minimum nonzero $\chi^2 \equiv \sum_{i=1}^{5}(M_{i,\text{exp}} - M_{i,\text{fit}})^2$. Units of χ^2 are in MeV^2. The favored range of $M_{5/2}$ is in the left-hand figure.

where ΣB is the binding energy of the (ss) diquark and b quark, estimated to be 83.6 MeV, and μ_R is the reduced mass of the ss–b system:

$$m_{ss} = 1098.8\ \text{MeV}, \qquad m_b = 5041.8\ \text{MeV},$$
$$\mu_R = 902.2\ \text{MeV}. \tag{13}$$

With these inputs one finds $\Delta E_{P-S} = 308$ MeV, implying $\bar{M} = 6370$ MeV. As we see above and below, under various assumptions the LHCb data imply values a bit lower than this.

V. EVALUATION OF PREDICTIONS FOR $\Omega_b = b(ss)$ STATES

In addition to the predictions of Ref. [3] for the hyperfine parameter c and the S-P splitting, we predicted other parameters for the $\Omega_b = bss$ states based on rescaling the fitted Ω_c values quoted in Eq. (10).

(a) The parameter a_1 was to be kept as in the css system, as it expresses the coefficient of $\boldsymbol{L} \cdot \boldsymbol{S}_{(ss)}$: $a_1[b(ss)] = a_1[c(ss)] = 26.95$ MeV. On the other hand, for $M_{5/2}$ around the favored value of 6358 MeV, $a_1 \simeq 10$ MeV.

(b) The spin-orbit parameter a_2 was expected to scale as the inverse of the heavy quark mass: $a_2[b(ss)] = (1708.8/5041.8)(25.74) = 8.72$ MeV, where we have taken the charm and bottom quark masses from Ref. [7]. Its value in the present fit for $M_{5/2} \simeq 6358$ MeV is about 6 MeV (see the left-hand panel of Fig. 1).

(c) The tensor force parameter b is found to be around 7 MeV, well within the range of ±20 MeV anticipated in [7].

VI. ALTERNATIVE INTERPRETATIONS

Predictions for the negative-parity Ω_b states were made by several authors [8–21], in papers prior to discovery of the excited Ω_c states, and by authors commenting on those states [22–32], including interpretations based on pentaquarks [33–35]. Since the discovery of the four narrow Ω_b states, several interpretations of them have been proposed [36–41]. These differ from one another in their J^P assignments and predicted widths. In Table II the first four columns denote states quoted in the (J, j) basis $(1/2, 0)$, (1/2, 1), (3/2, 1), (3/2, 1), (5/2, 2), where j is the total angular momentum (spin and orbital angular momentum L) of the ss diquark, while the last two columns refer to states quoted in the basis ${}^{2S+1}P_J = {}^2P_{1/2}$, ${}^2P_{3/2}$, ${}^4P_{1/2}$, ${}^4P_{3/2}$, ${}^4P_{5/2}$. Typical errors in predictions are 10 to 20 MeV, except for the QCD sum rule calculation [39], whose errors are of order 100 MeV. One should pay more attention to splitting among levels than their absolute values.

A general pattern emerges from these calculations. In the $j-j$ coupling scheme, the single state with $j = 0$ is deemed to be very wide (see Table III), and hence not observable in the current data set of LHCb [1]. The two states with $j = 1$ and the two with $j = 2$ are expected to be narrow, for the most part within the experimental resolution. This behavior is not seen in the case of the Ω_c states, where candidates for all five involving the spin-one ss pair in its ground state are seen [2].

TABLE II. Comparison of predicted J^P assignments of the narrow states decaying to $\Xi_b^0 K^-$ reported by LHCb [1]. Masses in MeV.

J^P	[36]	[37][a]	[39]	[40]	[41]	[32][b]
Coupling	$j-j$	$j-j$	$j-j$	$j-j$	$L-S$	$L-S$
$1/2^-$	6340[c]	6339[c]	6330[d]	6316[c,d]	6314	6305
$1/2^-$	6340	6330	[c]?	6316[d]	6330	6317
$3/2^-$	6340	6340	6316[d]	6340[d]	6339	6313
$3/2^-$	6350	6331	6350[d]	6330[d]	6342	6325
$5/2^-$	6360	6334	6340[d]	6350[d]	6352	6338

[a] Mass predictions taken from Ref. [8].

[b] Mass predictions taken from Ref. [31].

[c] State with $j = 0$ predicted to be very broad and not seen by LHCb.

[d] Experimental masses; proposed J^P assignments.

Even if one assumes the reason for seeing four rather than five excited Ω_b states is that the one with $j = 0$ is very broad, there is no unanimity on the order of the observed states. That is the question we address in considering the effects of spin-orbit, tensor force, and spin-spin couplings. We have found a consistent solution in which it is the state with $J^P = 5/2^-$ that is missing in the data.

We now repeat the exercise in which the states are described by $j-j$ coupling and it is the one with $j = 0$ whose mass we vary in order to determine the parameters a_1, a_2, b, c.

TABLE III. Comparison of predicted widths (in MeV) of the narrow states decaying to $\Xi_b \bar{K}$ in an S wave reported by LHCb [1].

J^P	[36]	[37]	[39]	[40]	[41]	[31]
Coupling	$j-j$	$j-j$	$j-j$	$j-j$	$L-S$	$L-S$
$1/2^-$	>1000	871[a]	[b]	126	0.78	0.50
$1/2^-$	0	⋯	[b]	⋯	3.18	1.14
$3/2^-$	0	⋯	[b]	⋯	1.74	2.79
$3/2^-$	0[c]	1.35[d]	[b]	2.2	0.58	0.62
$5/2^-$	0	2.98[e]	[b]	3.4	2.83	4.28

[a] For $\Omega_b(6316)$. (1057,1146,1224) MeV for $\Omega_b(6330, 6340, 6350)$.

[b] Observed states at (6316,6330,6340,6350) MeV assigned $J^P = (3/2^-, 1/2^-, 5/2^-, 3/2^-)$ with comparable pole strengths in QCD sum rules.

[c] Plus predicted $4.7^{+6.1}_{-2.9}$ MeV for decay to $\Xi_b \bar{K}$ in a D wave.

[d] For $\Omega_b(6340)$. (0.35,1.08,2.98) MeV for $\Omega_b(6316, 6330, 6350)$.

[e] For $\Omega_b(6350)$. (0.35,1.08,1.85) MeV for $\Omega_b(6316, 6330, 6340)$.

In order to determine mass splittings in the linearized $j-j$ coupling basis, we use lowest-order perturbation theory in the inverse of m_b [3]. (If we kept nondiagonal terms in this basis the outcome would be the same as in the $L-S$ coupling basis considered earlier.)

$$\Delta M\left(J=\frac{1}{2}, j=0\right)=-2a_1, \tag{14}$$

$$\Delta M\left(J=\frac{1}{2}, j=1\right)=-a_1-\frac{1}{2}a_2-b-\frac{1}{2}c, \tag{15}$$

$$\Delta M\left(J=\frac{3}{2}, j=1\right)=-a_1+\frac{1}{4}a_2+\frac{1}{2}b+\frac{1}{4}c, \tag{16}$$

$$\Delta M\left(J=\frac{3}{2}, j=2\right)=a_1-\frac{3}{4}a_2+\frac{3}{10}b-\frac{3}{4}c, \tag{17}$$

$$\Delta M\left(J=\frac{5}{2}, j=2\right)=a_1+\frac{1}{2}a_2-\frac{1}{5}b+\frac{1}{2}c. \tag{18}$$

The independent parameters are a_1, a_2+c, and b, so the five mass splittings obey two sum rules:

$$2\Delta M(1/2,1)+4\Delta M(3/2,1)=3\Delta M(1/2,0), \tag{19}$$

$$4\Delta M(3/2,2)+6\Delta M(5/2,2)=-5\Delta M(1/2,0), \tag{20}$$

where the first number refers to J and the second to j. We are assuming that the unseen state is the one with mass $M(J=1/2, j=0)$. Eliminating $\Delta M(1/2,0)$ from the above two equations and recalling that each $\Delta M\equiv M-\bar{M}$, we find an expression for $\bar{M}$ in terms of the four observed masses, whose value depends on the permutation of the masses assigned to each J^P level:

$$\bar{M}=\frac{1}{6}M(1/2,1)+\frac{1}{3}M(3/2,1)+\frac{1}{5}M(3/2,2)+\frac{3}{10}M(5/2,2). \tag{21}$$

We can now obtain $\Delta M(1/2,0)$ from Eq. (19), and find

$$M(1/2,0)=\frac{1}{3}[2M(1/2,1)+4M(3/2,1)]-\bar{M}. \tag{22}$$

The spin-dependent coefficients are

$$a_1=-\frac{1}{2}\Delta M(1/2,0), \tag{23}$$

$$a_2+c=\frac{1}{3}[3\Delta M(1/2,0)-\Delta M(1/2,1)+5\Delta M(5/2,2)], \tag{24}$$

$$b=\frac{5}{9}[3\Delta M(3/2,1)+\Delta M(3/2,2)-\Delta M(1/2,0)]. \tag{25}$$

TABLE IV. Parameters in MeV for all permutations of J^P assignments of excited Ω_b levels at 6316, 6330, 6339, and 6349 MeV.

Permutation	$\Delta M(1/2,0)$	$M(1/2,0)$	$\bar{M}$	a_1	a_2+c	b
1 2 3 4	−20.4	6315.2	6335.6	10.200	10.037	4.754
1 2 4 3	−18.6	6316.2	6334.6	9.183	−3.523	11.534
1 3 2 4	−10.4	6326.5	6336.9	5.181	18.402	6.845
1 3 4 2	−6.4	6328.5	6334.9	3.223	−7.705	19.899
1 4 2 3	2.5	6339.7	6337.2	−1.260	13.882	15.885
1 4 3 2	4.4	6340.7	6336.3	−2.201	1.335	22.159
2 1 3 4	−25.3	6307.9	6333.2	12.643	3.522	−11.535
2 1 4 3	−23.3	6308.9	6332.2	11.626	−10.038	−4.755
2 3 1 4	0.4	6336.8	6336.4	−0.194	24.917	−6.186
2 3 4 1	7.2	6340.2	6333.0	−3.618	−20.736	16.641
2 4 1 3	13.3	6350.0	6336.7	−6.635	20.397	2.854
2 4 3 1	18.1	6352.4	6334.3	−9.042	−11.696	18.901
3 1 2 4	−18.4	6314.5	6332.9	9.193	7.704	−19.899
3 1 4 2	−14.5	6316.4	6330.9	7.235	−18.403	−6.846
3 2 1 4	−2.7	6332.1	6334.8	1.374	20.735	−16.641
3 2 4 1	4.1	6335.5	6331.4	−2.050	−24.918	6.185
3 4 1 2	22.1	6357.5	6335.5	−11.027	12.033	0.763
3 4 2 1	25.0	6359.0	6334.0	−12.493	−7.514	10.536
4 1 2 3	−8.9	6322.6	6331.5	4.447	−1.336	−22.159
4 1 3 2	−7.0	6323.5	6330.6	3.506	−13.883	−15.886
4 2 1 3	6.7	6340.3	6333.5	−3.372	11.695	−18.901
4 2 3 1	11.6	6342.6	6331.0	−5.779	−20.398	−2.855
4 3 1 2	18.7	6352.4	6333.8	−9.332	7.513	−10.537
4 3 2 1	21.6	6353.9	6332.3	−10.798	−12.034	−0.764

Table IV lists all 24 permutations of the masses of the observed four levels with $(J,j) = (1/2,1), (3/2,1), (3/2,2), (5/2,2)$, obtaining parameters for each permutation. We denote the order in which the observed masses are monotonically increasing by the permutation 1 2 3 4. We then compare each set with values estimated in Ref. [3] by extrapolation from the excited Ω_c spectrum. Some notable features are the following:

(i) The values of a_1 are half or less that estimated by extrapolating from charm to bottom. It probably pays to choose the largest possible (positive) a_1.

(ii) The value of a_2 was estimated in [3] to be 8.72 MeV, while c was estimated to be small, less than a few MeV.

(iii) Reference [3] considered values of b lying within the range $-20 < b < 20$ MeV, satisfied by most sets in Table IV.

(iv) The value of $\bar{M}$ varies within a narrow range around 6335 MeV, to be compared with the crude estimate of 6362 MeV in Ref. [3], the central value of 6344 MeV obtained in Sec. III, and the value of 6370 MeV found in Sec. IV.

With these considerations the set labeled by the permutation 1234 seems the most satisfactory. The observed levels at 6316, 6330, 6340, and 6350 MeV then would correspond to the states with $(J,j) = (1/2,1), (3/2,1), (3/2,2), (5/2,2)$. respectively.

VII. CONCLUSIONS

We have interpreted the four narrow peaks seen by LHCb in the $\Xi_c^0 K^-$ mass distribution [1] as P-wave excitations of a spin-1 ss diquark with respect to a spin-1/2 b quark. While such a system is expected to have five states—two of spin 1/2, two of spin 3/2, and one of spin 5/2—we advance arguments in favor of the spin-5/2 state being missed. When the four observed levels are assigned $J^P = 1/2^-, 1/2^-, 3/2^-, 3/2^-$ in order of ascending mass, solutions for spin-dependent parameters are obtained for $6355.4\ \text{MeV} < M_{5/2} < 6382.5\ \text{MeV}$ and $6379.9\ \text{MeV} < M_{5/2} < 6406.9\ \text{MeV}$, with the lowest ~10 MeV of this range favored by consideration of the derived spin-dependent parameters.

An alternative explanation of the missing state offered by several authors [36,37,39,40] envisions the states as approximately diagonal in the (J,j) basis, where J is the total angular momentum and j is the ss-diquark's total (spin plus L) angular momentum. In this basis the $(1/2,0)$ state is predicted to be very wide and hence not seen by LHCb. In this case the most plausible set of spin-dependent parameters is obtained when the four observed levels are assigned $(J,j) = (1/2,1); (3/2,1); (3/2,2); (5/2,2)$ in order of ascending mass. Angular distributions of decay products should be able to distinguish between this scenario and the (favored) one in the preceding paragraph.

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Note added.—The molecular picture of excited Ω_b states offered in Ref. [38] proposes four states at 6405, 6427, 6465, and 6508 MeV, which they associate with (nonsignificant) enhancements in the LHCb spectrum [1] at 6402, 6427, 6468, and 6495 MeV. A QCD sum rule calculation [42] finds four excited states: $(1\text{P}, 1/2^-)$ at 6336 ± 183 MeV; $(2\text{S}, 1/2^+)$ at 6487 ± 187 MeV; $(1\text{P}, 3/2^-)$ at 6301 ± 193 MeV; and $(2\text{S}, 3/2^+)$ at 6422 ± 198 MeV. An unpublished undergraduate thesis [43] employs a simplified quark model to predict $M(\Omega_b(5/2,2))$ in the range of 6364 to 6372 MeV. A very recent analysis in the same spirit as ours predicts $M(1P, 5/2^-) \simeq 6352$ MeV [44].

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